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Hui et al.

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(54) **CRC INTERLEAVING PATTERN FOR POLAR CODES**

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(51) **Int. Cl.**

H03M 13/00	(2006.01)
H03M 13/13	(2006.01)
H03M 13/27	(2006.01)

(52) **U.S. Cl.**

CPC ***H03M 13/2792*** (2013.01); ***H03M 13/13*** (2013.01)

(58) **Field of Classification Search**

CPC .. H03M 13/2792; H03M 13/13; H03M 13/09; H03M 13/091; H03M 13/1137;

(Continued)

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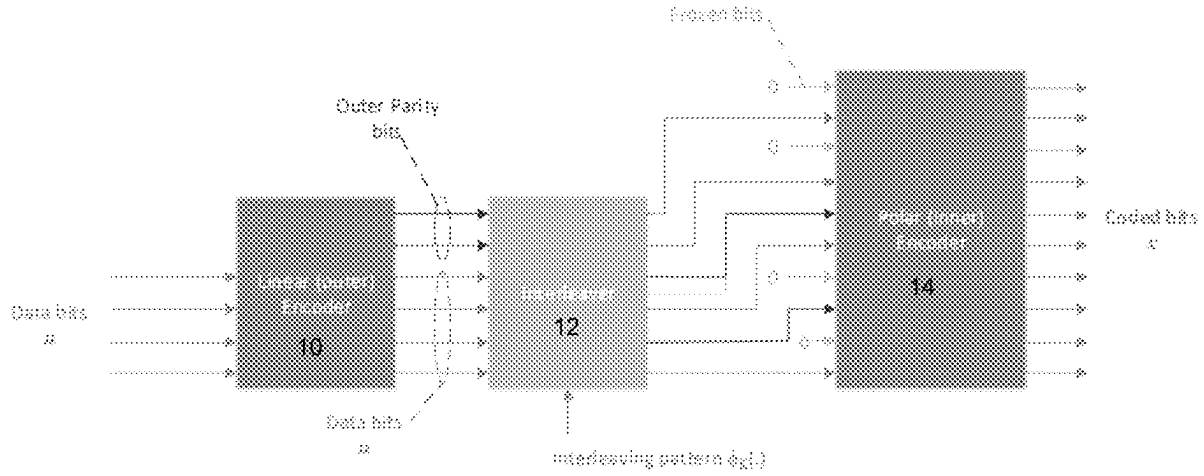
(57) **ABSTRACT**

According to some embodiments, a method of operation of a wireless transmitter in a wireless communication network comprises: encoding a set of information carrying data bits u of length K with a linear outer code to generate a set of outer parity bits p along with the data bits u; interleaving the set of outer parity bits p and the data bits u using a predetermined interleaving mapping function that depends on the number of data bits K and is operable to distribute some bits of the set of parity bits p in front of some data bits u; and encoding the interleaved bits using a Polar encoder to generate a set of encoded bits x. Various interleaving mapping functions are disclosed.

17 Claims, 10 Drawing Sheets



Related U.S. Application Data

continuation of application No. 16/645,720, filed as application No. PCT/IB2018/056983 on Sep. 12, 2018, now Pat. No. 11,165,442.

(60) Provisional application No. 62/559,991, filed on Sep. 18, 2017, provisional application No. 62/558,208, filed on Sep. 13, 2017, provisional application No. 62/557,715, filed on Sep. 12, 2017.

(58) **Field of Classification Search**
CPC H03M 13/118; H03M 13/2732; H03M 13/2757; H03M 13/2789; H03M 13/2742; H03M 13/2778
See application file for complete search history.

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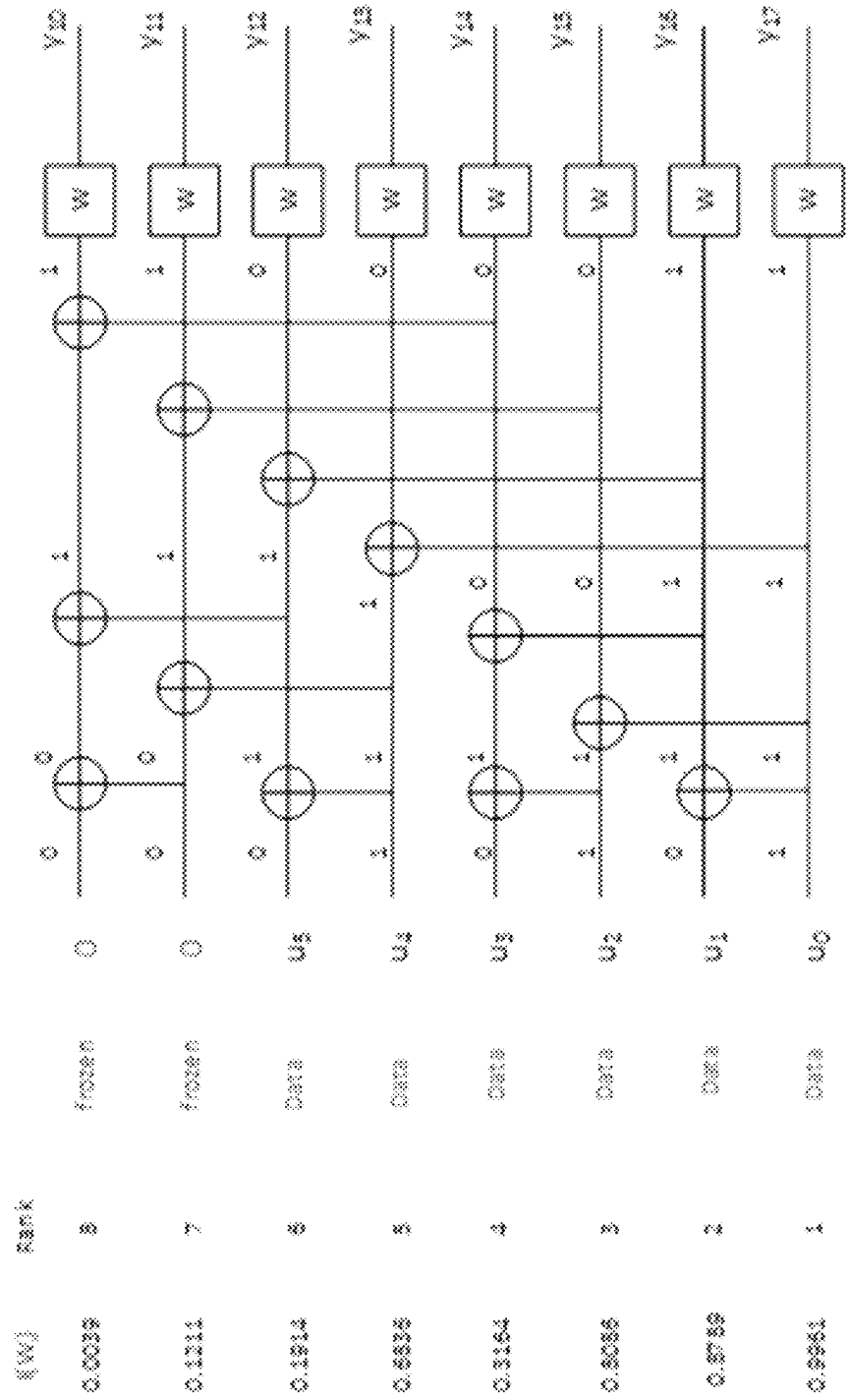
Y10
Y11
Y12
Y13
Y14
Y15
Y16
Y17
W
W
W
W
W
W
W
W
1
1
0
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0

