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### Augmented zero-knowledge proof with private membership test

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#### Abstract

A method for performing a privacy-preserving membership test includes performing an oblivious pseudo-random function (OPRF) protocol to determine a pseudo-random function (PRF) result based on an input from a proving device and a PRF key. The input indicates a user identity of a user associated with the providing device. The method further includes determining whether the user belongs to a verifier list associated with a verifier device based on testing membership of the user using the verifier list and the PRF result.

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## Background/Summary

CROSS-REFERENCE TO PRIOR APPLICATION (1) Priority is claimed to U.S. Provisional Application No. 63/411,658, filed on Sep. 30, 2022, the entire contents of which is hereby incorporated by reference herein.

### FIELD

(1) The present disclosure relates to a method, system, and computer-readable medium for an augmented zero-knowledge proof with private membership test.

### BACKGROUND

(2) A self-sovereign identity (SSI) system enables individuals to own and control their own identities and credentials in a decentralized manner. In these systems, user authentication is performed peer-to-peer between end users and verifiers, instead of relying solely on the service of credential issuers. This approach greatly enhances the privacy of users: not only can users decide what credentials to reveal to the verifier, they are also guaranteed that issuers cannot track their activities.

- (3) To concretely improve the privacy-preserving experience of users, which is one of the most relevant desiderata of SSI, efforts have been made to apply anonymous credentials to verifiable credentials. More specifically, anonymous credentials leverage zero-knowledge proofs (ZKP) protocols to enable users to prove possession of a valid credential, or selectively disclose the properties of their credential attributes, without revealing the credential itself.
- (4) While ZKP protocols guarantee user privacy during the user-authentication process, the privacy requirements of verifiers are often neglected. Among these requirements, private sanction-blacklist/whitelist matching is the most common use case that requires verifier privacy, which is difficult to perform using standard ZKP protocols.
- (5) In current realizations of user-authentication/authorization, the verifier's list is kept secret by the verifier, as it concerns the privacy of verifiers. Namely, the user presents their identity to the verifier, and the verifier locally checks the presented identity against its check list.
- (6) However, with anonymous credentials, users no longer share their credentials with the verifier: they only provide a cryptographic proof about their credentials possessing certain attributes, preventing verifiers from comparing user identities against their check list. Existing solutions for testing membership of anonymous credentials with respect to a verifier's check list require verifiers to disclose their check list (or at least some information about the lists), so that users are able to generate corresponding ZKP proofs to prove that they are not blacklisted or they are in the whitelist. Yet, such solutions are suitable only for use cases where no privacy is required for the verifier's check list, for example, the verifier applies membership test to check whether the user is from a European Union (EU) country. In fact, the state of the art only applies zero-knowledge membership tests on public sets. See, e.g., Benarroch, et al., "Zero-Knowledge Proofs for Set Membership: Efficient, Succinct, Modular", International Conference on Financial Cryptography and Data Security," In: Borisov, N., Diaz, C. (eds) Financial Cryptography and Data Security. FC 2021. Lecture Notes in Computer Science, vol 12674, pp. 393-414 (2021 Germany), the entire contents of which, are hereby incorporated by reference herein. However, there is currently no solution for privacy-preserving membership test on a private set that can be used to ensure the privacy of both the user and the verifier during the user authentication and authorization process.

## SUMMARY

- (7) In an embodiment, the present disclosure provides a method for performing a privacy-preserving membership test. An oblivious pseudo-random function (OPRF) protocol is performed to determine a pseudo-random function (PRF) result based on an input from a proving device and a PRF key. The input indicates a user identity of a user associated with the providing device. The method further includes determining whether the user belongs to a verifier list associated with a verifier device based on testing membership of the user using the verifier list and the PRF result.

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## Description

### BRIEF DESCRIPTION OF THE DRAWINGS

- (1) Subject matter of the present disclosure will be described in even greater detail below based on the exemplary figures. All features described and/or illustrated herein can be used alone or combined in different combinations. The features and advantages of various embodiments will become apparent by reading the following detailed description with reference to the attached drawings, which illustrate the following:
- (2) FIG. 1 illustrates a simplified block diagram depicting an exemplary computing environment according to an embodiment of the present disclosure;
- (3) FIG. 2 illustrates a privacy-preserving set membership combined with anonymous credential verification according to an embodiment of the present disclosure;
- (4) FIG. 3 illustrates a private set membership according to an embodiment of the present

disclosure;

(5) FIG. 4 illustrates a server-aided private and unlinkable membership test according to an embodiment of the present disclosure;

(6) FIG. 5 illustrates an oblivious pseudorandom function functionality with committed input and reverse roles according to an embodiment of the present disclosure;

(7) FIG. 6 illustrates an oblivious pseudorandom function evaluation with reverse roles according to an embodiment of the present disclosure; and

(8) FIG. 7 is a block diagram of an exemplary processing system, which can be configured to perform any and all operations disclosed herein.

#### DETAILED DESCRIPTION

(9) Aspects of the present disclosure provide a method to enhance an anonymous credential system with a privacy-preserving membership test. For instance, the method performs a privacy-preserving membership test between a Prover and a Verifier, and allows the Verifier to determine (e.g., learn) whether a credential holder's identity belongs to a verifier-held list without the need to disclose the user's identity to the Verifier and/or the Verifier's list to the credential holder. By performing the method, Verifiers are provided stronger privacy guarantees, which enhances anonymous credential systems.

(10) According to an aspect of the present disclosure, a method is provided that performs a privacy-preserving membership test in combination with anonymous credential verification. For instance, the method performs a privacy-preserving membership test, on a private set, for the identity associated to a verifiable anonymous credential. The method enables a Verifier to check whether a user (e.g., the Prover) participating in the authentication/authorization process belongs to the verifier's list while keeping the list private.

(11) Aspects of the present disclosure can be performed in several user-authentication/authorization applications. For instance, authorities and/or corporations (e.g., the Verifiers) may seek to screen users against their lists (e.g., blacklists and/or whitelists). Some relevant uses for the present invention may include, but are not limited to: During the check-in process with airlines, airline companies can have a “no-fly” list for unwelcoming customers. In many countries, in addition to an “unwelcoming-list”, casinos also have “self-exclusion” program that restricts the admission of certain customers. Border control police screens the passengers and vehicles according to a “watchlist”. Airlines or hotels check if the check-in guest is in their “VIP list” without leaking their VIP list to the users.

(12) According to a first aspect, the present disclosure provides a method for performing a privacy-preserving membership test. An oblivious pseudo-random function (OPRF) protocol is performed to determine a pseudo-random function (PRF) result based on an input from a proving device and a PRF key. The input indicates a user identity of a user associated with the providing device. The method further includes determining whether the user belongs to a verifier list associated with a verifier device based on testing membership of the user using the verifier list and the PRF result.

(13) According to a second aspect, the method according to the first aspect further comprises a list associated with a company is indicated by the verifier list, and the method further comprises that determining whether the user belongs to the verifier list comprises determining whether the user belongs to the list.

(14) According to a third aspect, the method according to the first or the second aspect further comprises performing a PRF evaluation on individuals indicated by the verifier list using the PRF key to determine a representation of the verifier list indicating a plurality of pseudo-random values, and the method further comprises that determining whether the user belongs to the verifier list comprises testing the membership of the user based on comparing the PRF result with the plurality of pseudo-random values indicated by the representation of the verifier list.

