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Vinod et al.

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(45) **Date of Patent: Aug. 19, 2025**

(54) **CONTROLLING SEARCH AGENTS TO PERFORM SEARCH WITH NOISY OBSERVATIONS AND PROBABILISTIC GUARANTEES**

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(73) Assignee: **Mitsubishi Electric Research Laboratories, Inc.**

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 182 days.

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(22) Filed: **Sep. 14, 2023**

(65) **Prior Publication Data**

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Related U.S. Application Data

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(51) **Int. Cl.**

G05D 1/00	(2024.01)
G06N 20/00	(2019.01)

(52) **U.S. Cl.**
CPC ***G05D 1/0088*** (2013.01); ***G06N 20/00*** (2019.01)

(58) **Field of Classification Search**
CPC G05D 1/0088; G05D 1/2464; G05D 1/43; G05D 1/648; G05D 1/6987;
(Continued)

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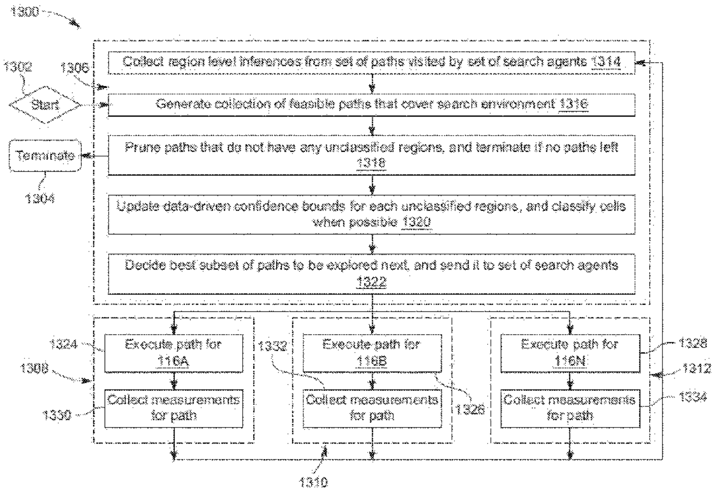
(57) **ABSTRACT**

A control system and a method for controlling search agents to perform search with noisy observations and probabilistic guarantees is provided. The control system collects confidence bounds of a probabilistic classification of at least one region within at least one path of a set of paths. The control system compares aggregations of the confidence bounds of the probabilistic classifications of each path of the set of paths based on the collected confidence bounds, a first path of a set of paths is selected, for visit by a first search agent based on the comparison. The control system commands the first search agent to visit the selected first path to collect measurements associated with each region within the selected first path. The control system updates the confidence bounds of the probabilistic classifications of each region within the selected first path based on the measurements associated with the corresponding regions.

20 Claims, 23 Drawing Sheets


1300
1302
1306
Start
Terminate
1304
Collect region level inferences from set of paths visited by set of search agents 1314
Generate collection of feasible paths that cover search environment 1316
Prune paths that do not have any unclassified regions, and terminate if no paths left 1318
Update data-driven confidence bounds for each unclassified regions, and classify cells when possible 1320
Decide best subset of paths to be explored next, and send it to set of search agents 1322
1324
1308
Execute path for 116A
Collect measurements for path
1330
1332
Execute path for 116B
Collect measurements for path
1326
Execute path for 116N
Collect measurements for path
1328
1312
1334
1310

(58) **Field of Classification Search**

CPC G05D 2105/55; G05D 2105/89; G05D 2107/21; G05D 2107/36; G06N 20/00; G06N 7/01

USPC 701/23

See application file for complete search history.

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2022/0026920	A1 *	1/2022	Ebrahimi Afrouzi	G06N 7/01
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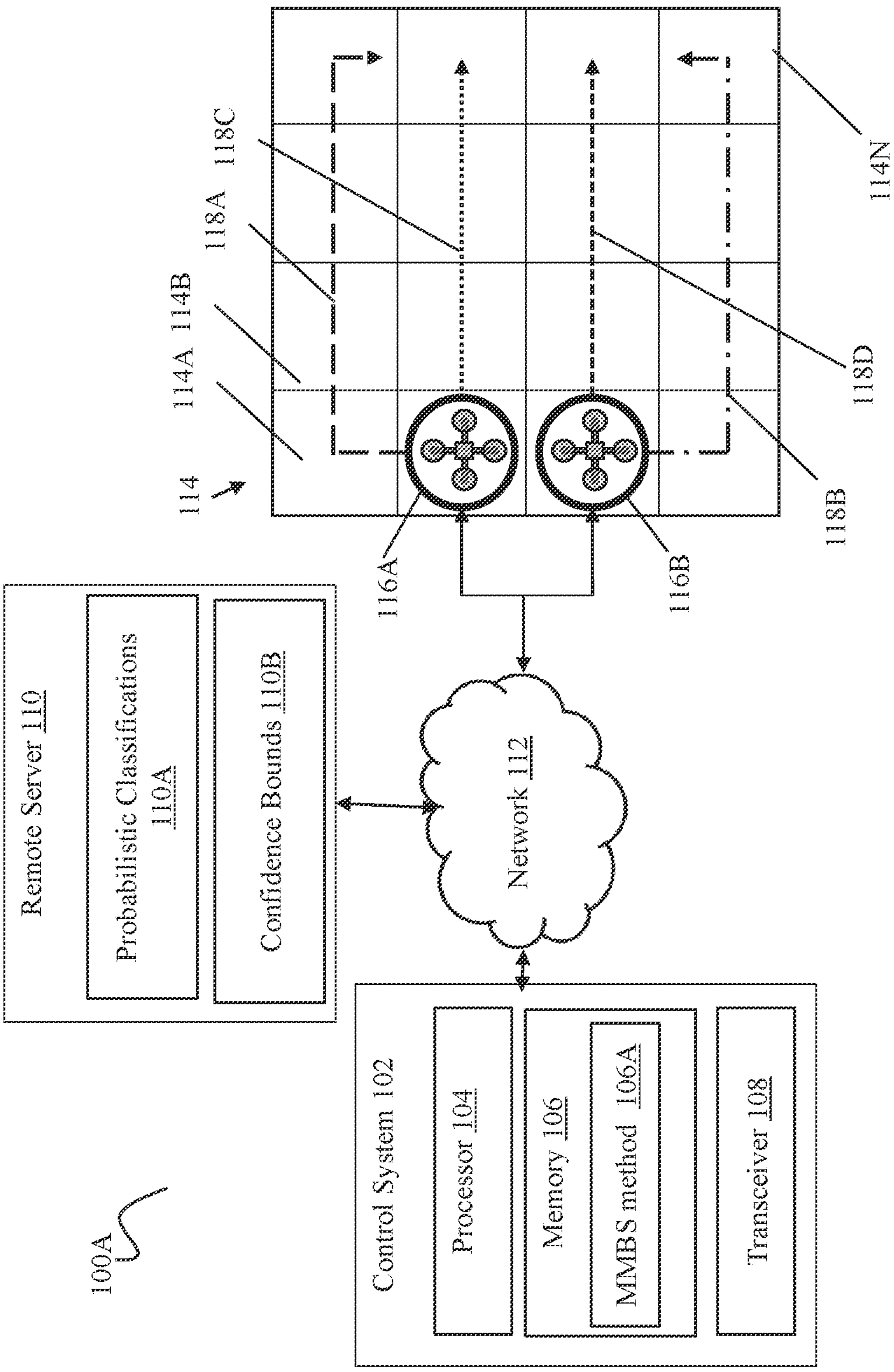

100A
Control System 102
Processor 104
Memory 106
MMBS method 106A
Transceiver 108
Remote Server 110
Probabilistic Classifications 110A
Confidence Bounds 110B
Network 112
114
114A
114B
118A
118C
114N
118D
118B
116A
116B

FIG. 1A

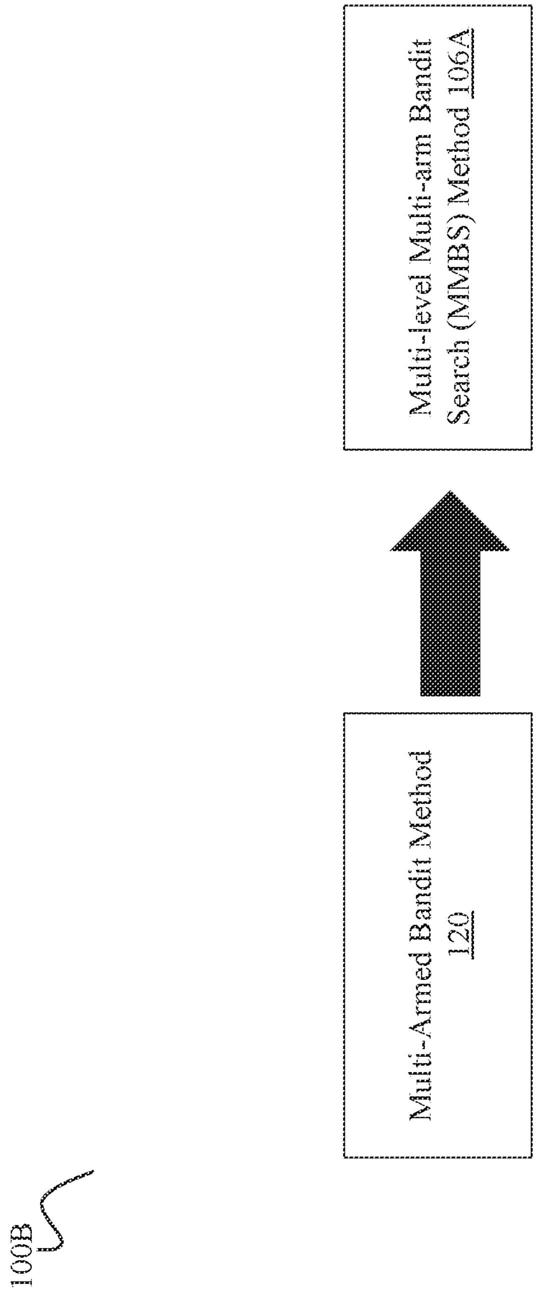

100B
Multi-Armed Bandit Method 120
Multi-level Multi-arm Bandit Search (MMBS) Method 106A

FIG. 1B

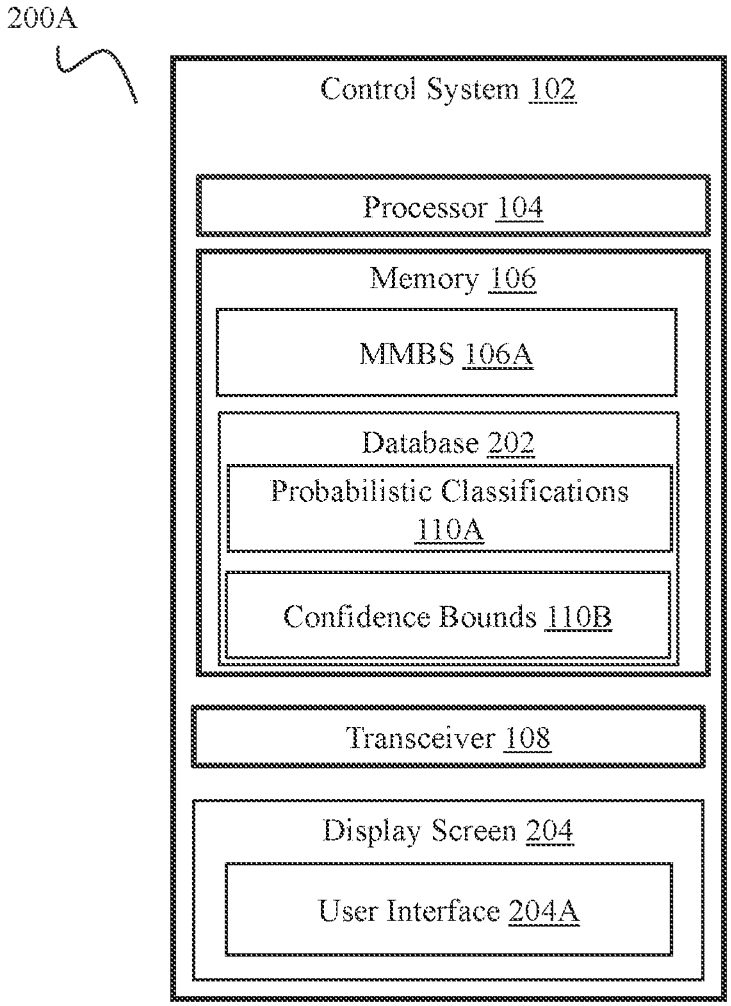

200A
Control System 102
Processor 104
Memory 106
MMBS 106A
Database 202
Probabilistic Classifications 110A
Confidence Bounds 110B
Transceiver 108
Display Screen 204
User Interface 204A

FIG. 2A

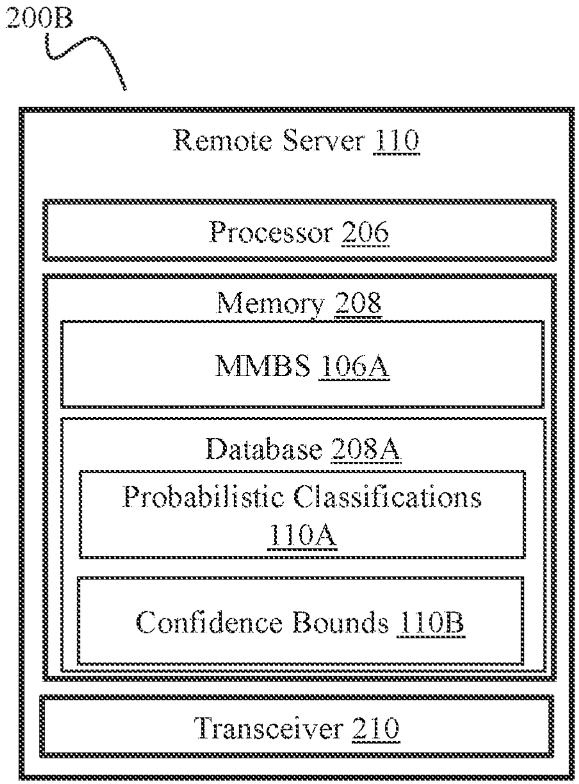

200B
Remote Server 110
Processor 206
Memory 208
MMBS 106A
Database 208A
Probabilistic Classifications 110A
Confidence Bounds 110B
Transceiver 210

FIG. 2B

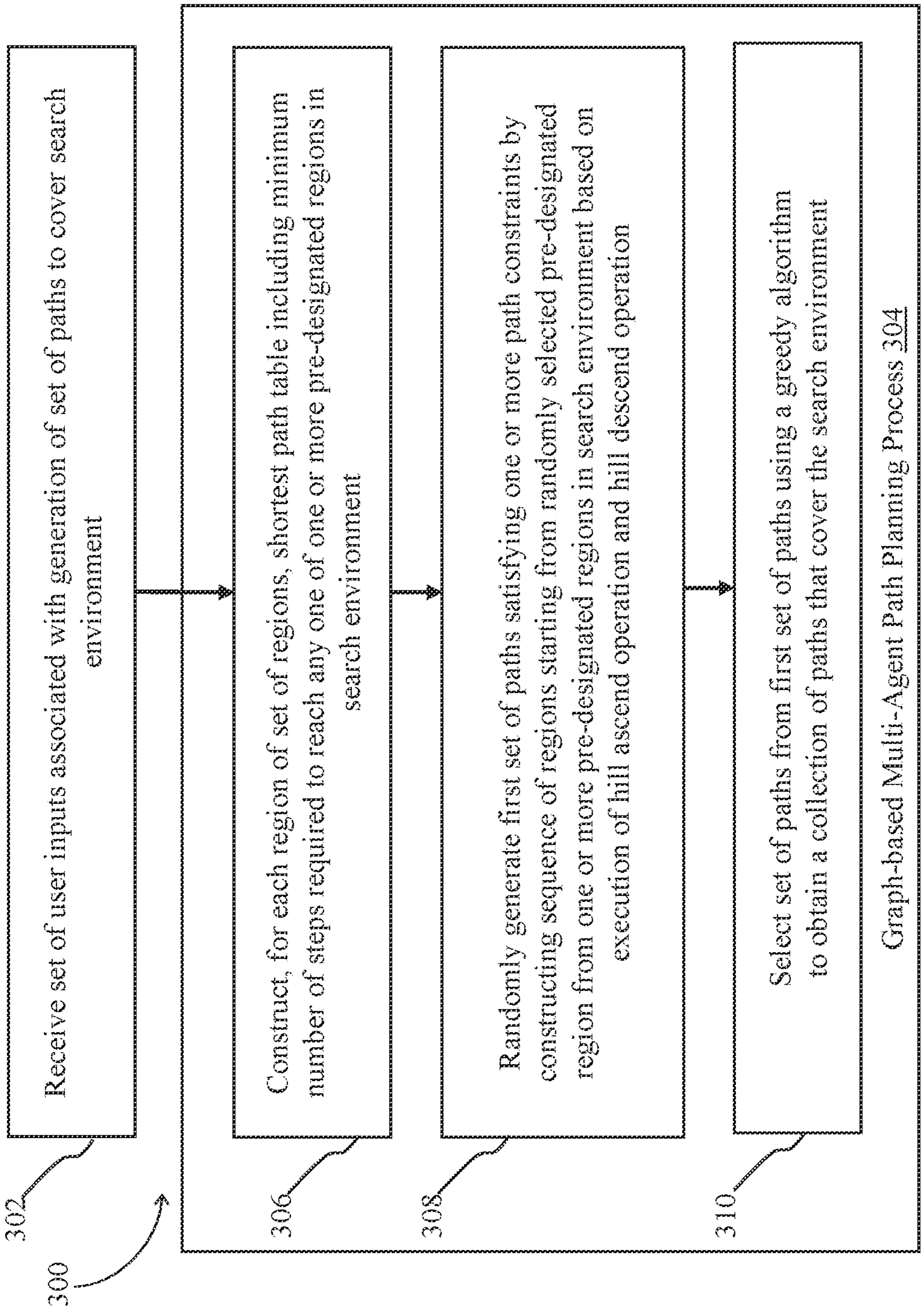

300
302
Receive set of user inputs associated with generation of set of paths to cover search environment
306
Construct, for each region of set of regions, shortest path table including minimum number of steps required to reach any one of one or more pre-designated regions in search environment
308
Randomly generate first set of paths satisfying one or more path constraints by constructing sequence of regions starting from randomly selected pre-designated region from one or more pre-designated regions in search environment based on execution of hill ascend operation and hill descend operation
310
Select set of paths from first set of paths using a greedy algorithm to obtain a collection of paths that cover the search environment
Graph-based Multi-Agent Path Planning Process 304

FIG. 3

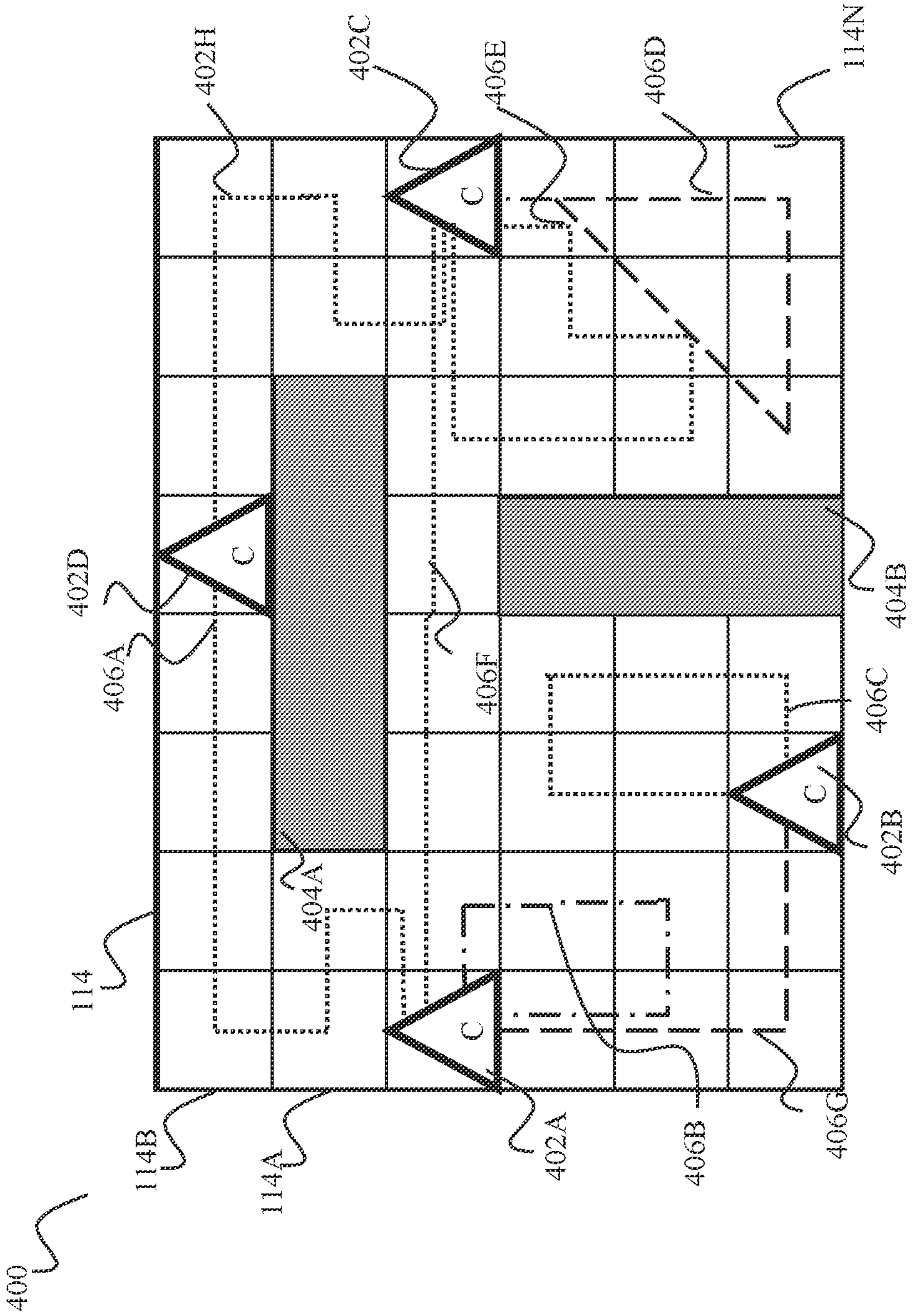

400
114
114B
114A
402A
406B
406G
404A
406A
402D
406F
402B
406C
404B
402H
402C
406E
406D
114N
C
C
C
C

FIG. 4

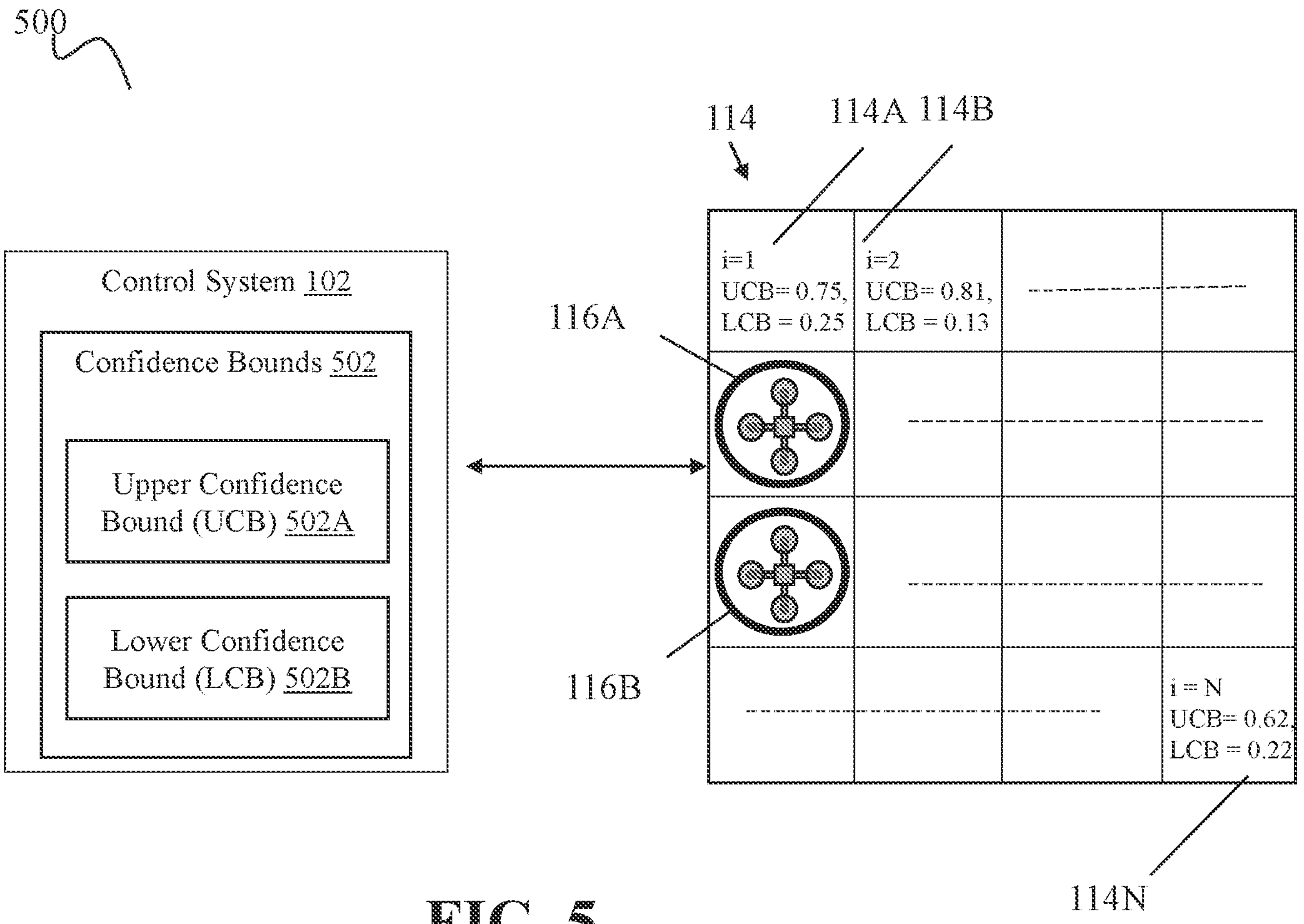

500
Control System 102
Confidence Bounds 502
Upper Confidence Bound (UCB) 502A
Lower Confidence Bound (LCB) 502B
114
114A 114B
i=1
UCB= 0.75,
LCB = 0.25
i=2
UCB= 0.81,
LCB = 0.13
116A
116B
i = N
UCB= 0.62,
LCB = 0.22
114N

