Tower of Hanoi with adjacency requirement (Ex. 8.1.18)

Let the pegs are lined up and labelled A, B and C. Now we only allow disk move between adjacent pegs, i.e. direct move from A to C is not allowed. Let a_n be the minimum # moves needed to transfer a tower of n disks from pole A to pole C.

(a) Find a_1, a_2, and a_3.

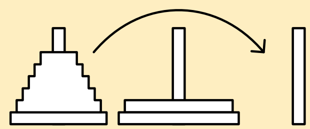
Solution

- $a_1 = 2$. Move disk from A to B and then to C.
- a_2:
  - Move disk 1 from A to C. (2)
  - Move disk 2 from A to B. (1)
  - Move disk 1 back from C to A. (2)
  - Move disk 2 from B to C. (1)
  - Move disk 1 from A to C. (2)

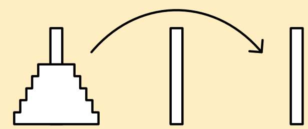
  So $a_2 = 8$.

Solution (cont)

- a_3:
  - Want to move disk 3 from A to B. So need to move things on top to C. ($a_2 = 8$; treat that as a "black box" because all the disks below is of larger size so the "small-on-large" constraint is not violated.)



Same as



  - Move disk 3 from A to B. (1)
  - Now want to move disk 3 from B to C. So need to move the 2 disks from C to A. (8; swap A and C in the above procedure)
  - Move disk 3 from B to C. (1)
  - Move the 2 disks from A to C. (8)
- $a_3 = 26$. Generalize it?

Tower of Hanoi with adjacency requirement (Ex. 8.1.18)

(c) Find a recurrence relation for a_k

Solution

- a_k: (Let T_k be the tower of the top k disks)
  - Want to move disk $\cancel{3}$ k from A to B. So need to move ~~the 2 disks~~ T_{k-1} to C. (a_{k-1} moves; treat transferring of T_{k-1} as a "black box")
  - Move disk k from A to B. (1)
  - Now want to move disk k from B to C. So need to move T_{k-1} from C to A. (a_{k-1} moves, by symmetry.)
  - Move disk k from B to C. (1)
  - Move T_{k-1} from A to C. (a_{k-1})
- So we have recurrence relation for a_k:

  $a_1 = 2$, $a_k = 3a_{k-1} + 2$ for $k \geq 2$.