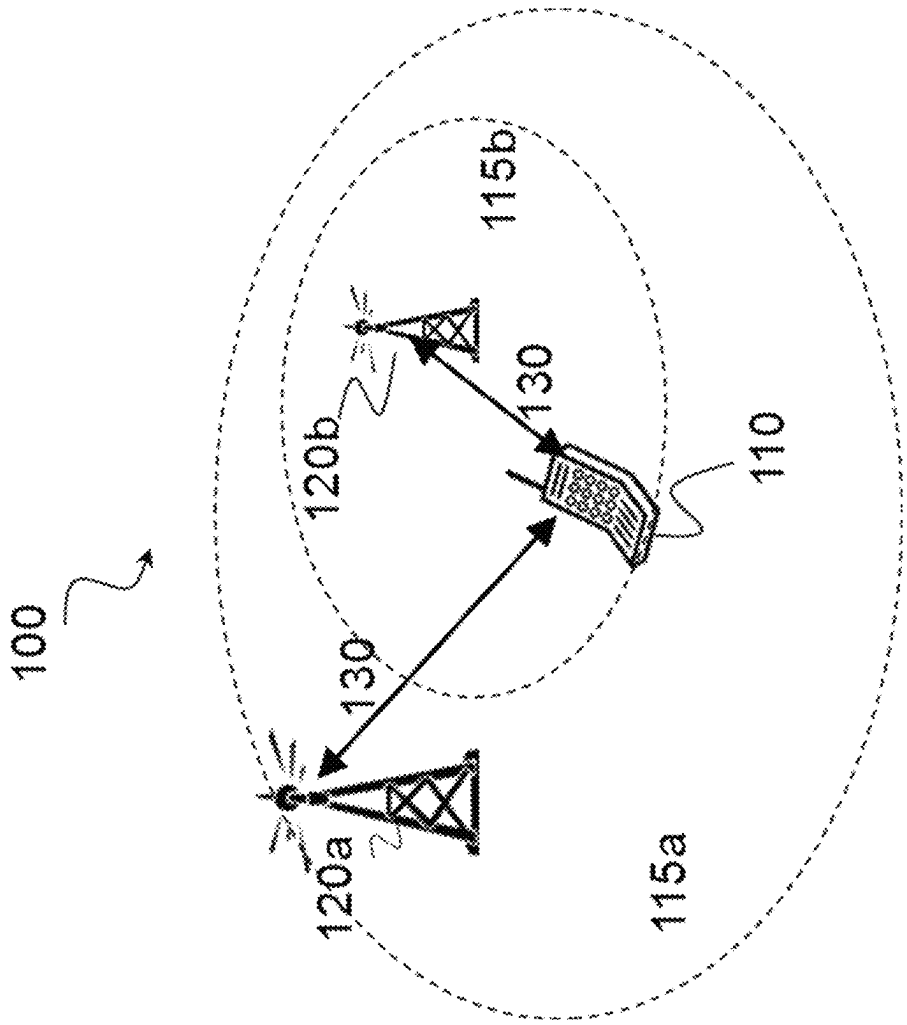

Fig. 2

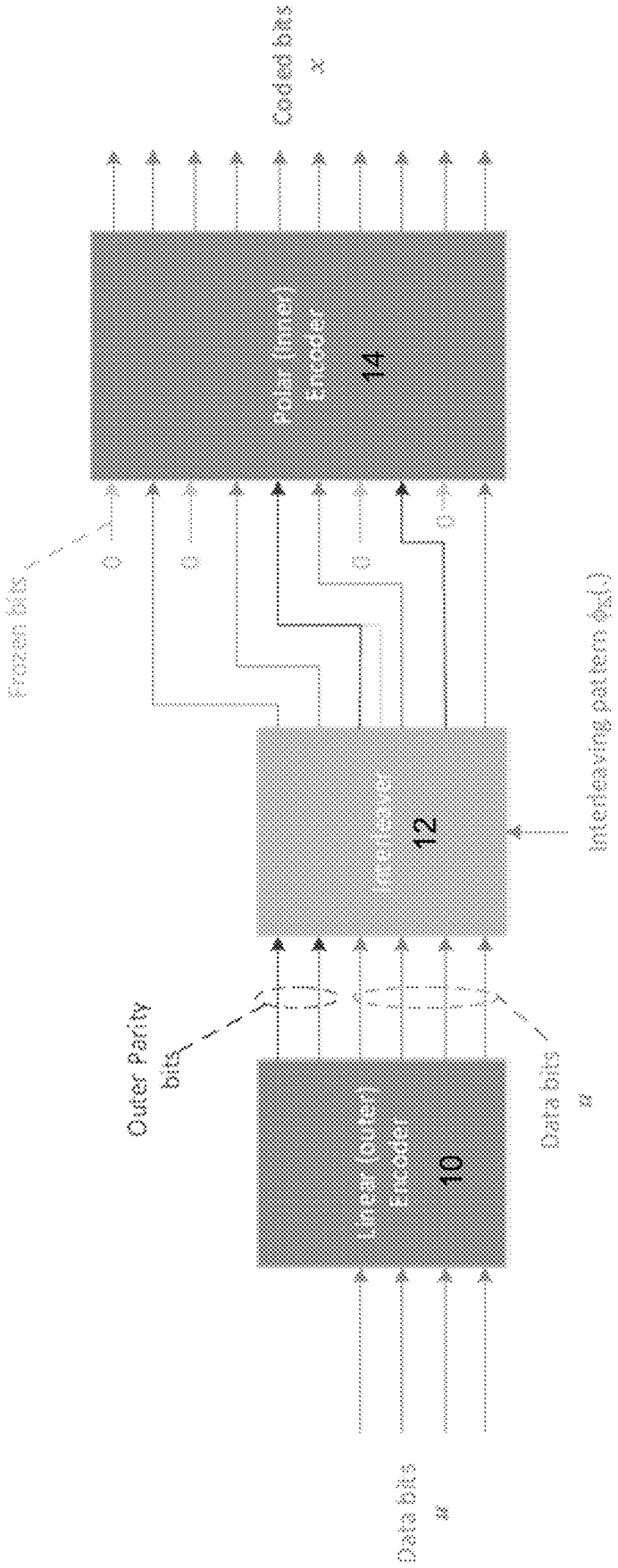
Data bits
u
Linear (Outer) Encoder
10
Outer Parity bits
Data bits
u
Interleaver
12
Interleaving pattern $\phi_{\pi}(\cdot)$
Frozen bits
0
0
0
0
Polar (Inner) Encoder
14
Coded bits
x

Fig. 3

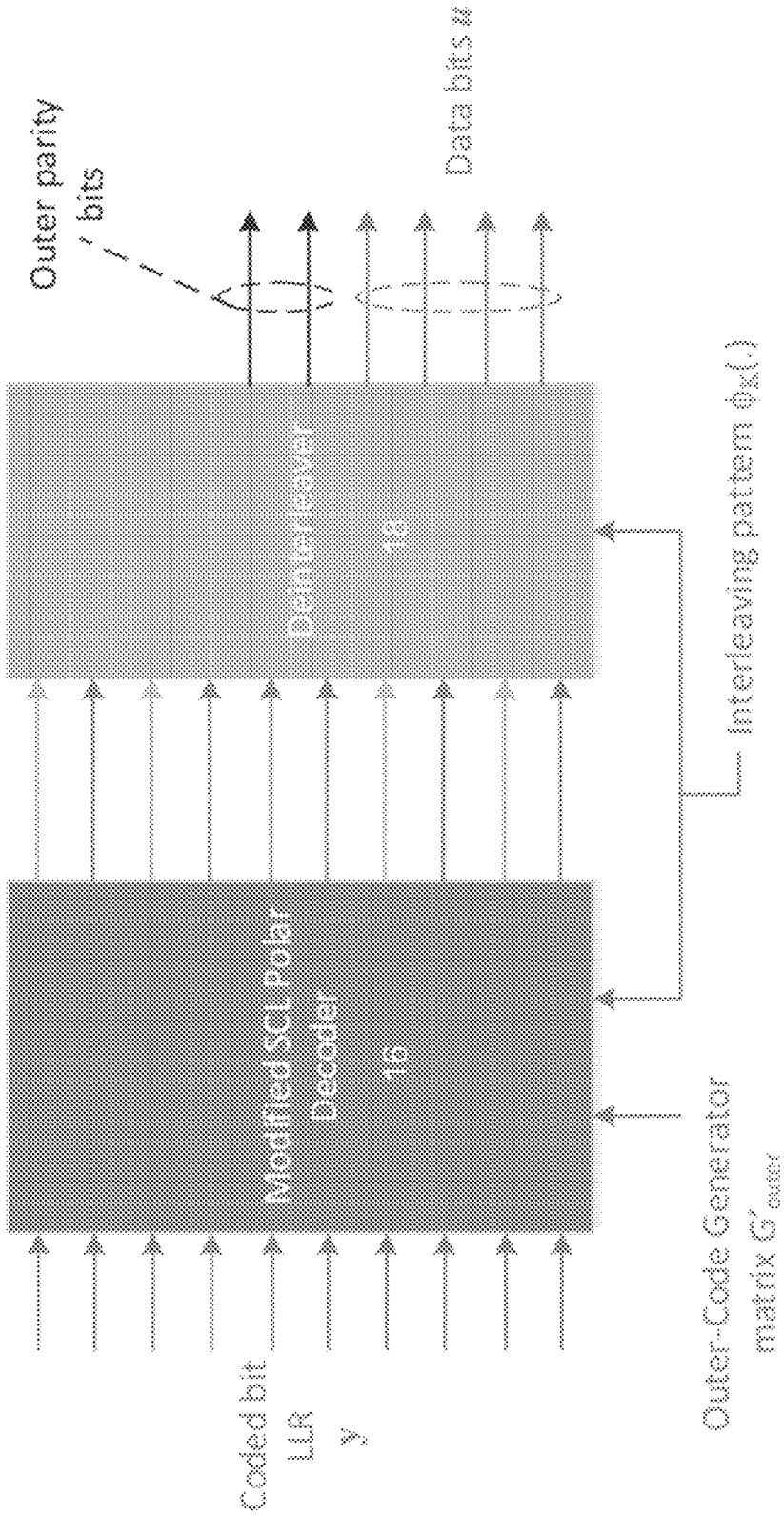
Coded bit LLR y
Modified SCL Polar Decoder 16
Deinterleaver 18
Outer parity bits
Data bits u
Interleaving pattern $\phi_K(.)$
Outer-Code Generator matrix G'_{outer}

