US 20250266983A1

(19) **United States**

(12) **Patent Application Publication** (10) **Pub. No.: US 2025/0266983 A1**

Najarian et al. (43) **Pub. Date: Aug. 21, 2025**

(54) **SYSTEMS AND METHODS FOR CLASSIFYING ENCRYPTED DATA USING AN ENCRYPTED MACHINE LEARNING MODEL**

(71) Applicant: **REGENTS OF THE UNIVERSITY OF MICHIGAN**, Ann Arbor, MI (US)

(72) Inventors: **Kayvan Najarian**, Northville, MI (US); **Alexander Wood**, Ann Arbor, MI (US); **Cristian Minoccheri**, Ann Arbor, MI (US); **Emily Wittrup**, Ann Arbor, MI (US); **Jonathan Gryak**, West Haven, CT (US); **Richard Wilson**, York (GB); **Delaram Kahrobaei**, Ann Arbor, MI (US)

(21) Appl. No.: **18/443,677**

(22) Filed: **Feb. 16, 2024**

Publication Classification

(51) **Int. Cl.**

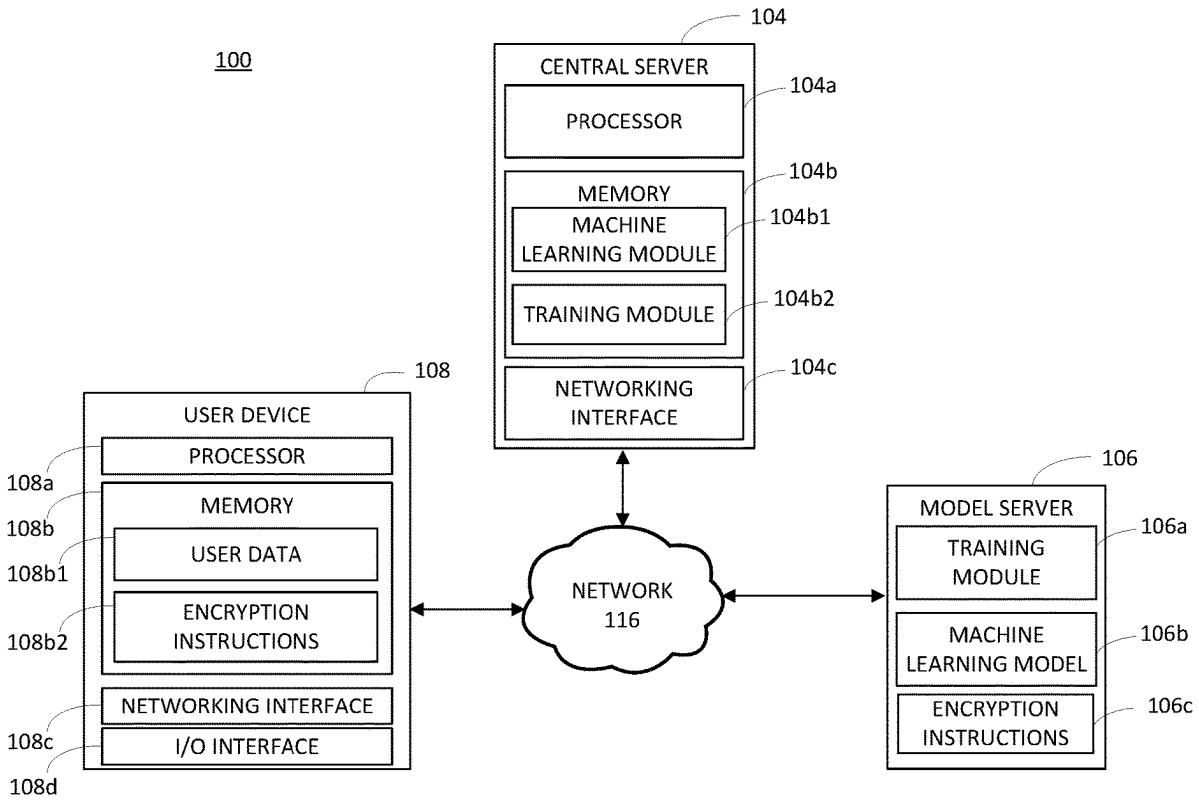
H04L 9/00	(2022.01)
G06N 20/00	(2019.01)
G06N 20/20	(2019.01)

(52) **U.S. Cl.**

CPC ***H04L 9/008*** (2013.01); ***G06N 20/00*** (2019.01); ***G06N 20/20*** (2019.01)

(57) **ABSTRACT**

Systems and methods for classifying encrypted data using an encrypted machine learning model are disclosed. An example method includes receiving, at one or more processors, encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique. The example method further includes analyzing, by the one or more processors executing an encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data. The example method further includes transmitting, by the one or more processors, the encrypted classification to a user computing device for decryption.



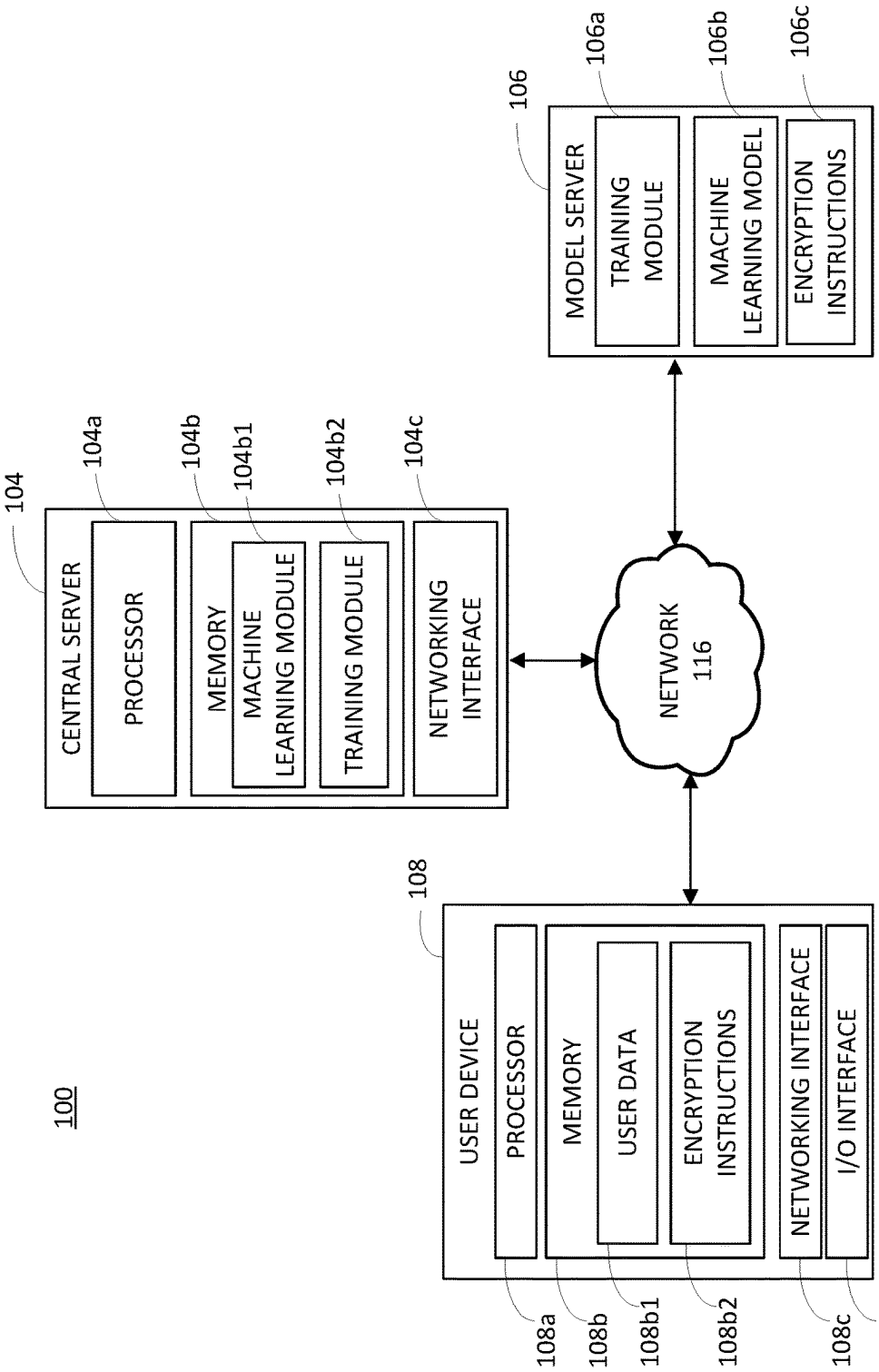

100
104
CENTRAL SERVER
PROCESSOR
MEMORY
MACHINE LEARNING MODULE
TRAINING MODULE
NETWORKING INTERFACE
104a
104b
104b1
104b2
104c
NETWORK 116
106
MODEL SERVER
TRAINING MODULE
MACHINE LEARNING MODEL
ENCRYPTION INSTRUCTIONS
106a
106b
106c
108
USER DEVICE
PROCESSOR
MEMORY
USER DATA
ENCRYPTION INSTRUCTIONS
NETWORKING INTERFACE
I/O INTERFACE
108a
108b
108b1
108b2
108c
108d

FIG. 1A

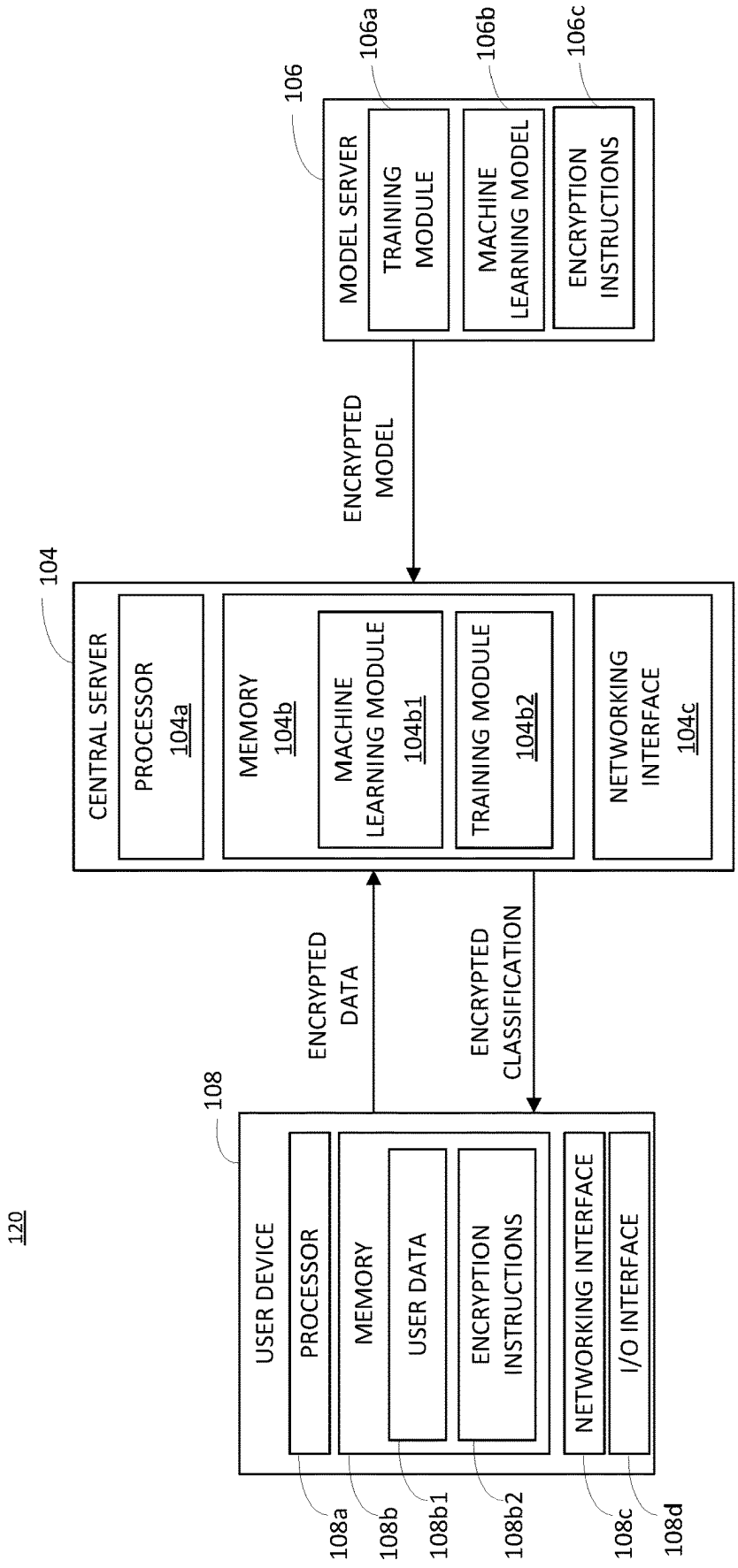

120
106
106a
106b
106c
MODEL SERVER
TRAINING MODULE
MACHINE LEARNING MODEL
ENCRYPTION INSTRUCTIONS
ENCRYPTED MODEL
104
CENTRAL SERVER
PROCESSOR 104a
MEMORY 104b
MACHINE LEARNING MODULE 104b1
TRAINING MODULE 104b2
NETWORKING INTERFACE 104c
ENCRYPTED DATA
ENCRYPTED CLASSIFICATION
108
USER DEVICE
PROCESSOR
MEMORY
USER DATA
ENCRYPTION INSTRUCTIONS
NETWORKING INTERFACE
I/O INTERFACE
108a
108b
108b1
108b2
108c
108d

