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(54) **VIRTUAL DISTILLATION FOR QUANTUM  
ERROR MITIGATION**

(52) **U.S. Cl.**

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(2022.01)

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(57)

**ABSTRACT**

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(60) Provisional application No. 63/112,593, filed on Nov.  
11, 2020.

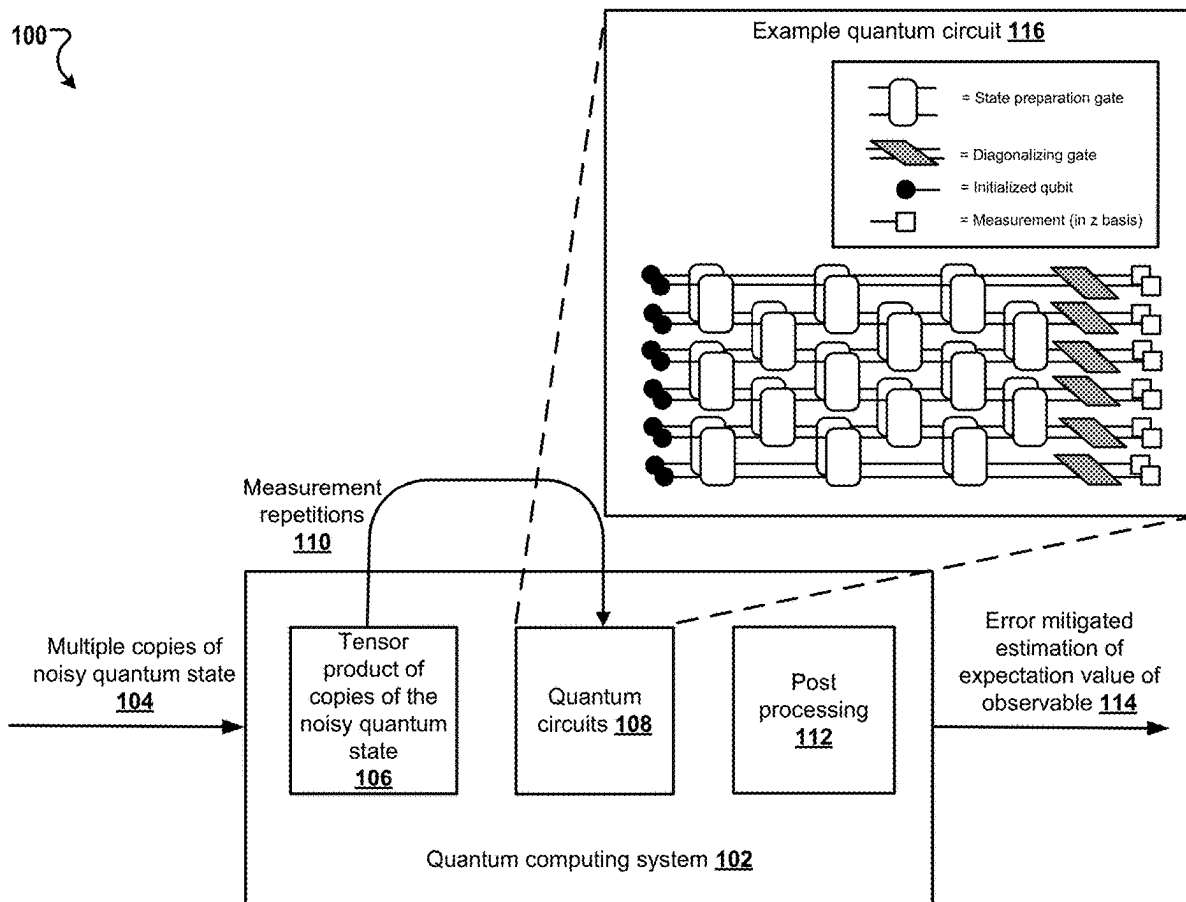
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Methods, systems and apparatus for determining an error-mitigated expectation value of a target observable with respect to a noisy quantum state. In one aspect a method includes obtaining multiple copies of the noisy quantum state; performing measurements on tensor products of M copies of the noisy quantum state to compute an expectation value of the target observable with respect to an entangled quantum state, wherein  $M \geq 1$  and eigenvalues corresponding to non-dominant eigenvectors of the noisy quantum state in the spectral decomposition of the entangled quantum state are suppressed exponentially in M; and using the computed expectation value of the target observable with respect to an entangled quantum state to determine the error-mitigated expectation value of the target observable with respect to the noisy quantum state.