(15) According to a fourth aspect, the method according to any of the first to third aspects further comprises that the PRF key is a verifier-held key that is stored in the verifier device, and the

method further comprises that performing the PRF on individuals comprises performing, by the verifier device, the PRF on individuals indicated by the verifier list using the PRF key to determine the representation of the verifier list indicating the plurality of pseudo-random values.

(16) According to a fifth aspect, the method according to any of the first to third aspects further comprises that the PRF key is a server-held key that is stored in a third-party server that is separate from the verifier device and the proving device, and the method further comprises that performing the PRF on individuals comprises performing a second PRF evaluation on the individuals indicated by the verifier list using the server-held key to determine the plurality of pseudo-random values indicated by the representation of the verifier list.

(17) According to a sixth aspect, the method according to any of the first, second, third, and fifth aspects further comprises that performing the OPRF protocol to determine the PRF result comprises obtaining, from the prover device, the PRF result and a commitment associated with the prover device, the method further comprises that the PRF result is based on the server-held key, and the method further comprises that determining whether the user belongs to the verifier list comprises performing a membership check based on comparing the PRF results received from the prover device and the plurality of pseudo-random values indicated by the representation of the verifier list.

(18) According to a seventh aspect, the method according to any of the first to sixth aspects further comprises obtaining a zero-knowledge proof of possession of credentials for the input from the proving device based on determining that the user belongs to the verifier list, and verifying the zero-knowledge proof.

(19) According to an eighth aspect, the method according to any of the first to seventh aspects further comprises that performing the OPRF protocol to determine the PRF result based on the input from the proving device and the PRF key comprises obtaining, from the prover device, the input and a zero-knowledge proof of the input, verifying correctness of the zero-knowledge proof of the input from the prover device, and determining the PRF result based on the input and the verification. The method further comprises that the input is a first ciphertext that is based on homomorphically encrypting credentials of the user.

(20) According to a ninth aspect, the method according to any of the first to eighth aspects further comprises that performing the OPRF to determine the PRF result based on the input from the proving device and the PRF key further comprises encrypting the PRF key using a public key of the prover device to determine the encrypted PRF key based on verifying the correctness of the zero-knowledge proof, and generating a second ciphertext based on the encrypted PRF key, the first ciphertext, and a randomly chosen blinding value. The method further comprises that determining the PRF result is based on the second ciphertext.

(21) According to a tenth aspect, the method according to any of the first to ninth aspects further comprises that generating the second ciphertext comprises multiplying the encrypted PRF key with the first ciphertext to determine an intermediate ciphertext, and blinding the intermediate ciphertext with the randomly chosen blinding value to generate the second ciphertext.

(22) According to an eleventh aspect, the method according to any of the first to tenth aspects further comprises that performing the OPRF protocol to determine the PRF result based on the input from the proving device and the PRF key further comprises obtaining, from the prover device, a partial PRF result and a zero-knowledge proof of the partial PRF result, and verifying the zero-knowledge proof of the partial PRF result. The method further comprises that the partial PRF result is based on decrypting the second ciphertext and hiding the decrypted second ciphertext by moving the decrypted second ciphertext into an exponent of the partial PRF result. The method further comprises that determining the PRF result is based on verifying the zero-knowledge proof of the partial PRF result.

(23) According to a twelfth aspect, the method according to any of the first to eleventh aspects further comprises that determining the PRF result comprises generating the PRF result based on

removing the randomly chosen blinding value from the exponent of the partial PRF result.

(24) According to a thirteenth aspect of the present disclosure, a system for performing a privacy-preserving membership test is provided, the system comprising one or more hardware processors, which, alone or in combination, are configured to provide for execution of the following steps: performing an oblivious pseudo-random function (OPRF) protocol to determine a PRF result based on an input from a proving device and a PRF key, wherein the input indicates a user identity of a user associated with the proving device; and determining whether the user belongs to a verifier list associated with a verifier device based on testing membership of the user using the verifier list and the PRF result.

(25) According to a fourteenth aspect, the system according to the thirteenth aspect further performs a PRF evaluation on individuals indicated by the verifier list using the PRF key to determine a representation of the verifier list indicating a plurality of pseudo-random values, and the system further determines whether the user belongs to the verifier list by testing the membership of the user based on comparing the PRF result with the plurality of pseudo-random values indicated by the representation of the verifier list.

(26) A fifteenth aspect of the present disclosure provides a tangible, non-transitory computer-readable medium having instructions thereon, which, upon being executed by one or more processors, provides for execution of the method according to any of the first to the twelfth aspects.

(27) For illustration of aspects of the present disclosure, a non-limiting example of an anonymous credential scenario involving a Prover and a Verifier is considered. For instance, a Prover (e.g., a user) wishes to authenticate to a Verifier (e.g., an authority or corporation) by proving possession of valid credentials in a privacy-preserving manner. A mechanism implemented according to an aspect of the present disclosure augments and enhances an anonymous credential system by enabling a private membership test allowing the Verifier to check whether the Prover's identity belongs to a checklist held by the Verifier. The membership test is private in the sense that the Prover's identity is not disclosed to the Verifier, and similarly, the Verifier's checklist is not disclosed to the Prover.

(28) FIG. 1 illustrates a simplified block diagram depicting an exemplary computing environment according to an embodiment of the present disclosure. FIG. 2 illustrates a privacy-preserving set membership combined with anonymous credential verification according to an embodiment of the present disclosure. For instance, FIG. 1 illustrates an exemplary computing environment that can perform the aforementioned scenario or embodiments and/or additional or alternative scenarios or embodiments. FIG. 2 provides a high-level illustration of performing the aforementioned scenario or embodiment.

(29) Referring to FIG. 1, the environment **100** includes a prover device **102** (e.g., a user device), a network **104**, a verifier system **106**, and a third-party server **108**. Although certain entities within environment **100** are described below and/or depicted in the FIGURES as being singular entities, it will be appreciated that the entities and functionalities discussed herein can be implemented by and/or include one or more entities.

(30) The entities within the environment **100** are in communication with other devices and/or systems within the environment **100** via the network **104**. The network **104** can be a global area network (GAN) such as the Internet, a wide area network (WAN), a local area network (LAN), or any other type of network or combination of networks. The network **104** can provide a wireline, wireless, or a combination of wireline and wireless communication between the entities within the environment **100**.

(31) The prover device **102** is a computing device that is operated by a user. For instance, the prover device **102** is associated with a Prover (e.g., a user or client) that seeks to authenticate their identity or another individual's identity with a Verifier. In some instances, as mentioned above, aspects of the present disclosure can be used to screen users against blacklists/whitelists. For example, during the check-in process for airlines or hotels, the user can use the prover device **102** to provide information to the verifier system **106** to authenticate their identity.

(32) The prover device **102** is and/or includes, but is not limited to, a desktop, laptop, tablet, mobile device (e.g., smartphone device, or other mobile device), server, computing system and/or other types of computing entities that generally comprises one or more communication components, one or more processing components, and one or more memory components.

