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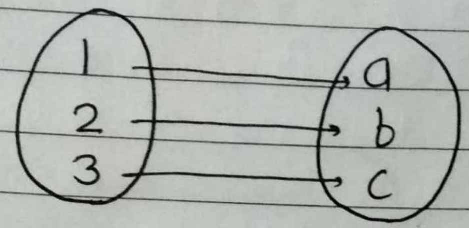
Tutorial 2 : Functions & Relations.

Functions are important part of Discrete Mathematics. Functions are rules assigned 1 input to 1 output. The function can be represented as: $A \to B$.

A is called domain of the func^n

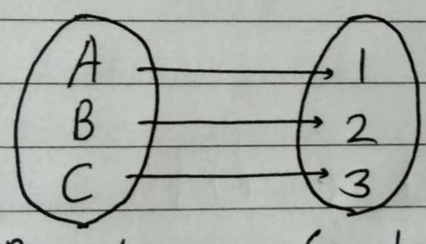
B is co-domain

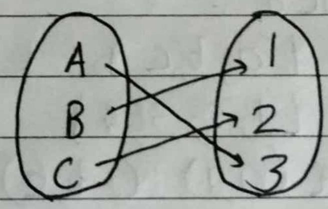


$f: A \to B$.

1) Injective func^n (one-one)

A func^n in which one element of domain set is connected to one element of co-domain set.





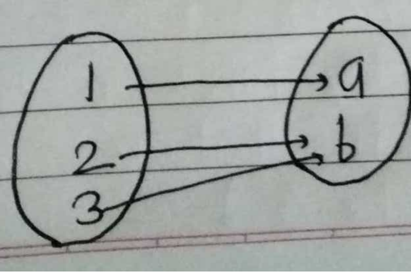
Domain — Co-domain

Pre-image — image

range.

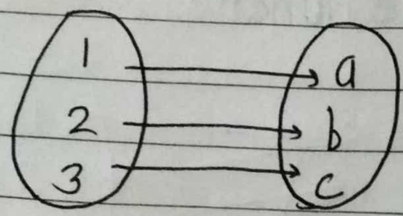
2) Surjective func^n (onto func^n)

A func^n in which every element of co-domain set has one pre-image.



3) Bijective func^n (one-one onto func^n)

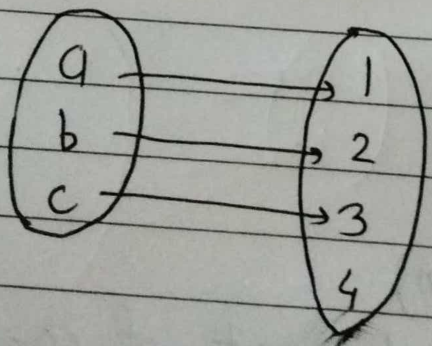
If the func^n f is one-one func^n & also it is onto so it is a bijective func^n.



4) Into func^n

A func^n in which there must be an element of co-domain y doesn't have a pre-image in domain.

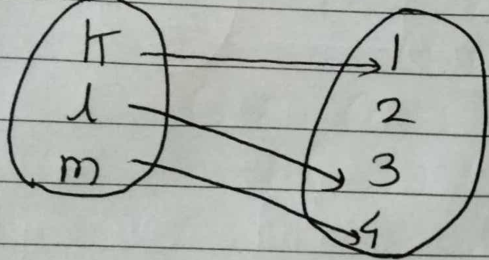
consider :
$A = \{a, b, c\}$
$B = \{1, 2, 3, 4\}$
$f = \{(a,1)\ (b,2)\ (c,3)\}$



5) One to one into func^n

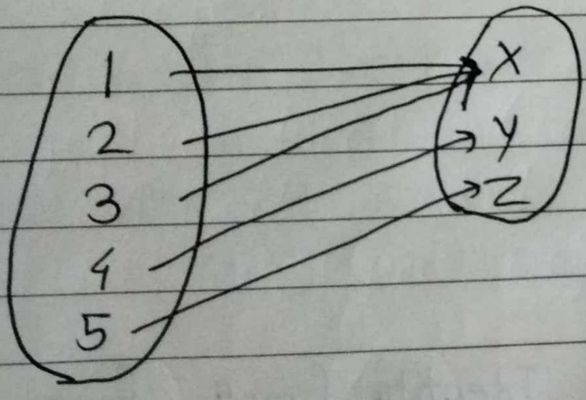
Let the func^n $x \to y$
func^n 'f' is called one to one & into func^n if different elements of x have different unique images of y.

$X = \{k, l, m\}$ $y = \{1, 2, 3, 4\}$
$f : X \to y$, $f = \{(k,1)\ (l,3)\ (m,4)\}$.



