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(54) **SYSTEM AND METHOD FOR PROCESSING A MULTI-DIMENSIONAL DATA OBJECT OBTAINED BY A METROLOGICAL INVESTIGATION**

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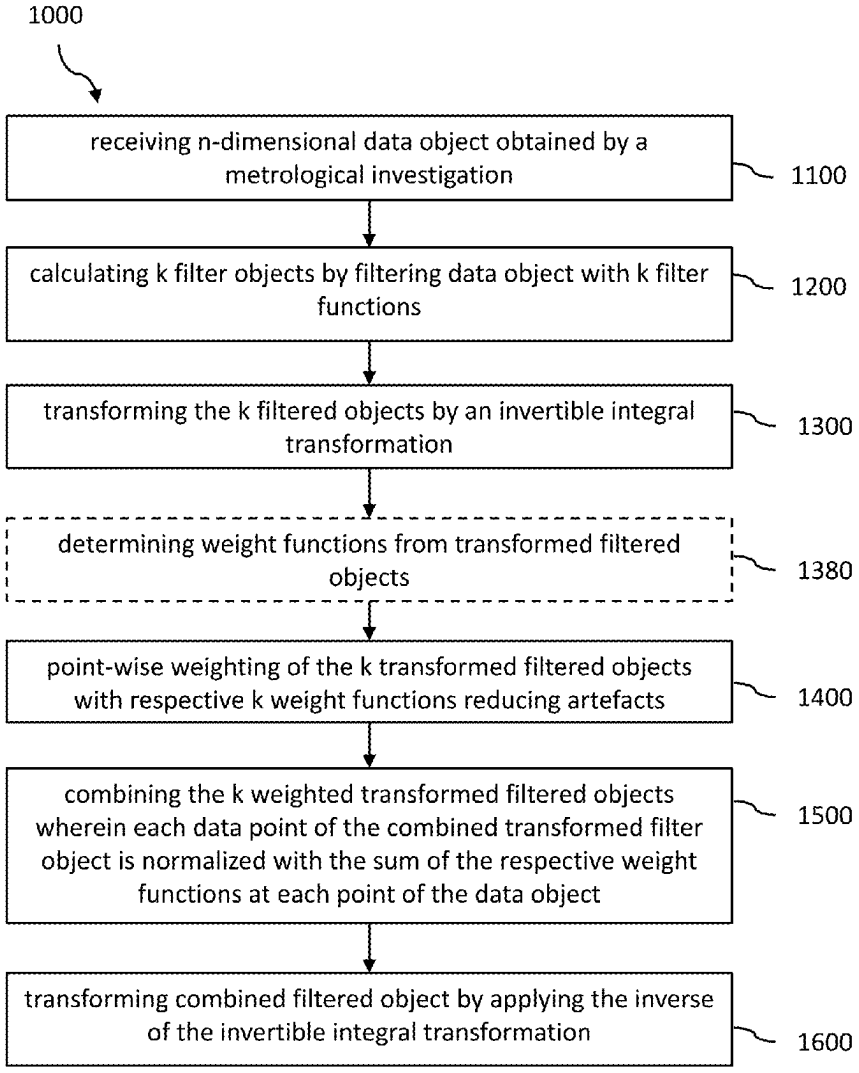
(57) **ABSTRACT**

A plurality of k filtered objects is calculated by filtering an n-dimensional data object of a metrological investigation received from a data source with k filter functions, wherein each filter function, in the space of the data object, is missing at least one of the n dimensions of the data object such that none of the n dimensions of the data object is missing in all filter functions. The k filtered objects are transformed by an invertible integral transformation into k transformed filtered objects followed by point-wise weighting of each of the k transformed filtered objects with a respective weight function resulting in k weighted transformed filtered objects. The k weighted transformed filtered objects are combined into a combined transformed filtered object, which is transformed by applying the inverse of the invertible integral transformation, into a weighted-filtered n-dimensional data object.



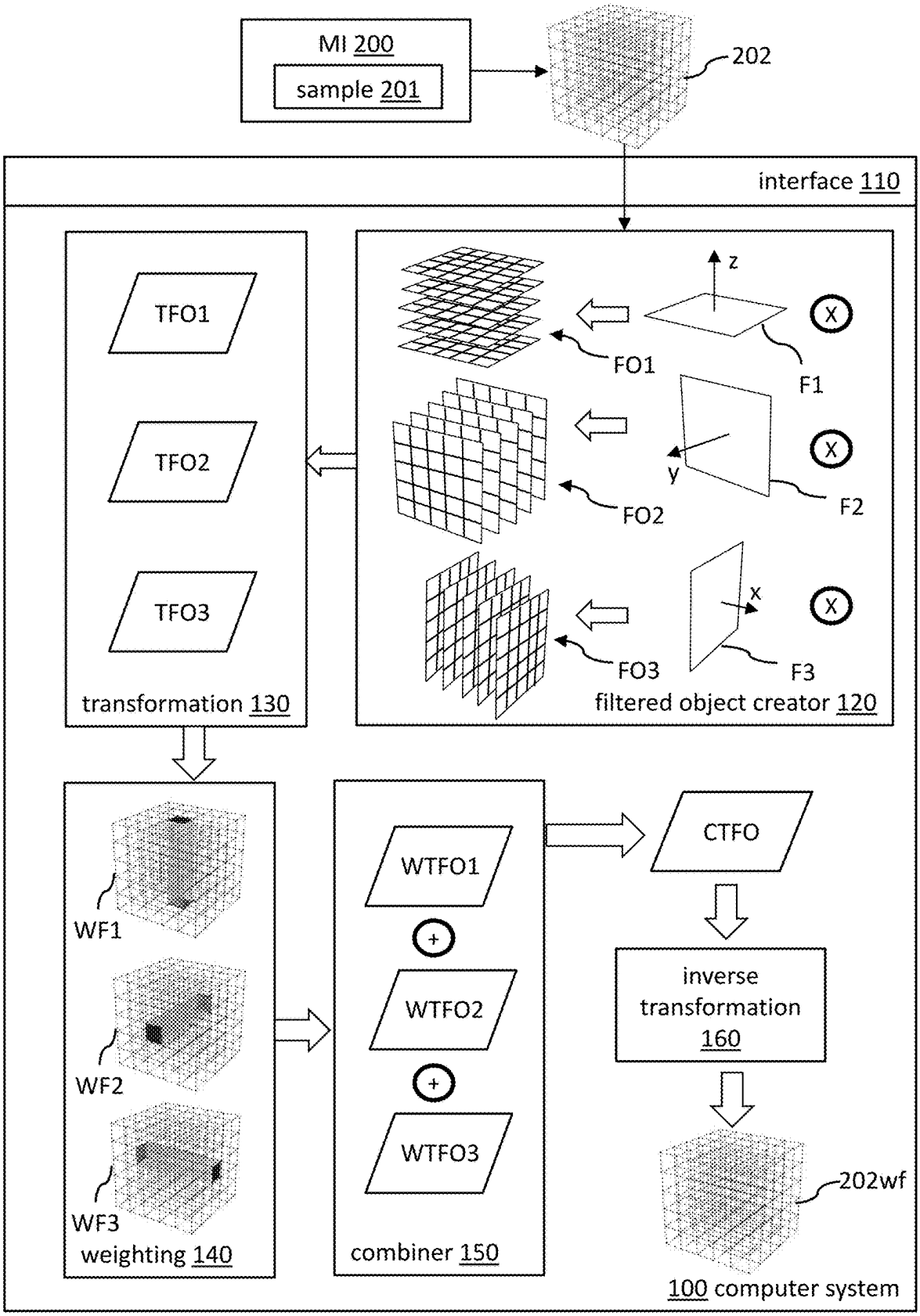

MI 200
sample 201
202
interface 110
TFO1
TFO2
TFO3
transformation 130
z
X
FO1
F1
y
FO2
F2
x
FO3
F3
filtered object creator 120
WF1
WF2
WF3
weighting 140
WTFO1
WTFO2
WTFO3
combiner 150
CTFO
inverse transformation 160
202wf
100 computer system

FIG. 1

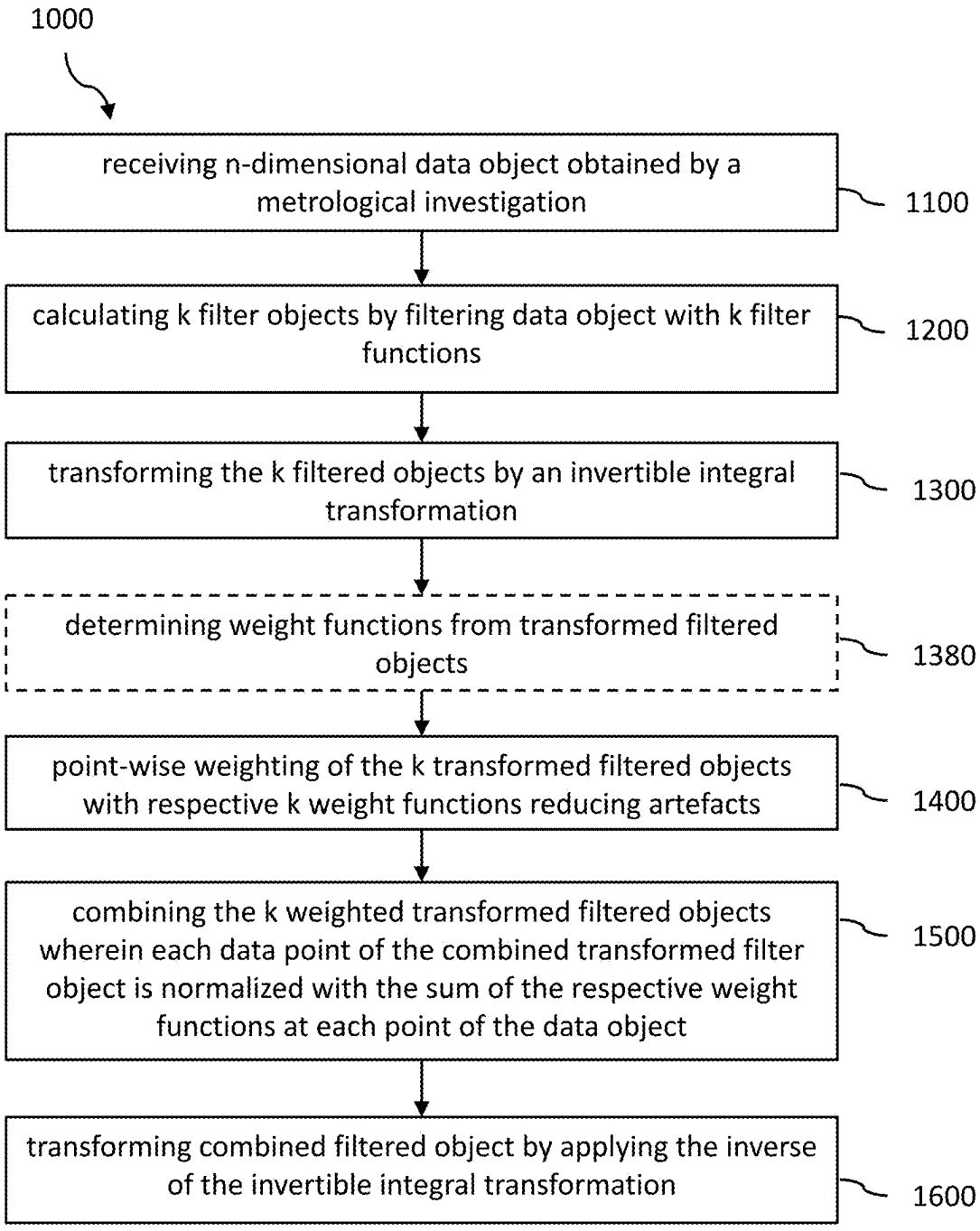

1000
receiving n-dimensional data object obtained by a metrological investigation
1100
calculating k filter objects by filtering data object with k filter functions
1200
transforming the k filtered objects by an invertible integral transformation
1300
determining weight functions from transformed filtered objects
1380
point-wise weighting of the k transformed filtered objects with respective k weight functions reducing artefacts
1400
combining the k weighted transformed filtered objects wherein each data point of the combined transformed filter object is normalized with the sum of the respective weight functions at each point of the data object
1500
transforming combined filtered object by applying the inverse of the invertible integral transformation
1600

FIG. 2

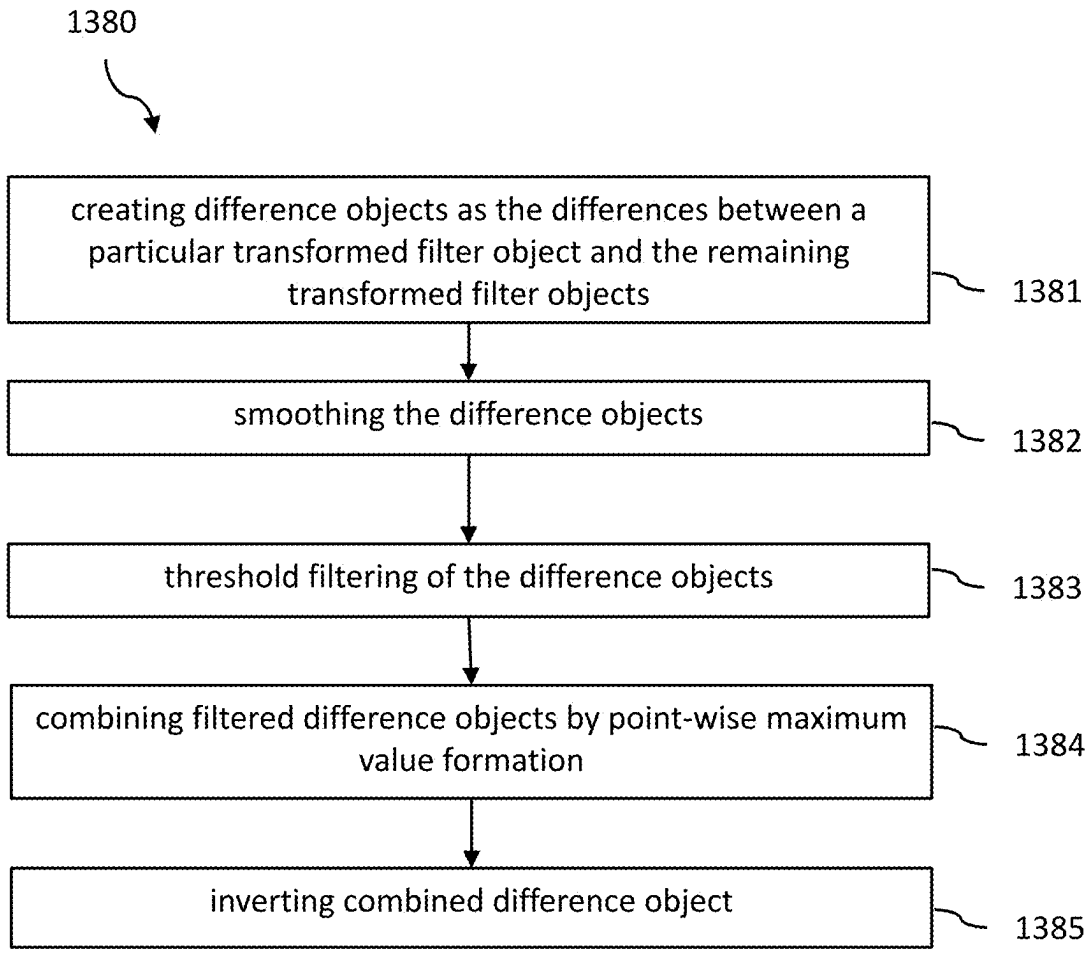

1380
creating difference objects as the differences between a particular transformed filter object and the remaining transformed filter objects
1381
smoothing the difference objects
1382
threshold filtering of the difference objects
1383
combining filtered difference objects by point-wise maximum value formation
1384
inverting combined difference object
1385

