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Assignment 01 : Complex Number.

1. Find the modulus & amplitude.

i) $\frac{1+i\sqrt{3}}{2}$

$\rightarrow$ $\frac{1}{2} + i\frac{\sqrt{3}}{2}.$

$x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$

$|z| = r = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{2} / \frac{1}{2}\right)$

$\theta = \tan^{-1}(\sqrt{3})$

$\theta = \frac{\pi}{3}$

ii) $\frac{1-i\sqrt{3}}{2}$

$\frac{1}{2} - i\frac{\sqrt{3}}{2}$

$x = \frac{1}{2}$, $y = -\frac{\sqrt{3}}{2}$

$|z| = r = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$

$\theta = \tan^{-1}(y/x)$

$\alpha = \tan^{-1}\left|\frac{-y}{x}\right|$

$\theta = 2\pi - \alpha$

$= 2\pi - \frac{\pi}{3}$

$= \frac{5\pi}{3}$

iii) $-1 - i\sqrt{3}$

$\rightarrow x = -1 \quad , \quad y = -\sqrt{3}$

$|z| = \sqrt{1+3} \quad = 2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\alpha = \tan^{-1}\left(\frac{-y}{-x}\right)$

$\theta = \pi + \alpha$

$= \pi + \frac{\pi}{3}$

$= \frac{4\pi}{3}$

iv) $\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$

$\rightarrow x = \frac{1}{\sqrt{2}}, \quad y = \frac{-1}{\sqrt{2}}$.

$|z| = r = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\alpha = \tan^{-1}\left(\frac{-y}{x}\right) = \tan^{-1}\left|\frac{-y}{x}\right|$

$\theta = 2\pi - \alpha$

$= 2\pi - \frac{\pi}{4}$

$= \frac{7\pi}{4}$.

2. Solve $\frac{[\cos 2\theta - i\sin 2\theta]^7 [\cos 3\theta + i\sin 3\theta]^{-5}}{[\cos 4\theta - i\sin 4\theta]^{12} [\cos 5\theta - i\sin 5\theta]^{-6}}$.

$\rightarrow \frac{[(\cos\theta + i\sin\theta)^{-2}]^7 [(\cos\theta + i\sin\theta)^3]^{-5}}{[(\cos\theta + i\sin\theta)^{-4}]^{12} [(\cos\theta + i\sin\theta)^{-5}]^{-6}}$.

$= \frac{(\cos\theta + i\sin\theta)^{-14} (\cos\theta + i\sin\theta)^{-15}}{(\cos\theta + i\sin\theta)^{-48} (\cos\theta + i\sin\theta)^{+30}}$

$= (\cos\theta + i\sin\theta)^{-14-15+48-30}$

$= (\cos\theta + i\sin\theta)^{-11}$

$\therefore \cos 11\theta - i\sin 11\theta$.

3. Show that the continued product of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3/4}$ is

$\rightarrow r = 1$

$\theta = \frac{\pi}{3}$

$\therefore \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{3/4}$.

$= \left[\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^3\right]^{1/4}$.

$= (\cos\pi + i\sin\pi)^{1/4}$.

$= [\cos(2k\pi + \pi) + i\sin(2k\pi + \pi)]^{1/4}$.

$= \cos\left(\frac{2k\pi + \pi}{4}\right) + i\sin\left(\frac{2k\pi + \pi}{4}\right)$

let $k = 0, 1, 2, 3$

$k = 0,\ x_1 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = e^{i(\pi/4)}$

$K=1, x_2 = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}, \quad e^{i(3\pi/4)}$

$K=2, x_3 = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}, \quad e^{i(5\pi/4)}$

$K=3, x_4 = \cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}, \quad e^{i(7\pi/4)}$

Continued product.

$e^{i\pi/4} \cdot e^{i3\pi/4} \cdot e^{i5\pi/4} \cdot e^{i7\pi/4} = e^{i4\pi} = 1$

$\therefore \quad (\cos 4\pi + i\sin 4\pi = 1)$

4. Find the all values of $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

→ Comparing with $x+iy$

$x = \frac{1}{2}, \; y = -\frac{\sqrt{3}}{2}.$

$|z| = r = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$

$\theta = \tan^{-1}(y/x)$

$\alpha = \tan^{-1}|-y/x|$

$= \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$$\therefore \left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)^{1/4}$$

$$\left[\cos\left(2k\pi + \frac{5\pi}{3}\right) + i\sin\left(2k\pi + \frac{5\pi}{3}\right)\right]^{1/4}$$

$$\left[\cos\left(\frac{6k\pi + 5\pi}{3}\right) + i\sin\left(\frac{6k\pi + 5\pi}{3}\right)\right]^{1/4}.$$

$$\cos\left(\frac{6k\pi + 5\pi}{12}\right) + i\sin\left(\frac{6k\pi + 5\pi}{12}\right)$$

let $k = 0, 1, 2, 3, \cancel{4}$

$k=0$ $x_1 = \cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}$, $e^{i\left(\frac{5\pi}{12}\right)}$

$k=1$ $x_2 = \cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}$, $e^{i\left(\frac{11\pi}{12}\right)}$

$k=2$, $x_3 = \cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}$, $e^{i\left(\frac{17\pi}{12}\right)}$

$k=3$ $x_4 = \cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}$, $e^{i\left(\frac{23\pi}{12}\right)}$.