6) Many to one func^n

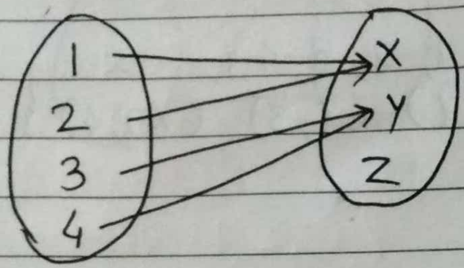
Let $f : x \to y$, 'f' said to be many to one func^n if there exist 2 or more different element in x having the same image in y.
consider $x = \{1, 2, 3, 4, 5\}$
$y = \{x, y, z\}$
$f = \{(1,x)\ (2,x)\ (3,x)\ (4,y)\ (5,z)\}$.



7) Many-one into func^n.

$f : x \rightarrow y$

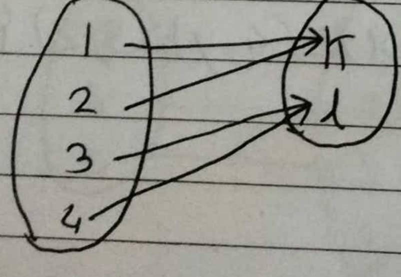
The 'f' is called Many-one into func^n if and only if it is both many-one & into func^n.



8) Many-one onto func^n.

The 'f' is many-one onto if it has both func^n many-one and onto func^n.

$x = \{1, 2, 3, 4\}$
$y = \{k, l\}$
$f = \{(1,k)\ (2,k)\ (3,l)\ (4,l)\}$.

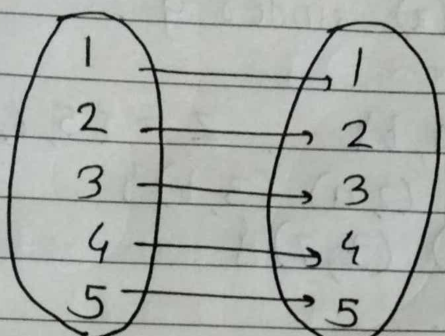


9) Identity func^n

The func^n 'f' is called Identity func^n if each element of set A has an image on itself that is $f(a) = a \ \forall\, a \in A$. (Denoted by 'I')

Ex. consider
$A = \{1, 2, 3, 4, 5\}$, $f : A \to A$
$f = \{(1,1) (2,2) (3,3) (4,4) (5,5)\}$

The func^n F is identity func^n as each element of A is mapped to itself.
A → one to one & onto.

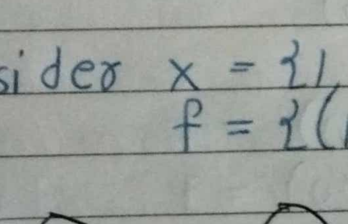


* Invertible or Inverse func^n.

The func^n $f : x \to y$ is Inversible if and only if it is a bijective func^n.
Consider the bijective func^n $f : x \to y$. As f is 1-1 ∴ each element of x correspondance to.
As f is onto ∴ there is no element of y which is not the image of x i.e range = co-domain of y

The inverse func^n of f exist if f^{-1} is func^n from y to x.

Consider $x = \{1, 2, 3\}$, $y = \{k, l, m\}$, $f : x \to y$
$f = \{(1,k) (2,m) (3,l)\}$.

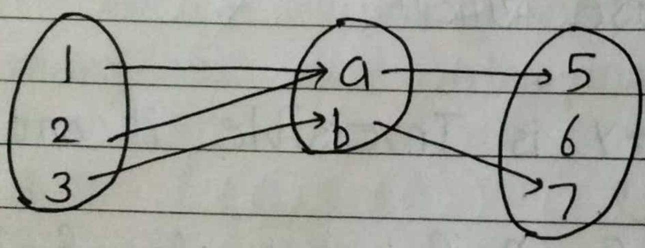


Composition of func^n :

Consider a func^n → $f: A \to B$ & $g: B \to C$
The composition of f with g is func^n from A to C
defined by $(g(f))(x) = g[f(x)]$

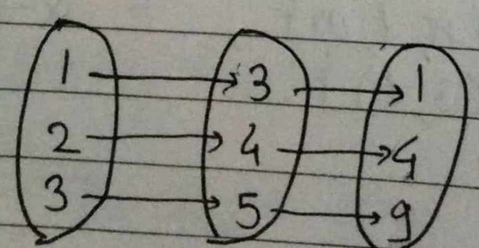
To find the composition of f & g, first find image of x under f & find f(x) under g.

① Let $X = \{1, 2, 3\}$, $Y = \{a, b\}$, $Z = \{5, 6, 7\}$
Consider a func^n $f = \{(1,a)\ (2,a)\ (3,b)\}$
$g = \{(a,5)\ (b,7)\}$.
Find composition of g(f).