(33) The verifier system **106** is a computing system that performs one or more functions described herein. For instance, the verifier system **106** is associated with a Verifier such as an authority (e.g., a government authority or entity) and/or corporation. The verifier system **106** is associated with (e.g., stores and/or retrieves) a list (e.g., a Verifier list) that indicates a list of individuals (e.g., users). In some variations, as mentioned above, aspects of the present disclosure can be used to screen users against a Verifier list such as a blacklist or whitelist. For example, the verifier system **106** can be associated with (e.g., owned, operated, and/or managed by) an airline or hotel company. The verifier system **106** includes a list of individuals or users that are part of a “no-fly” list or a “VIP list”. The verifier system **106** checks to confirm whether the user (e.g., the user operating the prover device **102**) is on the verifier list. In some examples, the verifier system **106** verifies whether the user operating the prover device **102** is on the list without the need to obtain the user's identity (e.g., the verifier system **106** obtaining or identifying the identity of the user) and/or provide the verifier's list to the prover device **102**, which allows the list to remain private.

(34) The verifier system **106** includes and/or is implemented using one or more computing devices, computing platforms, cloud computing platforms, systems, servers, and/or other apparatuses. In some instances, the verifier system **106** can be implemented as engines, software functions, and/or applications. For example, the functionalities of the verifier system **106** can be implemented as software instructions stored in storage (e.g., memory) and executed by one or more processors.

(35) The third-party system **108** is a computing system that performs one or more functions described herein. For instance, the third-party system **108** is associated with a third-party (e.g., a party that is separate from the Prover and the Verifier). The third-party system **108** is shown using a dotted box to indicate that the third-party system **108** is optional. When present, the third-party system **108** performs one or more functions described herein (e.g., described in FIG. 4 for aiding the oblivious membership test between the Prover and the Verifier in order to provide unlinkability of users).

(36) The third-party system **108** includes and/or is implemented using one or more computing devices, computing platforms, cloud computing platforms, systems, servers, and/or other apparatuses. In some instances, the third-party system **108** can be implemented as engines, software functions, and/or applications. For example, the functionalities of the third-party system **108** can be implemented as software instructions stored in storage (e.g., memory) and executed by one or more processors.

(37) Referring to FIG. 2, a high-level illustration **200** for privacy-preserving set membership is provided. For instance, a Prover **202** (e.g., the prover device **102** from FIG. 1) holding verifiable credentials **206**  $VC=(u, a)$  can authenticate to the Verifier **204** (e.g., the verifier device **106** from FIG. 1) by proving possession of verifiable credentials **206** that satisfy a certain public policy (e.g., validating a predicate  $Policy(u, a)$  where  $u$  denotes the user's identity and  $a$  denotes a set of verifiable attributes for identity  $u$ ). Before proceeding to the authentication/authorization step (e.g., the ZK protocol for policy verification **212**), the Verifier **204** performs a membership test **210** (e.g., an oblivious membership test) to check whether the user's identity  $u$  belongs to the Verifier's checklist  $L$  **208**.

(38) In some embodiments, the public policy can be a predicate that is known by all parties (e.g., “public” in the sense that it does not need to be kept private). For instance, a policy could be the person with the user identity ID is from a European Union (EU) country. Here, while the policy itself (e.g., being from an EU country) can be made public, the identity ID of the user is to be considered sensitive information, which is desired to be kept private. Formally, verifying a Policy can be defined as the process of evaluating a predicate over a (possibly private) input. Taking the

same example as above, a verifiable attribute can be an attribute such as “being from an EU country” or “holding German citizenship”. A verifiable attribute is one that can be verified by a designated party (e.g., a publicly verifiable attribute can be verified by anyone). For instance, presenting a valid passport issued from the German government would allow a person to prove, to a verifier **204**, that they hold German citizenship/that they are from an EU country.

(39) For instance, the Prover **202** can be the prover device **102**. The prover device **102** can include memory that stores the verifiable credentials **206**. In some instances, the prover device **102** receives (e.g., from the user and/or from another computing entity) the verifiable credentials **206**. The verifiable credentials **206** indicate  $u$ , which denotes the user's identity, and  $a$ , which denotes a set of verifiable attributes for identity  $u$ . The Verifier **204** can be the verifier device **106**. The verifier device **106** can include memory that stores the checklist **208**.

(40) At a first step, an oblivious membership test **210** is performed by the Prover **202**, the Verifier **204**, and/or another entity (e.g., the third-party system **108**). For instance, the checklist **208** from the Verifier **204** and the user identity  $u$  from the verifiable credentials **206** are used for the oblivious membership test **210**. The output from the oblivious membership test **210** indicates whether user's identity  $u$  belongs to the Verifier's checklist  $L$  **208** (e.g.,  $u \in L$  or  $u$  belongs to the set  $L$ ).

Subsequently, the zero knowledge (ZK) protocol (e.g., a ZKP protocol) for policy verification **212** is performed by the Prover **202**, the Verifier **204**, and/or the other entity. For instance, the verifiable credentials **206** indicating the user identity and verifiable attributes are used to validate a predicate  $\text{Policy}(u, a)$ . The ZK protocol for policy verification can be part of and/or be the authentication/authorization step described above.

(41) In some instances, an aspect of the present disclosure configured to realize the aforementioned oblivious membership test leverages (e.g., uses) a pseudorandom function (PRF), which can be evaluated obliviously. This type of function is referred to herein as an oblivious PRF (OPRF). For instance, at the first step, the oblivious membership test **210** uses an OPRF to determine whether the user's identity belongs to (e.g., is part of) the Verifier's checklist **208**.

(42) An OPRF is an interactive protocol allowing two parties, A and B, to evaluate a PRF on an input  $x$  held by A and a key  $k$  held by B, in such a way that A learns the result  $y = f.\text{sub}.k(x)$ , but learns nothing about the key, and B learns nothing. Below, party A is the “input holder” (e.g., the Prover **202**), and party B is the “key holder” (e.g., the Verifier **204** and/or a third-party system).

(43) A high-level idea implemented in aspects of the present disclosure is to apply the same pseudorandom transformation  $f.\text{sub}.k(\cdot)$  to the Prover-held input  $u$  and to the Verifier-held elements  $x.\text{sub}.i \in L$  (e.g., the elements or individuals  $x.\text{sub}.i$  that belongs to the checklist  $L$  **208**), and then let the Verifier check membership of the pseudo-random value  $y := f.\text{sub}.k(u)$ , derived from the Prover's identity  $u$ , with respect to the set of pseudo-random values  $\{f.\text{sub}.k(x.\text{sub}.i)\}.\text{sub}.x.\text{sub}.i.\text{sub}.\in L$  derived from the members  $x.\text{sub}.i$  of the checklist  $L$ . This allows the Verifier **204** to check if the Prover's **202** identity belongs to the checklist without leaking such identity.

(44) In the following, two non-limiting, exemplary embodiments implementing aspects of the present disclosure are described.