FIG. 1B

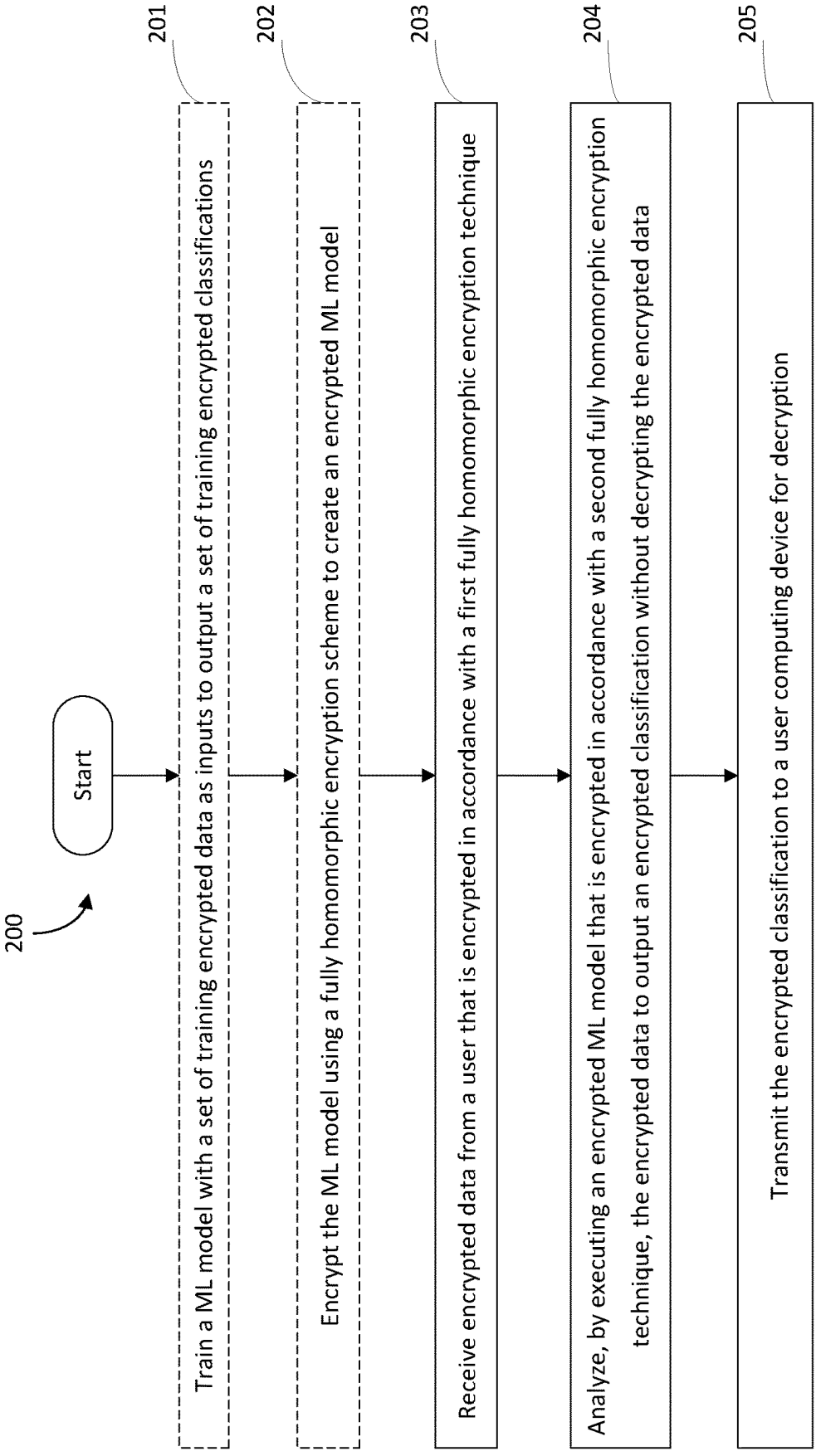

200
Start
201
Train a ML model with a set of training encrypted data as inputs to output a set of training encrypted classifications
202
Encrypt the ML model using a fully homomorphic encryption scheme to create an encrypted ML model
203
Receive encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique
204
Analyze, by executing an encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data
205
Transmit the encrypted classification to a user computing device for decryption

FIG. 2A

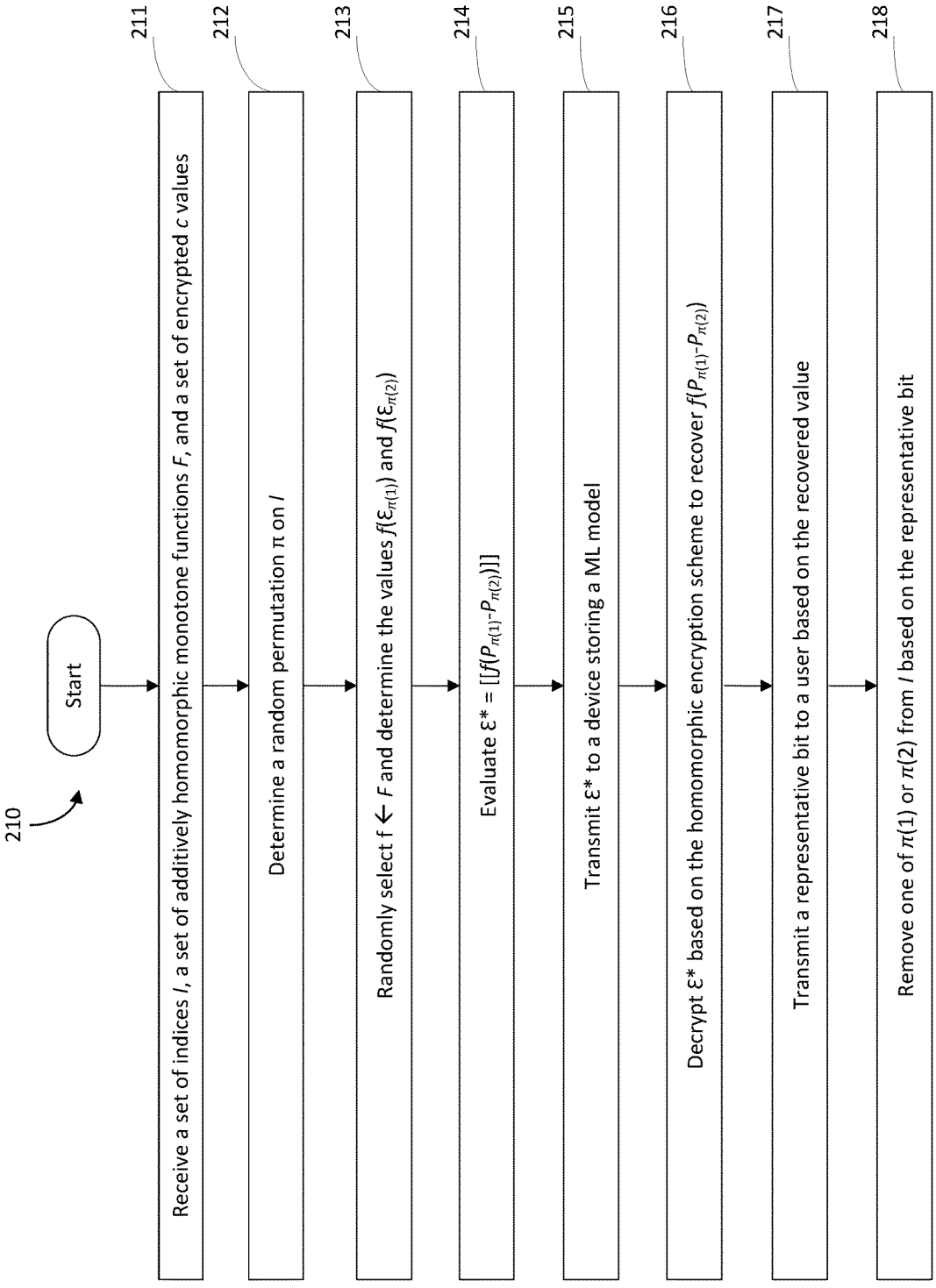

210
Start
211
Receive a set of indices I, a set of additively homomorphic monotone functions F, and a set of encrypted c values
212
Determine a random permutation π on I
213
Randomly select f ← F and determine the values $f(\varepsilon_{\pi(1)})$ and $f(\varepsilon_{\pi(2)})$
214
Evaluate $\varepsilon^* = [[f(P_{\pi(1)} - P_{\pi(2)})]]$
215
Transmit ε* to a device storing a ML model
216
Decrypt ε* based on the homomorphic encryption scheme to recover $f(P_{\pi(1)} - P_{\pi(2)})$
217
Transmit a representative bit to a user based on the recovered value
218
Remove one of $\pi(1)$ or $\pi(2)$ from I based on the representative bit

FIG. 2B

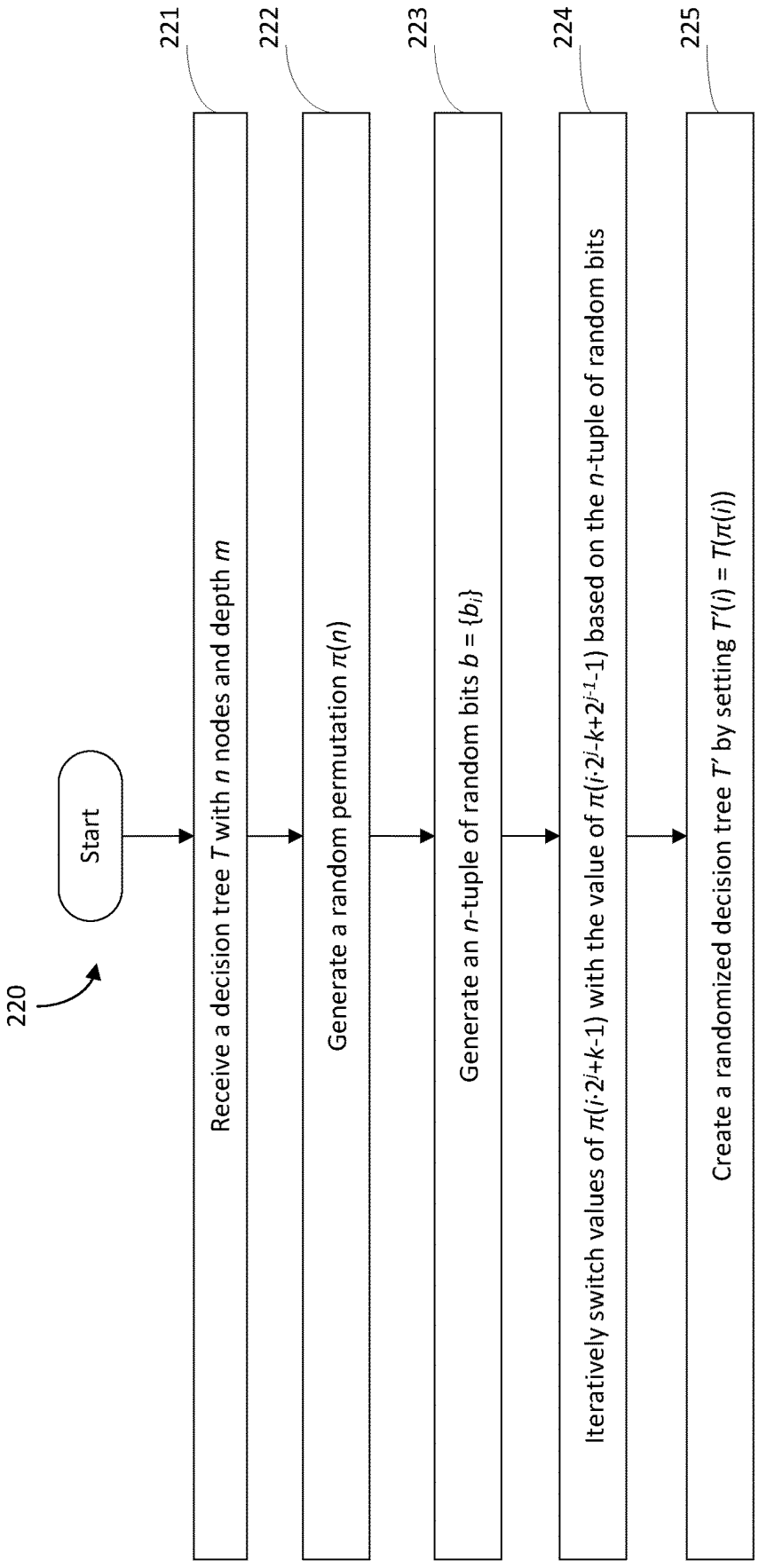

220
Start
221
Receive a decision tree T with n nodes and depth m
222
Generate a random permutation $\pi(n)$
223
Generate an n-tuple of random bits $b = \{b_i\}$
224
Iteratively switch values of $\pi(i \cdot 2^i + k - 1)$ with the value of $\pi(i \cdot 2^i - k + 2^{i-1} - 1)$ based on the n-tuple of random bits
225
Create a randomized decision tree T' by setting $T'(i) = T(\pi(i))$

FIG. 2C

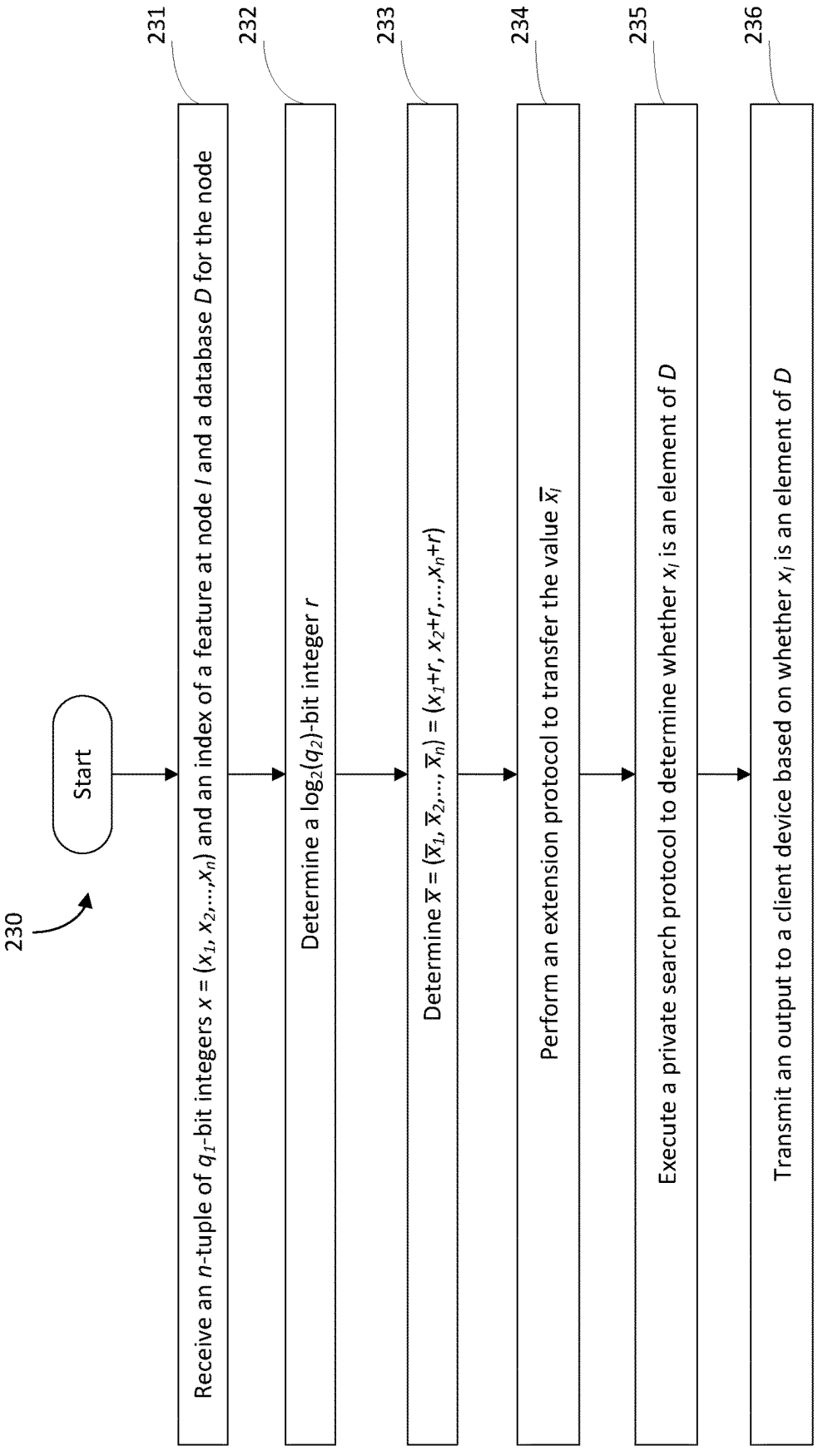

230
Start
231
Receive an n-tuple of q_1-bit integers $x = (x_1, x_2,...,x_n)$ and an index of a feature at node l and a database D for the node
232
Determine a $\log_2(q_2)$-bit integer r
233
Determine $\bar{x} = (\bar{x}_1, \bar{x}_2,..., \bar{x}_n) = (x_1+r, x_2+r,...,x_n+r)$
234
Perform an extension protocol to transfer the value $\bar{x}_l$
235
Execute a private search protocol to determine whether x_l is an element of D
236
Transmit an output to a client device based on whether x_l is an element of D

