EM – III : Assignment

INDEX

NAME. : Suraj. A. Sutar Std. : S.Y. Div. : C.S.E.

Roll No. : 27 School / College. :

Sr. No	TITLE	Date	Teacher's Sign / Remark
1	Assignment No. 1		
2	Assignment No. 2		
3	Assignment No. 3		

Assignment No. 3

Page No.
Date

Exercise -1

1) Find the L.T of $L^{-1}\left\{\frac{s+2}{s^2+4s+5}\right\}$

$\rightarrow L^{-1}\left\{\frac{s+2}{(s+2)+1}\right\}$

$L^{-1}\left\{\frac{s+2}{(s+2)+1}\right\} + 2L^{-1}\left\{\frac{1}{(s+2)+1}\right\}$

$= e^{-2t}\cos t + 2e^{-2t}\sin t$

2) Find $L^{-1}\left\{\frac{2s+3}{s^2+2s+2}\right\}$

$\rightarrow L^{-1}\left\{\frac{2s+3}{(s+1)+1}\right\}$

$2L^{-1}\left\{\frac{s}{(s+1)+1}\right\} + 3L^{-1}\left\{\frac{1}{(s+1)+1}\right\}$

$= 2e^{-t}\cos t + 3e^{-t}\sin t.$

$= e^{-t}(2\cos t + 3\sin t.$

3) Find $L^{-1}\left\{\frac{3s-7}{s^2-6s+8}\right\}$

$\rightarrow L^{-1}\left\{\frac{3s-7}{(s-3)-1}\right\}$

$$3L^{-1}\left\{\frac{s}{(s+1)-1}\right\} - 7L^{-1}\left\{\frac{1}{(s+1)-1}\right\}$$

$$= 3e^{-t}\cosh t - 7e^{-t}\sinh t.$$

$$= e^{-t}(3\cosh t - 7\sinh t).$$

4. Find $L^{-1}\left\{\frac{1}{(s+4)^4}\right\}$

$$\rightarrow e^{-4t} L^{-1}\left\{\frac{1}{s^4}\right\}$$

$$e^{-4t}\frac{t^3}{6}.$$

$$\therefore L^{-1}\left\{\frac{1}{(s+4)^4}\right\} = \frac{1}{6}e^{-4t}t^3.$$

5) Find $L^{-1}\left\{\log\left(1+\frac{4}{s^2}\right)\right\}$

$\rightarrow$

$$\phi(s) = \log\left(\frac{s^2+4}{s^2}\right)$$

$$= \log(s^2+4) - \log s^2$$

$$\phi'(s) = \frac{2s}{s^2+4} - \frac{2s}{s^2}$$

$$L^{-1}\{\phi'(s)\} = 2\cos 2t - 2$$

$$= 2(\cos 2t - 1)$$

$$L^{-1}\{\phi(s)\} = \frac{2}{t}(\cos 2t - 1)$$

6) Find $L^{-1}\left\{\cot^{-1}\left(\frac{s-2}{3}\right)\right\}$

$\rightarrow$

$$\phi(s) = \cot^{-1}\left(\frac{s-2}{3}\right)$$

$$\phi'(s) = \frac{1}{1+\left(\frac{s-2}{3}\right)^2}\frac{d}{ds}\left(\frac{s-2}{3}\right)$$

$$= \frac{1}{1+\left(\frac{s^2-4s+4}{9}\right)}\frac{1}{3}$$

$$= \frac{1}{\cancel{9+s^2-4s+4}/9}\frac{1}{3}$$

$$= \frac{\cancel{9}\,3}{s^2-4s+13}\frac{1}{\cancel{3}}$$

$$L^{-1}\{\phi'(s)\} = L^{-1}\left\{\frac{3}{(s-2)^2+9}\right\}$$

$$= e^{2t} \, L^{-1}\left\{\frac{3}{s^2+9}\right\}$$

$$= e^{2t} \sin 3t$$

$$L^{-1}\{\phi(s)\} = \frac{1}{t} e^{2t} \sin 3t$$

7) Find $L^{-1}\left\{ \tan^{-1}(s+1) \right\}$

$$\phi(s) = \tan^{-1}(s+1)$$

$$\phi'(s) = \frac{1}{1+(s+1)^2} \, \frac{d}{ds} s+1$$

$$= \frac{1}{s^2+2s+2}$$

$$L^{-1}\{\phi'(s)\} = L^{-1}\left\{\frac{1}{1+\cancel{(s+1)^2}\,(s+1)^2}\right\}$$