FIG. 3

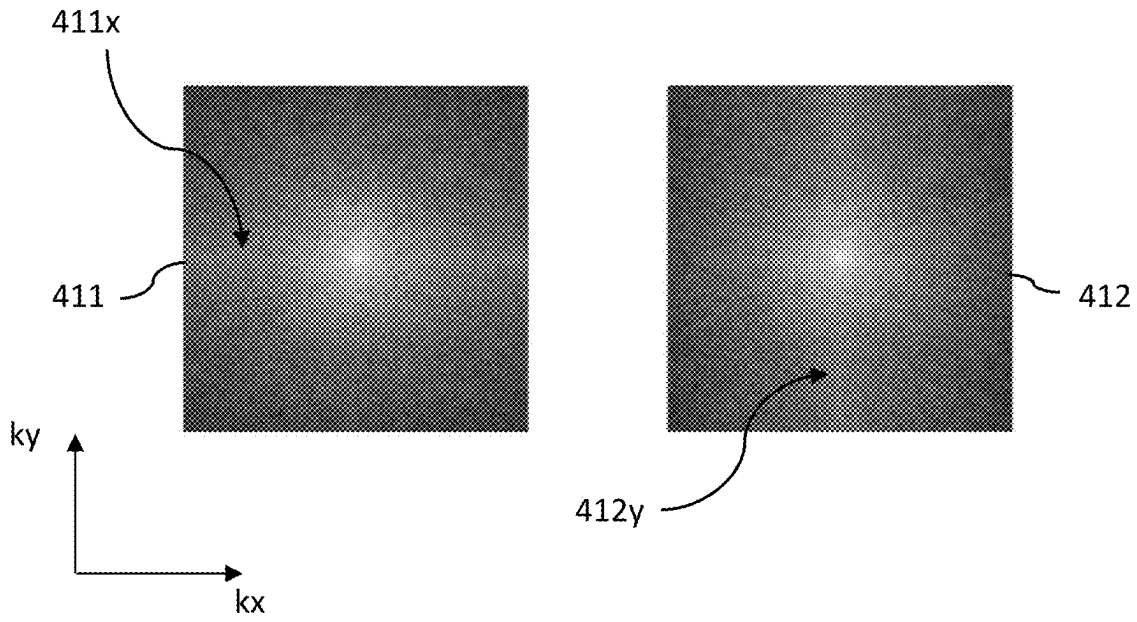


FIG. 4A

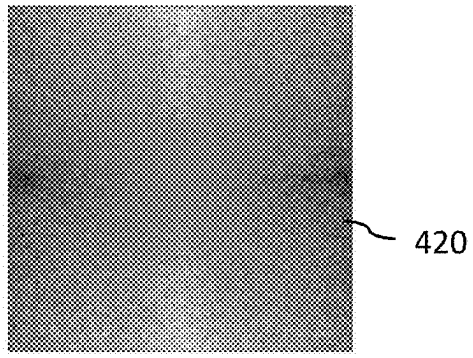


FIG. 4B

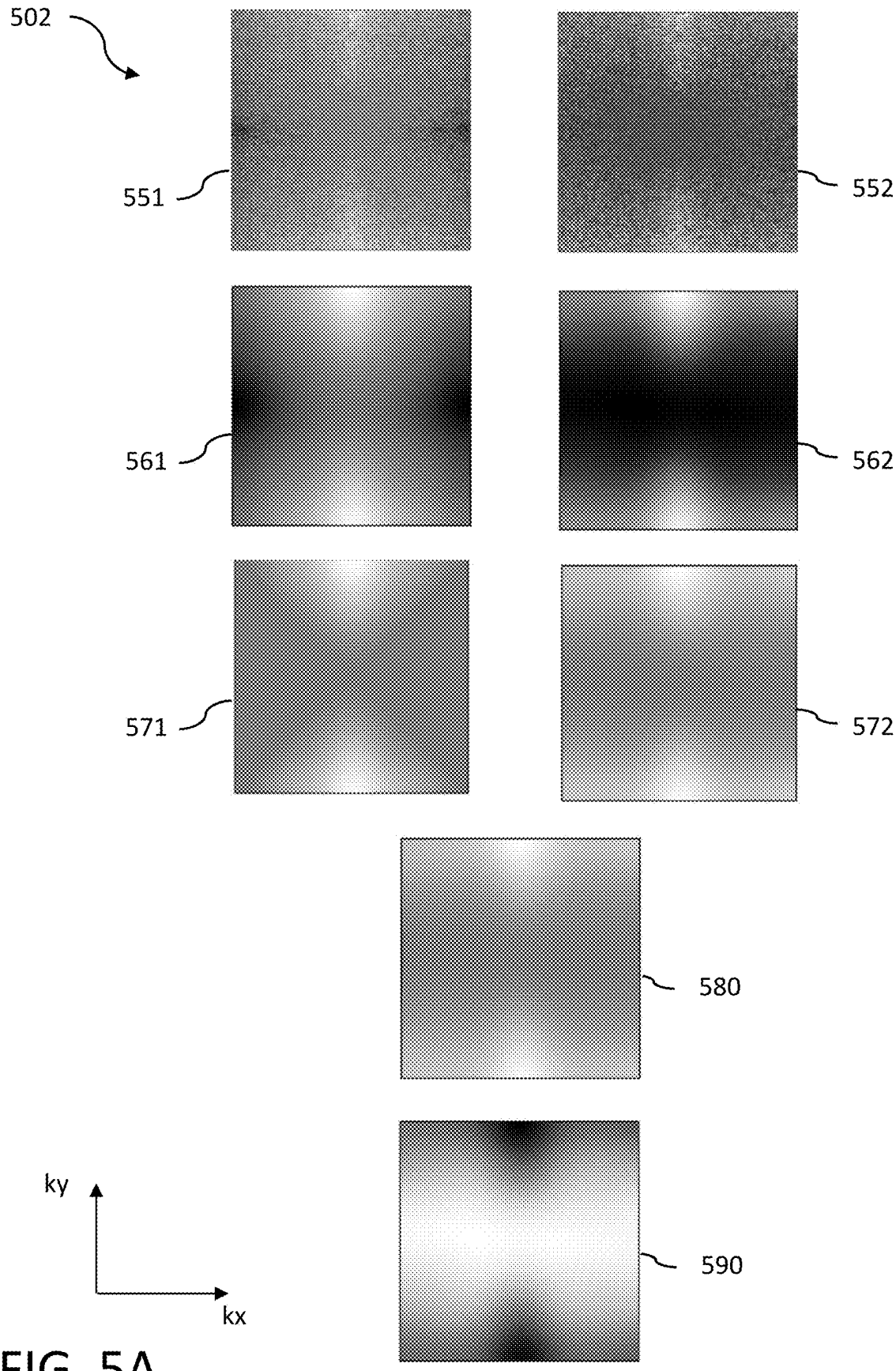


FIG. 5A

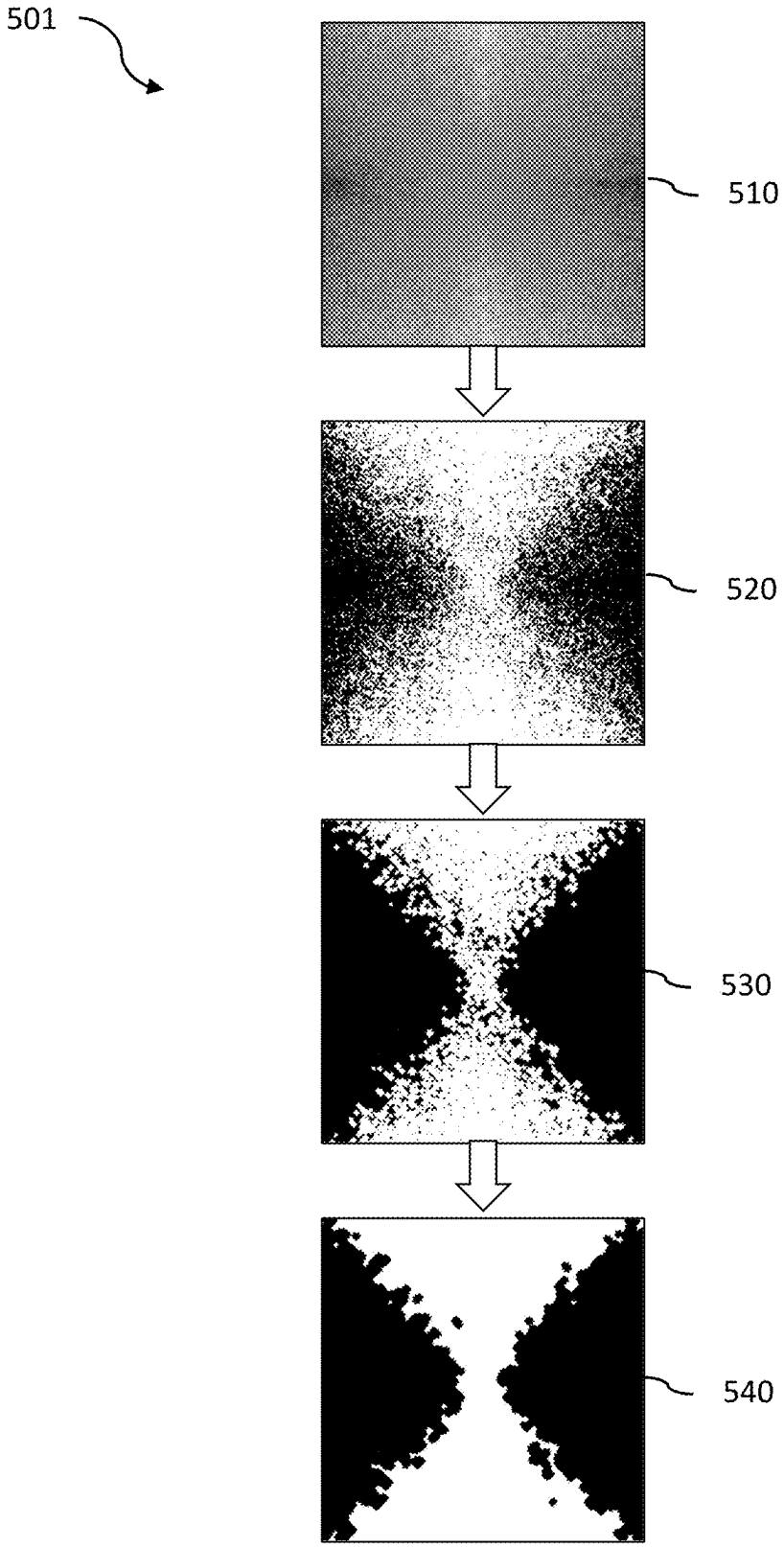


FIG. 5B

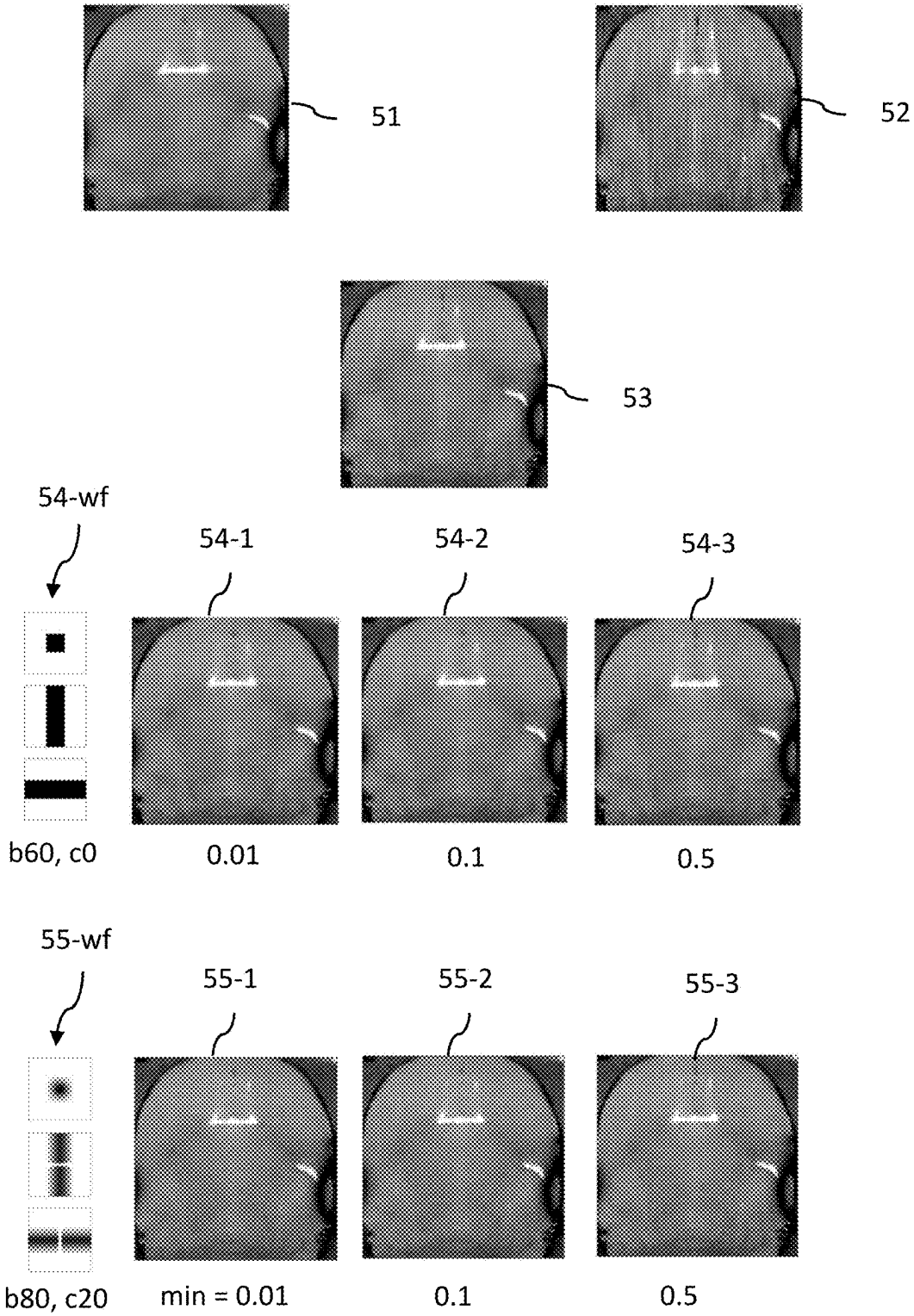

51
52
53
54-wf
54-1
54-2
54-3
b60, c0
0.01
0.1
0.5
55-wf
55-1
55-2
55-3
b80, c20
min = 0.01
0.1
0.5

