Tutorial 1

Sets : Set is an unordered collection of different element. A set can be written explicitely by listing its element by using set of brackets.

If the order of elements is changed or any element of set is repeated, it does not meet any changes in the set.

Ex. 1) Set of all positive integers
2) Set of all planets in the solar system.

Set Representation :
i) List representation
ii) Predicate representation.
iii) Missing element representation.

1) List Representation

Let us, suppose, we have a set A with elements 1, 2, 3, a, b
Generally, a set is represented by listing all the elements of it. Here, set A is represented by
$A = \{1, 2, 3, a, b\}$.

2) Predicate Representation :

In this representation, a set is defined by predicate, this representation is more convinient than list representation.

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Ex. $B = \{x \mid x$ is an odd positive integer$\}$.

Let us, suppose that $p(x)$ denotes x is an odd positive integer, then

$B = \{x \mid p(x)\}$

Note: For all x such that $p(x)$ is odd +ve int.

iii) Missing element representation.

Sometimes, it is convinient to represent sets by missing element representation.

$B = \{x \mid x$ is an odd +ve int$\}$
→ $B = \{1, 3, 5, 7, \ldots\ldots \}$ — infinite.
or
$B = \{1, 3, 5, 7, \ldots\ldots 13\}$ — finite.

Note: Some important sets.

N = Set of all natural numbers.
Z = Set of whole integer.
Z^+ = Set of whole +ve integer.
Q = Rational no's set
R = Set of all Real No.
W = set of all whole no.

Cardinality of Sets.

The cardinality of set S is denoted by $|S|$ is the no. of elements of a set. The no. is also reffered as cardinality no.

If set has infinite no of elements. Then it's cardinality is also ∞.

Ex. $|\{1, 2, 3, 4, 5\}| = 5$
$|\{1, 2, 3 \ldots\}| = \infty$

If there are two sets x & y.

i) $|x| = |y|$ It denotes 2 sets x & y having same Cardinality.
It occurs when, the no. of elements in x is exactly equal to the no. of elements in y.

ii) It denotes that set x is cardinality is less than y is cardinality. It occurs when the no. of element in x is less than that of y.

iii) $|x| \leq |y|$ and $|x| \geq |y|$ then $|x| = |y|$ known as equivalent sets.

Types of Set

Sets can be defined or classified into many types such as finite, infinite, subset, universal, proper, singleton.

i) Finte Sets :

The set which contains a define no of elements is called finite sets.

ii) Infinite sets :

A set which contains infinite no of elements is called infinite sets.

iii) Subsets :

A set x is subset of set Y written as $(x \subseteq y)$. If every element of x is an element of set Y.

$X = \{1, 2, 3, 4, 5\}$
$Y = \{1, 2\}$

Here, set Y is subset of x, has all the element of y is in the set of x.
$(Y \subseteq X)$

Let $X = \{1, 2, 3\}$
$Y = \{1, 2, 3\}$

$Y \subseteq X$

iv) Proper subset : The term proper subset can be defined as subset of but not equal to.
A set 'X' is proper subset of y defined as if every elements of X is a element of set Y and $|X| < |Y|$.

Ex. $X = \{1, 2, 3, 4, 5\}$
$Y = \{1, 2\}$
→ $Y \subset X$, y is proper subset of x.

v) Universal set (U) : It is collection of all elements. in a perticular container.

Ex. set of all animals on the earth

vi) Empty Set / Null Set ($\emptyset$) : The empty set is finite set. It denoted by null. Empty set contains no element.

Cardinality of empty set : 0 (zero elements)
Ex. $X = \{x / x \in N \text{ and } 7 < x < 8\} = 0$.

vii) Equal Set : If the content of two sets are same.

Ex : $A = \{1, 2, 6\}$
$B = \{6, 2, 1\}$

They are equal as every element of set A is an element of set B and the every element of set B is an element of set A.

viii) Equivalent Set : The cardinality of two set are same then it said to be equivalent set.