FIG. 5

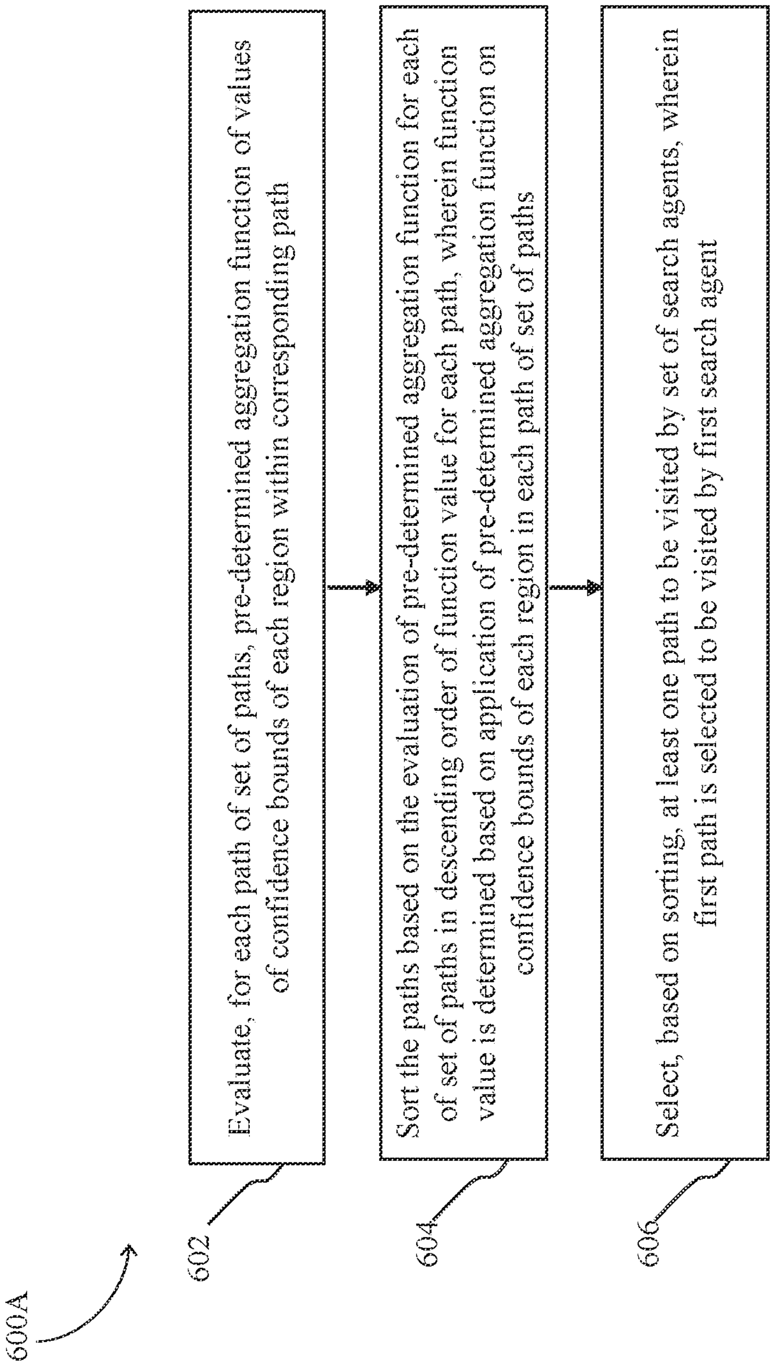

600A
602
Evaluate, for each path of set of paths, pre-determined aggregation function of values of confidence bounds of each region within corresponding path
604
Sort the paths based on the evaluation of pre-determined aggregation function for each of set of paths in descending order of function value for each path, wherein function value is determined based on application of pre-determined aggregation function on confidence bounds of each region in each path of set of paths
606
Select, based on sorting, at least one path to be visited by set of search agents, wherein first path is selected to be visited by first search agent

FIG. 6A

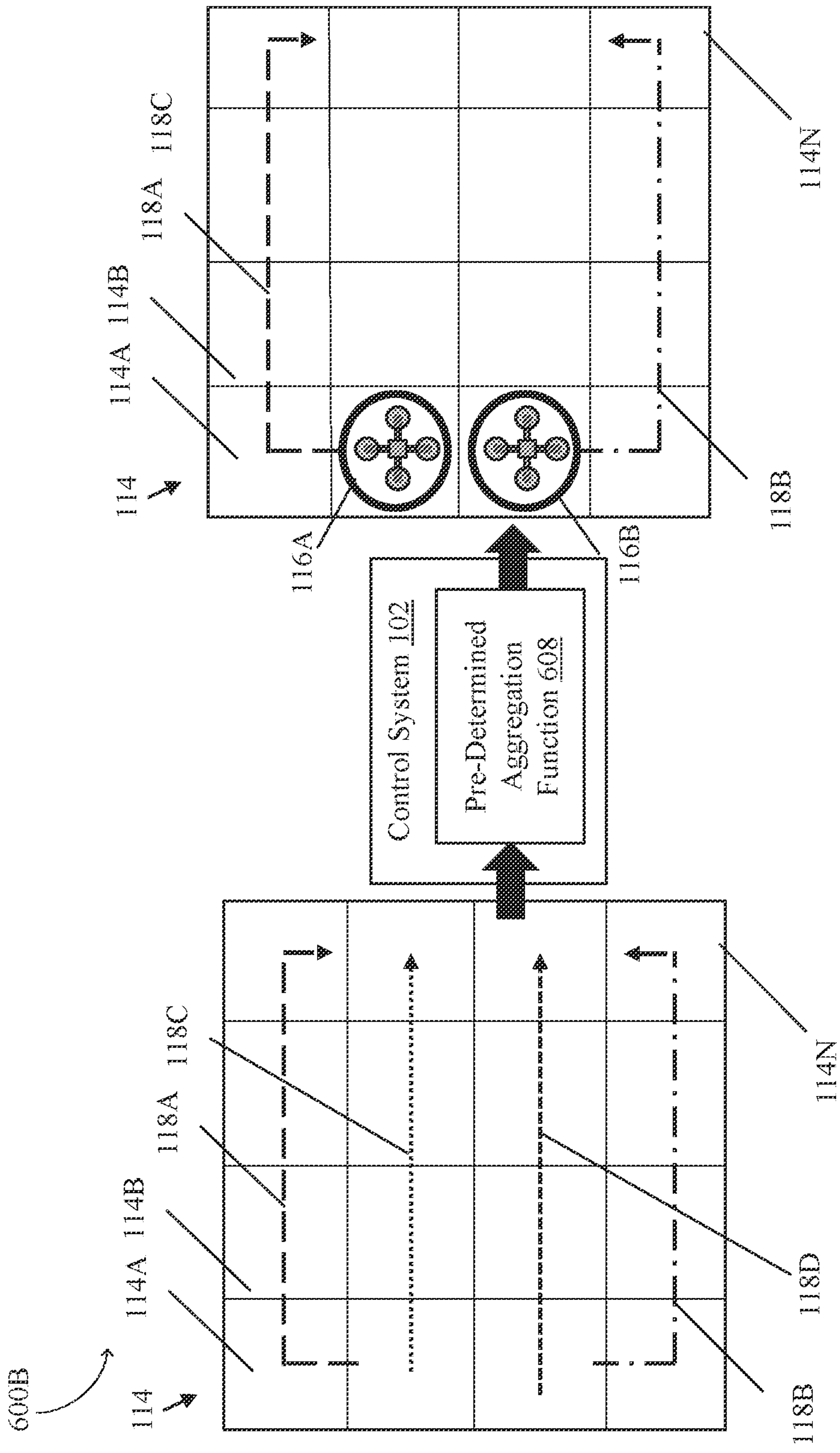

600B
114
114A
114B
118A
118C
114N
118D
118B
Control System 102
Pre-Determined Aggregation Function 608
116A
116B
114
114A
114B
118A
118C
114N
118B

FIG. 6B

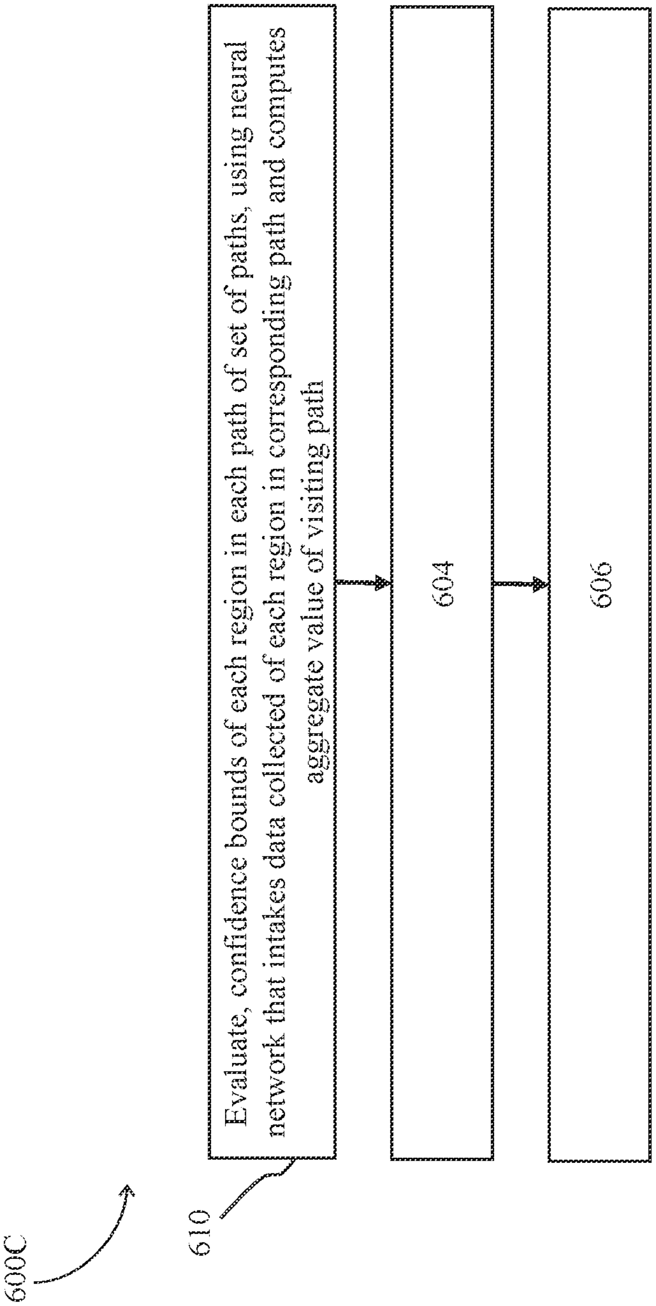

600C
610
Evaluate, confidence bounds of each region in each path of set of paths, using neural network that intakes data collected of each region in corresponding path and computes aggregate value of visiting path
604
606

FIG. 6C

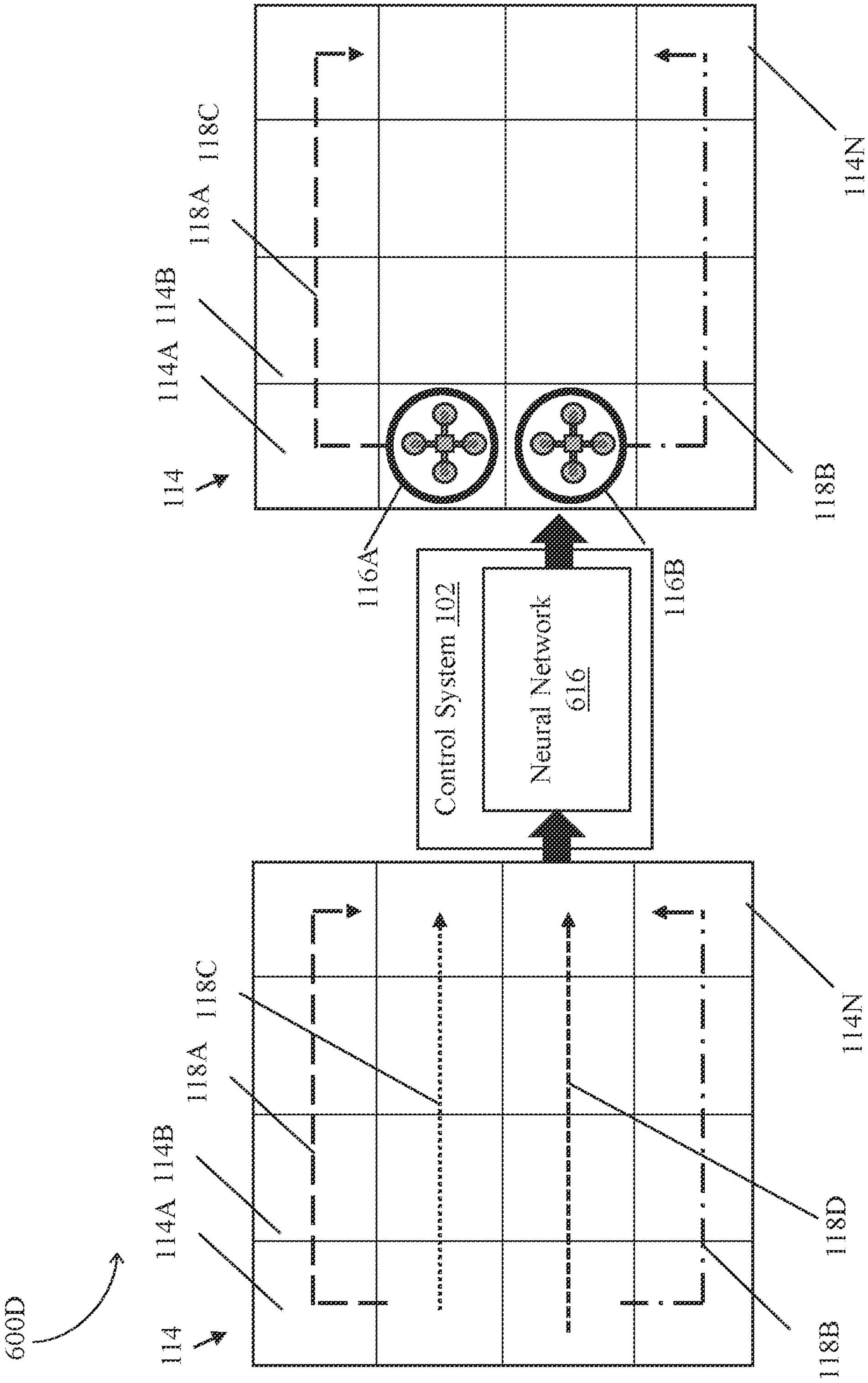

600D
114
114A
114B
118A
118C
114N
118B
118D
Control System 102
Neural Network 616
114
114A
114B
118A
118C
114N
116A
116B
118B

FIG. 6D

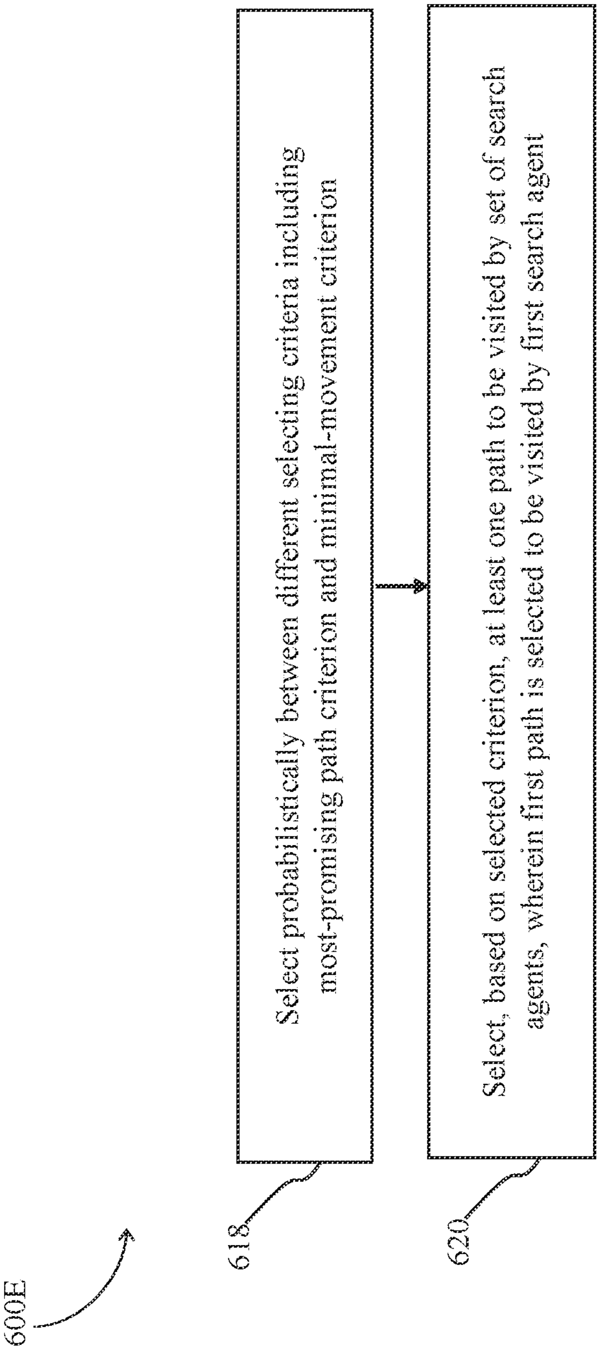

600E
618
Select probabilistically between different selecting criteria including most-promising path criterion and minimal-movement criterion
620
Select, based on selected criterion, at least one path to be visited by set of search agents, wherein first path is selected to be visited by first search agent

FIG. 6E

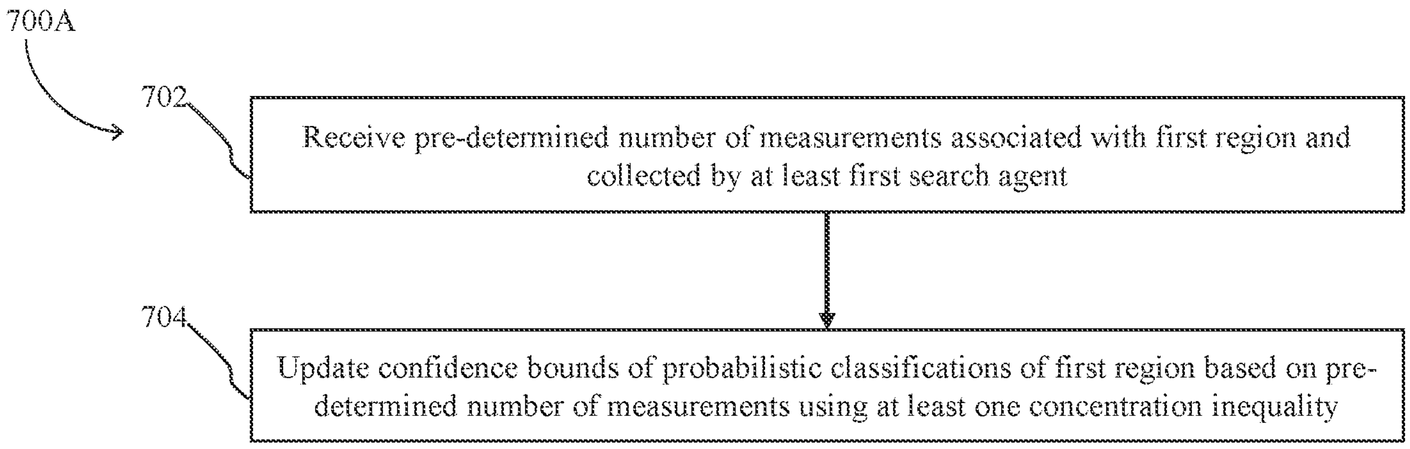

700A
702
Receive pre-determined number of measurements associated with first region and collected by at least first search agent
704
Update confidence bounds of probabilistic classifications of first region based on pre-determined number of measurements using at least one concentration inequality

FIG. 7A

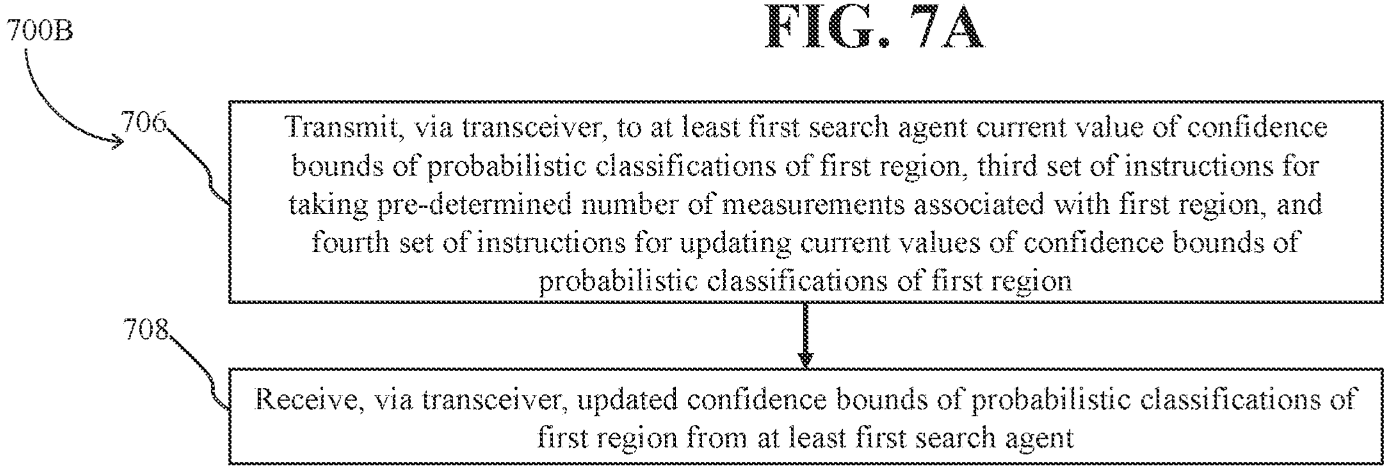

700B
706
Transmit, via transceiver, to at least first search agent current value of confidence bounds of probabilistic classifications of first region, third set of instructions for taking pre-determined number of measurements associated with first region, and fourth set of instructions for updating current values of confidence bounds of probabilistic classifications of first region
708
Receive, via transceiver, updated confidence bounds of probabilistic classifications of first region from at least first search agent

FIG. 7B

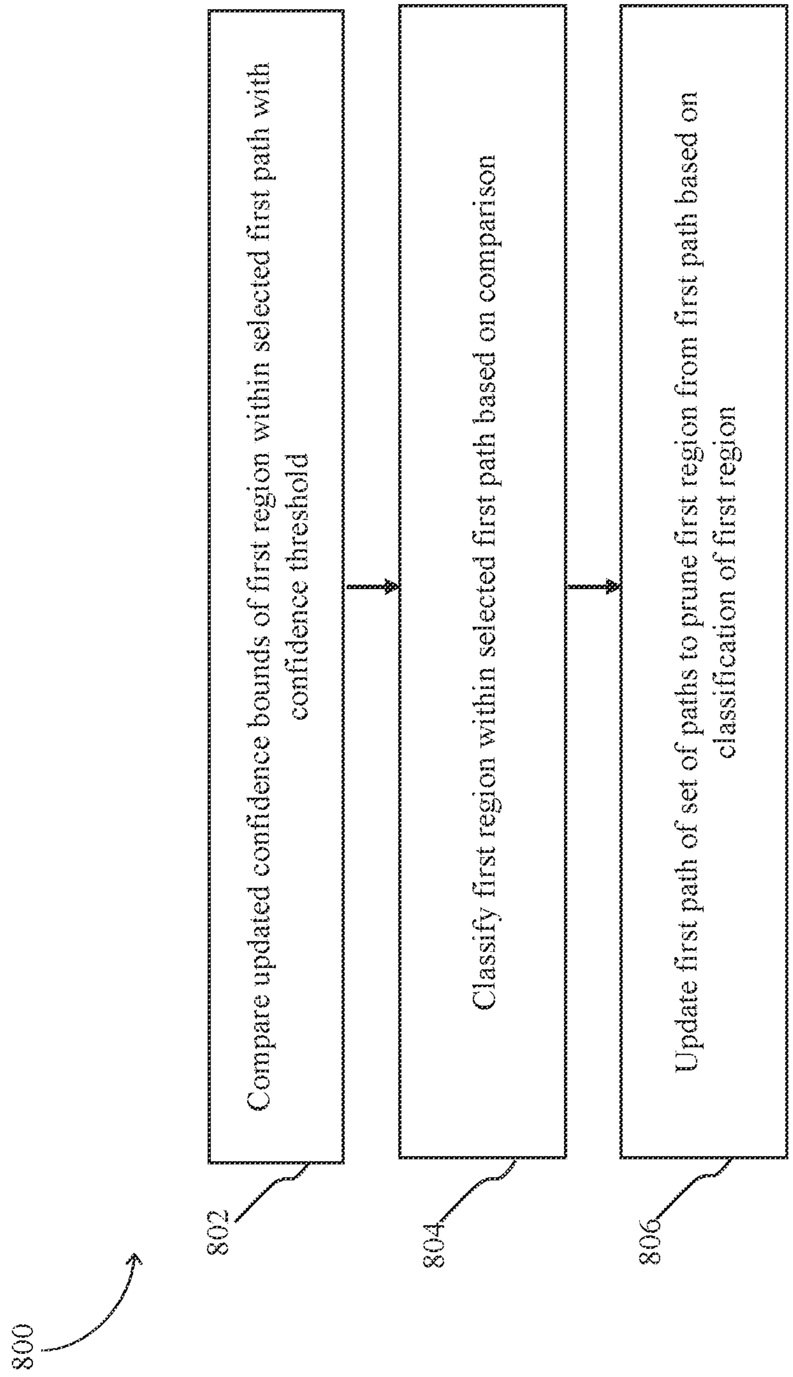

800
802
Compare updated confidence bounds of first region within selected first path with confidence threshold
804
Classify first region within selected first path based on comparison
806
Update first path of set of paths to prune first region from first path based on classification of first region