FIG. 2D

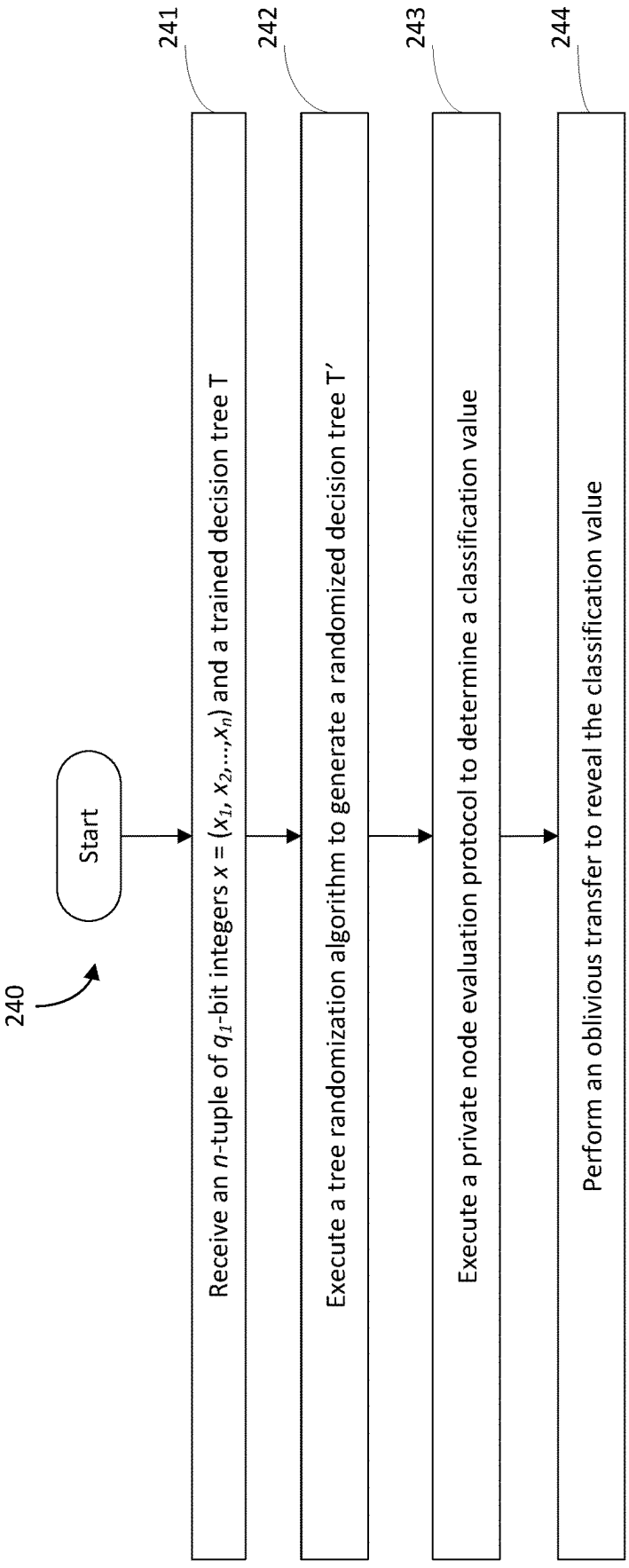

240
Start
241
Receive an n-tuple of q_1-bit integers $x = (x_1, x_2, \ldots, x_n)$ and a trained decision tree T
242
Execute a tree randomization algorithm to generate a randomized decision tree T'
243
Execute a private node evaluation protocol to determine a classification value
244
Perform an oblivious transfer to reveal the classification value

FIG. 2E

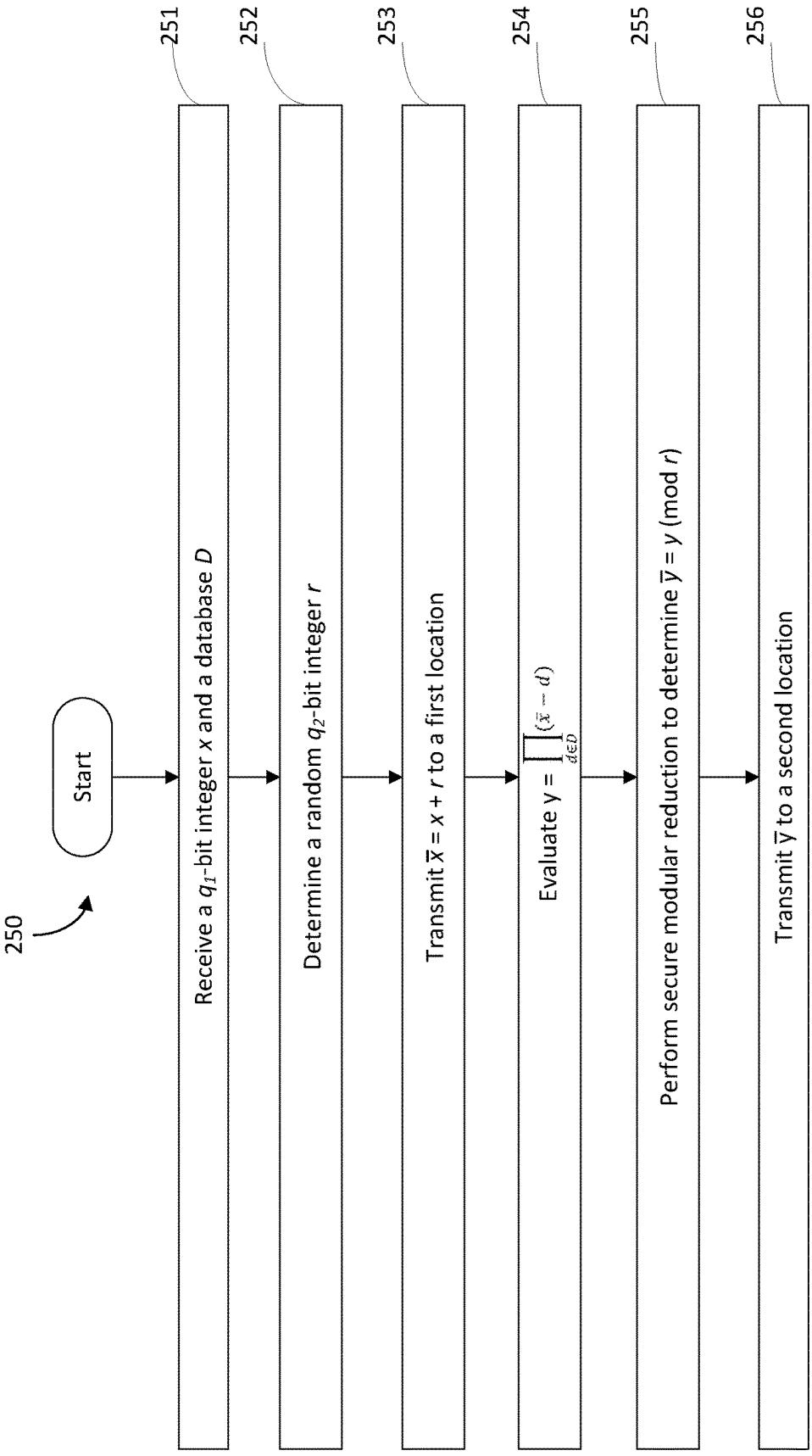

250
Start
251
Receive a q_1-bit integer x and a database D
252
Determine a random q_2-bit integer r
253
Transmit $\bar{x} = x + r$ to a first location
254
Evaluate $y = \prod_{d \in D} (\bar{x} - d)$
255
Perform secure modular reduction to determine $\bar{y} = y \pmod{r}$
256
Transmit $\bar{y}$ to a second location

FIG. 2F

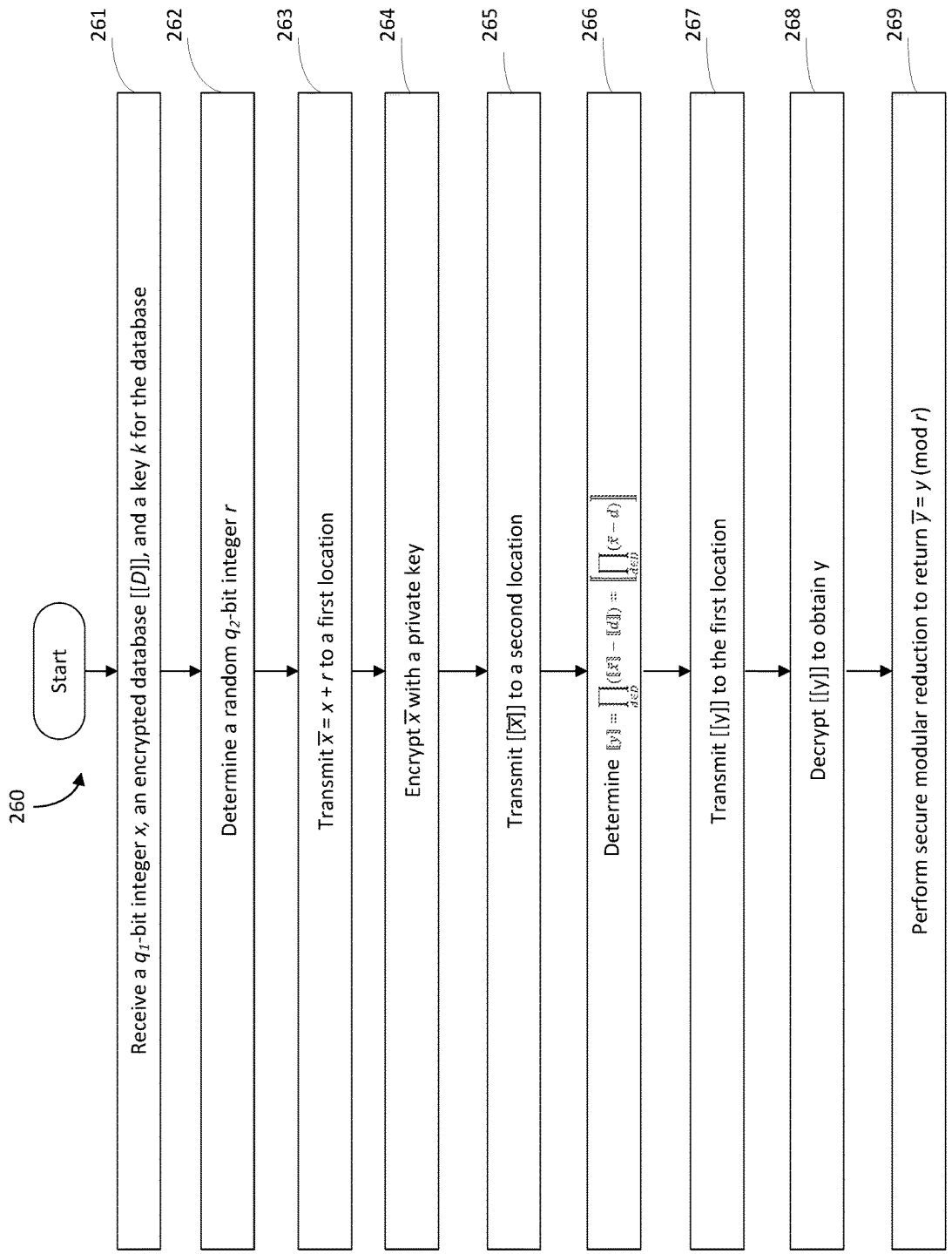

260
Start
261
Receive a q_1-bit integer x, an encrypted database $[[D]]$, and a key k for the database
262
Determine a random q_2-bit integer r
263
Transmit $\overline{x} = x + r$ to a first location
264
Encrypt $\overline{x}$ with a private key
265
Transmit $[[\overline{x}]]$ to a second location
266
Determine $[[y]] = \prod_{d \in D} ([[\overline{x}]] - [[d]]) = \left[\!\left[\prod_{d \in D} (\overline{x} - d)\right]\!\right]$
267
Transmit $[[y]]$ to the first location
268
Decrypt $[[y]]$ to obtain y
269
Perform secure modular reduction to return $\overline{y} = y \pmod{r}$

FIG. 2G

SYSTEMS AND METHODS FOR CLASSIFYING ENCRYPTED DATA USING AN ENCRYPTED MACHINE LEARNING MODEL

STATEMENT REGARDING FEDERALLY SPONSORED RESEARCH OR DEVELOPMENT

[0001] This invention was made with government support under 2209546 awarded by the National Science Foundation. The government has certain rights in the invention.

FIELD OF THE DISCLOSURE

[0002] The present disclosure relates generally to techniques for encrypted communications and, more particularly, to systems and methods for classifying encrypted data using an encrypted machine learning (ML) model.

BACKGROUND

[0003] The background description provided herein is for the purpose of generally presenting the context of the disclosure. Work of the presently named inventor, to the extent it is described in this background section, as well as aspects of the description that may not otherwise qualify as prior art at the time of filing, are neither expressly nor impliedly admitted as prior art against the present disclosure.

[0004] Private classification is a growing concern in medical applications as more assisted decision making and diagnosis systems become commercially available. However, owners of these systems generally do not want to share their models, and hospitals generally do not want to share their patients' data. Thus, without a secure ability to share models and data, such systems may experience delayed adoption, which can negatively impact the quality and timeliness of the corresponding medical care patients receive.

[0005] Therefore, there is a need for systems capable of performing private classification using an encrypted model to enable more secure and reliable decision making and diagnosis of various patient conditions.