FIG. 5C

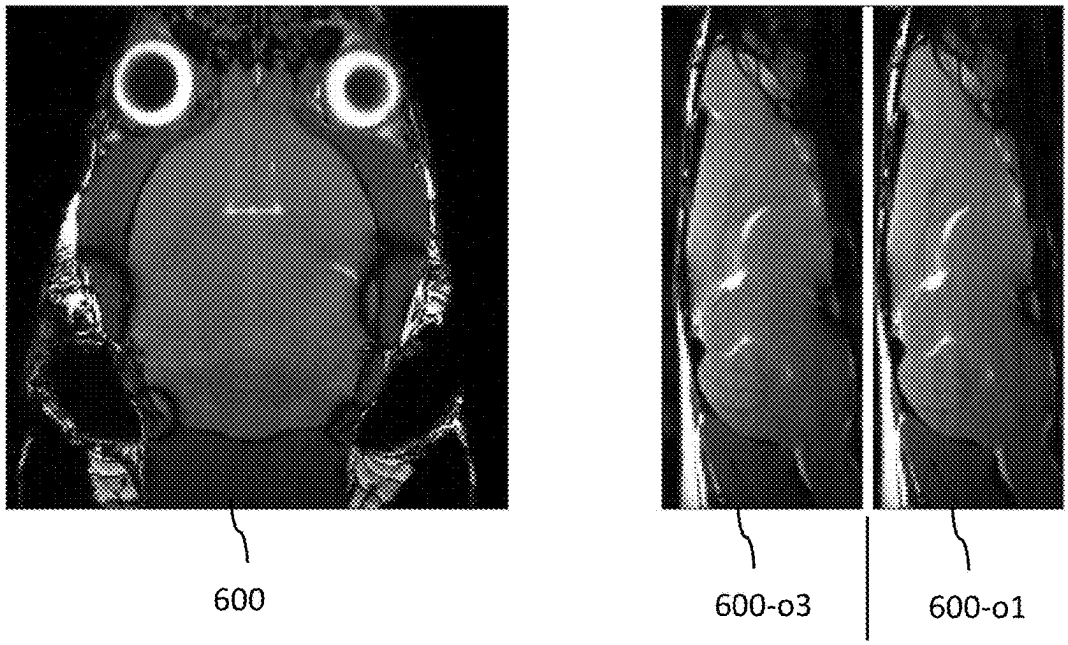


FIG. 6

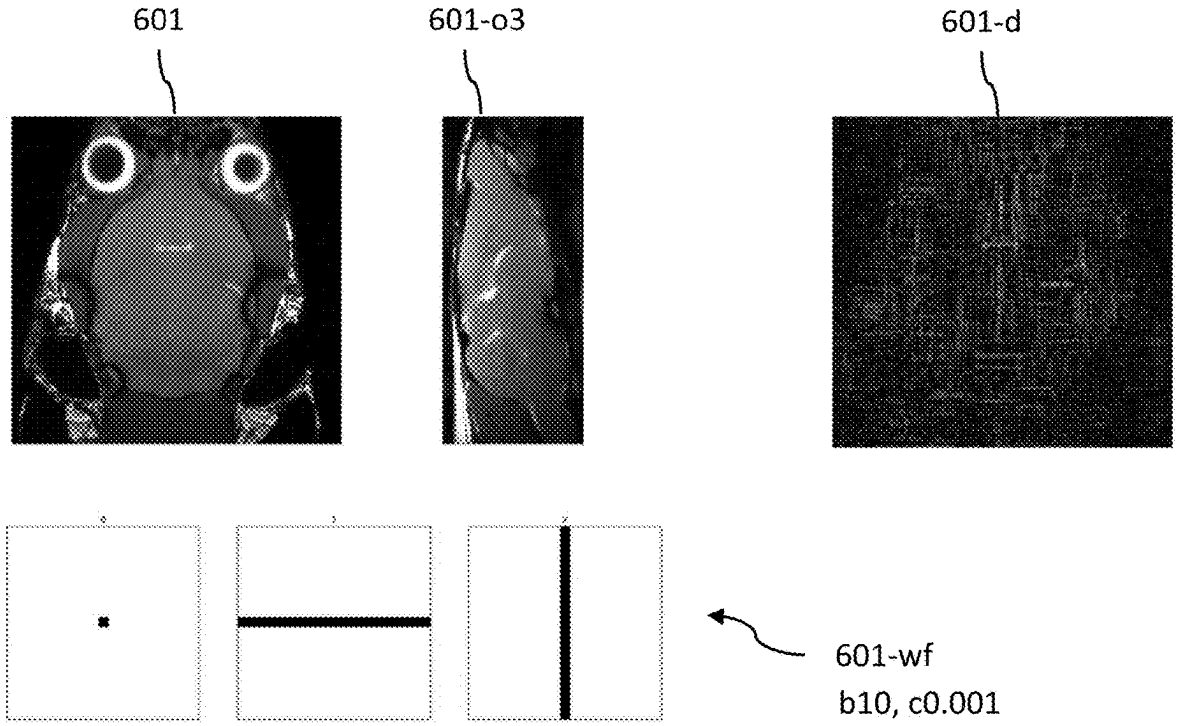


FIG. 6A

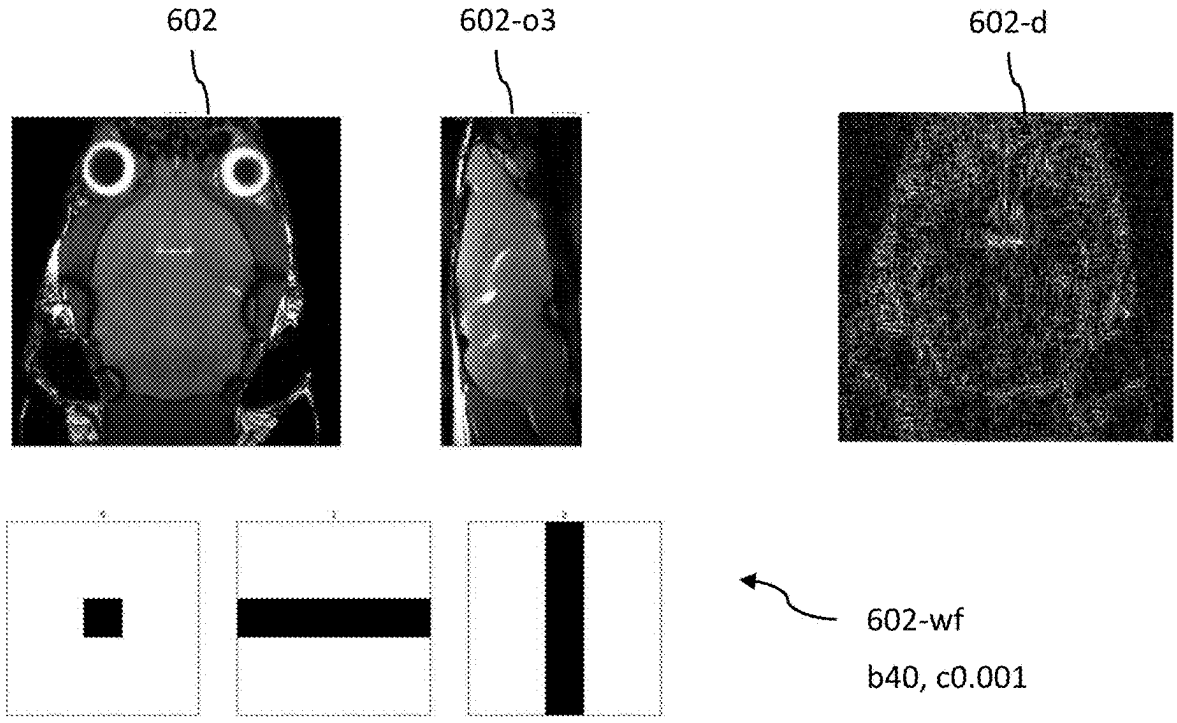

602
602-o3
602-d
602-wf
b40, c0.001

FIG. 6B

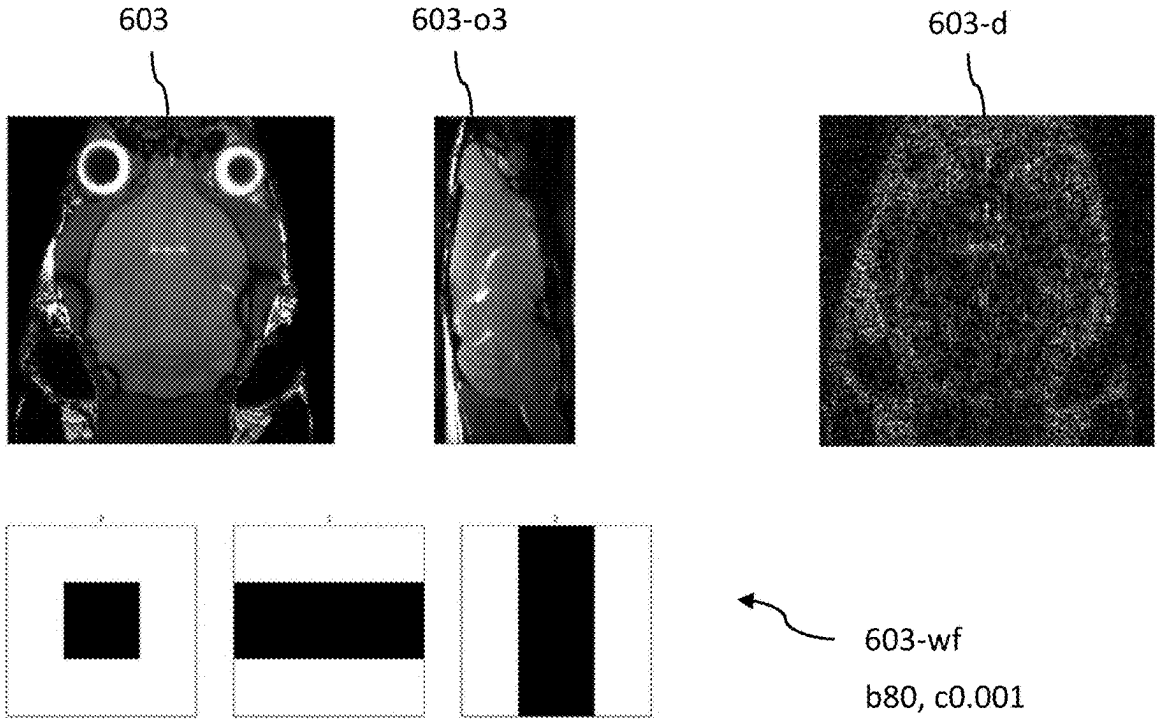

603
603-o3
603-d
603-wf
b80, c0.001

FIG. 6C

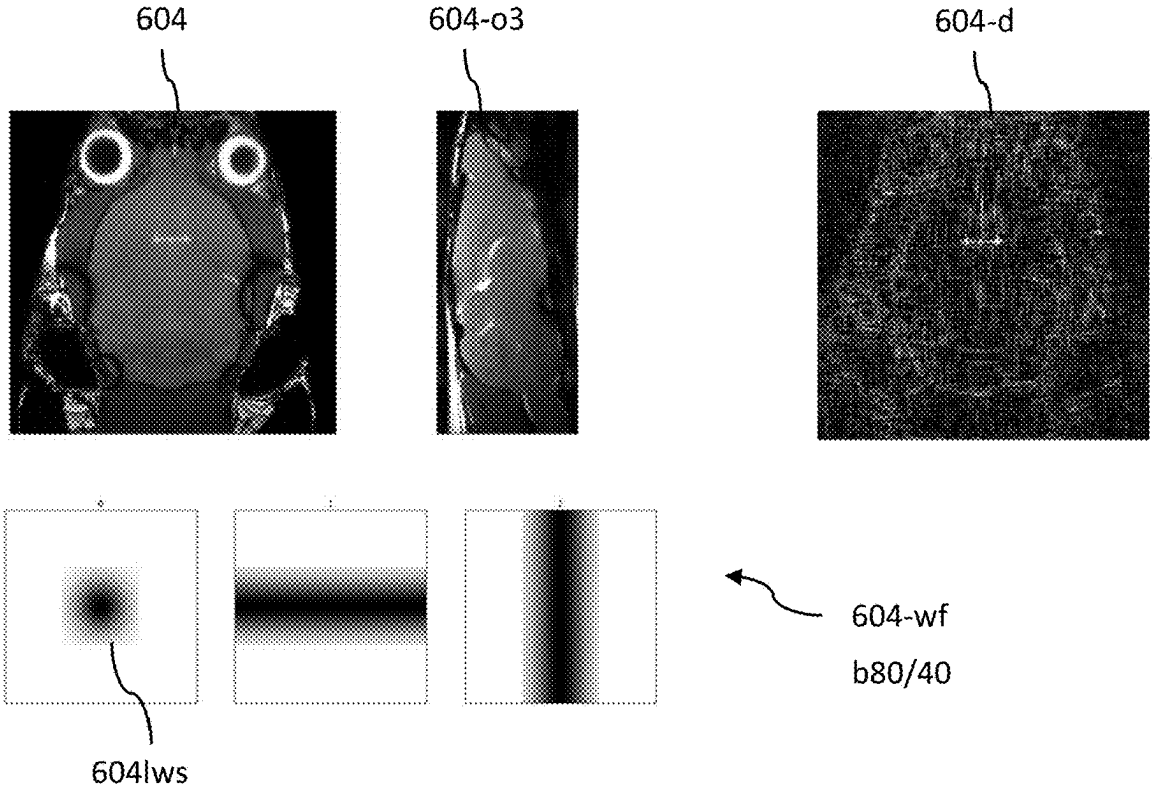

604
604-o3
604-d
604-wf
b80/40
604lws

FIG. 6D

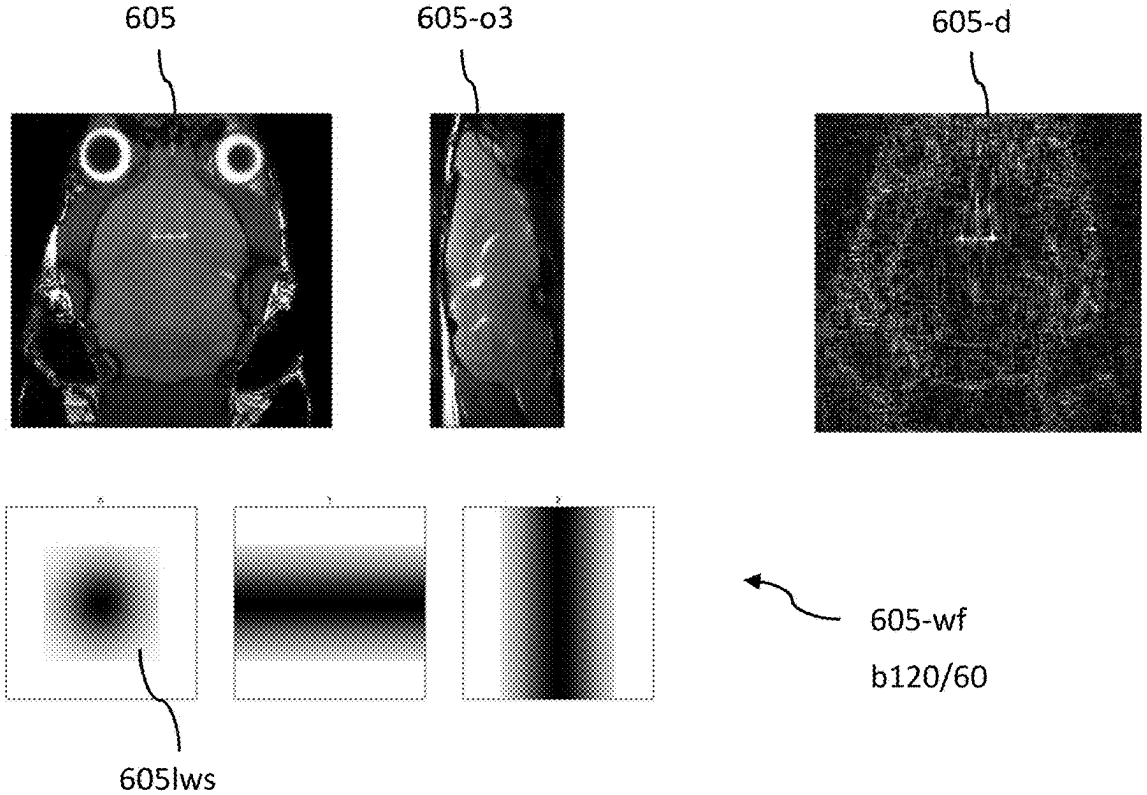

605
605-o3
605-d
605-wf
b120/60
605lws

FIG. 6E

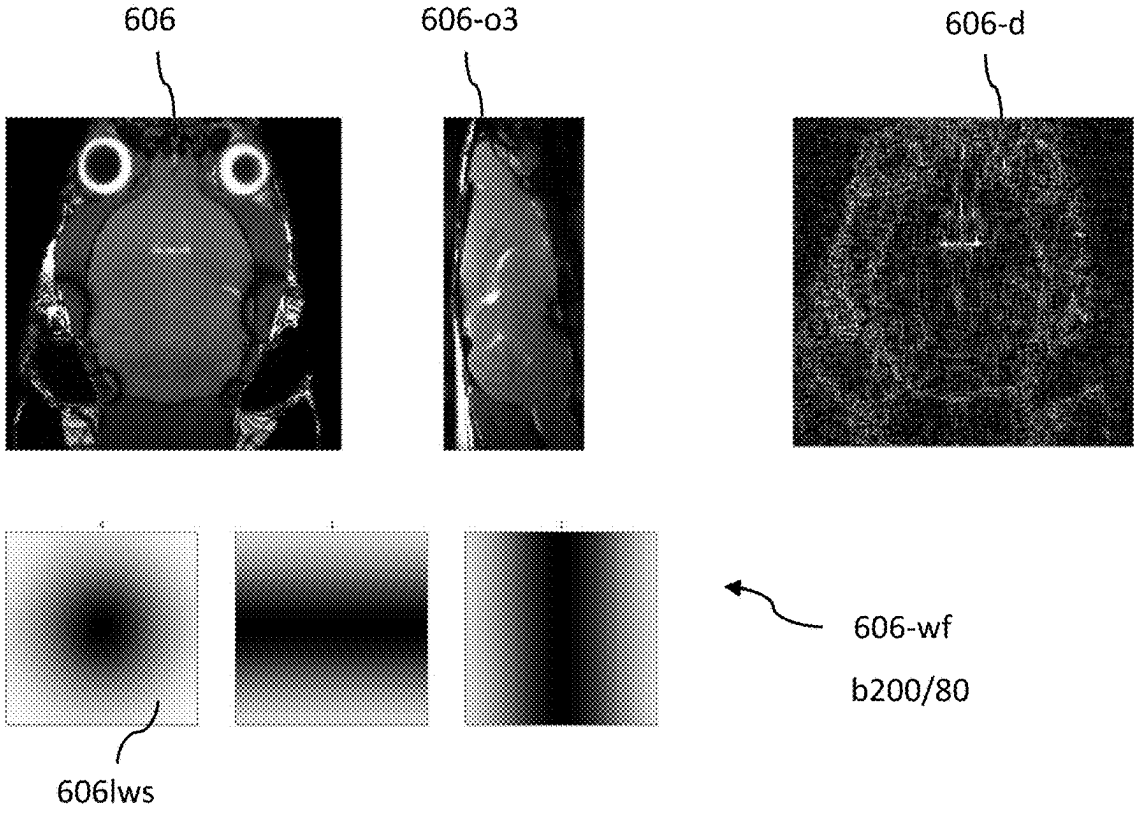

606
606-o3
606-d
606-wf
b200/80
606lws

FIG. 6F

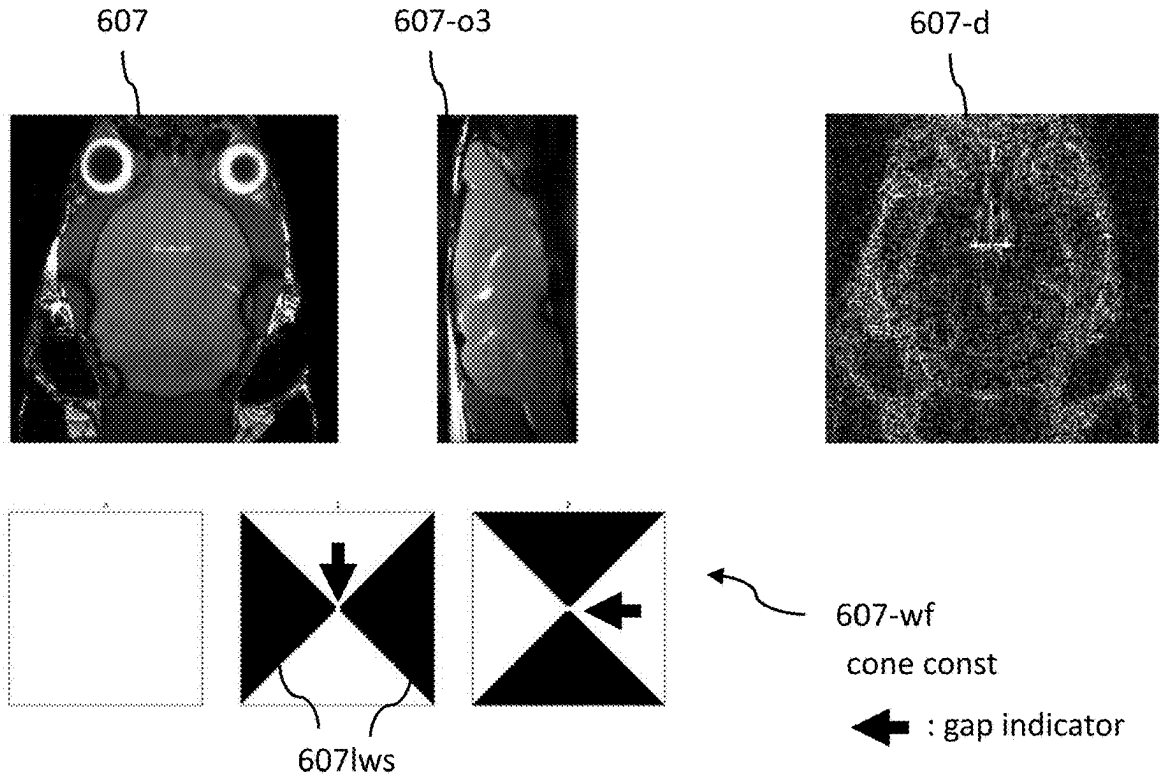

607
607-o3
607-d
607-wf
cone const
: gap indicator
607lws

FIG. 6G

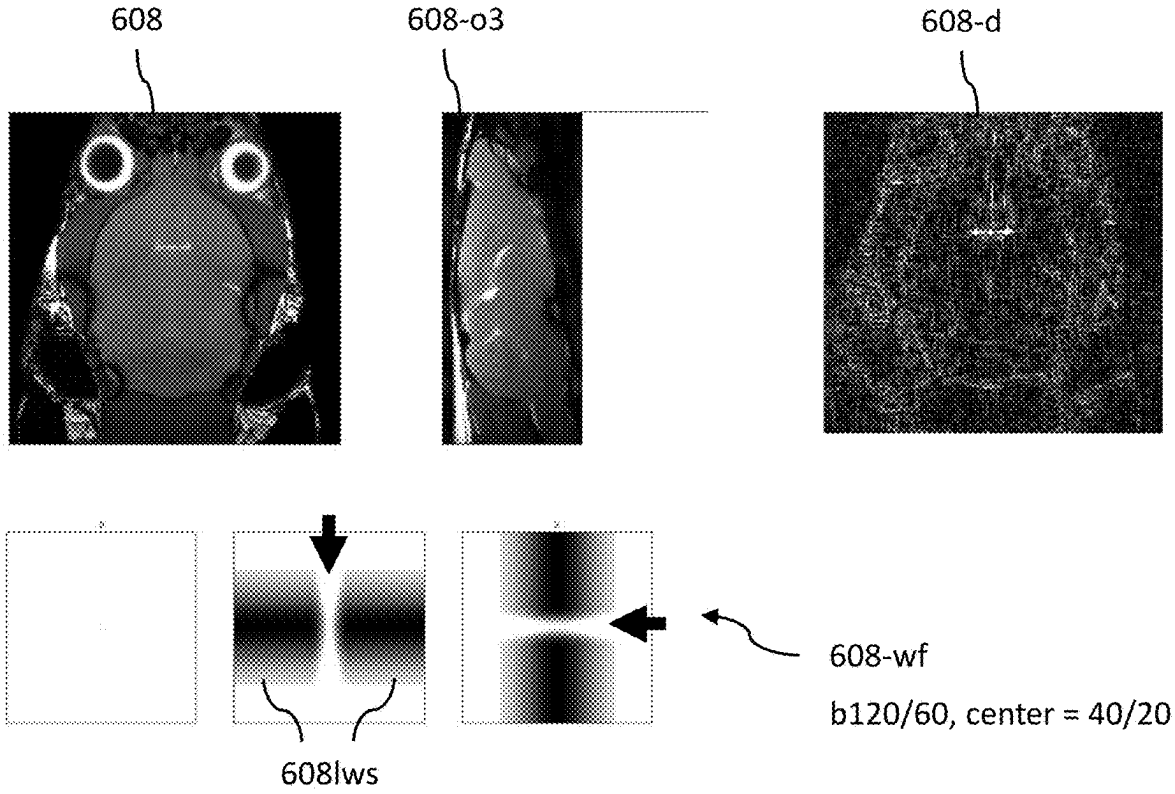

608
608-o3
608-d
608-wf
b120/60, center = 40/20
608lws

FIG. 6H

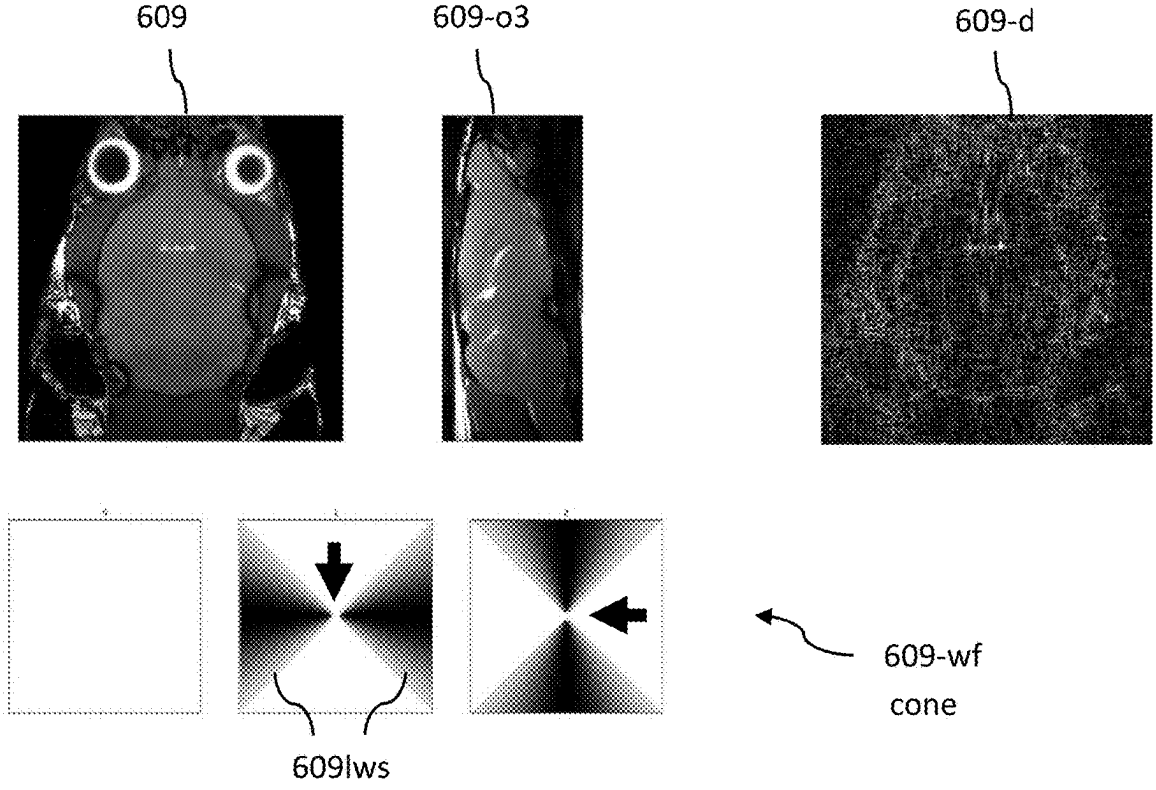

609
609-o3
609-d
609-wf
cone
609lws

FIG. 6I

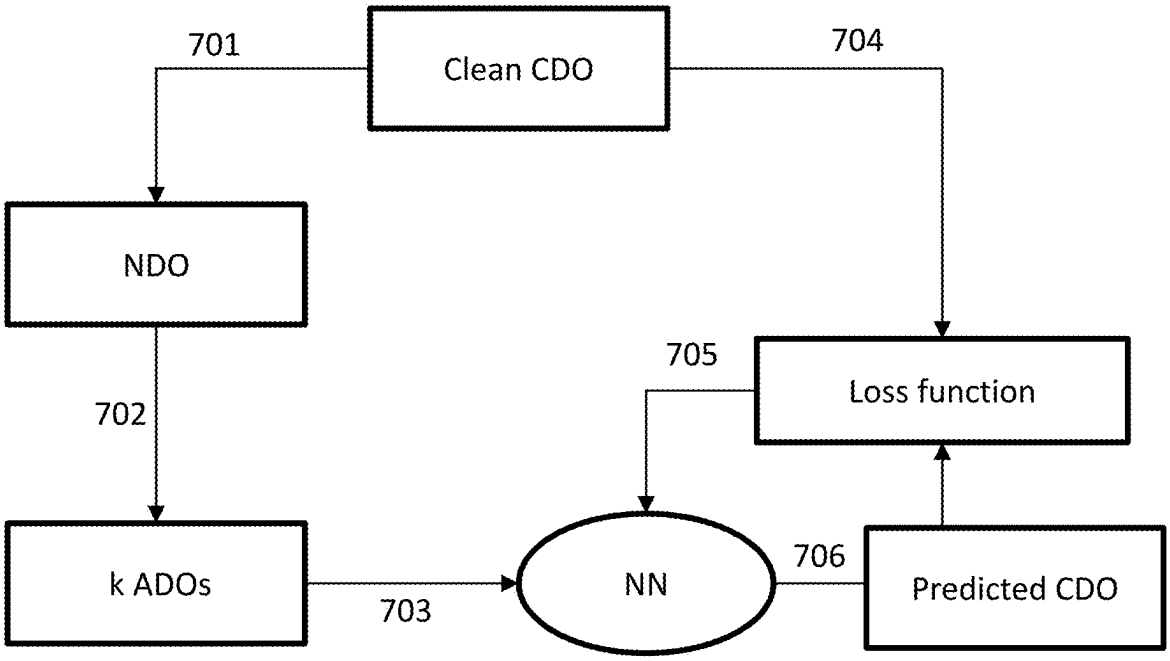

701
Clean CDO
704
NDO
705
Loss function
702
k ADOs
703
NN
706
Predicted CDO

FIG. 7A

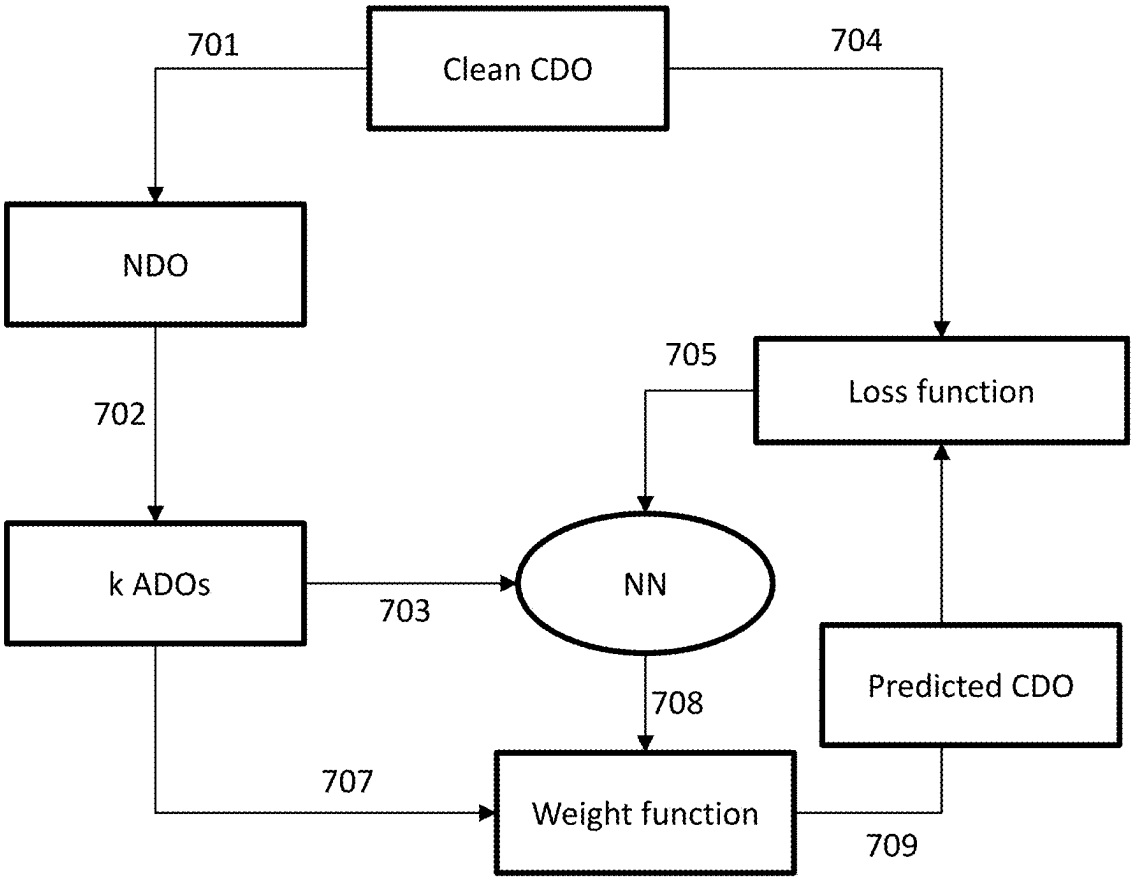

701
Clean CDO
704
NDO
705
Loss function
702
k ADOs
703
NN
Predicted CDO
708
707
Weight function
709

FIG. 7B

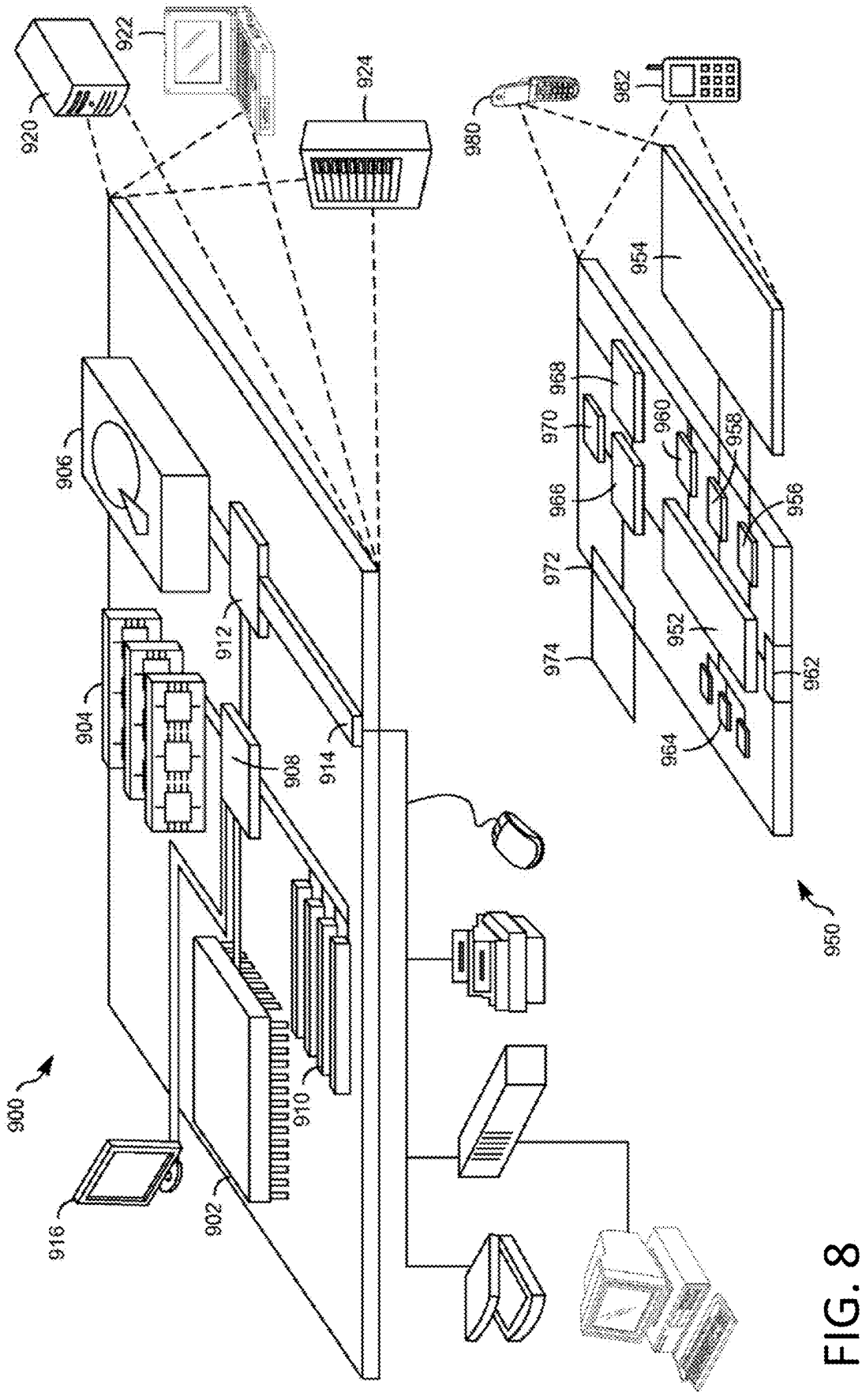

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FIG. 8

SYSTEM AND METHOD FOR PROCESSING A MULTI-DIMENSIONAL DATA OBJECT OBTAINED BY A METROLOGICAL INVESTIGATION

CROSS-REFERENCE TO RELATED APPLICATION

[0001] This application claims priority to EP Patent Application No. 24158833.4, filed on Feb. 21, 2024 and entitled "System and method for processing a multi-dimensional data object obtained by a metrological investigation," the disclosure of which is incorporated by reference herein in its entirety.

TECHNICAL FIELD

[0002] The present description generally relates to electronic data processing, and more particularly, relates to computer-implemented methods, computer program products and systems for processing an n-dimensional data object obtained by a metrological investigation with appropriate filter functions.

BACKGROUND

[0003] Multidimensional data objects obtained by a metrological investigation are often processed to improve the quality of the data object. For example, a three-dimensional image obtained by a three-dimensional imaging method from a respective sample typically includes some noise which advantageously is to be filtered out. Higher n-dimensional data objects with n>3 can also be generated by a metrological investigation. For example, an additional dimension can be the time. For example, a 3D-image of a beating heart may be recorded over time. In this case, the time becomes the fourth dimension of a 4-dimensional data object with each 3D-image being associated with a respective time stamp. Another example for providing higher n-dimensional data objects is 4D NMR spectroscopy. In 4D NMR spectroscopy, there is as plurality of pulses, as the experiment is repeated, and the pulse timings are systematically varied and the oscillations of the spin system are probed point by point in the time domain.

[0004] A prior art approach for filtering such n-dimensional data objects implements the filtering with n-dimensional filters which is very costly in terms of memory consumption and processor load. An example of multi-dimensional filtering is described in "Seven ways to improve example-based single image super resolution", Timofte et al., 2015, arXiv:1511.02228v1. This paper presents the following techniques to improve example-based single image super resolution (SR): 1) augmentation of data, 2) use of large dictionaries with efficient search structures, 3) cascading, 4) image self-similarities, 5) back projection refinement, 6) enhanced prediction by consistency check, and 7) context reasoning. For example, a 3D data object typically contains 32 to 128 2D partitions which increases the memory consumption for filtering 3D data objects by this factor compared to filtering 2D partitions. The size of the 3D object should not fall below a meaningful threshold of the data object to be filtered. This threshold is in the order of 32 2D partitions. For 3D filtering, additional data needs to be added in the edge portions of the 3D data object which further increases memory consumption. With a typical bit size of 32 or 64, GPUs with less than 32 GB memory are not able to perform filtering with such n-dimensional filters.