Ex. $\left.\begin{array}{l} A = \{1, 2, 3\} \\ B = \{4, 5, 6\} \end{array}\right\}$ 3 elements.

ix) ~~vii~~ Singleton Set :

Ex : $A = \{1, 2, 3\}$

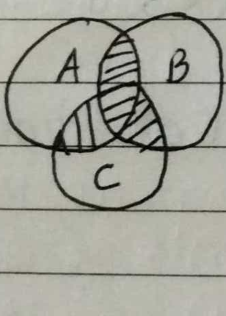
$\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}$

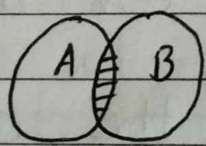
$p(A) = 8$ $\qquad 2^3 = 2 \times 2 \times 2 = 8.$

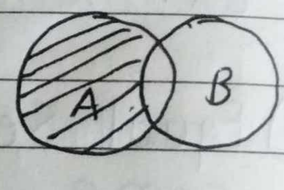
Venn diagram :

i) Venn diagram is made in 1880 by John Venn.

ii) It is schematic diagram, that represents all possible logic, relation between difference, mathematical set

① 

② 

③ 

Set operation : include set union, set intersection, set difference, compliment of set, Cartesian product

1) Set Union :

The union of sets A & B denoted by $A \cup B$ is a set of element which are in A & B or both A & B.

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

If $A = \{10, 11, 12, 13, 14\}$ & $B = \{13, 14, 15, 16\}$

then $A \cup B = \{10, 11, 12, 13, 14, 15, 16\}$.

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2) Set intersection :

- The set intersection of set A&B denoted by $(A \cap B)$ is the set of elements which are in both A&B. Hence,

$A \cap B = \{x / x \in A \text{ and } x \in B\}$

Ex. $A = \{11, 12, 13\}$

$B = \{13, 14, 15\}$

$A \cap B = \{13\}$

3) Set Difference / Relative compliment

- The set difference of set A&B denoted by $(A-B)$ is set of element which is in only A but not in B.

Hence,

$A - B = \{x / x \in A \text{ and } x \notin B\}$

Ex.

If $A = \{10, 11, 12, 13\}$

$B = \{13, 14, 15\}$ then

$(A-B) = \{14, 15\}$

Hence, we can see that $(A-B) \neq (B-A)$.

4) Compliment of set

The compliment of a set A denoted by A' is a set of elements which are not in set A which are A'

Hence,

$A' = \{x / x \notin A\}$

Most specially $A' = (U-A)$ when
U = Universal set which contains all elements.

Ex.
If $A = \{x / x$ belongs to set of odd int$\}$
then $A' = \{x / x$ does not belong to set of odd int$\}$

5) Cartesian Product / Cross product
→ The cartesian product of 'n' number of sets $A_1, A_2 \dots A_n$. denoted as $A_1 \times A_2 \times A_3 \dots A_n$ can be defined as all possible order page pairs $(x_1, x_2, x_3 \dots x_n)$ where $x_1 \in A_1$, $x_2 \in A_2$ $\dots x_n \in A_n$.

Take two sets.
$A = \{a, b\}$
$B = \{1, 2\}$
Caretsian product:
$A \times B = \{(a,1)\ (a,2)\ (b,1)\ (b,2)\}$
$B \times A = \{(1,a)\ (1,b)\ (2,a)\ (2,b)\}$

Properties of sets.
1) Commutative property : $A \cup B = B \cup A$
$A \cap B = B \cap A$

2) Associative property : $(A \cup B) \cup C = A \cup (B \cup C)$
$(A \cap B) \cap C = A \cap (B \cap C)$

3) Distributive property : $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4) Identity property : $A \cup \emptyset = A$

5) Compliement property : $A \cup A' = A$

6) Idempotent property : $A \cup A = A$
$A \cap A = A$.

De - morgan's law.