SUMMARY OF THE INVENTION

[0006] In some aspects, the techniques described herein relate to a system for classifying encrypted data using an encrypted machine learning (ML) model, the system including: a memory storing a set of computer-readable instructions including an encrypted ML model; and a processor interfacing with the memory, and configured to execute the set of computer-readable instructions to cause the system to: receive encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique, analyze, by executing the encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data, and transmit the encrypted classification to a user computing device for decryption.

[0007] In some aspects, the techniques described herein relate to a system, wherein the set of computer-readable instructions, when executed, further cause the system to: train a ML model with a set of training encrypted data as inputs to output a set of training encrypted classifications of the set of training encrypted data; and encrypt the ML model using the second fully homomorphic encryption technique to create the encrypted ML model.

[0008] In some aspects, the techniques described herein relate to a system, wherein encrypting the ML model further includes: encrypt one or more parameters of the ML model using the second fully homomorphic encryption technique.

[0009] In some aspects, the techniques described herein relate to a system, wherein the encrypted ML model includes: (i) a Naïve Bayes model, (ii) a Decision Tree, or (iii) a Random Forest model.

[0010] In some aspects, the techniques described herein relate to a system, wherein the first fully homomorphic encryption technique or the second fully homomorphic encryption technique further includes a private key encryption technique.

[0011] In some aspects, the techniques described herein relate to a system, wherein the first fully homomorphic encryption technique is different from the second fully homomorphic encryption technique.

[0012] In some aspects, the techniques described herein relate to a system, wherein the encrypted data from the user includes: (i) a set of encrypted values, (ii) a set of additively homomorphic monotone functions, (iii) a set of indexes, (iv) a q1-bit integer, or (v) an n-tuple q1-bit integer.

[0013] In some aspects, the techniques described herein relate to a system, wherein the set of computer-readable instructions, when executed, further cause the system to: receive a second set of encrypted data from a second user that is encrypted in accordance with the second fully homomorphic encryption technique, and wherein the second set of encrypted data includes: (i) a feature index at a node, (ii) a database, or (iii) a private encryption key for the database.

[0014] In some aspects, the techniques described herein relate to a system, wherein the set of computer-readable instructions, when executed, further cause the system to: produce an encrypted update to a ML model with a set of training encrypted data as inputs from one or more users; transmit the encrypted update to each user of the one or more users for decryption; and update the ML model using the encrypted update.

[0015] In some aspects, the techniques described herein relate to a system, wherein the set of computer-readable instructions, when executed, further cause the system to: receive, from each user of a plurality of users, a training dataset encrypted using an independently generated key pair, in accordance with a multi-key, multi-hop fully homomorphic encryption technique; train, with the training datasets, a ML model using a ML training technique; receive encrypted test datasets from one or more users of the plurality of users; generate, by executing the ML model, encrypted outputs for each encrypted test dataset; and cause user computing devices of each of the one or more users to participate in on-the-fly, multiparty computation to decrypt one or more respective encrypted outputs of the encrypted outputs by transmitting the encrypted outputs to each respective user of the one or more users.

[0016] In some aspects, the techniques described herein relate to a system, wherein the set of computer-readable instructions, when executed, further cause the system to: receive, from a new user, a new training dataset encrypted using a new independently generated key pair, in accordance with the multi-key, multi-hop fully homomorphic encryption

technique; and update the ML model with the new training dataset using the ML training technique without re-encrypting the training datasets.

[0017] In some aspects, the techniques described herein relate to a computer-implemented method for classifying encrypted data using an encrypted machine learning (ML) model, the method including: receiving, at one or more processors, encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique; analyzing, by the one or more processors executing an encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data; and transmitting, by the one or more processors, the encrypted classification to a user computing device for decryption.

[0018] In some aspects, the techniques described herein relate to a computer-implemented method, further including: training, by the one or more processors, a ML model with a set of training encrypted data as inputs to output a set of training encrypted classifications of the set of training encrypted data; and encrypting, by the one or more processors, the ML model using the second fully homomorphic encryption technique to create the encrypted ML model.

[0019] In some aspects, the techniques described herein relate to a computer-implemented method, wherein encrypting the ML model further includes: encrypting, by the one or more processors, one or more parameters of the ML model using the second fully homomorphic encryption technique.

[0020] In some aspects, the techniques described herein relate to a computer-implemented method, wherein the encrypted ML model includes: (i) a Naïve Bayes model, (ii) a Decision Tree, or (iii) a Random Forest model.

[0021] In some aspects, the techniques described herein relate to a computer-implemented method, wherein the first fully homomorphic encryption technique or the second fully homomorphic encryption technique further includes a private key encryption technique.

[0022] In some aspects, the techniques described herein relate to a computer-implemented method, wherein the first fully homomorphic encryption technique is different from the second fully homomorphic encryption technique.

[0023] In some aspects, the techniques described herein relate to a computer-implemented method, wherein the encrypted data from the user includes: (i) a set of encrypted values, (ii) a set of additively homomorphic monotone functions, (iii) a set of indexes, (iv) a q1-bit integer, or (v) an n-tuple q1-bit integer.

[0024] In some aspects, the techniques described herein relate to a computer-implemented method, further including: receiving, at the one or more processors, a second set of encrypted data from a second user that is encrypted in accordance with the second fully homomorphic encryption technique, and wherein the second set of encrypted data includes: (i) a feature index at a node, (ii) a database, or (iii) a private encryption key for the database.

[0025] In some aspects, the techniques described herein relate to a non-transitory computer readable medium including instructions for classifying encrypted data using an encrypted machine learning (ML) model that, when executed, may cause one or more processors to: receive encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique; analyze, by executing an encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data; and transmit the encrypted classification to a user computing device for decryption.

BRIEF DESCRIPTION OF THE DRAWINGS

[0026] The figures described below depict various aspects of the system and methods disclosed herein. It should be understood that each figure depicts an embodiment of a particular aspect of the disclosed system and methods, and that each of the figures is intended to accord with a possible embodiment thereof. Further, wherever possible, the following description refers to the reference numerals included in the following figures, in which features depicted in multiple figures are designated with consistent reference numerals.

[0027] FIG. **1**A illustrates an example environment for classifying encrypted data using an encrypted ML model, in accordance with various aspects disclosed herein.

[0028] FIG. **1**B illustrates an example system classifying encrypted data using an encrypted ML model including components of the example environment of FIG. **1**A, and in accordance with various aspects disclosed herein.

[0029] FIGS. **2**A-**2**G illustrate example methods for classifying encrypted data using an encrypted ML model, in accordance with various aspects disclosed herein.

DETAILED DESCRIPTION

[0030] As previously mentioned, conventional diagnostic and decision-making techniques can suffer from security issues due to a lack of encryption and/or otherwise secure data transmission between data owners (e.g., hospitals) and model owners. The techniques of the present disclosure overcome issues associated with conventional techniques by facilitating fully encrypted data transmission and processing between data owners and model owners. More specifically, the present disclosure introduces a system including a data owner providing encrypted data as inputs to an encrypted ML model that outputs an encrypted classification, wherein the encryption of all data, models, and classifications is performed and/or otherwise in accordance with one or more fully homomorphic encryption (FHE) techniques. This system/architecture collectively eliminates and/or reduces the drawbacks of conventional techniques by creating a more secure data transmission and processing pipeline that ensures the privacy of data owners and model owners alike.

[0031] Thus, in accordance with the above, and with the disclosure herein, the present disclosure includes improvements in computer functionality or in improvements to other technologies at least because the disclosure describes that, e.g., a server (e.g., a central server, model server), or otherwise computing device (e.g., a user computing device), is improved where the intelligence or predictive ability of the server and/or computing device is enhanced by a trained and encrypted ML model. These models, executing on the server and/or user computing device, are able to accurately and securely determine data classifications for encrypted data received from a user (e.g., data owner). That is, the present disclosure describes improvements in the functioning of the computer itself or "any other technology or technical field" because a server and/or user computing device, is enhanced with the trained and encrypted ML model to securely determine data classifications by process-

ing the encrypted data to output an encrypted classification that improves a user's ability to evaluate/analyze the encrypted data without compromising the security of the data. This improves over the prior art at least because existing systems lack such encrypted data interpretation and/or encrypted classification output functionality and are generally unable to securely analyze input data to output classifications and/or are unable to analyze encrypted data and/or output an encrypted classification for decryption only by the data owner.

[0032] As mentioned, the model(s) may be trained using machine learning and may utilize machine learning during operation. Therefore, in these instances, the techniques of the present disclosure may further include improvements in computer functionality or in improvements to other technologies at least because the disclosure describes such models being trained with a plurality of training data (e.g., 10,000 s of training data corresponding to encrypted data, etc.) to output the encrypted classification(s) configured to improve the user/operator's ability to evaluate/analyze the encrypted data without compromising the security of the data.

[0033] Moreover, the present disclosure includes effecting a transformation or reduction of a particular article to a different state or thing, e.g., transforming or reducing the data security of encrypted data analysis from a non-optimal or error state (e.g., data requiring decryption prior to analysis) to an optimal state (e.g., analyzing encrypted data while encrypted) by training a ML model with encrypted training data to output encrypted training classifications.

[0034] Still further, the present disclosure includes specific features other than what is well-understood, routine, conventional activity in the field, or adding unconventional steps that demonstrate, in various embodiments, particular useful applications, e.g., analyzing, by the one or more processors executing an encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data, and/or transmitting, by the one or more processors, the encrypted classification to a user computing device for decryption, among others.

[0035] To provide a general understanding of the system (s)/components utilized in the techniques of the present disclosure, FIGS. **1**A and **1**B illustrate, respectively, an example environment **100** for classifying encrypted data using an encrypted ML model and an example system **120** classifying encrypted data using an encrypted ML model including components of the example environment **100**. Accordingly, FIG. **1**A provides a general overview of the example environment **100** components, and FIG. **1**B illustrates the components and their respective functions in greater detail. Moreover, it should be appreciated that the example environment **100** may include more/fewer components and/or the example system **120** may include communications between any suitable combination/configuration of the components from the example environment **100**.

[0036] In any event, FIG. **1**A illustrates an example environment **100** for classifying encrypted data using an encrypted ML model, in accordance with various aspects disclosed herein. It should be appreciated that the example environment **100** is merely exemplary and that alternative or additional embodiments are envisioned.

[0037] In reference to FIG. **1**A, the example environment **100** may be a distributed computing environment where the encrypted data and encrypted ML model are located/stored in any suitable location(s) and may be transmitted across a network **116** to the various components, as necessary. In particular, the example environment **100** includes a central server **104**, a model server **106**, and a user device **108**. Broadly, the user device **108** may store and transmit encrypted user data **108***b***1** to the central server **104** across the network **116**, and the central server **104** may also receive an encrypted, trained ML model **106***b* from the model server **106**. Upon receipt of both the encrypted user data **108***b***1** and the encrypted, trained ML model **106***b*, the central server **104** may output encrypted classifications and/or any other suitable information and transmit the output(s) to the user device **108**. The user device **108** may, for example, display the data output from the central server **104** for viewing by a user, such as the user that provided and/or authorized the encrypted data to be transmitted to the central server **104** for analysis by the encrypted ML model **106***b*. However, in certain embodiments, the user device **108** may transmit data directly to the model server **106** for analysis using the ML model **106***b*.

[0038] While in various examples herein a central server is shown, the techniques herein may be implemented in a distributed processing model, such as what is termed a federated computational model. For example, computations performed by FHE trained ML models herein may be performed, in some architectures, across decentralized devices or servers. In some such examples, each distributed device or server may include an FHE trained ML model and a central server may coordinate operations across these distributed devices. That is, while centralized data consumption examples are described and illustrated, in some implementations, fully homomorphic encrypted data may be retained locally at a user device or server, and a central server may deploy instantiations of the fully homomorphically encrypted machine learning models to that user device or server or consuming the data without decryption.

[0039] Further, as referenced herein, "encrypted data" may be or include data representative of and/or otherwise corresponding to (i) a set of encrypted values, (ii) a set of additively homomorphic monotone functions, (iii) a set of indexes, (iv) a q_1-bit integer, or (v) an n-tuple q_1-bit integer. Further, the "training data" or "training encrypted data" may be or include similar data values as the encrypted data. Moreover, while the data and actions described herein may be referenced in the context of medical data, it should be appreciated that this is for the purposes of discussion only. Thus, the data and actions described herein may apply to any suitable field and/or types of data.

[0040] Generally speaking, the central server **104** may include a processor **104***a*, a memory **104***b*, and a networking interface **104***c*. The memory **104***b* may include a ML module **104***b***1** and a training module **104***b***2**. The model server **106** may include a training module **106***a*, a ML model **106***b*, and encryption instructions **106***c*. The user device **108** may include a processor **108***a*, a memory **108***b*, a networking interface **108***c*, and an input/output (I/O) interface **108***d*. The memory **108***b* may include user data **108***b***1** and encryption instructions **108***b***2**.

[0041] The user device **108** may store user data **108***b***1** that may be sensitive and/or otherwise should not be transmitted outside of the user device **108** while unencrypted (e.g.,

patient medical data). Accordingly, the user device **108** may also store encryption instructions **108***b***2** configured to encrypt the user data **108***b***1** prior to transmitting the user data **108***b***1** to the central server **104** for subsequent analysis. More specifically, the encryption instructions **108***b***2** may include instructions configured to encrypt the user data **108***b***1** in accordance with a fully homomorphic encryption technique.

[0042] The user device **108** may transmit the encrypted user data **108***b***1** to the central server **104**, where the central server **104** may apply/utilize the ML module **104***b***1** and/or the training module **104***b***2** to the encrypted user data **108***b***1**. The central server **104** may also receive an encrypted ML model **106***b* from the model server **106**, and the central server **104** may then input the encrypted user data **108***b***1** into the encrypted ML model **106***b*. In particular, the central server **104** may execute the encrypted ML model **106***b* via the ML module **104***b***1** to analyze the encrypted user data **108***b***1** and generate encrypted classification(s) as output.

[0043] The model server **106** may include a training module **106***a*, a ML model **106***b*, and encryption instructions **106***c*. The training module **106***a* may utilize input training data (e.g., sets of training encrypted data) to train the ML model **106***b* to generate the training outputs (e.g., training sets of classifications). The ML model **106***b* may generally implement and be trained using machine learning (ML) techniques and may include instructions configured to determine a classification corresponding to encrypted user data **108***b***1**. The encryption instructions **106***c* may generally include instructions configured to encrypt the ML model **106***b* prior to transmitting the ML model **106***b* to the central server **104** for subsequent execution. More specifically, the encryption instructions **106***c* may include instructions configured to encrypt the ML model **106***b* in accordance with a fully homomorphic encryption technique, for example, by encrypting the weights, parameters, and/or other suitable values associated with the ML model **106***b*. In certain embodiments, the encryption instructions **106***c* may encrypt the ML model **106***b* in accordance with a fully homomorphic encryption technique that is different from the fully homomorphic encryption technique used to encrypt the user data **108***b***1**.