FIG. 8

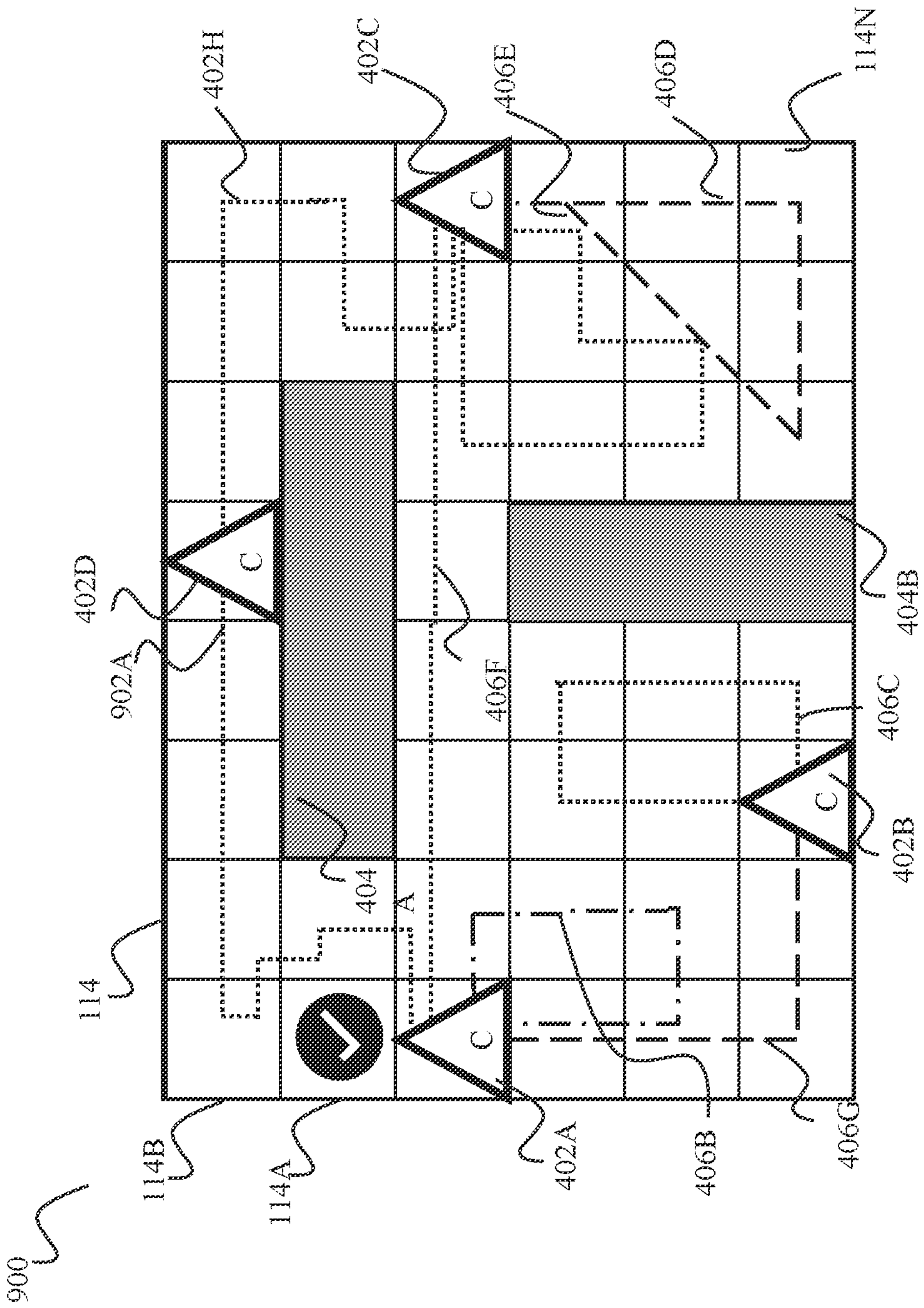

900
114B
114A
114
402D
902A
402H
402C
406E
406D
114N
C
C
C
C
A
404
406F
404B
406C
402B
406G
406B
402A

FIG. 9

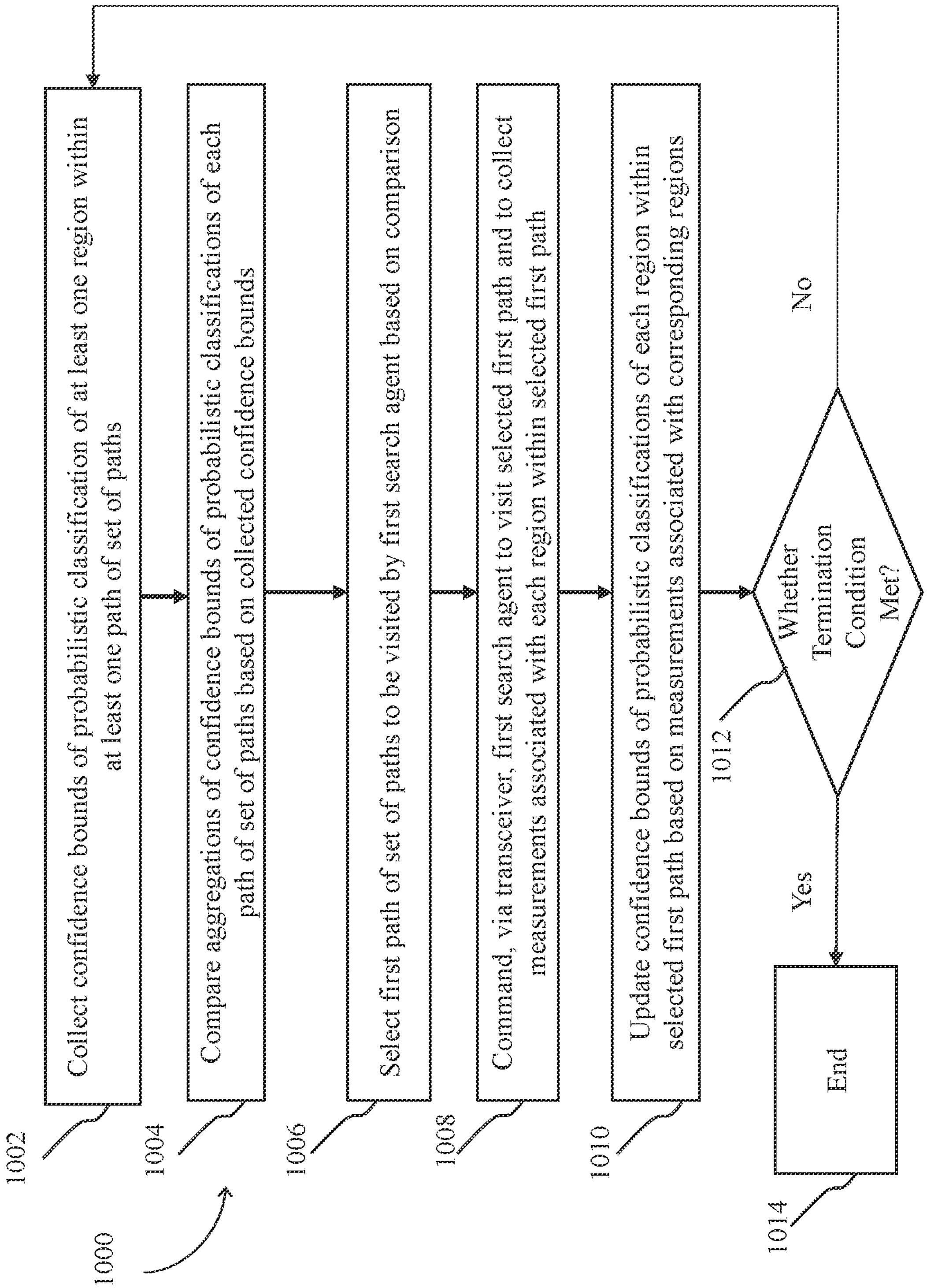

1000
1002
Collect confidence bounds of probabilistic classification of at least one region within at least one path of set of paths
1004
Compare aggregations of confidence bounds of probabilistic classifications of each path of set of paths based on collected confidence bounds
1006
Select first path of set of paths to be visited by first search agent based on comparison
1008
Command, via transceiver, first search agent to visit selected first path and to collect measurements associated with each region within selected first path
1010
Update confidence bounds of probabilistic classifications of each region within selected first path based on measurements associated with corresponding regions
1012
Whether Termination Condition Met?
No
Yes
1014
End

FIG. 10

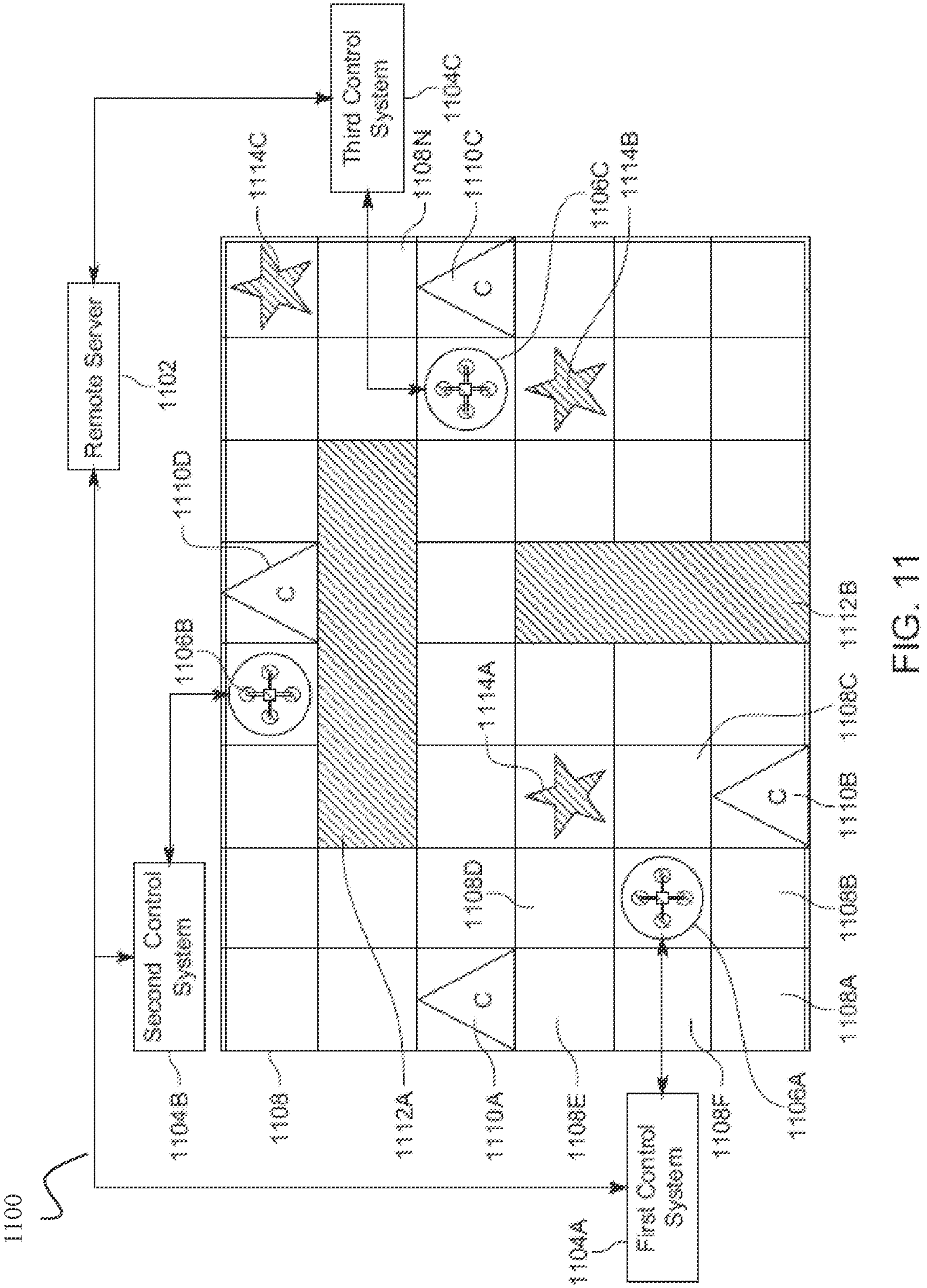

1100
Remote Server
1102
Third Control System
1104C
Second Control System
1104B
First Control System
1104A
1108
1108A
1108B
1108C
1108D
1108E
1108F
1108N
1106A
1106B
1106C
1110A
1110B
1110C
1110D
C
C
C
C
1112A
1112B
1114A
1114B
1114C

FIG. 11

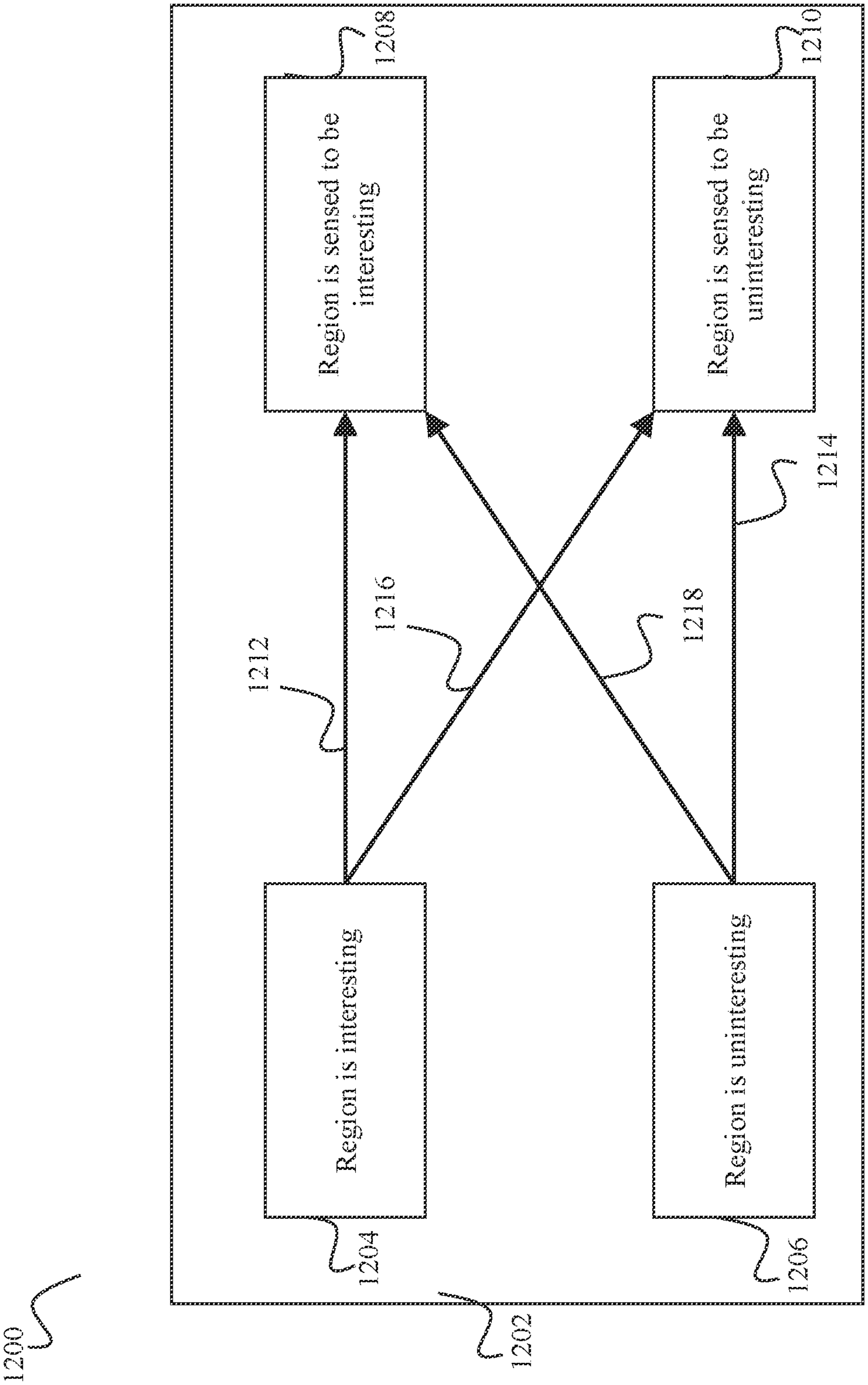

1200
1202
Region is interesting
1204
Region is uninteresting
1206
Region is sensed to be interesting
1208
Region is sensed to be uninteresting
1210
1212
1214
1216
1218

FIG. 12

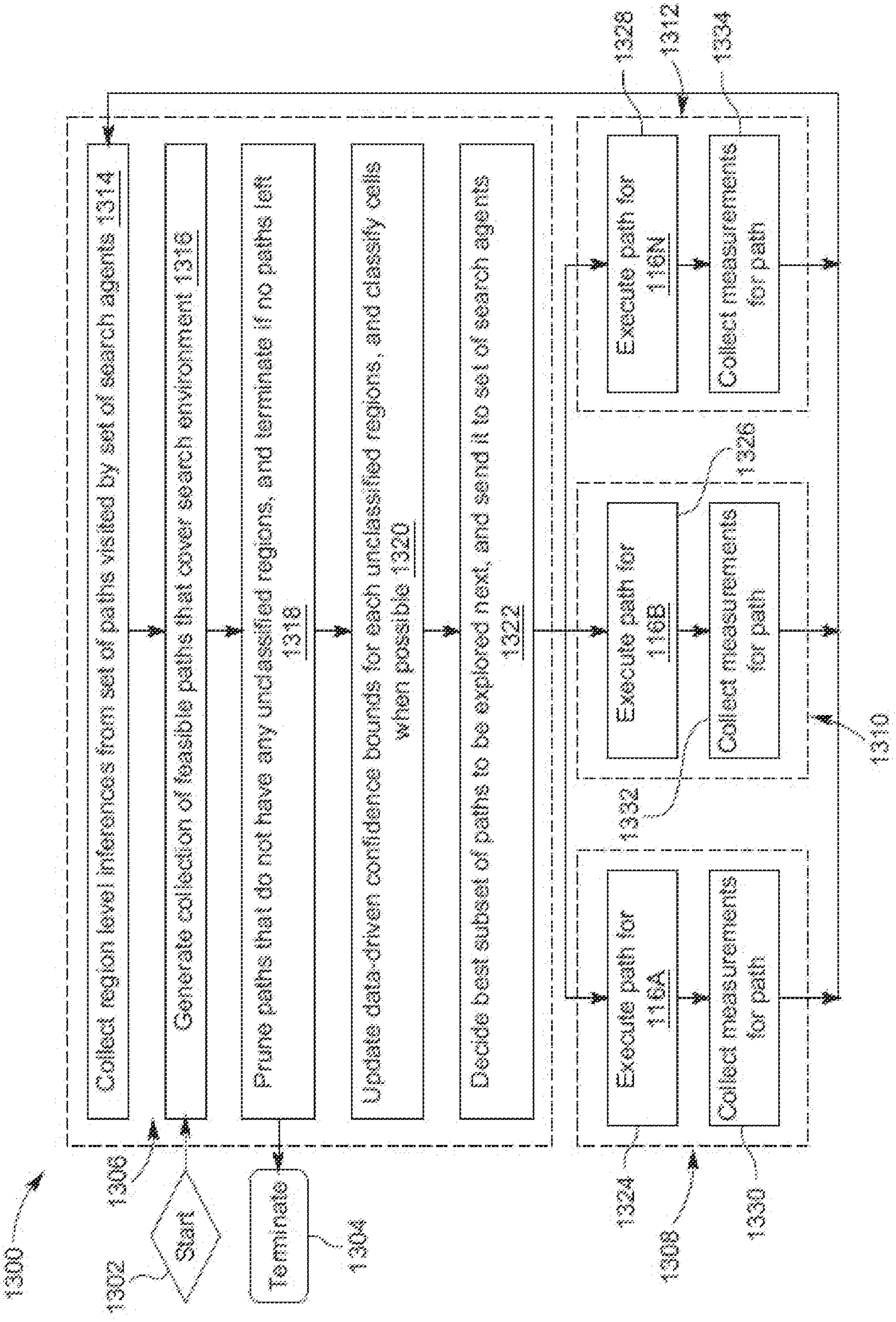

1300
1302
Start
1304
Terminate
1306
Collect region level inferences from set of paths visited by set of search agents 1314
Generate collection of feasible paths that cover search environment 1316
Prune paths that do not have any unclassified regions, and terminate if no paths left 1318
Update data-driven confidence bounds for each unclassified regions, and classify cells when possible 1320
Decide best subset of paths to be explored next, and send it to set of search agents 1322
1308
1324
Execute path for 116A
1330
Collect measurements for path
1310
1326
Execute path for 116B
1332
Collect measurements for path
1312
1328
Execute path for 116N
1334
Collect measurements for path

FIG. 13

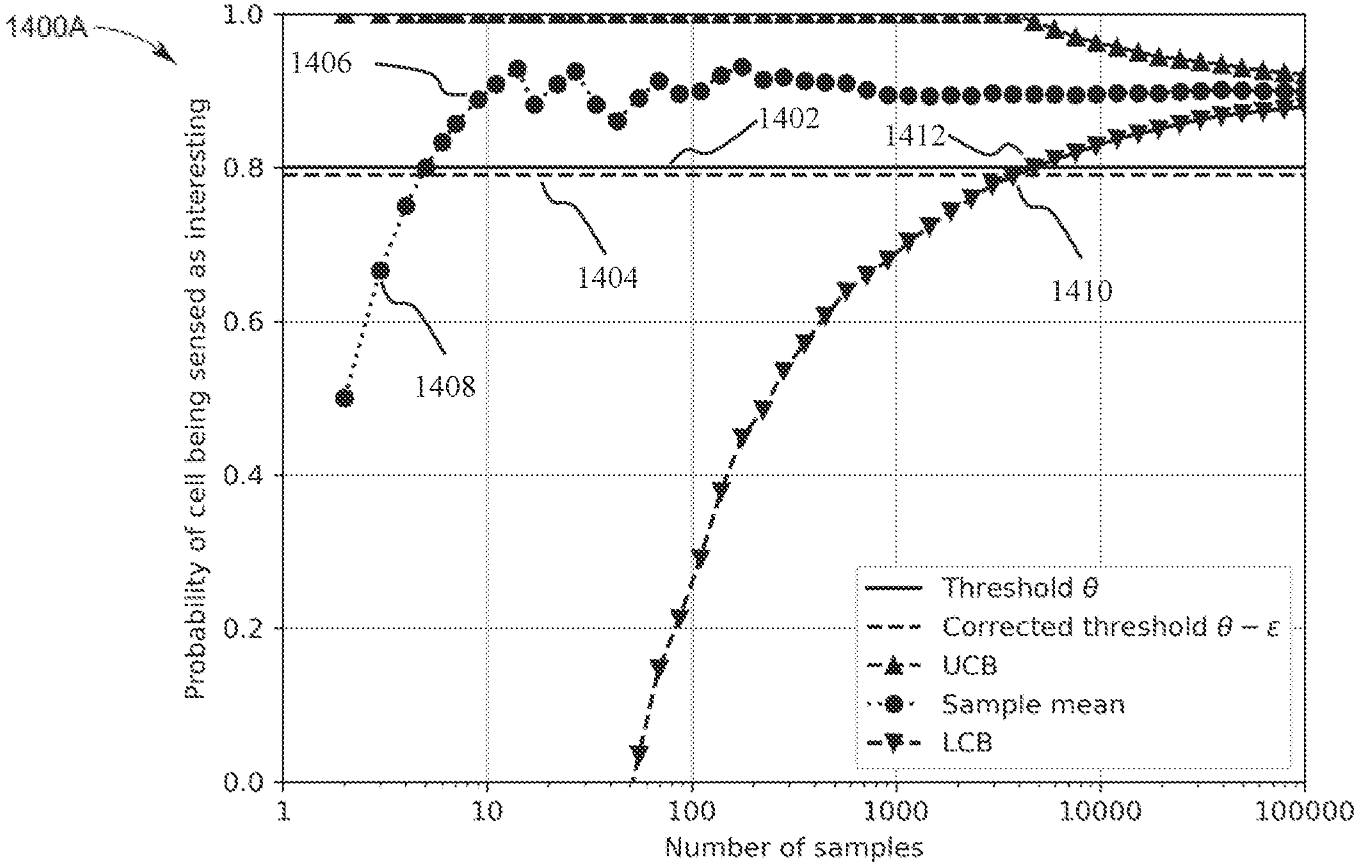

1400A
1406
1402
1412
1404
1410
1408
1.0
0.8
0.6
0.4
0.2
0.0
Probability of cell being sensed as interesting
1
10
100
1000
10000
100000
Number of samples
Threshold θ
Corrected threshold θ − ε
UCB
Sample mean
LCB