#### Embodiment #1

(45) In the first embodiment, a mechanism (e.g., process and/or method) is implemented according to aspects of the present disclosure that realizes an oblivious membership test **210** by letting the Prover **202** and the Verifier **204** operate an OPRF protocol with an inverted outcome as the conventional OPRF to evaluate  $f.\text{sub}.k(u)$ . Here,  $f$  is an agreed-upon PRF (e.g., the PRF agreed-upon by the Prover **202** and the Verifier **204**),  $k$  is a PRF key held by the Verifier **204**, and  $u$  is an encoding of the Prover's identity from the verifiable credentials **206** to prevent the Verifier **204** from learning the identity of the user. The OPRF protocol is inverted as for a conventional OPRF, the Prover **202** (e.g., the input holder of  $u$ ) receives an output. For the inverted OPRF protocol, the Verifier **204** receives the output. The protocol (e.g., the process or method) terminates or ends with



the Prover **202** learning nothing and the Verifier **204** obtaining an output. The output includes a PRF value  $y=f.sub.k(u)$  as well as a commitment to the Prover's input  $c=com(u)$ . The Verifier **204** uses the PRF key  $k$  to evaluate elements of the checklist  $L$ , obtaining a representation  $Y=\{f.sub.k(x.sub.i):x.sub.i\in L\}$  of pseudo-random values derived from the members  $x.sub.i$ , where each value  $f.sub.k(x.sub.i)$  is the pseudo-random value associated to  $x.sub.i$  under key  $k$ . The Verifier **204** then checks if the user's input belongs to the list by testing whether the PRF value  $y$  belongs to the representation  $Y$  of the list (e.g., the blacklist). Additionally, and/or alternatively, the commitment  $c=com(u)$  allows to cryptographically bind the output  $y$  of the OPRF protocol to the credentials employed in the authentication protocol, thereby enforcing honest behavior of the Prover **202**. The commitment  $c$  (e.g., a Pedersen commitment) is a cryptographic primitive that allows one to commit to a chosen value while keeping it hidden to others (e.g., a party cannot change the value or statement after they have committed to it).

(46) A method **300** according to the above-described embodiment, embedded with a ZK authentication system for anonymous credentials, is illustrated in FIG. 3. For instance, FIG. 3 illustrates a private set membership according to an embodiment of the present disclosure. The Prover **202** with the verifiable credentials **206** and the Verifier **204** with the check list **208** are shown.

(47) At the first step **302**, the Verifier **204** performs local PRF evaluation on the Verifier's checklist **208** to determine a representation of the checklist  $Y(L)$ . For instance, the representation may be determined by:  $Y(L) \leftarrow \{f.sub.k(x.sub.i)\}.sub.x.sub.i.sub.\in L$ . For instance, the Verifier **204** determines the representation of the checklist based on the inverted OPRF protocol to evaluate function  $f.sub.k$ , the PRF key  $k$ , and/or the checklist  $L$ .  $x.sub.i$  indicates the entries (e.g., users or individuals) within the checklist (e.g.,  $x.sub.i\in L$ ).

(48) At the second step **304**, the Prover **202** and the Verifier **204** perform oblivious evaluation of the Prover's identity. For example, the Verifier **204** checks the membership of  $y:=f.sub.k(u)$ , which is described above, and determines the commitment  $c=com(u)$ . The Verifier **204** further checks whether the membership  $y$  belongs to the checklist representation  $Y(L)$  (e.g.,  $y\in Y(L)$ , where  $y$  is determined based on the PRF  $f.sub.k(\cdot)$  and  $Y(L)$  is the representation of the checklist that is determined using the PRF key  $k$  at the first step **302**). This is described in further detail below in FIG. 6.

(49) At the third step **306**, the Prover **202** performs the zero-knowledge proof (ZKP) of possession of the credentials for the committed input. For example, the Prover **202** determines the ZKP  $\pi$  based on:

$$\pi \leftarrow \text{ZKPoK}\{(u,\omega,a)|c=com(u;\omega)\wedge \text{Policy}(u,a)\}$$

where the ZKP  $\pi$  allows the Prover **202** to prove possession of an identity  $u$ , with attribute  $a$ , which fulfils a public policy  $\text{Policy}(u, a)$ , as well as to prove knowledge of a value  $\omega$  such that  $c$  is a commitment to the identity  $u$  under randomness  $\omega$ . The ZKP  $\pi$  binds the input  $u$ , used by the Prover **202** as their identity in the membership test protocol, to the actual Prover's **202** identity.

(50) The Prover **202** then provides the ZKP  $\pi$  to the Verifier **204**. At the fourth step **308**, the Verifier **204** performs credential verification with respect to (w.r.t.) the committed input.

(51) The above-described OPRF protocol method **300** prevents the Verifier **204** from learning the identity  $u$  of the Prover **202**. However, the Verifier **204** can detect whether the same user requests authentication/authorization multiple times, because the value  $y$  obviously computed in the second step **304** is the same across all protocol executions involving the same user. This protocol is suitable for applications that do not require unlinkability of users (e.g., whether the same user executed the protocol or method **300** multiple times).

Embodiment #2

(52) In the second embodiment, a mechanism is implemented according to an aspect of the present disclosure that leverages a third-party Server or system (e.g., the third-party system **108** shown in FIG. 1) to aid the oblivious membership test between Prover **202** and Verifier **204** in order to

provide unlinkability of users. The third-party server acts as an intermediary between the Prover **202** and the Verifier **204**, and is in charge of selecting the PRF key  $k$  for the oblivious evaluations of PRF values. The Verifier **204** interacts with the third-party server to obviously compute  $\{f.\text{sub}.k(x.\text{sub}.i):x.\text{sub}.i \in L\}$  (e.g., the OPRF function described above  $f.\text{sub}.k$  for an input  $x.\text{sub}.i$  and whether the input belongs to the checklist  $L$  **208**), and likewise the Prover **202** interacts with the third-party system to obviously compute  $f.\text{sub}.k(u)$ . The third-party system uses a uniformly chosen PRF key  $k$  for each protocol interaction. In this way, the third-party system can no longer link users by comparing values  $y$  across various interactions.

(53) A method **400** according to the above-described embodiment, embedded with a ZK authentication system for anonymous credentials, is illustrated in FIG. **4**. FIG. **4** illustrates a server-aided private and unlinkable membership test according to an embodiment of the present disclosure. For instance, FIG. **4** shows the Prover **202** with the verifiable credentials **206** and the Verifier **204** with the check list **208**. Additionally, FIG. **4** shows the aiding server **402** (e.g., the third-party system **108** shown in FIG. **1**), which includes and/or utilizes a fresh PRF key  $k$  **404**.

(54) At the first step **406**, an oblivious evaluation on the verifier's checklist **208** is performed by the aiding server **402** and the verifier **204**. For instance, the aiding server **402** and the verifier **204** determines the representation of the verifier's checklist  $Y(L)$  using a fresh PRF key  $k$  **404** (e.g.,  $Y(L) \leftarrow \{f.\text{sub}.k(x.\text{sub}.i)\}.\text{sub}.x.\text{sub}.i.\text{sub}.\in L$ ). For example, the first step **406** is similar to the first step **302** of FIG. **3** except the oblivious evaluation on the Verifier's checklist is performed by the aiding server **402** and the Verifier **204** rather than the Verifier **204** themselves. Further, the first step **406** uses the key  $k$  **404** that belongs to the aiding server **402** rather than the Verifier **204**.