Fig. 4

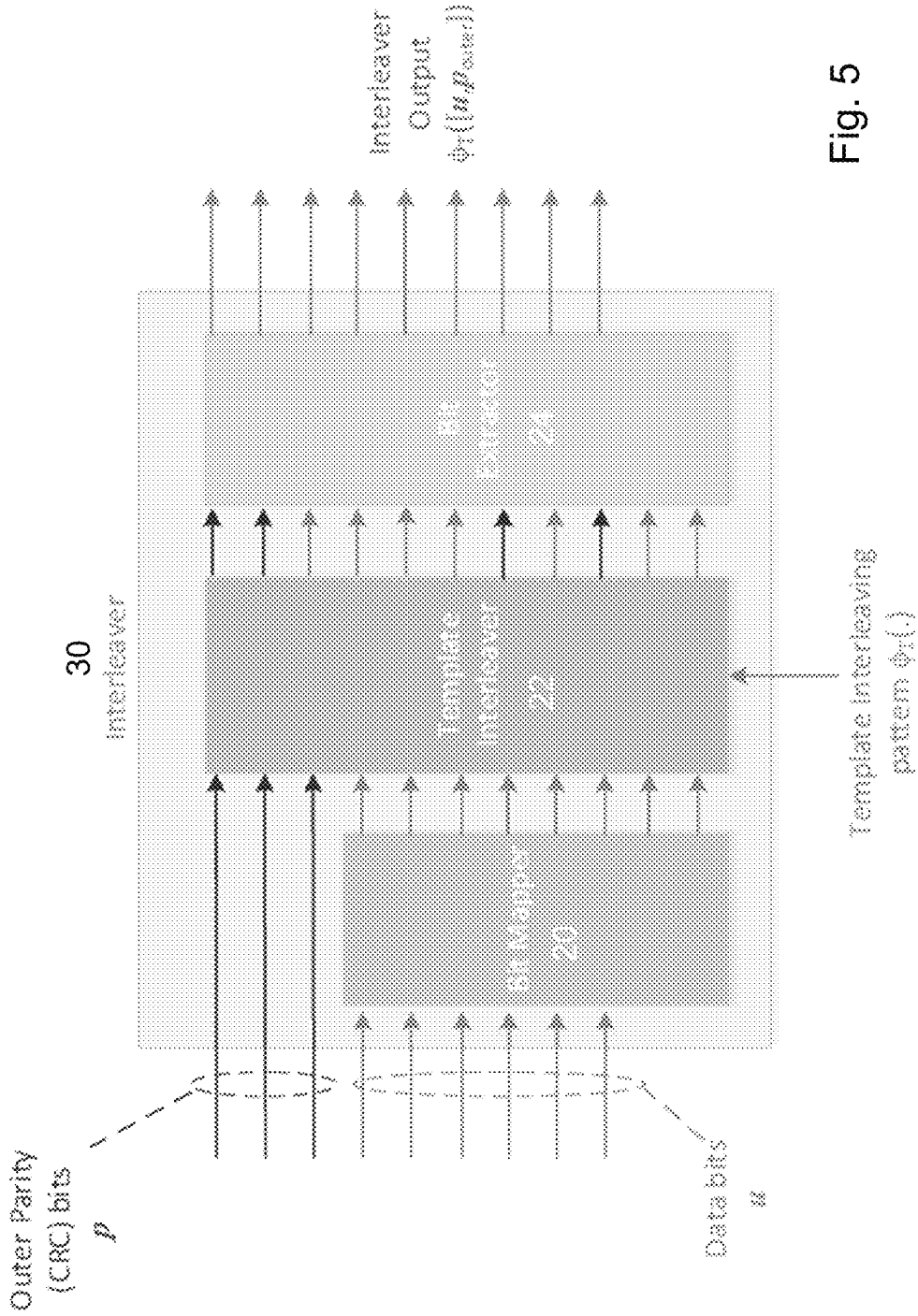
Outer Parity
(CRC) bits
p
Data bits
u
30
Interleaver
Bit Manipulator
28
Template
Interleaver
22
Bit
Collector
24
Template interleaving
pattern Φ-(.)
Interleaver
Output
Φ-(u,p_out)

Fig. 5

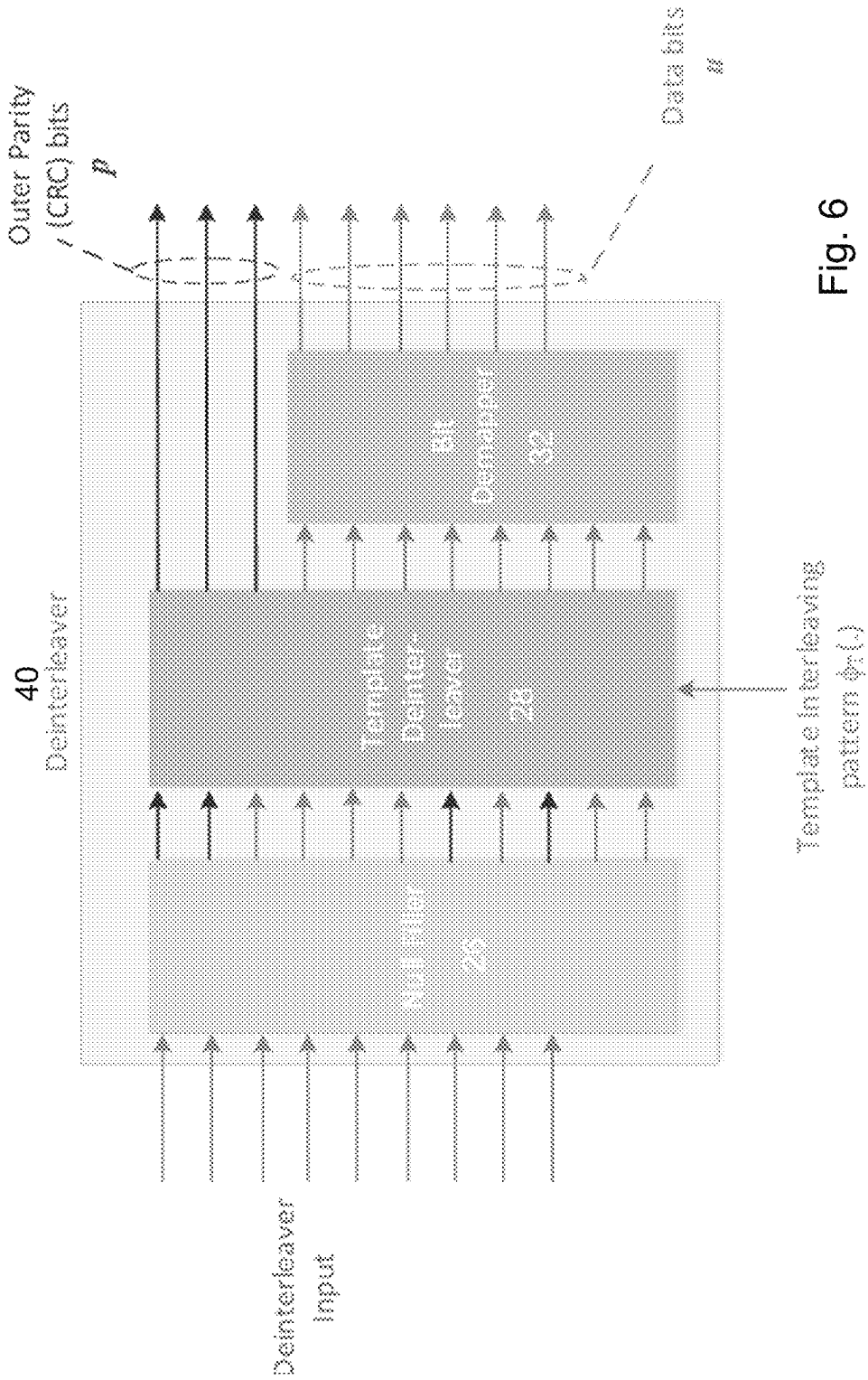
40
Deinterleaver
Outer Parity
(CRC) bits
p
Data bits
a
Bit
Deinterleaver
22
Template
Deinterleaver
28
Multiplexer
26
Deinterleaver
Input
Template interleaving
pattern $\phi_T(.)$

Fig. 6

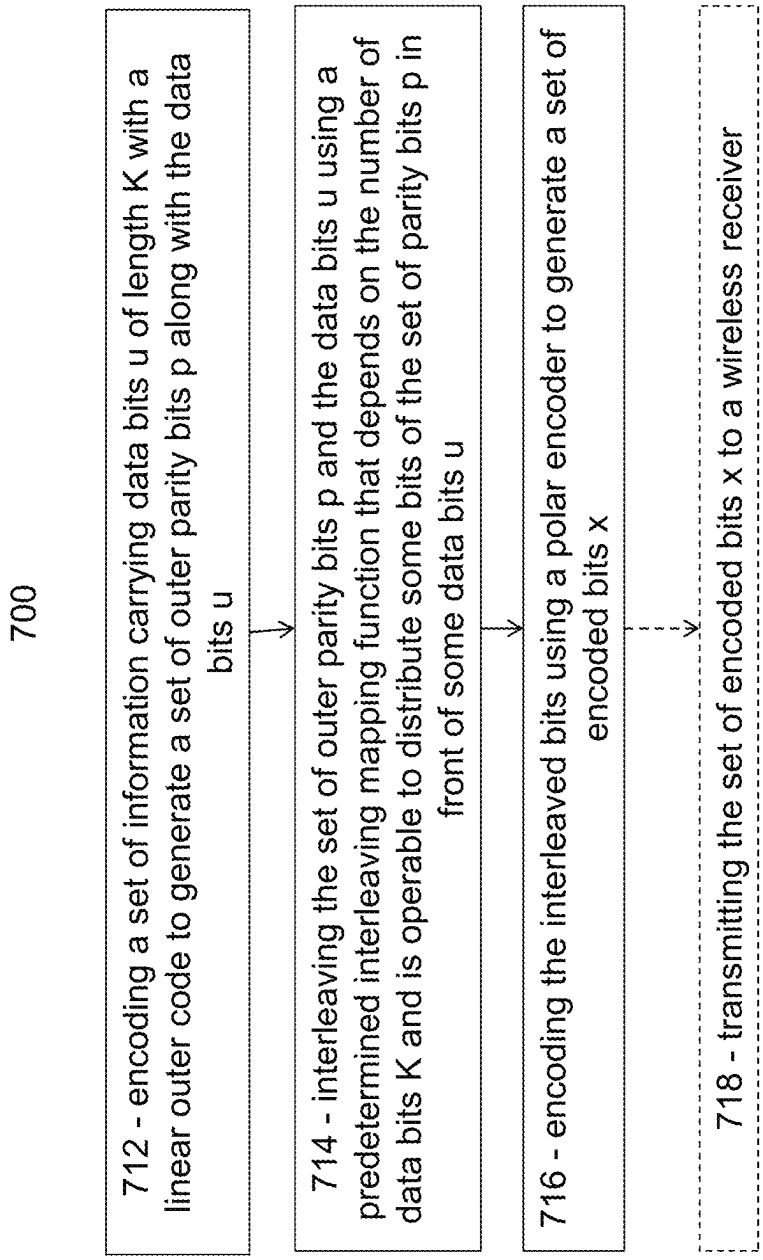
700
712 - encoding a set of information carrying data bits u of length K with a linear outer code to generate a set of outer parity bits p along with the data bits u
714 - interleaving the set of outer parity bits p and the data bits u using a predetermined interleaving mapping function that depends on the number of data bits K and is operable to distribute some bits of the set of parity bits p in front of some data bits u
716 - encoding the interleaved bits using a polar encoder to generate a set of encoded bits x
718 - transmitting the set of encoded bits x to a wireless receiver