[0044] The central server **104** may include a ML module **104***b***1** and a training module **104***b***2** that may generally be configured to collectively train, re-train, and/or update the ML model **106***b*. The ML module **104***b***1** may receive input encrypted user data **108***b***1** (e.g., data from a hospital) and may execute the encrypted ML model **106***b* to generate encrypted classification(s). The training module **104***b***2** may utilize new input data (e.g., new encrypted data) to re-train/update the ML model **106***b* after training at the model server **106** to generate outputs (e.g., classifications) based on the prior training dataset and the new input data.

[0045] The user device **108** may generally be a mobile device, laptop/desktop computer, wearable device, and/or any other suitable computing device that may enable a user to view the data transmitted from the central server **104**. The user device **108** may include one or more processors **108***a*, a memory **108***b*, a networking interface **108***c*, and an input/output (I/O) interface **108***d*. The user device **108** may receive the encrypted classification(s) from the central server **104** via the networking interface **108***c*, decrypt the encrypted classification(s), and may display the decrypted classification(s) to the user via a display or other output device that is included as part of the I/O interface **108***d*. In certain embodiments, the user may interact with the user device **108** to view various aspects of the classification(s) received from the central server **104**, communicate with the central server **104**, and/or perform other actions (e.g., contacting a patient) in response to receiving the user interaction.

[0046] To provide a better understanding of the communications and general functionality of the components of the example environment **100**, FIG. **1**B illustrates an example system **120** for classifying encrypted data (e.g., user data **108***b***1**) using an encrypted ML model (e.g., ML model **106***b*) including components of the example environment **100** of FIG. **1**A, and in accordance with various aspects disclosed herein. Generally, example system **120** of FIG. **1**B includes the central server **104** receiving data transmissions from the model server **106** and the user device **108** and generating and transmitting outputs to the model server **106** and the user device **108** based on those data transmissions.

[0047] For example, the model server **106** may transmit an encrypted ML model **106***b* to the central server **104**, and the user device **108** may transmit encrypted data **108***b***1** to the central server **104**. The central server **104** may analyze the encrypted data **108***b***1** using the encrypted ML model **106***b* to generate an encrypted classification, which the central server **104** may transmit to the user device **108** for decryption, in accordance with the encryption instructions **108***b***2**.

[0048] In this manner, each party to the data transmissions/analysis may benefit from the execution of such transmission/analysis without exposing and/or otherwise risking their contributed data to any other participating party. Namely, the user of the user device **108** (e.g., a hospital, a physician, business owner, etc.) may securely transmit their user data **108***b***1** to the central server **104** to receive a classification without the central server **104** owner/operator and/or the model server **106** owner/operator receiving an unencrypted/decrypted version of the user data **108***b***1** or the resulting classification. Similarly, the owner/operator of the model server **106** may securely transmit their ML model **106***b* to the central server **104** to continually update/re-train/train and/or otherwise improve the ML model **106***b* without the central server **104** owner/operator and/or the user of the user device **108** receiving an unencrypted/decrypted version of the ML model **106***b* (e.g., unencrypted/decrypted model parameters, weights, etc.).

[0049] Moreover, while the techniques described herein may apply to a wide variety of scenarios involving sensitive, encrypted data and/or models, a few illustrative example scenarios may provide further clarification regarding the types of interactions/actions taking place, as described herein.

[0050] For example, in a first scenario, a hospital H may have a private database of patient data, D, which cannot be shared with an external entity. A research center C would like to use D to construct a machine learning model (e.g., ML model **106***b*), for example to build a function F to calculate a risk assessment score or perform actuarial analyses. In practice, C could be a research center, another hospital, a pharmaceutical company, an insurance company, and/or any other third party. Additionally, C may be reluctant to share the function, F, with H. Such a scenario can occur quite often, as researchers find it difficult to train generalizable models due to the lack of public data.

[0051] In a second scenario, C may have a collection of multivariate functions F_i: $x^n \rightarrow Y$, representing a specific machine learning algorithm (e.g., ML model **106***b*), while H has inputs $x_1, x_2, \ldots, x_n$ (e.g., user data **108***b***1**) representing a patient x's medical data. The set of known functions, $F=\{Fi\}_{i=1}^{k}$ are expected to predict a diagnosis or prognosis for x, for example, the chances of patient x having Parkinson's, cancer, a heart attack, etc.

[0052] The difference between this second scenario and the first scenario is that, in the second scenario, F is assumed to be a known function (e.g., it is known how to calculate Glasgow Coma Scale (GCS) using eye, verbal, and motor responses). Regardless, the example system **120** may be configured to apply F in either scenario on both encrypted and unencrypted data to generate outputs. These outputs may be or include, for example, "health metrics", "severity scores", other clinical functions typically computed as a linear combination of privacy-protected factors, such as quantitative clinical or physiological patient data with a set of weights (coefficients) and/or any other suitable classifications corresponding to the input data.

[0053] As previously mentioned, and in very general terms, a potential exchange between the hospital H (e.g., user of user device **108**) and a research center C (e.g., owner/operator of model server **106**) may proceed in the following manner. H may encrypt data, $x_1, x_2, \ldots, x_n$, of a particular patient x with a fully homomorphic encryption technique E and may send the encrypted values $\varepsilon(x_1), \varepsilon(x_2), \ldots, \varepsilon(x_n)$ to C. C may apply the private function F to the encrypted data, thereby computing $F(E(x_1), \varepsilon(x_2), \ldots, \varepsilon(x_n))$. Due to the fully homomorphic property of E, this computation is equal to $\varepsilon(F(x_1, x_2, \ldots, x_n))$. Next, C may send this result to H, who may decrypt the value and thus recover $F(x_1, x_2, \ldots, x_n)$. H may then send this decrypted value back to C. Based on the received evaluation of F, H may have a diagnosis or risk assessment for patient x. Thus, C never learns the patient x's data and H never learns the function F.

[0054] In any event, the processor **104***a* may interface with the memory **104***b* to access/execute the ML module **104***b***1**, the training module **104***b***2**, and/or any other instructions stored therein (e.g., an operating system). For example, the processor **104***a* may access the ML module **104***b***1** to execute the encrypted ML model **106***b* configured to receive encrypted data (e.g., user data **108***b***1**) as input and to output an encrypted classification based on the encrypted data, as discussed herein. The processor **104***a* may also access the training module **104***b***2** to update, re-train, and/or train the encrypted ML model **106***b* based on encrypted data (e.g., user data **108***b***1**) received from the user device **108** that the training module **104***b***2** may use, in conjunction with the encrypted classification output by the encrypted ML model **106***b* to generate/apply an encrypted update. It should be appreciated that one or more other applications are envisioned. Moreover, it should be understood that any processor (e.g., processor **104***a*, **108***a*), user interface (e.g., I/O interface **108***d*), and/or memory (e.g., memory **104***b*, **108***b*) referenced herein may include one or more processors, one or more user interfaces, and/or one or more memories.

[0055] The user device **108** and the model server **106** may execute the encryption instructions **108***b***2**, **106***c* to encrypt/decrypt the data transmitted to the central server **104** and/or received from the central server **104**. For example, the user device **108** may execute the encryption instructions **108***b***2** to encrypt the user data **108***b***1** using a first fully homomorphic encryption technique prior to transmitting the user data **108***b***1** to the central server **104** and to decrypt the encrypted classification received from the central server **104**. In certain embodiments, the encrypted data from the user may include: (i) a set of encrypted values, (ii) a set of additively homomorphic monotone functions, (iii) a set of indexes, (iv) a q_1-bit integer, or (v) an n-tuple q_1-bit integer.

[0056] Moreover, while various examples are described herein of performing FHE on user data, it will be appreciated that in any of the examples herein the FHE may be performed on a subset of the user data, wherein the subset is to be consumed in accordance with the methods and techniques herein. In some examples, the user data may be encrypted using an FHE technique, and the data sent to the ML model trained and/or encrypted in accordance with the techniques herein may include other data that is not encrypted by an FHE technique. In either case, the accompanying data not encrypted by an FHE technique may include metadata (such as, inter alia, data indicating that an FHE technique was used), data encrypted using non-FHE techniques, and/or non-encrypted data.

[0057] The model server **106** may execute the encryption instructions **106***c* to encrypt the ML model **106***b* using a second fully homomorphic encryption technique prior to transmitting the ML model **106***b* to the central server **104** and to decrypt any updates to the ML model **106***b* or other data received from the central server **104**. In certain embodiments, the first fully homomorphic encryption technique may be different from the second fully homomorphic encryption technique. Of course, it should be appreciated that, in some embodiments, the first fully homomorphic encryption technique may be identical to the second fully homomorphic encryption technique.

[0058] Regardless, the memories **104***b*, **108***b* may include one or more forms of volatile and/or non-volatile, fixed and/or removable memory, such as read-only memory (ROM), electronic programmable read-only memory (EPROM), random access memory (RAM), erasable electronic programmable read-only memory (EEPROM), and/or other hard drives, flash memory, MicroSD cards, and others.

[0059] The example system **120** may further include a user interface (e.g., I/O interface **108***d*) configured to present/receive information to/from a user. The user interface included as part of the I/O interface **108***d* may include a display screen (not shown) and I/O components (e.g., ports, capacitive or resistive touch sensitive input panels, keys, buttons, lights, LEDs). According to some embodiments, a user may access the example system **120** via the user interface to review outputs from the central server **104** (e.g., encrypted ML model **106***b*), make various selections, and/or otherwise interact with the example system **120**.

[0060] In some aspects, the example system **120** may perform the functionalities as discussed herein as part of a "cloud" network or may otherwise communicate with other hardware or software components within the cloud to send, retrieve, or otherwise analyze data. Thus, it should be appreciated that the example system **120** may be in the form of a distributed cluster of computers, servers, machines, or the like. In this implementation, a user may utilize the example system **120** as part of an on-demand cloud computing platform. Accordingly, when the user interfaces with the example system **120** (e.g., by interacting with an input component of the I/O interface **108***d*), the example system

120 may actually interface with one or more of a number of distributed computers, servers, machines, or the like, to facilitate the described functionalities.

[0061] In certain aspects, components of the example system **120** may communicate and interface with an external server and/or external devices (e.g., user device **108**) via a network(s) (e.g., network **116**). The network(s) **116** used to connect the components of the example system **120** to one another and/or components of the example system **120** to external server(s)/device(s) may support any type of data communication via any standard or technology (e.g., GSM, CDMA, TDMA, WCDMA, LTE, EDGE, OFDM, GPRS, EV-DO, UWB, Internet, IEEE 802 including Ethernet, WiMAX, Wi-Fi, Bluetooth, and others). Moreover, the external server(s)/device(s) may include a memory as well as a processor, and the memory may store an operating system capable of facilitating the functionalities as discussed herein as well as the ML module **104***b***1**, the training module **104***b***2**, encryption instructions **106***c*, **108***b***2**, and/or any other suitable instructions described herein or combinations thereof.

[0062] Additionally, it is to be appreciated that a computer program product in accordance with an aspect may include a computer usable storage medium (e.g., standard random access memory (RAM), an optical disc, a universal serial bus (USB) drive, or the like) having computer-readable program code embodied therein, wherein the computer-readable program code may be adapted to be executed by the processor(s) **104***a*, **108***a* (e.g., working in connection with the corresponding operating system(s)) to facilitate the functions as described herein. In this regard, the program code may be implemented in any desired language, and may be implemented as machine code, assembly code, byte code, interpretable source code or the like (e.g., via Golang, Python, Scala, C, C++, Java, Actionscript, Objective-C, Javascript, CSS, XML). In some aspects, the computer program product may be part of a cloud network of resources.

[0063] Moreover, to facilitate the encrypted data processing described herein, the training module **106***a*, **104***b***2** may be configured to utilize artificial intelligence (AI) and/or machine learning (ML) techniques to train the ML model **106***b*. The training module **106***a*, **104***b***2** may generally employ supervised or unsupervised machine learning techniques, which may be followed or used in conjunction with reinforced or reinforcement learning techniques. As noted above, in some embodiments, the central server **104**, the model server **106**, and/or other computing device(s) may be configured to implement machine learning, such that the central server **104**, the model server **106**, and/or other computing device(s) "learns" to analyze, organize, and/or process data through the ML model **106***b* without being explicitly programmed. Thus, the training module **106***a*, **104***b***2** may train the ML model **106***b* to automatically analyze encrypted data (e.g., user data **108***b***1**) from a user device (e.g., user device **108**), and thereby enable the central server **104**, the model server **106**, and/or other computing device(s) to automatically process encrypted data without requiring manual intervention.

[0064] In some embodiments, at least one of a plurality of machine learning methods and algorithms may be applied, which may include but are not limited to: linear or logistic regression, instance-based algorithms, regularization algorithms, decision trees, Bayesian networks, naïve Bayes algorithms, cluster analysis, association rule learning, neural networks (e.g., convolutional neural networks (CNN), deep learning neural networks, combined learning module or program), deep learning, combined learning, reinforced learning, dimensionality reduction, support vector machines, k-nearest neighbor algorithms, random forest algorithms, gradient boosting algorithms, Bayesian program learning, voice recognition and synthesis algorithms, image or object recognition, optical character recognition, natural language understanding, and/or other ML programs/algorithms either individually or in combination. In various embodiments, the implemented machine learning methods and algorithms are directed toward at least one of a plurality of categorizations of machine learning, such as supervised learning, unsupervised learning, and reinforcement learning. Of course, it should be appreciated that the machine learning models and techniques utilized herein may be or include any suitable models or techniques, such as variations CNNs (e.g., residual neural network or attention-based network) and/or recurrent neural networks (RNNs) (e.g., long short-term memory networks, gated recurrent units (GRUs), transformer networks, and/or other networks with/without an attention mechanism).

[0065] In one embodiment, the training module **106***a*, **104***b***2** may employ supervised learning techniques, which involves identifying patterns in existing data to make predictions about subsequently received data. Specifically, the training module **106***a*, **104***b***2** may "train" the ML model **106***b* using training data, which includes example inputs (e.g., reference/training encrypted data) and associated example outputs (e.g., corresponding training encrypted classifications). Based upon the training data, the training module **106***a*, **104***b***2** may cause the ML model **106***b* to generate a predictive function which maps outputs to inputs and may utilize the predictive function to generate machine learning outputs based upon data inputs. The example inputs and example outputs of the training data may include any of the data inputs or machine learning outputs described above. In the example embodiment, a processing element may be trained by providing it with a large sample of data with known characteristics or features.

[0066] In another embodiment, the training module **106***a*, **104***b***2** may employ unsupervised learning techniques, which involves finding meaningful relationships in unorganized data. Unlike supervised learning, unsupervised learning does not involve user-initiated training based upon example inputs with associated outputs. Rather, in unsupervised learning, the training module **106***a*, **104***b***2** may cause the ML model **106***b* to organize unlabeled data according to a relationship determined by at least one machine learning method/algorithm employed by the training module **106***a*, **104***b***2**. Unorganized data may include any combination of data inputs and/or machine learning outputs as described above.