FIG. 14A

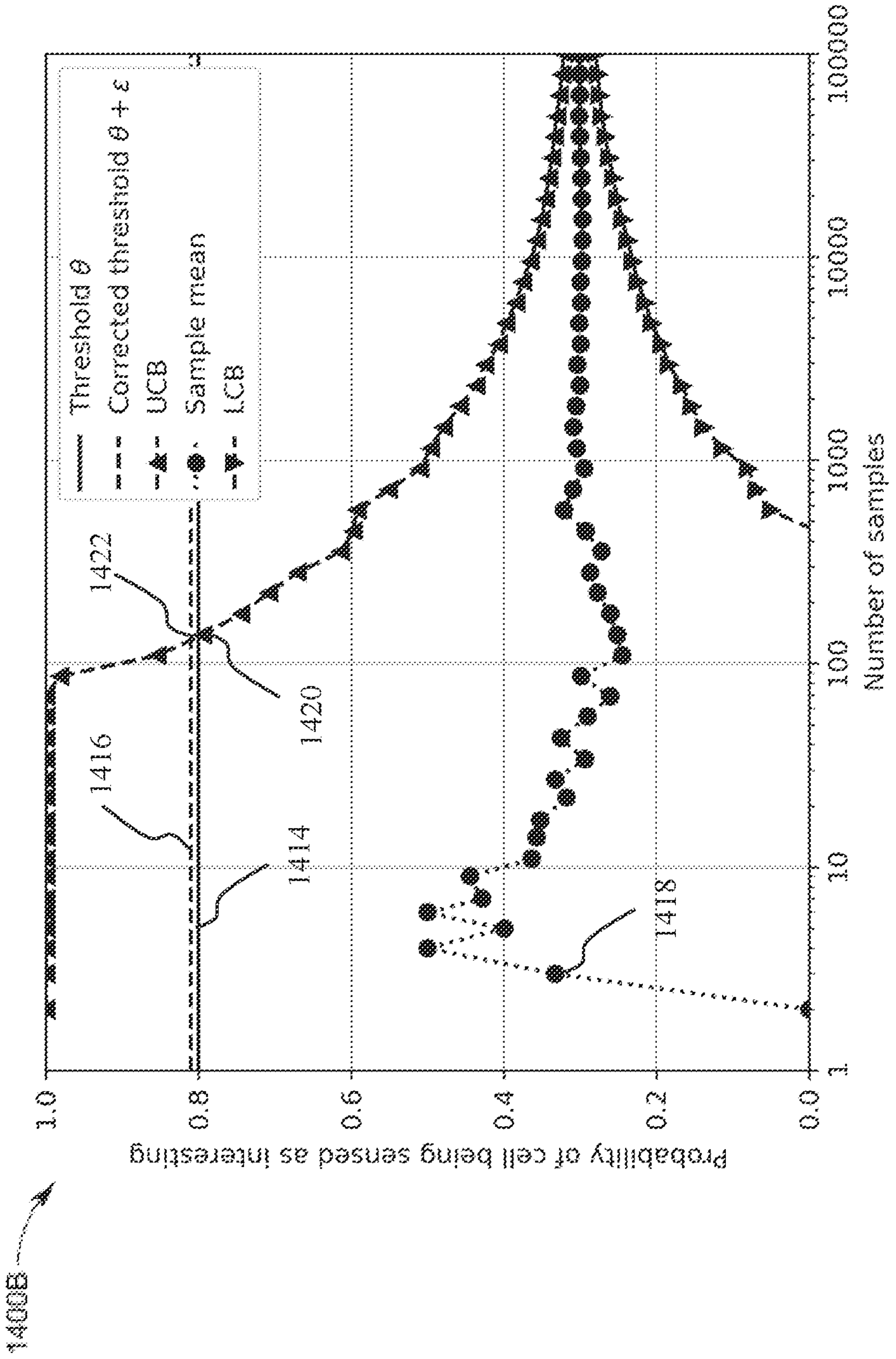
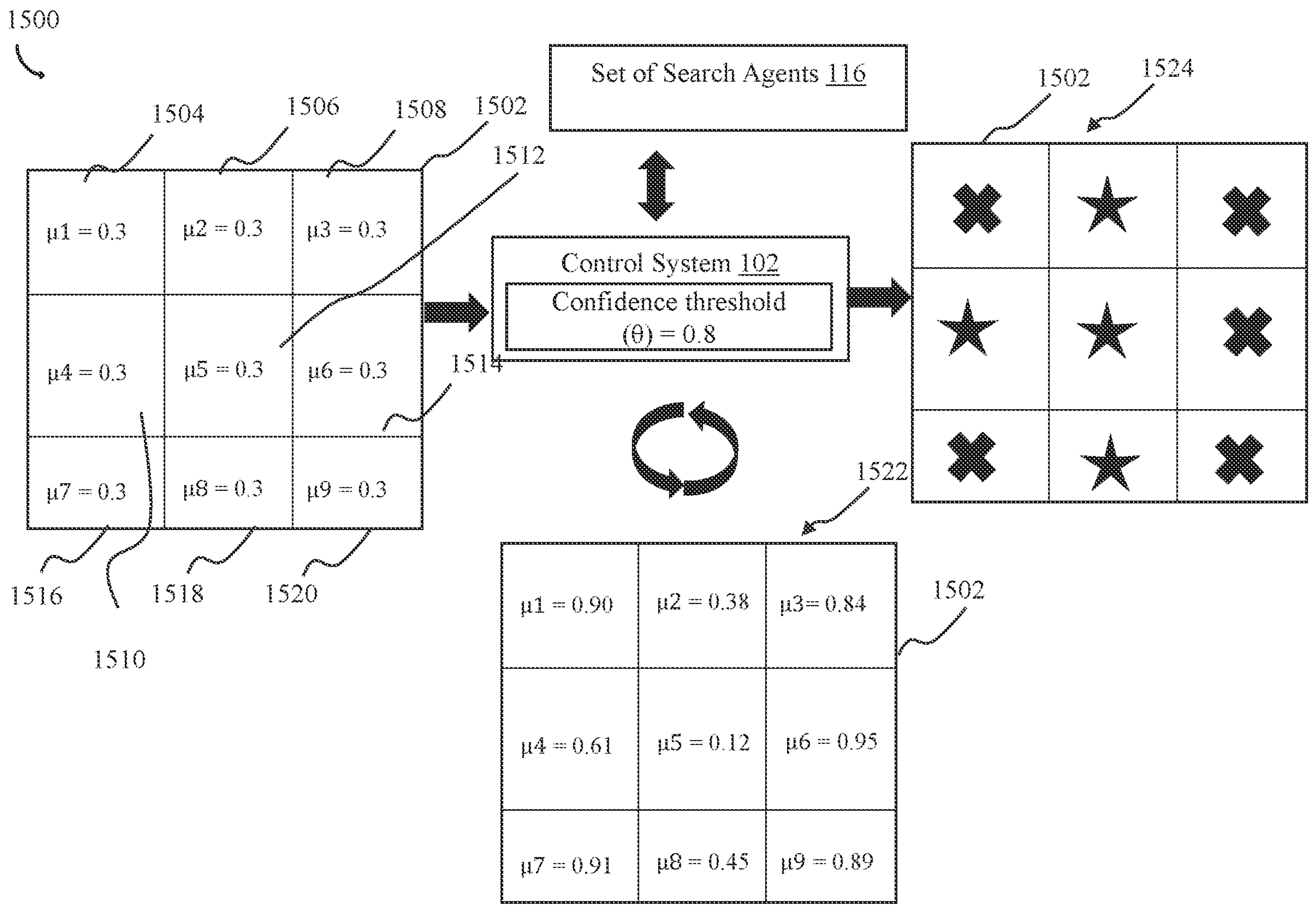

1400B
Probability of cell being sensed as interesting
1.0
0.8
0.6
0.4
0.2
0.0
Number of samples
1
10
100
1000
10000
100000
Threshold θ
Corrected threshold $\theta + \varepsilon$
UCB
Sample mean
LCB
1414
1416
1418
1420
1422

FIG. 14B



μ1 = 0.3	μ2 = 0.3	μ3 = 0.3
μ4 = 0.3	μ5 = 0.3	μ6 = 0.3
μ7 = 0.3	μ8 = 0.3	μ9 = 0.3

μ1 = 0.90	μ2 = 0.38	μ3= 0.84
μ4 = 0.61	μ5 = 0.12	μ6 = 0.95
μ7 = 0.91	μ8 = 0.45	μ9 = 0.89

FIG. 15

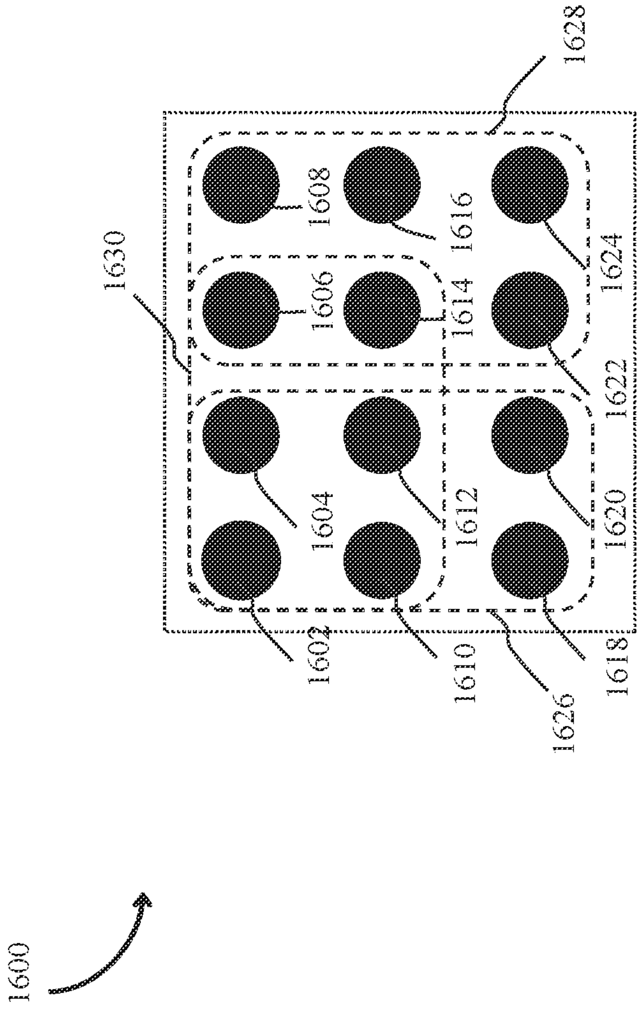

1600
1602
1604
1606
1608
1610
1612
1614
1616
1618
1620
1622
1624
1626
1628
1630

FIG. 16

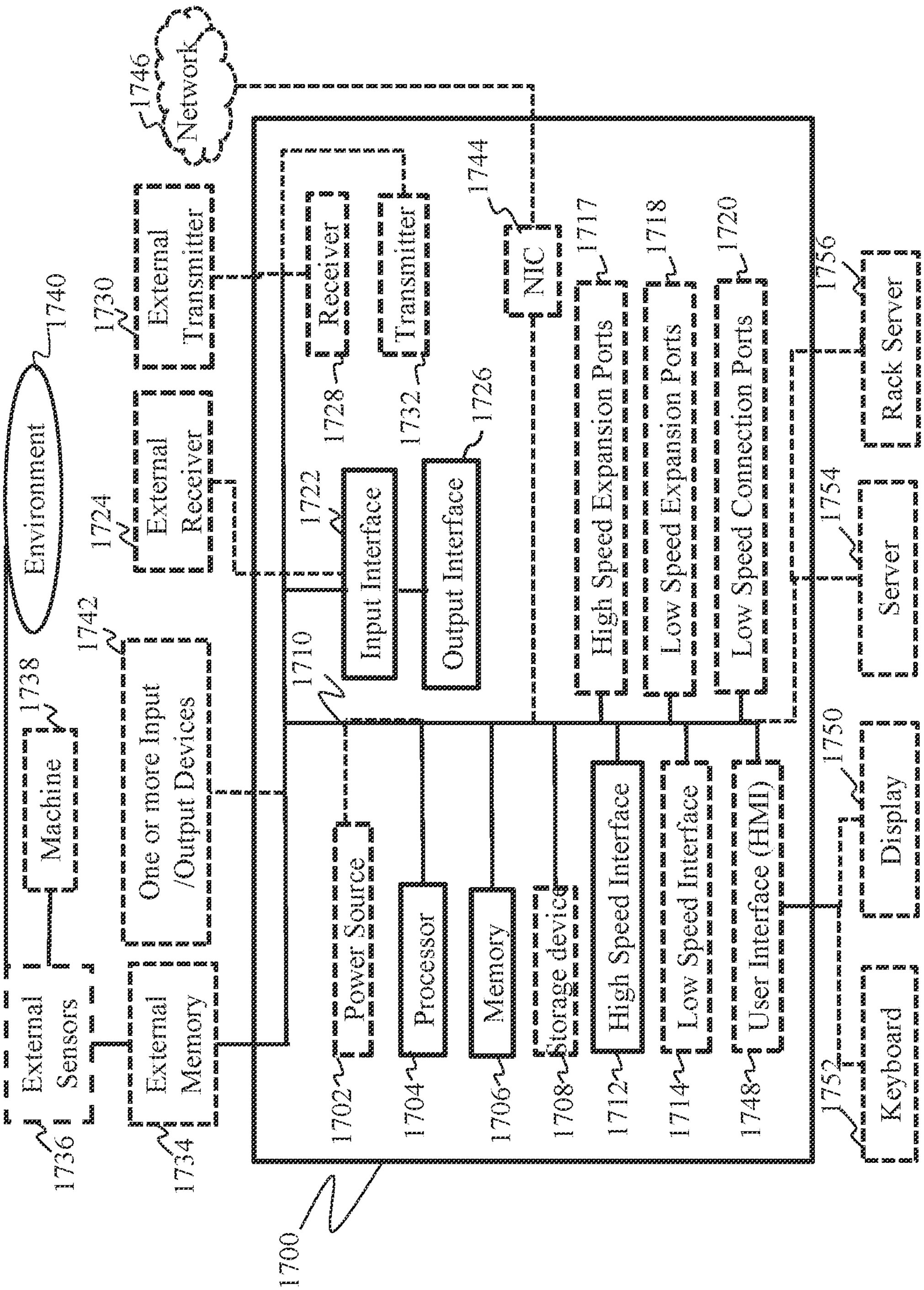

Environment
1740
Machine
1738
External Sensors
1736
External Memory
1734
One or more Input /Output Devices
1742
External Receiver
1724
External Transmitter
1730
Network
1746
1700
Power Source
1702
Processor
1704
Memory
1706
Storage device
1708
1710
High Speed Interface
1712
Low Speed Interface
1714
User Interface (HMI)
1748
Input Interface
1722
Output Interface
1726
Receiver
1728
Transmitter
1732
NIC
1744
High Speed Expansion Ports
1717
Low Speed Expansion Ports
1718
Low Speed Connection Ports
1720
Keyboard
1752
Display
1750
Server
1754
Rack Server
1756

FIG. 17

CONTROLLING SEARCH AGENTS TO PERFORM SEARCH WITH NOISY OBSERVATIONS AND PROBABILISTIC GUARANTEES

TECHNICAL FIELD

The disclosure relates generally to control applications, and more particularly to a control system and a method to control search agents to perform search with noisy observations and probabilistic guarantees.

BACKGROUND

With advancements in the field of machine learning, several types of algorithms are being used to solve a variety of real-life (or practical) problems optimally in a feasible amount of time. Examples of such real-life problems include, but are not limited to, a search-and-rescue operation, a ready-to-harvest tree detection for an agriculture application, a determination of congested areas in a city for a traffic monitoring application, an environmental monitoring application, a wildlife monitoring application, a disaster management application, and the like. Usually, such real-life problems are constrained to a physical environment (also referred to as a search environment) and traditional methods for solving such problems may be time-consuming, and cumbersome. For example, consider the ready-to-harvest tree detection problem to determine whether an apple orchard is ready for harvesting; such a problem may be specific to the apple orchard (i.e. the physical environment). Traditional methods for detecting whether the apple orchard is ready for harvesting typically involve people going to the apple orchard and manually inspecting the apple orchard to determine whether the apple orchard is ready for harvesting or not.

With the advent of time and the capabilities of computer systems, several computer-implemented techniques in combination with new devices (such as aerial vehicles) have emerged that are used to solve such real-life problems. For example, an autonomous multi-agent search technique for searching objects/phenomena of interest over large areas is crucial in the above-mentioned real-life applications. Regarding the above example of detecting whether the apple orchard is ready for harvesting, a set of search agents (e.g., aerial vehicles/drones) may fly in the apple orchard (or the search environment) to determine various parameters such as, but not limited to, a yield of one or more apple trees in the apple orchard, health of one or more apple trees in the apple orchard, and the like. As another example, in the case of the search-and-rescue operation where multiple humans are trapped on rooftops after a calamity (such as severe flooding) in a geographical area, the objective may be to identify areas using the set of search agents where a rescue team must be sent. Similar applications may be considered in infrastructure monitoring and wildfire rescue.

However, such computer-implemented techniques also have their limitations/constraints. For example, due to several types of constraints such as a cost constraint and a weight constraint, the set of search agents deployed in the apple orchard (or any physical environment) may only be able to carry low-cost and light sensors that may yield noisy data. Also, the economic costs associated with sensing by the set of search agents (e.g., battery energy for movement and sensing) must be reduced.

In some scenarios, autonomous monitoring systems are used to track spatial changes in the physical environment and other resources based on data collected by the sensors integrated within the set of search agents. However, the autonomous monitoring systems also encounter a problem that the set of search agents (e.g., aerial vehicles) must search over a pre-specified bounded area to identify regions of interest.

To solve the above-mentioned problem, the search environment is usually partitioned into a set of regions and a search agent of the set of search agents collects measurements and labels each region as an interesting region or an uninteresting region. For example, in the case of the search-and-rescue problem, the region where humans are detected (or where humans are likely to be found) may be classified as the interesting region, and the region where humans are not detected (where humans are less likely to be found) may be classified as the uninteresting region.

As discussed above, due to the cost constraint and weight constraint associated with the set of search agents, the set of search agents deployed in the search environment may only be able to carry low-cost and light sensors that usually result in noisy observation data. Given the stochastic nature of the sensors, it may be impractical to predict the amount of data needed to separate the interesting regions from the uninteresting regions. Consequently, the set of search agents must actively decide when and where to collect data based on current available data, while ensuring that deployment decisions are feasible for the set of search agents. Additionally, it may be important to consider constraints on the set of search agents arising from physical limitations like dynamics and energy limitations.

Autonomous monitoring has been an area of active research that spans several research communities including robotics, perception, learning, and control. However, existing approaches do not tackle all aspects of the above-mentioned problems. They may require improved sensing, may not have finite-time guarantees on the search performance, and/or may have high economic costs of search associated with movement and sensing. Other strategies may require prior knowledge of the total number of interesting regions to solve the multi-agent search problem. For example, some methods allow visiting each region multiple times allowing to reduce the amount of data collected at each visit. Other methods may prescribe collecting all necessary data to classify the visited region according to the object of the search at once. Examples of these search methods include a collaborative sensor network method, a branch and bound method, and a multi-arm bandit-based search method. However, these methods are based on some assumptions that may not be valid for some real-life applications. Specifically, the branch and bound method assumes the knowledge of probability distribution of the occurrence of targets (i.e., objects of interest) in the set of regions. Similarly, the collaborative sensor network method assumes the distribution of the target is Gaussian which allows using the collected data to estimate the parameters of a Gaussian distribution.

Currently, the "multi-arm bandit-based search" method includes strategies to identify maximal or top-k interesting regions in the set of regions (or a grid). However, the "multi-arm bandit-based search" method requires prior knowledge of a total number of interesting regions in the search environment to solve the considered multi-agent search problem and does not explicitly consider the physical limitations of the set of search agents.

Another existing solution to the autonomous monitoring problem is a "label-then-move" search. In the "label-then-move" search, at least one of the set of search agents keeps collecting enough data at a region until it is confident enough to label the region as an interesting region or an uninteresting region. The search agent may move to the next region only after labeling a current region as the interesting region or the uninteresting region. Since the "label-then-move" search ignores the data collected online when deciding on locations to sense, it may spend a significant amount of time labeling uninteresting regions and hence, may not be well suited for time-sensitive applications (such as the search-and-rescue problem).

Therefore, there is a requirement for systems and methods that aim to solve the challenges identified above.

SUMMARY

It is an object of the present disclosure to provide a method and a control system that addresses the above-mentioned problems associated with multiple search agents. Specifically, it is an object of the present disclosure to provide a method and a control system that focus on classifying each region of a set of regions in a search environment as interesting or uninteresting without any prior knowledge of the total number of interesting regions and/or uninteresting regions, explicitly considering the physical limitations and constraints of the set of search agents and provides output within the feasible amount of time so that the disclosed method may be used in real-time scenarios.

It is an object of some embodiments to disclose a control system, a remote server, and a method for controlling a set of search agents to perform classification of each region of the set of regions in the search environment using noisy observations and probabilistic guarantees. As used herein, the search agent may be an autonomous vehicle (say a drone), a mobile robot, an aerial drone, a ground vehicle, an aerial vehicle, a water surface vehicle, or an underwater vehicle. Additionally or alternatively, it is an object of some embodiments to disclose a data-driven method for multi-agent search under noisy observations allowing controlling of the set of search agents to move over the search environment partitioned into the set regions and to classify each region of the set of regions into one of the interesting region or the uninteresting region based on the data (or measurements) collected by one or more sensors installed within each search agent of the set of search agents. The collected data (or measurements) may be noisy due to the use of low-cost and low-weight systems (or perception systems) used in the search agents. Additionally, or alternatively, it is an object of some embodiments to co-ordinate the set of search agents to minimize the time required to identify all the interesting regions and the economic costs associated with moving the search agents (for example battery/fuel usage), despite the noise present in the collected data.

Furthermore, it is an object of some embodiments to provide a search method for controlling the search agents under noisy observation that may not rely on assumptions regarding the spatial distribution of one or more objects of interest or the sensing quality of one or more sensors. An example of such a method is a search method based on a bandit-based search algorithm, such as a "multi-arm bandit" algorithm. The "multi-arm bandit" algorithm may use collected data to tackle online, sequential decision-making problems. Specifically, given a choice between a fixed set of options (arms) and an unknown stochastic reward function that returns a reward upon execution of an option (pulling an arm), the "multi-arm bandit" algorithms may identify a sequence of execution of arms such that some desired criterion is satisfied (for example, minimize the regret of not executing the optimal option). Another variant of the "multi-arm bandit" algorithm known as a "thresholding multi-arm bandit" algorithm is a special class of "multi-arm bandit" algorithms that classify the arms into "good" and "bad" arms based on a user-specified threshold and online data. However, these algorithms assume that it may be possible to investigate any choice at any point in time.

The "multi-arm bandit" algorithms, in general, may synthesize a data-driven upper confidence bound on the return for each choice and then select choices based on the highest upper confidence bounds. The "multi-arm bandit" algorithm may not make any assumptions about the distribution of the objects of interest in the search environment and may provide a probabilistic performance guarantee for search exploration without any assumption(s). However, employing the current versions of the "multi-arm bandit" algorithm for the current search operations may be impractical because a direct application of the "thresholding multi-arm bandit" algorithm may force the search agents to traverse the environment without any regard for the economic costs and/or other physical constraints associated with the set of search agents. Consequently, the search agents may even run out of fuel or battery if the set of search agents may follow the search directly following a prescription of the bandit algorithms.

To that end, it is another object of some embodiments to adapt the search operations using the "multi-arm bandit" algorithm to consider the specifics of moving the search agents under the fuel constraints and dynamics constraints associated with the search agents.

Some embodiments may be based on an observation that the simplicity of another search method referred to herein as a "label-then-move" search, inadvertently and even accidentally results in the probabilistic performance guarantees of the search that consider specifics of moving the set of search agents. This may be because the "label-then-move" search approach controls the search agent to collect all necessary data (or measurements) to classify the region to avoid visiting the same region multiple times and controls the search agents to go to the neighboring region only after the classification of the current region is done, thereby reducing the travel cost of the search agent for going from one region to another.

As such, the "label-then-move" search may be a naive search algorithm that traverses the search environment and moves out of each region only after classifying it. Such an approach may incur low economic costs since it may make the search agents move minimally. But the "label-then-move" search approach may ignore the data collected online in deciding which region to sense next. Subsequently, the "label-then-move" search may take a long time to identify all the interesting regions.

Some embodiments are based on the intuition that a combination of the "multi-arm bandit" search method and the "label-then-move" search method may get synergy in reducing both the time and cost of the search operation. However, while each of the search methods may provide probabilistic performance guarantees, the combination of these methods may break it.

Some embodiments are based on the realization that the combination of these methods should also be probabilistic to preserve the probabilistic performance guarantees. For example, some embodiments switch between these search strategies based on repeated tosses of a biased coin. By tuning the bias of the coin and using a specific form of the upper confidence bound in the "thresholding multi-arm bandit" algorithm, some embodiments improve the performance of the combination of these two search algorithms over the performance of each search algorithm. Some embodiments obtain upper bounds on the performance of the searches by determining finite upper bounds on the time taken to complete the search, the time taken to label all interesting regions, and the costs incurred during the search according to some embodiments.

Some embodiments of the present disclosure may be based on a recognition that it may be possible to design a set of paths that may traverse the search environment such that the search agent may follow each of the designed set of paths and may have enough fuel (and/or battery) to finish the selected path. For example, the search environment may be partitioned into a set of paths (also referred to as a grid of cells), and the set of paths may be designed to traverse through each region of the set of regions (or cells) in the search environment, such that each path may start and end at a pre-designated region (such as a charging station) and may have a length feasible to be traversed without a need for intermediate charging. However, the problem of determining such a set of paths to cover all the regions in the search environment and subjected to one or more constraints may be a variant of the well-known problem in operations research, called the "multi-depot, fuel-constrained, multiple-vehicle routing" problem. The "multi-depot, fuel-constrained, multiple-vehicle routing" problem may be an NP-hard problem, which may necessitate a need for one or more approximation algorithms. The NP-hard problem may correspond to a class of hard problems that can be verified in polynomial time.

Some embodiments of the present disclosure may be based on the recognition that a collection of paths to solve the "multi-depot, fuel-constrained, multiple-vehicle routing problem" sub-optimally may be computed using a combination of dynamic programming and a sub-modular optimization. The proposed approach may scale well with an increase in the count of the set of search agents and the number of charging stations, covers all the regions (or the cells) in the search environment, and respects the fuel constraints associated with the set of search agents.

In comparison to the traditional techniques that may not have finite-time guarantees on the search performance or may have high economic costs of search associated with movement and sensing or require prior knowledge of the total number of interesting regions in the search environment, the disclosed control system classifies each region in the search environment effectively, and efficiently in the feasible amount of time and with probabilistic guarantees. Therefore, the disclosed control system overcomes all the limitations of the traditional techniques that are known in the art.

It is an object of some embodiments to disclose a control system to control a movement of at least a first search agent of a set of search agents in a search environment partitioned into a set of regions and to classify each region of the set of regions based on measurements collected by the first search agent. The control system includes a transceiver configured to exchange data with the set of search agents over a wired or wireless communication channel. The control system further includes a memory configured to store executable instructions specifying an operation of a multi-level multi-arm bandit search (MMBS) method. The MMBS method specifies a first set of instructions for individual probabilistic classifications of each region of the set of regions based on the measurements associated with the corresponding region, and a second set of instructions for visiting, by the set of search agents, each path of a set of paths formed from the set of regions based on comparing aggregations of the probabilistic classifications of each path of the set of paths. Each path of the set of paths may include at least two regions of the search environment. The control system further includes a processor coupled with the executable instructions, when executed by the processor, causes an iterative execution of the MMBS method until a termination condition is met. An iteration of the MMBS method causes the control system to collect confidence bounds of the probabilistic classification of at least one region within at least one path of the set of paths. The control system further compares aggregations of the confidence bounds of the probabilistic classifications of each path of the set of paths based on the collected confidence bounds. The control system further selects a first path of the set of paths to be visited by the first search agent based on the comparison. The control system further commands the first search agent to visit the selected first path and to collect measurements associated with each region within the selected first path. The control system may command the first search agent via the transceiver. The control system further updates the confidence bounds of the probabilistic classifications of each region within the selected first path based on the measurements associated with the corresponding regions.

In some embodiments, the control system may be configured to receive a set of user inputs associated with generation of the set of paths to cover the search environment. The control system may be further configured to generate the set of paths to cover the search environment based on the received set of user inputs.