Fig. 7

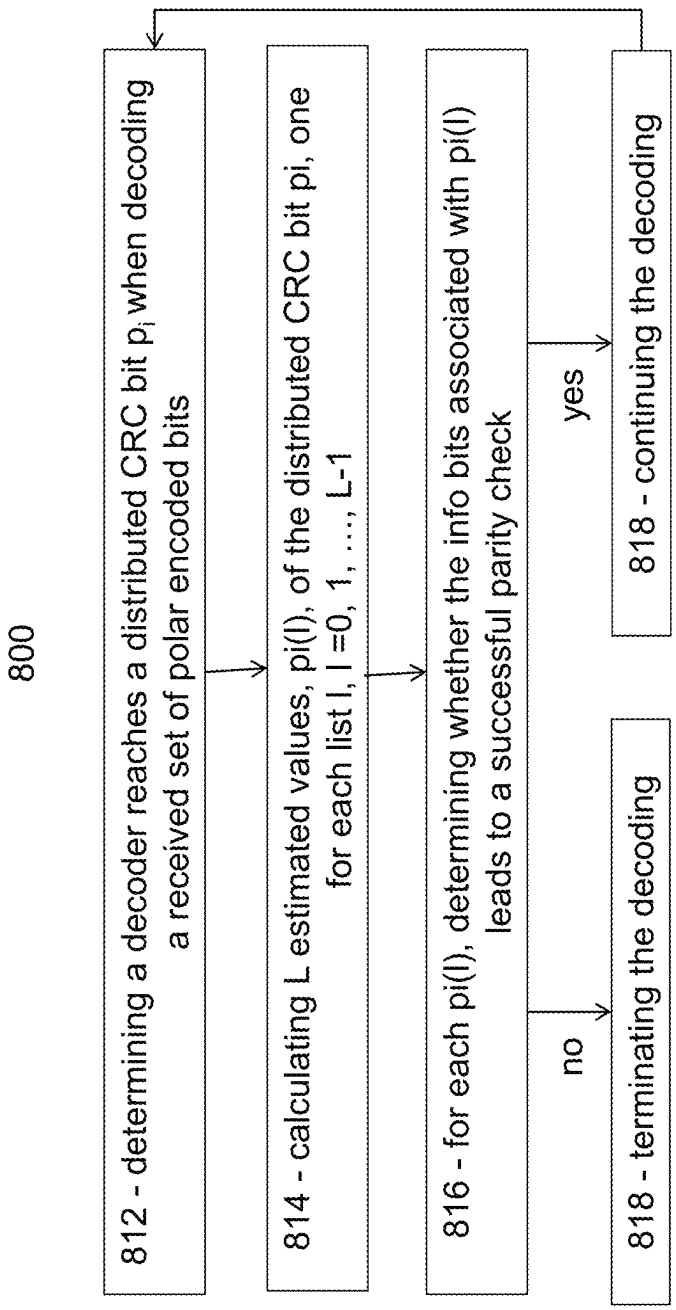
800
812 - determining a decoder reaches a distributed CRC bit p_i when decoding a received set of polar encoded bits
814 - calculating L estimated values, pi(l), of the distributed CRC bit pi, one for each list l, l =0, 1, …, L-1
816 - for each pi(l), determining whether the info bits associated with pi(l) leads to a successful parity check
yes
no
818 - continuing the decoding
818 - terminating the decoding

Fig. 8

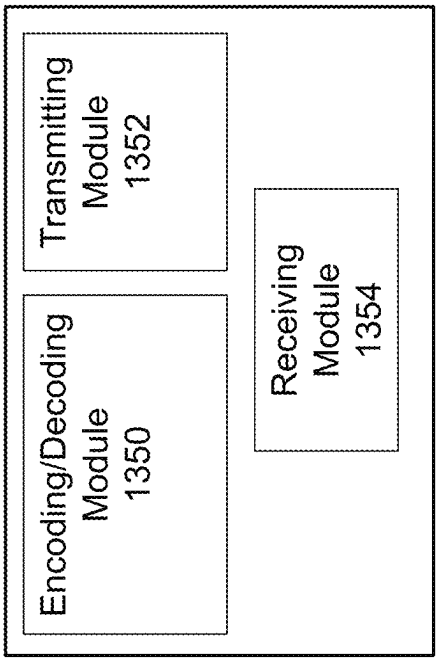
Encoding/Decoding Module 1350
Transmitting Module 1352
Receiving Module 1354

Fig. 9B

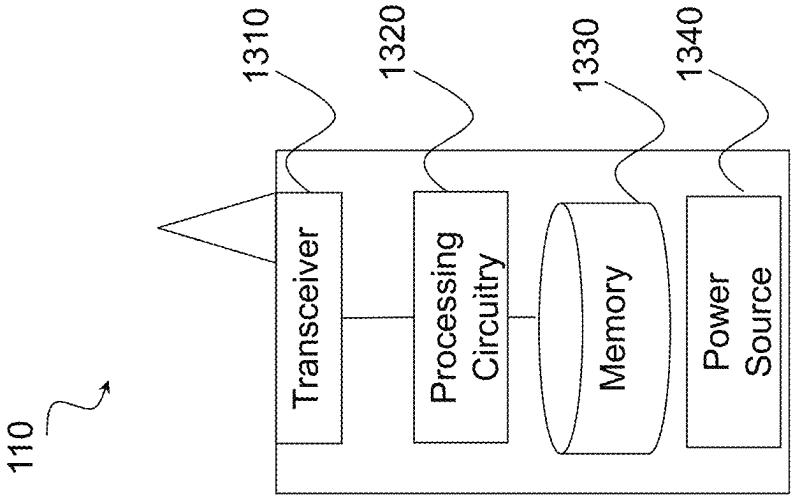
110
Transceiver
1310
Processing Circuitry
1320
Memory
1330
Power Source
1340