[0067] In yet another embodiment, the training module **106***a*, **104***b***2** may employ reinforcement learning techniques, which involves optimizing outputs based upon feedback from a reward signal. Specifically, the training module **106***a*, **104***b***2** may cause the ML model **106***b* to receive a user-defined reward signal definition, receive a data input, utilize a decision-making model to generate a machine learning output based upon the data input, receive a reward signal based upon the reward signal definition and the machine learning output, and alter the decision-making

model so as to receive a stronger reward signal for subsequently generated machine learning outputs. Of course, other types of machine learning techniques may also be employed, including deep or combined learning techniques.

[0068] After training, the ML model **106***b* and/or other machine learning programs (or information generated by such machine learning programs) may be used to evaluate additional data. Such data may be and/or may be related to the encrypted data (e.g., user data **108***b***1**) and/or other data that was not included in the training dataset. The trained machine learning programs (or programs utilizing models, parameters, or other data produced through the training process) may accordingly be used for determining, assessing, analyzing, predicting, estimating, evaluating, or otherwise processing new data not included in the training dataset. Such trained machine learning programs (e.g., trained ML model **106***b*) may, therefore, be used to perform part or all of the analytical functions of the methods described elsewhere herein.

[0069] It is to be understood that supervised machine learning and/or unsupervised machine learning may also comprise retraining, relearning, or otherwise updating models with new, or different, information, which may include information received, ingested, generated, or otherwise used over time. Further, it should be appreciated that, as previously mentioned, the training module **106***a*, **104***b***2** may train the ML model **106***b* to output encrypted classifications and/or any other values or combinations thereof using artificial intelligence (e.g., a ML model of the ML model **106***b*) or, in alternative aspects, without using artificial intelligence.

[0070] Moreover, although the methods described elsewhere herein may not directly mention machine learning techniques, such methods may be read to include such machine learning for any determination or processing of data that may be accomplished using such techniques. In some aspects, such machine learning techniques may be implemented automatically upon occurrence of certain events or upon certain conditions being met. In any event, use of machine learning techniques, as described herein, may begin with training a machine learning program, or such techniques may begin with a previously trained machine learning program.

[0071] In particular, the training module **106***a* may train the ML model **106***b* using any suitable ML technique with a set of training encrypted data as inputs to output a set of training encrypted classifications of the set of training encrypted data. Further, once the training module **106***a* has trained the ML model **106***b*, the model server **106** may execute the encryption instructions **106***c* to encrypt the trained ML model **106***b* using a fully homomorphic encryption technique to create an encrypted, trained ML model **106***b*.

[0072] In certain embodiments, the model server **106** may only transmit the ML model **106***b* to the central server **104** after the ML model **106***b* has been trained by the training module **106***a* and encrypted in accordance with the encryption instructions **106***c*. Further, in certain embodiments, the encrypted, trained ML model **106***b* may be updated/re-trained/trained by the training module **104***b***2** of the central server **104**. In some embodiments, the fully homomorphic encryption techniques described herein to encrypt the ML model **106***b*, the user data **108***b***1**, and/or any other data may include a private key encryption technique.

[0073] However, in some embodiments, the encrypted, trained ML model **106***b* may be updated/re-trained/trained by the training module **106***a* of the model server **106**. For example, the central server **104** may execute the ML module **104***b***1** and generate/produce an encrypted update to the ML model **106***b* with a set of training encrypted data as inputs from one or more users (e.g., using one or more user devices **108**). The central server **104** may transmit the encrypted update to each user and may also transmit the encrypted update to the model server **106**, where the ML model **106***b* may be updated with the encrypted update by the training module **106***a*. In certain embodiments, the model server **106** may decrypt the encrypted update prior to training the ML model **106***b*. In some embodiments, the central server **104** may apply and/or otherwise update the ML model **106***b* using the encrypted update.

[0074] Furthermore, the model server **106** and/or the central server **104** may train the ML model **106***b* in a manner that enables new users to contribute to the training of the ML model **106***b* without affecting the encrypted data that has been used previously to train the ML model **106***b*. The central server **104** and/or model server **106** may receive from each user a training dataset encrypted using an independently generated key pair, in accordance with a multi-key, multi-hop fully homomorphic encryption technique. The central server **104** and/or model server **106** may use the training datasets to train the ML model **106***b* using a ML training technique. The central server **104** may then receive encrypted test datasets from each user and generate encrypted outputs for each encrypted test dataset using the ML model **106***b*. The central server **104** may then cause the user devices (e.g., user device **108**) of each of the users to participate in on-the-fly, multiparty computation to decrypt encrypted outputs by transmitting the encrypted outputs to each respective user. After this training process is completed, the central server **104** may receive, from a new user, a new training dataset encrypted using a new independently generated key pair, in accordance with the multi-key, multi-hop fully homomorphic encryption technique. The central server **104** and/or the model server **106** may update the ML model **106***b* with the new training dataset using a ML technique without re-encrypting the training datasets originally used to train the ML model **106***b*. Thus, the example system **120** may continually update/re-train the ML model **106***b* without requiring re-encryption/bootstrapping of the previous encrypted datasets, and without requiring any keys to be replaced/updated.

[0075] Additionally, or alternatively, the example system **120** may allow direct transmissions between the user device **108** and the model server **106** without compromising the security of data transmitted and/or stored on/by the user device **108** and/or the model server **106**. As an example, the example system **120** may enable a fully homomorphic private classification technique using a Naïve Bayes technique. Broadly speaking, such a classification technique may include a model owner/operator creating a learned/trained model using a Naïve Bayes technique, the model owner/operator encrypting the learned/trained model using a fully homomorphic private key encryption technique, and the data owner publishing the encrypted learned/trained model. The classification technique may further include a data owner calculating encrypted class probabilities based on their pri-

vate data, and the data owner determining a final classification using a privacy-preserving argmax protocol, as described herein.

[0076] More specifically, and as an initial matter, assume that a data owner (e.g., utilizing user device **108**) intends to classify their vector X which contains q features based on a learned model w owned by a classification model owner/operator (e.g., utilizing model server **106**). The group of classes G may include r distinct classes, $G_1 \ldots G_r$. Importantly, during this classification technique, the classification model owner/operator should learn no unnecessary information about the input data (e.g., user data **108***b***1**) provided by the data owner, and the data owner should learn nothing but the predicted class index of X.

[0077] Regardless, there may be a certain amount of information which the data owner must know in order to carry out the classification technique. Namely, the data owner must know that the data vector X has p features and that there are exactly r classes. However, the data owner should not be able to deduce any information about the conditional class probabilities associated with the q features or the r class probabilities.

[0078] Because the classification model owner/operator has already computed/generated a learned/trained model, the classification model owner/operator may prepare two tables. First, the classification model owner/operator may prepare table P represented as a column vector of degree r where $P_i=P_r(G=G_i)$, the prior probability on class G_i. Next the classification model owner/operator may prepare a table T as an r×q matrix where entry T_{ij} represents $P_r(X=X_i|G=G_i)$.

[0079] Accordingly, the private, fully homomorphic Naïve Bayes classification technique may proceed with the classification model owner/operator preparing the tables P and T and publishing their encryptions, $[\![P]\!]$ and $[\![T]\!]$. For each class G_i for i from 1 to r, the data owner may compute

$$[\![Pr(G_i|X)]\!] = [\![Pr(G_i)]\!] \cdot \prod_{j=1}^{p} [\![Pr(X_j|G_i)]\!] = \left[\!\!\left[P_i \cdot \prod_{j=1}^{p} T_{ij}\right]\!\!\right], \quad (1)$$

and the data owner may then compute

$$i = \underset{1 \le i \le r}{\operatorname{argmax}} [\![Pr(G_i|X)]\!] \quad (2)$$

using, for example, the private argmax protocol described herein in reference to FIG. **2**B.

[0080] Continuing this Naïve Bayes classification technique example, and as mentioned, there are two parties to consider when discussing the security of this protocol: the privacy of the data owner's information (e.g., user data **108***b***1**) as well as the privacy of the classification model owner's learned/trained model (e.g., ML model **106***b*). The privacy of the learned/trained model may be derived from the security of the encryption technique used. The privacy of the data owner's information, however, relates to the argmax protocol referenced in equation (2).

[0081] A private argmax protocol using public-key encryption allows the data owner to mask their value with random noise encrypted under the model owner's public key. However, this approach does not work with private-key encryption because the data owner cannot encrypt random noise. To overcome these challenges with traditional private-key encryption, the techniques described herein enable data owners to compute argmax in a private-key FHE protocol.

[0082] Such a private-key FHE protocol may include denoting $P_i=Pr(G_i|X)$, $\varepsilon_1=[\![P_i]\!]$, and the set of all encrypted probabilities E; as E. The protocol may include the data owner computing argmax by sending the set of encrypted probability values E to M. The model owner may then decrypt each value and transmit the index of the highest value to the data owner using an asymmetric encryption protocol to hide the value of the index from an cavesdropper. While the data owner has learned nothing about the encrypted model, the model owner learns not only which class the data owner's datapoint belongs to, but also the exact probabilities for each class.

[0083] Additionally, or alternatively, the data owner's device (e.g., user device **108**) may perform a permutation π on the class probabilities then transmit x (E) to the model owner. In this embodiment, the model owner's device (e.g., model server **106**) may decrypt the values, determine which value is largest, and transmit the index associated with that largest value to the data owner, who reverses the permutation x to determine the class (or classification). Importantly, in this embodiment, it is not necessary for the example system **120** to hide the value of the index sent to the data owner (e.g., to the user device **108**) from any cavesdroppers in this scenario, as the permutation a randomized the value. However, while the protocol of this embodiment prevents the model owner from determining which probability is associated with each class, the protocol does not prevent the model owner from learning the class probabilities themselves.

[0084] Accordingly, the data owner device (e.g., user device **108**) may attempt to hide the class probabilities by breaking down the computation of argmax to a series of comparisons. In particular, the user device **108** may randomly select two encrypted probabilities ε_i, $\varepsilon_j \in \varepsilon$ and transmit a value represented by

$$\mathcal{E}* = \mathcal{E}_i - \mathcal{E}_j = [\![P_i - P_j]\!] \quad (3)$$

to the model server **106**. When the model server **106** decrypts ε* the model server **106** may also determine the value $P_i-P_j \varepsilon(-p/, p/2]$, but may not have and/or otherwise determine either of the probabilities themselves. If this value is negative, the model server **106** may transmit the bit b=0 to the user device **108**, and the model server **106** may transmit the bit b=1 otherwise. In this manner, the model server **106** may be able to determine which value was larger based on the initial selection of P_i and P_j and may be able to rule out the smaller of the two values. The procedure may be repeated until only one value remains.

[0085] Because the value represented by equation (3) sent to the model server **106** may be randomly ordered, publishing the bit 0 or 1 will also appear random to any observer, such that this bit publishing/transmission does not compromise the privacy of the data owner. However, the model server **106** recovers some partial information about the user data **108***b***1** stored on the user device **108**. In particular, the model server **106** may recover a collection of r values representing a difference between pairs of the posterior probabilities for different i∈[r]. While permuting the elements keeps the model server **106** from determining which

pairs of elements correspond to which values, it is not impossible for the model server **106** to determine partial information about the user data **108***b***1** based on this information.

[0086] To avoid this, the user device **108** may mask the value given by the difference between each pair of probabilities. Namely, the user device **108** may take advantage of the fully homomorphic properties of this protocol by constructing a family F of additively homomorphic, monotone functions that commute with encryption. Briefly, a function f: R→R commutes with encryption if $f(\llbracket m \rrbracket)=\llbracket f(m) \rrbracket$. The additively homomorphic property of f guarantees that $f(m+n)=f(m)+f(n)$. With the family of functions F, the user device **108** may execute a privacy-preserving algorithm for argmax, as described herein, for example, in reference to FIG. **2**B.

[0087] During execution of such an algorithm the model server **106** may collect r values representing the result of a monotone function applied to the difference between random pairs of the posterior probabilities. Advantageously, the application of an unknown monotone function to this difference prevents the model server **106** from determining partial information from the decrypted value, thereby increasing the overall privacy/security of the user data **108***b***1**.

[0088] Of course, this argmax protocol and Naïve Bayes classification technique described in the previous example may represent a portion of the potential protocols, techniques, and/or other algorithms that are contemplated as part of the techniques described herein. For example, FIGS. **2**C and **2**E describe decision tree randomization/classification algorithms, FIG. **2**D describes a private node evaluation protocol, FIG. **2**E describes a third-party private search (TPPS) algorithm, and FIG. **2**G describes a TPPS over an encrypted database algorithm.

[0089] To provide more detail regarding many of the processes described herein (e.g., training the ML model **106***b*, encrypting data/models, generating encrypted classifications, etc.) FIGS. **2**A-**2**G illustrate example methods for classifying encrypted data using an encrypted ML model, in accordance with various aspects disclosed herein. These example methods may describe many different methods for encrypting and implementing various ML models to perform such encrypted data classification. However, it should be understood that these example methods are for the purposes of discussion only, and that other methods utilizing other ML techniques and/or encryption techniques are contemplated herein.

[0090] FIG. **2**A illustrates an example method **200** for classifying encrypted data using an encrypted ML model, in accordance with various aspects disclosed herein. For case of discussion, many of the various actions included in the example methods **200**, **210**, **220**, **230**, **240**, **250**, **260** may be described herein as performed by or with the use of a processor (e.g., processors **104***a*, **108***a*). However, it is to be appreciated that the various actions included in the example methods **200**, **210**, **220**, **230**, **240**, **250**, **260** may be performed by, for example, any suitable processing device (e.g., central server **104**, model server **106**, user device **108**) executing the training module **106***a*, **104***b***2**, the ML model **106***b*, **104***b***1**, the encryption instructions **106***c*, **108***b***2**, and/or other suitable modules/models/applications or combinations thereof.

[0091] At block **201**, the example method **200** optionally includes training a ML model with a set of training encrypted data as inputs to output a set of training encrypted classifications. The example method **200** may further optionally include encrypting the ML model using a fully homomorphic encryption technique to create an encrypted ML model (block **202**). The example method **200** may further include receiving encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique (block **203**).

[0092] In some examples, the example method **200** further includes receiving encrypted data from a user device and determining, at the recipient device (e.g., central server **104**, model server **106**), that the received data has been encrypted using an FHE scheme/technique or that at least a portion of the received data has been encrypted using an FHE technique. In some such examples, the user device may send metadata and/or other data identifying that an FHE technique was used to encrypt the received data. In certain examples, the recipient device may determine that an FHE technique was used to encrypt the received data by analyzing data included as part of the received data indicating that such an FHE technique was used.