De - morgan's law is applicable in relating the union & intersection of two sets via complement There are two laws under De- morgan's law.

1) De - morgan's law of union :

It states that the compliment of the union of two sets is equal to the intersection of the complement of individual sets.
Mathematically expressed as

$(A \cup B)' = A' \cap B'$

2) De - morgan's law of intersection :

It states that the complement of intersection of two sets is equal to the union of the compleme of individual sets.

$(A \cap B)' = A' \cup B'$.

Logical Connectivity :

Logical connectivity can be described as the operators that can be used to connect one or more propositions or predicate logic. On the basis of input logic & connectivity which is used to connect the proposition, so that, we can get resultant logic.

Five Basic Connectivities

1) Negation $\sim$ Not
2) Conjunction $\wedge$ AND
3) Disjunction $\vee$ OR
4) Conditional $\rightarrow$ If then
5) Biconditional $\leftrightarrow$ If & only if.

1) Negation

The symbol $\sim$ is used to indicate the negation if there is proposition (p) then negation of 'p' will also proposition which contains the following property

1) When P is true, then negation of p is false
2) When P is false, then negation of p is true.

Truth Table :

p	$\sim p$
T	F
F	T

Ex. p : I am a good student
$\sim p$: I am not a good student.

2) Conjunction :

Conjunction is indicated by the symbol '$\wedge$'
If there are 2 ~~composi~~ proposition then $p \wedge q$ also be a proposition.

i) When p & q are true, then the conjunction will be true.
ii) In other cases, conjunction will be false.

Truth Table :

p	q	$\wedge$
T	T	T
T	F	F
F	T	F
F	F	F

3) Disjunction :

Disjunction is denoted by V (OR). If there are 2 proposition P & Q then disjunction will be also proposition.

i) when P & Q are false, then disjunction is also false
ii) when any of P or Q is true, then disjunction is true.

Truth Table :

P	Q	V
T	T	T
T	F	T
F	T	T
F	F	F

4) Conditional Statement

The conditional propositional is also known as implication proposition & is indicated by symbol '→'. If there are 2 proposition P & Q then implication also be proposition.

i) If there is a proposition that has the form of 'if P then Q' then that type of proposition will be known as conditional proposition.

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ii) When P is false or p&q are true then the implication of them will be true.

iii) When P is true & q is false then the implication will be false

Truth Table.

p	q	→
T	T	T
T	F	F
F	T	T
F	F	T

Ex.
If I learn very hard, then I will get good marks in the exam.

5) Biconditional Statement:

The Biconditional proposition is also known as Biimplication proposition. It is denoted by '↔'. If there are 2 proposition p&q then the biconditional also be a proposition.

i) When both p&q are true and p&q are false then proposition will be True.
ii) If other cases, the biconditional will be false.

Truth Table :

p	q	$\leftrightarrow$
T	T	T
T	F	F
F	T	F
F	F	T

Ex. I will go to beach if & only if it is sunny

Tautologies :

Tautologies is a formula which is always true for every value of its propositional variable.

Ex. Prove $[(A \to B) \land A] \to B$ is tautologiey.

A	B	$A \to B$		$(A \to B) \land A$	$[(A \to B) \land A] \to B$
T	T	T		T	T
T	F	F		F	T
F	T	T		F	T
F	F	T		F	T

As we can see that all values are true hence it is a tautology.

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* Contradiction : The contradiction is a formula which is always false for every value.

Ex. Prove $(A \vee B) \wedge [(\sim A) \wedge (\sim B)]$

A	B	$\sim A$	$\sim B$	$A \vee B$	$(\sim A) \wedge (\sim B)$	$(A \vee B) \wedge [\sim A \wedge \sim B]$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

* Contingency : is a formula which has both, some true and some false, for every value of its propositional variable.

A	B	$\sim A$	$A \vee B$	$(A \vee B) \wedge (\sim A)$
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Inverse :

The inverse of conditional statement is the negation of both the hypothesis & conclusion.