$$= e^{-t} \sin t$$

Exercise 2

1) Find $L^{-1}\left\{\frac{s}{s^2+5s+6}\right\}$

$\rightarrow L^{-1}\left\{\frac{s}{(s+3)(s+2)}\right\} = \frac{A}{s+3} + \frac{B}{s+2}$ — (i)

multi eqⁿ (i) by $(s+3)(s+2)$

$s = (s+2)A + (s+3)B$ — (ii)

put $s=-2$ in eqⁿ (ii)
$-2 = B$
put $s=-3$ in eqⁿ (ii)
$-3 = -1A$
$A = 3$

put the values in eqⁿ (i)

$L^{-1}\left\{\frac{s}{(s+3)(s+2)}\right\} = 3L^{-1}\left\{\frac{1}{s+3}\right\} - 2L^{-1}\left\{\frac{1}{s+2}\right\}$

$= 3e^{-3t} - 2e^{-2t}$

2) Find $L^{-1}\left\{\frac{s^2+11}{s^3+3s^2+2s}\right\}$

$\rightarrow$

$s^3+3s^2+2s = s(s^2+3s+2) = s(s+2)(s+1)$

$L^{-1}\left\{\frac{s^2+11}{s(s+2)(s+1)}\right\} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1}$ — (i)

Date

Multiply the eq^n by $s(s+2)(s+1)$

$$s^2+11 = (s+2)(s+1)A + s(s+1)B + s(s+2)C \quad - (ii)$$

put $s=0$ in eq^n (ii)

$$11 = 2A$$

$$A = \frac{11}{2}$$

put $s=-2$

$$15 = 1B$$

$$B = 15$$

put $s = -1$

$$12 = -1C.$$

$$C = -1$$

put the values in eq^n (i)

$$L^{-1}\left\{\frac{s^2+11}{s(s+2)(s+1)}\right\} = L^{-1}\left\{\frac{11}{2s} + \frac{15}{s+2} \cancel{+} - \frac{12}{s+1}\right\}$$

$$= \frac{11}{2}L^{-1}\left\{\frac{1}{s}\right\} + 15L^{-1}\left\{\frac{1}{s+2}\right\} - 12L^{-1}\left\{\frac{1}{s+1}\right\}$$

$$= \frac{11}{2} + 15e^{-2t} - 12e^{-t}$$

3) Find $L^{-1}\left\{\dfrac{s^2}{(s^2+1)(s^2+4)}\right\}$

$$\rightarrow \frac{s^2}{(s^2+1)(s^2+4)} = \frac{As+B}{(s^2+1)} + \frac{Cs+D}{s^2+4}.$$

$$s^2 = (s^2+4)(As+B) + (s^2+1)(Cs+D)$$

$$s^2 = As^3 + Bs^2 + 4As + 4B + Cs^3 + Ds^2 + Cs + D.$$

$$s^2 = s^3(A+C) + s^2(B+D) + s(4A+C) + 4B + D.$$

$A+C = 0$ — 1

$B+D = 1$ — 2

$4A+C = 0$ — 3

$4B+D = 0$ — 4

substract eq^n 1 & eq^n 3

$$\begin{array}{r} 4A + C = 0 \\ -\quad A + C = 0 \\ \hline \end{array}$$

$\therefore A = 0$

$\therefore C = 0$

substract eq^n 2 & eq^n 4

$$\begin{array}{r} 4B + D = 0 \\ -\quad B + D = 1 \\ \hline 3B = -1 \end{array}$$

$\therefore B = \dfrac{-1}{3}$

put B in ②

$\frac{-1}{3} + D = 1$

$D = \frac{4}{3}$

Now, putting the values

$$\frac{s^2}{(s^2+1)(s^2+4)} = \frac{0-\frac{1}{3}}{(s^2+1)} + \frac{0+\frac{4}{3}}{s^2+4}$$

$$L^{-1}\left\{\frac{s^2}{(s^2+1)(s^2+4)}\right\} = \frac{-1}{3} L^{-1}\left\{\frac{1}{s^2+1}\right\} + \frac{2}{3}\left\{\frac{2}{s^2+4}\right\}$$

$$= \frac{-1}{3}\sin t + \frac{2}{3}\sin 2t$$

4) Find $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$

$\rightarrow$ $$\frac{5s+3}{(s-1)(s^2+2s+5)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+2s+5} \quad -①$$