[0005] Another prior art approach uses lower-dimensional filters to filter the n-dimensional data object. However, in this implementation, information of higher dimensions is not used and lost resulting in significantly lower quality of the filtered data object compared to multi-dimensional filtering. An example of lower-dimensional filtering (i.e., the dimensionality of the filter is lower than the dimensionality of the data object) is described in "Dose reduction and image enhancement in micro-CT using deep learning", Muller et al., 2023, DOI: 10.1002/mp.16385. This paper describes a method used in micro-computed tomography (CT) to acquire high resolution anatomical images of rodents. The example in FIG. **1**(*a*) of Muller illustrates 2D-Net1 as a 2D neural network for filtering for a 3D data object. This filtering algorithm only takes into account two neighboring data layers of the 3D object and fully filters only in two dimensions of the data object. The other dimension of the 3D data object is not taken into account in its full extent when filtering the 3D data object.

SUMMARY

[0006] There is therefore a need for improved data processing of multi-dimensional data objects with filtering functions that achieve high-quality filtering of the multi-dimensional data object in all dimensions of the data object while, at the same time, keeping memory consumption and processor load at levels significantly (at least more than one order of magnitude) below the levels required for multi-dimensional filtering with filters having the same dimensionality as the multi-dimensional data object. Order of magnitude is used herein in the meaning of the following Merriam-Webster Dictionary definition: a range of magnitude extending from some value to ten times that value.

[0007] For example, when comparing the herein disclosed approach with the 3D filtering prior art approach also disclosed in Muller et al. on page 4, the processing of an entire multi-dimensional data object at once using this 3D filtering method requires memory resources at a factor of approximately 500 higher than when using the herein disclosed approach. When comparing it to the above mentioned 2D filtering method of Muller, the herein disclosed approach performs lower-dimensional filtering independently in all dimensions of the data object. With regard to the 3D data object of Muller, the herein disclosed approach performs filtering in all three dimensions taking into account all layers of the data object in each dimension. In other words, although only lower dimensional filters are used, the information of all dimensions is used when generating a weighted-filtered n-dimensional data object from the original n-dimensional data object. This finally leads to balanced denoising in all dimensions.

[0008] This technical problem is solved by features of the independent claims implementing a computer-implemented method, a computer program product and a computer system for processing an n-dimensional data object of a metrological investigation. In the computer-implemented method embodiment, the n-dimensional data object is received from a data source providing the n-dimensional data object as measurement result obtained by the metrological investigation. Typically, the data object has at least three dimensions. For example, the data object may originate from any of the following metrological investigations: magnetic resonance

imaging, magnetic resonance spectroscopy, computed tomography, positron emission tomography or optical imaging. These metrological investigation examples all generate 3D images. However, the n-dimensional data object may comprise further dimensions, such as additional dimensions for the spatial direction, the time, a diffusion direction, a spectroscopic dimension (i.e., a direction in the multidimensional data object along which different frequencies of a respective sample are quantified), etc.

[0009] Then, k filtered objects are calculated by filtering the n-dimensional data object with k filter functions. Thereby, each filter function has a dimensionality dF with dF<n. In other words, the filter functions have a lower dimensionality than the data object. For example, in case of a three-dimensional image data object in the (x, y, z)-space, the filter functions can be two-dimensional kernel functions in the space of the data object. In general, a kernel filter is a type of kernel-based smoothing method which is often applied to an input vector, time series or matrix to generate the smoothed version of the input sequence. A specific type of kernel can be used for a smoothing process. The smoothing process involves computation of a convolution between the input sequence and the kernel. Thereby, each filter function misses at least one of the n dimensions of the data object such that none of the n dimensions of the data object is missing in all filter functions. Advantageously, the filtering of the data object results in denoising of the data object.