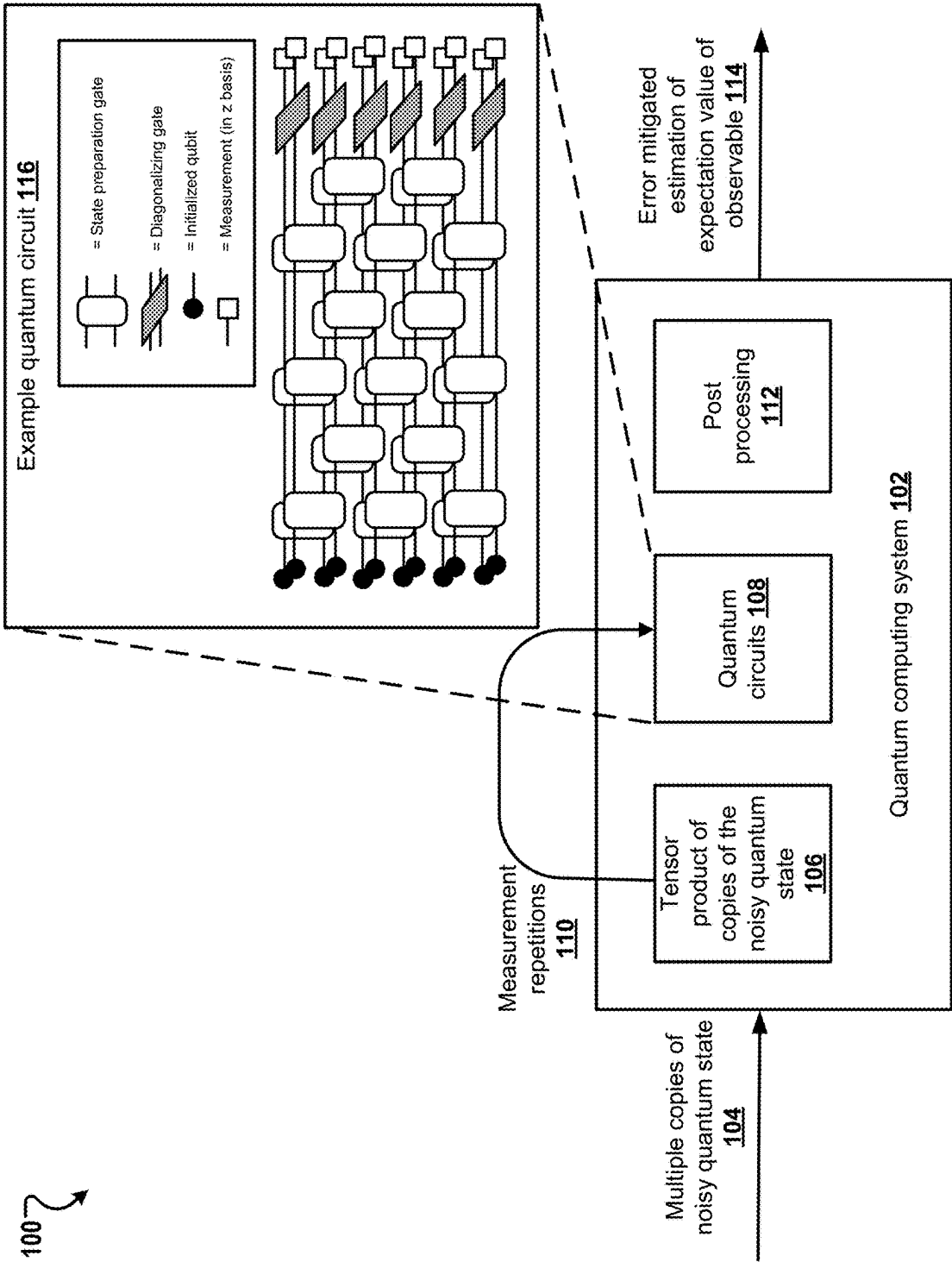


FIG. 1

200 ↷

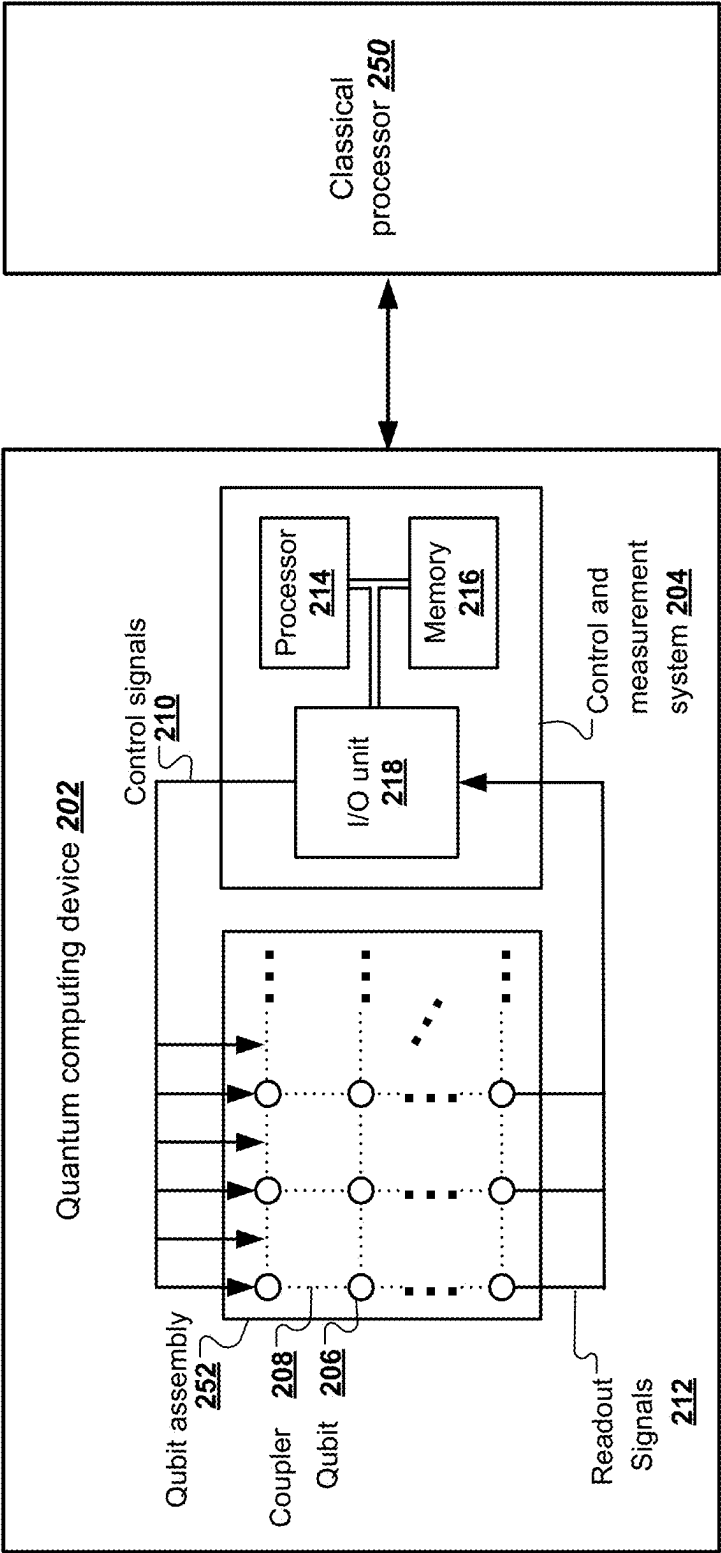


FIG. 2

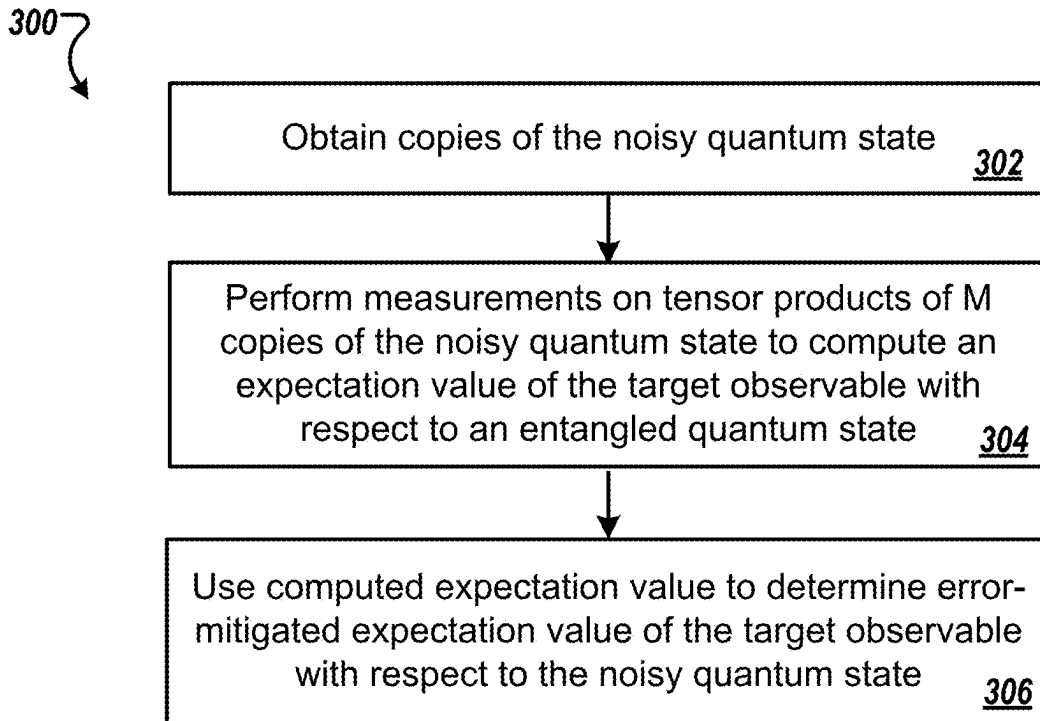


FIG. 3

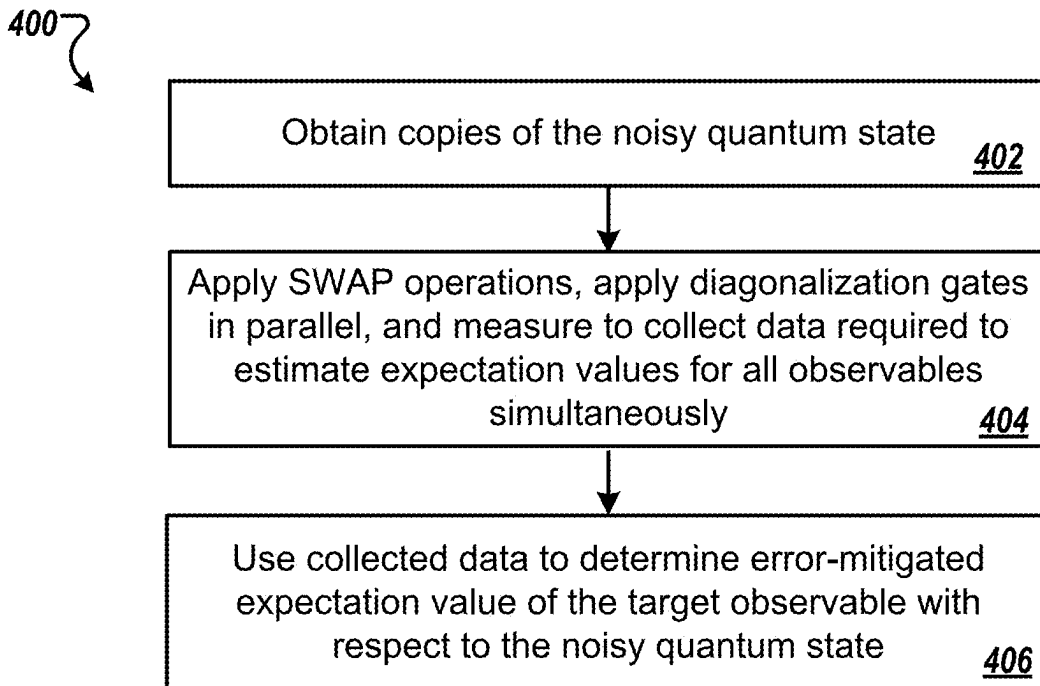


FIG. 4

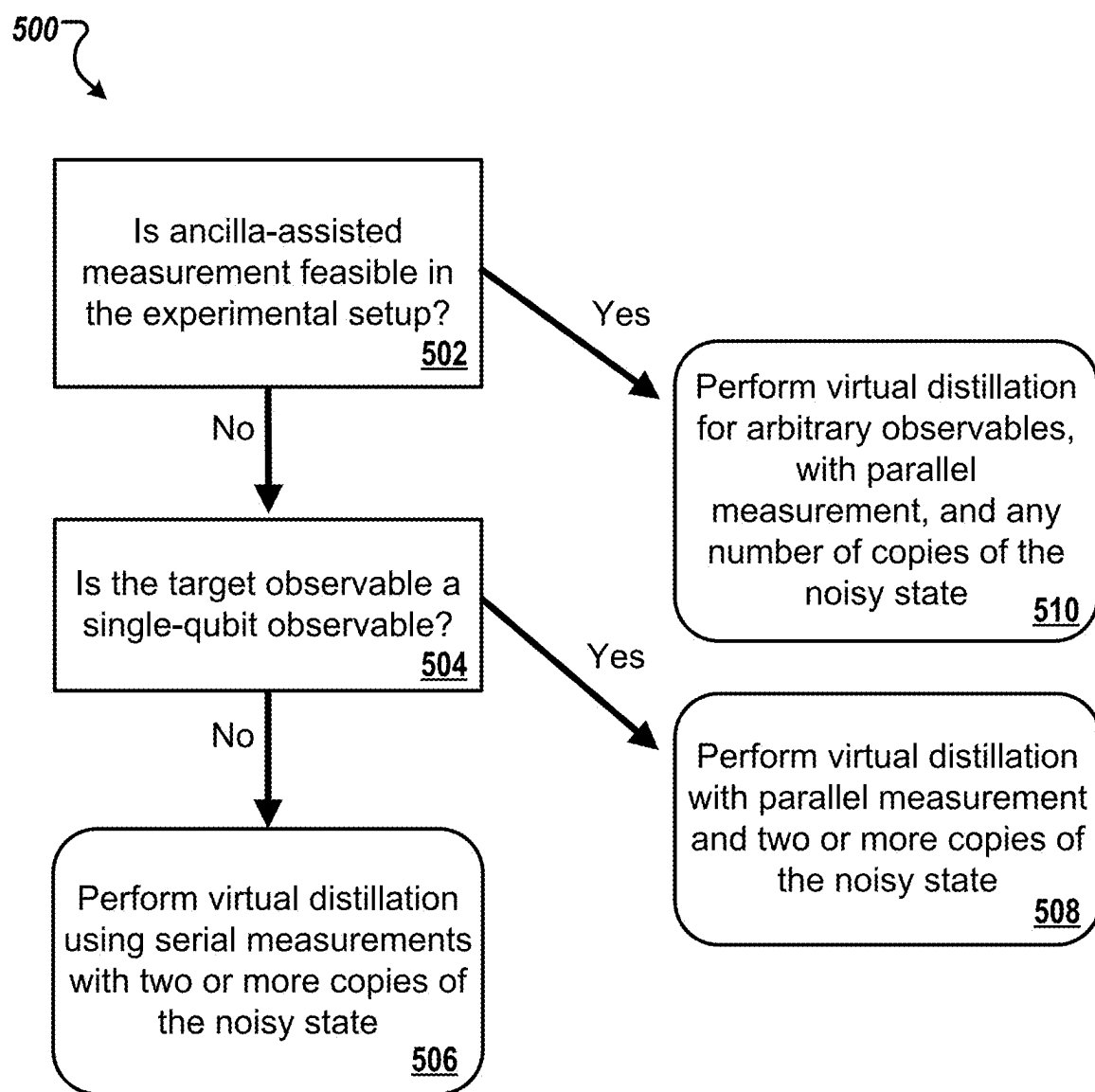


FIG. 5

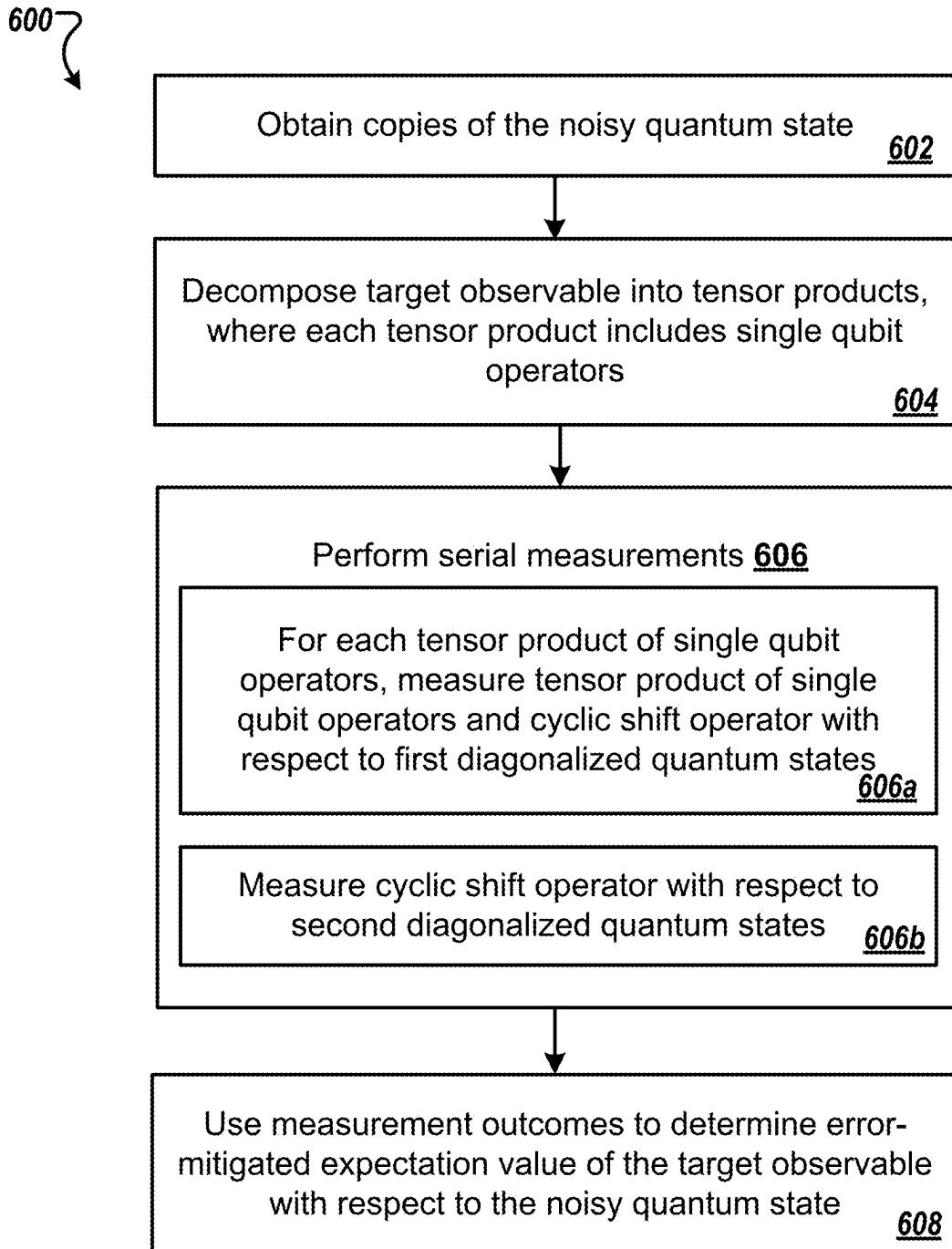


FIG. 6

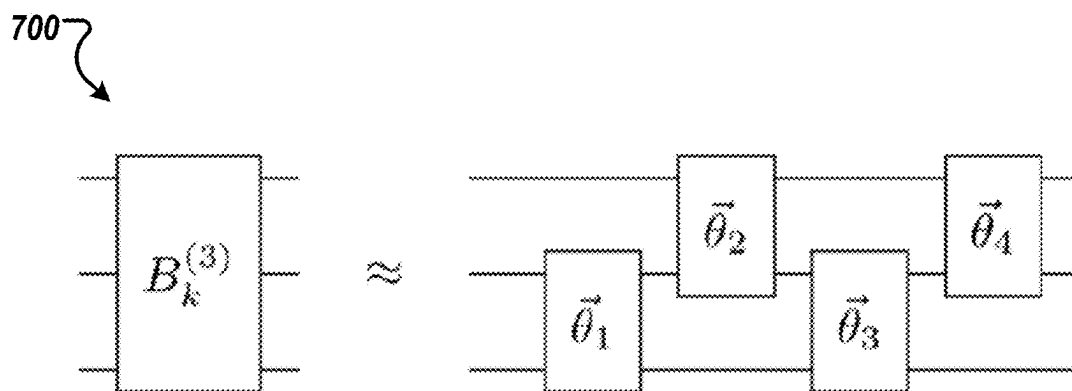


FIG. 7

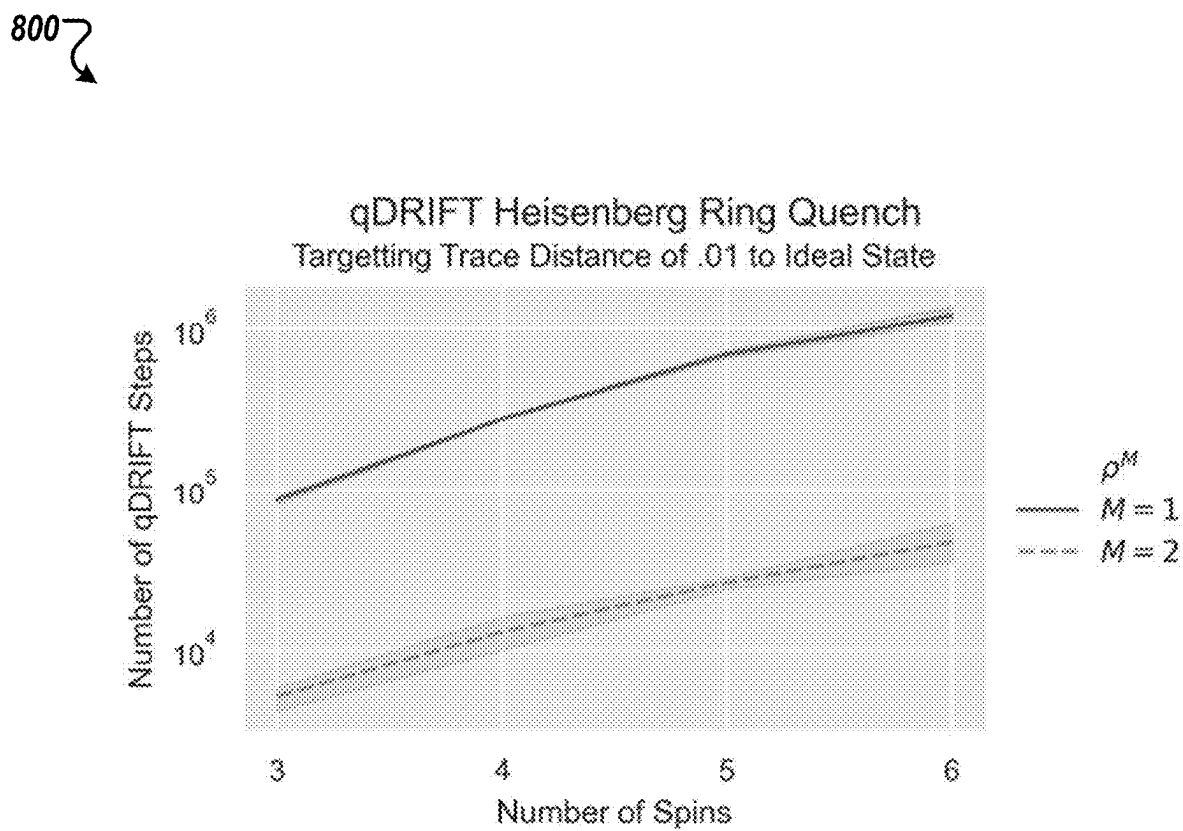


FIG. 8

## VIRTUAL DISTILLATION FOR QUANTUM ERROR MITIGATION

### CROSS-REFERENCE TO RELATED APPLICATIONS

**[0001]** This application is a continuation application and claims priority under 35 U.S.C. § 120 to U.S. patent application Ser. No. 18/036,252, filed on May 10, 2023, which is a National Stage Application under 35 U.S.C. § 371 and claims the benefit of International Application No. PCT/US2021/058958, filed on Nov. 11, 2021, which claims the benefit of priority to U.S. Provisional Patent Application No. 63/112,593, filed on Nov. 11, 2020, the entirety of each are incorporated herein by reference.

### BACKGROUND

**[0002]** This specification relates to quantum computing.

**[0003]** Quantum error correction codes are used in quantum computing to protect quantum information from errors due to decoherence and other quantum noise. Quantum error correction codes spread information encoded in one logical qubit onto a highly entangled state of several physical qubits. For example, the Shor code is a quantum error correction code that can correct phase flip and bit flip errors using a 9 qubit circuit that requires 8 ancillary qubits to correct 1 qubit.

### SUMMARY

**[0004]** This specification describes techniques for mitigating errors in noisy quantum computations using virtual quantum state distillation.

**[0005]** In general, one innovative aspect of the subject matter described in this specification can be implemented in methods for determining an error-mitigated expectation value of a target observable with respect to a noisy quantum state, the methods including: obtaining multiple copies of the noisy quantum state; performing measurements on tensor products of M copies of the noisy quantum state to compute an expectation value of the target observable with respect to an entangled quantum state, wherein  $M \geq 1$  and eigenvalues corresponding to non-dominant eigenvectors of the noisy quantum state in the spectral decomposition of the entangled quantum state are suppressed exponentially in M; and using the computed expectation value of the target observable with respect to an entangled quantum state to determine the error-mitigated expectation value of the target observable with respect to the noisy quantum state.

**[0006]** Other implementations of this aspect include corresponding classical and quantum computer systems, apparatus, and computer programs recorded on one or more computer storage devices, each configured to perform the actions of the methods. A system of one or more quantum and classical computers can be configured to perform particular operations or actions by virtue of having software, firmware, hardware, or a combination thereof installed on the system that in operation causes or cause the system to perform the actions. One or more computer programs can be configured to perform particular operations or actions by virtue of including instructions that, when executed by data processing apparatus, cause the apparatus to perform the actions.

**[0007]** The foregoing and other implementations can each optionally include one or more of the following features,

alone or in combination. In some implementations noise experienced by each copy of the noisy quantum state comprises a same form and strength.

**[0008]** In some implementations the entangled quantum state is given by  $\rho^M/\text{Tr}(\rho^M)$ , wherein  $\rho$  represents the noisy quantum state.

**[0009]** In some implementations the method further comprises performing measurements on tensor products of M copies of the noisy quantum state to compute an expectation value of the identity operator with respect to an entangled quantum state, wherein using the computed expectation value of the target observable with respect to the entangled quantum state to determine the error-mitigated expectation value of the target observable with respect to the noisy quantum state comprises dividing i) the expectation value of the target observable with respect to the entangled quantum state by ii) the expectation value of the identity operator with respect to an entangled quantum state.

**[0010]** In some implementations wherein performing measurements on tensor products of M copies of the noisy quantum state to compute expectation values of the target observable with respect to an entangled quantum state comprises: for each of a first number of measurement repetitions, measuring a product of i) the target observable and ii) a cyclic shift operator with respect to a tensor product of M copies of the noisy quantum state to obtain a first measurement outcome; computing, using the first measurement outcomes, an expectation value of the product of i) the target observable and ii) a cyclic shift operator with respect to the tensor product of M copies of the noisy quantum state; and normalizing the expectation value of the product of i) the target observable and ii) a cyclic shift operator with respect to the tensor product of M copies of the noisy quantum state to obtain the error-mitigated expectation value of the target observable with respect to the noisy quantum state.