If the statement "if p then q" then inverse will be "if not p, then not q".

- $p \rightarrow q$
- $\sim p \rightarrow \sim q$.

Converse :

The converse of conditional statement is computed by interchanging the hypothesis & conclusion.

- $p \rightarrow q$
- $q \rightarrow p$

Contrapositive :

The contrapositive of the condition is computed by interchanging the hypothesis and the conclusion of inverse statement.

- $p \rightarrow q$
- $\sim q \rightarrow \sim p$

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① "If the weather is sunny, then I will go to school".

p : The weather is sunny
q : I will go to school.

Inverse : $p \to q$: $\sim p \to \sim q$.
"If the weather is not sunny, then I will not go to school".

Converse : $q \to p$
"If I will go to school, then the weather is sunny".

Contrapositive : $\sim q \to \sim p$.
"If I will not go to school, then the weather is not sunny".

② "If there is rainy weather, then I will go outside to enjoy it.".

P: There is rainy weather.
q : I will go outside to enjoy it.

Inverse : If there is not rainy weather, then I will not go outside to enjoy it.

Converse : If ~~there~~ I will go outside to enjoy it, then there is rainy weather.

Contrapositive : If I will not go outside to enjoy it, then there is not rainy weather.

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Q.1 Negation of following propositions.

i) Today is Thursday
→ Today is not Thursday

ii) ~~New~~ There is no pollution in New Jersy.
→ New Jersy is polluted.

iii) $2+1=3$
→ $2+1 \neq 3$

iv) The summer in Maine is hot and sunny.
→ The summer in Maine is not hot and not sunny.

Q.2 p : I bought a lottery ticket this week.
q : I won the million dollar jackpot on Friday.
Express following propositions as an English statement

a) $\sim p$
b) $p \vee q$
c) $p \rightarrow q$
d) $p \wedge q$
e) $p \leftrightarrow q$
f) $\sim p \rightarrow \sim q$
g) $\sim p \wedge \sim q$
h) $\sim p \vee (p \wedge q)$.

→

a) I did not buy a lottery ticket this week.

b) Either I bought a lottery ticket this week or I won the million dollar jackpot on Friday.

c) If I bought a lottery ticket this week, then I won the million dollar jackpot on Friday.

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d) I bought a ~~to~~ lottery ticket this week and I won the million dollar jackpot on Friday.

e) I bought a lottery ticket this week if and only if I won the million dollar jackpot on Friday.

f) If I did not buy a lottery ticket this week then I did not win the million dollar jackpot on Friday.

g) I did not buy a lottery ticket this week and I did not win the million dollar jackpot.

h) Either I did not buy a lottery this week, else I did buy one and won the million dollar jackpot on Friday.

Q.3 p : The election is decided
q : The votes have been counted.
Express :

a) $\sim p$	e) $\sim q \rightarrow \sim p$
b) $p \vee q$	f) $\sim p \rightarrow \sim q$
c) $\sim p \wedge q$	g) $p \leftrightarrow q$
d) $q \rightarrow p$	h) $\sim q \vee (\sim p \wedge q)$

a) The election is not decided.

b) The election is decided or the votes have been counted

c) The election is not ~~decided~~ decided & the votes have been counted.

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d) If the votes have been counted, then the election is decided.

e) If the votes have not been counted, then the election is not decided.

f) If the election is not decided, then the votes have not been counted.

g) The election is decided if and only if the votes have been counted.

h) ~~The electio~~ Either the votes have not been counted, or else the election is not decided and the votes have been counted.

Q.4 p : It is below freezing
q : It is snowing.

a) It is below freezing and snowing.
→ $p \wedge q$

b) It is below freezing but not snowing.
→ $p \wedge \sim q$

c) It is not below freezing and not snowing.
→ $\sim p \wedge \sim q$.

d) It is either snowing or below freezing.
→ $p \vee q$.