[0010] For the three-dimensional image data object example this implies that there are two or three two-dimensional filter functions. For example, for three two-dimensional filter functions, the first filter function may be a 2D kernel in the x-y-plane, the second filter function may be a 2D kernel in the x-z-plane, and the third filter function may be a 2D kernel in the y-z-plane. In principle, further 2D filter functions can be defined for the x-y plane, such as functions in the y-x-plane or the −x-y-plane. But these functions are not relevant for the herein disclosed approach because the missing dimension for these three functions is always the z-dimension. The lower-dimensional filter functions are applied to layers of the data object along the corresponding missing dimension. In the 3D image example, the 2D kernel filter function in the x-y-plane is independently applied to each x-y layer of the 3D image along the z-dimension. Similarly, the other kernel functions are independently applied to the respective layers along the x- and y-dimensions. As a result of the filtering, for the 3D image, three filtered objects are generated by applying the three 2D-filter functions to the 3D image, wherein each of the filtered objects includes a plurality of filtered layers in parallel to the plane of the respective filter function. Using lower dimensional filter functions significantly reduces the memory consumption and CPU/GPU load of the filtering calculation compared to filter functions having the dimensionality of the data object. Applying a 3D-filter function would consume significantly more computing resources than the 2D-filter functions. However, using lower-dimensional filter functions typically results in artefacts in the filtered objects. This effect is due to the use of lower-dimensional filter functions which are not able to remove noise from the data object in the respective missing dimension(s). Typically, any n-dimensional data object obtained by a metrological investigation includes some noise which is caused by respective measurement methods. In the example of a 3D-image, such artefacts are visible as lines or stripe-like artefacts in the direction of the respective missing dimension(s), that is the dimension(s) in which no filtering occurred.

[0011] The following steps of the computer-implemented method allow to significantly reduce such artefacts so that a high-quality weighted-filtered n-dimensional data object is obtained when applying the method.

[0012] The k filtered objects are then transformed by an invertible integral transformation (i.e., an integral transform for which an inverse transform exists) from the data object space into k transformed filtered objects in a transformation space. The invertible integral transformation (sometimes also referred to as "invertible transformation", "inverse integral transformation" or "invertibility of integral transform") may be a Fourier transform, a wavelet transform or a cosine transform. For example, when using a Fourier transform, the transformation space is the Fourier space. In the transformation space, the artefacts can be localized dependent on the missing dimension(s). Thereby, transformation into the transformation space, in which the artefacts have different localizations, allows to make use of data redundancies to reduce or even suppress the artefacts in their respective localizations.

[0013] Then, k weight functions are applied to the k transformed filtered objects, respectively, for point-wise weighting of the data points of the transformed filtered objects resulting in k weighted transformed filtered objects. The point-wise weighting allows to reduce (attenuate) the artefacts created by the calculating (filtering) step. Thereby, the weight functions are configured such that data points in the transformed filtered objects which belong to sections of the transformation space that contain artefacts are underweighted compared to data points which are in sections of the transformation space that are free of artefacts. In other words, data points in artefact-containing sections are weighted with lower weight than data points in artefact-free sections. Thereby, there must not be any data point in the transformation space where all weight functions are 0. In other words, the sum of all weight function values for any data point is always greater than 0. In one implementation, each weight function may have only values smaller than or equal to 1 and greater than 0. In another implementation, each weight function may have only two values greater than 0.