Multiply eq^n ① by $(s-1)(s^2+2s+5)$

$5s+3 = (s^2+2s+5)A + (s-1)\,Bs+C$

$5s+3 = As^2+2As+5A + Bs^2 - Bs + Cs - C \quad -②$

$5s+3 = s^2(A+B) + s(2A-B+C) + 5A-C$

$A+B = 0 \quad -①$

$2A-B+C = 5 \quad -②$, $5A-C = 3$

$2A \overline{+} B + 0C$
$A + B + 0C = 0$

$A + B + 0C = 0$
$5A - 0B + C = 5$

$2A - B + C = 5$
$A + B + 0C = 0$

$$5s + 3 = (s^2 + 2s + 5)A + (s-1)(Bs + C) \quad - (2)$$

put $s = +1$

$8 = 8A$

$\therefore A = 1$

$5s + 3 = s^2 + 2s + 5 + Bs^2 - Bs + Cs - C$

$5s + 3 = s^2 + Bs^2 + 2s - Bs + Cs - C + 5$

$5s + 3 = s^2(1 + B) + s(2 - B + C) + 1(-C + 5)$

comparing on both sides

$-C + 5 = 3$

$\therefore C = -2$

$2 - B + C = 5$

$\therefore B = -1$

put the values of A, B & C in eq^n (i)

$$\frac{5s+3}{(s-1)(s^2+2s+5)} = \frac{1}{s-1} + \frac{-(s+2)}{(s+1)^2+4}$$

Taking L.T on both sides.

Page No.
Date

$$L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\} = L^{-1}\left\{\frac{1}{s-1}\right\} + L^{-1}\left\{\frac{-(s+1)+3}{(s+1)^2+2^2}\right\}$$

$$= L^{-1}\left\{\frac{1}{s-1}\right\} - L^{-1}\left\{\frac{(s+1)}{(s+1)^2+2^2}\right\}$$

$$+ 3L^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\}$$

$$= e^{t} - e^{-t}\cos 2t + \frac{3}{2}e^{-t}\sin 2t$$

5) $L^{-1}\left\{\frac{s^2}{(s+4)^3}\right\}$

→ Let $\frac{s^2}{(s+4)^3} = \frac{A}{(s+4)} + \frac{B}{(s+4)^2} + \frac{C}{(s+4)^3}$ — ①

Multiply eq^n ① by $(s+4)^3$

$s^2 = (s+4)^2A + (s+4)B + C$ — ②

put $s=-4$ in eq^n ②

$\therefore C = 16$

$s^2 = (s+4)^2A + Bs + 4B + 16$

$= As^2 + 2As + 16A + Bs + 4B + 16$

$\therefore A = 1$ — ③

put $A=1$, $C=16$ & $s=0$

$0 = 16 + 4B + 16$ in ③

$-32 = 4B$

$\therefore B = -8$

put values of A, B & C in eqⁿ ①

$$\frac{s^2}{(s+4)^3} = \frac{1}{s+4} - \frac{8}{(s+4)^2} + \frac{16}{(s+4)^3}$$

$$L^{-1}\left\{\frac{s^2}{(s+4)^3}\right\} = L^{-1}\left\{\frac{1}{s+4}\right\} - 8L^{-1}\left\{\frac{1}{(s+4)^2}\right\} + 16L^{-1}\left\{\frac{1}{(s+4)^3}\right\}$$

$$= e^{-4t} - 8e^{-4t}\cdot t + 8e^{-4t}\cdot t^2$$

$$\therefore L^{-1}\left\{\frac{s^2}{(s+4)^3}\right\} = e^{-4t}\left(1 - 8t + 8t^2\right).$$

Page No.
Date

Exercise 3.

1) Find I.L.T by using convolution theorem.

1) $L^{-1}\left\{\frac{s^2}{(s^2+2^2)^2}\right\}$

$\rightarrow L^{-1}\left\{\frac{s}{s^2+2^2} \cdot \frac{s}{s^2+2^2}\right\}$

$\phi_1(s) = \frac{s}{s^2+2^2} \rightarrow L^{-1}\{\phi_1(s)\} = L^{-1}\left\{\frac{s}{s^2+2^2}\right\}$

$= \cos 2t$

replace t by u.