**[0011]** In some implementations normalizing the expectation value of the target observable comprises: for each of a second number of measurement repetitions, measuring the cyclic shift operator with respect to a tensor product of M copies of the noisy quantum state to obtain a second measurement outcome; computing, using the second measurement outcomes, an expectation value of the cyclic shift operator with respect to a tensor product of M copies of the noisy quantum state; and dividing the expectation value of the product of i) the target observable and ii) a cyclic shift operator with respect to the tensor product of M copies of the noisy quantum state by the expectation value of the cyclic shift operator with respect to the tensor product of M copies of the noisy quantum state.

**[0012]** In some implementations performing measurements on tensor products of M copies of the noisy quantum state comprises performing parallel measurements.

**[0013]** In some implementations  $M \geq 2$  and the target observable acts on a single qubit.

**[0014]** In some implementations performing measurements on tensor products of M copies of the noisy quantum state comprises, for each of K measurement repetitions: applying a diagonalization operator to a tensor product of M copies of the noisy quantum state to obtain an evolved quantum state, wherein the diagonalization operator diagonalizes i) a cyclic shift operator and ii) a product of a symmetrized version of the target observable and the cyclic shift operator, and measuring a product of the target observ-



able and the cyclic shift operator with respect to the evolved quantum state to obtain a respective measurement outcome for each qubit in the evolved quantum state.

**[0015]** In some implementations the method further comprises computing an expectation value of the product of the target observable and the cyclic shift operator with respect to the evolved quantum state using the measurement outcomes; computing an expectation value of the cyclic shift operator with respect to the evolved quantum state using the measurement outcomes; and dividing the expectation value of the product of the target observable and the cyclic shift operator with respect to the evolved quantum state by the expectation value of the cyclic shift operator with respect to the evolved quantum state.

**[0016]** In some implementations application of the cyclic shift operator couples qubits in a first copy of the noisy quantum state to corresponding qubits in each other copy of the noisy quantum state.

**[0017]** In some implementations performing measurements on tensor products of  $M$  copies of the noisy quantum state comprises performing serial measurements.

**[0018]** In some implementations  $M \geq 2$  and the target observable acts on two or more qubits, the target observable comprising multiple tensor products of one-qubit operators.

**[0019]** In some implementations performing measurements on tensor products of  $M$  copies of the noisy quantum state comprises: for each tensor product of one-qubit operators: for each of a first number of measurement repetitions: applying a first diagonalization operator to a tensor product of the  $M$  copies of the noisy quantum state to obtain an evolved quantum state, wherein the first diagonalization operator diagonalizes a product of i) the tensor product of one-qubit operators and ii) the cyclic shift operator, and measuring a product of the tensor product of one-qubit operators and the cyclic shift operator with respect to the evolved quantum state to obtain a respective first measurement outcome for each qubit in the evolved quantum state; for each of a second number of measurement repetitions: applying a second diagonalization operator to a tensor product of the  $M$  copies of the noisy quantum state to obtain a second evolved quantum state, wherein the second diagonalization operator diagonalizes the cyclic shift operator, and measuring the cyclic shift operator with respect to the second evolved quantum state to obtain a respective second measurement outcome for each qubit in the second evolved quantum state.

**[0020]** In some implementations the method further comprises computing an expectation value of a product of the target observable and the cyclic shift operator using the first measurement outcomes; computing an expectation value of the cyclic shift operator using the second measurement outcomes; and dividing the expectation value of the product of the target observable and the cyclic shift operator by the expectation value of the cyclic shift operator.

**[0021]** In some implementations performing measurements on tensor products of  $M$  copies of the noisy quantum state to compute an expectation value of the target observable with respect to an entangled quantum state comprise performing ancilla-assisted measurements.

**[0022]** In some implementations the target observable comprises an arbitrary observable.

**[0023]** In some implementations  $M \geq 2$  and performing ancilla assisted measurements comprises, for each of multiple measurement repetitions: preparing two quantum reg-

isters, where each quantum register comprises the noisy quantum state and an ancilla qubit prepared in the 0 state; performing a non-destructive measurement of a cyclic shift operator, comprising: applying a Hadamard gate to the ancilla qubit, applying a the cyclic shift operator to the two quantum registers conditioned on the ancilla qubit being in the 1 state, and measuring the ancilla qubit in the X basis to obtain a first respective measurement outcome; and measuring qubits in the two quantum registers with respect to a symmetrized version of the target observable to obtain respective second measurement outcomes.

**[0024]** In some implementations the method further comprises computing an expectation value of the target observable using the second measurement outcomes; and computing an expectation value of the cyclic shift operator using the first measurement outcomes; and dividing the expectation value of the target observable by the expectation value of the cyclic shift operator.

**[0025]** The subject matter described in this specification can be implemented in particular ways so as to realize one or more of the following advantages.