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e) If it is below freezing, it is also snowing.

f) It is either below freezing or it is snowing, but it is not snowing if it is below freezing.

g) That is below freezing is necessary and sufficient for it to be snowing.

e) $\Rightarrow p \rightarrow q$
f) $= (p \vee q) \wedge (p \rightarrow \sim q)$
g) $= p \leftrightarrow q$.

Q.5 p : You have the flu.
q : You miss the final examination
r : You pass the course.

a) $p \rightarrow q$
→ If you have flu, then you miss the final examination.

b) $\sim q \leftrightarrow r$
→ You will pass the course if and only if don't miss the final examination.

c) $q \rightarrow \sim r$
→ If you miss the final examination, then you will not pass the course

d) $p \vee q \vee r$
→ You have the flu or you miss the final examination or you pass the course

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e) $(p \rightarrow \sim r) \vee (q \rightarrow \sim r)$

→ If you either have the flu or miss the final examination, ~~or~~ then you will not pass the course (or) If you have the flu, then you do not pass the course or if you miss the final examination then you do not pass the course or both.

f) $(p \wedge q) \vee (\sim q \wedge r)$.

→ Either you have the flu and miss the final examination, or you don't miss the final examination and pass the course.

Q.6 p : You drive over 65 miles per hour
q : You get a speeding ticket. ~~if you~~

a) You do not drive over 65 miles per hour.

→ $\sim p$

b) If You drive over 65 miles per hour, then you will not get a speeding ticket.

→ $p \rightarrow \sim q$.

c) You drive over 65 miles per hour, but you do not get a speeding ticket.

→ $p \wedge \sim q$.

d) You will get a speeding ticket if you drive over 65 miles per hour

→ $p \rightarrow q$.

e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.
→ $p \to q$.

f) You get a speeding ticket, but you do not drive over 65 miles per hour.
→ $q \land \sim p$

g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.
→ $p \to q$.

Q.7 p : You get an A on the final exam.
q : You do every exercise in this book.
r : You get an A in this class.

~~a) You get an A in this class, but you do not do every exercise in this book,~~

a) You get an A in this class, but you do not do every exercise in this book.
→ $r \land \sim q$.

b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
→ $p \land q \land r$.

c) To get an A in this class, it is necessary for you to get an A on the final.
→ $r \to p$.

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d. You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

→ $p \wedge \sim q \wedge r$.

e. Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class

→ $(p \wedge q) \rightarrow r$.

f. You will get A in this class if and only if you either do every exercise in this book or you get an A on the final.

→ $r \leftrightarrow (p \vee q)$.

good.

Mam
12/9/23.

Well formed formula (wff)

WFF is a predicate holding any of the following:

i) All propositionals constant & propositional variable are WFF

ii) If x is a variable and y is WFF, $\forall x y$ and $\exists x y$ are also WFF

iii) Truth value and False value are WFF.

iv) Each atomic formula is wff.

v) All connetivities connecting wff are wff.

Quantifiers :

The variable of predicate is quantified by Quantifiers. They are two types of quantifiers in predicate logic.

1) Universal Quantifiers '$\forall$' for all

2) Extential Quantifiers '$\exists$' their exist, some, few

1) Universal Quantifiers

Universal Q. states that the statement within its scope are true for every value of its specific variable.

It is denoted by '$\forall$'. Ex. $\forall x\, P(x)$

→ for every value of x, $P(x)$ is true.

E.g ① 'Man is mortal'.

$\forall x\, P(x)$: where $P(x)$ is a predicate which denotes x is mortal & the universe discourse is all man.

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2) Extential Quantifiers :

Extential Q. states the statement within its scope are true for some values of specific variabl
It is denoted by '$\exists$'.

Representation : $\exists x\, P(x)$
Read as : For some values of x. $p(x)$ is true
E.g Some people are dishonest
$\exists x\, P(x)$.