$\therefore L^{-1}\{\phi_1(s)\} = \cos 2u.$

$\phi_2(s) = \frac{s}{s^2+2^2} \rightarrow L^{-1}\{\phi_2(s)\} = \cos 2t$

put $t = t-u$

$\therefore L^{-1}\{\phi_2(s)\} = \cos 2u - 2t$

$L^{-1}\{\phi_1(s), \phi_2(s)\} = \int_0^t \cos 2u \cos 2t - 2u \, du.$

$= \frac{1}{2}\int_0^t \cos(2u+2t-2u) + \cos(2u-2t+2u)$

$= \frac{1}{2}\int_0^t \cos 2t + \cos 4u - 2t$

$= \frac{1}{2}\left[2u\cos 2t + \frac{1}{4}\sin 2t - \frac{1}{4}\sin(2t)\right]$

Page No.
Date

$$L^{-1}\{\phi_1(s)\,\phi_2(s)\} = \frac{1}{2}\left[2t.\cos 2t + \frac{1}{4}\sin 2t\right]$$

$$= \frac{1}{4}\left[\sin 2t + 2t.\cos 2t\right]$$

2) $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$

$\rightarrow L^{-1}\left\{\frac{1}{s(s^2+4)}\right\} = L^{-1}\left\{\frac{1}{s}\;\frac{1}{s^2+4}\right\}$

$\phi_1(s) = \frac{1}{s} \rightarrow L^{-1}\{\phi_1(s)\} = L^{-1}\left\{\frac{1}{s}\right\} = 1$

$\phi_2(s) = \frac{1}{s^2+2^2} \longrightarrow L^{-1}\{\phi_2(s)\} = L^{-1}\left\{\frac{1}{s^2+2^2}\right\} = \frac{\sin 2t}{2}.$

put $t = t-u$.

$\therefore \frac{\sin 2(t-u)}{2} = \frac{\sin 2t-2u}{2}$

$$L^{-1}\{\phi_1(s)\,\phi_2(s)\} = \int_0^t f_1(u)\,.f_2(t-u)\,du$$

$$= \int_0^t 1\cdot \frac{\sin 2t-2u}{2}\,du$$

$$= \frac{1}{2}\int_0^t \sin 2t-2u\,.\,du$$

$$= \frac{1}{2}\left[\frac{\cos 2t-2u}{2}\right]_0^t$$

$$= \frac{1}{2}\left[\frac{1}{2} - \frac{\cos 2t}{2}\right]$$

$$L^{-1}\{\phi_1(s)\,\phi_2(s)\} = \frac{1}{4}[1-\cos 2t]$$

3] By convolution theorem $L^{-1}\left\{\frac{(s+3)^2}{(s^2+6s+18)^2}\right\}$

→

$$L^{-1}\left\{\frac{s+3}{[(s+3)^2+9]^2}\right\} = e^{-3t}\left\{\frac{s^2}{(s^2+3^2)^2}\right\}$$

$$= e^{-3t}\left\{\frac{s}{s^2+3^2}\cdot\frac{s}{s^2+3^2}\right\} \quad -①$$

$\therefore\ \phi_1(s) = \frac{s}{s^2+3^2}$

Taking L.T on both sides.

$L^{-1}\{\phi_1(s)\} = \cos 3t$

put $t=u$

$\phi_2(s) = \frac{s}{s^2+3^2}$

$$L^{-1}\{\phi_2(s)\} = L^{-1}\left\{\frac{s}{s^2+3^2}\right\} = \cos 3t$$

put $t = t-u$

$$L^{-1}\{\phi_2(s)\} = \cos 3t - 3u .$$

$$L^{-1}\{\phi_1(s)\ \phi_2(s)\} = \int_0^t f_1(u) . f_2(t-u)\, du$$

$$= \int_0^t \cos 3u \ \cos 3t - 3u .$$

$$= \frac{1}{2}\int_0^t \cos(3u + 3t - 3u) + \cos(3u - 3t + 3u) du.$$

$$= \frac{1}{2}\int_0^t \cos 3t + \cos(6u - 3t)\, du .$$

$$= \frac{1}{2}\left[u \cdot \cos 3t + \frac{1}{6} \sin 6u - 3t\right]_0^t$$

$$= \frac{1}{2}\left[t . \cos 3t + \frac{1}{6} \sin 6t - 3t + \frac{1}{6} \sin 3t\right]$$

$$L^{-1}\{\phi_1(s) . \phi_2(s)\} = \frac{1}{2}\left[t . \cos 3t + \cancel{2}\frac{\sin 3t}{\cancel{6}\, 3}\right]$$

$$= \frac{1}{6}\left[3t . \cos 3t + \sin 3t\right]$$

By eq^n 1

$$= \frac{1}{6} e^{-3t} \left[3t \cos 3t + \sin 3t\right].$$

4) $L^{-1}\left\{\frac{1}{(s-3)(s+4)^2}\right\}$

→ $\phi_1(s) = \frac{1}{(s+4)^2}$

$L^{-1}\{\phi_1(s)\} = e^{-4t}\cdot t$

put $t = u$.