Fig. 9A

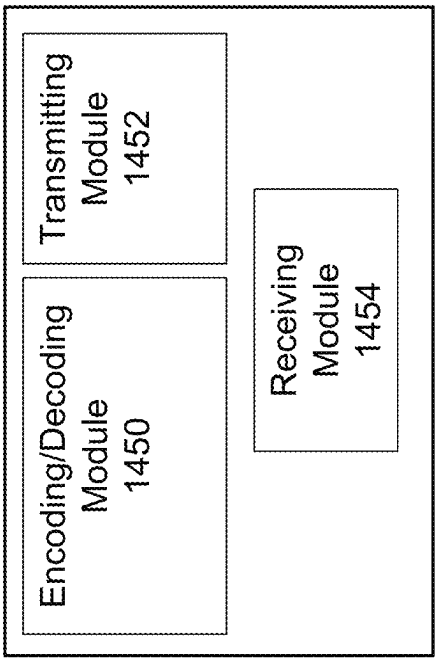

Fig. 10B

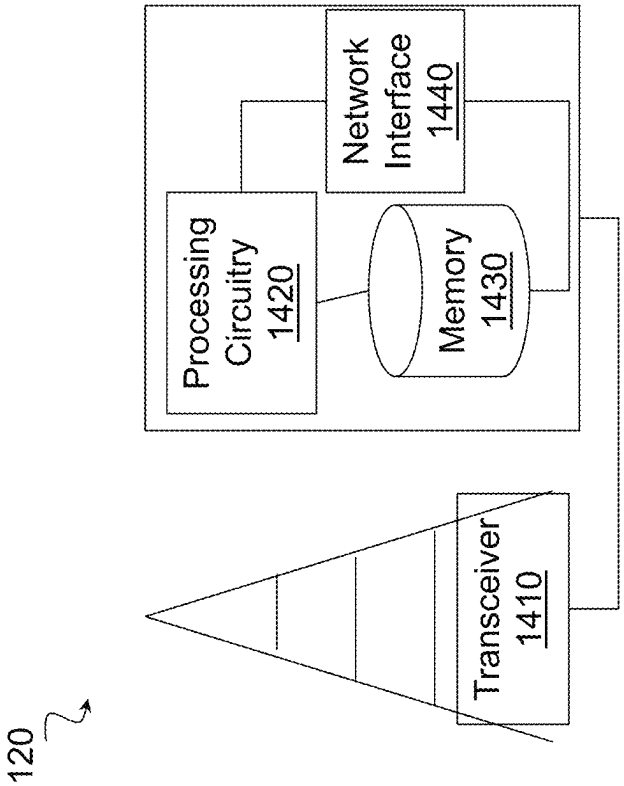

Fig. 10A

CRC INTERLEAVING PATTERN FOR POLAR CODES

PRIORITY

This application is a continuation under 35 U.S.C. § 120, of U.S. patent application Ser. No. 17/514,249 filed on Oct. 29, 2021, which issued as U.S. patent Ser. No. 11/936,399 on Mar. 19, 2024, which is a continuation under 35 U.S.C. § 120, of U.S. patent application Ser. No. 16/645,720 filed on Mar. 9, 2020, which issued as U.S. patent Ser. No. 11/165,442 on Nov. 2, 2021, which nonprovisional application is a U.S. National Stage Filing under 35 U.S.C. § 371 of International Patent Application Serial No. PCT/IB2018/056983 filed Sep. 12, 2018 and entitled "CRC INTERLEAVING PATTERN FOR POLAR CODES" which claims priority to: U.S. Provisional Patent Application No. 62/557,715 filed Sep. 12, 2017; U.S. Provisional Patent Application No. 62/558,208 filed Sep. 13, 2017; and U.S. Provisional Patent Application No. 62/559,991 filed Sep. 18, 2017 all of which are hereby incorporated by reference in their entirety.

TECHNICAL FIELD

Particular embodiments are directed to wireless communications and, more particularly, to cyclic redundancy check (CRC) interleaving patterns for polar codes.

INTRODUCTION

Polar codes, proposed by E. Arikan, "Channel Polarization: A Method for Constructing Capacity-Achieving Codes for Symmetric Binary-Input Memoryless Channels," IEEE Transactions on Information Theory, vol. 55, pp. 3051-3073, July 2009, are a class of constructive coding schemes that achieve the symmetric capacity of the binary-input discrete memoryless channels under a low-complexity Successive Cancellation (SC) decoder. The finite-length performance of polar codes under SC, however, is not competitive compared to other modem channel coding schemes such as low-density parity-check (LDPC) codes and Turbo codes. An SC list (SCL) decoder is proposed in I. Tal and A. Vardy, "List Decoding of Polar Codes," Proceedings of IEEE Symp. Inf. Theory, pp. 1-5, 2011, that approaches the performance of optimal maximum-likelihood (ML) decoder. By concatenating a simple cyclic redundancy check (CRC) coding, the performance of a concatenated polar code is competitive with that of well-optimized LDPC and Turbo codes. As a result, polar codes are being considered as a candidate for future fifth generation (5G) wireless communication systems.

Polar coding transforms a pair of identical binary-input channels into two distinct channels of different qualities, one better and one worse than the original binary-input channel. Repeating such a pair-wise polarizing operation on a set of $N=2^n$ independent uses of a binary-input channel results in a set of 2^n "bit-channels" of varying qualities. Some of the bit channels are nearly perfect (i.e., error free) while the rest of them are nearly useless (i.e., totally noisy). Polar coding uses the nearly perfect channel to transmit data to the receiver and sets the input to the useless channels to have fixed or frozen values (e.g., 0) known to the receiver. For this reason, the input bits to the nearly useless and the nearly perfect channel are commonly referred to as frozen bits and non-frozen (or information) bits, respectively. Only the non-frozen bits are used to carry data in a polar code. Loading the data into the proper information bit locations has direct impact on the performance of a polar code. The set of information bit locations is commonly referred to as an information set. An illustration of the structure of a length-8 polar code is illustrated in FIG. **1**.

Although the original polar code, as proposed by Arikan, was proven to be capacity achieving with a low-complexity successive cancellation (SC) decoder, the finite-length performance of polar codes under SC is not competitive compared to other modem channel coding schemes such LDPC and Turbo codes. A more complex decoder, the SC list (SCL) decoder, is proposed in I. Tal and A. Vardy, "List Decoding of polar codes," in Proceedings of IEEE Symp. Inf. Theory, pp. 1-5, 2011, where a list of more than one surviving decision paths is maintained in the decoding process, but the resulting performance is still unsatisfactory. Tal et al. further proposed that by concatenating a linear outer code, a cyclic redundancy check (CRC) code, with the original polar code as inner code, the outer code can be used to check if any of the candidate paths in the list is correctly decoded. Such a two-step decoding process significantly improves the performance and makes polar codes competitive with that of well-optimized LDPC and Turbo codes. However, the two-step decoding process is in general sub-optimal because each step does not account for the structure of the other (inner or outer) code.

The two-step decoding process also increases the decoding latency as the outer decoder typically needs to wait for the inner decoder to finish decoding before it operates. To compensate for the delay incurred by extra processing, methods of improving the average decoding latency are needed. One method is to try to terminate the decoding when one of decoded CRC bits is found to be inconsistent with the previously decoded information bits that the CRC bit depends upon. However, this method is not effective when all CRC bits are attached at the end of the code block.

SUMMARY

The embodiments described herein include applying a bit-interleaver with a specific interleaving pattern between a linear outer code (e.g., a cyclic redundancy check (CRC) code) and a polar inner code. The interleaving pattern enables the decoder to achieve early termination of the decoding when some of the CRC bits that are encountered early in the successive decoding process are used for early error detection, while maintaining a low false-alarm rate (FAR). In addition, the interleaver also enables some of the parity bits generated by the outer code to be used earlier to positively influence the decisions made in a modified successive cancellation list (SCL) decoder for the polar inner code. This facilitates a single-step decoding for the overall concatenated code that accounts for the structure of the outer code and thus outperforms its two-step counterpart.

According to some embodiments, a method of operation of a wireless transmitter in a wireless communication network comprises: encoding a set of information carrying data bits u of length K with a linear outer code to generate a set of outer parity bits p along with the data bits u; interleaving the set of outer parity bits p and the data bits u using a predetermined interleaving mapping function that depends on the number of data bits K and is operable to distribute some bits of the set of parity bits p in front of some data bits u; and encoding the interleaved bits using a polar encoder to generate a set of encoded bits x. The method may further comprise transmitting the set of encoded bits x to a wireless receiver.

In particular embodiments the predetermined interleaving mapping function comprises a template interleaver for a largest value of K, referred to a K_{max}, and the template interleaver comprises a high-index bit mapper wherein the K data bits are loaded at the high-index positions of the input of the template interleaver, where $u=[u_0, u_1, \ldots, u_{K-1}]$ and the input of the template interleaver, denoted by $v=[v_0, v_1, \ldots, v_{K_{max}-1}]$, is given by the following bit mapping:

$$v_i = \begin{cases} u_{i-K_{max}+K} & K_{max} - K \le i < K_{max} \\ p_{i-K_{max}} & i \ge K_{max} \\ \text{NULL} & \text{otherwise} \end{cases}.$$

In some embodiments the template interleaver comprises a low-index bit mapper wherein: the K data bits are loaded at the low-index positions of the input of the template interleaver in reverse, where the input of the template interleaver is given by the following bit mapping:

$$v_i = \begin{cases} u_{K-1-i} & 0 \le i < K \\ p_{i-K_{max}} & i \ge K_{max} \\ \text{NULL} & \text{otherwise} \end{cases}.$$

According to some embodiments, a wireless transmitter comprises processing circuitry. The processing circuitry is operable to: encode a set of information carrying data bits u of length K with a linear outer code to generate a set of outer parity bits p along with the data bits u; interleave the set of outer parity bits p and the data bits u using a predetermined interleaving mapping function that depends on the number of data bits K and is operable to distribute some bits of the set of parity bits p in front of some data bits u; and encode the interleaved bits using a polar encoder to generate a set of encoded bits x. The processing circuitry may be further operable to transmit the set of encoded bits x to a wireless receiver.

In particular embodiments, the wireless transmitter comprises as wireless device (e.g., user equipment) or a base station (e.g., gNB).

According to some embodiments, a method of operation of a wireless receiver in a wireless communication network comprises: determining a decoder reaches a distributed CRC bit p_i when decoding a received set of polar encoded bits; calculating L estimated values, $p_i(\ell)$, of the distributed CRC bit p_i, one or each list ℓ, $\ell=0, 1, \ldots, L-1$; for each $p_i(\ell)$, determining whether the info bits associated with $p_i(\ell)$ leads to a successful parity check; and upon determining there is no successful parity check for each $p_i(\ell)$, terminating the decoding; and upon determining there exists a successful parity check, continuing the decoding.

According to some embodiments, a method of operation of a wireless receiver in a wireless communication network comprises: determining a decoder reaches a distributed CRC bit p_i when decoding a received set of polar encoded bits, wherein the distributed CRC bit p_i is masked by a bit q_i, and becomes: $w_i=(p_i+q_i) \bmod 2$; calculating L estimated values, $w_i(\ell)$, of the distributed CRC bit p_i, one for each list ℓ, $\ell=0, 1, \ldots, L-1$; for each $w_i(\ell)$, removing the mask, $p_i=(w_i+q_i) \bmod 2$; for each $p_i(\ell)$, determining whether the info bits associated with $p_i(\ell)$ leads to a successful parity check; upon determining there is no successful parity check for each $p_i(\ell)$, terminating the decoding; and upon determining there exists a successful parity check, continuing the decoding.

In particular embodiments, the decoding comprises a template deinterleaver and a bit demapper which performs the inverse of the bit mapping of any of the interleaving functions described herein.