**[0026]** Performing meaningful calculations using near-term quantum computers can be challenging because of the relatively high error rates of these devices. While quantum error correction promises to enable quantum computation with arbitrarily small levels of noise, the computational overhead incurred is too large to be currently practical. The presently described error mitigation technique enhances the quality of a noisy computation without the large overhead of traditional quantum error correction. The error mitigation strategy is capable of reducing the impact of stochastic errors arising from noise, e.g., on a near-term device, as well as stochastic errors inherent to randomized quantum algorithms implemented on an error-free device. In addition, since the error mitigation strategy is particularly suited to implementations on near term devices, e.g., NISQ devices, the error mitigation strategy can be applied in a wide range of technical applications, including simulations of molecules and materials, optimization processes, and quantum neural networks.

**[0027]** The details of one or more implementations of the subject matter of this specification are set forth in the accompanying drawings and the description below. Other features, aspects, and advantages of the subject matter will become apparent from the description, the drawings, and the claims.

## BRIEF DESCRIPTION OF THE DRAWINGS

**[0028]** FIG. 1 shows a block diagram of an example system performing virtual distillation to mitigate errors when calculating an expectation value of an observable with respect to a noisy quantum state.

**[0029]** FIG. 2 shows an example quantum computing system.

**[0030]** FIG. 3 is a flowchart of an example process for determining an error-mitigated expectation value of a target observable with respect to a noisy quantum state.

**[0031]** FIG. 4 is a flowchart of an example process for performing virtual distillation with measurement by diagonalization.

**[0032]** FIG. 5 is a flowchart of an example process for selecting a virtual distillation protocol.

**[0033]** FIG. 6 is a flowchart of an example process for virtual distillation with serial measurements on two or more

copies of the noisy quantum state, where the target observable acts on two or more qubits.

[0034] FIG. 7 shows an example ansatz for numerically optimizing a diagonalization gate

[0035] FIG. 8 is a plot showing qDRIFT coherent cost reduction through virtual distillation in the Heisenberg model.

[0036] Like reference numbers and designations in the various drawings indicate like elements.

#### DETAILED DESCRIPTION

[0037] This specification describes techniques for mitigating errors in a noisy quantum computation. In particular, the presently described techniques enable an efficient approximation of a computational output that would be produced in the absence of noise. Expectation values of an observable with respect to an approximately purified version of a noisy quantum state are reconstructed without explicitly preparing the purified quantum state. This approach is referred to herein as “virtual distillation,” where the word “virtual” is used to emphasize that the purified version of the quantum state is not actually prepared (as in conventional distillation schemes.) More specifically, collective measurements of  $M$  copies of the noisy quantum state  $\rho$  are used to measure expectation values of an observable with respect to an entangled quantum state represented as:

$$\frac{\rho^M}{\text{Tr}(\rho^M)} = \frac{\sum_i p_i^M |i\rangle\langle i|}{\sum_i p_i^M}, \quad (1)$$

where  $\rho = \sum_i p_i |i\rangle\langle i|$

is a spectral decomposition (e.g. density matrix) of the noisy quantum state  $\rho$  and  $\text{Tr}$  denotes the trace operation. Under this approach, the relative weights  $p_i^M$  of the non-dominant eigenvectors are suppressed exponentially in  $M$ . This represents an improvement over other approaches that demand that the approximately purified state is prepared explicitly, which achieve a suppression that is only linear in  $M$  in the general case. Accordingly, virtual distillation approximates a corrected expectation value (e.g., an expectation value where incoherent errors have been suppressed) of the observable  $O$  as a trace of a product of the observable and  $M$  copies of the spectral decomposition of the noisy quantum state, divided by a trace of  $M$  copies of the spectral decomposition of the noisy quantum state, e.g.

$$\langle O \rangle_{\text{corrected}} := \frac{\text{Tr}(O \rho^M)}{\text{Tr}(\rho^M)}. \quad (2)$$

[0038] The resulting estimator converges exponentially quickly towards the closest pure state to  $\rho$  as  $M$  is increased. This is in contrast to other approaches that only achieve a linear convergence to a corrected state. The presently described techniques therefore provide a technical improvement over these other approaches, since the corrected state can be obtained in less time.

[0039] In this specification the operations described are operations that act on multiple copies of the same noisy state. It is assumed that the noise experienced by the separate

copies has the same form and strength. If this assumption is relaxed, then an effective state that corresponds to the product of the density matrices of the individual copies is still measured so long as the copies are not entangled prior to virtual distillation. The letter  $N$  indicates the number of qubits in an individual system and the letter  $M$  indicates the number of copies (which is sometimes referred to as subsystems). Superscripts with parentheses indicate an operator that acts on multiple systems. For example, the cyclic shift operator between  $M$  copies is denoted by  $S^{(M)}$ . Bolded superscripts without parentheses denote which copy an operator/observable acts on, e.g.,  $O^1$  indicates the observable  $O$  acting on subsystem **1**. Superscripts without a bold-faced font or parentheses indicate exponentiation as usual. Subscripts on an operator generally indicate which qubit within a system the operator acts on. The exception is when the subscript is being used more generically as an index in a summation, which to the skilled person will be clear from the context and the presence of the  $2$  symbol.

#### Example Operating Environment

[0040] FIG. 1 shows a block diagram **100** of an example system performing virtual distillation to mitigate errors when calculating an expectation value of an observable with respect to a noisy quantum state. A quantum computing system **102** obtains multiple copies of a noisy quantum state **104**. For example, the quantum computing system **102** can include a quantum computer that is performing a quantum computation. Part of the quantum computation can include measuring an output quantum state to determine properties of the quantum state, e.g., an expectation value of a target observable, and obtain a result of the quantum computation. In the absence of noise, the output quantum state would be error-free. Therefore, an error-free expectation value could be obtained through repeated preparation and measurement of the output quantum state. However, realistically the quantum computation performed by the quantum computer is likely to be a noisy quantum computation, e.g., in cases where the quantum computer is a NISQ device. Therefore, the output quantum state is typically a noisy quantum state and an expectation value obtained through repeated preparation and measurement of the output quantum state will be noisy. The quantum computer must therefore correct the expectation value in order to obtain a more accurate and meaningful result of the quantum computation.

[0041] To compute a corrected expectation value the quantum computer performs collective measurements on  $M$  copies of the noisy quantum state  $\rho$  to determine expectation values of the target observable with respect to the entangled quantum state given by Equation (1) above. That is, the quantum computer determines the expectation value given by Equation (2) above.

[0042] In order to compute the numerator and denominator of Equation (2), the following equality is used

$$\text{Tr}(O \rho^M) = \text{Tr}(O^i S^{(M)} \rho^{\otimes M}). \quad (3)$$

[0043] In Equation (3),  $O^i$  indicates the observable  $O$  acting on (an arbitrary) subsystem  $i$  and  $S^{(M)}$  indicates the cyclic shift operator on  $M$  systems, i.e.,

$$O^j := \mathbb{I} \otimes \mathbb{I} \dots O \dots \mathbb{I}, \quad (4)$$

$$S^{(M)} |\psi_1\rangle \otimes |\psi_2\rangle \dots |\psi_M\rangle := |\psi_2\rangle \otimes |\psi_3\rangle \dots |\psi_1\rangle$$

[0044] This identity can be proven by expanding the right-hand side, keeping track of the indices. Without loss of generality the index  $i$  can be chosen to equal 1, yielding

$$\begin{aligned} \text{Tr}(O^1 S^{(M)} \rho^{\otimes M}) &= \sum_{i_1, i_2, \dots, i_M, j_1, j_2, \dots, j_M, k} O_{k, j_1} \delta_{j_2, i_1} \dots \delta_{j_1, i_M} \rho_{i_1, k} \dots \rho_{i_M, j_M} \\ &= \sum_{i_1, i_2, \dots, i_M, k} \rho_{i_1, k} O_{k, i_M} \rho_{i_M, i_{M-1}} \dots \rho_{i_2, i_1} \\ &= \text{Tr}(O \rho^M) \end{aligned} \quad (5)$$

[0045] Setting  $O$  as equal to the identity operator  $\mathbb{I}$  gives an equivalent expression for the denominator of Equation (2), i.e.,  $\text{Tr}(S^{(M)} \rho^{\otimes M}) = \text{Tr}(\rho^M)$ .

[0046] Therefore, to compute the numerator and denominator of Equation (2) the quantum computer is configured to perform operations to compute the quantities  $\text{Tr}(O^1 S^{(M)} \rho^{\otimes M})$  and  $\text{Tr}(S^{(M)} \rho^{\otimes M})$ . These operations include preparing a tensor product **106** of  $M$  copies of the noisy quantum state, i.e., the state  $\rho^{\otimes M}$  and applying a quantum circuit **108** to the tensor product **106**, e.g., a quantum circuit designed to implement a measurement of a product of the target observable and the cyclic shift operator  $OS^{(M)}$  with respect to the tensor product **106** or a quantum circuit designed to implement a measurement of the cyclic shift operator with respect to the tensor product **106**. After multiple measurement repetitions **110**, the quantum computer can post process **112** the obtained measurement outcomes to determine an expectation value of a product of the target observable and the cyclic shift operator with respect to the tensor product of  $M$  copies of the noisy quantum state, i.e.  $\text{Tr}(O^1 S^{(M)} \rho^{\otimes M})$ , and a corresponding normalization factor, i.e.  $\text{Tr}(S^{(M)} \rho^{\otimes M})$ . The normalization factor may correspond to the expectation value of the identity operator with respect to an entangled quantum state. The error mitigated estimation of the expectation value of the observable **114** can be obtained by dividing the expectation value of a product of the target observable and the cyclic shift operator with respect to the tensor product of  $M$  copies of the noisy quantum state by the normalization factor, as given by Equation (2).

[0047] As discussed above, the quantum circuits **108** that the quantum computer applies to the tensor products **106** include gates that either diagonalize or represent the cyclic shift operator  $S^{(M)}$ . In some implementations the quantum circuits **108** can include additional quantum gates to optimize the evaluation of the numerator and denominator of Equation (2). For example, in example quantum circuit **116** the quantum circuit further includes a single layer of two-qubit diagonalizing gates **118** which can be applied in parallel, as described in more detail below with reference to FIG. 5. Generally, the quantum circuits **108** can vary depending on various factors such as the number of copies of the noisy state included in the tensor product **106**, the type of target observable, e.g., whether the target observable is a single qubit operator or multi-qubit operator, and whether additional quantum computing resources are available to the quantum computer, e.g., whether ancilla-assisted measure-

ment is feasible in the experimental setup. Variants of virtual distillation are described in more detail below with reference to FIG. 5.

[0048] FIG. 2 depicts an example quantum computing system **200** for performing virtual distillation to mitigate errors when calculating an expectation value of an observable with respect to a noisy quantum state. The example quantum computing system **200** is an example of a system

implemented as classical and quantum computer programs on one or more classical computers and quantum computing devices in one or more locations, in which the systems, components, and techniques described herein can be implemented.

[0049] The example quantum computing system **200** includes a quantum computing device **202**. The quantum computing device **202** can be used to perform the quantum computation operations described in this specification according to some implementations. The quantum computing device **202** is intended to represent various forms of quantum computing devices. In some implementations the quantum computing device **202** is a noisy quantum computing device, e.g., a NISQ device. The components shown here, their connections and relationships, and their functions, are exemplary only, and do not limit implementations of the inventions described and/or claimed in this specification.

[0050] The example quantum computing device **202** includes a qubit assembly **252** and a control and measurement system **204**. The qubit assembly includes multiple qubits, e.g., qubit **206**, that are used to perform algorithmic operations or quantum computations. While the qubits shown in FIG. 2 are arranged in a rectangular array, this is a schematic depiction and is not intended to be limiting. The qubit assembly **252** also includes adjustable coupling elements, e.g., coupler **208**, that allow for interactions between coupled qubits. In the schematic depiction of FIG. 2, each qubit is adjustably coupled to each of its four adjacent qubits by means of respective coupling elements. However, this is an example arrangement of qubits and couplers and other arrangements are possible, including arrangements that are non-rectangular, arrangements that allow for coupling between non-adjacent qubits, and arrangements that include adjustable coupling between more than two qubits.

[0051] Each qubit can be a physical two-level quantum system or device having levels representing logical values of 0 and 1. The specific physical realization of the multiple qubits and how they interact with one another is dependent on a variety of factors including the type of the quantum computing device included in example system **200** or the type of quantum computations that the quantum computing device is performing. For example, in an atomic quantum computer the qubits may be realized via atomic, molecular or solid-state quantum systems, e.g., hyperfine atomic states. As another example, in a superconducting quantum com-

puter the qubits may be realized via superconducting qubits or semi-conducting qubits, e.g., superconducting transmon states. As another example, in a NMR quantum computer the qubits may be realized via nuclear spin states.