[0093] The example method **200** may further include analyzing, by executing an encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data (block **204**). The example method **200** may further include transmitting the encrypted classification to a user computing device for decryption (block **205**).

[0094] In some embodiments, encrypting the ML model may further include encrypting one or more parameters of the ML model using the second fully homomorphic encryption technique. In some embodiments, the ML model may include (i) a Naïve Bayes model, (ii) a Decision Tree, and/or (iii) a Random Forest model.

[0095] In some embodiments, the fully homomorphic encryption technique further includes a private key encryption technique. In some embodiments, the first fully homomorphic technique is different from the second fully homomorphic encryption technique. In some embodiments, the encrypted data from the user includes: (i) a set of encrypted values, (ii) a set of additively homomorphic monotone functions, (iii) a set of indexes, (iv) a q_1-bit integer, and/or (v) an n-tuple q_1-bit integer.

[0096] In some embodiments, the example method **200** further includes receiving a second set of encrypted data from a second user that is encrypted in accordance with the second fully homomorphic encryption technique, and the second set of encrypted data includes (i) a feature index at a node, (ii) a database, and/or (iii) a private encryption key for the database.

[0097] In some embodiments, the example method **200** further includes producing an encrypted update to a ML model with a set of training encrypted data as inputs from one or more users; transmitting the encrypted update to each user of the one or more users for decryption; and updating the ML model using the encrypted update.

[0098] In some embodiments, the example method **200** further includes receiving, from each user of a plurality of users, a training dataset encrypted using an independently generated key pair, in accordance with a multi-key, multi-hop fully homomorphic encryption technique. The example

method **200** may further include training, with the training datasets, a ML model using a ML training technique and receiving encrypted test datasets from one or more users of the plurality of users. The example method **200** may further include generating, by executing the ML model, encrypted outputs for each encrypted test dataset, and causing user computing devices of each of the one or more users to participate in on-the-fly, multiparty computation to decrypt one or more respective encrypted outputs of the encrypted outputs by transmitting the encrypted outputs to each respective user of the one or more users.

[0099] In some embodiments, the example method **200** further includes receiving, from a new user, a new training dataset encrypted using a new independently generated key pair, in accordance with the multi-key, multi-hop fully homomorphic encryption technique; and updating the ML model with the new training dataset using the ML technique without re-encrypting the training datasets.

[0100] FIG. **2**B illustrates another example method **210** for classifying encrypted data using an encrypted ML model, in accordance with various aspects disclosed herein. Generally speaking, the example method **210** may represent a privacy preserving argmax algorithm for generating classifications of input user data.

[0101] At block **211**, the example method **210** includes receiving a set of indices/={1, 2, . . . , c}, a set of additively homomorphic monotone functions F, and a set of c values that may be encrypted by the model server **106** using a private key. The example method **210** may further include determining a random permutation x on/(block **212**). The example method **210** may further include randomly selecting a function $f \rightarrow F$ and determining the values $f(\varepsilon_{\pi(1)}$ and $f(\varepsilon_{\pi(2)})$ (block **213**). The example method **210** may further include evaluating

$$\mathcal{E}* = [\![f(P\pi(1) - P\pi(2))]\!] \quad (4)$$

based on the additive homomorphic properties of the encryption technique (block **214**).

[0102] The example method **210** may further include transmitting ε^* of equation (4) to a device storing a ML model (e.g., model server **106**) (block **215**). The example method **210** may further include decrypting ε^* of equation (4) based on the homomorphic encryption technique to recover $f(P_{\pi(1)}-P_{\pi(2)})$ (block **216**). The example method **210** may further include transmitting a representative bit (e.g., b=1 or b=0) to a user based on the recovered value (block **217**). Further, based on the representative bit transmitted to the user, the example method **210** may include removing one of $\pi(1)$ or $\pi(2)$ from/(block **218**).

[0103] It should be appreciated that the example method **210** may include executing any suitable number of the actions therein any suitable number of times and/or in any suitable combination. For example, any/all/some of the actions associated with blocks **211** through **218** may be iteratively performed any suitable number of times, such as until $|I| \leq 1$.

[0104] FIG. **2**C illustrates another example method **220** for classifying encrypted data using an encrypted ML model, in accordance with various aspects disclosed herein. Generally speaking, the example method **220** may represent a tree randomization algorithm for generating classifications of input user data.

[0105] At block **221**, the example method **220** may include receiving a decision tree T with n nodes and depth m. The example method **220** may further include generating a random permutation $\pi(n)$ (block **222**). The example method **220** may further include generating an n-tuple of random bits $b=\{b_i\}$ (block **223**). These random bits may generally denote whether a decision made by a node will be reversed.

[0106] If the node is reversed, the subtrees stemming from the children nodes may be swapped. In other words, and as indicated at block **224**, the example method **220** may further include iteratively switching values of $\pi(i \cdot 2^j+k-1)$ with the value of $\pi(i \cdot 2^j-k+2^{j-1}-1)$ based on the n-tuple of random bits. The example method **220** may further include creating a randomized decision tree T' by setting $T'(i)=T(\pi(i))$ (block **225**).

[0107] It should be appreciated that the example method **220** may include executing any suitable number of the actions therein any suitable number of times and/or in any suitable combination. For example, any/all/some of the actions associated with blocks **221** through **225** may be iteratively performed any suitable number of times, such as while i ranges from 1 to n.

[0108] FIG. **2**D illustrates another example method **230** for classifying encrypted data using an encrypted ML model, in accordance with various aspects disclosed herein. Generally speaking, the example method **230** may represent a private node evaluation protocol for generating classifications of input user data.

[0109] At block **231**, the example method **230** may include receiving an n-tuple of q_1-bit integers $x=(x_1, x_2, \ldots, x_n)$ and an index of a feature at node/and a database D for the node. The n values referenced in this example method **230** may correspond to, for example, n features used within the encrypted ML model. The example method **230** may further include determining a $\log_2(q_2)$-bit integer r (block **232**), which may correspond to parameters of a TTPS protocol, as described herein.

[0110] The example method **230** may further include determining $\bar{x}=(\overline{x_1}, \overline{x_2}, \ldots, x_n)=(x_1+r, x_2+r, \ldots, x_n+r)$ (block **233**). The example method **230** may further include performing an extension protocol to transfer the value $\overline{x_1}$ from the model owner (e.g., model server **106**) to the data owner (e.g., user device **108**) (block **234**). The value $I \in \{1, 2, \ldots, n\}$ may be the index of the decision variable on the node. The example method **230** may further include executing a private search protocol to determine whether x_1 is an element of D (i.e., whether $x_1 \in D$) (block **235**). For example, the example method **230** may include executing a TTPS to determine whether $x_1 \in D$.

[0111] The example method **230** may further include transmitting an output to a client device (e.g., user device **108**) based on whether x, is an element of D (block **236**). For example, the example method **230** may include transmitting an output of 1 if $x_1 \in D$. By contrast, the example method **230** may include transmitting an output of 0 if x_1 is not an element of D.

[0112] It should be appreciated that the example method **230** may include executing any suitable number of the actions therein any suitable number of times and/or in any suitable combination.

[0113] FIG. **2**E illustrates another example method **240** for classifying encrypted data using an encrypted ML model, in accordance with various aspects disclosed herein. Generally speaking, the example method **240** may represent a private decision tree classification algorithm for generating classifications of input user data.

[0114] Prior to performing the actions of the example method **240**, a data model owner may train (e.g., via model server **106**) a binary decision tree, T, under any suitable algorithm, such as the classification and regression tree (CART) algorithm. This binary decision tree T may be a complete tree of depth n consisting of 2^n-1 nodes. In certain embodiments, some of these nodes may be dummy nodes. Each decision node may be denoted by ti with a corresponding decision function f_i for i from 1 to $2^{n-1}-1$. The leaf nodes may be denoted by t_i with class assignment c_i for i from 2^n-1 to 2^n-1. The model owner may store this tree on a server (e.g., model server **106**), and the data owner may have a data point with n features, $x=(x_1, x_2, \ldots, x_n)$.

[0115] The data owner may desire to classify the data point using the tree, T, so the user device **108** may compute a binary decision function f_i on each decision node t_i to determine the tree path. The user device **108** may then also retrieve a class assignment c_i which corresponds to the determined tree path.

[0116] To achieve several of the privacy benefits described herein, each step in this procedure must be randomized, as illustrated in the example method **240** of FIG. **2**E. Thus, after receiving the n-tuple of q_1-bit integers $x=(x_1, x_2, \ldots, (n)$ and a trained decision tree T (block **241**), the example method **240** may include executing a tree randomization algorithm to generate a randomized decision tree T' (block **242**).

[0117] Furthermore, the data owner may require a privacy-preserving method of determining the relevant path in the randomized decision tree, T'. In certain embodiments, every node in the randomized decision tree T' must be queried to avoid enabling the model server (e.g., model server **106**) being able to determine the path followed based on the data owner queries, and thereafter leverage a colluding server to determine the relevant classification.

[0118] In any event, the example method **240** may further include executing a private node evaluation protocol to determine a classification value (block **243**). For example, the example method **240** may include utilizing a TPPS protocol to perform classification, and thereby yield the classification value. The example method **240** may further include performing an oblivious transfer to reveal the classification value (block **244**).

[0119] It should be appreciated that the example method **240** may include executing any suitable number of the actions therein any suitable number of times and/or in any suitable combination. For example, any/all/some of the actions associated with blocks **241** through **244** may be iteratively performed any suitable number of times, such as while node i is an element of the randomized decision tree, T'.

[0120] FIG. **2**F illustrates another example method **250** for classifying encrypted data using an encrypted ML model, in accordance with various aspects disclosed herein. Generally speaking, the example method **250** may represent a TPPS algorithm for generating classifications of input user data.

[0121] At block **251**, the example method **250** includes receiving a q_1-bit integer x and a database D. In certain embodiments, the database D may include entries of size at most q_1 bits each. The example method **250** may further include determining a random q_2-bit integer r (block **252**). The example method **250** may further include transmitting $\bar{x}=x+r$ to a first location (e.g., central server **104**, model server **106**) (block **253**).

[0122] The example method **250** may further include evaluating

$$y = \prod_{d \in D} (\bar{x} - d) \tag{5}$$

(block **254**). The example method **250** may further include performing secure modular reduction to determine $\bar{y}=y \pmod r$ based on equation (5) (block **255**). The example method **250** may further include transmitting $\bar{y}$ to a second location (e.g., user device **108**). Namely, if $\bar{y}=0$, then the example method **250** may include transmitting the value/classification 1 to the second location. By contrast, if $\bar{y}\neq 0$, then the example method **250** may include transmitting the value/classification 0 to the second location.

[0123] It should be appreciated that the example method **240** may include executing any suitable number of the actions therein any suitable number of times and/or in any suitable combination.

[0124] FIG. **2**G illustrates another example method **260** for classifying encrypted data using an encrypted ML model, in accordance with various aspects disclosed herein. Generally speaking, the example method **260** may represent a TPPS over an encrypted database algorithm for generating classifications of input user data.

[0125] At block **261**, the example method **260** may include receiving a q_1-bit integer x, an encrypted database $[\![D]\!]$ encrypted under a fully homomorphic encryption technique, and a key k for the database (block **261**). The example method **260** may further include determining a random q_2-bit integer r (block **262**). The example method **260** may further include transmitting $\bar{x}=x+r$ to a first location (e.g., central server **104**, model server **106**) (block **263**).

[0126] At block **264**, the example method **260** may include encrypting $\bar{x}$ with a private key. The example method **260** may further include transmitting the encrypted x, $[\![x]\!]$ to a second location (e.g., central server **104**, model server **106**) (block **265**). The processors at the second location may then determine

$$[\![y]\!] = \prod_{d \in D} ([\![\bar{x}]\!] - [\![d]\!]) = \left[\!\!\left[\prod_{d \in D} (\bar{x} - d) \right]\!\!\right] \tag{6}$$

(block **266**). The example method **260** may further include transmitting $[\![y]\!]$ from equation (6) to the first location (block **267**). The example method **260** may further include decrypting $[\![y]\!]$ to obtain y (block **268**). The example method **260** may further include performing secure modular reduction to return $\bar{y}=y \pmod r$ (block **269**). Namely, if $\bar{y}=0$, then the example method **260** may include transmitting the value/classification 1 to a third location (e.g., user device **108**). By contrast, if $\bar{y}\neq 0$, then the example method **260** may include transmitting the value/classification 0 to the third location.

[0127] It should be appreciated that the example method **260** may include executing any suitable number of the actions therein any suitable number of times and/or in any suitable combination.

Aspects of the Disclosure

[0128] 1. A system for classifying encrypted data using an encrypted machine learning (ML) model, the system comprising: a memory storing a set of computer-readable instructions including an encrypted ML model; and a processor interfacing with the memory, and configured to execute the set of computer-readable instructions to cause the system to: receive encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique, analyze, by executing the encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data, and transmit the encrypted classification to a user computing device for decryption.

[0129] 2. The system of aspect 1, wherein the set of computer-readable instructions, when executed, further cause the system to: train a ML model with a set of training encrypted data as inputs to output a set of training encrypted classifications of the set of training encrypted data; and encrypt the ML model using the second fully homomorphic encryption technique to create the encrypted ML model.

[0130] 3. The system of aspect 2, wherein encrypting the ML model further comprises: encrypt one or more parameters of the ML model using the second fully homomorphic encryption technique.

[0131] 4. The system of any of aspects 1 through 3, wherein the ML model comprises: (i) a Naïve Bayes model, (ii) a Decision Tree, or (iii) a Random Forest model.

[0132] 5. The system of any of aspects 1 through 4, wherein the fully homomorphic encryption technique further includes a private key encryption technique.

[0133] 6. The system of any of aspects 1 through 5, wherein the first fully homomorphic technique is different from the second fully homomorphic encryption technique.

[0134] 7. The system of any of aspects 1 through 6, wherein the encrypted data from the user comprises: (i) a set of encrypted values, (ii) a set of additively homomorphic monotone functions, (iii) a set of indexes, (iv) a q_1-bit integer, or (v) an n-tuple q_1-bit integer.

[0135] 8. The system of any of aspects 1 through 7, wherein the set of computer-readable instructions, when executed, further cause the system to: receive a second set of encrypted data from a second user that is encrypted in accordance with the second fully homomorphic encryption technique, and wherein the second set of encrypted data comprises: (i) a feature index at a node, (ii) a database, or (iii) a private encryption key for the database.

[0136] 9. The system of any of aspects 1 through 8, wherein the set of computer-readable instructions, when executed, further cause the system to: produce an encrypted update to a ML model with a set of training encrypted data as inputs from one or more users; transmit the encrypted update to each user of the one or more users for decryption; and update the ML model using the encrypted update.