$g(1) = 5$
$g(2) = 5$
$g(3) = 7$

② Let $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$ $C = \{1, 4, 9\}$
Find the composition of g (f(1))



$g(f(1)) = 1$
$g(f(3)) = 9$

Relations.

Relation or binary relation from set A to B is subset of A x B which can be defined as
$aRb \leftrightarrow (a,b) \in R \leftrightarrow R(a,b)$.

The binary relation are on single set A a subset of A x A. For two distinct set A & B with cardinality m & n. The max cardinality of relation R from A to B is mn.

Domain and Range.

If there are two sets A & B and relation from A to B is R(a,b) then domain is defined as the set $\{a \mid (a,b) \in R$ for some b in B$\}$ and range is defined as $\{b \mid (a,b) \in R$ for some a in A$\}$.

Types of Relation :

① Empty relation : A relation R on set A is called empty if the set A is empty set.

② Full relation : A binary relation 'R' on set A & B is called Full if A x B.

③ Reflexiv relation : A relation R on set A is called reflexiv if $(a,a) \in R$ holds for every element $a \in A$
i.e if set $A = \{a, b\}$ then
$R = \{(a,a)\ (b,b)\}$ is reflexiv relation.

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$A = \{1, 2, 3\}$

$R = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$

→ It is Reflexiv

$R = \{(1,1)\ (1,2)\ (1,3)\ (2,2)\ (2,3)\}$

→ Not Reflexiv.

$R = \{(1,1)\ (1,2)\ (2,2)\ (3,3)\ (3,2)\}$

→ Reflexiv.

④ Irreflexiv relation : A relation R on set A is called irreflexiv if the set $A = \{a, b\}$ then $R = \{(a,b)\ (b,a)\}$

i.e. $x \notin x\ \forall x \in A$.

$A = \{1, 2, 3\}$

$R = \{ (1,1)\ (1,2)\ (1,3)$
$(2,1)\ (2,2)\ (2,3)$
$(3,1)\ (3,2)\ (3,3)$

i) $R = \{(1,1)\ (2,1)\}$ → Not reflexiv

ii) $R = \{(1,2)\ (1,3)\ (2,1)\}$ → irreflexiv

iii) $R = \{(1,1)\ (2,2)\ (3,1)\}$ → Not reflexiv not irreflexiv

⑤ Symmetric relation : A relation R on set A is called symmetric if $(b,a) \in R$ holds when $(a,b) \in R$

i.e. the relation $R = \{(4,5)\ (5,4)\ (6,5)\ (5,6)\}$ on set $A = \{4,5,6\}$ is symmetric.

i.e ① xRy then yRx $\forall$ $(x,y) \in A$
② $(x,y) \in R$ then $(y,x) \in R$ $\forall$ $(x,y) \in A$

$R = \{(1,2)\ (1,3)\ (2,1)\ (2,2)\ (3,1)\}$
It is symmetric.

$R = \{(2,3)\ (3,2)\ (1,1)\}$
Symmetric.

$R = \{(1,2)\ (2,1)\ (1,3)\}$
It is not symmetric.

⑥ Anti-Symmetric :
The relation 'R' on set A is said to be anti-symmetric if xRy and yRx if $x=y$ $\forall x \in A$.

i.e. $(x,y) \in R$ then $(y,x) \notin R$ only if $x=y$ $\forall x,y \in A$.

$R = \{(1,2)\ (2,2)\ (2,1)\}$
Not Anti-symmetric.

$R = \{(1,1)\ (2,2)\ (3,1)\}$
Anti symmetric.

⑦ Asymmetric.
A relation R on set A is said to be asymmetric if xRy & yRx then $x=y$ $\forall$ $x,y \in A$.
i.e. if $(x,y) \in R$ then $(y,x) \in R$ $\forall (x,y) \in A$.
$A = \{1,2,3\}$, $R = \{(1,2)\ (2,3)\ (1,1)\} \rightarrow$ Not
$R = \{(1,2)\ (2,3)\ (1,3)\} \rightarrow$ Yes

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⑧ Transitive Relations :

- xRy, yRz then xRz.

if $(x,y) \in R$ & $(y,z) \in R$ then $(x,z) \in R$.

$A = \{1, 2, 3\}$
$R = \{(1,2)\ (2,3)\ (3,1)\}$ ✓
$R = \{(1,1)\ (1,2)\ (2,1)\}$ ✓
$R = \{(1,2)\ (2,3)\}$ X
$R = \{\}$ ✓

⑨ Equivalence Relation

A relation R on set A is said to be equivalent if it is reflexiv, symmetric and Transitive.

$A = \{1, 2, 3\}$
$R = \{(1,1)\ (2,2)\ (3,3)\}$ ✓
$R = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,1)\}$ ✓
$R = \{\}$ X