**[0052]** In some implementations a quantum computation can proceed by preparing the qubits in an initial state, e.g., resetting each qubit or performing quantum operations on the qubits to obtain a target quantum state, and applying a sequence of unitary operators to the initial state. Applying a unitary operator to a quantum state can include applying a corresponding sequence of quantum logic gates to the qubits. Example quantum logic gates include single-qubit gates, e.g., Pauli-X, Pauli-Y, Pauli-Z (also referred to as X, Y, Z), Hadamard and S gates, two-qubit gates, e.g., SWAP gates, controlled-X, controlled-Y, controlled-Z (also referred to as CX, CY, CZ), and gates involving three or more qubits, e.g., Toffoli gates. The quantum logic gates can be implemented by applying control signals **210** generated by the control and measurement system **204** to the qubits and to the couplers.

**[0053]** For example, in some implementations the qubits in the qubit assembly **252** can be frequency tuneable. In these examples, each qubit can have associated operating frequencies that can be adjusted through application of voltage pulses via one or more drive-lines coupled to the qubit. Example operating frequencies include qubit idling frequencies, qubit interaction frequencies, and qubit readout frequencies. Different frequencies correspond to different operations that the qubit can perform. For example, setting the operating frequency to a corresponding idling frequency may put the qubit into a state where it does not strongly interact with other qubits, and where it may be used to perform single-qubit gates. As another example, in cases where qubits interact via couplers with fixed coupling, qubits can be configured to interact with one another by setting their respective operating frequencies at some gate-dependent frequency detuning from their common interaction frequency. In other cases, e.g., when the qubits interact via tuneable couplers, qubits can be configured to interact with one another by setting the parameters of their respective couplers to enable interactions between the qubits and then by setting the qubit's respective operating frequencies at some gate-dependent frequency detuning from their common interaction frequency. Such interactions may be performed in order to perform multi-qubit gates.

**[0054]** The type of control signals **210** used depends on the physical realizations of the qubits. For example, the control signals may include RF or microwave pulses in an NMR or superconducting quantum computer system, or optical pulses in an atomic quantum computer system.

**[0055]** A quantum computation can be completed by measuring the states of the qubits, e.g., using a quantum observable such as X or Z, using respective control signals **210**. The measurements cause readout signals **212** representing measurement results to be communicated back to the measurement and control system **204**. The readout signals **212** may include RF, microwave, or optical signals depending on the physical scheme for the quantum computing device and/or the qubits. For convenience, the control signals **210** and readout signals **212** shown in FIG. 2 are depicted as addressing only selected elements of the qubit assembly (i.e. the top and bottom rows), but during operation the control signals **210** and readout signals **212** can address each element in the qubit assembly **252**.

**[0056]** The control and measurement system **204** is an example of a classical computer system that can be used to perform various operations on the qubit assembly **252**, as described above, as well as other classical subroutines or computations. The control and measurement system **204** includes one or more classical processors, e.g., classical processor **214**, one or more memories, e.g., memory **216**, and one or more I/O units, e.g., I/O unit **218**, connected by one or more data buses. The control and measurement system **204** can be programmed to send sequences of control signals **210** to the qubit assembly, e.g. to carry out a selected series of quantum gate operations, and to receive sequences of readout signals **212** from the qubit assembly, e.g. as part of performing measurement operations.

**[0057]** The processor **214** is configured to process instructions for execution within the control and measurement system **204**. In some implementations, the processor **214** is a single-threaded processor. In other implementations, the processor **214** is a multi-threaded processor. The processor **214** is capable of processing instructions stored in the memory **216**.

**[0058]** The memory **216** stores information within the control and measurement system **204**. In some implementations, the memory **216** includes a computer-readable medium, a volatile memory unit, and/or a non-volatile memory unit. In some cases, the memory **216** can include storage devices capable of providing mass storage for the system **204**, e.g. a hard disk device, an optical disk device, a storage device that is shared over a network by multiple computing devices (e.g., a cloud storage device), and/or some other large capacity storage device.

**[0059]** The input/output device **218** provides input/output operations for the control and measurement system **204**. The input/output device **218** can include D/A converters, A/D converters, and RF/microwave/optical signal generators, transmitters, and receivers, whereby to send control signals **210** to and receive readout signals **212** from the qubit assembly, as appropriate for the physical scheme for the quantum computer. In some implementations, the input/output device **218** can also include one or more network interface devices, e.g., an Ethernet card, a serial communication device, e.g., an RS-232 port, and/or a wireless interface device, e.g., an 802.11 card. In some implementations, the input/output device **218** can include driver devices configured to receive input data and send output data to other external devices, e.g., keyboard, printer and display devices.

**[0060]** Although an example control and measurement system **204** has been depicted in FIG. 2, implementations of the subject matter and the functional operations described in this specification can be implemented in other types of digital electronic circuitry, or in computer software, firmware, or hardware, including the structures disclosed in this specification and their structural equivalents, or in combinations of one or more of them.

**[0061]** The example system **200** further includes an example classical processor **250**. The classical processor **250** can be used to perform classical computation operations described in this specification according to some implementations. In FIG. 2 the classical processor **250** is shown as separate to the quantum computing device **202**, however in some implementations the classical processor **250** can be included in the quantum computing device **202**, e.g., in the processor **214**.

### Programming the Hardware: Virtual Distillation

**[0062]** FIG. 3 is a flowchart of an example process 300 for determining an error-mitigated expectation value  $\langle O \rangle_{corrected}$  of a target observable  $O$  with respect to a noisy quantum state  $\rho$ . For convenience, the process 300 will be described as being performed by a system of one or more classical and quantum computing devices located in one or more locations. For example, the quantum computing device 200 of FIG. 2, appropriately programmed in accordance with this specification, can perform the process 300.

**[0063]** The system obtains multiple of the noisy quantum state  $\rho$  (step 302). For example, the system can perform quantum computations to prepare the copies of the noisy quantum state or retrieve the copies of the noisy quantum state from a quantum memory. In some implementations the noise experienced by each copy of the noisy quantum state has a same form and strength. The number of copies can be chosen based on a variety of factors including the capabilities of the quantum device performing the process 300, the type of target observable, and/or a target accuracy of the output error-mitigated expectation value.

**[0064]** The system performs measurements on tensor products of  $M \geq 1$  copies of the noisy quantum state, e.g.,  $\rho^{\otimes M}$  to compute an expectation value of the target observable with respect to an entangled quantum state (step 304). For example, the system computes  $\text{Tr}(O^i S^{(M)} \rho^{\otimes M})$  as given by Equation (3) above. The entangled quantum state is a quantum state where eigenvalues corresponding to non-dominant eigenvectors of the noisy quantum state in the spectral decomposition of the entangled quantum state are suppressed exponentially in  $M$ . For example, the entangled quantum state can be the quantum state  $\rho_M$  given by Equation (1) above.

**[0065]** To perform the measurements, the system can, for each of a first number of measurement repetitions, measure a product of i) the target observable and ii) a cyclic shift operator (e.g., measure  $O^i S^{(M)}$ ) with respect to a tensor product of  $M$  copies of the noisy quantum state  $\rho^{\otimes M}$  to obtain a first measurement outcome. The system can then use the first measurement outcomes to compute the expectation value of the product of the target observable and the cyclic shift operator with respect to  $\rho^{\otimes M}$ , e.g., compute  $\text{Tr}(O^i S^{(M)} \rho^{\otimes M})$ .

**[0066]** As described above with reference to Equation (3), the expectation value of the product of the target observable and the cyclic shift operator with respect to  $\rho^{\otimes M}$  is equal to the expectation value of the target observable with respect to the entangled quantum state. Therefore, at step 304 the system computes the expectation value of the target observable with respect to the entangled quantum state without having to explicitly prepare the entangled quantum state.

**[0067]** The system uses the computed expectation value of the product of the target observable and the cyclic shift operator with respect to  $\rho^{\otimes M}$  to determine the error-mitigated expectation value of the target observable with respect to the noisy quantum state (step 306). For example, the system can normalize the expectation value of the product of the target observable and the cyclic shift operator with respect to  $\rho^{\otimes M}$  by dividing the expectation value by an expectation value of the cyclic shift operator with respect to  $\rho^{\otimes M}$ . The normalized expectation value provides an approximation to the error-mitigated expectation value of the target observable with respect to the noisy quantum state, as described above with reference to Equations (1)-(3).

**[0068]** To compute the expectation value of the cyclic shift operator with respect to  $\rho^{\otimes M}$  the system performs a similar process to that described at step 304, where the observable is set to equal the identity operator. That is, for each of a second number of measurement repetitions, the system measures the cyclic shift operator with respect to a tensor product of  $M$  copies of the noisy quantum state ( $\rho^{\otimes M}$ ) to obtain a second measurement outcome. The system can then use the second measurement outcomes to compute the expectation value of the cyclic shift operator with respect to the quantum state  $\rho^{\otimes M}$ , e.g.,  $\text{Tr}(S^{(M)} \rho^{\otimes M})$ .

**[0069]** Example process 300 describes a general application of virtual distillation. However, in some implementations the quantities in the numerator and denominator of Equation 2 can be evaluated in a number of different ways. For example, in some implementations steps of the process can be combined and/or implemented in parallel. Example variations of example process 300 are described in detail below with reference to FIG. 5.

### Programming the Hardware: Virtual Distillation with Measurement by Diagonalization

**[0070]** FIG. 4 is a flowchart of an example process 400 for performing virtual distillation with measurement by diagonalization. For convenience, the process 400 will be described as being performed by a system of one or more classical and quantum computing devices located in one or more locations. For example, the quantum computing device 200 of FIG. 2, appropriately programmed in accordance with this specification, can perform the process 400.

**[0071]** Example process 400 is particularly suitable for implementations where  $M=2$  and the target observable  $O$  acts on a single qubit. For convenience, example process 400 is described for the case where the target observable  $O$  is a Pauli-Z operator acting on a single qubit. However, example process 400 can also be applied to other single-qubit observables, e.g., by applying appropriate single-qubit rotations before the virtual distillation procedure. For example, the Pauli-X operator can be measured using the procedure for measuring a Pauli-Z operator after applying Hadamard gates to the appropriate qubit in each copy.

**[0072]** The system obtains multiple copies of the noisy quantum state  $\rho$  (step 402). Step 402 is similar to step 302 of example process 300 and for brevity, details are not repeated.

**[0073]** The system performs measurements on tensor products of 2 copies of the noisy quantum state, e.g.,  $\rho^{\otimes 2}$  to compute an expectation value of the target observable with respect to an entangled quantum state. At step 304 of example process 300, the relation in Equation (3) was used to compute the required expectation values. In example process 400, rather than using the relation in Equation (3) directly, the system determines a symmetrized version of the target observable is instead defined

$$O^{(M)} = \frac{1}{M} \sum_{i=1}^M O^i. \quad (6)$$

**[0074]** For the specific case that the target observable  $O$  is a Pauli-Z operator acting on a single qubit, this means that

$$Z_k^{(2)} = \frac{1}{2}(Z_k^1 + Z_k^2). \quad (7)$$

[0075] Equation (3) can then be used to show that

$$\frac{\text{Tr}(O \rho^M)}{\text{Tr}(\rho^M)} = \frac{\text{Tr}(O^{(M)} S^{(M)} \rho^{\otimes M})}{\text{Tr}(S^{(M)} \rho^{\otimes M})} \quad (8)$$

[0076] Using the symmetrized observable is advantageous because

$$[O^{(M)}, S^{(M)}] = 0, \quad (9)$$

or in this case  $[Z_k^{(2)}, S^{(2)}] = 0$ .

[0077] Both  $S^{(2)}$  and  $Z_k^{(2)}$  factorize into tensor products of operators that act separately on each pair of qubits, where the  $i$ th pair consists of the  $i$ th qubit from each system. Therefore,  $S^{(2)}$  and  $Z_k^{(2)}$  can be simultaneously diagonalized using an operator that factorizes with the same structure. The two-qubit unitary that performs this diagonalization on the  $i$ th pair is denoted as  $B_i^{(2)}$ . A matrix representation for this gate is given below

$$B_i^{(2)} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

[0078] where the choice of phases for the matrix elements can vary. The operator  $B^{(2)}$  can then be defined as

$$B^{(2)} := \bigotimes_{i=1}^M B_i^{(2)}. \quad (11)$$

[0079] As desired, this unitary diagonalized the individual factors that make up the observables

$$B^{(2)} S_i^{(2)} B^{(2)\dagger} \rightarrow \frac{1}{2}(1 + Z_i^1 - Z_i^2 + Z_i^1 Z_i^2), \quad (12)$$

$$B^{(2)} Z_k^{(2)} S_k^{(2)} B^{(2)\dagger} \rightarrow \frac{1}{2}(Z_k^1 + Z_k^2). \quad (13)$$

[0080] This diagonalization is particularly easy to implement when each qubit from the first copy of  $\rho$  is adjacent to the corresponding qubit from the second copy.