$\therefore e^{-4u}\cdot u$.

$\phi_2(s) = \frac{1}{s-3}$

Taking L.T on b.s.

$L^{-1}\{\phi_2(s)\} = L^{-1}\{1/s^3\} = e^{-3t}$

put $t = t-u$.

$L^{-1}\{\phi_2(s)\} = e^{3t-3u}$.

$L^{-1}\{\phi_1(s)\cdot\phi_2(s)\} = \int_0^t f_1(u)\, f_2(t-u)\, du$

$= \int_0^t e^{-4u}\cdot u\cdot e^{3t-3u}\, du$.

$= \int_0^t u\cdot e^{3t-7u}\, du$.

$= \left[u\cdot\frac{e^{3t-7u}}{-7} - \frac{e^{3t-7u}}{49}\right]_0^t$

$$= \left[t \cdot \frac{e^{-4t}}{7} - \frac{e^{-4t}}{49} \right] - \left[0 - \frac{e^{3t}}{49} \right]$$

$$= -7t \cdot \frac{e^{-4t}}{49} - \frac{e^{-4t}}{49} + \frac{e^{3t}}{49}$$

$$= \frac{1}{49} \left(e^{3t} - e^{-4t} - 7t \cdot e^{-4t} \right)$$

Exercise 4.

1) Using L.T. solve the following differential equation with the given condition.

1) $(D^2+4D+3)y = e^{-t}$ $\quad y(0) = y'(0) = 0$

$\rightarrow (D^2+4D+3) \cdot y = e^{-t} \rightarrow \quad y''(t) + 4y'(t) + 3y(t) = e^{-t}$

$L\{y''(t)\} + 4L\{y'(t)\} + 3L\{y(t)\} = L\{e^{-t}\}$.

$s^2 y(s) - sy(0) - y'(0) + 4(sy(s) - y(0)) + 3y(s) = \frac{1}{s+1}$

$s^2 y(s) + 4\{s y(s) + 3y(s) = \frac{1}{s+1}$

$(s^2+4s+3)\, y(s) = \frac{1}{s+1}$

$(s+1)(s+3)\, y(s) = \frac{1}{s+1}$

$\therefore y(s) = \frac{1}{(s+1)^2(s+3)}$

$$\frac{1}{(s+1)^2(s+3)} = \frac{A}{(s+3)} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2} \quad -①$$

Multiply the eq^n by $(s+3)(s+1)^2$.

$1 = (s+1)^2 A + (s+1)(s+3) B + (s+3) C \quad -②$

put $s=-1$

$1 = 2C$

$\therefore C = \frac{1}{2}$

Page No. | Date

put $s = -3$ in eq^n ②

$1 = 4A + 0 + 0$

$\therefore A = \frac{1}{4}$

put $A = \frac{1}{4}$, $C = \frac{1}{2}$ and $s = 0$

$1 = \frac{1}{4} + 3B + \frac{3}{2}$

$1 = \frac{7}{4} + 3B$

$1 - \frac{7}{4} = 3B$

$\frac{-3}{4} = 3B$

$\therefore B = \frac{-3}{12} = -\frac{1}{4}$.

put the values of A, B, C in eq^n ①

$$\frac{1}{(s+3)(s+1)^2} = \frac{1}{4}\frac{1}{(s+3)} - \frac{1}{4}\frac{1}{(s+1)} + \frac{1}{2}\frac{1}{(s+1)^2}$$

Taking I.L.T on both sides.

$$L^{-1}\left\{\frac{1}{(s+3)(s+1)^2}\right\} = \frac{1}{4}L^{-1}\left\{\frac{1}{s+3}\right\} - \frac{1}{4}L^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{(s+1)^2}\right\}$$

$$= \frac{1}{4}e^{-3t} - \frac{1}{4}e^{-t} + \frac{1}{2}e^{-t}t.$$

Page No.
Date

2] $\left(\frac{dy}{dt} + 3y + 2 \int_0^t y\,dt \right) = t$ with $y=0$ at $t=0$ $[y(0)=0]$