[0137] 10. The system of any of aspects 1 through 9, wherein the set of computer-readable instructions, when executed, further cause the system to: receive, from each user of a plurality of users, a training dataset encrypted using an independently generated key pair, in accordance with a multi-key, multi-hop fully homomorphic encryption technique; train, with the training datasets, a ML model using a ML training technique; receive encrypted test datasets from one or more users of the plurality of users; generate, by executing the ML model, encrypted outputs for each encrypted test dataset; and cause user computing devices of each of the one or more users to participate in on-the-fly, multiparty computation to decrypt one or more respective encrypted outputs of the encrypted outputs by transmitting the encrypted outputs to each respective user of the one or more users.

[0138] 11. The system of aspect 10, wherein the set of computer-readable instructions, when executed, further cause the system to: receive, from a new user, a new training dataset encrypted using a new independently generated key pair, in accordance with the multi-key, multi-hop fully homomorphic encryption technique; and update the ML model with the new training dataset using the ML technique without re-encrypting the training datasets.

[0139] 12. A computer-implemented method for classifying encrypted data using an encrypted machine learning (ML) model, the method comprising: receiving, at one or more processors, encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique; analyzing, by the one or more processors executing an encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data; and transmitting, by the one or more processors, the encrypted classification to a user computing device for decryption.

[0140] 13. The computer-implemented method of aspect 12, further comprising: training, by the one or more processors, a ML model with a set of training encrypted data as inputs to output a set of training encrypted classifications of the set of training encrypted data; and encrypting, by the one or more processors, the ML model using the second fully homomorphic encryption technique to create the encrypted ML model.

[0141] 14. The computer-implemented method of aspect 13, wherein encrypting the ML model further comprises: encrypting, by the one or more processors, one or more parameters of the ML model using the second fully homomorphic encryption technique.

[0142] 15. The computer-implemented method of any of aspects 12 through 14, wherein the ML model comprises: (i) a Naïve Bayes model, (ii) a Decision Tree, or (iii) a Random Forest model.

[0143] 16. The computer-implemented method of any of aspects 12 through 15, wherein the fully homomorphic encryption technique further includes a private key encryption technique.

[0144] 17. The computer-implemented method of any of aspects 12 through 16, wherein the first fully homomorphic technique is different from the second fully homomorphic encryption technique.

[0145] 18. The computer-implemented method of any of aspects 12 through 17, wherein the encrypted data from the user comprises: (i) a set of encrypted values, (ii) a set of additively homomorphic monotone functions, (iii) a set of indexes, (iv) a q_1-bit integer, or (v) an n-tuple q_1-bit integer.

[0146] 19. The computer-implemented method of any of aspects 12 through 18, further comprising: receiving, at the

one or more processors, a second set of encrypted data from a second user that is encrypted in accordance with the second fully homomorphic encryption technique, and wherein the second set of encrypted data comprises: (i) a feature index at a node, (ii) a database, or (iii) a private encryption key for the database.

[0147] 20. A non-transitory computer readable medium comprising instructions for classifying encrypted data using an encrypted machine learning (ML) model that, when executed, may cause one or more processors to: receive encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique; analyze, by executing an encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data; and transmit the encrypted classification to a user computing device for decryption.

Additional Considerations

[0148] Throughout this specification, plural instances may implement components, operations, or structures described as a single instance. Although individual operations of one or more methods are illustrated and described as separate operations, one or more of the individual operations may be performed concurrently, and nothing requires that the operations be performed in the order illustrated. Structures and functionality presented as separate components in example configurations may be implemented as a combined structure or component. Similarly, structures and functionality presented as a single component may be implemented as separate components. These and other variations, modifications, additions, and improvements fall within the scope of the subject matter herein.

[0149] Additionally, certain embodiments are described herein as including logic or a number of routines, subroutines, applications, or instructions. These may constitute either software (e.g., code embodied on a machine-readable medium or in a transmission signal) or hardware. In hardware, the routines, etc., are tangible units capable of performing certain operations and may be configured or arranged in a certain manner. In example embodiments, one or more computer systems (e.g., a standalone, client or server computer system) or one or more hardware modules of a computer system (e.g., a processor or a group of processors) may be configured by software (e.g., an application or application portion) as a hardware module that operates to perform certain operations as described herein.

[0150] In various embodiments, a hardware module may be implemented mechanically or electronically. For example, a hardware module may comprise dedicated circuitry or logic that is permanently configured (e.g., as a special-purpose processor, such as a field programmable gate array (FPGA) or an application-specific integrated circuit (ASIC)) to perform certain operations. A hardware module may also comprise programmable logic or circuitry (e.g., as encompassed within a general-purpose processor or other programmable processor) that is temporarily configured by software to perform certain operations. It will be appreciated that the decision to implement a hardware module mechanically, in dedicated and permanently configured circuitry, or in temporarily configured circuitry (e.g., configured by software) may be driven by cost and time considerations.

[0151] Accordingly, the term "hardware module" should be understood to encompass a tangible entity, be that an entity that is physically constructed, permanently configured (e.g., hardwired), or temporarily configured (e.g., programmed) to operate in a certain manner or to perform certain operations described herein. Considering embodiments in which hardware modules are temporarily configured (e.g., programmed), each of the hardware modules need not be configured or instantiated at any one instance in time. For example, where the hardware modules comprise a general-purpose processor configured using software, the general-purpose processor may be configured as respective different hardware modules at different times. Software may accordingly configure a processor, for example, to constitute a particular hardware module at one instance of time and to constitute a different hardware module at a different instance of time.

[0152] Hardware modules can provide information to, and receive information from, other hardware modules. Accordingly, the described hardware modules may be regarded as being communicatively coupled. Where multiple of such hardware modules exist contemporancously, communications may be achieved through signal transmission (e.g., over appropriate circuits and buses) that connects the hardware modules. In embodiments in which multiple hardware modules are configured or instantiated at different times, communications between such hardware modules may be achieved, for example, through the storage and retrieval of information in memory structures to which the multiple hardware modules have access. For example, one hardware module may perform an operation and store the output of that operation in a memory device to which it is communicatively coupled. A further hardware module may then, at a later time, access the memory device to retrieve and process the stored output. Hardware modules may also initiate communications with input or output devices, and can operate on a resource (e.g., a collection of information).

[0153] The various operations of the example methods described herein may be performed, at least partially, by one or more processors that are temporarily configured (e.g., by software) or permanently configured to perform the relevant operations. Whether temporarily or permanently configured, such processors may constitute processor-implemented modules that operate to perform one or more operations or functions. The modules referred to herein may, in some example embodiments, comprise processor-implemented modules.

[0154] Similarly, the methods or routines described herein may be at least partially processor-implemented. For example, at least some of the operations of a method may be performed by one or more processors or processor-implemented hardware modules. The performance of certain of the operations may be distributed among the one or more processors, not only residing within a single machine, but also deployed across a number of machines. In some example embodiments, the processor or processors may be located in a single location (e.g., within a home environment, an office environment or as a server farm), while in other embodiments the processors may be distributed across a number of locations.

[0155] The performance of certain of the operations may be distributed among the one or more processors, not only residing within a single machine, but also deployed across a number of machines. In some example embodiments, the

one or more processors or processor-implemented modules may be located in a single geographic location (e.g., within a home environment, an office environment, or a server farm). In other example embodiments, the one or more processors or processor-implemented modules may be distributed across a number of geographic locations.

[0156] Unless specifically stated otherwise, discussions herein using words such as "processing," "computing," "calculating," "determining," "presenting," "displaying," or the like may refer to actions or processes of a machine (e.g., a computer) that manipulates or transforms data represented as physical (e.g., electronic, magnetic, or optical) quantities within one or more memories (e.g., volatile memory, non-volatile memory, or a combination thereof), registers, or other machine components that receive, store, transmit, or display information.

[0157] As used herein any reference to "one embodiment," "one aspect," "an aspect," or "an embodiment" means that a particular element, feature, structure, or characteristic described in connection with the embodiment is included in at least one embodiment/aspect. The appearances of the phrase "in one embodiment" or "in one aspect" in various places in the specification are not necessarily all referring to the same embodiment/aspect.

[0158] Some embodiments/aspects may be described using the expression "coupled" and "connected" along with their derivatives. For example, some embodiments/aspects may be described using the term "coupled" to indicate that two or more elements are in direct physical or electrical contact. The term "coupled," however, may also mean that two or more elements are not in direct contact with each other, but yet still co-operate or interact with each other. The embodiments/aspects are not limited in this context.

[0159] As used herein, the terms "comprises," "comprising," "includes," "including," "has," "having" or any other variation thereof, are intended to cover a non-exclusive inclusion. For example, a process, method, article, or apparatus that comprises a list of elements is not necessarily limited to only those elements but may include other elements not expressly listed or inherent to such process, method, article, or apparatus. Further, unless expressly stated to the contrary, "or" refers to an inclusive or and not to an exclusive or. For example, a condition A or B is satisfied by any one of the following: A is true (or present) and B is false (or not present), A is false (or not present) and B is true (or present), and both A and B are true (or present).

[0160] In addition, use of the "a" or "an" are employed to describe elements and components of the embodiments/aspects herein. This is done merely for convenience and to give a general sense of the description. This description, and the claims that follow, should be read to include one or at least one and the singular also includes the plural unless it is obvious that it is meant otherwise.

[0161] While the present invention has been described with reference to specific examples, which are intended to be illustrative only and not to be limiting of the invention, it will be apparent to those of ordinary skill in the art that changes, additions and/or deletions may be made to the disclosed embodiments/aspects without departing from the spirit and scope of the invention.

[0162] The foregoing description is given for clearness of understanding; and no unnecessary limitations should be understood therefrom, as modifications within the scope of the invention may be apparent to those having ordinary skill in the art.

What is claimed:

1. A system for classifying encrypted data using an encrypted machine learning (ML) model, the system comprising:

a memory storing a set of computer-readable instructions including an encrypted ML model; and

a processor interfacing with the memory, and configured to execute the set of computer-readable instructions to cause the system to:

receive encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique,

analyze, by executing the encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data, and

transmit the encrypted classification to a user computing device for decryption.

2. The system of claim **1**, wherein the set of computer-readable instructions, when executed, further cause the system to:

train a ML model with a set of training encrypted data as inputs to output a set of training encrypted classifications of the set of training encrypted data; and

encrypt the ML model using the second fully homomorphic encryption technique to create the encrypted ML model.

3. The system of claim **2**, wherein encrypting the ML model further comprises:

encrypt one or more parameters of the ML model using the second fully homomorphic encryption technique.

4. The system of claim **1**, wherein the encrypted ML model comprises: (i) a Naïve Bayes model, (ii) a Decision Tree, or (iii) a Random Forest model.

5. The system of claim **1**, wherein the first fully homomorphic encryption technique or the second fully homomorphic encryption technique further includes a private key encryption technique.

6. The system of claim **1**, wherein the first fully homomorphic encryption technique is different from the second fully homomorphic encryption technique.

7. The system of claim **1**, wherein the encrypted data from the user comprises: (i) a set of encrypted values, (ii) a set of additively homomorphic monotone functions, (iii) a set of indexes, (iv) a q_1-bit integer, or (v) an n-tuple q_1-bit integer.

8. The system of claim **1**, wherein the set of computer-readable instructions, when executed, further cause the system to:

receive a second set of encrypted data from a second user that is encrypted in accordance with the second fully homomorphic encryption technique, and

wherein the second set of encrypted data comprises: (i) a feature index at a node, (ii) a database, or (iii) a private encryption key for the database.

9. The system of claim **1**, wherein the set of computer-readable instructions, when executed, further cause the system to:

produce an encrypted update to a ML model with a set of training encrypted data as inputs from one or more users;

transmit the encrypted update to each user of the one or more users for decryption; and

update the ML model using the encrypted update.

10. The system of claim **1**, wherein the set of computer-readable instructions, when executed, further cause the system to:

receive, from each user of a plurality of users, a training dataset encrypted using an independently generated key pair, in accordance with a multi-key, multi-hop fully homomorphic encryption technique;

train, with the training datasets, a ML model using a ML training technique;

receive encrypted test datasets from one or more users of the plurality of users;

generate, by executing the ML model, encrypted outputs for each encrypted test dataset; and

cause user computing devices of each of the one or more users to participate in on-the-fly, multiparty computation to decrypt one or more respective encrypted outputs of the encrypted outputs by transmitting the encrypted outputs to each respective user of the one or more users.

11. The system of claim **10**, wherein the set of computer-readable instructions, when executed, further cause the system to:

receive, from a new user, a new training dataset encrypted using a new independently generated key pair, in accordance with the multi-key, multi-hop fully homomorphic encryption technique; and

update the ML model with the new training dataset using the ML training technique without re-encrypting the training datasets.

12. A computer-implemented method for classifying encrypted data using an encrypted machine learning (ML) model, the method comprising:

receiving, at one or more processors, encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique;

analyzing, by the one or more processors executing an encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data; and

transmitting, by the one or more processors, the encrypted classification to a user computing device for decryption.

13. The computer-implemented method of claim **12**, further comprising:

training, by the one or more processors, a ML model with a set of training encrypted data as inputs to output a set of training encrypted classifications of the set of training encrypted data; and

encrypting, by the one or more processors, the ML model using the second fully homomorphic encryption technique to create the encrypted ML model.

14. The computer-implemented method of claim **13**, wherein encrypting the ML model further comprises:

encrypting, by the one or more processors, one or more parameters of the ML model using the second fully homomorphic encryption technique.

15. The computer-implemented method of claim **12**, wherein the encrypted ML model comprises: (i) a Naïve Bayes model, (ii) a Decision Tree, or (iii) a Random Forest model.

16. The computer-implemented method of claim **12**, wherein the first fully homomorphic encryption technique or the second fully homomorphic encryption technique further includes a private key encryption technique.

17. The computer-implemented method of claim **12**, wherein the first fully homomorphic encryption technique is different from the second fully homomorphic encryption technique.

18. The computer-implemented method of claim **12**, wherein the encrypted data from the user comprises: (i) a set of encrypted values, (ii) a set of additively homomorphic monotone functions, (iii) a set of indexes, (iv) a q_1-bit integer, or (v) an n-tuple q_1-bit integer.

19. The computer-implemented method of claim **12**, further comprising:

receiving, at the one or more processors, a second set of encrypted data from a second user that is encrypted in accordance with the second fully homomorphic encryption technique, and

wherein the second set of encrypted data comprises: (i) a feature index at a node, (ii) a database, or (iii) a private encryption key for the database.

20. A non-transitory computer readable medium comprising instructions for classifying encrypted data using an encrypted machine learning (ML) model that, when executed, may cause one or more processors to:

receive encrypted data from a user that is encrypted in accordance with a first fully homomorphic encryption technique;

analyze, by executing an encrypted ML model that is encrypted in accordance with a second fully homomorphic encryption technique, the encrypted data to output an encrypted classification without decrypting the encrypted data; and

transmit the encrypted classification to a user computing device for decryption.

* * * * *