In some embodiments, a first user input of the set of user inputs corresponds to a path length for each path of the set of paths. The path length for each path corresponds to a maximum number of regions in each path to be traversed by at least one search agent of the set of search agents.

In some embodiments, each path of the set of paths starts or ends at any pre-designated region of one or more pre-designated regions. At least one pre-designated region is located in a region inside the search environment, and at least one pre-designated region is located outside of the search environment.

In some embodiments, the pre-designated region corresponds to one of an energy refueling station, a calibration station, a service station, or a docking station. Each search agent of the set of search agents corresponds to one of an autonomous vehicle, a mobile robot, an aerial drone, a ground vehicle, an aerial vehicle, a water surface vehicle, or an underwater vehicle.

In some embodiments, the control system may be configured to generate the set of paths using a graph-based multi-agent path planning process that minimizes a first set of paths into the set of the paths covering the search environment subjected to one or more path constraints.

In some embodiments, the control system may be configured to construct, for each region of the set of regions, a shortest path table comprising a minimum number of steps required to reach any one of the one or more pre-designated regions in the search environment, where the shortest path table is constructed using one or more graph-based shortest path planning processes. The control system may be further configured to randomly generate the first set of paths satisfying the one or more path constraints by constructing a sequence of regions starting from a randomly selected pre-designated region from the one or more pre-designated regions in the search environment and executing hill ascend operation according to the shortest path table for a first half of the path length and further executing hill descend operation according to the shortest path table for a second half of the path length to reach one of the one or more pre-designated regions. The control system may be configured to select the set of paths from the first set of paths further based on the execution of the hill ascend operation and the hill descend operation.

In some embodiments, the control system may be configured to evaluate, for each path of the set of paths, a pre-determined aggregation function of the values of the confidence bounds of each region within the corresponding path. The control system may be further configured to sort the evaluation of the pre-determined aggregation function for each of the set of paths in descending order of a function value for each path. The function value is determined based on an application of the pre-determined aggregation function on the confidence bounds of each region in each path of the set of paths. The control system may be further configured to select, based on the sorting, at least one path to be visited by the set of search agents. The first path is selected to be visited by the first search agent.

In some embodiments, the control system may be configured to evaluate the confidence bounds of each region in each path of the set of paths, using a neural network that intakes the confidence bounds of each region in the corresponding path. The control system may be configured to return an assignment of the set paths to be executed by each search agent of the set search agents based on the evaluation. The first path is assigned to the first search agent. The control system may be configured to select, based on the assignment, at least one path to be visited by the set of search agents. The first path is selected to be visited by the first search agent based on the assignment.

In some embodiments, the control system may be configured to select probabilistically between different selecting criteria including a most-promising path criterion and a minimal-movement criterion. The control system may be further configured to select, based on the selected criterion, at least one path to be visited by the set of search agents. The first path may be selected to be visited by the first search agent.

In some embodiments, the probability of selection may be a biased probability.

In some embodiments, the selection of at least one path to be visited by the set of search agents is performed over multiple control steps dependent on a number of the regions in the search environment. A bias of the biased probability may vary between at least some control steps.

In some embodiments, the control system may be configured to update the bias based on the likelihood of interest at different regions of the set of regions in the search environment.

In some embodiments, the control system may be configured to update the confidence bounds of the different regions with a neural network trained with machine learning to produce the biased probability.

In some embodiments, the control system may be configured to select the first path and a second path from the set of paths based on the aggregations of the probabilistic classifications of each of the set of paths. The control system may be configured to control the movement of the first search agent and a second search agent of the set of search agents concurrently on the selected first path and the second path to take measurements associated with the corresponding regions within the first path and the second path.

In some embodiments, the control system may be configured to receive a pre-determined number of measurements associated with the first region and collected by at least the first search agent and update the confidence bounds of the probabilistic classifications of the first region based on the pre-determined number of measurements using at least one concentration inequality.

In some embodiments, the control system may be configured to transmit, via the transceiver, to at least the first search agent: a current value of the confidence bounds of the probabilistic classifications of the first region, a third set of instructions for taking a pre-determined number of measurements associated with the first region, and a fourth set of instructions for updating the current values of the confidence bounds of the probabilistic classifications of the first region. The control system may be further configured to receive, via the transceiver, the updated confidence bounds of the probabilistic classifications of the first region from at least the first search agent.

In some embodiments, the control system may be configured to compare the updated confidence bounds of a first region within the selected first path with a confidence threshold. The control system may be configured to classify the first region within the selected first path based on the comparison and update the first path of the set of paths to prune the first region from the first path based on the classification of the first region.

In some embodiments, each region of the set of regions is classified with a first label or a second label, and wherein the termination condition is met when each region of the set of regions is classified with one of the first label or the second label.

Another embodiment discloses a method for controlling a movement of at least a first search agent of a set of search agents in a search environment partitioned into a set of regions and classifying each region of the set of regions based on measurements collected by the first search agent comprising an iterative execution of a multi-level multi-arm bandit search (MMBS) method until a termination condition is met, wherein the MMBS method specifies a first set of instructions for individual probabilistic classifications of each region of the set of regions based on the measurements associated with the corresponding region, and a second set of instructions for visiting, by the set of search agents, each path of a set of paths formed from the set of regions based on comparing aggregations of the probabilistic classifications of different paths, wherein each path of the set of paths comprises at least two regions of the search environment, and wherein the iterative execution of MMBS includes collecting confidence bounds of the probabilistic classification of at least one region within at least one path of the set of paths. The method further includes comparing aggregations of the confidence bounds of each path of the set of paths based on the collected confidence bounds. The method further includes selecting a first path of the set of paths to be visited by the first search agent based on the comparison. The method further includes commanding, via a transceiver configured to exchange data with the set of search agents over a wired or wireless communication channel, the first search agent to visit the selected first path and to collect measurements associated with each region within the selected first path. The method further includes updating the confidence bounds of the probabilistic classifications of each region within the selected first path based on the measurements associated with the corresponding regions.

BRIEF DESCRIPTION OF THE DRAWINGS

The presently disclosed embodiments will be further explained with reference to the attached drawings. The drawings shown are not necessarily to scale, with emphasis instead generally being placed upon illustrating the principles of the presently disclosed embodiments.

FIG. **1**A is a diagram that illustrates a network environment for controlling search agents to perform search with noisy observations and probabilistic guarantees, in accordance with an embodiment of the disclosure.

FIG. **1**B depicts a diagram to illustrate a difference between the multi-arm bandit search method and a multi-level multi-arm bandit search (MMBS) method, in accordance with an embodiment of the disclosure.

FIG. **2**A is an exemplar block diagram of the control system of FIG. **1**A, in accordance with an embodiment of the disclosure.

FIG. **2**B is an exemplar block diagram of the remote server of FIG. **1**A, in accordance with an embodiment of the disclosure.

FIG. **3** is a flowchart that illustrates an exemplar method for the generation of a set of paths, in accordance with an embodiment of the disclosure.

FIG. **4** is a diagram that depicts a generated set of paths in the search environment, in accordance with an embodiment of the disclosure.

FIG. **5** is a diagram that depicts confidence bounds associated with each region of the set of regions of the search environment, in accordance with an embodiment of the disclosure.

FIG. **6**A is a flowchart that illustrates an exemplar first method for the selection of a path from the set of paths to be visited by at least one search agent, in accordance with an embodiment of the disclosure.

FIG. **6**B depicts a diagram to illustrate an exemplar scenario for selecting paths to be visited by at least one search agent, in accordance with an embodiment of the disclosure.

FIG. **6**C is a flowchart that illustrates an exemplar second method for the selection of a path from the set of paths to be visited by at least one search agent, in accordance with an embodiment of the disclosure.

FIG. **6**D depicts a diagram to illustrate an exemplar scenario for selecting paths to be visited by at least one search agent, in accordance with an embodiment of the disclosure.

FIG. **6**E is a flowchart that illustrates an exemplar third method for the selection of a path from the set of paths to be visited by at least one search agent, in accordance with an embodiment of the disclosure.

FIG. **7**A is a flowchart that illustrates an exemplar first method for updating confidence bounds of at least one region of the set of regions of the search environment, in accordance with an embodiment of the disclosure.

FIG. **7**B is a flowchart that illustrates an exemplar second method for updating confidence bounds of at least one region of the set of regions of the search environment, in accordance with an embodiment of the disclosure.

FIG. **8** is a flowchart that illustrates an exemplar method for pruning a classified region from a corresponding path, in accordance with an embodiment of the disclosure.

FIG. **9** is a diagram that illustrates an updated set of paths in the search environment based on the classification of the first region, in accordance with an embodiment of the disclosure.

FIG. **10** is a flowchart that illustrates an exemplar method for controlling search agents to perform search with noisy observations and probabilistic guarantees, in accordance with an embodiment of the disclosure.

FIG. **11** is a network environment for a remote server controlling search agents to perform a search with noisy observations and probabilistic guarantees, in accordance with an embodiment of the disclosure.

FIG. **12** illustrates the sensing assumptions for the classification of regions within the search environment, in accordance with an embodiment of the disclosure.

FIG. **13** is a flowchart that illustrates an exemplar method for controlling search agents to perform search with noisy observations, in accordance with an embodiment of the disclosure.

FIG. **14**A is a graph that illustrates a convergence of the confidence bounds with an increasing number of realizations for a region that may have a high likelihood of being sensed, in accordance with an embodiment of the disclosure.

FIG. **14**B is a graph that illustrates a convergence of the confidence bounds with an increasing number of realizations for a region that may have a low likelihood of being sensed, in accordance with an embodiment of the disclosure.

FIG. **15** illustrates a diagram of an exemplar scenario for classifying the set of regions in the search environment, in accordance with an embodiment of the disclosure.

FIG. **16** illustrates an exemplar application of a multi-level multi-arm bandit search (MMBS) method, in accordance with an embodiment of the disclosure.

FIG. **17** illustrates a schematic diagram of a computing device that may be used for implementing the control system or the remote server, in accordance with an embodiment of the disclosure.

DETAILED DESCRIPTION

In the following description, for purposes of explanation, numerous specific details are set forth in order to provide a thorough understanding of the present disclosure. It will be apparent, however, to one skilled in the art that the present disclosure may be practiced without these specific details. In other instances, apparatuses and methods are shown in block diagram form only in order to avoid obscuring the present disclosure.

As used in this specification and claims, the terms "for example," "for instance," and "such as," and the verbs "comprising," "having," "including," and their other verb forms, when used in conjunction with a listing of one or more components or other items, are each to be construed as open ended, meaning that that the listing is not to be considered as excluding other, additional components or items. The term "based on" means at least partially based on. Further, it is to be understood that the phraseology and terminology employed herein are for the purpose of the description and should not be regarded as limiting. Any heading utilized within this description is for convenience only and has no legal or limiting effect.

FIG. **1**A is a diagram that illustrates a network environment **100**A for controlling search agents to perform search with noisy observations and probabilistic guarantees, in accordance with an embodiment of the disclosure. With reference to FIG. **1**A, there is shown the network environment **100**A. The network environment **100**A includes a control system **102** that may further include a processor **104**, a memory **106**, and a transceiver **108**. With reference to FIG. **1**A, there is further shown a remote server **110**, a search environment **114**, and a network **112**. The search environment **114** may be partitioned into a set of regions, such as a region **114**A, a region **114**B, and a region **114**N (hereinafter referred to as the set of regions **114**A-**114**N). There is further shown a set of search agents **116** and a set of paths **118**. The set of search agents **116** may include, but is not limited to, a first search agent **116**A, and a second search agent **116**B. Similarly, the set of paths may include, but is not limited to, a first path **118**A, a second path **118**B, a third path **118**C, and a fourth path **118**D.

The control system **102** may include suitable logic, circuitry, interfaces, and/or code that may be configured to control the set of search agents **116** to perform search with noisy observations and probabilistic guarantees in the search environment **114**. The control system **102** may be configured to iteratively execute a multi-level multi-arm bandit search (MMBS) method **106**A for classifying each of the set of regions **114**A-**114**N. Examples of the control system **102** may include, but are not limited to, a computing device, a mainframe machine, a server, a computer workstation, a smartphone, a cellular phone, a mobile phone, a gaming device, a consumer-electronic (CE) device and/or any other device with device control capabilities.

The processor **104** may comprise suitable logic, circuitry, and interfaces that may be configured to execute instructions stored in the memory **106**. The processor **104** may be implemented based on a number of processor technologies known in the art. Examples of the processor **104** may include, but are not limited to, a Graphical Processing Unit (GPU), a co-processor, a Central Processing Unit (CPU), an x86-based processor, a Reduced Instruction Set Computing (RISC) processor, an Application-Specific Integrated Circuit (ASIC) processor, a Complex Instruction Set Computing (CISC) processor, and a combination thereof.

The memory **106** may include suitable logic, circuitry, and/or interfaces that may be configured to store the program instructions executable by the processor **104**. Specifically, the memory **106** may store executable instructions specifying an operation of the MMBS method **106**A. Examples of implementation of the memory **106** may include, but are not limited to, Random Access Memory (RAM), Read Only Memory (ROM), Electrically Erasable Programmable Read-Only Memory (EEPROM), Hard Disk Drive (HDD), a Solid-State Drive (SSD), a CPU cache, and/or a Secure Digital (SD) card.

The MMBS method **106**A may be used to classify each region of the set of regions **114**A-**114**N of the search environment **112** in a feasible amount of time and with probabilistic guarantees. The MMBS method **116**A may classify each region based on noisy measurements collected by the set of search agents **116**. Moreover, the disclosed MMBS method **106**A may select the next regions to be visited by the set of search agents **116** based on the confidence bounds of for each region **114**A-**114**N, constructed using the collected noisy measurements. Moreover, the disclosed MMBS method **106**A may control the movement of the set of search agents from one region to another region in the same path or from one region in one path to another region in another path. Therefore, the disclosed MMBS method **116**A may not restrict the movement of the set of search agents **116** in a single path only. Also, the disclosed MMBS method **106**A may probabilistically select between different selecting criteria including a most-promising path criterion and a minimal-movement criterion to classify the region and to further decide the next region to be visited by the set of search agents **116**.

In an embodiment, the MMBS method **106**A may specify a first set of instructions for individual probabilistic classifications **110**A of each region of the set of regions **114**A-**114**N based on measurements associated with the corresponding region, and a second set of instructions for visiting, by the set of search agents **116**, each path of the set of paths **118** formed from the set of regions **114**A-**114**N based on comparing aggregations of the probabilistic classifications **110**A of each path of the set of paths **118**. Each path of the set of paths **118** may formed from at least two regions of the search environment **114**. The MMBS method **116**A may be iteratively executed by the control system **102** until each region of the set of regions **114**A-**114**N may be classified.

The transceiver **108** may include suitable logic, circuitry, and/or interfaces that may be configured to exchange data between the control system **102**, the remote server **110**, and the set of search agents **116**. In an embodiment, the transceiver **108** may be configured to exchange data between the control system **102**, the remote server **110**, and the set of search agents **116** over a wired or wireless communication channel. Examples of the transceiver **108** may include, but are not limited to, a wireless transceiver, a radio frequency (RF) transceiver, an ethernet transceiver, and a fiber-optic transceiver.

The remote server **110** may include suitable logic, circuitry, and interfaces, and/or code that may be configured to control the set of search agents **116**. The remote server **110** may be further configured to determine probabilistic classifications **110**A, and confidence bounds **110**B of at least one region of the set of regions **114**A-**114**N or at least one path of the set of paths **118**. The remote server **110** may be implemented as a cloud server and may execute operations through web applications, cloud applications, HTTP requests, repository operations, file transfer, and the like. Other example implementations of the remote server **110** may include, but are not limited to, a database server, a file server, a web server, a media server, an application server, a mainframe server, or a cloud computing server.

In at least one embodiment, the remote server **110** may be implemented as a plurality of distributed cloud-based resources using several technologies that are well known to those ordinarily skilled in the art. A person with ordinary skill in the art will understand that the scope of the disclosure may not be limited to the implementation of the remote server **110** and the control system **102** as two separate entities. In an embodiment, the remote server **110** may provide instructions to the control system **102** to perform the operations for the classification of each region in the search environment **114**. In certain embodiments, the functionalities of the remote server **110** can be incorporated in its entirety or at least partially in the control system **102**, without a departure from the scope of the disclosure.

The network **112** may include a communication medium through which the control system **102**, the remote server **110**, and the set of search agents **116** may communicate with each other. The network **112** may be one of a wired connection or a wireless connection. Examples of the network **112** may include, but are not limited to, the Internet, a cloud network, a Wireless Fidelity (Wi-Fi) network, a Personal Area Network (PAN), a Local Area Network (LAN), or a Metropolitan Area Network (MAN). Various devices in the network environment **100**A may be configured to connect to the network **112** in accordance with various wired and wireless communication protocols. Examples of such wired and wireless communication protocols may include, but are not limited to, at least one of a Transmission Control Protocol and Internet Protocol (TCP/IP), User Datagram Protocol (UDP), Hypertext Transfer Protocol (HTTP), File Transfer Protocol (FTP), Zig Bee, EDGE, IEEE 802.11, light fidelity (Li-Fi), 802.16, IEEE 802.11s, IEEE 802.11g, multi-hop communication, wireless access point (AP), device to device communication, cellular communication protocols, and Bluetooth (BT) communication protocols.

The search environment **114** may correspond to a portion of the physical environment (or geographical space) in a real-world environment. The search environment **114** may be associated with a problem statement that is being solved. For example, in case of a problem detecting whether the apple orchard is ready for harvesting, the search environment **114** may correspond to the apple orchard, and each region **114**A-**114**N may correspond to the apple tree. As another example, in the scenario of the search-and-rescue operation where multiple humans are trapped on rooftops after a calamity, the search environment **114** may correspond to a geographical area where the calamity might have happened, and each region **114**A-**114**N may be various rooftops in the region.

In an embodiment, the search environment **114** may be divided into the set of regions **114**A-**114**N. Each region may be of a pre-determined shape and of pre-determined dimensions. In some embodiments, each region of the set of regions **114**A-**114**N may be of the same shape and similar dimensions. In another embodiment, each region of the set of regions **114**A-**114**N may be of different shapes and different dimensions. In an embodiment, the shape and the dimensions may be provided as a user input. Based on the user input, the control system **102** may be configured to partition the search environment **114** into the set of regions **114**A-**114**N.

Each search agent of the set of search agents **116** may include suitable logic, circuitry, and interfaces, and/or code that may be configured to search the search environment **114** and take measurements from the corresponding region in which the search agent is deployed. In an embodiment, each search agent of the set of search agents **116** may be equipped with one or more sensors that may be configured to capture data associated with each region and transmit the captured data, as measurements, to the control system **102** or the remote server **110**. As discussed above, the one or more sensors may be low-cost noisy sensors that may capture noisy data. Further, each search agent may have fuel or power constraints such that each search agent may work until the search agent runs out of fuel or power. Examples of each search agent of the set of search agents **116** may include, but are not limited to, an autonomous vehicle, a mobile robot, an aerial drone, a ground vehicle, an aerial vehicle, a water surface vehicle, or an underwater vehicle. As shown in the FIG. **1**A, the set of search agents **116** may be the aerial drone and may include the first search agent **116**A, and the second search agent **116**B. In an embodiment, the set of search agents **116**A may also be referred to as a search team.

Each path of the set of paths **118** may correspond to a route or a track that may have to be traversed by the at least one search agent of the set of search agents **116**. Each path of the set of paths may be generated based on one or more path constraints and the fuel or power constraints of the set of search agents **116**. Each path may include a minimum of two regions. The set of paths **118** may be generated in such a way that the whole search environment **114** is covered. As shown in the FIG. **1**A, the set of paths **118** may include the first path **118**A, the second path **118**B, the third path **118**C, and the fourth path **118**D. In an embodiment, each path of the set of paths **118** may also be referred to as a feasible path and the set of paths **118** may be referred to as a set of feasible paths. Details about the generation of the set of paths **118** are provided, for example, in FIG. **2**.

In operation, the search environment **114** may be portioned into the set of regions **114**A-**114**N. Each region of the set of regions **114**A-**114**N may correspond to a portion of the search environment **114**. The control system **102** may be configured to control the movement of at least the first search agent **116**A of the set of search agents **116** in the search environment **114** partitioned into the set of regions **114**A-**114**N and to classify each region of the set of regions **114**A-**114**N based on measurements collected by the first search agent **116**A. The control system **102** may include the transceiver **108** that may be configured to exchange data with the set of search agents **116** over the network **112**. The control system **102** may further include memory **106** configured to store executable instructions specifying the operation of the MMBS method **106**A. The MMBS method **106**A may specify the first set of instructions for individual probabilistic classifications **110**A of each region of the set of regions **114**A-**114**N based on the measurements associated with the corresponding region, and the second set of instructions for visiting, by the set of search agents **116**, each path of the set of paths **118** formed from the set of regions **114**A-**114**N based on comparing aggregations of the probabilistic classifications **110**A of each path of the set of paths **118**. The control system **102** may further include the processor **104**. The processor **104** may be configured to cause an iterative execution of the MMBS method **106**A until the termination condition is met. Each iteration of the MMBS method **106**A may cause the control system **102** to collect measurements of a subset of the regions **114**A-**114**N to construct confidence bounds **110**B. The confidence bounds **110**B may include an upper confidence bound and a lower confidence bound. In general, the upper confidence bound, and the lower confidence bound may be used to measure an uncertainty that may be associated with the measurements. The upper and the lower confidence bounds may provide a range within which a true value is likely to fall. Each region of the set of regions **114**A-**114**N has a corresponding true value that may refer to the a priori unknown likelihood of being assigned with a label or classified into a specific category. For example, in case of a problem detecting whether the apple orchard is ready for harvesting, the true value associated with the region **114**A is the apriori unknown likelihood of apple tree in region **114**A is sensed to be ready for harvesting. The confidence bounds **110**B may represent a range of probabilities assigned to each classification and further provide an estimation of the uncertainty associated with the classification. The upper confidence bound may represent an upper limit of this range, giving an upper limit estimate of a confidence score. On the other hand, the lower confidence bound may represent a lower limit of the range, giving the lowest possible value of the confidence score.

The probabilistic classification **110**A may refer to assigning a probability to different classes or categories to each region of the set of regions **114**A-**114**N. Instead of simply assigning a single class label to a given input, the probabilistic classification **110**A may provide a probability distribution over all possible classes. In an embodiment, the control system **102** may be configured to determine the confidence bounds **110**B of the probabilistic classification **110**A of at least one region within at least one path of the set of paths **118** based on one or more measurements taken by at least the first search agent **116**A of the set of search agents **116**. In another embodiment, the control system **102** may be configured to collect the confidence bounds of the probabilistic classification **110**A of at least one region within at least one path of the set of paths **118** from the remote server **110** or at least one of the set of search agents **116**.

The processor **104** may be further configured to compare aggregations of the confidence bounds of the probabilistic classifications **110**A of each path of the set of paths **118**. In an embodiment, the processor **102** may be further configured to compare aggregations of the confidence bounds of the probabilistic classifications **110**A of each path of the set of paths **118** based on the collected confidence bounds **110**B. The aggregation of the confidence bounds **110**B may correspond to combining the confidence bounds of each region of each path of the set of paths **118** in the search environment **114** using a pre-determined aggregation function. Such pre-determined aggregation function may include, but are not limited to, an addition function, an average function, a maximum function, or a minimum function. Once the aggregation of the confidence bounds **110**B is determined for each path, the processor **104** may be configured to compare the determined aggregations. Details about the aggregation of the confidence bounds **110**B are provided, for example, in FIG. **6**A, FIG. **6**B, FIG. **6**C, and FIG. **6**D.

The processor **104** may be further configured to select the first path **118**A of the set of paths **118** to be visited by the first search agent **116**A based on the comparison. In an example, the path for which the addition of the confidence bounds **110**B yields a maximum value may be selected as the first path **118**A. In another embodiment, the path for which the average of the confidence bounds yields **110**B the maximum value may be selected as the first path **118**A. Details about the selection of the first path **118**A are provided, for example, in FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, and FIG. **6**E.

After the selection of the first path **118**A, the processor **104** may be configured to command the first search agent **116**A to visit the selected first path **118**A. The processor **104** may further command the first search agent **116**A to collect measurements associated with each region within the selected first path **118**A. In an embodiment, the first search agent **116**A may be commanded via the transceiver **108**. The measurements associated with each region within the selected first path **118**A may be captured by the one or more sensors embedded in the first search agent **116**A. Examples of such measurements may include image data, sensor data, and the like. The measurements may vary according to the problem statement being solved. As a first example, if the problem being solved is the search-and-rescue operation, the measurements may correspond to one or more images captured by at least one image sensor embedded in the first search agent **116**A.

In an embodiment, the processor **104** may be configured to select the first path **118**A and the second path **118**B from the set of paths **118** based on the aggregations of the probabilistic classifications of each of the set of paths **118**. The processor **104** may be further configured to control the movement of the first search agent **116**A and the second search agent **116**B of the set of search agents **116** concurrently in the selected first path **118**A and the second path **118**B to take measurements associated with the corresponding regions within the first path **118**A and the second path **118**B.

The processor **104** may be further configured to update the confidence bounds **110**B of the probabilistic classifications **110**A of each region within the selected first path **118**A based on the measurements associated with the corresponding regions. Specifically, the processor **104** may be further configured to increase (and/or decrease) the value of the confidence bounds **110**B of the probabilistic classifications **110**A of each region within the selected first path **118**A. With reference to the first example, if the captured image includes humans, then the confidence bound of the corresponding region may be increased.

The processor **104** may be further configured to compare the updated confidence bounds **110**B of a first region (say the first region **114**A) within the first path **118**A with a confidence threshold. The confidence threshold may correspond to a pre-determined user-specified threshold value for the classification of the first region **114**A with a first label and a second label. In case the first label is assigned to the first region **114**A, it may be deemed that the first region **114**A is classified as the interesting region. In case the second label is assigned to the first region **114**A, it may be deemed that the first region **114**A is classified as the uninteresting region. The processor **104** may be further configured to classify the first region **114**A within the first path **118**A based on the comparison. For example, the control system **102** may classify the first region **114**A as interesting with a high probability whenever the lower confidence bound may be greater than the confidence threshold. Similarly, the control system **102** may classify the first region **114**A as uninteresting with a high probability whenever the upper confidence bound may be less than the confidence threshold. Details about the classification are provided below, for example, in FIG. **8**, FIG. **13**, FIG. **14**A, FIG. **14**B, and FIG. **15**.