According to some embodiments, a wireless receiver comprises processing circuitry. The processing circuitry is operable to: determine a decoder reaches a distributed CRC bit p_i when decoding a received set of polar encoded bits; calculate L estimated values, $p_i(\ell)$, of the distributed CRC bit p_i, one for each list ℓ, $\ell=0, 1, \ldots, L-1$; for each $p_i(\ell)$, determine whether the info bits associated with $p_i(\ell)$ leads to a successful parity check; and upon determining there is no successful parity check for each $p_i(\ell)$, terminate the decoding; and upon determining there exists a successful parity check, continue the decoding.

According to some embodiments, a wireless receiver comprises processing circuitry. The processing circuitry is operable to: determine a decoder reaches a distributed CRC bit p_i when decoding a received set of polar encoded bits, wherein the distributed CRC bit p_i is masked by a bit q_i, and becomes: $w_i=(p_i+q_i) \bmod 2$; calculate L estimated values, $w_i(\ell)$, of the distributed CRC bit p_i, one for each list ℓ, $\ell=0, 1, \ldots, L-1$; for each $w_i(\ell)$, remove the mask, $p_i=(w_i+q_i) \bmod 2$; for each $p_i(\ell)$, determine whether the info bits associated with $p_i(\ell)$ leads to a successful parity check; upon determining there is no successful parity check for each $p_i(\ell)$, terminate the decoding; and upon determining there exists a successful parity check, continue the decoding.

According to some embodiments, a wireless transmitter comprises an encoding module (**1350**, **1450**). The encoding module is operable to: encode a set of information carrying data bits u of length K with a linear outer code to generate a set of outer parity bits p along with the data bits u; interleave the set of outer parity bits p and the data bits u using a predetermined interleaving mapping function that depends on the number of data bits K and is operable to distribute some bits of the set of parity bits p in front of some data bits u; and encode the interleaved bits using a polar encoder to generate a set of encoded bits x.

According to some embodiments, a wireless receiver comprises a decoding module (**1350**, **1450**). The decoding module is operable to: determine a decoder reaches a distributed CRC bit p_i when decoding a received set of polar encoded bits: calculate L estimated values, $p_i(\ell)$, of the distributed CRC bit p_i, one for each list ℓ, $\ell=0, 1, \ldots, L-1$; for each $p_i(\ell)$, determine whether the info bits associated with $p_i(\ell)$ leads to a successful parity check; and upon determining there is no successful parity check for each $p_i(\ell)$, terminate the decoding; and upon determining there exists a successful parity check, continue the decoding.

According to some embodiments, a wireless receiver comprises a decoding module (**1350**, **1450**). The decoding module is operable to: determine a decoder reaches a distributed CRC bit p_i when decoding a received set of polar encoded bits, wherein the distributed CRC bit p_i is masked by a bit q_i, and becomes: $w_i=(p_i+q_i) \bmod 2$; calculate L estimated values, $w_i(\ell)$, of the distributed CRC bit p_i, one for each list ℓ, $\ell=0, 1, \ldots, L-1$; for each $w_i(\ell)$, remove

the mask, $p_i=(w_i+q_i)$ mod 2; for each $p_i(\ell)$, determine whether the info bits associated with $p_i(\ell)$ leads to a successful parity check; upon determining there is no successful parity check for each $p_i(\ell)$, terminate the decoding; and upon determining there exists a successful parity check, continue the decoding.

In particular embodiments, the wireless receiver comprises as wireless device (e.g., user equipment) or a base station (e.g., gNB).

Also disclosed is a computer program product. The computer program product comprises instructions stored on non-transient computer-readable media which, when executed by a processor, perform the steps of encoding a set of information carrying data bits u of length K with a linear outer code to generate a set of outer parity bits p along with the data bits u; interleaving the set of outer parity bits p and the data bits u using a predetermined interleaving mapping function that depends on the number of data bits K and is operable to distribute some bits of the set of parity bits p in front of some data bits u; and encoding the interleaved bits using a Polar encoder to generate a set of encoded bits x. The instructions may further perform the step of transmitting the set of encoded bits x to a wireless receiver.

Another computer program product comprises instructions stored on non-transient computer-readable media which, when executed by a processor, perform the steps of determining a decoder reaches a distributed CRC bit p_i when decoding a received set of polar encoded bits; calculating L estimated values, $p_i(\ell)$, of the distributed CRC bit p_i, one for each list ℓ, $\ell=0, 1, \ldots, L-1$; for each $p_i(\ell)$, determining whether the info bits associated with $p_i(\ell)$ leads to a successful parity check; and upon determining there is no successful parity check for each $p_i(\ell)$, terminating the decoding; and upon determining there exists a successful parity check, continuing the decoding.

Another computer program product comprises instructions stored on non-transient computer-readable media which, when executed by a processor, perform the steps o determining a decoder reaches a distributed CRC bit p_i when decoding a received set of polar encoded bits, wherein the distributed CRC bit p_i is masked by a bit q_i, and becomes: $w_i=(p_i+q_i)$ mod 2; calculating L estimated values, $w_i(\ell)$, of the distributed CRC bit p_i, one for each list ℓ, $\ell=0, 1, \ldots, L-1$; for each $w_i(\ell)$, removing the mask, $p_i=(w_i+q_i)$ mod 2; for each $p_i(\ell)$, determining whether the info bits associated with $p_i(\ell)$ leads to a successful parity check; upon determining there is no successful parity check for each $p_i(\ell)$, terminating the decoding; and upon determining there exists a successful parity check, continuing the decoding.

Particular embodiments may include some, all, or none of the following advantages. For example, particular interleaving patterns enable the decoder to take early termination of decoding if the decoded info bits in each candidate of the decoding list are not consistent with the decoded value of a CRC bit. This reduces the overall decoding latency. With existing methods, the entire length-K_{CRC} vector of CRC bits are typically used in CRC checking after all information bits are decoded. With particular embodiments described herein, CRC checking can be done bit-by-bit for each individual CRC bit during the successive cancellation list decoding. The interleaving patterns strike a balance between early termination gain in decoding and the false-alarm probability (i.e., the probability of falsely accepting an incorrectly decoded code block).

Particular embodiments also facilitate a single-step decoding process for the concatenation of any linear outer code and a polar inner code through a judicious design of the interleaver, as opposed to a two-step decoding process where the inner polar code is first decoded followed by the decoding of the outer code. Such a single step decoding jointly accounts the structure of the polar inner code and the linear outer (e.g., CRC) code and thus improves the performance, compared to the two-step solution.

BRIEF DESCRIPTION OF THE DRAWINGS

For a more complete understanding of the embodiments and their features and advantages, reference is now made to the following description, taken in conjunction with the accompanying drawings, in which:

FIG. **1** illustrates an example of a polar code structure with N **8**;

FIG. **2** is a block diagram illustrating an example wireless network, according to a particular embodiment;

FIG. **3** is a block diagram illustrating the encoder structure of an interleaved concatenated Polar code, according to a particular embodiment;

FIG. **4** is a block diagram illustrating a one-step decoder structure of an interleaved concatenated Polar code, according to a particular embodiment;

FIG. **5** is a block diagram illustrating a template interleave of a fixed size K_{max}, according to a particular embodiment;

FIG. **6** is a block diagram illustrating the structure of a deinterleaver, according to a particular embodiment;

FIG. **7** is a flow diagram illustrating an example method in a wireless transmitter, according to particular embodiments;

FIG. **8** is a flow diagram illustrating an example method in a wireless receiver, according to particular embodiments;

FIG. **9**A is a block diagram illustrating an example embodiment of a wireless device;

FIG. **9**B is a block diagram illustrating example components of a wireless device;

FIG. **10**A is a block diagram illustrating an example embodiment of a network node; and

FIG. **10**B is a block diagram illustrating example components of a network node.

DETAILED DESCRIPTION

Polar codes achieve the symmetric capacity of the binary-input discrete memoryless channels under a low-complexity successive cancellation (SC) decoder. However, the finite-length performance of polar codes under SC is not competitive compared to other modem channel coding schemes such as low-density parity-check (LDPC) codes and Turbo codes. An SC list (SCL) decoder approaches the performance of optimal maximum-likelihood (ML) decoder. By concatenating a simple cyclic redundancy check (CRC) coding, the performance of a concatenated polar code is competitive with that of well-optimized LDPC and Turbo codes. As a result, polar codes may be used for fifth generation (5G) wireless communication systems.

By concatenating a linear outer code, such as a CRC code, with the original polar code as inner code, the outer code can be used to check if any of the candidate paths in the list is correctly decoded. The two-step decoding process significantly improves the performance, however, it is in general

sub-optimal because each step does not account for the structure of the other (inner or outer) code.

The two-step decoding process also increases the decoding latency as the outer decoder typically needs to wait for the inner decoder to finish decoding before it operates. To compensate for the delay incurred by extra processing, particular embodiments improve the average decoding latency.

Particular embodiments described herein obviate the problems described above and include applying a bit-interleaver with a specific interleaving pattern between a linear outer code (e.g., a CRC code) and a polar inner code. The interleaving pattern enables the decoder to achieve early termination of the decoding when some of the CRC bits that are encountered early in the successive decoding process are used for early error detection, while maintaining a low false-alarm rate (FAR). In addition, the interleaver also enables some of the parity bits generated by the outer code to be used earlier to positively influence the decisions made in a modified SCL decoder for the polar inner code. This facilitates a single-step decoding for the overall concatenated code that accounts for the structure of the outer code and thus outperforms its two-step counterpart.