(55) At the second step **408**, an oblivious evaluation on the Prover's identity is performed by the Prover **202** and the aiding server **402**. For example, the oblivious evaluation includes the Prover **202** and the aiding server **402** using the key **404** and the user input  $u$  indicating an encoding of the user's identity. Based on the oblivious evaluation on the Prover's identity, the result of the OPRF  $y$  (e.g.,  $y := f.\text{sub}.k(u)$ ) and the commitment  $c = \text{com}(u)$  are determined. The output  $y$  and the commitment  $c$  are provided by the aiding server **402** to the Verifier **204**. For instance, the second step **408** is similar to the second step **304** of FIG. **3** except the oblivious evaluation on the Prover's identity is performed by the Prover **202** and the Aiding Server **402**. Further, the second step **408** uses the key  $k$  **404** that belongs to the aiding server **402** rather than the Verifier **204**.

(56) At the third step **410**, the Verifier **204** performs the membership check (e.g.,  $y \in Y(L)$ ). For instance, after receiving the result of the OPRF  $y$ , the Verifier **204** determines whether  $y$  belongs to the representation of the Verifier's checklist  $Y(L)$  that was determined at the first step **406**. Based on the determination, the Verifier **204** determines whether the user belongs to the checklist  $L$  **208** (e.g., whether the user is on the “no-fly list” or “VIP list”).

(57) At the fourth step **412**, a zero-knowledge proof of possession of valid credentials is performed by the Prover **202** and the Verifier **204** (e.g.,  $\text{ZKPoK}\{(u, \omega, a) | c = \text{com}(u; \omega) \wedge \text{Policy}(u, a)\}$ ). The fourth step **412** can function similarly to step **306** of FIG. **3**. For instance, the ZPK allows the Prover **202** to prove possession of an identity  $u$ , with attribute  $a$ , which fulfils a public policy  $\text{Policy}(u, a)$ , as well as to prove knowledge of a value  $\omega$  such that  $c$  is a commitment to the identity  $u$  under randomness  $\omega$ . The ZKP  $\pi$  binds the input  $u$ , used by the Prover **202** as their identity in the membership test protocol, to the actual Prover's **202** identity.

(58) In some embodiments, state of the art designs for OPRF protocols are not suitable for the scenario considered in the present disclosure. In typical OPRF applications, it is the input-holder that obtains the protocol output (e.g., the result of the OPRF is obtained by the Prover **202**), while the oblivious membership testing for the present disclosure requires the key-holder (e.g., the Verifier **204** in the first embodiment and the third-party or aiding server **402** in the second embodiment) to learn the output  $f.\text{sub}.k(u)$  along with a commitment  $\text{com}(u)$  to the Prover's identity.

(59) Aspects of the present disclosure rely on a new OPRF functionality with reverse roles (e.g., the

input-holder learns nothing and the key-holders receive the output), which is disclosed by the present invention and described in further detail below. FIG. 5 illustrates an oblivious pseudorandom function functionality with committed input and reverse roles according to an embodiment of the present disclosure. For instance, FIG. 5 describes a realization of the modified OPRF functionality that is used in one or more embodiments of the present disclosure in further detail. For instance, the OPRF functionality **500** includes the OPRF function **504** (e.g.,  $f.\text{sub.k}(x)$ ). An input  $x$  **502** and the key  $k$  **506** are provided to the OPRF function **504**. The output **508** of the OPRF function **504** is  $f.\text{sub.k}(x)$  and a commitment  $\text{com}(x)$ .

(60) For instance, to realize the desired OPRF functionality, the present disclosure defines an adapted version of the protocol specified by Jarecki and Liu (Stanislaw Jarecki and Xiaomin Liu, "Efficient Oblivious Pseudorandom Function with Applications to Adaptive OT and Secure Computation of Set Intersection," TCC 2009: 577-594, the entire contents of which are hereby incorporated by references) for the evaluation of the Dodis-Yampolskiy PRF, defined by:

$$(61) f_k(x) = \begin{cases} g^{\frac{1}{k+x}} & \text{if } \text{gdc}(x+k, n) \neq 1 \\ 1 & \text{otherwise,} \end{cases},$$

where  $\text{gdc}$  denotes the greatest common divisor,  $n$  is a safe Rivest-Shamir-Adleman (RSA) modulus,  $g$  is a group element of  $\text{custom character.sub.n}$ , the set of integers modulo  $n$ , and the key  $k$  is a randomly chosen element in  $\text{custom character*.sub.n}$ . For example, referring to FIGS. **3** and **4**, the OPRF protocol used in steps **302**, **304**, **406**, and **408** can be the OPRF protocol described in FIG. 5. For instance, at step **304**, the input  $u$ , which indicates the user's identity, is the input  $x$  shown in FIG. 5. Similarly, the key  $k$  from FIG. **3** is the key  $k$  shown in FIG. 5. Then, the output of the OPRF protocol **508** shown in FIG. 5 is a commitment and  $y$ , which is  $y:=f.\text{sub.k}(u)$  and described above.

(62) The protocol (e.g., method **600**) is depicted in FIG. 6, described below, and is used by the Prover **202** and the Verifier **204**. FIG. 6 illustrates an oblivious pseudorandom function evaluation with reverse roles according to an embodiment of the present disclosure. For instance, the method **600** includes the input-holder that is the Prover **202** with the input  $x$  **602** and the key-holder that is the Verifier **204** with the key  $k$  **604**. In some instances, the method **600** provides an instantiation of the abstract functionality described in FIG. 5. For instance, FIG. 5 specifies the input and expected output of each participant and method **600** describes concrete steps that allows the participants to derive the expected outputs. Method **600** can be part of methods **300** and **400** described above (e.g., a sub-component of the methods or protocols **300** and **400** described in FIGS. **3** and **4**).

(63) The protocol (e.g., method **600**) uses the following two cryptographic schemes. The first scheme is an additively homomorphic encryption scheme HE with verifiable encryption and decryption. This can be instantiated using the Cramer-Shoup encryption scheme. The second scheme is a commitment scheme  $\text{com}$  that supports zero knowledge proofs of knowledge (ZKPoK) of committed values. This can be instantiated using Pedersen commitments.


(64) The Prover **202** and the Verifier **204** share the following common inputs: a safe RSA modulus  $n$  and a group element  $g$  of a multiplicative group, such that  $\text{ord}(g)=n$  (e.g., the ordinal of the group element  $g$  returns the RSA modulus  $n$ ).

(65) The oblivious evaluation protocol **600** comprises the following steps. At a first set of steps **606-612**, the Prover **202** generates a key pair  $(\text{pk.sub.P}, \text{sk.sub.P})$  for an additively homomorphic encryption scheme, encrypts its input under  $\text{pk.sub.P}$  deriving ciphertext  $C.\text{sub.1}$ , generates a commitment  $c.\text{sub.x}$  to the input, and generates a ZKP  $\pi.\text{sub.1}$  to prove knowledge of the input and that all values have been honestly generated. The Prover **202** sends  $\text{pk.sub.P}$ ,  $C.\text{sub.1}$ ,  $c.\text{sub.x}$ ,  $\pi.\text{sub.1}$  to the Verifier **204** and the Verifier **204** verifies the proof  $\pi.\text{sub.1}$ .