FIG. **1**B depicts a diagram to illustrate a difference between the multi-arm bandit search method and a multi-level multi-arm bandit search (MMBS) method, in accordance with an embodiment of the disclosure. FIG. **1**B is explained in conjunction with elements from FIG. **1**A. With reference to FIG. **1**B, there is shown a diagram **100**B.

Traditionally, a multi-armed bandit search method **120** may be a problem-solving framework in the field of machine learning as well as optimization and may be used in situations where there is a need to balance exploration (trying different options to learn about their rewards) and exploitation (choosing the best-known option to maximize cumulative rewards).

Specifically, the multi-armed bandit search method **120** may be used to solve a multi-armed bandit problem that may be a classical sequential decision-making problem in which the set of search agents **116** may be faced with a set of "arms" (or actions), each with an unknown and stochastic reward distribution. The goal of the set of search agents **116** may be to maximize a cumulative reward obtained over the series of actions, while also learning about the reward distribution of each arm through exploration.

In comparison with the traditional multi-armed bandit search method **120** which can classify each region of the set of regions **114**A-**114**N without any probabilistic and time-frame guarantees, the disclosed MMBS method **106**A may be able to classify each region of the set of regions **114**A-**114**N of the search environment **112** in a feasible amount of time along with probabilistic guarantees. Also, the disclosed MMBS method **106**A may control the movement of the set of search agents from one region to another region in the same path or from one region in one path to another region in another path whereas the traditional multi-armed bandit search method **120** may restrict the movement of the set of search agents **116** in a single path only.

FIG. **2**A is an exemplary block diagram of the control system of FIG. **1**A, in accordance with an embodiment of the disclosure. FIG. **2**A is explained in conjunction with elements from FIG. **1**A and FIG. **1**B. With reference to FIG. **2**A, there is shown a block diagram **200**A of the control system **102**. The control system **102** may include the processor **104**, the memory **106**, and the transceiver **108**. With reference to FIG. **2**A, there is further shown a database **202**, and a display screen **204** that may include a user interface

204A. The processor **202** may be communicatively coupled to the database **202**, the display screen **204**, the memory **106**, and the transceiver **108**.

The database **202** may include suitable logic, circuitry, code, and/or interfaces that may be configured to store the confidence bounds **110**B and the probabilistic classifications **110**A for each region and each path in the search environment **114**. In another embodiment, the database **202** may store program instructions to be executed by the control system **102**. In another embodiment, the database **202** may store the set of paths **118** in the search environment **114**. Example implementations of the database **202** may include, but are not limited to, a centralized database, a distributed database, a no structured query language (NoSQL) database, a cloud database, a relational database, a network database, an object-oriented database, and a hierarchical database.

The display screen **204** may comprise suitable logic, circuitry, and interfaces that may be configured to display assigned labels to each region of the set of regions **114**A-**114**N. In an embodiment, the display screen **204** may further display the confidence bounds **110**B of each region in the search environment **114**. The touch screen may be at least one of a resistive touch screen, a capacitive touch screen, or a thermal touch screen. The display screen **204** may be realized through several known technologies such as, but not limited to, at least one of a Liquid Crystal Display (LCD) display, a Light Emitting Diode (LED) display, a plasma display, or an Organic LED (OLED) display technology, or other display devices. In accordance with an embodiment, the display screen **204** may refer to a display screen of a head-mounted device (HMD), a smart-glass device, a see-through display, a projection-based display, an electro-chromic display, or a transparent display.

The user interface **204**A may be configured as a medium for a user to interact with the control system **102**. The user interface **204**A may be a dynamic interface that may change according to the configuration of the control system **102**. In some embodiments, the control system **102** may receive a set of user inputs from the user of the control system via the user interface **204**A.

FIG. **2**B is an exemplary block diagram of the remote server of FIG. **1**A, in accordance with an embodiment of the disclosure. FIG. **2**B is explained in conjunction with elements from FIG. **1**A, FIGS. **1**B, and **2**A. With reference to FIG. **2**B, there is shown a block diagram **200**B of the remote server **110**. The remote server **110** may include a processor **206**, the memory **208**, a database **208**A, and the transceiver **210**. The processor **206** may be communicatively coupled to the memory **208**, and the transceiver **210**.

The processor **206** may comprise suitable logic, circuitry, and interfaces that may be configured to execute instructions stored in the memory **208** for controlling search agents to perform search with noisy observations and probabilistic guarantees. The processor **206** may be implemented based on a number of processor technologies known in the art. Examples of the processor **206** may include, but are not limited to, the GPU, the co-processor, the CPU, the x86-based processor, the RISC processor, the ASIC processor, the CISC processor, and a combination thereof.

The memory **208** may include suitable logic, circuitry, and/or interfaces that may be configured to store the program instructions executable by the processor **206**. Specifically, the memory **208** may store executable instructions specifying an operation of the MMBS method **106**A. In an embodiment, the memory **208** may include the database **208**A. The database **208**A may be configured to store the probabilistic classification **110**A and the confidence bounds **110**B. In an embodiment, the memory **208** may be similar to the memory **106** of the control system **102**. Examples of implementation of the memory **208** may include, but are not limited to, the RAM, the ROM, the EEPROM, the HDD, the SSD, the CPU cache, and/or the SD card.

The transceiver **210** may include suitable logic, circuitry, and/or interfaces that may be configured to exchange data between the remote server **110**, the control system **102**, and the set of search agents **116**. In an embodiment, the transceiver **210** may be configured to exchange data between the remote server **110**, the control system **102**, and the set of search agents **116** over a wired or wireless communication channel. Examples of the transceiver **210** may include, but are not limited to, a wireless transceiver, a radio frequency (RF) transceiver, an ethernet transceiver, and a fiber-optic transceiver.

FIG. **3** is a flowchart **300** that illustrates an exemplary method for the generation of the set of paths **118**, in accordance with an embodiment of the disclosure. FIG. **3** is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, and FIG. **2**B. With reference to FIG. **3**, there is shown the flowchart **300**. The operations of the exemplary method may be executed by any computing system, for example, by the control system **102** or the processor **104**, or the remote server **110** of FIG. **1**A. The operations of the flowchart **300** may start at **302**.

At **302**, a set of user inputs may be received. In an embodiment, the processor **104** may be configured to receive the set of user inputs associated with generation of the set of paths to cover the search environment **114**. Specifically, the set of user inputs may correspond to one or more path constraints. The set of user inputs may include, but are not limited to, a first user input, a second user input, and a third user input.

The first user input may correspond to a path length for each path for each of the set of paths **118**. The path length for each path may correspond to a maximum number of regions in each path that maybe traversed by at least one search agent of the set of search agents **116** while satisfying its physical constraints, wherein the physical constraints can be due to the motion constraints of the search agents and limited on-board energy available for the search agents.

The second user input may be indicative of a starting region and an ending region of each path of the set of paths **118**. In an embodiment, each path of the set of paths **118** may start and/or end at a pre-designated region of one or more pre-designated regions. In an embodiment, at least one pre-designated region may be located inside the search environment **114**. In another embodiment, at least one pre-designated region may be located outside of the search environment **114**. The pre-designated region may correspond to one of an energy refueling station, a calibration station, a service station, or a docking station. The energy refueling station may correspond to a charging station or a fuel station where the set of search agents **116** may recharge their batteries or refuel themselves. The calibration station may correspond to a region where one or more parameters or one or more sensors associated with the set of search agents **116** may be calibrated. The service station may correspond to a region where the set of search agents **116** may be serviced or repaired. The docking station may correspond to a region where the set of search agents **116** may dock when they are not in use.

The third user input may be indicative of one or more restricted regions that must not be included in any path of the set of paths **118**. These regions could be obstacles in the search environment, gps-denied environments, or regions that are known to be uninteresting from the beginning.

In an embodiment, the control system **102** may be configured to receive the set of user inputs from the user of the control system **102** via the user interface **204**A of the display screen **204**.

The received set of user inputs may be provided as an input to a graph-based multi-agent path planning process **304**. In an embodiment, the processor **104** may be configured to transmit the received set of user inputs to the graph-based multi-agent path planning process **304** to generate the set of paths **118** to cover the search environment **114**. The graph-based multi-agent path planning process **304** may include the operations **306**, **308**, and **310** as described below.

At **306**, a shortest path table may be constructed. In an embodiment, the processor **104** may be configured to construct a shortest path table for each region of the set of regions **114**A-**114**N in the search environment **114**. The shortest path table may include a number that may be assigned to each region (i) of the set of regions **114**A-**114**N. The assigned number may indicate a minimum of steps required to reach any one of the one or more pre-designated regions in the search environment **114** from the corresponding associated with the shortest path table. For example, if the shortest path connecting the pre-designated region to the current region has a length **2**, then the current region may be assigned with the number ‘2’. It may be noted that the minimum number of steps may be between 0 (at the charging station) to [T/2], where T is the path length specified in the first user input of the set of user inputs. In an embodiment, the shortest path table may be constructed using one or more graph-based shortest path planning processes (or techniques). Examples of the one or more graph-based shortest path planning processes may include, but are not limited to, dynamic programming algorithms, a breadth-first search algorithm, or an A* algorithm.

In an embodiment, the processor **104** may be configured to compute the path table using the dynamic programming and using the following recursion for computing functions V_k:

$$G \rightarrow \left[0, \left\lfloor \frac{T}{2} \right\rfloor\right],$$

where G is the set of N regions. The processor **104** may set $V_0(i)=0$ if the region i correspond to any one of the one or more pre-designated regions, otherwise, $V_0(i)=+\infty$ (in implementation a large number).

The processor **104** may further compute V_k using the following dynamic programming recursion for

$$k = \left\{1, 2, \ldots, \left\lfloor \frac{T}{2} \right\rfloor\right\}:$$

$$V_k(i) = 1 + \min_{i^+ \in Neighbor(i)} V_{k-1}(i^+),$$

for every region, i in the set of regions **114**A-**114**N that may not be covered by an obstacle with Neighbor(i) may be the set of regions **114**A-**114**N that may be visited by at least one search agent at region i. By definition, Neighbor(i) is always a subset of G. Some embodiments are based on the realization that the set Neighbor(i) can encode any physical constraints on the motion of the search agents **116**. As discussed above, the processor **104** may set $V_0(i)=0$ if the region i may correspond to the pre-designated region and set $V_0(i)=+\infty$ (in implementation a very large number) for the remaining regions (i.e. the regions covered by an obstacle). The desired shortest path table map may be given by

$$V_{\left\lfloor \frac{T}{2} \right\rfloor}.$$

At **308**, a first set of paths may be generated. In an embodiment, the processor **104** may be configured to randomly generate the first set of paths that may satisfy the one or more path constraints by constructing a sequence of regions starting from a randomly selected pre-designated region of the one or more pre-designated regions in the search environment **122** based on execution of hill ascend operation and hill descend operation.

The processor **104** may be further configured to randomly generate the first set of paths (also referred to as C feasible paths) by randomly choosing any pre-designated region, denoted by R, and executing a hill ascend operation according to the shortest path table for a first half of the path length. The hill ascend operation may refer to an optimization technique that may be used to find the optimal values for the parameters of a model. The goal is to climb the “hill” of the objective function

$$\text{(here, } V_{\left\lfloor \frac{T}{2} \right\rfloor}\text{)}$$

by moving towards the highest point for maximization. Specifically, the processor **104** may execute the hill ascend operation for

$$\left\lfloor \frac{T}{2} \right\rfloor$$

steps while breaking ties randomly, where at each iteration it appends to the currently constructed path (originating from R) a region in the set Neighbor(i) where i is its current location that has the highest value in

$$V_{\left\lfloor \frac{T}{2} \right\rfloor}.$$

The processor **104** may further execute a hill descend operation according to the shortest path table for a second half of the path length to reach one of the one or more pre-designated regions. By construction, all these paths may satisfy one or more path constraints, including the requirements that it originates and terminates in one of the pre-designated regions while ensuring that the path does not fly over the obstacle regions and the path length is no longer than the maximum path length T.

At **310**, the set of paths **118** (or a set of feasible paths) may be selected. In an embodiment, the processor **104** may be configured to select the set of paths **118** from the first set of paths based on the execution of the hill ascend operation and the hill descend operation. Specifically, the processor **104** may be configured to use one or more greedy algorithms to select the set of paths **118** that may cover the search environment **114**. Specifically, the control system **102** may sequentially select feasible paths from the first set of paths constructed at block **306** that may have the least overlap with the selected set of paths. Such one or more greedy algorithms may be known to return a minimal collection of paths that cover the search environment **114** with bounded sub-optimality. In an embodiment, the processor **104** may be configured to select the minimal number of paths, as the set of paths **118** that together cover the search environment **114**, from the first set of paths using existing algorithms for set-cover problems like the greedy algorithms. Specifically, the greedy algorithm starts with a second set of paths initialized with a randomly chosen path from the first set of paths, and then repeats the following steps until a termination criterion is met—first, evaluate the overlap among each one of paths in the first set to the cover generated by the paths in second set, second, remove the path in the first set that has the least overlap from the first set, and third, add the removed path to the second set. A commonly used termination criterion for the greedy algorithm is that either the first set is empty or the collection of paths in the second set together covers the search environment. Details about the selected set of paths are provided, for example, in FIG. **4**.

FIG. **4** is a diagram that depicts the generated set of paths in the search environment, in accordance with an embodiment of the disclosure. FIG. **4** is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**, and FIG. **3**. The path length of each path may be set as eight (8) or the path length may include at most eight (8) regions. With reference to FIG. **4**, there is shown a diagram **400** that depicts the search environment **114** partitioned into the set of regions **114**A-**114**N. With reference to FIG. **4**, there is further shown a first pre-designated region **402**A, a second pre-designated region **402**B, a third pre-designated region **402**C, and a fourth pre-designated region **402**D (hereinafter collectively referred to as one or more pre-designated regions **402**). There is further shown a first restricted region **404**A, and a second restricted region **404**B (hereinafter collectively referred to as one or more restricted regions **404**). There is further shown a first path **406**A, a second path **406**B, a third path **406**C, a fourth path **406**D, a fifth path **406**E, a sixth path **406**F, a seventh path **406**G, and an eighth path **406**H (hereinafter collectively referred to as a set of paths **406**) that cover the search environment **114**.

As discussed above, each of the one or more pre-designated regions **402** may correspond to a region from where each path of the set of paths **406** may start and/or end. Each of the one or more pre-designated regions **402** may correspond to one of the energy refueling station, the calibration station, the service station, or the docking station from the set of search agents **116**. Each search agent of the set of search agents **116** may be charged, calibrated (or sensor calibration), serviced, or docked at any one of the one or more pre-designated regions **402**.

Each of the one or more restricted regions **404** may correspond to a region where any search agent of the set of search agents **116** may not be allowed to visit or must not visit. As an example, the one or more restricted regions may correspond to a military installation. Other examples of restricted regions **404** may include obstacles like tall buildings or trees and GPS-denied areas in the search environment.

Each path of the set of paths **406** may correspond to a combination of at least two regions and may be formed based on the one or more path constraints. The one or more path constraints may include the path length for each path of the set of paths **406**, each path of the set of paths **406** must start and/or ends at any pre-designated region of one or more pre-designated regions **402**, and any path must not include the one or more restricted regions **404**. Based on the one or more path constraints, the processor **104** may be configured to generate the set of paths **406** to cover the search environment **114**. As discussed above, the set of paths **406** may include the first path **406**A, the second path **406**B, the third path **406**C, the fourth path **406**D, the fifth path **406**E, the sixth path **406**F, the seventh path **406**G, and the eighth path **406**H. The set of paths may also be referred to as the set of feasible paths or feasible paths, hereinafter.

The first path **406**A may originate from the first pre-designated region **402**A and may terminate at the fourth pre-designated region **402**D and may have the path length of 8. The second path **406**B may originate from the first pre-designated region **402**A and may terminate at the first pre-designated region **402**A and may have the path length of 6. The second path **406**B is illustrated in a dashed-dotted line to distinguish it from others. The third path **406**C may originate and terminate at the second pre-designated region **402**B and may have the path length of 6. The fourth path **406**D may originate and terminate at the third pre-designated region **402**C and may have the path length of 8. The fourth path **406**D is illustrated in a dashed line to distinguish it from others. The fifth path **406**E may originate and terminate at the third pre-designated region **402**C and may have the path length of 8. The sixth path **406**F may originate from the first pre-designated region **402**A and may terminate at the third pre-designated region **402**C and may have the path length of 7. The seventh path **406**G may originate from the first pre-designated region **402**A and may terminate at the second pre-designated region **402**B and may have the path length of 5. The seventh path **406**G is illustrated in a dashed line to distinguish it from others. The eighth path **406**H may originate from the fourth pre-designated region **402**D and may terminate at the third pre-designated region **402**C and may have a path length of 7.

In an embodiment, the set of paths **406** may be dependent on locations of the one or more pre-designated regions **402** and locations of the one or more restricted regions **404**. It may also be possible that for a given configuration of the one or more pre-designated regions **402**, the one or more restricted regions **404**, and a maximum path length T, there may be no set of feasible paths **406** that may cover the entire search environment **114**. In such a situation, the one or more pre-designated regions **402** may be updated at appropriate locations to rectify such a situation. For example, the control system **102** may be configured to update the locations of one or more pre-designated regions **402** within (or outside) the search environment **114** to determine the set of paths **406**. Alternatively, the search environment could be shrunk to only include regions that are covered by at least one path identified in **310**.

FIG. **5** is a diagram that depicts confidence bounds associated with each region of the set of regions of the search environment, in accordance with an embodiment of the disclosure. FIG. **5** is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, and FIG. **5**. With reference to FIG. **5**, there is shown a diagram **500** that includes the control system **102**, the search environment **114** partitioned into the set of regions **114A**-**114N**, and the set of search agents **116**. There is further shown confidence bounds **502** associated with each region (i) of the set of regions **114A**-**114N**. The confidence bounds **502** may include an upper confidence bound (UCB) **502**A, and a lower confidence bound (LCB) **502**B.

As discussed in FIG. **1**A, the upper confidence bound (UCB) **502**A may correspond to a statistical method that may be used to estimate an upper limit or maximum value of potential outcomes with a certain level of confidence. The lower confidence bound (LCB) **502**B may be the opposite of UCB **502**A and may be used to estimate the lower limit or minimum value with a certain level of confidence. Together, the UCB **502**A and the LCB **502**B may help determine a range within which the true value of a variable lies, allowing decision-makers to make informed choices considering the level of uncertainty involved.

While any valid confidence bound suffices, the control system **102** (or the remote server **110**) may use the following bounds to bound the unknown Bernoulli parameter μ given M realizations $\{u_j\}_{j=1}^{M}$ of a Bernoulli variable for the search environment **114** with N regions. The UCB **502**A and the LCB **502**B may be calculated using equation (1) and equation (2) respectively.

$$UCB = \frac{\sum_{j=1}^{M} u_j}{M} + 2\sqrt{\frac{2\ \log(\log_2(2M)) + \log(12N/\delta)}{2M}} \tag{1}$$

$$LCB = \frac{\sum_{j=1}^{M} u_j}{M} - 2\sqrt{\frac{2\ \log(\log_2(2M)) + \log(12N/\delta)}{2M}} \tag{2}$$

It may be noted that the UCB **502**A and the LCB **502**B may be constructed using the sample mean of the M realizations and a correction term that may approach zero as M increases. These constructed bounds are such that upon termination, the proposed approach, which classified regions using these bounds and confidence threshold, returns a valid classification of the search environment **114** with a probability no smaller than 1-δ, where δ may be the probability of unreliability. Typically, the chosen probability of unreliability δ may be set to a small number.

In the apple orchard example described above, each region is an apple tree, and the Bernoulli variable associated with the region can be used to model the output of the noisy measurement when a search agent visits the tree. The noisy measurement could say that the tree is ready for harvesting or not with some unknown probability μ, the true mean of the Bernoulli variable. To classify the tree as ready for harvesting, it suffices to have the true mean above a confidence threshold. However, since the true mean is unknown, we use upper confidence bound and lower confidence bounds as a surrogate for the true mean to arrive at a valid classification. Specifically, by when the upper confidence bound is below a confidence threshold, it is evident that the true mean is also below the confidence threshold with high probability, indicating that the tree is not ready for harvesting with high probability. On the other hand, when the lower confidence bound is above a confidence threshold, it is evident that the true mean is also above the confidence threshold with high probability, indicating that the tree is ready for harvesting with high probability. Here, M in (1) and (2) refers to the number of visits of the particular apple tree by the search team, and $\{u_j\}_{j=1}^{M}$ indicate the binary measurements associated with each visit by the search team where u_j is one (1) when the search team measured it as ready-to-harvest, and u_j is zero (0) otherwise.

The control system **102** may be configured to collect the confidence bounds **502** of the probabilistic classification **110**A of each region of the set of regions **114**A-**114**N. Specifically, the control system **102** may be configured to collect the confidence bounds **502** of the probabilistic classification **110**A of at least one region within at least one path of the set of paths **118**. As shown in FIG. **5**, each region (i) may have the associated UCB **502**A and LCB **502**B at any point of time based on the measurements collected so far. For example, in the first region **114**A (when i=1), the value of UCB **502**A may be 0.75, and the value of LCB **502**B may be 0.125. As another example, the value of UCB **502**A and the value of LCB **502**B for the second region (when i=2) may be 0.81 and 0.13, respectively. Similarly, the UCB **502**A and the LCB **502**B associated with the Nth region **114**N (when i=N) may be 0.62 and 0.22, respectively.

In an embodiment, the control system **102** may be configured to assign an initial value of each of the UCB **502**A and the LCB **502**B to each region of the set of regions **114**A-**114**N. These initial values could be used to encode prior information about the search environment **116**, or could be The values of the UCB **502**A and the LCB **502**B may be updated iteratively based on the measurements collected by the set of search agents **116** as described in the above figures.

FIG. **6**A is a flowchart **600**A that illustrates an exemplary first method for the selection of a path from the set of paths **118** to be visited by at least one search agent, in accordance with an embodiment of the disclosure. FIG. **6**A is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, and FIG. **5**. With reference to FIG. **6**A, there is shown the flowchart **600**A. The operations of the exemplary method may be executed by any computing system, for example, by the control system **102** or the processor **104**, or the remote server **110** of FIG. **1**A. The operations of the flowchart **600**A may start at **602**.

At **602**, a pre-determined aggregation function may be evaluated. In an embodiment, the processor **104** may be configured to evaluate the pre-determined aggregation function of the values of the confidence bounds **502** of each region within the corresponding path. In an embodiment, the processor **104** may be configured to evaluate the pre-determined aggregation function for each path of the set of paths **118** in the search environment **114**. The pre-determined aggregation function (also referred to as a summary function or a combining function) may be a type of function that may calculate a single value from the values of the confidence bounds **502** of each region within the corresponding path. In an embodiment, the pre-determined aggregation function may correspond to at least one of an addition function, an average function, a maximum function, or a minimum function. The sum function may calculate a total value of the confidence bounds **502** of each region within the corresponding path. The average function calculates an average value of the confidence bounds **502** of each region within the corresponding path. The maximum and minimum functions return the largest and smallest values of the confidence bounds **502** of each region within the corresponding path, respectively. In an embodiment, the sum function may be used only if the path length of each path of the set of paths **118** is same. Otherwise, the sum function may generate results that may be biased towards the paths that have greater path length as compared to other paths. For example, if the path length of a first path is 6 and the path length of all other paths is less than 6, then the result of the sum function will always be biased towards the first path because the sum value for the first path may be greater than the sum value of all the other paths.

At **604**, the evaluation of the pre-determined aggregation function for each of the set of paths **118** may be sorted. In an embodiment, the processor **104** may be configured to sort the evaluation of the pre-determined aggregation function for each of the set of paths **118** in a descending order of a function value for each path of the set of paths **118**. In an embodiment, the function value may be determined based on an application of the pre-determined aggregation function on the confidence bounds **502** of each region in each path of the set of paths **118**. By way of example, if a count of the set of paths **118** is 11, then there may be 11 functional values that may be sorted in the descending order.

At **606**, at least one path may be selected for at least one search agent. In an embodiment, the processor **104** may be configured to select at least one path to be visited by the set of search agents **116**. The selection of the at least one path may be based on the sorting of the evaluation of the pre-determined aggregation function. In an embodiment, the processor **104** may be configured to select as many paths from the top of the sorted list as a count of the set of search agents **116**. For example, if there are 3 search agents in the set of search environment **114**, then the first 3 paths may be selected from the top of the sorted list. The selected top 3 paths may be randomly assigned to any search agent of the set of search agents **116**. In an embodiment, the first path may be selected to be visited by the first search agent **116**A based on the sorting. Control may pass to the end.

In an embodiment, the selection of at least one path to be visited by the set of search agents **116** various over each iteration of **600**A. Specifically, after execution of the paths, the confidence bounds associated with each region may change, which in turn affects the aggregate value in **602**, the sorting arrangement in **604**, and the selected paths in **606** at the next iteration. Since the updates can be done immediately or over multiple iterations to give time for the search team to upload the measurements to the remote server, and the updates themselves depend on the collected date, the sequence of selected paths is stochastic and depends on a variety of factors including the search environment **114**, the path length T, the locations of the pre-designated areas **402**, the restricted regions **404**, and the data collected by the search team.

FIG. **6**B depicts a diagram to illustrate an exemplary scenario **600**B for selecting paths to be visited by at least one search agent, in accordance with an embodiment of the disclosure. FIG. **6**B is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, and FIG. **6**A. With reference to FIG. **6**B, there is shown an exemplary scenario **600**B. The exemplary scenario **600**B may include the control system **102**, the search environment **114** partitioned into a set of regions **114**A-**114**N, and the set of search agents **116** including the first search agent **116**A, and the second search agent **116**B. With reference to FIG. **6**B, there is further shown the set of paths **118** including the first path **118**A, the second path **118**B, the third path **118**C and the fourth path **118**D.

The system **102** may be configured to evaluate a pre-determined aggregation function **608** of the values of the confidence bounds of each region within the corresponding path. The pre-determined aggregation function **608** may be evaluated for each path of the set of paths **118**. As discussed above, the pre-determined aggregation function **608** may correspond to at least one of the addition function, the average function, the maximum function, or the minimum function. For example, if the pre-determined aggregation function **608** is the average function, the control system **102** may be configured to evaluate the average of the values of the confidence bounds of each region within each path of the set of paths **118**. Based on the evaluation, it may be determined that the first path **118**A may have the highest (or maximum) average value, the second path **118**B may have a second highest (or second maximum) average value, the third path **118**B may have a third highest (or third maximum) average value, and the fourth path **118**D may have a fourth highest (or fourth maximum) average value. Since the first path **118**A may have the highest average value, the control system **102** may assign the first path **118**A to the first search agent **116**A. In another embodiment, the control system **102** may be further configured to assign the second path **118**B to the second search agent **116**B as the average value for the second path **118**B may be maximum after the first path **118**A.