[0014] Weight functions may be represented by a neural network which has been trained by providing n-dimensional data objects as output data and generating n-dimensional data objects as input data from the output data by simulating the artefacts to be removed by the filtering, and then filtering them with the k filter functions. In other words, there are n-dimensional clean, noise-free data objects (CDO) without artefacts, and a set of k lower-dimensional filters designed to remove such noise, e.g., Gaussian noise. These CDO are used as output data. Noise data objects (NDO) are generated by adding noise to the CDO. These NDO are filtered by the set of filters, generating ADO (data objects with residual artefacts resulting from the lower dimensionality of the filters compared to the NDO). The neural network is trained with the ADO as an input and CDO as an output. Alternatively, the neural network is trained with the transformed ADO as an input and the transformed CDO as an output. Alternatively, the neural network is trained to provide a n-dimensional weighting function with the transformed input data objects as an input and output data objects as an

output. In this alternative implementation, weight functions are generated by a neural network.

[0015] In one embodiment, the filter functions are (n−1)-dimensional, wherein the invertible integral transformation is a Fourier transform or a cosine transform and the weight functions applied to the transformed filtered objects each extend along the direction corresponding to the dimension missing in the previously applied filter functions. Thereby, each weight function has a minimum along a direction R corresponding to a missing dimension in relation to values present transversely to the direction R. For example, 2D filter functions may be applied to a 3D data object. The invertible integral transformation can be a Fourier transform or a cosine transform and the weight functions applied to the transformed filtered objects each extend along the direction corresponding to the dimension missing in the previously applied 2D filter functions. Thereby, each weight function has a minimum along the direction corresponding to the missing dimension in relation to values present transversely to said direction. In case the invertible integral transformation is a Fourier transform, each weight function has a rotational symmetry along said direction.

[0016] In one embodiment, each weight function is mirror symmetrical in a plane perpendicular to said direction. The k weight functions can be identical except for a change of coordinates and direction dependent compression or extension factors. In other words, if the size or number of data points—of the transformed filtered objects varies in respective dimensions (i.e., different expansion in the respective directions), the weight function needs to be adjusted accordingly. That is, the k weight functions may vary by rotation and compression/stretching factors in the transformation space. In one implementation, the k weight functions can be inverse Gaussian functions with an offset such that all values are greater than zero.

[0017] In one embodiment, each weight function can be determined from transformed filtered objects. In a first step, k−1 difference objects are created by computing the differences between a particular transformed filtered object and the remaining k−1 transformed filtered objects. Then, the difference objects are smoothed (e.g., by convolution with a Gaussian function over a range of 3-15 data points or fitting with an analytical function). Threshold filtering is applied to the k−1 smoothened difference objects by replacing negative values by 0. The filtered k−1 difference objects are then combined into a combined difference object by a point-wise maximum value formation. Finally, the combined difference object is inverted.

[0018] In one implementation, a morphological operation may be applied to the data points of the difference objects. A person skilled in the art is familiar with the basic morphological operators (operations) erosion, dilation, opening and closing as described for example in Image Analysis and Mathematical Morphology by Jean Serra, ISBN 0-12-637240-3 (1982) or Image Analysis and Mathematical Morphology, Volume 2: Theoretical Advances by Jean Serra, ISBN 0-12-637241-1 (1988).

[0019] The k weighted transformed filtered objects are then combined into a combined transformed filtered object wherein each data point of the combined transformed filtered object is normalized with the sum of the respective weight functions at each point. That is, the combination of the weighted transformed filtered objects includes adding the various weighted transformed filtered objects and normalizing the added values such that the combined transformed filtered object has values which are of the same order of magnitude as the values of the weighted transformed filtered objects.

[0020] The combined filtered object is then transformed back into the n-dimensional object space by applying the inverse of the invertible integral transformation to the combined filtered object. The resulting n-dimensional data object corresponds to a weighted-filtered data object which is improved in quality versus the original data object obtained from the metrological investigation.

[0021] By using lower-dimensional filters with the herein disclosed steps for reducing the impact of artefacts caused by such lower-dimensional filters, a high-quality filtering method is provided which can be executed on any computing device (e.g., a standard personal computer) because memory consumption and CPU/GPU load are significantly lower compared to methods using filters with the same dimension as the data object.

[0022] A further embodiment is a computer program product for processing an n-dimensional data object of a metrological investigation. The program, when loaded into a memory of a computing device and executed by at least one processor of the computing device, causes the at least one processor to execute the steps of the computer-implemented method as claimed herein.

[0023] Further aspects of the description will be realized and attained by means of the elements and combinations particularly depicted in the appended claims. It is to be understood that both, the foregoing general description and the following detailed description are exemplary and explanatory only and are not restrictive of the description.

SHORT DESCRIPTION OF THE FIGURES

[0024] FIG. **1** shows a block diagram of a computer system for processing an n-dimensional data object obtained by a metrological investigation, according to an embodiment;

[0025] FIG. **2** is a simplified flowchart of a computer-implemented method for processing such n-dimensional data object according to an embodiment;

[0026] FIG. **3** is a simplified flowchart with sub-steps of an optional step of said method for automatically determining weight functions according to an embodiment;

[0027] FIG. **4**A illustrates an example of two transformed filtered object layers for two dimensions;

[0028] FIG. **4**B illustrates an example of a difference object derived from two transformed filtered object layers;

[0029] FIG. **5**A illustrates an example for determining a weight function from transformed filtered objects;

[0030] FIG. **5**B illustrates an example using morphological operations to determine a weight function from a difference object;

[0031] FIG. **5**C illustrates filtering results for a three-dimensional data object for two different types of weightings with three different weighting values each;

[0032] FIG. **6** shows examples of lower dimensional filtering results without using weight functions;

[0033] FIGS. **6**A to **6**I illustrate filter results for an example 3D image showing the head of a mouse with each figure using a different weight function;

[0034] FIGS. **7**A, **7**B illustrate example embodiments where weight functions may be represented or generated by a neural network; and

[0035] FIG. **8** is a diagram that shows an example of a generic computer device and a generic mobile computer device, which may be used with the techniques described herein.

DETAILED DESCRIPTION

[0036] FIG. **1** shows a block diagram of an example embodiment of a computer system **100** for processing an n-dimensional data object **202** obtained by a metrological investigation. FIG. **2** is a simplified flowchart illustrating an embodiment of a computer-implemented method **1000** for processing such n-dimensional data object **202**. The method **1000** may be executed by the computer system **1000** when loading a corresponding computer program into a memory of said system and processing the program with one or more processors of the system. FIG. **1** will be described in the context of FIG. **2**. Therefore, the following description refers to reference numbers of FIG. **1** and FIG. **2**.

[0037] The computer system **100** is communicatively coupled via interface **110** to a data source **200** to receive **1100** the n-dimensional data object **202** as a result of a metrological investigation MI of a particular sample **201**. For example, the data object **202** can be obtained by Positron-Emission-Tomography (PET), Computer-Tomography (CT), MR-Imaging (MRI), or the like. However, the metrological investigation is not limited to imaging methods (e.g., 4D NMR spectroscopy). In the example of FIG. **1**, data object **202** is a 3D cube (e.g., the three dimensions of a 3D image with corresponding voxels as data points). However, additional dimensions may be part of n-dimensional data object **202**, such as for example, the time, a diffusion direction, a dimension representing multiple data receivers, a spectroscopic dimension, etc. The 3D example will be used throughout the description of FIG. **1**. However, a person skilled in the art is able to transfer the underlying concept to data objects having more than three dimensions.

[0038] A filtered object creator module **120** of system **100** is adapted to calculate (step **1200**) k filtered objects FO**1** to FO**3** by filtering the n-dimensional data object **202** with k filter functions F**1** to F**3**. Thereby, each filter function has a dimension dF with dF<n. Each filter function F**1**, F**2**, F**3**, in the space of the data object **202**, is missing at least one of the n dimensions x, y, z of the data object such that none of the n dimensions of the data object is missing in all filter functions. For example, for a 3D object, a filter function may be a 2D filter or a 1D filter. For a 4D object, a filter function can be one-dimensional, two-dimensional or three-dimensional, and so on. In the example, the filtered object FO**1** is obtained by applying the 2D filter function F**1** (e.g., a 2D kernel function) to each x-y layer of the data object **202** (i.e., the data points of the data object in layers that are parallel to the x-y plane). The result is a stack of filtered x-y layers which are stacked on top of each other in the z-direction. For FO**1**, no filtering is performed in z-direction leading to artefacts in the filtered object. As explained in more detail in the context of FIG. **4**A, such artefacts can easily be localized in a transformation space (e.g., K-space). The filtered object FO**2** is obtained in the same manner but with the x-z layers of data object **202** being filtered with the filter function F**2** in the x-z plane. The filtered layers of FO**2** are stacked on top of each other in the y-direction. Finally, the filtered object FO**3** is obtained in the same manner but with the y-z layers of data object **202** being filtered with the filter function F**3** in the y-z plane. The filtered layers of FO**3** are stacked on top of each other in the x-direction. For higher dimensional data objects, filters for combinations sub-dimensions will lead to more complex filtered objects. For example, when adding a time dimension to data object **202**, a 2D filter function complying with the above constraints for filter functions can include the same filter functions as in the 3D example. In addition, a filter function may be defined in the x-time plane or the y-time plane, and so on. 3D filter functions can also be used—for example, a filter function in the 3D x-y-z "plane" with the filtered 3D-"layers" (volumes) being stacked on top of each other in the time-direction, or filtered x-y-t volumes being stacked in the z-dimension, etc.

[0039] The k filtered objects FO**1** to FO**3** are then transformed **1300** by an invertible integral transformation **130** into k transformed filtered objects TFO**1** to TFO**3**. For example, the invertible integral transformation may be a Fourier transformation, a wavelet transformation or a cosine transformation. For example, when using a Fourier transformation, the transformation space is the Fourier space (K-space). A person skilled in the art may use other transformations as long as the respective transformation space allows the localization of artefacts which are introduced into the filtered objects by the filtering.

[0040] Turning briefly to FIG. **4**A, two layers (cross-sections) **411**, **412** of transformed filtered objects (based on a 3D data object) are illustrated (after a Fourier transformation) in K-space. Transformed filtered layer **411** shows an artefact **411***x* which extends in the kx-direction. This results from a 2D filter function in the y-z plane which leads to such artefacts in the unfiltered x-direction. Transformed filtered layer **412** shows an artefact **412***y* which extends in the ky-direction. This results from a 2D filter function in the x-z plane which leads to such artefacts in the unfiltered y-direction.

[0041] The kx and ky axes of K-space correspond to the horizontal (x-) and vertical (y-) axes of the original 3D data object. The kx- and ky-axes, however, represent spatial frequencies in the x- and y-directions rather than positions. As known by a skilled person, the individual points (kx,ky) in K-space do not correspond one-to-one with individual data points (x,y) (e.g., pixels) in the image. Each K-space point contains spatial frequency and phase information about every pixel in the final image. Conversely, each pixel in the image maps to every point in K-space.

[0042] A weighting module **140** of system **100** performs data-point-wise weighting **1400** of the k transformed filtered objects TFO**1** to TFO**3** with respective k weight functions WF**1** to WF**3** to reduce artefacts from the calculating (filtering) step **1200** resulting in k weighted transformed filtered objects WTFO**1** to WTFO**3**. In other words, each of the transformed filtered objects is weighted with a respective weight function. In the example, three weight functions WF**1** to WF**3** are illustrated in K-space. In each weight function, a grey shaded cuboid indicates K-space points with weight values smaller than 1 and greater than 0. All remaining points in the respective weight function have values of 1. When applying (multiplying) the weight functions to the respective transformed filtered objects, the values of the transformed filtered objects in the section of the respective cuboids are scaled down. As the cuboids are located such that they overlap with artefacts caused by the lower-dimensional filtering, the artefacts are reduced compared to the remaining points of the transformed filtered objects. For example, transformed filtered object TFO**1** was filtered in

the x-y plane. Therefore, the filtering artefacts appear as an artefact in the z-direction. Weight function WF**1** with lower weight values in the cuboid in z direction (and centered in the x-y plane) is applied to TFO**1** and reduces the respective artefacts by multiplying the section including the artefacts with lower weight values (<1) whereas the non-artefact sections are multiplied with a weight value of 1 (i.e., not changing the original values). The same mechanism is used when applying WF**2** to TFO**2** to reduce artefacts in the y-direction and when applying WF**3** to TFO**3** to reduce artefacts in the x-direction. The values of a weight function are advantageously greater than 0 and smaller than or equal to 1. In one implementation, each weight function may only include two different values greater than 0 (e.g., 0.1 and 1). Different implementations for weight functions with respective filtering results are discussed in more detail in FIG. **5**C and FIGS. **6**A to **6**I.

[0043] A combiner module **150** then combines **1500** the k weighted transformed filtered objects WTFO**1** to WTFO**3** into a combined transformed filtered object CTFO. Thereby, each data point of the combined transformed filtered object is normalized with the sum of the respective weight functions at each point. In the example, when combining **1500** the weighted transformed filtered objects, the weighted values of WTFO**1**, WTFO**2** and WTFO**3** are added at each data point in K-space. The data values of the combined transformed filtered object CTFO result from adding the values of all three transformed filtered objects at each K-space data point. Assuming that a data point in the upper left corner of the transformed data objects TFO**1** to TFO**3** is weighted with 1 by each of the weight functions WF**1** to WF**3**, the respective sum value in the combined CTFO is divided by 3 (normalization). Assuming that the weight values in the grey cuboids of the weight functions are 0.3, then a data point in the intersection of all three weight functions would be normalized by dividing the respective sum value in the combined CTFO by 0.9. Because of the normalization it must be guaranteed that at least one of the weight functions has a value greater than 0 at each data point in K-space to avoid a division by 0. Advantageously, all values of all weight functions are greater than 0.

[0044] Finally, system **100** transforms **1600** the combined filtered object TCFO by applying the inverse **160** of the invertible integral transformation into a weighted-filtered n-dimensional data object **202***wf*. This weighted-filtered data object **202***wf* includes only a low level of artefacts which result from using resource-saving lower-dimensional filtering while still taking into account the information of all dimensions of the n-dimensional data object **202**.

[0045] In one embodiment, the weighting module **140** of system **100** can also determine **1380** the weight functions from the transformed filtered objects. FIG. **3** shows the sub-steps of the optional determining step **1380** of FIG. **2** which are now described in view of the example shown in FIG. **5**A.

[0046] Firstly, k−1 difference objects are created 1381 by forming the respective differences between a particular transformed filtered object and the remaining k−1 transformed filtered objects (e.g., the two differences TFO**1**-TFO**2** and TFO**1**-TFO**3**). FIG. **4**B shows an example of a difference object **420** which is the result of the difference between the transformed filtered object **412** and the transformed filtered object **411** of FIG. **4**A.

[0047] FIG. **5**A illustrates the development stages **502** in the automatic determining of a weight function by way of a particular example. Each stage is shown as an image row in FIG. **5**A. The first row includes layers (cross-sections) of two difference objects **551**, **552** which were created from respective transformed filtered objects. Each image shows the central x-y-layer of the respective difference object in its current processing stage.