(66) For instance, as shown at step **606**, the Prover **202** generates the key pair  $\text{pk.sub.P}$ ,  $\text{sk.sub.P}$  for an additively homomorphic encryption scheme HE such as the Cramer-Shoup encryption scheme. As such, the key pairs  $(\text{pk.sub.P}, \text{sk.sub.P})$  are generated based on HE.Gen. At step **608**, the Prover

**202** encrypts its input  $x$  using its public key  $pk.sub.P$  to determine (e.g., generate) the ciphertext  $C.sub.1$ . Therefore, the ciphertext  $C.sub.1$  is determined using an HE encryption based on the public key and input (e.g.,  $C.sub.1 \leftarrow HE.Enc.sub.pk.sub.P(x)$ ). At step **610**, the Prover **202** generates a commitment  $c.sub.x$  based on the input  $x$  and randomness  $\omega$  (e.g.,  $c.sub.x \leftarrow com(x; \omega)$ ). At step **612**, the Prover **202** generates a ZKP  $\pi.sub.1$  to prove knowledge of the input. For instance,  $\pi.sub.1 \leftarrow ZKPoK\{(x, \omega) | C.sub.1 \in HE.Enc.sub.pk.sub.P(x) \wedge c.sub.x = com(x; \omega)\}$  provides a ZKP of knowledge of input  $x$  and randomness  $\omega$  for the commitment. Then, the Prover **202** provides the public key  $pk.sub.P$ , the ciphertext  $C.sub.1$ , the commitment  $c.sub.x$ , and the ZKP  $\pi.sub.1$  to the Verifier **204**. The Verifier **204** verifies  $\pi.sub.1$ . For instance, the Verifier **204** verifies  $\pi.sub.1$  based on the ZKP. For example, the Verifier **204** may use a function  $Verify(\pi.sub.1)$  that outputs “accept” or “reject” as a verdict on the correctness of the proof  $\pi.sub.1$ .

(67) At a second set of steps **614** and **616**, after verifying the proof  $\pi.sub.1$ , the Verifier **204** computes an encryption of its key under  $pk.sub.P$ , multiplies the generated ciphertext with  $C.sub.1$ , and blinds the resulting ciphertext by raising it to a randomly chosen exponent  $r$ . The Verifier **204** finally sends the aggregated and blind ciphertext  $C.sub.2$  to the Prover **202**.

(68) For instance, after verifying the proof  $\pi.sub.1$  at step **612**, the Verifier **204** generates the ciphertext  $C.sub.2$  based on the received ciphertext  $C.sub.1$  from the Prover **202** and the encryption of its key  $k$  using the public key  $pk.sub.K$  from the Prover **202** (e.g.,  $HE.Enc.sub.pk.sub.P(k)$ ). For instance, the Verifier **204** multiplies the received ciphertext with the encryption of its key under the public key, and blinds the resulting ciphertext by raising it to a randomly chosen exponent  $r$  (e.g.,  $(C.sub.1.Math.HE.Enc.sub.pk.sub.P(k)).sup.r$ ) to determine the second ciphertext  $C.sub.2$ . The Verifier **204** chooses the exponent  $r$  in  by means of a uniformly random selection process. The Verifier **204** provides the second ciphertext  $C.sub.2$  to the Prover **202**.

(69) At the third set of steps **618-622**, the Prover **202** decrypts  $C.sub.2$ , computes  $W=g.sup.w$  where  $w$  is the inverse mod  $n$  of the decrypted value, and computes a ZKP  $\pi.sub.2$  to prove correct decryption and honest generation of  $W$ . The Prover **202** sends the group element  $W$  and the proof  $\pi.sub.2$  to the Verifier **204**. For instance, in some examples, the Prover **202** decrypts the Verifier-provided ciphertext, hides the resulting plaintext by moving it to the exponent, and generates a zero-knowledge proof of the correctness of the computation.

(70) For instance, the Prover **202** determines  $z$ , which is the decryption of the second ciphertext  $C.sub.2$ . For example, at step **618**, the Prover **202** determines  $z$  based on the decryption of  $C.sub.2$  using the secret key  $sk.sub.P$  (e.g.,  $z \leftarrow HE.Dec.sub.sk.sub.P(C.sub.2)$ ). At step **620**, the Prover **202** determines  $w$  based on an inverse modulus (mod)  $n$  (e.g., a safe RSA modulus) and  $z$ , which is the decryption of  $C.sub.2$  using the secret key  $sk.sub.P$ . For instance, the Prover **202** determines the inverse mod  $n$  of value  $z$ , i.e.,  $w \leftarrow z.sup.-1 \text{ mod } n$ . Using the so-derived value  $w$ , the Prover **202** further computes  $W=g.sup.w$  by raising the public group element  $g$  to the exponent  $w$ , obtaining a new group element  $W$ . At step **622**, the Prover **202** determines the proof  $\pi.sub.2$ , which is sent to the Verifier **204** along with the group element  $W$ . For instance, the Prover **202** determines the proof  $\pi.sub.2$  based on the secret key  $sk.sub.P$ , the decryption of the second ciphertext  $z$ , and the group element  $w$ . For instance, the Prover **202** computes a ZKP  $\pi.sub.2 \leftarrow ZKPoK\{(sk.sub.P, z, w) | (sk.sub.P, pk.sub.P) \in HE.Gen \wedge z = HE.Dec.sub.sk.sub.P(C.sub.2) \wedge W = g.sup.w \wedge zw = 1 \text{ mod } n\}$ , which allows the Prover **202** to prove knowledge of the decryption key  $sk.sub.P$  associated to the public key  $pk.sub.P$ , and knowledge of values  $z$  and  $w$  such that  $z$  is the decryption of ciphertext  $C.sub.2$  under key  $sk.sub.P$ ,  $w$  is the discrete logarithm of group element  $W$ , and  $z$  and  $w$  are inverse modulo  $n$ . After, the Prover **202** provides group element  $W$  and the proof  $\pi.sub.2$  to the Verifier **204**.

(71) At the fourth set of steps **624-628**, after verifying the proof  $\pi.sub.2$ , the Verifier **204** raises  $W$  to its own blinding exponent  $r$  obtaining  $y=W.sup.r$ , and outputs the PRF value  $y$  and the previously obtained commitment  $c.sub.x$ . For example, in some instances, the Verifier **204** verifies the correctness of the Prover-provided proof, removes the blinding value from the exponent of the

partial PRF value provided by the prover, and returns the final PRF value and the Prover-provided commitment.

(72) For instance, at step **624**, the Verifier **204** obtains the proof  $\pi$ .sub.2 and verifies the proof. At step **626**, the Verifier **204** then determines  $y$  based on the group element  $W$  and the randomly chosen exponent  $r$ . For instance,  $y$ , which is the output of the PRF function described above, is determined based on  $y \leftarrow W.\text{sup}.r$ . At step **628**, the Verifier **204** outputs  $y$  and the previously obtained commitment  $c$ .sub. $x$ .

(73) FIG. 7 is a block diagram of an exemplary processing system, which can be configured to perform any and all operations disclosed herein. For instance, a processing system **700** can include one or more processors **702**, memory **704**, one or more input/output devices **706**, one or more sensors **708**, one or more user interfaces **710**, and one or more actuators **712**. Processing system **700** can be representative of each computing system disclosed herein.