The interleaving patterns strike a balance between early termination gain in decoding and the false-alarm probability (i.e., the probability of falsely accepting an incorrectly decoded code block). Particular embodiments also facilitate a single-step decoding process for the concatenation of any linear outer code and a polar inner code through a judicious design of the interleaver, as opposed to a two-step decoding process where the inner polar code is first decoded followed by the decoding of the outer code. Such a single step decoding jointly accounts for the structure of the polar inner code and the linear outer (CRC) code and thus improves the performance, compared to the two-step solution.

The following description sets forth numerous specific details. It is understood, however, that embodiments may be practiced without these specific details. In other instances, well-known circuits, structures and techniques have not been shown in detail in order not to obscure the understanding of this description. Those of ordinary skill in the art, with the included descriptions, will be able to implement appropriate functionality without undue experimentation.

References in the specification to "one embodiment," "an embodiment," "an example embodiment," etc., indicate that the embodiment described may include a particular feature, structure, or characteristic, but every embodiment may not necessarily include the particular feature, structure, or characteristic. Moreover, such phrases are not necessarily referring to the same embodiment. Further, when a particular feature, structure, or characteristic is described in connection with an embodiment, it is submitted that it is within the knowledge of one skilled in the art to implement such feature, structure, or characteristic in connection with other embodiments, whether or not explicitly described.

Particular embodiments are described with reference to FIGS. **2**-**10**B of the drawings, like numerals being used for like and corresponding parts of the various drawings. Long term evolution (LTE) and NR are used throughout this disclosure as an example cellular system, but the ideas presented herein may apply to other wireless communication systems as well.

FIG. **2** is a block diagram illustrating an example wireless network, according to a particular embodiment. Wireless network **100** includes one or more wireless devices **110** (such as mobile phones, smart phones, laptop computers, tablet computers, MTC devices, or any other devices that can provide wireless communication) and a plurality of network nodes **120** (such as base stations, eNodeBs, gNBs, etc.). Wireless device **110** may also be referred to as a UE. Network node **120** serves coverage area **115** (also referred to as cell **115**).

In general, wireless devices **110** that are within coverage of network node **120** (e.g., within cell **115** served by network node **120**) communicate with network node **120** by transmitting and receiving wireless signals **130**. For example, wireless devices **110** and network node **120** may communicate wireless signals **130** containing voice traffic, data traffic, and/or control signals. A network node **120** communicating voice traffic, data traffic, and/or control signals to wireless device **110** may be referred to as a serving network node **120** for the wireless device **110**. Communication between wireless device **110** and network node **120** may be referred to as cellular communication. Wireless signals **130** may include both downlink transmissions (from network node **120** to wireless devices **110**) and uplink transmissions (from wireless devices **110** to network node **120**).

Each network node **120** may have a single transmitter or multiple transmitters for transmitting signals **130** to wireless devices **110**. In some embodiments, network node **120** may comprise a multi-input multi-output (MIMO) system. Wireless signal **130** may comprise one or more beams. Particular beams may be beamformed in a particular direction. Each wireless device **110** may have a single receiver or multiple receivers for receiving signals **130** from network nodes **120** or other wireless devices **110**. Wireless device **110** may receive one or more beams comprising wireless signal **130**.

Wireless signals **130** may be transmitted on time-frequency resources. The time-frequency resources may be partitioned into radio frames, subframes, slots, and/or mini-slots. Network node **120** may dynamically schedule subframes/slots/mini-slots as uplink, downlink, or a combination uplink and downlink. Different wireless signals **130** may include different transmission processing times.

Network node **120** may operate in a licensed frequency spectrum, such as an LTE spectrum. Network node **120** may also operate in an unlicensed frequency spectrum, such as a 5 GHz Wi-Fi spectrum. In an unlicensed frequency spectrum, network node **120** may coexist with other devices such as IEEE 802.11 access points and terminals. To share the unlicensed spectrum, network node **120** may perform listen-before-talk (LBT) protocols before transmitting or receiving wireless signals **130**. Wireless device **110** may also operate in one or both of licensed or unlicensed spectrum and in some embodiments may also perform LBT protocols before transmitting wireless signals **130**. Both network node **120** and wireless device may also operate in licensed shared spectrum.

For example, network node **120***a* may operate in a licensed spectrum and network node **120***b* may operate in an unlicensed spectrum. Wireless device **110** may operate in both licensed and unlicensed spectrum. In particular embodiments, network nodes **120***a* and **120***b* may be configurable to operate in a licensed spectrum, an unlicensed spectrum, a licensed shared spectrum, or any combination. Although the coverage area of cell **115***b* is illustrated as included in the coverage area of cell **115***a*, in particular embodiments the coverage areas of cells **115***a* and **115***b* may overlap partially or may not overlap at all.

In particular embodiments, wireless device **110** and network nodes **120** may perform carrier aggregation. For example, network node **120***a* may serve wireless device **110** as a PCell and network node **120***b* may serve wireless device **110** as a SCell. Network nodes **120** may perform self-

scheduling or cross-scheduling. If network node **120***a* is operating in licensed spectrum and network node **120***b* is operating in unlicensed spectrum, network node **120***a* may provide license assisted access to the unlicensed spectrum (i.e., network node **120***a* is a LAA PCell and network node **120***b* is a LAA SCell).

In particular embodiments, wireless signals **130** may be encoded using a polar code. For example, wireless device **110** and/or network node **120** may use a polar code for encoding wireless signal **130**. In some embodiments, the encoding may include an interleaver. The interleaver is described in more detail with respect to FIGS. **3-8**.

In wireless network **100**, each network node **120** may use any suitable radio access technology, such as long term evolution (LTE), LTE-Advanced, UMTS, HSPA, GSM, cdma2000, NR, WiMax, WiFi, and/or other suitable radio access technology. Wireless network **100** may include any suitable combination of one or more radio access technologies. For purposes of example, various embodiments may be described within the context of certain radio access technologies. However, the scope of the disclosure is not limited to the examples and other embodiments could use different radio access technologies.

As described above, embodiments of a wireless network may include one or more wireless devices and one or more different types of radio network nodes capable of communicating with the wireless devices. The network may also include any additional elements suitable to support communication between wireless devices or between a wireless device and another communication device (such as a landline telephone). A wireless device may include any suitable combination of hardware and/or software. For example, in particular embodiments, a wireless device, such as wireless device **110**, may include the components described with respect to FIG. **9**A below. Similarly, a network node may include any suitable combination of hardware and/or software. For example, in particular embodiments, a network node, such as network node **120**, may include the components described with respect to FIG. **10**A below.

Particular embodiments include a judiciously designed interleaver, with specific interleaver patterns, between a linear outer code and a polar inner code. The interleaving patterns move the CRC bits to the beginning of the code blocks so that the decoding process may be terminated earlier to reduce average latency if the decoded values of some CRC bits are inconsistent with the values of the corresponding information bits. These interleaving patterns are designed in such a way that the early termination gain is maximized while maintain a low FAR.

According to particular embodiments of the interleaver, a single-pass or single-step decoding is performed using a slightly modified SCL polar decoder to jointly take advantage of the structure of both the inner and outer coder. The interleaver enables some of the parity bits generated by the outer code to be used earlier to positively influence the decisions made in a modified SCL decoder for the polar inner code. This facilitates a single-step decoding for the overall concatenated code while outperforming its two-step counterpart.

Particular embodiments include an interleaver with a specific interleaving pattern, for each possible number of information bits, between a linear outer code, such as a CRC code, and a polar inner code. The interleaver distributes some CRC bits in front of some information bits. This enables a SCL decoder to terminate the decoding process when the decoded value of any of these CRC bits is not consistent with the information bits that the CRC bits depend upon for every candidate in the list Interleaving of CRC bits also facilitates list decoding for the inner polar code that accounts for the dependency structure of the parity bits and the data bits from the outer code. An example is illustrated in FIG. **3**.

FIG. **3** is a block diagram illustrating the encoder structure of an interleaved concatenated polar code, according to a particular embodiment. The information-carrying data bits u of length K are first encoded by a linear outer encoder **10** (the linear outer code is typically a CRC code) to generate a number of outer parity bits p along with the data bits u. All the bits x_{outer}=[u|p] are interleaved at interleaver **12** and put into polar inner encoder along with the frozen bits to form the input to polar inner encoder **14**, which generates the overall coded bits x. Interleaver **12** operates based on a predetermined interleaving mapping $\phi_K(\cdot)$, which depends on the number of data bits, K.

FIG. **4** is a block diagram illustrating a one-step decoder structure of an interleaved concatenated polar code, according to a particular embodiment. At the receiver, the input log-likelihood ratios (LLRs) y of the coded bits are first decoded using modified SCL polar decoder **16**, whose outputs are then passed through deinterleaver **18** that extracts the decoded data bits. Deinterleaver **18** depends on the interleaving mapping $\phi_K(\cdot)$ used in the encoder **14** in a straightforward manner. The operations of the modified SCL polar decoder is similar to an ordinary SCL polar decoder except that whenever an outer parity bit is reached, as indicated by the interleaving mapping $\phi_K(\cdot)$, during the successive decoding process, its value is computed based on the previous data bits as indicated by the corresponding columns of the generating matrix G_o of the outer code.

The size of the interleaver described with respect to FIG. **3**, for example, generally depends on the number K of data bits as well as the number n_{CRC} of CRC bits. To ease implementation, particular embodiments implement a single template interleaver $\phi_T(\cdot)$ at the largest possible value of K, dented by K_{max}, and then use a subset of this template interleaver to implement the interleaver needed for any given value of K in FIG. **3**.