[0048] The second row shows corresponding layers of smoothed difference objects **561**, **562** which result from smoothing **1382** the corresponding transformed filtered objects **551**, **552**. For example, smoothing may be implemented by convolution with a Gaussian function over a range of 3-15 data points or fitting with an analytical function. A skilled person may use other well-known smoothing algorithms.

[0049] In the third row, threshold filtering **1383** was applied to the smoothed k−1 difference objects in that negative values are replaced by 0.

[0050] In the fourth row, the filtered k−1 difference objects **571**, **572** are combined **1384** by a point-wise maximum value formation into a combined difference object **580**. For the illustrated cross-section images **572** and **580** (FIG. **5**A only illustrates cross-sections in the kx-ky plane) look equal because the values of image **572** are greater for all points in the cross-section than in image **571**. This may be different in other cross-sections.

[0051] In the fifth (last) row, the combined difference object is pointwise inverted **1385** resulting in the determined weight function **590**. Pointwise inverted in this context means pointwise inverse value formation. Large values in image **580** occur in areas with artefacts. To reduce/suppress these large values, the corresponding weight function values have to be small. In the example, the following pointwise inversion was applied to the data point values v_p **580** of the combined difference object **580** to arrive at the data point values v_p **590** of the corresponding weight function **590**:

$$v_p590 = 1 - v_p580/v_{max}580$$

where a particular data point value v_p **580** is normalized by dividing that value through the maximum value v_{max} **580** of all data points of the combined difference object **580**.

An alternative pointwise inversion is:

$$v_p590 = 1/v_p580$$

To avoid division by zero, zero values are replaced by small positive values.

For better visibility of the weight function (image **590**) the grey scale values represent normalized values where black pixels represent the value '0' and white pixels represent the value '1'. For the other images in FIG. **5**A, the value '0' is represented by a mid-grey value. Black pixels represent the respective negative maximum absolute value and white pixels the respective positive maximum absolute value.

[0052] The example of FIG. **5**A illustrates the automatic determination of one layer of weight function **590** in K-space which is adapted to reduce artefacts of the transformed filtered object that served as minuend for the cre-

ation **1381** of the difference objects. When looking at the example of FIG. **1**, WF**1** can be determined from the difference objects TFO**1**-TFO**2** and TFO**1**-TFO**3**. In more general, when generating a weight function for a particular transformed filtered object, step **1381** creates a plurality of difference objects with the particular transformed filtered object being the minuend in each respective subtraction of two transformed filtered objects.

[0053] FIG. **5**B illustrates an alternative to smoothing steps **561**, **562** of FIG. **5**A to automatically generate appropriate weight functions from respective transformed filtered objects by using morphological operations. In this alternative implementation, a difference object **510** is created from two transformed filtered objects. A threshold object **520** is generated by assigning all values of the difference object equal to or greater than a predefined threshold value to a predefined high-weight value (white pixels in object **520**), and assigning all values of the difference object smaller than the predefined threshold value to a predefined low-weight value (black pixels). The object **530** shows the result after application of a morphological closing operation to threshold object **520**. In the closing operation, high-weight values (white pixels) which are in section dominated by low-weight values (black pixels) are switched to low-weight values (black pixels). The object **540** shows the result after application of a morphological opening operation to object **530**. In the opening operation, low-weight values (black pixels) which are in section dominated by high-weight values (white pixels) are switched to high-weight values (white pixels). Object **540** can then be processed in accordance with threshold filtering **571**, **572**, point-wise maximum value formation **580**, and pointwise inversion **590** as described above.

[0054] FIG. **5**C illustrates a filtering quality comparison for two different types of weight functions applied to a real 3D image obtained by magnetic resonance imaging. The 3D image shows a section of the head of a mouse. Image **51** is a layer of a filtered object which was obtained by applying a 2D filter function to the filtered object in the plane of image **51**. Image **52** shows the same section of the mouse head in a filtered object which was filtered with a 2D filter function orthogonal to the plane of image **51**. The dark stripe-like artefacts are caused by using the lower-dimensional orthogonal filter functions. Image **53** was generated by adding the filtered objects of all three dimensions and averaging the corresponding voxel values. That is, the added values for each voxel are divided by the number of filtered objects (e.g., divided by 3 in the case of the 3D image). It can be seen that the artefacts in image **53** are significantly reduced compared to image **52** but still significantly higher than in image **51**. It is therefore desirable to further improve the quality of the filtering in all dimensions so that the final filtered 3D image has a quality similar to the quality of the 2D filtered image **51**. In other words, the artefacts caused by filtering of the data object can be considered as noise and the weight functions are used for denoising the filtered 3D object.

[0055] The images **54-1** to **54-3** were generated by using the three weight functions **54**-*wf* which correspond to the type of weight functions as shown in FIG. **1**. The three schematic views of weight functions **54**-*wf* (and of all other similar schematic views of weight functions herein) show cross-sections through the 3D filter function in each dimension. The shown cross-sections cut the weight functions in half with regard to the respective dimensions. For example, the lower view may be the cross-section cutting the weight function in half orthogonal to the y-dimension. The middle view may be the cross-section orthogonal to the x-dimension, and the upper view may be the cross-section orthogonal to the z-dimension. The width b**60** of the black sections indicating the sections with low-weight values in the weight function is 60 pixels and the section is not interrupted (indicated by c**0**). For image **54-1**, the weighting value in the black sections is set to 0.01 whereas the weighting value in the white sections is set to 1. Similarly, for image **54-2**, the weighting value in the black sections is set to 0.1, and for image **54-3** to 0.5 The best improvement vs. the unweighted filtered image **53** is achieved with the weighting value 0.01 in image **54-1** although all three weighting values provide a similar level of quality which is still quite below the 2D filter quality shown in image **51**.

[0056] The images **55-1** to **55-3** were generated by using the three weight functions **55**-*wf* which differ from the weight functions **54**-*wf* in that the width b**80** of the low-weight sections is 80 pixels and the weight values gradually decrease towards the center of the respective low-weight section. Further, the low-weight sections are cut apart in the middle and show a gap of 20 pixels between the two low-weight sub-sections. For image **55-1**, the weighting value in the black sections is set to 0.01 whereas the weighting value in the white sections is set to 1. Similarly, for image **55-2**, the weighting value in the black sections is set to 0.1, and for image **55-3** to 0.5 Again, the best improvement vs. the unweighted filtered image **53** is achieved with the weighting value 0.01 in image **55-1**. But even image **55-3** still shows a significant denoising improvement vs. image **53**. Remarkably, image **55-1** already comes quite close to the quality of the 2D filtered image **51**.

[0057] In general, the lower the weight values are chosen in the low-weight sections of the weight functions, the higher is the quality of the resulting 3D image (weighted lower-dimensional filtered). Nevertheless, weight values up to 0.5 can still achieve improved results compared to an unweighted lower-dimensional filtered image.

[0058] FIG. **6** shows a cross-section image **600** of a 3D image of the head of a mouse. The eyes of the mouse are clearly visible in the upper section of image **600**. Image **600** is the result of a combination of three filtered objects where each filtered object was generated by a lower dimensional filtering process along another dimension of the underlying unfiltered 3D image object (as described in the context of FIG. **1**). Cross-section image **600**-*o***3** shows a cross-section orthogonal to cross-section image **600**. Image **600**-*o***3** has also been filtered along all three dimensions of the original 3D image (not shown). Cross-section image **600**-*o***1** shows the same section of the mouse head, no filtering was applied along a respective dimension. Therefore, the stripe-like artefacts are much more visible than in image **600**-*o***3** where filtering occurred in all three dimensions. Nevertheless, images **600** and **600**-*o***3** still show artefacts which can be significantly reduced by applying appropriate weight functions to the corresponding transformed filtered objects.

[0059] FIGS. **6**A to **6**I illustrate denoising improvements for different pre-defined weight functions (i.e., weight functions not automatically generated based on transformed filtered objects). Each figure includes an image **60*** (*:=1 to 9) showing a cross-section of a weighted filtered image where weight functions **60***-*wf* have been applied to the

respective transformed filtered objects. Thereby, the cross-sections images **60*** correspond to the unweighted filtered image **600** of FIG. **6**. Images **60***-*o***3** are weighted filtered cross-sections orthogonal to the respective cross-sections **60***. Images **60***-*d* are difference images illustrating the difference between image **600** and the respective image **60***. That is, such difference visualizes the improvement (denoising) over the unweighted filtered objects which is achieved by applying the respective weight functions **60***-*wf*.

[0060] FIGS. **6**A, **6**B, **6**C use weight functions **601**-*wf*, **602**-*wf* and **603**-*wf* which are all of a first type with a band (black sections) of a given width and a constant weight value defining the low-weight sections of the weight functions. The constant weight value c is 0.001 and the widths are: **601**-*wf:* 10 pixels, **602**-*wf:* 40 pixels and **603**-*wf:* 80 pixels. The differences observed in images **602**-*d,* 603-d are greater than in image **601**-*d* which indicates that the weight function **601**-*wf* provides the worst weighted filtering result for this weight function type.

[0061] FIGS. **6**D, **6**E, **6**F show weight functions **604**-*wf*, **605**-*wf*, **606**-*wf* of a second type. The low-weight sections **604***lws*, **605***lws* and **606***lws* have continuously decreasing weight values towards the center of the weight function (orthogonal to the direction of the respective missing dimension of the previously applied filter function). Thereby, the weight values show rotational symmetry along the respective missing dimension (as illustrated by the corresponding left weight functions of **604**-*wf*, **605**-*wf*, **606**-*wf*). For the weight functions **604**-*wf*, a total width of 80 pixels is used for defining the low-weight section where a circle with a radius of 20 pixels around the center in the direction of missing dimensions includes the lowest constant weight values. For the weight functions **605**-*wf*, a total width of 120 pixels is used for the low-weight section where a circle with a radius of 30 pixels around the center in the respective missing dimensions includes the lowest constant weight values in the low-weight section. For the weight functions **606**-*wf*, a total width of 200 pixels is used for the low-weight section (the entire width of the weight function) where a circle with a radius of 40 pixels around the center in the respective missing dimensions includes the lowest constant weight values in the low-weight section. Again, the implementation with the smaller width (b**80**) shows a smaller difference 604-d over the unweighted filtered image **600**.

[0062] In general, the second type of weight functions is applicable to transformed filtered objects generated by Fourier transformation or cosine transformation where each transformed filtered object extends along one or more directions corresponding to the dimensions missing in the respective previously applied filter functions. The values of the corresponding weight functions then have a minimum along a direction R corresponding to a missing dimension in relation to values present transversely to the direction R. In the example, weight functions **604**-*wf*, **605**-*wf*, **606**-*wf* have a rotational symmetry along the direction R and the minimum weight values are at the center axes of the respective low-weight cylindric sections.

[0063] FIGS. **6**G, **6**H, **6**I show weight functions **607**-*wf*, **608**-*wf*, **609**-*wf* of a third type. The low-weight sections **604***lws*, **605***lws* and **606***lws* show a gap in the middle (indicated by thick black arrows as gap indicators). Because of the gap, the weight functions have no low weight values between the respective parts of the low-weight sections. For this reason, the cross-sections of the left weight functions of **607**-*wf*, **608**-*wf*, **609**-*wf* do not show any low-weight sections.

[0064] In the example of FIG. **6**G, weight functions **607**-*wf* have low-weight sections **607***lws* with two cone-shaped parts where the tips of the cones point towards the center of the weight function and, from there, extend to two opposite edges of the respective weight functions. In this example, the weight values inside the low-weight section are constant (illustrated by the black filling).

[0065] In the example of FIG. **6**I, a similarly cone-shaped weight functions **609**-*wf* are used. However, the weight values inside the two parts of the low-weight section **609***lws* are not constant. Rather, they continuously decrease from the outer surface of the cones towards the center-axes of the cones.

[0066] In the example of FIG. **6**H, the low-weight sections **608***lws* of the weight functions **608**-*wf* have two parts which have a cylindrical shape with curved caps at the inner end of the respective cylinder. In the example, the 3D data object was a matrix of the size 200×200×80. Therefore, the applied weight function is not the same in all dimensions. The inner end is the end which points towards the center of the respective weight function. In the example, the cylinders have a radius of 60 pixels in the larger dimension and 30 in the smaller dimension where the weight values continuously decrease towards the center axes of the cylinder. The gaps between the two cylindric parts are 20 pixels in the smaller dimension and 40 pixels in the larger dimension.

[0067] From the weight functions of the third type, in the example, weight function **607**-*wf* results in difference object **607**-*d* showing the greatest difference over image **600**. That is, weight function **607**-*wf* appears to provide the best weighted filtering result of the three examples with weight functions of the third type.

[0068] FIGS. **7**A, **7**B illustrate example embodiments where weight functions may be represented (FIG. **7**A) or generated (FIG. **7**B) by a neural network NN which has been trained by providing **704** n-dimensional clean data objects CDO as output data (training reference or ground truth) and generating n-dimensional data objects as input data from the output data by simulating **701** the artefacts to be removed by the filtering, and then filtering **702** them with the k filter functions. In other words, there are n-dimensional clean data objects CDO without artefacts and noise, and a set of k dF (lower)-dimensional filters designed to remove such noise, e.g., Gaussian noise. These CDO objects are used as output data. Noise data objects NDO are generated by adding noise to the CDO objects. These NDO objects are filtered **702** by the set of filters, generating k ADO objects (filtered data objects with residual artefacts resulting from the lower dimensionality of the filters compared to the NDO). In the implementation of FIG. **7**A, the neural network NN is trained **703** with the ADO objects as an input and the CDO objects as an output wherein training feedback **705** is provided by a loss function which is determined by using the clean CDO objects and predicted CDO objects (predicted **706** by the neural network NN). The weight functions are then represented by the trained neural network NN. Alternatively, in FIG. **7**B, the neural network NN is trained **703** with the transformed ADO objects as an input and the transformed clean CDO objects as an output (ground truth). The neural network may be trained to provide **708** a n-dimensional weighting function with the transformed input

data objects as an input **707** and output data objects as predicted CDOs (by applying **709** the weight function to the respective ADOs). In this alternative implementation, weight functions are generated **708** by the neural network NN.

[0069] FIG. **8** is a diagram that shows an example of a generic computer device **900** and a generic mobile computer device **950**, which may be used with the techniques described here. Computing device **900** is intended to represent various forms of digital computers, such as laptops, desktops, workstations, personal digital assistants, servers, blade servers, mainframes, and other appropriate computers. Advantageously, device **900** has a GPU adapted to process machine learning algorithms. Generic computer device **900** may correspond to the computer system **100** of FIG. **1**. Computing device **950** is intended to represent various forms of mobile devices, such as personal digital assistants, cellular telephones, smart phones, and other similar computing devices. For example, computing device **950** may be used as a GUI frontend for a user to visualize the weight-filtered n-dimensional objects for the user. The components shown here, their connections and relationships, and their functions, are meant to be exemplary only, and are not meant to limit implementations described and/or claimed in this document.

[0070] Computing device **900** includes a processor **902**, memory **904**, a storage device **906**, a high-speed interface **908** connecting to memory **904** and high-speed expansion ports **910**, and a low speed interface **912** connecting to low speed bus **914** and storage device **906**. Each of the components **902**, **904**, **906**, **908**, **910**, and **912**, are interconnected using various busses, and may be mounted on a common motherboard or in other manners as appropriate. The processor **902** can process instructions for execution within the computing device **900**, including instructions stored in the memory **904** or on the storage device **906** to display graphical information for a GUI on an external input/output device, such as display **916** coupled to high speed interface **908**. In other implementations, multiple processing units and/or multiple buses may be used, as appropriate, along with multiple memories and types of memory. Also, multiple computing devices **900** may be connected, with each device providing portions of the necessary operations (e.g., as a server bank, a group of blade servers, or a processing device).