(74) Processors **702** can include one or more distinct processors, each having one or more cores. Each of the distinct processors can have the same or different structure. Processors **702** can include one or more central processing units (CPUs), one or more graphics processing units (GPUs), circuitry (e.g., application specific integrated circuits (ASICs)), digital signal processors (DSPs), and the like. Processors **702** can be mounted to a common substrate or to multiple different substrates.

(75) Processors **702** are configured to perform a certain function, method, or operation (e.g., are configured to provide for performance of a function, method, or operation) at least when one of the one or more of the distinct processors is capable of performing operations embodying the function, method, or operation. Processors **702** can perform operations embodying the function, method, or operation by, for example, executing code (e.g., interpreting scripts) stored on memory **704** and/or trafficking data through one or more ASICs. Processors **702**, and thus processing system **700**, can be configured to perform, automatically, any and all functions, methods, and operations disclosed herein. Therefore, processing system **700** can be configured to implement any of (e.g., all of) the protocols, devices, mechanisms, systems, and methods described herein.

(76) For example, when the present disclosure states that a method or device performs task “X” (or that task “X” is performed), such a statement should be understood to disclose that processing system **900** can be configured to perform task “X”. Processing system **700** is configured to perform a function, method, or operation at least when processors **702** are configured to do the same.

(77) Memory **704** can include volatile memory, non-volatile memory, and any other medium capable of storing data. Each of the volatile memory, non-volatile memory, and any other type of memory can include multiple different memory devices, located at multiple distinct locations and each having a different structure. Memory **704** can include remotely hosted (e.g., cloud) storage.

(78) Examples of memory **704** include a non-transitory computer-readable media such as RAM, ROM, flash memory, EEPROM, any kind of optical storage disk such as a DVD, a Blu-Ray® disc, magnetic storage, holographic storage, a HDD, a SSD, any medium that can be used to store program code in the form of instructions or data structures, and the like. Any and all of the methods, functions, and operations described herein can be fully embodied in the form of tangible and/or non-transitory machine-readable code (e.g., interpretable scripts) saved in memory **704**.

(79) Input-output devices **706** can include any component for trafficking data such as ports, antennas (i.e., transceivers), printed conductive paths, and the like. Input-output devices **706** can enable wired communication via USB®, DisplayPort®, HDMI®, Ethernet, and the like. Input-output devices **706** can enable electronic, optical, magnetic, and holographic, communication with suitable memory **706**. Input-output devices **706** can enable wireless communication via WiFi®, Bluetooth®, cellular (e.g., LTE®, CDMA®, GSM®, WiMax®, NFC®), GPS, and the like. Input-output devices **706** can include wired and/or wireless communication pathways.

(80) Sensors **708** can capture physical measurements of environment and report the same to processors **702**. User interface **710** can include displays, physical buttons, speakers, microphones,

keyboards, and the like. Actuators **712** can enable processors **702** to control mechanical forces.

(81) Processing system **700** can be distributed. For example, some components of processing system **700** can reside in a remote hosted network service (e.g., a cloud computing environment) while other components of processing system **700** can reside in a local computing system.

Processing system **700** can have a modular design where certain modules include a plurality of the features/functions shown in FIG. 7. For example, I/O modules can include volatile memory and one or more processors. As another example, individual processor modules can include read-only-memory and/or local caches.

(82) According to a first aspect of the present disclosure, a method is provided that provides the following. First, in some embodiments, aspects of the present disclosure construct in the Dodis-Yampolskiy pseudo-random function an oblivious PRF protocol with reversed roles by leveraging the symmetry between the key and inputs. Second, in some embodiments, aspects of the present disclosure leverage an intermediate server (e.g., the third-party system **108** and/or the aiding server **402**) to provide unlinkability of users across different executions of a private membership test protocol. Third, in some embodiments, aspects of the present disclosure test private membership based on an oblivious PRF with reversed roles. For instance, aspects of the present disclosure test the private membership in such a way that the private identity is cryptographically bound to a verifiable anonymous credential.

(83) According to a second aspect of the present disclosure, a method for an oblivious OPR protocol with reversed roles is provided. For instance, at a first step, the Prover (e.g., the Prover **202**) homomorphically encrypts and commits to an input, and generates a zero-knowledge proof of knowledge of the input and correctness of the computation. At a second step, the Verifier (e.g., the Verifier **204**) verifies the correctness of the Prover-provided proof, encrypts a PRF key, combines the ciphertext with the Prover-provided ciphertext and blinds the resulting ciphertext with a randomly chosen blinding value. At a third step, the Prover decrypts the Verifier-provided ciphertext, hides the resulting plaintext by moving it to the exponent, and generates a zero-knowledge proof of the correctness of the computation. At a fourth step, the Verifier verifies the correctness of the Prover-provided proof, removes the blinding value from the exponent of the partial PRF value provided by the prover, and returns the final PRF value and the Prover-provided commitment.

(84) According to a third aspect of the present disclosure, a method for private set membership is provided (e.g., the first embodiment described above). For instance, at a first step, the Verifier locally evaluates a pseudo-random function on input the identities in the checklist and a Verifier-held key. At a second step, the Prover and the Verifier obliviously evaluates a pseudo-random function on input the Prover's identity and the Verifier-held key, using a committed-input oblivious PRF protocol with reversed roles (e.g., the second aspect of the present disclosure). At a third step, the Verifier tests membership of the output obtained in the second step with respect to (w.r.t.) the pseudo-random values obtained in the first step. At a fourth step, the Prover generates a zero-knowledge proof of possession of credentials for the input committed to in the second step. At a fifth step, the Verifier verifies the proof obtained from the Prover in the fourth step.

(85) According to a fourth aspect, a method for private set membership with unlinkability is provided using an aiding server (e.g., the second embodiment described above). For instance, at a first step, the Server and the Verifier obliviously evaluate a pseudo-random function on input the identities in the checklist and a Server-held key. At a second step, the Prover and the Server obliviously evaluate a pseudo-random function on input the Prover's identity and the Server-held key, using a committed-input OPRF protocol with reversed roles. At a third step, the Verifier tests membership of the output obtained in the second step with respect to (w.r.t.) the pseudorandom values obtained in the first step. At the fourth step, the Prover generates a zero-knowledge proof of possession of credentials for the input committed to in the second step. At the fifth step, the Verifier verifies the proof obtained from the Prover in the fourth step.



(86) According to a fifth aspect, a method may be executed using any combination of steps listed above in conjunction with the first through fifth aspects.

(87) According to a sixth aspect of the present disclosure, a non-transitory computer readable medium is provided, having stored thereon instructions for performing a method according to at least one of the first through fifth aspects when executed by one or more processors/computers.

(88) According to a seventh aspect, a device is provided comprising pressing circuitry (e.g., a central processing unit coupled to memory), which is configured to execute a method according to at least one of the first through fifth aspects.

(89) In some embodiments, the OPRF protocol (e.g., the protocol described above such as methods **300**, **400**, and/or **600**) is an interactive protocol between two participants that allows a prover and a verifier to perform an oblivious evaluation of a pseudo-random function, obtaining value  $f_{\text{sub.k}}(x)$ , based on a prover-held input  $x$  and a verifier-held PRF key  $k$ . The PRF function is a function used by the OPRF protocol as described above. For instance, the OPRF denotes the interactive protocol. Other terms (e.g., PRF result  $y$ , PRF key  $k$ , PRF input  $x$ ) refer to the PRF function.