[0081] The procedure for measuring the observables required to estimate the numerator and denominator of Equation (8) then reduces to applying a single layer of  $N$  two-qubit gates in parallel and measuring in the computational basis (step 404). In fact, because  $B^{(2)}$  diagonalizes  $Z_k^{(2)} S^{(2)}$  for all  $N$  values of  $k$ , the data required to estimate the error mitigated expectation values for all  $N$  of the operators  $Z_k$  is collected simultaneously. By applying the

appropriate single qubit rotations before performing virtual distillation, an arbitrary single-qubit observable on each qubit can be accessed.

[0082] In order to develop some intuition, it is helpful to express  $\rho^{\otimes 2}$  using a spectral decomposition of  $\rho$  and consider two separate components of the resulting sum,

$$\begin{aligned} \rho^{\otimes 2} &= \sum_{ij} p_i p_j |i\rangle\langle i| \otimes |j\rangle\langle j| \\ &= \sum_i p_i^2 |i\rangle\langle i| \otimes |i\rangle\langle i| + \sum_{i \neq j} p_i p_j |i\rangle\langle i| \otimes |j\rangle\langle j|. \end{aligned} \quad (14)$$

[0083] The calculation of measurement probabilities and expectation values is a linear operation on the density matrix; these two components can therefore be considered separately. The component of the state with  $i=j$  is in the  $+1$  eigenspace of  $S^{(2)}$  and leads to measurements of  $S^{(2)}$  which yield the  $+1$  eigenvalue with probability  $p = \sum_i p_i^2 = \text{Tr}(\rho^2)$ . In the case where  $i \neq j$ ,  $|i\rangle\langle i| \otimes |j\rangle\langle j|$  is an even superposition of symmetric and anti-symmetric states,

$$|i\rangle\langle i| \otimes |j\rangle\langle j| = \frac{1}{2}(|i\rangle\langle i| \otimes |j\rangle\langle j| + |j\rangle\langle j| \otimes |i\rangle\langle i|) + \frac{1}{2}(|i\rangle\langle i| \otimes |j\rangle\langle j| - |j\rangle\langle j| \otimes |i\rangle\langle i|). \quad (15)$$

[0084] For this component of the state, measurements of  $S^{(2)}$  yield  $+1$  and  $-1$  with equal probability and  $\langle S^{(2)} \rangle = 0$ . Combining these two cases gives the expected equality  $\text{Tr}(S^{(2)} \rho^{\otimes 2}) = \text{Tr}(\rho^2)$ . Measurements of  $S^{(2)} O^{(2)}$  follow a similar pattern.

[0085] It is interesting to contrast this behavior with the stabilizer theory of quantum error correction. In the stabilizer formalism, errors are detected by projecting through measurement into the  $-1$  eigenspace of one or more symmetries. In the presently described approach, the errors are equally supported on the eigenspaces of the symmetry that is measured.

[0086] Returning to FIG. 4, to perform the measurements on the tensor products of 2 copies of the noisy quantum state to compute an expectation value of the target observable with respect to an entangled quantum state, for each of multiple measurement repetitions, the system applies a diagonalization operator to a tensor product of  $M$  copies of the noisy quantum state to obtain an evolved quantum state, where the diagonalization operator diagonalizes i) the cyclic shift operator and ii) a product of a symmetrized version of the target operator and the cyclic shift operator. The system then measures a product of the target observable and the cyclic shift operator with respect to the evolved quantum state to obtain a respective measurement outcome for each qubit in the two copies of the noisy quantum state.

[0087] The system then uses the measurement outcomes to compute the expectations values given by Equation (8), and in turn the error-mitigated expectation value of the target observable with respect to the noisy quantum state (step 406). For example, the system can compute an expectation value of the product of the target observable and the cyclic shift operator with respect to  $\rho^{\otimes M}$  using the measurement outcomes, compute an expectation value of the cyclic shift operator with respect to  $\rho^{\otimes M}$  using the measurement outcomes, divide the expectation value of the product of the

target observable and the cyclic shift operator with respect to  $\rho^{\otimes M}$  by the expectation value of the cyclic shift operator with respect to  $\rho^{\otimes M}$ .

[0088] An example algorithm for performing virtual distillation with measurement by diagonalization with  $M=2$  and  $O=Z_i$  is given below:

[0089] Input: A number of measurement repetitions  $K$ ,  $2K$  copies of the  $N$ -qubit noisy quantum state  $\rho$  (provided two at a time).

[0090] Output: An error-mitigated estimate of  $\langle Z_i \rangle$  for each qubit in  $\rho$ ;

$$\langle Z_i \rangle_{corrected} \approx \frac{\text{Tr}(Z_i \rho^2)}{\text{Tr}(\rho^2)}.$$

[0091] Set  $E_i=0$  for each qubit  $i \in 1, \dots, N$ .

[0092] Set  $D=0$ .

[0093] for  $k \in K$  do

[0094] Perform any SWAP operations necessary to make it possible to couple each qubit in the first copy of  $\rho$  with the corresponding qubit in the second copy.

[0095] Apply the two-qubit gate  $B_i^{(2)}$  defined in Equation (10) between each qubit  $i$  in the first copy and the corresponding qubit in the second copy.

[0096] Measure both states in the computational basis.

[0097] Let  $z_i^1$  and  $z_i^2$  denote the measurement outcomes for the  $i$ th qubits in the first and second copies of  $\rho$  respectively.

[0098] for  $i \in 1, \dots, N$  do

$$E_i += \frac{1}{2^N} (z_i^1 + z_i^1) \prod_{j \neq i} (1 + z_j^1 - z_j^1 + z_j^1 z_j^1)$$

end for

$$D += \frac{1}{2^N} \prod_{j=1, \dots, N} (1 + z_j^1 - z_j^1 + z_j^1 z_j^1)$$

[0099] end for

[0100] return  $\{ \langle Z \rangle_{corrected} := E_i/D \}$

Programming the Hardware: Selecting Different Virtual Distillation Protocols

[0101] FIG. 5 is a flowchart of an example process 500 for selecting a virtual distillation protocol. For convenience, the process 500 will be described as being performed by a system of one or more classical and quantum computing devices located in one or more locations. For example, the quantum computing device 200 of FIG. 2, appropriately programmed in accordance with this specification, can perform the process 500.

[0102] The system determines whether ancilla-assisted measurement is feasible in the available experimental setup (step 502). In response to determining that ancilla-assisted measurement is not feasible, the system determines whether the target observable is a single qubit observable or not (step 504). In response to determining that the target observable is not a single qubit observable, the system performs a virtual distillation protocol with serial measurements on two or more copies of the noisy quantum state (step 506).

[0103] Measuring observables with support on more than one qubit can be challenging. The challenge arises due to the use of Equation 8, in particular the choice to use the symmetrized version of an observable (defined in Equation 6.) Using the symmetrized version of a multi-qubit observable means that it is not possible to perform the required diagonalization using a tensor product of separate unitaries across each pair of qubits. As an example, consider the observable  $O=Z_i Z_j$  (the analysis is equally valid for any other operator composed of a tensor product of (more than one) single-qubit Pauli operators.) Taking the product of the symmetrized observable and the swap operator for  $M=2$  yields

$$O^{(2)} S^{(2)} = \frac{1}{2} (Z_i^1 Z_j^1 + Z_i^2 Z_j^2) S^{(2)}. \quad (16)$$

[0104] This operator does not factorize into a tensor product of operators with support on the individual pairs of qubits, nor can it be diagonalized by an operator that factors this way.

[0105] However, instead of using Equation (8) to determine the corrected expectation value of  $O$ , the non-symmetrized form introduced in Equation (2) can be used. Returning to the example where  $O=Z_i Z_j$ , the task is that the numerator and denominator of the below equation is to be computed:

$$\frac{\text{Tr}(Z_i^1 Z_j^1 S^{(2)} \rho^{\otimes 2})}{\text{Tr}(S^{(2)} \rho^{\otimes 2})}. \quad (17)$$

[0106] Unlike the symmetrized observable of Equation (16), the operator  $Z_i^1 Z_j^1 S^{(2)}$  factorizes into a tensor product over the  $N$  pairs of qubits (a pair being one qubit from the first system and the corresponding qubit from the second system).  $Z_i^1 Z_j^1 S^{(2)}$  is not Hermitian, but because it is unitary, the quantity  $\text{Tr}(Z_i^1 Z_j^1 S^{(2)} \rho^{\otimes 2})$  can be estimated by applying a circuit to diagonalize it and measuring in the computational basis. Since  $Z_i^1 Z_j^1 S^{(2)}$  factorizes into a product of two-qubit operators, the circuit that diagonalizes it does as well. The details of the diagonalization depends upon the operator to be measured.

[0107] Because  $Z_i^1 Z_j^1$  does not commute with  $S^{(2)}$ , the numerator and denominator of Equation (17) cannot be simultaneously computed. It is also not possible to measure the corrected expectation value corresponding to different choices of  $i$  and  $j$  simultaneously. More generally, it is not possible to measure any single tensor product of one-qubit operators at a time, regardless of the number of qubits it acts on.

[0108] FIG. 6 is a flowchart of an example process 600 for virtual distillation with serial measurements on two or more copies of the noisy quantum state, where the target observable acts on two or more qubits. For convenience, the process 600 will be described as being performed by a system of one or more classical and quantum computing devices located in one or more locations. For example, the quantum computing device 200 of FIG. 2, appropriately programmed in accordance with this specification, can perform the process 600. Some of the steps described below are

similar to steps described with respect to example process 300 and for brevity, details are not repeated.

[0109] The system obtains multiple copies of the noisy quantum state (step 602). The system decomposes the target observable into multiple tensor products, where each tensor product includes single-qubit operators (step 604). The system performs serial measurements on tensor products of M copies of the noisy quantum state (step 606). For each of tensor product of single qubit operators, the system performs a first number of measurement repetitions. Each measurement repetition includes applying a first diagonalization operator to a tensor product of the M copies of the noisy quantum state to obtain a first evolved quantum state, where the first diagonalization operator diagonalizes a product of i) the tensor product of one-qubit operators and ii) the cyclic shift operator, and measures a product of the tensor product of one-qubit operators and the cyclic shift operator with respect to the first evolved quantum state to obtain a respective first measurement outcome for each qubit in the first evolved quantum state (step 606a).

[0110] In addition, for each of a second number of measurement repetitions, the system applies a second diagonalization operator to a tensor product of the M copies of the noisy quantum state to obtain a second evolved quantum state, where the second diagonalization operator diagonalizes the cyclic shift operator. The system measures the cyclic shift operator with respect to the second evolved quantum state to obtain a respective second measurement outcome for each qubit in the second evolved quantum state (step 606b).

[0111] The system uses the obtained measurement outcomes to compute the error-mitigated expectation value of the target observable with respect to the noisy quantum state (step 608). For example, the system can compute an expectation value of the product of the target observable and the cyclic shift operator with respect to  $\rho^{\otimes M}$  using the first measurement outcomes, compute an expectation value of the cyclic shift operator with respect to  $\rho^{\otimes M}$  using the second measurement outcomes, and divide the expectation value of the product of the target observable and the cyclic shift operator with respect to  $\rho^{\otimes M}$  by the expectation value of the cyclic shift operator with respect to  $\rho^{\otimes M}$ .

[0112] Returning to FIG. 5, in response to determining that the target observable is a single qubit observable, the system performs a virtual distillation protocol with parallel measurements on two or more copies of the noisy quantum state (step 508).

[0113] For example, the system can perform virtual distillation with measurement by diagonalization with M=2, as described in example process 400 above. As another example, the system can perform virtual distillation with measurement by diagonalization with M>2 by generalizing example process 400 to higher powers of p.

[0114] Like the swap operator, the cyclic shift operator between M N-qubit systems,  $S^{(M)}$ , factorizes into a tensor product of N M-qubit gates. Specifically, it factorizes into the tensor product of N single-qubit cyclic shift operators. The symmetrized operator

$$Z_k^{(M)} = \frac{1}{M} \sum_{i=1}^M Z_k^i$$

commutes with the operator  $S^{(M)}$ . Therefore,  $Z_k^{(M)} S^{(M)}$  and  $S^{(M)}$  are simultaneously diagonalizable even though  $S^{(M)}$  is

unitary but not Hermitian for M>2. Because  $Z_k^{(M)}$  and  $S^{(M)}$  both factorize into tensor products over N M-tuples of qubits, the unitary that diagonalizes these operators then factorizes into a tensor product of M-qubit operators in the same way. Accordingly, the process 400 described above with respect to FIG. 4 can be applied.