[0071] The memory **904** stores information within the computing device **900**. In one implementation, the memory **904** is a volatile memory unit or units. In another implementation, the memory **904** is a non-volatile memory unit or units. The memory **904** may also be another form of computer-readable medium, such as a magnetic or optical disk.

[0072] The storage device **906** is capable of providing mass storage for the computing device **900**. In one implementation, the storage device **906** may be or contain a computer-readable medium, such as a floppy disk device, a hard disk device, an optical disk device, or a tape device, a flash memory or other similar solid state memory device, or an array of devices, including devices in a storage area network or other configurations. A computer program product can be tangibly embodied in an information carrier. The computer program product may also contain instructions that, when executed, perform one or more methods, such as those described above. The information carrier is a computer- or machine-readable medium, such as the memory **904**, the storage device **906**, or memory on processor **902**.

[0073] The high speed controller **908** manages bandwidth-intensive operations for the computing device **900**, while the low speed controller **912** manages lower bandwidth-intensive operations. Such allocation of functions is exemplary only. In one implementation, the high-speed controller **908** is coupled to memory **904**, display **916** (e.g., through a graphics processor or accelerator), and to high-speed expansion ports **910**, which may accept various expansion cards (not shown). In the implementation, low-speed controller **912** is coupled to storage device **906** and low-speed expansion port **914**. The low-speed expansion port, which may include various communication ports (e.g., USB, Bluetooth, Ethernet, wireless Ethernet) may be coupled to one or more input/output devices, such as a keyboard, a pointing device, a scanner, or a networking device such as a switch or router, e.g., through a network adapter.

[0074] The computing device **900** may be implemented in a number of different forms, as shown in the figure. For example, it may be implemented as a standard server **920**, or multiple times in a group of such servers. It may also be implemented as part of a rack server system **924**. In addition, it may be implemented in a personal computer such as a laptop computer **922**. Alternatively, components from computing device **900** may be combined with other components in a mobile device (not shown), such as device **950**. Each of such devices may contain one or more of computing device **900**, **950**, and an entire system may be made up of multiple computing devices **900**, **950** communicating with each other.

[0075] Computing device **950** includes a processor **952**, memory **964**, an input/output device such as a display **954**, a communication interface **966**, and a transceiver **968**, among other components. The device **950** may also be provided with a storage device, such as a microdrive or other device, to provide additional storage. Each of the components **950**, **952**, **964**, **954**, **966**, and **968**, are interconnected using various buses, and several of the components may be mounted on a common motherboard or in other manners as appropriate.

[0076] The processor **952** can execute instructions within the computing device **950**, including instructions stored in the memory **964**. The processor may be implemented as a chipset of chips that include separate and multiple analog and digital processing units. The processor may provide, for example, for coordination of the other components of the device **950**, such as control of user interfaces, applications run by device **950**, and wireless communication by device **950**.

[0077] Processor **952** may communicate with a user through control interface **958** and display interface **956** coupled to a display **954**. The display **954** may be, for example, a TFT LCD (Thin-Film-Transistor Liquid Crystal Display) or an OLED (Organic Light Emitting Diode) display, or other appropriate display technology. The display interface **956** may comprise appropriate circuitry for driving the display **954** to present graphical and other information to a user. The control interface **958** may receive commands from a user and convert them for submission to the processor **952**. In addition, an external interface **962** may be provide in communication with processor **952**, so as to enable near area communication of device **950** with other devices. External interface **962** may provide, for example, for wired commu-

nication in some implementations, or for wireless communication in other implementations, and multiple interfaces may also be used.

[0078] The memory **964** stores information within the computing device **950**. The memory **964** can be implemented as one or more of a computer-readable medium or media, a volatile memory unit or units, or a non-volatile memory unit or units. Expansion memory **984** may also be provided and connected to device **950** through expansion interface **982**, which may include, for example, a SIMM (Single In Line Memory Module) card interface. Such expansion memory **984** may provide extra storage space for device **950**, or may also store applications or other information for device **950**. Specifically, expansion memory **984** may include instructions to carry out or supplement the processes described above, and may include secure information also. Thus, for example, expansion memory **984** may act as a security module for device **950**, and may be programmed with instructions that permit secure use of device **950**. In addition, secure applications may be provided via the SIMM cards, along with additional information, such as placing the identifying information on the SIMM card in a non-hackable manner.

[0079] The memory may include, for example, flash memory and/or NVRAM memory, as discussed below. In one implementation, a computer program product is tangibly embodied in an information carrier. The computer program product contains instructions that, when executed, perform one or more methods, such as those described above. The information carrier is a computer- or machine-readable medium, such as the memory **964**, expansion memory **984**, or memory on processor **952**, that may be received, for example, over transceiver **968** or external interface **962**.

[0080] Device **950** may communicate wirelessly through communication interface **966**, which may include digital signal processing circuitry where necessary. Communication interface **966** may provide for communications under various modes or protocols, such as GSM voice calls, SMS, EMS, or MMS messaging, CDMA, TDMA, PDC, WCDMA, CDMA2000, or GPRS, among others. Such communication may occur, for example, through radio-frequency transceiver **968**. In addition, short-range communication may occur, such as using a Bluetooth, WiFi, or other such transceiver (not shown). In addition, GPS (Global Positioning System) receiver module **980** may provide additional navigation- and location-related wireless data to device **950**, which may be used as appropriate by applications running on device **950**.

[0081] Device **950** may also communicate audibly using audio codec **960**, which may receive spoken information from a user and convert it to usable digital information. Audio codec **960** may likewise generate audible sound for a user, such as through a speaker, e.g., in a handset of device **950**. Such sound may include sound from voice telephone calls, may include recorded sound (e.g., voice messages, music files, etc.) and may also include sound generated by applications operating on device **950**.

[0082] The computing device **950** may be implemented in a number of different forms, as shown in the figure. For example, it may be implemented as a cellular telephone **980**. It may also be implemented as part of a smart phone **982**, personal digital assistant, or other similar mobile device.

[0083] Various implementations of the systems and techniques described here can be realized in digital electronic circuitry, integrated circuitry, specially designed ASICs (application specific integrated circuits), computer hardware, firmware, software, and/or combinations thereof. These various implementations can include implementation in one or more computer programs that are executable and/or interpretable on a programmable system including at least one programmable processor, which may be special or general purpose, coupled to receive data and instructions from, and to transmit data and instructions to, a storage system, at least one input device, and at least one output device.

[0084] These computer programs (also known as programs, software, software applications or code) include machine instructions for a programmable processor, and can be implemented in a high-level procedural and/or object-oriented programming language, and/or in assembly/machine language. As used herein, the terms "machine-readable medium" and "computer-readable medium" refer to any computer program product, apparatus and/or device (e.g., magnetic discs, optical disks, memory, Programmable Logic Devices (PLDs)) used to provide machine instructions and/or data to a programmable processor, including a machine-readable medium that receives machine instructions as a machine-readable signal. The term "machine-readable signal" refers to any signal used to provide machine instructions and/or data to a programmable processor.

[0085] To provide for interaction with a user, the systems and techniques described here can be implemented on a computer having a display device (e.g., a CRT (cathode ray tube) or LCD (liquid crystal display) monitor) for displaying information to the user and a keyboard and a pointing device (e.g., a mouse or a trackball) by which the user can provide input to the computer. Other kinds of devices can be used to provide for interaction with a user as well; for example, feedback provided to the user can be any form of sensory feedback (e.g., visual feedback, auditory feedback, or tactile feedback); and input from the user can be received in any form, including acoustic, speech, or tactile input.

[0086] The systems and techniques described here can be implemented in a computing device that includes a back end component (e.g., as a data server), or that includes a middleware component (e.g., an application server), or that includes a front end component (e.g., a client computer having a graphical user interface or a Web browser through which a user can interact with an implementation of the systems and techniques described here), or any combination of such back end, middleware, or front end components. The components of the system can be interconnected by any form or medium of digital data communication (e.g., a communication network). Examples of communication networks include a local area network ("LAN"), a wide area network ("WAN"), and the Internet.

[0087] The computing device can include clients and servers. A client and server are generally remote from each other and typically interact through a communication network. The relationship of client and server arises by virtue of computer programs running on the respective computers and having a client-server relationship to each other.

[0088] A number of embodiments have been described. Nevertheless, it will be understood that various modifications may be made without departing from the spirit and scope of the description.

[0089] In addition, the logic flows depicted in the figures do not require the particular order shown, or sequential

order, to achieve desirable results. In addition, other steps may be provided, or steps may be eliminated, from the described flows, and other components may be added to, or removed from, the described systems. Accordingly, other embodiments are within the scope of the following claims.

1. A computer-implemented method for processing an n-dimensional data object of a metrological investigation, comprising:

a) receiving the n-dimensional data object from a data source providing the n-dimensional data object as measurement result of the metrological investigation;

b) calculating a plurality of k filtered objects by filtering the n-dimensional data object with k filter functions, wherein each filter function has a dimension dF with dF<n, and each filter function, in a space of the n-dimensional data object, is missing at least one of the n dimensions of the n-dimensional data object such that none of the n dimensions of the n-dimensional data object is missing in all filter functions;

c) transforming the k filtered objects by an invertible integral transformation into k transformed filtered objects;

d) point-wise weighting of each of the k transformed filtered objects with a respective weight function to reduce artefacts from the calculating operation b) resulting in k weighted transformed filtered objects;

e) combining the k weighted transformed filtered objects into a combined transformed filtered object wherein each data point of the combined transformed filtered object is normalized with a sum of the respective weight functions at the respective data point; and

f) transforming the combined filtered object, by applying an inverse of the invertible integral transformation, into a weighted-filtered n-dimensional data object.

2. The method of claim **1**, wherein each weight function is determined from the transformed filtered objects comprising:

i) creating k−1 difference objects by forming the differences between a particular transformed filtered object and remaining k−1 transformed filtered objects;

ii) smoothing the difference objects;

iii) threshold filtering of the smoothed k−1 difference objects by replacing negative values by 0;

iv) combining the filtered k−1 difference objects by a point-wise maximum value formation into a combined difference object; and

v) pointwise inverting the combined difference object.

3. The method of claim **2**, wherein a morphological operation is applied to the data points of the difference objects created in operation (i).

4. The method of claim **1**, wherein k=n−1 and dF=n−1.

5. The method of claim **1**, wherein each weight function only has values smaller than or equal to 1 and greater than 0.

6. The method of claim **1**, wherein each weight function has only two values greater than 0.

7. The method of claim **1**, wherein a particular dimension of the data object is selected from: a spatial direction, a time, a diffusion direction, a dimension representing multiple data receivers, and a spectroscopic dimension.

8. The method of claim **1**, wherein the data object originates from magnetic resonance imaging, magnetic resonance spectroscopy, computed tomography, positron emission tomography or optical imaging.

9. The method of claim **1**, wherein the filtering of the data object comprises denoising.

10. The method of claim **1**, wherein the invertible integral transformation is a Fourier transform, a wavelet transform or a cosine transform.

11. The method of claim **1**, wherein the weight functions are represented by a neural network which has been trained by providing n-dimensional data objects as output data and generating n-dimensional data objects as input data from the output data by simulating the artefacts to be removed by the filtering, and then filtering them with the k filter functions.

12. The method of claim **1**, wherein the filter functions are n−1-dimensional, wherein the invertible integral transformation is a Fourier transform or a cosine transform and the weight functions applied to the transformed filtered objects each extend along a direction corresponding to the dimension missing in previously applied filter functions, and each weight function having a minimum along a direction R corresponding to a missing dimension in relation to values present transversely to the direction R.

13. The method of claim **12**, wherein the invertible integral transformation is a Fourier transform and each weight function has a rotational symmetry along the direction R.

14. A computer program product storing a program for processing an n-dimensional data object of a metrological investigation, the program, when loaded into a memory of a computing device and executed by at least one processor of the computing device, causes the at least one processor to:

a) receive the n-dimensional data object from a data source providing the n-dimensional data object as measurement result of the metrological investigation;

b) calculate a plurality of k filtered objects by filtering the n-dimensional data object with k filter functions, wherein each filter function has a dimension dF with dF<n, and each filter function, in a space of the n-dimensional data object, is missing at least one of the n dimensions of the n-dimensional data object such that none of the n dimensions of the n-dimensional data object is missing in all filter functions;

c) transform the k filtered objects by an invertible integral transformation into k transformed filtered objects;

d) point-wise weight each of the k transformed filtered objects with a respective weight function to reduce artefacts from the calculating operation b) resulting in k weighted transformed filtered objects;

e) combine the k weighted transformed filtered objects into a combined transformed filtered object wherein each data point of the combined transformed filtered object is normalized with a sum of the respective weight functions at the respective data point; and

f) transform the combined filtered object, by applying an inverse of the invertible integral transformation, into a weighted-filtered n-dimensional data object.

15. The computer program product of claim **14**, wherein each weight function is determined from the transformed filtered objects comprising:

i) creating k−1 difference objects by forming the differences between a particular transformed filtered object and remaining k−1 transformed filtered objects;

ii) smoothing the difference objects;

iii) threshold filtering of the smoothed k−1 difference objects by replacing negative values by 0;

iv) combining the filtered k−1 difference objects by a point-wise maximum value formation into a combined difference object; and

v) pointwise inverting the combined difference object.

16. The computer program product of claim **14**, wherein the weight functions are represented by a neural network which has been trained by providing n-dimensional data objects as output data and generating n-dimensional data objects as input data from the output data by simulating the artefacts to be removed by the filtering, and then filtering them with the k filter functions.

17. The computer program product of claim **14**, wherein the filter functions are

n−1-dimensional, wherein the invertible integral transformation is a Fourier transform or a cosine transform and the weight functions applied to the transformed filtered objects each extend along a direction corresponding to the dimension missing in previously applied filter functions, and each weight function having a minimum along a direction R corresponding to a missing dimension in relation to values present transversely to the direction R.

18. A computer system adapted for processing an n-dimensional data object of a metrological investigation, comprising at least one processor and at least one memory, wherein the memory stores instructions which, when loaded into the at least one processor, cause the at least one processor to:

a) receive the n-dimensional data object from a data source providing the n-dimensional data object as measurement result of the metrological investigation;

b) calculate a plurality of k filtered objects by filtering the n-dimensional data object with k filter functions, wherein each filter function has a dimension dF with dF<n, and each filter function, in a space of the n-dimensional data object, is missing at least one of the n dimensions of the n-dimensional data object such that none of the n dimensions of the n-dimensional data object is missing in all filter functions;

c) transform the k filtered objects by an invertible integral transformation into k transformed filtered objects;

d) point-wise weight each of the k transformed filtered objects with a respective weight function to reduce artefacts from the calculating operation b) resulting in k weighted transformed filtered objects;

e) combine the k weighted transformed filtered objects into a combined transformed filtered object wherein each data point of the combined transformed filtered object is normalized with a sum of the respective weight functions at the respective data point; and

f) transform the combined filtered object, by applying an inverse of the invertible integral transformation, into a weighted-filtered n-dimensional data object.

19. The system of claim **18**, wherein each weight function is determined from the transformed filtered objects comprising:

i) creating k−1 difference objects by forming the differences between a particular transformed filtered object and remaining k−1 transformed filtered objects;

ii) smoothing the difference objects;

iii) threshold filtering of the smoothed k−1 difference objects by replacing negative values by 0;

iv) combining the filtered k−1 difference objects by a point-wise maximum value formation into a combined difference object; and

v) pointwise inverting the combined difference object.

20. The system of claim **18**, wherein the weight functions are represented by a neural network which has been trained by providing n-dimensional data objects as output data and generating n-dimensional data objects as input data from the output data by simulating the artefacts to be removed by the filtering, and then filtering them with the k filter functions.

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