① Suppose, $p(x)$ indicates a predicate where "x must take an electronic course" and $Q(x)$ also indicates that a predicate where "x is an electrical student"

- Find Universal Quantifiers

Solution :
Suppose the students are from xyz college for both predicate, the universe of discourse will be all xyz students.
The statement can be "Every electrical student must take an electronic course".
→ $\forall x\, (Q(x) \Rightarrow P(x))$.

② Everybody must take an electronic course or be an electrical student".
→ $\forall x\, (Q(x) \vee P(x))$

GCD - Greatest Common Divisior

i) GCD(12, 33)

	12	33
Divisior	1, 2, 3, 4, 6, 12	1, 3, 11, 13
Common Divisor	1, 3	
GCD	3	

ii) GCD (25, 150)

	25	150
Divisior	1, 5, 25	1, 3, 5, 10, 15, 25, 75, 150
Common Divisicor	1, 5, 25	
GCD	25	

Euclid's Algorithm for finding GCD.

① First Method

i) Find GCD (750, 950)

Q	A	B	R
1	900	750	150
5	750	150	0
x	150	0	

ii) GCD (12, 33)

Q	A	B	R
2	33	12	9
1	12	9	3
3	9	3	0
x	3	0	

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② Second Method.

Prerequisite : a>b
Euclid GCD (a, b)

```
        if b = 0 then
            return a;
        else
            return Euclid_GCD (b, a mod b);
```

i) GCD (50, 12)
a = 50, b = 12.

GCD (a, b) = GCD (b, a mod b)
GCD (50, 12) = GCD (12, 50 mod 12) = GCD (12, 2)
GCD (12, 2) = GCD (2, 12 mod 2) = GCD (2, 0).

∴ GCD (50, 12) = 2.

ii) GCD (529, 123)

GCD (529, 123) = GCD (123, 529 mod 123) = GCD (123, 37)
GCD (123, 37) = GCD (37, 123 mod 37) = GCD (37, 12)
GCD (37, 12) = GCD (12, 37 mod 12) = GCD (12, 1)
GCD (12, 1) = GCD (1, 12 mod 1) = GCD (1, 0)

∴ GCD (529, 123) = 1.

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Mathematical Induction:

Basis Step : We verify $P(1)$ is true.

Inductive Step : We show that the conditional statement $P(K) \to P(K+1)$ is true for all +ve int K.

① If n is a positive int, then show that

$$1+2+\cdots+n = \frac{n(n+1)}{2}$$

Basis step : $P(1)$ is true $n=1$

$$1 = \frac{1(1+1)}{2}$$

$$1 = 1$$

$$L.H.S = R.H.S$$

Inductive step :

$$1+2+3+\cdots+K = \frac{K(K+1)}{2}$$

$P(P+1)$ is true, namely that

$$1+2\cdots+K+(K+1) = \frac{K+1((K+1)+1)}{2}$$

$$R.H.S = \frac{(K+1)(K+2)}{2}.$$

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$$L.H.S = \frac{K(K+1)}{2} + (K+1)$$

$$= \frac{K(K+1) + 2(K+1)}{2}$$

$$= \frac{(K+1)(K+2)}{2}$$

Hence proved $L.H.S = R.H.S.$

ii) $1 + 3 + 5 + \dots + (2K-1) = K^2$

Basis Step : $P(1)$ is true $K = 1$

$$(2-1) = (1)^2$$
$$1 = 1$$
$$L.H.S = R.H.S.$$

Inductive Step :

$1 + 3 + 5 + \dots + (2K-1) = K^2$

Division Algorithm.

When an int is divided by a positive int, there is quotient & remainder as the division algorithm shows :

Theorem : Let 'a' be an int and 'd' be a +ve int then there are unique int q & r, $0 \le r \le d$ such that $a = dq + r$.

101 / 11

$\rightarrow q = 9, \quad r = 2$

$101 = 9 \times 11 + 2$

$101 = 101$