[0115] Similar concerns about determining corrected expectation values of multi-qubit observables described above with reference to step 506 for the two-copy (M=2) case apply to this generalized protocol. The tools developed so far allow for simultaneous estimation of  $\text{Tr}(Z_k \rho^M)$  for all values of m and also  $\text{Tr}(\rho^M)$ . If  $\text{Tr}(P \rho^M)$  is to be reconstructed for some multi-qubit Pauli operator P, a generalization of Equation (17) could be used, but this would be limited to measuring the operators required for one particular P at a time.

[0116] For the specific case of M=3, a corresponding circuit can be optimized to simultaneously diagonalize  $Z_k^{(3)}$ ,  $S_k^{(3)}$  and  $S_k^{(3)}$  (enabling an application of example process 400 with M=3). FIG. 7 shows an example ansatz for numerically optimizing a diagonalization gate  $B_k^{(3)}$  (e.g., a gate satisfying properties similar to those satisfied by the gate defined in Equation (10)) that diagonalizes  $Z_k^{(3)} S_k^{(3)}$  and  $S_k^{(3)}$ . In FIG. 7,  $\theta_i$  represents two-qubit gate parameters for an i-th arbitrary two-qubit gate. Parameters for the four two-qubit gates that allow for an error (measured in the Frobenius norm of the difference between the exact and approximate matrices) of approximately 5E-5 when the following equations are used

$$\begin{aligned} B_k^{(3)} S_k^{(3)} B_k^{(3)\dagger} &\rightarrow \frac{1}{8} (2 + (-3 - \sqrt{3}i) Z_k^1 + \\ &\quad (3 - \sqrt{3}i) Z_k^1 Z_k^2 + (3 - \sqrt{3}i) Z_k^3 + 2i\sqrt{3} Z_k^1 Z_k^3 + (3 - \sqrt{3}i) Z_k^2 Z_k^3), \\ B_k^{(3)} S_k^{(3)} B_k^{(3)\dagger} &\rightarrow \frac{1}{3} (Z_k^1 + Z_k^2 + Z_k^3). \end{aligned}$$

[0117] Returning to FIG. 5, in response to determining at step 502 that ancilla-assisted measurement is feasible, the system performs virtual distillation for arbitrary observables, with parallel measurement on any number M of copies of the noisy quantum state (step 510).

[0118] When performing virtual distillation with ancilla-assisted measurement, the form of the observables being measured is not restricted and simultaneous measurement of operators acting on overlapping subsets of qubits is possible. Therefore, virtual distillation with ancilla-assisted measurement is compatible with known techniques for efficiently measuring a large collection of commuting operators.

[0119] To perform virtual distillation with ancilla-assisted measurement on M=2 copies of the noisy quantum state, the system can approximate the numerator and denominator of Equation 8 using a non-destructive measurement of the swap operator  $S^{(2)}$ . For each of multiple measurement repetitions, the system prepares (or otherwise obtains) two quantum registers, where each register is prepared in the noisy quantum state  $\rho$  and includes an ancilla qubit prepared in the 0 state. The system performs a non-destructive measurement of  $S^{(2)}$ , e.g., using a swap or Hadamard test. Specifically, the system applies a Hadamard gate to the ancilla qubit, applies a cyclic shift operator to the two registers conditioned on the ancilla qubit being in the 1 state, and measures the ancilla qubit in the X basis to obtain a first measurement outcome.



Because  $S^{(2)}$  factorizes into a tensor product of two-qubit swap gates, its controlled version likewise factorizes into a series of  $N$  Fredkin (controlled-swap) gates. In some implementations compiling this circuit may necessitate some extra steps (such as expanding the single ancilla qubit into a GHZ state using a series of CNOT gates) in order to deal with the restricted connectivity of a near-term device.

**[0120]** Under the assumption that the target observable can be symmetrized, as described above with reference to example process **400**, the system can then measure the product  $O^{(2)}S^{(2)}$  by measuring the qubits in the two registers with respect to the target observable  $O^{(2)}$  to obtain respective second measurement outcomes. The system can then use the first measurement outcomes and the second measurement outcomes from the different measurement repetitions to compute the error-mitigated expectation value of the target observable with respect to the noisy quantum state. In particular, the system can compute an expectation value of the target observable using the second measurement outcomes, compute an expectation value of the cyclic shift operator using the first measurement outcomes, and divide the expectation value of the target observable by the expectation value of the cyclic shift operator.

**[0121]** This protocol does not require a separate estimation of  $\text{Tr}(\rho O)$ . Furthermore, measurements of  $O$  on both copies of  $p$  can be used and the numerator and denominator of Equation (8) can be estimated simultaneously, leading to a relatively sample-efficient scheme.

**[0122]** For virtual distillation with ancilla-assisted measurement on  $M > 2$  copies of the noisy quantum state,  $S^{(N)}$  is not Hermitian for  $N > 2$ . However, the natural generalization to the above strategy still works as expected. Specifically, a controlled version of the cyclic shift operator  $S^{(N)}$  can be used to sample an observable whose expectation value is equal to  $\text{Re}(\text{Tr}(S^{(N)}\rho^{\otimes N}))$ . Because the symmetrized observable  $O^{(N)}$  commutes with  $S^{(N)}$ , it also commutes with the observable measured by this generalization of the swap test. Therefore, an observable whose expectation value is  $\text{Tr}(S^{(N)}O^{(N)}\rho^{\otimes N})$  can be sampled from by first performing the higher-order swap test and then a measurement of  $O^{(N)}$ .

#### Mitigating Algorithmic Errors

**[0123]** Most error mitigation methods focus on the reduction of errors caused by imperfections in a device implementation, such as decoherence or control errors. Some of these techniques can be applied to algorithmic errors incurred during otherwise noise-free implementations of randomized algorithms. Developments in Hamiltonian simulation have led to the development of randomized evolution methods such as qDRIFT, randomized Trotter, and combinations thereof, which have benefits in some situations over their deterministic counterparts. As these methods are randomized, they output mixed states rather than pure states, even in the absence of noise. Moreover, they depend on an approximation parameter with a natural limit in which they converge to the pure state generated by exact evolution. As described below, virtual distillation applied to qDRIFT can suppress this deviation from the exact evolution. For the particular model system considered below, it is found that virtual distillation can reduce the coherent space-time volume required to reach a particular accuracy threshold by a factor of 8 or more compared with the standard qDRIFT.

**[0124]** The qDRIFT method simulates time evolution under a Hamiltonian  $H$  by constructing product formulae

using a randomized selection rule. Terms are chosen from  $H$  at random, with a selection probability proportional to their interaction strength in the Hamiltonian. The system is then evolved forwards in time under this Hamiltonian term, for a fixed time step. This process is repeated a number of times, generating a product formula that provides an approximation to the time evolution operator. When averaged over the classical randomness (in the choice of interaction terms), qDRIFT generates a quantum channel that closely approximates the exact evolution more closely than the individual product formulae. Importantly, unlike most deterministic Trotter methods, the scaling of this approach does not depend explicitly on the number of terms in the Hamiltonian, but rather than 1-norm of the coefficients.

**[0125]** Consider a Hamiltonian that can be decomposed as  $H = \sum_i h_i H_i$ , where all  $h_i$  are can be made real and positive by absorbing signs into  $H_i$ , and the spectral norm of  $H_i$  is bounded by 1. Defining  $\lambda = \sum_i h_i$  the diamond norm distance between the qDRIFT channel and the true time evolution is bounded by  $\epsilon = 2\lambda^2 t^2 / \eta$  where  $n$  is the number of qDRIFT selection steps performed to generate each instance of the qDRIFT channel, and hence controls the amount of coherent evolution required. As  $n$  increases, the resulting quantum channel converges to the unitary corresponding to the exact evolution.

**[0126]** The presently described virtual distillation technique can reduce the coherent space-time volume required, by reducing this factor  $n$  required to achieve the same error in practice. In particular, virtual distillation can impact the number of coherent steps,  $\eta$ , required to reach a target accuracy. For this, a Heisenberg Hamiltonian with up to 6 qubits per copy is considered. The Hamiltonian is given by

$$H = \sum_{i=1}^N X_i X_{i+1} + Y_i Y_{i+1} + Z_i Z_{i+1} + h_i Z_i$$

where  $h_i \in \{-h, h\}$  are randomly chosen Z magnetic field strengths, and periodic boundary conditions are applied such that site  $N+1$  is site 1. For the presently described studies a time evolution length of  $t=N$  is considered and let  $h=1$ . The number of coherent qDRIFT steps required to achieve a trace distance of 0:01 to the ideal state for evolutions under such a Heisenberg model are numerically investigated.

**[0127]** FIG. 7 shows results of this analysis. In particular, FIG. 7 shows qDRIFT coherent cost reduction through virtual distillation in the Heisenberg model. The number of coherent qDRIFT steps required to reach a target trace distance, with and without virtual distillation using 2 copies, is shown. It can be seen that there is a consistent reduction of at least 16× in the number of required steps. When accounting for the over-head of using two copies, this amounts to a 8× reduction in the coherent space-time volume used to reach the same error rate. These results show that the use of the presently described error mitigation techniques can provide practical algorithmic improvements for real systems, especially in the NISQ regime. For example, by incorporating the presently described error mitigation techniques algorithms can produce more accurate results with reduced overhead, e.g., the overhead associated with quantum error correction codes.

**[0128]** Implementations of the digital and/or quantum subject matter and the digital functional operations and

quantum operations described in this specification can be implemented in digital electronic circuitry, suitable quantum circuitry or, more generally, quantum computational systems, in tangibly-embodied digital and/or quantum computer software or firmware, in digital and/or quantum computer hardware, including the structures disclosed in this specification and their structural equivalents, or in combinations of one or more of them. The term “quantum computational systems” may include, but is not limited to, quantum computers, quantum information processing systems, quantum cryptography systems, or quantum simulators.

**[0129]** Implementations of the digital and/or quantum subject matter described in this specification can be implemented as one or more digital and/or quantum computer programs, i.e., one or more modules of digital and/or quantum computer program instructions encoded on a tangible non-transitory storage medium for execution by, or to control the operation of, data processing apparatus. The digital and/or quantum computer storage medium can be a machine-readable storage device, a machine-readable storage substrate, a random or serial access memory device, one or more qubits, or a combination of one or more of them. Alternatively or in addition, the program instructions can be encoded on an artificially-generated propagated signal that is capable of encoding digital and/or quantum information, e.g., a machine-generated electrical, optical, or electromagnetic signal, that is generated to encode digital and/or quantum information for transmission to suitable receiver apparatus for execution by a data processing apparatus.

**[0130]** The terms quantum information and quantum data refer to information or data that is carried by, held or stored in quantum systems, where the smallest non-trivial system is a qubit, i.e., a system that defines the unit of quantum information. It is understood that the term “qubit” encompasses all quantum systems that may be suitably approximated as a two-level system in the corresponding context. Such quantum systems may include multi-level systems, e.g., with two or more levels. By way of example, such systems can include atoms, electrons, photons, ions or superconducting qubits. In many implementations the computational basis states are identified with the ground and first excited states, however it is understood that other setups where the computational states are identified with higher level excited states are possible. The term “data processing apparatus” refers to digital and/or quantum data processing hardware and encompasses all kinds of apparatus, devices, and machines for processing digital and/or quantum data, including by way of example a programmable digital processor, a programmable quantum processor, a digital computer, a quantum computer, multiple digital and quantum processors or computers, and combinations thereof. The apparatus can also be, or further include, special purpose logic circuitry, e.g., an FPGA (field programmable gate array), an ASIC (application-specific integrated circuit), or a quantum simulator, i.e., a quantum data processing apparatus that is designed to simulate or produce information about a specific quantum system. In particular, a quantum simulator is a special purpose quantum computer that does not have the capability to perform universal quantum computation. The apparatus can optionally include, in addition to hardware, code that creates an execution environment for digital and/or quantum computer programs, e.g., code that

constitutes processor firmware, a protocol stack, a database management system, an operating system, or a combination of one or more of them.

**[0131]** A digital computer program, which may also be referred to or described as a program, software, a software application, a module, a software module, a script, or code, can be written in any form of programming language, including compiled or interpreted languages, or declarative or procedural languages, and it can be deployed in any form, including as a stand-alone program or as a module, component, subroutine, or other unit suitable for use in a digital computing environment. A quantum computer program, which may also be referred to or described as a program, software, a software application, a module, a software module, a script, or code, can be written in any form of programming language, including compiled or interpreted languages, or declarative or procedural languages, and translated into a suitable quantum programming language, or can be written in a quantum programming language, e.g., QCL or Quipper.