(90) The methods and methodologies provide technical advancements to the state of the art which enables membership tests on anonymous credentials with a private blacklist held by the Verifier. Thus the mechanisms provide privacy protection not offered in the state of the art. For example, all existing solutions require the Verifier to make the blacklist public. Accordingly, aspects of the present invention can be implemented to provide an enhanced SSI solution that provides privacy for corporations and authorities.

(91) While subject matter of the present disclosure has been illustrated and described in detail in the drawings and foregoing description, such illustration and description are to be considered illustrative or exemplary and not restrictive. Any statement made herein characterizing the invention is also to be considered illustrative or exemplary and not restrictive as the invention is defined by the claims. It will be understood that changes and modifications may be made, by those of ordinary skill in the art, within the scope of the following claims, which may include any combination of features from different embodiments described above.

(92) The terms used in the claims should be construed to have the broadest reasonable interpretation consistent with the foregoing description. For example, the use of the article “a” or “the” in introducing an element should not be interpreted as being exclusive of a plurality of elements. Likewise, the recitation of “or” should be interpreted as being inclusive, such that the recitation of “A or B” is not exclusive of “A and B,” unless it is clear from the context or the foregoing description that only one of A and B is intended. Further, the recitation of “at least one of A, B and C” should be interpreted as one or more of a group of elements consisting of A, B and C, and should not be interpreted as requiring at least one of each of the listed elements A, B and C, regardless of whether A, B and C are related as categories or otherwise. Moreover, the recitation of “A, B and/or C” or “at least one of A, B or C” should be interpreted as including any singular entity from the listed elements, e.g., A, any subset from the listed elements, e.g., A and B, or the entire list of elements A, B and C.

## Claims

1. A method for performing a privacy-preserving membership test, comprising: performing an oblivious pseudo-random function (OPRF) protocol to determine a pseudo-random function (PRF) result based on an input from a proving device and a PRF key, wherein the input indicates a user identity of a user associated with the proving device; and determining whether the user belongs to a verifier list associated with a verifier device based on testing membership of the user using the verifier list and the PRF result.
2. The method of claim 1, wherein the verifier list indicates a list associated with a company, and

wherein determining whether the user belongs to the verifier list comprises determining whether the user belongs to the list.

3. The method of claim 1, further comprising: performing a PRF evaluation on individuals indicated by the verifier list using the PRF key to determine a representation of the verifier list indicating a plurality of pseudo-random values, and wherein determining whether the user belongs to the verifier list comprises testing the membership of the user based on comparing the PRF result with the plurality of pseudo-random values indicated by the representation of the verifier list.

4. The method of claim 3, wherein the PRF key is a verifier-held key that is stored in the verifier device, and wherein performing the PRF on individuals comprises performing, by the verifier device, the PRF on individuals indicated by the verifier list using the PRF key to determine the representation of the verifier list indicating the plurality of pseudo-random values.

5. The method of claim 3, wherein the PRF key is a server-held key that is stored in a third-party server that is separate from the verifier device and the proving device, and wherein performing the PRF on individuals comprises: performing a second PRF evaluation on the individuals indicated by the verifier list using the server-held key to determine the plurality of pseudo-random values indicated by the representation of the verifier list.

6. The method of claim 5, wherein performing the OPRF protocol to determine the PRF result comprises obtaining, by the verifier device and from the prover device, the PRF result and a commitment associated with the prover device, wherein the PRF result is based on the server-held key, and wherein determining whether the user belongs to the verifier list comprises performing, by the verifier device, a membership check based on comparing the PRF results received from the prover device and the plurality of pseudo-random values indicated by the representation of the verifier list.

7. The method of claim 1, further comprising: based on determining that the user belongs to the verifier list, obtaining, by the verifier device, a zero-knowledge proof of possession of credentials for the input from the proving device; and verifying, by the verifier device, the zero-knowledge proof.

8. The method of claim 1, wherein performing the OPRF protocol to determine the PRF result based on the input from the proving device and the PRF key comprises: obtaining, by the verifier device and from the prover device, the input and a zero-knowledge proof of the input, wherein the input is a first ciphertext that is based on homomorphically encrypting credentials of the user; verifying, by the verifier device, correctness of the zero-knowledge proof of the input from the prover device; and determining, by the verifier device, the PRF result based on the input and the verification.

9. The method of claim 8, wherein performing the OPRF to determine the PRF result based on the input from the proving device and the PRF key further comprises: based on verifying the correctness of the zero-knowledge proof, encrypting the PRF key using a public key of the prover device to determine the encrypted PRF key; generating a second ciphertext based on the encrypted PRF key, the first ciphertext, and a randomly chosen blinding value, and wherein determining the PRF result is based on the second ciphertext.

10. The method of claim 9, wherein generating the second ciphertext comprises: multiplying the encrypted PRF key with the first ciphertext to determine an intermediate ciphertext; and blinding the intermediate ciphertext with the randomly chosen blinding value to generate the second ciphertext.

11. The method of claim 9, wherein performing the OPRF protocol to determine the PRF result based on the input from the proving device and the PRF key further comprises: obtaining, by the verifier device and from the prover device, a partial PRF result and a zero-knowledge proof of the partial PRF result, wherein the partial PRF result is based on decrypting the second ciphertext and hiding the decrypted second ciphertext by moving the decrypted second ciphertext into an exponent of the partial PRF result; and verifying the zero-knowledge proof of the partial PRF result, wherein



determining the PRF result is based on verifying the zero-knowledge proof of the partial PRF result.

12. The method of claim 11, wherein determining the PRF result comprises generating the PRF result based on removing the randomly chosen blinding value from the exponent of the partial PRF result.

13. A system for performing a privacy-preserving membership test, the system comprising one or more hardware processors, which, alone or in combination, are configured to provide for execution of the following steps: performing an oblivious pseudo-random function (OPRF) protocol to determine a PRF result based on an input from a proving device and a PRF key, wherein the input indicates a user identity of a user associated with the proving device; and determining whether the user belongs to a verifier list associated with a verifier device based on testing membership of the user using the verifier list and the PRF result.

14. The system of claim 13, wherein the one or more hardware processors, which, alone or in combination, are configured to further provide for execution of the following step: performing a PRF evaluation on individuals indicated by the verifier list using the PRF key to determine a representation of the verifier list indicating a plurality of pseudo-random values, and wherein determining whether the user belongs to the verifier list comprises testing the membership of the user based on comparing the PRF result with the plurality of pseudo-random values indicated by the representation of the verifier list.

15. A tangible, non-transitory computer-readable medium having instructions thereon which, upon being executed by one or more processors, alone or in combination, provide for execution of a method for secure aggregation using public randomness comprising the following steps: performing an oblivious pseudo-random function (OPRF) protocol to determine a PRF result based on an input from a proving device and an PRF key, wherein the input indicates a user identity of a user associated with the proving device; and determining whether the user belongs to a verifier list associated with a verifier device based on testing membership of the user using the verifier list and the PRF result.

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