$\rightarrow y'(t) + 3y(t) + 2 \int_0^t y\,dt = t$

$L\{y'(t)\} + 3\,L\{y(t)\} + 2\,\frac{y(s)}{s} = \frac{1}{s^2}$

$s \cdot y(s) + 3y(s) + 2y(s) = \frac{1}{s}$

$y(s)\ (s+5) = \frac{1}{s}$

$y(s) = \frac{1}{s(s+5)}$

Let $\frac{1}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5}$ — ①

Multiply eqⁿ ① by $s(s+5)$

$1 = (s+5)A + (s)B$

put $s=0$

$1 = 5A$

$\therefore A = \frac{1}{5}$.

put $s=-5$

$1 = -5B$

$\therefore B = \frac{-1}{5}$.

Page No.
Date

put values of A & B in eq^n ①

$$\frac{1}{s(s+5)} = \frac{1}{5}\frac{1}{s} - \frac{1}{5}\frac{1}{(s+5)}$$

$$L^{-1}\left\{\frac{1}{s(s+5)}\right\} = \frac{1}{5} L^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{5} L^{-1}\left\{\frac{1}{s+5}\right\}$$

$$= \frac{1}{5} - \frac{1}{5}e^{-5t}.$$

$$\therefore L^{-1}\{y(s)\} = \frac{1}{5}(1 - e^{-5t}).$$

3) $y'' - 2y' + y = e^t$ — $y(0) = 2,\ y'(0) = -1$

→ $L\{y''(t)\} - 2L\{y'(t)\} + L\{y(t)\} = L\{e^t\}.$

$$(s^2y(s) - sy(0) - y'(0)) - 2(sy(0) - y(0) + y(s) = \frac{1}{s-1}).$$

By given conditions.

$$s^2y(s) - 2s + 1 - 2sy(s) + 4 + y(s) = \frac{1}{s-1}$$

$$(s^2 - 2s + 1)\,y(s) = \frac{1}{s-1} + 2s - 5.$$

$$y(s) = \frac{2s-5}{(s-1)^2} + \frac{1}{(s-1)^3}.$$

$\therefore L^{-1}\{y(s)\} = L^{-1}\left\{\frac{2(s-1)-3}{(s-1)^2}\right\} + L^{-1}\left\{\frac{1}{(s-1)^3}\right\}$

$= e^t L^{-1}\left\{\frac{2s-3}{(s^2-1)^2}\right\} + e^t L^{-1}\left\{\frac{1}{s^3}\right\}$

$= e^t(2-3t) + e^t \cdot \frac{1}{2}t^2.$

$\therefore L^{-1}\{y(s)\} = e^t\left(2 - 3t + \frac{1}{2}t^2\right).$

4) $(D^2-2D+1)x = e^t$, $x=2$ $Dx=-1$ at $t=0$.

$\rightarrow$

$x'' - 2x' + x = e^t$

$L\{x''(t)\} - 2L\{x'(t)\} + L\{x(t)\} = L\{e^t\}$

$(s^2x(s) - sx(0) - x'(0)) - 2(sx(s) - x(0)) + x(s) = \frac{1}{s-1}$

$(s^2-2s+1)\ x(s) - 2s+5 = \frac{1}{s-1}$

$(s-1)^2\ x(s) = \frac{1}{s-1} + 2s-5.$

$x(s) = \frac{1}{(s-1)^3} + \frac{2s-5}{(s-1)^2}.$

Page No.
Date

$$L^{-1}\{x(s)\} = L^{-1}\left\{\frac{1}{(s-1)^3}\right\} + L^{-1}\left\{\frac{2(s-1)-3}{(s-1)^2}\right\}$$

$$= e^t \cdot \frac{t^2}{2} + e^t\left[L^{-1}\left\{\frac{2s}{s^2}\right\} - 3L^{-1}\left\{\frac{1}{s^2}\right\}\right]$$

$$= e^t \cdot \frac{t^2}{2} + e^t(2-3t)$$

$$\therefore L^{-1}\{x(s)\} = e^t \cdot \frac{t^2}{2} + 2e^t - 3te^t.$$