**[0132]** A digital and/or quantum computer program may, but need not, correspond to a file in a file system. A program can be stored in a portion of a file that holds other programs or data, e.g., one or more scripts stored in a markup language document, in a single file dedicated to the program in question, or in multiple coordinated files, e.g., files that store one or more modules, sub-programs, or portions of code. A digital and/or quantum computer program can be deployed to be executed on one digital or one quantum computer or on multiple digital and/or quantum computers that are located at one site or distributed across multiple sites and interconnected by a digital and/or quantum data communication network. A quantum data communication network is understood to be a network that may transmit quantum data using quantum systems, e.g. qubits. Generally, a digital data communication network cannot transmit quantum data, however a quantum data communication network may transmit both quantum data and digital data.

**[0133]** The processes and logic flows described in this specification can be performed by one or more programmable digital and/or quantum computers, operating with one or more digital and/or quantum processors, as appropriate, executing one or more digital and/or quantum computer programs to perform functions by operating on input digital and quantum data and generating output. The processes and logic flows can also be performed by, and apparatus can also be implemented as, special purpose logic circuitry, e.g., an FPGA or an ASIC, or a quantum simulator, or by a combination of special purpose logic circuitry or quantum simulators and one or more programmed digital and/or quantum computers.

**[0134]** For a system of one or more digital and/or quantum computers to be “configured to” perform particular operations or actions means that the system has installed on it software, firmware, hardware, or a combination of them that in operation cause the system to perform the operations or actions. For one or more digital and/or quantum computer programs to be configured to perform particular operations or actions means that the one or more programs include instructions that, when executed by digital and/or quantum data processing apparatus, cause the apparatus to perform the operations or actions. A quantum computer may receive instructions from a digital computer that, when executed by

the quantum computing apparatus, cause the apparatus to perform the operations or actions.

**[0135]** Digital and/or quantum computers suitable for the execution of a digital and/or quantum computer program can be based on general or special purpose digital and/or quantum processors or both, or any other kind of central digital and/or quantum processing unit. Generally, a central digital and/or quantum processing unit will receive instructions and digital and/or quantum data from a read-only memory, a random access memory, or quantum systems suitable for transmitting quantum data, e.g. photons, or combinations thereof.

**[0136]** The essential elements of a digital and/or quantum computer are a central processing unit for performing or executing instructions and one or more memory devices for storing instructions and digital and/or quantum data. The central processing unit and the memory can be supplemented by, or incorporated in, special purpose logic circuitry or quantum simulators. Generally, a digital and/or quantum computer will also include, or be operatively coupled to receive digital and/or quantum data from or transfer digital and/or quantum data to, or both, one or more mass storage devices for storing digital and/or quantum data, e.g., magnetic, magneto-optical disks, optical disks, or quantum systems suitable for storing quantum information. However, a digital and/or quantum computer need not have such devices.

**[0137]** Digital and/or quantum computer-readable media suitable for storing digital and/or quantum computer program instructions and digital and/or quantum data include all forms of non-volatile digital and/or quantum memory, media and memory devices, including by way of example semiconductor memory devices, e.g., EPROM, EEPROM, and flash memory devices; magnetic disks, e.g., internal hard disks or removable disks; magneto-optical disks; CD-ROM and DVD-ROM disks; and quantum systems, e.g., trapped atoms or electrons. It is understood that quantum memories are devices that can store quantum data for a long time with high fidelity and efficiency, e.g., light-matter interfaces where light is used for transmission and matter for storing and preserving the quantum features of quantum data such as superposition or quantum coherence.

**[0138]** Control of the various systems described in this specification, or portions of them, can be implemented in a digital and/or quantum computer program product that includes instructions that are stored on one or more non-transitory machine-readable storage media, and that are executable on one or more digital and/or quantum processing devices. The systems described in this specification, or portions of them, can each be implemented as an apparatus, method, or system that may include one or more digital and/or quantum processing devices and memory to store executable instructions to perform the operations described in this specification.

**[0139]** While this specification contains many specific implementation details, these should not be construed as limitations on the scope of what may be claimed, but rather as descriptions of features that may be specific to particular implementations. Certain features that are described in this specification in the context of separate implementations can also be implemented in combination in a single implementation. Conversely, various features that are described in the context of a single implementation can also be implemented in multiple implementations separately or in any suitable

sub-combination. Moreover, although features may be described above as acting in certain combinations and even initially claimed as such, one or more features from a claimed combination can in some cases be excised from the combination, and the claimed combination may be directed to a sub-combination or variation of a sub-combination.

**[0140]** Similarly, while operations are depicted in the drawings in a particular order, this should not be understood as requiring that such operations be performed in the particular order shown or in sequential order, or that all illustrated operations be performed, to achieve desirable results. In certain circumstances, multitasking and parallel processing may be advantageous. Moreover, the separation of various system modules and components in the implementations described above should not be understood as requiring such separation in all implementations, and it should be understood that the described program components and systems can generally be integrated together in a single software product or packaged into multiple software products.

**[0141]** Particular implementations of the subject matter have been described. Other implementations are within the scope of the following claims. For example, the actions recited in the claims can be performed in a different order and still achieve desirable results. As one example, the processes depicted in the accompanying figures do not necessarily require the particular order shown, or sequential order, to achieve desirable results. In some cases, multitasking and parallel processing may be advantageous.

What is claimed is:

1. A method performed by a system comprising a classical processor and a quantum computer, the method comprising:
  - determining an error-mitigated expectation value of a target observable with respect to a noisy quantum state, wherein the target observable is supported on multiple qubits and comprises one or more tensor products of single-qubit operators, the determining comprising:
    - obtaining, by the quantum computer, multiple copies of the noisy quantum state;
    - performing, by the quantum computer, measurements on tensor products of two copies of the noisy quantum state to compute an expectation value, with respect to an entangled quantum state, of a product of the target observable in non-symmetrized form and a cyclic shift operator, wherein: the entangled quantum state is given by  $\rho^2/\text{Tr}(\rho^2)$  with  $\rho$  representing the noisy quantum state, the target observable in non-symmetrized form acts only on a first copy of the noisy quantum state in each tensor product of the two copies of the noisy quantum state, and the cyclic shift operator acts on both copies of the noisy quantum state in each tensor product; and
    - dividing, by the classical processor, the computed expectation value of the product of the target observable in non-symmetrized form and a cyclic shift operator by a normalization factor to determine the error-mitigated expectation value of the target observable with respect to the noisy quantum state.
2. The method of claim 1, wherein the noisy quantum state comprises errors resulting from noise on the quantum computer and the error-mitigated expectation value of the target observable with respect to the noisy quantum state approximates an expectation value of the target observable

computed by the quantum computer in an absence of the noise and errors on the quantum computer.

3. The method of claim 1, wherein performing the measurements on the tensor products of the two copies of the noisy quantum state comprises:

for each tensor product of single-qubit operators:

for each of a first number of measurement repetitions:

applying a first diagonalization operator to a tensor product of the two copies of the noisy quantum state to obtain an evolved quantum state, wherein the first diagonalization operator diagonalizes a product of i) the tensor product of one-qubit operators and ii) the cyclic shift operator, and

measuring a product of the tensor product of one-qubit operators and the cyclic shift operator with respect to the evolved quantum state to obtain a respective first measurement outcome for the repetition of the first number of repetitions for each qubit in the evolved quantum state.

4. The method of claim 3, wherein the measuring is performed in the computational basis.

5. The method of claim 3, further comprising:

for each of a second number of measurement repetitions:

applying a second diagonalization operator to a tensor product of the two copies of the noisy quantum state to obtain a second evolved quantum state, wherein the second diagonalization operator diagonalizes the cyclic shift operator, and

measuring the cyclic shift operator with respect to the second evolved quantum state to obtain a respective second measurement outcome for the repetition of the second number of repetitions for each qubit in the second evolved quantum state.

6. The method of claim 5, further comprising:

computing the expectation value of the product of the target observable in non-symmetrized form and the cyclic shift operator using the first measurement outcomes obtained from the first number of measurement repetitions;

computing an expectation value of the cyclic shift operator using the second measurement outcomes obtained from the second number of measurement repetitions; and

dividing the expectation value of the product of the target observable in non-symmetrized form and the cyclic shift operator by the expectation value of the cyclic shift operator.

7. The method of claim 1, wherein noise experienced by each copy of the noisy quantum state comprises a same form and strength.

8. The method of claim 1, wherein the normalization factor comprises an expectation value of the cyclic shift operator with respect to the tensor product of two copies of the noisy quantum state.

9. The method of claim 1, wherein performing measurements on the tensor products of the two copies of the noisy quantum state comprises performing serial measurements.

10. The method of claim 1, wherein the measurements on the tensor products of the two copies of the noisy quantum state exclude ancilla-assisted measurements.

11. An apparatus comprising:

one or more classical processors; and

one or more quantum computing devices in data communication with the one or more classical processors, wherein the quantum computing hardware comprises: one or more qubit registers, each qubit register comprising one or more qubits, and

a plurality of control devices configured to operate the one or more qubit registers;

wherein the apparatus is configured to perform operations comprising:

determining an error-mitigated expectation value of a target observable with respect to a noisy quantum state, wherein the target observable is supported on multiple qubits and comprises one or more tensor products of single-qubit operators, the determining comprising:

obtaining, by the quantum computer, multiple copies of the noisy quantum state;

performing, by the quantum computer, measurements on tensor products of two copies of the noisy quantum state to compute an expectation value, with respect to an entangled quantum state, of a product of the target observable in non-symmetrized form and a cyclic shift operator, wherein: the entangled quantum state is given by  $\rho^2/\text{Tr}(\rho^2)$  with  $\rho$  representing the noisy quantum state, the target observable in non-symmetrized form acts only on a first copy of the noisy quantum state in each tensor product of the two copies of the noisy quantum state, and the cyclic shift operator acts on both copies of the noisy quantum state in each tensor product; and dividing, by the classical processor, the computed expectation value of the product of the target observable in non-symmetrized form and a cyclic shift operator by a normalization factor to determine the error-mitigated expectation value of the target observable with respect to the noisy quantum state.

12. The apparatus of claim 11, wherein the noisy quantum state comprises errors resulting from noise on the quantum computer and the error-mitigated expectation value of the target observable with respect to the noisy quantum state approximates an expectation value of the target observable computed by the quantum computer in an absence of the noise and errors on the quantum computer.

13. The apparatus of claim 11, wherein performing the measurements on the tensor products of the two copies of the noisy quantum state comprises:

for each tensor product of single-qubit operators:

for each of a first number of measurement repetitions:

applying a first diagonalization operator to a tensor product of the two copies of the noisy quantum state to obtain an evolved quantum state, wherein the first diagonalization operator diagonalizes a product of i) the tensor product of one-qubit operators and ii) the cyclic shift operator, and

measuring a product of the tensor product of one-qubit operators and the cyclic shift operator with respect to the evolved quantum state to obtain a respective first measurement outcome for the repetition of the first number of repetitions for each qubit in the evolved quantum state.

14. The apparatus of claim 13, wherein the measuring is performed in the computational basis.

15. The apparatus of claim 13, further comprising:

for each of a second number of measurement repetitions:

applying a second diagonalization operator to a tensor product of the two copies of the noisy quantum state to obtain a second evolved quantum state, wherein the second diagonalization operator diagonalizes the cyclic shift operator, and

measuring the cyclic shift operator with respect to the second evolved quantum state to obtain a respective second measurement outcome for the repetition of the second number of repetitions for each qubit in the second evolved quantum state.

**16.** The apparatus of claim **15**, further comprising:

computing the expectation value of the product of the target observable in non-symmetrized form and the cyclic shift operator using the first measurement outcomes obtained from the first number of measurement repetitions;

computing an expectation value of the cyclic shift operator using the second measurement outcomes obtained from the second number of measurement repetitions; and

dividing the expectation value of the product of the target observable in non-symmetrized form and the cyclic shift operator by the expectation value of the cyclic shift operator.

**17.** The apparatus of claim **11**, wherein noise experienced by each copy of the noisy quantum state comprises a same form and strength.

**18.** The apparatus of claim **11**, wherein the normalization factor comprises an expectation value of the cyclic shift operator with respect to the tensor product of two copies of the noisy quantum state.

**19.** The apparatus of claim **11**, wherein performing measurements on the tensor products of the two copies of the noisy quantum state comprises performing serial measurements.

**20.** The apparatus of claim **11**, wherein the measurements on the tensor products of the two copies of the noisy quantum state exclude ancilla-assisted measurements.

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