In an embodiment, after a first epoch, the control system **102** may be configured to select a next region to visited of the set of regions **114A-114N**. The next region may lie within the same path as that of a current region of the set of regions **114A-114N**. In another embodiment, the next region may lie in a different path as that of the current region of the set of regions **114A-114N**.

FIG. **6**C is a flowchart **600**C that illustrates an exemplary second method for the selection of a path from the set of paths **118** to be visited by at least one search agent, in accordance with an embodiment of the disclosure. FIG. **6**C is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, and FIG. **6**B. With reference to FIG. **6**C, there is shown the flowchart **600**C. The operations of the exemplary method may be executed by any computing system, for example, by the control system **102** or the processor **104**, or the remote server **110** of FIG. **1**A. The operations of the flowchart **600**C may start at **610**.

At **610**, the confidence bounds **502** of each region in each path of the set of paths **118** may be evaluated. In an embodiment, the confidence bounds **502** of each region in each path of the set of paths **118** may be evaluated using a neural network. The neural network may output the confidence bounds **502** of each region in the corresponding path of the set of paths **118**. The neural network may be trained using machine learning to assign an aggregate value for the confidence bound **502** based on the data collected for each region within the path. Specifically, the weights of the neural network are trained by minimizing a loss function that encodes various desirable behaviors in the generation of the aggregate value, including providing higher evaluations for more informative paths, aggregating the confidence bounds of the regions, etc.

FIG. **6**D depicts a diagram to illustrate an exemplary scenario **600**D for selecting paths to be visited by at least one search agent, in accordance with an embodiment of the disclosure. FIG. **6**D is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, and FIG. **6**C. With reference to FIG. **6**D, there is shown an exemplary scenario **600**D. The exemplary scenario **600**D may include the control system **102**, the search environment **114** partitioned into a set of regions **114A-114**N, and the set of search agents **116** including the first search agent **116**A, and the second search agent **116**B. With reference to FIG. **6**D, there is further shown a neural network **616**, and the set of paths **118** including the first path **118**A, the second path **118**B, the third path **118**C and the fourth path **118**D.

The neural network **616** may be a computational network or a system of artificial neurons, arranged in a plurality of layers, as nodes. The plurality of layers of the neural network **616** may include an input layer, one or more hidden layers, and an output layer. Each layer of the plurality of layers may include one or more nodes (or artificial neurons). Outputs of all nodes in the input layer may be coupled to at least one node of hidden layer(s). Similarly, inputs of each hidden layer may be coupled to outputs of at least one node in other layers of the neural network **616**. Outputs of each hidden layer may be coupled to inputs of at least one node in other layers of the neural network **616**. Node(s) in the final layer may receive inputs from at least one hidden layer to output a result. The number of layers and the number of nodes in each layer may be determined from hyper-parameters of the neural network **616**. Such hyper-parameters may be set before or while training the neural network **616** on a training dataset.

Each node of the neural network **616** may correspond to a mathematical function (e.g., a sigmoid function or a rectified linear unit) with a set of parameters, tunable during training of the network. The set of parameters may include, for example, a weight parameter, a regularization parameter, and the like. Each node may use the mathematical function to compute an output based on one or more inputs from nodes in other layer(s) (e.g., previous layer(s)) of the neural network **616**. All or some of the nodes of the neural network **616** may correspond to a same or a different same mathematical function.

In training of the neural network **616**, one or more parameters of each node of the neural network **616** may be updated based on whether an output of the final layer for a given input (from the training dataset) matches a correct result based on a loss function for the neural network **616**. The above process may be repeated for the same or a different input until a minima of loss function may be achieved, and a training error may be minimized. Several methods for training are known in art, for example, gradient descent, stochastic gradient descent, batch gradient descent, gradient boost, meta-heuristics, and the like.

The neural network **616** may include electronic data, such as, for example, a software program, code of the software program, libraries, applications, scripts, or other logic or instructions for execution by a processing device, such as processor **104**. The neural network **616** may include code and routines configured to enable a computing device, such as the control system **102** to perform one or more operations. Additionally or alternatively, the neural network **616** may be implemented using hardware including a processor, a microprocessor (e.g., to perform or control performance of one or more operations), a field-programmable gate array (FPGA), or an application-specific integrated circuit (ASIC). Alternatively, in some embodiments, the neural network **616** may be implemented using a combination of hardware and software.

The system **102** may be configured to evaluate the confidence bounds **502** of each region in each path of the set of paths **118** using the neural network **616**. In an embodiment, the confidence bounds **502** of each region in each path of the set of paths **118** may be provided as an input to the neural network **616**. The neural network **616** may intake the confidence bounds **502** of each region in the corresponding path of the set of paths **118**.

FIG. **6**E is a flowchart **600**E that illustrates an exemplary third method for the selection of a path from the set of paths **118** to be visited by at least one search agent, in accordance with an embodiment of the disclosure. FIG. **6**E is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, and FIG. **6**D. With reference to FIG. **6**E, there is shown a flowchart **600**E. The operations of the exemplary method may be executed by any computing system, for example, by the control system **102** or the processor **104** of FIG. **1**A. The operations of the flowchart **600**E may start at **618**.

At **618**, a selection between a most-promising path criterion and a minimal-movement criterion may be made. In an embodiment, the processor **104** may be configured to probabilistically select between different selecting criteria including the most-promising path criterion and a minimal-movement criterion. The most-promising path criterion may correspond to the selection of the most promising path from the set of paths **118** to be traversed by at least one search agent of the set of search agents **116**. In an embodiment, the most-promising path criterion may be similar to an execution of a multi-arm bandit (MAB) algorithm, where the path is selected based on an aggregate function of the upper confidence bounds. Since the goal of some embodiments is to quickly identify the interesting regions, selecting paths with the high upper confidence bounds introduces optimism in the face of uncertainty that enables providing probabilistic guarantees of performance.

The minimal-movement criterion may correspond to the execution of a label-then-move search. The label-then-move search may be a naive search algorithm that may traverse the search environment **114** and moves out of each region only after classifying it. But, such an approach may require minimal movements from the search agents. A drawback of label-then-move search is that it may ignore the data collected online in deciding where to sense next. Subsequently, the label-then-move search may take a long time to identify all the interesting regions. Some embodiments are based on the intuition that a combination of the multi-arm bandit search method and the label-then-move search method may get synergy in reducing both the time and cost of the search operation.

In an embodiment, the probability of selection between the most-promising path criterion and the minimal-movement criterion may be a biased probability. The biased probability may refer to a situation where the likelihood of certain outcomes or events is intentionally or unintentionally skewed in favor of one particular outcome. In an embodiment, the designer may choose the probability of selection to be biased towards either the most-promising path criterion, the minimal-movement criterion, or be unbiased.

In an embodiment, a bias of the biased probability varies between at least some control steps. The processor **104** may be further configured to update the bias based on the likelihood of interest at different regions of the set of regions **114**A-**114**N in the search environment **114**.

At **620**, at least one path to be visited by the set of search agents **116** may be selected. In an embodiment, the processor **104** may be configured to select the at least one path to be visited by the set of search agents **116** based on the selected criterion. In case of the most-promising path criterion, the selected path may correspond to the path for which the probability of classification of at least one region may be higher as compared to other paths. In another embodiment, if the minimum-movement criterion is selected, the path that corresponds to a nearest path with minimum cost constraints may be selected. In an embodiment, the processor **104** may be further configured to update the confidence bounds **502** of the different regions with a neural network that may be trained with machine learning to produce the biased probability.

FIG. **7**A is a flowchart **700**A that illustrates an exemplary first method for updating confidence bounds **502** of at least one region of the set of regions **114**A-**114**N of the search environment **114**, in accordance with an embodiment of the disclosure. FIG. **7**A is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, and FIG. **6**D.

With reference to FIG. **7**A, there is shown the flowchart **700**A. The operations of the exemplary method may be executed by any computing system, for example, by the control system **102** or the processor **104**, or the remote server **110** of FIG. **1**A. The operations of the flowchart **700**A may start at **702**.

At **702**, a pre-determined number of measurements may be received. In an embodiment, the processor **104** may be configured to receive the pre-determined number of measurements associated with the first region **114**A within the selected first path **118**A. The pre-determined number of measurements may correspond to a minimum number of measurements that may be required to update the confidence bounds **502** associated with the first region **114**A. In an embodiment, the pre-determined number of measurements may be based on a type of problem statement and may change based on the problem statement. For example, for a first problem statement, the pre-determined number of measurements may be ten whereas for a second problem statement, the pre-determined number of measurements may be 25. In an embodiment, the pre-determined number of measurements may be collected by the first search agent **116**A of the set of search agents **116**. In another embodiment, the pre-determined number of measurements may be collected by the search agents other than the first search agent **116**A in the set of search agents **116** or a combination thereof.

As discussed above, each measurement of the pre-determined number of measurements may correspond to sensor data that may be captured by the one or more noisy sensors embedded in the set of search agents **116**. For example, in the case of the search-and-rescue operations, the measurement may correspond to an image of the corresponding region captured by an image sensor embedded within the set of search agents **116**.

At **704**, the confidence bounds **502** of the probabilistic classification **110**A of the first region **114**A may be updated. In an embodiment, the processor **104** may be configured to update the confidence bounds **502** of the probabilistic classifications **110**A of the first region **114**A based on the collected pre-determined number of measurements. As discussed above, the confidence bounds **502** of the first region **114**A may include the upper confidence bound **502**A and the lower confidence bound **502**B.

In an embodiment, the confidence bounds **502** of the first region **114**A may be updated using at least one concentration inequality. The concentration inequality may refer to a mathematical concept that may provide bounds on the variability of a random variable around its mean. In an embodiment, the confidence bounds **502** of the first region **114**A may be updated using the bounds given in (1) and (2). In another embodiment, the confidence bounds **502** of the first region **114**A may be updated using the Hoeffding's bound that may characterize an interval around the mean constructed using the measurements that may be guaranteed to contain the true mean with a user-specified confidence probability. In another embodiment, other concentration inequalities such as Chernoff bound, Azuma's inequality, McDiarmid's inequality, Bennett's inequality, Bernstein inequalities, and the like may be used to update the confidence bounds **502** of the first region **114**A.

FIG. **7**B is a flowchart **700**B that illustrates an exemplary second method for updating confidence bounds **502** of at least one region of the set of regions **114**A-**114**N of the search environment **114**, in accordance with an embodiment of the disclosure. FIG. **7**B is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, and FIG. **7**A. With reference to FIG. **7**B, there is shown the flowchart **700**B. The operations of the exemplary method may be executed by any computing system, for example, by the control system **102** or the processor **104**, or the remote server **110** of FIG. **1**A. The operations of the flowchart **700**B may start at **706**.

At **706**, the processor **104** may be configured to transmit a current value of the confidence bounds **502** of the probabilistic classifications **110**A of the first region **114**A, a third set of instructions for taking a pre-determined number of measurements of the first region **114**A, and a fourth set of instructions for updating the current values of the confidence bounds **502** of the probabilistic classifications **110**A of the first region **114**A to at least the first search agent **116**A. In an embodiment, the current value, the third set of instructions, and the fourth set of instructions may be transmitted via the transceiver **108**.

Based on the reception of the third set of instructions, the first search agent **116**A of the set of search agents **116** may be configured to capture the pre-determined set of measurements of the first region **114**A within the first path **118**A of the set of paths **118**. Details about the pre-determined set of measurements are provided, for example, in FIG. **7**A.

After taking the pre-determined number of measurements of the first region **114**A, the first search agent **116**A may be configured to update the confidence bounds **502** of the probabilistic classifications **110**A of the first region **114**A based on the received current value of the confidence bounds **502**, the pre-determined number of measurements, and the received fourth set of instructions. As discussed above, the set of instructions for updating the current values of the confidence bounds **502** of the probabilistic classifications **110**A of the first region **114**A may include instructions to use at least one concentration inequality (such as (1) and (2), or the Hoeffding's bound) to update the current values of the confidence bounds **502**. Details about the concentration inequality are provided, for example, in FIG. **5**A.

After updating the confidence bounds **502** of the first region **114**A, the first search agent **116**A may be configured to transmit the updated confidence bounds **502** to the control system **102**.

At **708**, the updated confidence bounds **502** of the probabilistic classifications **110**A of the first region **114**A may be received. In an embodiment, the processor **104** may be configured to receive the updated confidence bounds **502** of the probabilistic classifications **110**A of the first region **114**A from the first search agent **116**A via the transceiver **108**.

FIG. **8** is a flowchart **800** that illustrates an exemplary method for classifying at least one region of the set of regions **114**A-**114**N of the search environment **114** and pruning the classified region from the corresponding path, in accordance with an embodiment of the disclosure. FIG. **8** is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, FIG. **7**A, and FIG. **7**B. With reference to FIG. **8**, there is shown the flowchart **800**. The operations of the exemplary method may be executed by any computing system, for example, by the control system **102** or the processor **104**, or the remote server **110** of FIG. **1**A. The operations of the flowchart **800** may start at **802**.

At **802**, the updated confidence bounds **502** of the first region **114**A may be compared with a confidence threshold. In an embodiment, the processor **104** may be configured to compare the updated confidence bounds **502** of the first region **114**A within the selected first path **118** with the confidence threshold. The updated confidence bounds **502** of the first region **114**A may be determined after the pre-determined number of measurements associated with the first region **114**A may be received. Details about the updated confidence bounds **502** are provided, for example, in FIG. **7**A, and FIG. **7**B.

At **804**, the first region **114**A within the selected first path **118**A may be classified. In an embodiment, the processor **104** may be configured to classify the first region **114**A within the first path **118**A based on the comparison. The first region **114**A may be classified as an interesting region or an uninteresting region. Specifically, the control system **102** may be configured to classify the first region **114**A as interesting with a high probability whenever the lower confidence bound may be greater than the confidence threshold. Similarly, the control system **102** may classify the first region **114**A as uninteresting with a high probability whenever the upper confidence bound may be less than the confidence threshold. Details about the classification of the first region **114**A are provided, for example, in FIG. **8** and FIG. **13**.

At **806**, the first path **118**A may be updated. In an embodiment, the processor **104** may be configured to update the first path **118**A. The first path **118**A may be updated to prune the first region **114**A from the first path **118**A based on the classification of the first path as the interesting region or the uninteresting region. In case, the first region **114**A is not classified as the interesting region or the uninteresting region, the control system **102** may be configured to again repeat the steps described in FIG. **7**A or FIG. **7**B until the first region **114**A is classified as the interesting region or the uninteresting region.

In an embodiment, despite pruning regions that are classified, the set of paths remain unchanged and only the aggregation function in **602** is updated to exclude classified regions. Alternatively, the set of paths are updated as explained in FIG. **9**.

FIG. **9** is a diagram that shows an updated set of paths in the search environment **114** based on the classification of the first region, in accordance with an embodiment of the disclosure. FIG. **9** is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, FIG. **7**A, FIG. **7**B, and FIG. **8**. With reference to FIG. **9**, there is shown a diagram **400** that depicts search environment **114** that may be partitioned into the set of regions **114A**-**114**N. With reference to FIG. **9**, there is further shown the one or more pre-designated regions **402**, the one or more restricted regions **404**, and an updated first path **902**A of the set of paths **406**.

In an embodiment, the processor **104** may be configured to prune the first region **114**A from the first path **902**A. As shown in FIG. **9**, the updated first path **902**A may originate from the first pre-designated region **402**A and may terminate at the fourth pre-designated region **402**D and may have the path length of 7. Similarly, each region that may be classified may be pruned from their corresponding paths. The method may end when each region is classified as the interesting region or the uninteresting region. Specifically, the termination condition may be met when each region of the set of regions **114A**-**114**N may be classified as the interesting region or the uninteresting region.

FIG. **10** is a flowchart **1000** that illustrates an exemplary method for controlling search agents to perform search with noisy observations and probabilistic guarantees, in accordance with an embodiment of the disclosure. FIG. **10** is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, FIG. **7**A, FIG. **7**B, FIG. **8**, and FIG. **9**. With reference to FIG. **10**, there is shown the flowchart **1000**. The operations of the exemplary method may be executed by any computing system, for example, by the control system **102** or the processor **104**, or the remote server **110** of FIG. **1**A. The operations of the flowchart **1000** may start at **1002**.

At **1002**, the confidence bounds **502** of the probabilistic classification **110**A of at least one region within at least one path of the set of paths **118** may be collected. In an embodiment, the processor **104** may be configured to collect the confidence bounds **502** of the probabilistic classification **110**A of at least one region within at least one path of the set of paths **118**. In an embodiment, the confidence bounds **502** may be collected from the set of search agents **116**. In another embodiment, the processor **104** may be configured to collect the measurements captured by the set of search agents **116** and determine the confidence bounds **502** of the probabilistic classification **110**A of at least one region within at least one path of the set of paths **118**. Details about the confidence bounds **502** are provided, for example, in FIG. **1**A, and FIG. **5**.

At **1004**, aggregations of the confidence bounds **502** may be compared. In an embodiment, the processor **104** may be configured to compare the aggregations of the confidence bounds **502** of the probabilistic classifications **110**A of each path of the set of paths **118** based on the collected confidence bounds **502**. In an embodiment, the aggregation of the confidence bounds **502** may be determined based on the application of the pre-determined aggregation function on the set of confidence bounds **502**. Details about the comparison of aggregations of the confidence bounds **502** are provided, for example, in FIG. **5**.

At **1006**, the first path **118**A of the set of paths **118** may be selected. In an embodiment, the processor **104** may be configured to select the first path **118**A of the set of paths **118** to be visited by the first search agent **116**A based on the comparison. In an embodiment, the selection of the first path **118**A may be based on a sorted arrangement of the evaluation of the pre-determined aggregation function along each path. In another embodiment, the selection of the first path **118**A may be based on a sorted arrangement of the evaluation of the path by a neural network. In another embodiment, the selection of the first path **118**A may be based on probabilistic selection between different selecting criteria. Details about the selection of the first path **118**A are provided, for example, in FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, and FIG. **6**E.

In an embodiment, the processor **104** may be configured to select the first path **118**A and the second path **118**B from the set of paths **118** based on the aggregations of the probabilistic classifications of each of the set of paths **118**. The processor **104** may be further configured to control the movement of the first search agent **116**A and the second search agent **116**B of the set of search agents **116**B concurrently on the selected first path **118**A and the second path **118**B to take measurements associated with the corresponding regions within the first path **118**A and the second path **118**B.

At **1008**, the first search agent **116**A may be commanded. In an embodiment, the processor **104** may be configured to command, via the transceiver **108**, the first search agent **116**A to visit the selected first path **118**A and to collect measurements associated with each region within the selected first path **118**A. The collected measurements to be collected may be based on the problem statement. Details about the measurements are provided, for example, in FIG. **7**A.

At **1010**, the confidence bounds **502** may be updated. In an embodiment, the processor **104** may be configured to update the confidence bounds **502** of the probabilistic classifications **110**A of each region within the selected first path **118**A based on the measurements associated with the corresponding regions. Details about updating the confidence bounds **502** are provided, for example, in FIG. **7**A and FIG. **7**B.

At **1012**, it may be determined whether a termination condition is met or not. The termination condition may correspond to the classification of each region of the set of regions **114**A-**114**N in the search environment **114**. In case each region is classified, it may be deemed that the termination condition is met, and the control may pass to end **1014**. Otherwise, the control may pass to **1002**.

FIG. **11** is a network environment for a remote server controlling search agents to perform a search with noisy observations and probabilistic guarantees, in accordance with an embodiment of the disclosure. FIG. **11** is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, FIG. **7**A, FIG. **7**B, FIG. **8**, FIG. **9**, and FIG. **10**. With reference to FIG. **10**, there is shown a network environment **1100** that may include a remote server **1102**, a first control system **1104**A, a second control system **1104**B, a first control system **1104**C (hereinafter referred to as a set of control systems **1104**). controlling a first search agent **1106**A, a second search agent **116**B, a third search agent **116**C (hereinafter referred to as a set of search agents **1106**) deployed in a search environment **1108** partitioned into a set of regions **1108**A-**1108**N.

In an embodiment, the search environment **1108** may further include a first pre-designated region **1110**A, a second pre-designated region **1110**B, a third pre-designated region **1110**C, and a fourth pre-designated region **1110**D (hereinafter referred to as one or more pre-designated regions **1110**). There is further shown a first restricted region **1112**A, and a second restricted region **1112**B (hereinafter referred to as one or more restricted regions **1112**), and a first interesting region **1114**A, a second interesting region **1114**B, and a third interesting region **1114**C (hereinafter referred to as one or more interesting regions **1114**).

The remote server **1102** may be an exemplary embodiment of the remote server **110** of FIG. **1**A. Each of the set of control systems **1104** may be an exemplary embodiment of the control system **102** of FIG. **1**A. The set of search agents **1106** may be an exemplary embodiment of the set of search agents **116** of FIG. **1**A. The search environment **1108** may be an exemplary embodiment of the search environment **114** of FIG. **1**A. The set of regions **1108**A-**1108**N may be an exemplary embodiment of the set of regions **114**A-**114**N of FIG. **1**A. The one or more pre-designated regions **1110** and the one or more restricted regions **1112** may be an exemplary embodiment of the one or more restricted regions **402** and the one or more restricted regions of **404** FIG. **4** respectively.

The set of control system **1104** may include the first control system **1104**A controlling the first search agent **1106**A of the set of search agents **1106**. Similarly, the set of control systems **1104** may further include the second control system **1104**B and the third control system **1104**C controlling the second search agent **1106**B and the third search agent **1106**C of the set of search agents **1106**. The one or more pre-designated regions **1110** may include the first pre-designated region **1110**A, the second pre-designated region **1110**B, the third pre-designated region **1110**C, and the fourth pre-designated region **1110**D. The one or more restricted regions **1112** may include the first restricted region **1112**A, and the second restricted region **1112**B. Similarly, the one or more interesting regions **1114** may include the first interesting region **1114**A, the second interesting region **1114**B, and the third interesting region **1114**C.

In an embodiment, the remote server **1102** may be configured to partition the search environment **1108** in the set of regions **1108**A-**1108**N. In an embodiment, each region of the set of regions **1108**A-**1108**N may be of the same dimensions and same shapes. In another embodiment, each region of the set of regions **1108**A-**1108**N may be of different dimensions and different shapes. Each of the one or more pre-designated regions **1110** may correspond to an energy refueling station, a calibration station, a service station, or a docking station. Each of the one or more restricted regions **1112** may have to be avoided by the set of search agents **1106** and the one or more interesting regions **1114** may be classified as interesting regions by the remote server **1102** or by at least one control system of the set of control system **1104**. In another embodiment, the one or more interesting regions **1114** may correspond to a region that may include at least one object of interest. In an embodiment, if the region includes at least one object of interest, the region may be deemed as an interesting region whereas if the region does not include at least one object of interest, then the region may be deemed as an uninteresting region. For example, in the case of search-and-rescue operations, the object of interest may be animated objects such as humans, or animals.

In an embodiment, the following modeling assumptions may be made to make the search problem tractable and practical.

It may be assumed that the search environment **1108** may be described by the set of regions **1108**A-**1108**N.

Each of the set of search agents **1106** may be constrained to move only to their neighboring grid cells (or regions) at any given time. For example, the first search agent **1106**A may be able to move to regions **1108**A, **1108**B, **1108**C, **1108**D, **1108**E, or **1108**F or to the second pre-designated region **1110**B or to any interesting region of the one or more interesting regions **1114**.

The movement of each of the set of search agents **1106** due to the energy requirements and no-fly-zone requirements may be constrained to the set of paths that includes a sequence of regions to visit, that:

a. Originate from one of the one or more pre-designated regions **1110**, and
b. Terminate at one of the one or more pre-designated regions **1110**, and
c. Are no longer than a user-specified maximum path length ‘T’ and
d. Do not pass through the one or more restricted regions **1112**.
e. Each path of the set of paths that satisfies the requirements 3a, 3b, 3c, and 3d may be referred to as a feasible path and may be included in the set of paths **114**.

The set of search agents **1106** may communicate easily with the remote server **1102** or their corresponding control system of the set of control systems **1104**.

FIG. **12** illustrates the sensing assumptions for the classification of regions within the search environment, in accordance with an embodiment of the disclosure. FIG. **11** is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, FIG. **7**A, FIG. **7**B, FIG. **8**, FIG. **9**, FIG. **10**, and FIG. **11**. With reference to FIG. **12**, there is shown a diagram **1200** of a sensing block **1202** integrated within each of the set of control systems **1104** or the remote server **1102**. Each region (or the grid cell) visited by the set of search agents **1106** may be either an interesting region or an uninteresting region. In an embodiment, each region of the set of regions **1108**A-**1108**N may be of a pre-determined dimension. The one or more noisy sensors embedded within the set of search agents **1106** may be controlled by the remote server **1102** via the set of control systems **1104** to capture measurements associated with the corresponding region and transmit the captured measurements to the remote server **1102** via the set of search agents **1106**. The remote server **110** may classify each region of the set of regions **1108**A-**1108**N in the search environment **1108** as the interesting region or the uninteresting region based on the captured measurements and/or the confidence bounds constructed.

For example, in the search-and-rescue operation, the one or more noisy sensors may include the image capture sensor that may be installed on at least one of the set of search agents **1106**. The remote server **110** may control the image capture sensor to capture one or more images (i.e. the measurements) of the region upon visiting the corresponding region. The region may be interesting (humans that need rescuing are present in the area) **1204** or uninteresting (no humans that need rescuing are present in the area) **1206**.

In an embodiment, the remote server **1102** may be configured to execute an image classification algorithm to determine whether one or more humans may be visible in the captured one or more images, and consequently decide if the corresponding region may be sensed to be interesting **1208** or uninteresting **1210**. In another embodiment, the image classification algorithm may be executed on the set of search agents **1106**. However, the captured one or more images may be of a low resolution due to cost and weight requirements on the image capture sensor on the set of search agents **1106**, and additionally, the image classification algorithm used may produce erroneous results. Consequently, the sensing block **1202** may provide an accurate inference **1214** and **1212** or an inaccurate inference **1216** and **1218**.

It may be hard to obtain the likelihood of the occurrences of **1212**, **1214**, **1216**, and **1218**, since the likelihood of the occurrences may depend on a variety of factors including a quality of the image capture sensor and an accuracy of the image classification algorithm. It may be noted that some of the works known in the art assume the knowledge of such likelihoods, whereas the present disclosure focuses on the case where these likelihoods are apriori unknown.