5. Solve the eqⁿ $x^7 + x^4 + x^3 + 1 = 0$

$\rightarrow$ $x^4(x^3+1) + 1(x^3+1) = 0$

$(x^3+1)+(x^4+1) = 0$

$\therefore\ x^3 = -1 \quad \& \quad x^4 = -1$

$x = (-1)^{1/3} \quad \& \quad x = (-1)^{1/4}$

$(-1) = \cos\pi + i\sin\pi$

for $x = (-1)^{1/3}$

$= (\cos\pi + i\sin\pi)^{1/3}.$

$= [\cos(2k\pi + \pi) + i\sin(2k\pi+\pi)]^{1/3}$

$= \cos\left(\frac{2k\pi+\pi}{3}\right) + i\sin\left(\frac{2k\pi+\pi}{3}\right)$

let $k = 0, 1, 2$

$k=0, \quad x_1 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}, \quad e^{i(\pi/3)}$

$k=1, \quad x_2 = \cos\frac{3\pi}{3} + i\sin\frac{3\pi}{3}, \quad e^{i(3\pi/3)}$

$k=2, \quad x_3 = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}, \quad e^{i(5\pi/3)}$

for $x = (-1)^{1/4}.$

$= \cos\left(\frac{2k\pi+\pi}{4}\right) + i\sin\left(\frac{2k\pi+\pi}{4}\right)$

let $k = 0, 1, 2, 3$.

$k=0$, $x_1 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$, $e^{i(\frac{\pi}{4})}$

$k=1$, $x_2 = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}$, $e^{i(\frac{3\pi}{4})}$

$k=2$, $x_3 = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}$, $e^{i(\frac{5\pi}{4})}$

$k=3$, $x_4 = \cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}$, $e^{i(\frac{7\pi}{4})}$

6. Find the continuous product of $(i)^{2/3}$

$\rightarrow$ $i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$.

$$\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{2/3}$$

$$= \left[\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^2\right]^{1/3}.$$

$$= \left[\left(\cos\frac{2\pi}{2} + i\sin\frac{2\pi}{2}\right]^{1/3}.$$

$$= \left[\{\cos(2k\pi + \pi) + i\sin(2k\pi + \pi)\right]^{1/3}$$

$$= \cos\left(\frac{2k\pi + \pi}{3}\right) + i\sin\left(\frac{2k\pi+\pi}{3}\right)$$

let $k = 0, 1, 2$

$x_1 = e^{i(\pi/3)}$
$x_2 = e^{i(3\pi/3)}$
$x_3 = e^{i(5\pi/3)}$

$$e^{i(5\pi/3)} \cdot e^{i(3\pi/3)} \cdot e^{i(\pi/3)} = e^{i(\pi/3 + 3\pi/3 + \frac{5\pi}{3})}$$

$= e^{i3\pi}$
$= \cos 3\pi + i\sin 3\pi$
$= -1.$

7. Determine whether following function is analytic if so find its derivative.
i) $\sinh z$

7. Separate into real & imaginary ~~of~~ part.

1) $\cos^{-1}(3i/4)$

$\rightarrow$ $\cos^{-1}(3i/4) = x + iy$

$3i/4 = \cos(x+iy)$

$0 + 3i/4 = \cos x \cdot \cos iy - \sin x \sin iy.$

$= \cos x \cosh y - i\sin x \sinh y.$

R.P $= \cos x \cosh y = 0$

$\cos x = 0$

$x = \frac{\pi}{2}$

I.P =

$3/4 = -\sin x \sinh y$

$3/4 = -\sin \pi/2 \sinh y.$

$3/4 = -(1) \sin hy.$

$\therefore \sinh y = -3/4.$

$y = \sinh^{-1}(-3/4)$

$= -\sinh^{-1}(3/4)$

$$y = -\log\left(\frac{3}{4} + \sqrt{\left(\frac{3}{4}\right)^2 + 1}\right)$$

$$= -\log\left(\frac{3}{4} + \sqrt{\frac{9}{16} + 1}\right)$$

$$= -\log\left(\frac{3}{4} + \frac{5}{4}\right)$$

$$= -\log 2.$$

$\therefore \cos^{-1}(3i/4) = \frac{\pi}{2} - i\log 2.$

2) $\cos^{-1}(ix) = \alpha + i\beta$

$\Rightarrow ix = \cos(\alpha + i\beta)$

$0 + ix = \cos\alpha \cos i\beta - \sin\alpha \sin i\beta$

$0 + ix = \cos\alpha \cosh\beta - i\sin\alpha \sinh\beta$

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R.P $= \cos\alpha \cosh\beta = 0$

$\cos\alpha = 0$

$\alpha = \frac{\pi}{2}$

I.P :

$x = -\sin\alpha \sinh\beta$

$x = -\sin\frac{\pi}{2} \sinh\beta$

$x = -(1) \sinh\beta$

$\therefore \sinh\beta = -x$

$\beta = \sinh^{-1}(-x)$

$= -\sinh^{-1}(x)$.

$y = -\log(x + \sqrt{x^2+1})$

Q.8 Find the real & imaginary part of $\log(x+iy)$.

→ let $z = \log(x+iy)$ where

$x = r\cos\theta$,

$y = r\sin\theta$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

by putting the values, we get,

$z = \log(r\cos\theta + ir\sin\theta)$

$= \log r(\cos\theta + i\sin\theta)$

$= \log r \cdot e^{i\theta}$.

$z = \log r + \log e^{i\theta}$

$= \log(x^2+y^2)^{1/2} + i\theta$.

$$\therefore z = \frac{1}{2} \log (x^2 + y^2) + i \tan^{-1} (y/x)$$