It may also be assumed that whether the region may be sensed (or classified) to be interesting or uninteresting may be based on a Bernoulli random variable associated with that region. The Bernoulli random variable may be a random variable that may return either 0 or 1. The remote server **1102** may denote a likelihood of returning 1 by a Bernoulli parameter $\mu \in [0, 1]$. The Bernoulli random variable may be considered as a mathematical model of a coin toss that may be biased to yield head (return 1) with probability μ. It may be also known that the mean of the Bernoulli random variable may also be μ.

It may be assumed that each region i in the search environment **1108** may have a Bernoulli random variable attached to it. Additionally, it may be further assumed that the remote server **1102** may not know μ_i for each region i. Instead, the remote server **1102** assumes that the sensing block **1202** provides realizations of the Bernoulli random variable associated with the region i whenever the set of search agents **1106** visits the corresponding region 'i'.

Therefore, the task of identifying interesting regions in the search environment **1108** may be casted as identifying a first set of regions for which the value of μ_i may be above a user-defined pre-specified confidence threshold θ.

FIG. **13** is a flowchart **1300** that illustrates an exemplary method for controlling search agents to perform search with noisy observations, in accordance with an embodiment of the disclosure. FIG. **13** is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, FIG. **7**A, FIG. **7**B, FIG. **8**, FIG. **9**, FIG. **10**, FIG. **11**, and FIG. **12**. With reference to FIG. **13**, there is shown the flowchart **1300**. The operations of the exemplary method may be executed by any computing system, for example, by the control system **102** or the processor **104**, or the remote server **110** of FIG. **1**A. The operations may start at block **1302** and may terminate at block **1304**. The flowchart **1300** may include multiple blocks such as a first block **1306**, and a set of blocks that may include a block **1308**, a block **1310**, and a block **1312**.

The first block **1306** corresponds to the algorithmic steps that may be performed by the remote server **110**, and each of the set of blocks (or the block **1308**, **1310**, and **1312**) corresponds to the algorithmic steps that may be performed by the set of search agents **1106** using their respective set of control systems **1104**.

Each of the set of blocks (or the blocks **1308**, **1310**, and **1312**) may expect an assignment of a (possibly unique) feasible path to each of the set of search agents **1106** respectively from the first block **1306**. As discussed above, the feasible paths may correspond to the paths that satisfy requirements 3a, 3b, 3c, and 3d mentioned in the description of FIG. **11**. Upon reception of the feasible path, the set of control systems **1104** may be configured to command the set of search agents **1106** to traverse the feasible path by visiting at least one region in the feasible path in an order (or sequence) that may be prescribed by the feasible paths in **1324**, **1326**, and **1328**. The set of control systems **1104** may be assumed to be well-designed and may be capable of handling waypoint tracking tasks as well as collision avoidance with other search agents. Details about the assignment of the paths are provided, for example, in FIGS. **6**A, **6**C, and **6**E.

The set of search agents **1106** at each visited region may collect a user-specified number of observations or inferences using the sensing block **1202** at **1330**, **1332**, and **1334**. Specifically, the sensing block **1302** may return a realization of the corresponding Bernoulli random variable (tosses of the corresponding biased coin) that may be stored by the set of control systems **1104**, to be subsequently sent back to the remote server **1102**.

By design, the feasible paths may terminate at the one or more pre-designated regions **1110**. The set of control systems **1104** may further control the set of search agents **1106** to charge the set of search agents **1106** and wait for the new feasible path to be assigned by the remote server **1102**.

In an embodiment, the remote server **1102** may be configured to store a set of lists to execute the algorithm in the first block **1306**. The set of lists may include a first set of unclassified regions, a second set of regions that may be classified as interesting, and a third set of regions that may be classified as uninteresting.

Next, at **1318**, given the set of paths, the remote server **1102** may be configured to prune away paths that may include only classified regions. Any path that only has the classified regions may be deemed redundant and may be removed from consideration without affecting the performance of the set of search agents **1106**. Consequently, if all paths in the collection are removed, the algorithm in the block **1306** terminates. Details about the pruning of the paths are provided, for example, in FIG. **8**.

Assuming that there are non-trivial paths left in the set of paths, the algorithm in the block **1306** proceeds to construct data-driven confidence bounds **502** for each region at **1320**. The confidence bounds **502** may correspond to scalar values that may provide optimistic and pessimistic estimates of the Bernoulli parameter μ_i associated with the region i in the feasible paths. The optimistic estimates may be referred to as the upper bounds on μ_i, and may be constructed using the past observations at the respective regions. Such optimistic estimates may be referred to as an upper confidence bounds (UCB) **502**A and may be denoted by UCB_i for the region (i). Similarly, the pessimistic estimates are lower bounds on μ_i, and are also constructed using the past observations at the respective regions. Such pessimistic estimates may be referred to as a lower confidence bounds (LCB) **502**B and may be denoted by LCB_i for the region (i). Examples of these confidence bounds include (1) and (2), or any other confidence bound constructed from concentration inequalities like the Hoeffding's bound.

The remote server **1102** may be configured to classify the region as interesting with a high probability whenever $LCB_i \geq \theta - \in$, for a small tolerance $\in > 0$. Similarly, the remote server **1102** may be configured to classify the region as uninteresting with a high probability whenever $UCB_i \leq \theta + \in$, for a small tolerance $\in > 0$. It may be an object of some embodiments to construct these bounds using data and the tolerances. In some embodiments, the bounds $\theta - \in$ and $\theta + \in$ are confidence thresholds as discussed in FIG. **1**A, FIG. **5**, and FIG. **8**. Additional details about the confidence bounds are provided, for example, in FIG. **5**.

At block **1322**, the remote server **1102** may be configured to identify the best subset of paths to assign to the search team (the set of search agents **1106**) to execute. Specifically, at block **1322**, the remote server **1102** aggregates the UCB_i among the feasible paths and selects the top 'N' paths, for a search team of 'N' search agents. One option to aggregate these upper confidence bounds **502** may be to consider the maximum among the regions in a path. The remote server **1102** further transmits information about these paths to the respective search agent for execution.

The control system **102** may command the set of search agents to visit the selected top 'N' paths and collect the measurements associated with each region within the selected first path at block **1314**. The control system **102** may be configured to iteratively repeat the algorithm from block **1316** until each regions in the search environment is classified as either the interesting region or the uninteresting region.

FIG. **14**A is a graph that illustrates a convergence of the confidence bounds **502** with an increasing number of realizations for a region that may have a high likelihood of being sensed, in accordance with an embodiment of the disclosure. FIG. **14**A is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, FIG. **7**A, FIG. **7**B, FIG. **8**, FIG. **9**, FIG. **10**, FIG. **11**, FIG. **12**, and FIG. **13**. With reference to FIG. **14**A, there is shown a graph **1400**A.

In an embodiment, the remote server **1102** may set a confidence threshold $\theta = 0.8$, a tolerance $\in = 0.01$, a probability of unreliability $\delta = 10^{-4}$, and a number of regions to be $N = 100$. Consequently, while **1402** may correspond to the confidence threshold θ, the remote server **1102** may be configured to use $\theta - \in$, the corrected confidence threshold **1404**, to enable probabilistic guarantees of performance.

The remote server **1102** may be configured to set the apriori unknown Bernoulli parameter $\mu = 0.9$, which implies that the region corresponds to the interesting region. The sample mean **1406** may converge towards μ as the number of samples increases. However, the sample mean may be an unreliable estimator of whether the region may be interesting or not. For example, at **1408**, the remote server **1102** may incorrectly classify the region as uninteresting since the sample mean may be below θ. Specifically, after collecting a few thousand of samples (beyond **1410**), it may be deemed confidently that the region may be interesting despite the uncertainty in the data. It may be noted that the LCB crosses the threshold θ at **1412**, but the remote server **1102** may declare the region as interesting earlier, when LCB crosses the corrected threshold $\theta - \in$ at **1410** to achieve probabilistic guarantees of performance. While reducing the tolerance ∈ yields more accurate classification, it comes at the cost of requiring more samples.

FIG. **14**B is a graph that illustrates a convergence of the confidence bounds **502** with an increasing number of realizations for a region that may have a low likelihood of being sensed, in accordance with an embodiment of the disclosure. FIG. **14**B is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**, FIG. **3**, FIG. **4**, FIG. **5**A, FIG. **5**B, FIG. **6**, FIG. **7**, FIG. **8**A, FIG. **8**B, FIG. **8**C, FIG. **9**, FIG. **10**, FIG. **11**, FIG. **12**, FIG. **13**, and FIG. **14**A. With reference to FIG. **14**B, there is shown a graph **1400**B.

In an embodiment, the remote server **1102** may set a confidence threshold $\theta = 0.8$, tolerance $\in = 0.01$, probability of unreliability $\delta = 10^{-4}$, and a number of regions to be $N = 100$. Consequently, while **1414** may correspond to the confidence threshold θ, the remote server **1102** may be configured to use $\theta + \in$, the corrected confidence threshold **1416**, to enable probabilistic guarantees of performance.

The remote server **1102** may be configured to set the apriori unknown Bernoulli parameter $\mu = 0.3$, which implies that the region may be uninteresting. The sample mean **1418**, as expected, may converge to μ as the number of samples increases. However, the sample mean on its own may be an unreliable estimator of whether the region may be interesting as evidenced in FIG. **14**A. It may be noted that the UCB crosses the threshold θ at **1420**, but the remote server **1102** may be configured to declare the region as uninteresting early after the UCB crosses the corrected threshold $\theta + \in$ at **1422** to achieve probabilistic guarantees of performance.

In another embodiment, the remote server **1102** may be configured to use the Hoeffding's inequality to generate a separate set of upper and lower confidence bounds. In another embodiment, to ensure the application of the bandit algorithm, the remote server **1102** may require that the set of search agents **116** execute each feasible path as a round trip. This ensures that the distribution of the search agents at each charging station remains unchanged.

FIG. **15** depicts a diagram to illustrate an exemplary scenario for classifying the set of regions in the search environment, in accordance with an embodiment of the disclosure. FIG. **15** is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, FIG. **7**A, FIG. **7**B, FIG. **8**, FIG. **9**, FIG. **10**, FIG. **11**, FIG. **12**, FIG. **13**, FIG. **14**A, and FIG. **14**B. With reference to FIG. **15**, there is shown an exemplary scenario **1500**. The exemplary scenario **1500** may include the control system **102**, a search environment **1502** which may be portioned into a set of regions. The set of regions may include a first region **1504**, a second region **1506**, a third region **1508**, a fourth region

1510, a fifth region **1512**, a sixth region **1514**, a seventh region **1516**, an eighth region **1518**, and a ninth region **1520**.

In an embodiment, the control system **102** may assign a value of 0.3 to the Bernoulli parameter (μ) associated with each of the set of regions as described in FIG. **11**. The control system may command the set of search agents **116** to collect measurements iteratively. With each measurement, the control system **102** may be configured to update the value of the Bernoulli parameter (μ) associated with each of the set of regions. An intermediate stage **1522** of the search environment with updated Bernoulli parameter (μ) of each region is shown in FIG. **15**. For example, the value of the Bernoulli parameter (μ_1) associated with the first region **1504** may be updated from the initialized value of 0.3 to 0.8.

The control system **102** may iteratively perform the MMBS method until each region of the set of regions is classified. To classify each region, the control system **102** may compare the Bernoulli parameter (μ) associated with each of the set of regions with the confidence threshold (θ) that may be set to 0.8, as described in FIG. **11**. The control system **102** may further classify each region of the set of regions based on the classification.

With reference to FIG. **15**, there is further shown a final stage **1524** when each of the set of regions is classified as the interesting region or the uninteresting region. The interesting regions may be marked with a star symbol whereas the uninteresting regions may be marked with a cross symbol. Specifically, the second region **1506**, the fourth region **1510**, the fifth region **1512**, and the eighth region **1518** may be classified as the interesting regions whereas the others are classified as uninteresting regions.

FIG. **16** depicts a diagram to illustrate a difference between an application of the multi-arm bandit search method and a multi-level multi-arm bandit search (MMBS) method, in accordance with an embodiment of the disclosure. FIG. **16** is explained in conjunction with elements from FIG. **1**A, FIG. **1**B, FIG. **2**A, FIG. **2**B, FIG. **3**, FIG. **4**, FIG. **5**, FIG. **6**A, FIG. **6**B, FIG. **6**C, FIG. **6**D, FIG. **7**A, FIG. **7**B, FIG. **8**, FIG. **9**, FIG. **10**, FIG. **11**, FIG. **12**, FIG. **13**, FIG. **14**A, FIG. **14**B and FIG. **15**. With reference to FIG. **16**, there is shown a search environment **1600**.

Specifically, it may be assumed that **1602** to **1624** may be a set of choices corresponding to the regions that may be available to a multi-arm bandit search method (algorithm). In accordance with an embodiment of the present disclosure, such choices may correspond to the set of regions that may have to be classified by the set of search agents **1106** based on their spatial proximity, and the remote server **1102** may decide the region to visit at every decision epoch.

Traditional multi-arm bandit search methods may assume that the execution of a choice does not affect the choice at the next decision epoch. For example, the traditional multi-arm bandit algorithm may assume that choice **1624** may be executed in the next time step after choosing **1602**. However, in a search problem using mobile sensors with dynamics like the set of search agents **106**, it may be physically impossible to reach **1624** after visiting **1602** before the next decision epoch. Ignoring such constraints arising from the dynamics of the mobile sensors may result in suboptimal decision-making.

In some embodiments of the present disclosure, it may be realized that the choices may be grouped together into groups **1626**, **1628**, and **1630** such that no choice may be left out. According to an embodiment of the present disclosure, the above-mentioned choices may correspond to a set of paths that may pass through a sequence of regions and may be physically admissible to be executed by the search team. Instead of deciding which region **1602** to **1624** to visit, the remote server **1102** may choose which path **1626**, **1628**, and/or **1630** to visit. However, the decision to declare if any region from **1602** to **1624** may be classified as interesting or uninteresting may be still performed at the region level by the remote server **1102**.

FIG. **17** shows a schematic diagram of a computing device that may be used for implementing the control system **102** or the remote server **110**, in accordance with an embodiment of the disclosure. The computing device **1700** includes a power source **1702**, a processor **1704**, a memory **1706**, and a storage device **1708**, all connected to a bus **1710**. Further, a high-speed interface **1712**, a low-speed interface **1714**, high-speed expansion ports **1716**, and low-speed connection ports **1718**, may be connected to the bus **1710**. In addition, a low-speed expansion port **1720** may be in connection with the bus **1710**. Further, an input interface **1722** may be connected via the bus **1710** to an external receiver **1724** and an output interface **1726**. A receiver **1728** may be connected to an external transmitter **1730** and a transmitter **1732** via the bus **1710**. Also connected to the bus **1710** may be an external memory **1734**, external sensors **1736**, machine(s) **1738**, and an environment **1740**. Further, one or more external input/output devices **1742** may be connected to the bus **1710**. A network interface controller (NIC) **1744** may be adapted to connect through the bus **1710** to a network **1746**, wherein data or other data, among other things, may be rendered on a third-party display device, third-party imaging device, and/or third-party printing device outside of the computing device **1700**.

The memory **1706** may store instructions that are executable by the computing device **1700** and any data that may be utilized by the methods and systems of the present disclosure. The memory **1706** may include random access memory (RAM), read-only memory (ROM), flash memory, or any other suitable memory systems. The memory **1706** may be a volatile memory unit or units, and/or a non-volatile memory unit or units. The memory **1706** may also be another form of computer-readable medium, such as a magnetic or optical disk.

The storage device **1708** may be adapted to store supplementary data and/or software modules used by the computer device **1700**. The storage device **1708** may include a hard drive, an optical drive, a thumb-drive, an array of drives, or any combination thereof. Further, the storage device **1708** may contain a computer-readable medium, such as a floppy disk device, a hard disk device, an optical disk device, or a tape device, a flash memory or other similar solid-state memory device, or an array of devices, including devices in a storage area network or other configurations. Instructions may be stored in an information carrier. The instructions, when executed by one or more processing devices (for example, the processor **1704**), perform one or more methods, such as those described above.

The computing device **1700** may be linked through the bus **1710**, optionally, to a display interface or user Interface (HMI) **1748** adapted to connect the computing device **1700** to a display **1750** and a keyboard **1752**, wherein the display **1750** may include a computer monitor, camera, television, projector, or mobile device, among others. In some implementations, the computer device **1700** may include a printer interface to connect to a printing device, wherein the printing device may include a liquid inkjet printer, solid ink printer, large-scale commercial printer, thermal printer, UV printer, or dye-sublimation printer, among others.

The high-speed interface **1712** manages bandwidth-intensive operations for the computing device **1700**, while the low-speed interface **1714** manages lower bandwidth-intensive operations. Such allocation of functions may be an example only. In some implementations, the high-speed interface **1712** may be coupled to the memory **1706**, the user interface (HMI) **1748**, and to the keyboard **1752**, and the display **1750** (e.g., through a graphics processor or accelerator), and to the high-speed expansion ports **1716**, which may accept various expansion cards via the bus **1710**. In an implementation, the low-speed interface **1714** may be coupled to the storage device **1708** and the low-speed expansion ports **1718**, via the bus **1710**. The low-speed expansion ports **1718**, which may include various communication ports (e.g., USB, Bluetooth, Ethernet, wireless Ethernet) may be coupled to the one or more input/output devices **1742**. The computing device **1700** may be connected to a server **1754** and a rack server **1756**. The computing device **1700** may be implemented in several different forms. For example, the computing device **1700** may be implemented as part of the rack server **1756**.

Many modifications and other embodiments of the disclosure set forth herein will come to mind to one skilled in the art to which these disclosures pertain having the benefit of the teachings presented in the foregoing descriptions and the associated drawings. It is to be understood that the disclosures are not to be limited to the specific embodiments disclosed and that modifications and other embodiments are intended to be included within the scope of the appended claims. Moreover, although the foregoing descriptions and the associated drawings describe example embodiments in the context of certain example combinations of elements and/or functions, it should be appreciated that different combinations of elements and/or functions may be provided by alternative embodiments without departing from the scope of the appended claims. In this regard, for example, different combinations of elements and/or functions than those explicitly described above are also contemplated as may be set forth in some of the appended claims. Although specific terms are employed herein, they are used in a generic and descriptive sense only and not for purposes of limitation.

The invention claimed is:

1. A method implemented by one of a control system or a remote server for controlling a movement of at least a first search agent of a set of search agents in a search environment partitioned into a set of regions and classifying each region of the set of regions based on measurements collected by the first search agent, comprising:

an iterative execution of a multi-level multi-arm bandit search (MMBS) method until a termination condition is met, wherein the MMBS method specifies a first set of instructions for individual probabilistic classifications of each region of the set of regions based on the measurements associated with the corresponding region, and a second set of instructions for visiting, by the set of search agents, each path of a set of paths formed from the set of regions based on comparing aggregations of the probabilistic classifications of different paths, wherein each path of the set of paths comprises at least two regions of the search environment, and wherein the iterative execution of MMBS comprises:

collecting confidence bounds of the probabilistic classification of at least one region within at least one path of the set of paths;

comparing aggregations of the confidence bounds of the probabilistic classifications of each path of the set of paths based on the collected confidence bounds;

selecting a first path of the set of paths to be visited by the first search agent based on the comparison;

commanding, via a transceiver configured to exchange data with the set of search agents over a wired or wireless communication channel, the first search agent to visit the selected first path and to collect measurements associated with each region within the selected first path;

updating the confidence bounds of the probabilistic classifications of each region within the selected first path based on the measurements associated with the corresponding regions; and

controlling the movement of at least the first search agent of a set of search agents.

2. A control system to control a movement of at least a first search agent of a set of search agents in a search environment partitioned into a set of regions and to classify each region of the set of regions based on measurements collected by the first search agent, comprising:

a transceiver configured to exchange data with the set of search agents over a wired or wireless communication channel;

a memory configured to store executable instructions specifying an operation of a multi-level multi-arm bandit search (MMBS) method, wherein the MMBS method specifies a first set of instructions for individual probabilistic classifications of each region of the set of regions based on the measurements associated with the corresponding region, and a second set of instructions for visiting, by the set of search agents, each path of a set of paths formed from the set of regions based on comparing aggregations of the probabilistic classifications of each path of the set of paths, wherein each path of the set of paths comprises at least two regions of the search environment; and

a processor coupled with the executable instructions, when executed by the processor, causes an iterative execution of the MMBS method until a termination condition is met, wherein an iteration of the MMBS method causes the control system to:

collect confidence bounds of the probabilistic classification of at least one region within at least one path of the set of paths;

compare aggregations of the confidence bounds of the probabilistic classifications of each path of the set of paths based on the collected confidence bounds;

select a first path of the set of paths to be visited by the first search agent based on the comparison;

command, via the transceiver, the first search agent to visit the selected first path and to collect measurements associated with each region within the selected first path;

update the confidence bounds of the probabilistic classifications of each region within the selected first path based on the measurements associated with the corresponding regions; and

control the movement of at least the first search agent of the set of search agents.

3. The control system of claim **2**, wherein the processor is further configured to:

receive a set of user inputs associated with generation of the set of paths to cover the search environment; and

generate the set of paths to cover the search environment based on the received set of user inputs.

4. The control system of claim **3**, wherein a first user input of the set of user inputs corresponds to a path length for each path of the set of paths, and wherein the path length for each path corresponds to a maximum number of regions in each path that maybe traversed by at least one search agent of the set of search agents while satisfying its physical constraints, wherein the physical constraints can be due to the motion constraints of the search agents and limited on-board energy available for the search agents.

5. The control system of claim **3**, wherein each path of the set of paths starts or ends at a pre-designated region of one or more pre-designated regions, and wherein at least one pre-designated region is located inside the search environment, and at least one pre-designated region is located outside of the search environment.

6. The control system of claim **5**, wherein each of the one or more pre-designated regions corresponds to one of: an energy refueling station, a calibration station, a service station, or a docking station, and wherein each search agent of the set of search agents corresponds to one of: an autonomous vehicle, a mobile robot, an aerial drone, a ground vehicle, an aerial vehicle, a water surface vehicle, or an underwater vehicle.

7. The control system of claim **3**, wherein the processor is further configured to generate the set of paths using a graph-based multi-agent path planning process that shrinks a first set of paths into the set of the paths covering the search environment subjected to one or more path constraints.

8. The control system of claim **7**, wherein to perform the graph-based multi-agent path planning process, the processor is configured to:

construct, for each region of the set of regions, a shortest path table comprising a minimum number of steps required to reach any one of the one or more pre-designated regions in the search environment, where the shortest path table is constructed using one or more graph-based shortest path planning processes;

randomly generate the first set of paths satisfying the one or more path constraints by constructing a sequence of regions starting from a randomly selected pre-designated region from the one or more pre-designated regions in the search environment and executing a hill ascend operation according to the shortest path table for a first half of the path length and further executing a hill descend operation according to the shortest path table for a second half of the path length to reach one of the one or more pre-designated regions; and

select the set of paths from the first set of paths further based on the execution of the hill ascend operation and the hill descend operation.

9. The control system of claim **2**, wherein the processor is further configured to:

evaluate, for each path of the set of paths, a pre-determined aggregation function of the values of the confidence bounds of each region within the corresponding path;

sort the evaluation of the pre-determined aggregation function for each of the set of paths in descending order of a function value for each path, wherein the function value is determined based on an application of the pre-determined aggregation function on the confidence bounds of each region in each path of the set of paths; and

select, based on the sorting, at least one path to be visited by the set of search agents, wherein the first path is selected to be visited by the first search agent.

10. The control system of claim **2**, wherein the processor is further configured to:

evaluate, confidence bounds of each region in each path of the set of paths, using a neural network that intakes the data collected of each region in the corresponding path and computes an aggregate value of visiting the path;

return an assignment of the set paths to be executed by each search agent of the set search agents based on the evaluation, wherein the first path is assigned to the first search agent; and

select, based on the assignment, at least one path to be visited by the set of search agents, wherein the first path is selected to be visited by the first search agent based on the assignment.

11. The control system of claim **2**, wherein the processor is further configured to:

select probabilistically between different selecting criteria including a most-promising path criterion and a minimal-movement criterion; and

select, based on the selected criterion, at least one path to be visited by the set of search agents, wherein the first path is selected to be visited by the first search agent.

12. The control system of claim **11**, wherein the probability of selection is a biased probability.

13. The control system of claim **12**, wherein the selection of at least one path to be visited by the set of search agents is performed over multiple control steps dependent on a number of the regions in the search environment, and wherein a bias of the biased probability varies between at least some control steps.

14. The control system of claim **13**, wherein the processor is further configured to update the bias based on the likelihood of interest at different regions of the set of regions in the search environment.

15. The control system of claim **14**, wherein the processor is further configured to use a neural network trained that takes as input the set of regions, set of paths, data collected at each region to produce the biased probability.

16. The control system of claim **2**, wherein the processor is further configured to:

select the first path and a second path from the set of paths based on the aggregations of the probabilistic classifications of each of the set of paths; and

control the movement of the first search agent and a second search agent of the set of search agents concurrently on the selected first path and the second path to take measurements associated with the corresponding regions within the first path and the second path.

17. The control system of claim **2**, wherein to update the confidence bounds of the probabilistic classifications of a first region within the selected first path, the processor is configured to

receive a pre-determined number of measurements associated with the first region and collected by at least the first search agent; and

update the confidence bounds of the probabilistic classifications of the first region based on the pre-determined number of measurements using at least one concentration inequality.

18. The control system of claim **2**, wherein to update the confidence bounds of the probabilistic classifications of a first region within the selected first path, the processor is configured to:

transmit, via the transceiver, to at least the first search agent: a current value of the confidence bounds of the probabilistic classifications of the first region, a third set of instructions for taking a pre-determined number of measurements associated with the first region, and a fourth set of instructions for updating the current values of the confidence bounds of the probabilistic classifications of the first region; and

receive, via the transceiver, the updated confidence bounds of the probabilistic classifications of the first region from at least the first search agent.

19. The control system of claim **2**, wherein the processor is further configured to:

compare the updated confidence bounds of a first region within the selected first path with a confidence threshold;

classify the first region within the selected first path based on the comparison; and

update the first path of the set of paths to prune the first region from the first path based on the classification of the first region.

20. The control system of claim **2**, wherein each region of the set of regions is classified with a first label or a second label, and wherein the termination condition is met when each region of the set of regions is classified with one of the first label or the second label.

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