FIG. **5** is a block diagram illustrating a template interleaver of a fixed size K_{max}, according to a particular embodiment. Interleaver **30** comprises bit mapper **20**, template interleaver **22**, and bit extractor **24**. Bit mapper **20** maps K data bits into certain input positions of template interleaver **22** of size K_{max}. Bit mapper $\mu_K(\cdot)$ depends on K. The n_{crc} CRC bits are mapped into other input positions. The rest of the input positions are filled with NULL. Template interleaver **22** re-orders the data bits, CRC bits, and NULLs. Bit extractor **24** removes the NULLs from the output of template interleaver **22** to form the output of interleaver **30**.

The design of the template interleaver is tied to that of the bit mapper. The following are two examples of a bit mapper. A high-index bit mapper loads the K data bits at the high-index positions of the input of the template interleaver. Specifically, let u=[u_0, u_1, . . . , u_{K-1}] be the data bits. Then the input of the template interleaver, denoted by v=[v_0, v_1, $v_{K_{max}-1}$], is given by the following bit mapping.

$$v_i = \begin{cases} u_{i-K_{max}+K} & K_{max} - K \le i < K_{max} \\ p_{i-K_{max}} & i \ge K_{max} \\ \text{NULL} & \text{otherwise} \end{cases}$$

FIGS. **5** and **6** describe the interleaving and de-interleaving operations for a high-index bit mapper.

Another example includes a low-index bit mapper. The low-index bit mapper loads the K data bits at the low-index positions of the input of the template interleaver in a reversed manner. Specifically, the input of the template interleaver is given by the following bit mapping

$$v_i = \begin{cases} u_{K-1-i} & 0 \le i < K \\ p_{i-K_{max}} & i \ge K_{max} \\ \text{NULL} & \text{otherwise} \end{cases}$$

FIG. **6** is a block diagram illustrating the structure of a deinterleaver, according to a particular embodiment. Deinterleaver **40** includes null filler **26**, template deinterleaver **28**, and bit demapper **32**. The example deinterleaver illustrates the corresponding inverse operations in the deinterleaver of the decoder illustrated in FIG. **4**.

After polar decoding, the output of the polar decoder is input to deinterleaver **40**. Null filler **26** fills the input with NULLs according to the same NULL positions as used in bit extractor **24** in FIG. **5**. Template deinterleaver **40** deinterleaves the null-filled sequence.

Part of the output of the template deinterleaver forms the decoded outer (CRC) parity bits and part of the output is passed through bit demapper **32**, which performs the inverse operation of the bit mapping performed in FIG. **4**. The output of bit demapper **32** is the decoded data bits. In the case of the outer code being a CRC code used for error detection, the decoded data bits and decoded outer CRC parity bits are used subsequently to check if the CRC passes to detect whether an error has occurred.

Particular embodiments may include any of a number of judiciously designed interleaving patterns for the template interleaver from which the corresponding interleaver for each possible number of information bits can be derived. Particular patterns maximize the potential reduction in decoding latency through earlier termination while maintaining the FAR.

Listed below are some example interleaving patterns for the template interleaver, each associated with a particular bit mapper described above. In all cases, the following two CRC polynomials are used as examples.

$$g_{crc}(D) = D^{24} + D^{23} + D^{21} + D^{20} + D^{17} + D^{15} + D^{13} + D^{12} + D^{8} + D^{4} + D^{2} + D + 1$$

$$g_{crc}(D) = D^{19} + D^{16} + D^{14} + D^{13} + D^{12} + D^{10} + D^{8} + D^{7} + D^{4} + D^{3} + 1$$

Because the template interleaver $\phi_T(\cdot)$ is a mapping from integers to integers, it can be equivalently described using an integer sequence, denoted by ϕ_T. The indices corresponding to the CRC bits are underlined.

As examples, value of K_{max} is assumed to be in the set {53, 72, 140, 160, 200}, which are values likely to be used in 5G-NR systems, to generate the following interleaving patterns.

1. For high-index bit mapper for K_{max}=53
   (a) ϕ_T=[1 4 5 8 10 11 14 15 16 20 24 26 28 30 31 35 44 45 46 48 51 52 58 12 19 21 22 25 32 33 37 38 39 47 68 7 9 17 18 27 41 50 61 0 34 36 43 53 40 49 54 29 42 64 13 65 2 62 23 55 69 71 3 56 57 67 6 59 60 63 66 70];
   (b) ϕ_T=[1 4 5 8 10 11 14 15 16 20 24 26 28 30 31 35 44 45 46 48 51 52 58 12 19 21 22 25 32 33 37 38 39 47 68 7 9 17 18 27 41 50 61 0 2 3 6 13 23 29 34 36 40 42 43 49 53 54 55 56 57 59 60 62 63 64 65 66 67 69 70 71],
   (c) ϕ_T=[1 4 5 8 10 11 14 15 16 20 24 26 28 30 31 35 44 45 46 48 51 52 58 12 19 21 22 25 32 33 37 38 39 47 68 7 9 17 18 27 41 50 61 0 34 36 43 53 2 3 6 13 23 29 40 42 49 54 55 56 57 59 60 2 63 64 65 66 67 69 70 71];
   (d) ϕ_T=[1 4 5 8 10 11 14 15 16 20 24 26 28 30 31 35 44 45 46 48 51 52 58 12 19 21 22 25 32 33 37 38 39 47 68 7 9 17 18 27 41 50 61 0 34 36 43 53 40 49 54 2 3 6 13 23 29 42 55 56 57 59 60 62 63 64 65 66 67 69 70 71];
2. For low-index bit mapper for K_{max}=53
   (a) ϕ_T=[0 1 4 6 7 8 17 21 22 24 26 28 32 36 37 38 41 42 44 47 48 51 58 5 13 14 15 19 20 27 30 31 33 40 68 2 11 25 34 35 43 45 61 9 16 18 52 53 3 12 54 10 23 64 39 65 50 62 29 55 69 71 49 56 57 67 46 59 60 63 66 70];
   (b) ϕ_T=[51 48 47 44 42 41 38 37 36 32 28 26 24 22 21 17 8 7 6 4 1 0 58 40 33 31 30 27 20 19 15 14 13 5 68 45 43 35 34 25 11 2 61 52 18 16 9 53 12 3 54 23 10 64 39 65 50 62 29 55 69 71 49 56 67 57 46 59 66 60 70 63];
   (c) ϕ_T=[0 1 4 6 7 8 17 21 22 24 26 28 32 36 37 38 41 42 44 47 48 51 58 5 13 14 15 19 20 27 30 31 33 40 68 2 11 25 34 35 43 45 61 3 9 10 12 16 18 23 29 39 46 49 50 52 53 54 55 56 57 59 60 62 63 64 65 66 67 69 70 71];
   (d) ϕ_T=[51 48 47 44 42 41 38 37 36 32 28 26 24 22 21 17 8 7 6 4 1 0 58 40 33 31 30 27 20 19 15 14 13 5 68 45 43 35 34 25 11 2 61 3 9 10 12 16 18 23 29 39 46 49 50 52 53 54 55 56 57 59 60 62 63 64 65 66 67 69 70 71];
   (e) ϕ_T=[0 1 4 6 7 8 17 21 22 24 26 28 32 36 37 38 41 42 44 47 48 51 58 5 13 14 15 19 20 27 30 31 33 40 68 2 11 25 34 35 43 45 61 9 16 18 52 53 3 10 12 23 29 39 46 49 50 54 55 56 57 59 60 62 63 64 65 66 67 69 70 71];
   (f) ϕ_T=[51 48 47 44 42 41 38 37 36 32 28 26 24 22 21 17 8 7 6 4 1 0 58 40 33 31 30 27 20 19 15 14 13 5 68 45 43 35 34 25 11 2 61 52 18 16 9 53 3 10 12 23 29 39 46 49 50 54 55 56 57 59 60 62 63 64 65 66 67 69 70 71];
   (g) ϕ_T=[0 1 4 6 7 8 17 21 22 24 26 28 32 36 37 38 41 42 44 47 48 51 58 5 13 14 15 19 20 27 30 31 33 40 68 2 11 25 34 35 43 45 61 9 16 18 52 53 3 12 54 10 23 29 39 46 49 50 55 56 57 59 60 62 63 64 65 66 67 69 70 71];
   (h) ϕ_T=[51 48 47 44 42 41 38 37 36 32 28 26 24 22 21 17 8 7 6 4 1 0 58 40 33 31 30 27 20 19 15 14 13 5 68 45 43 35 34 25 11 2 61 52 18 16 9 53 12 3 54 10 23 29 39 46 49 50 55 56 57 59 60 62 63 64 65 66 67 69 70 71];
3. For high-index bit mapper for K_{max}=72
   (a) ϕ_T=[3 4 8 11 13 14 17 18 23 30 31 34 38 40 41 43 44 45 47 49 51 52 56 57 58 63 64 65 66 71 87 5 7 9 12 16 20 24 27 29 33 35 39 50 54 67 70 77 0 15 26 28 36 37 46 60 69 80 1 19 55 59 68 73 32 53 61 84 10 62 72 21 81 48 83 22 76 42 88 2 74 75 25 82 85 6 78 79 86 89 90];
   (b) ϕ_T=[3 4 8 11 13 14 17 18 23 30 31 34 38 40 41 43 44 45 47 49 51 52 56 57 58 63 64 65 66 71 87 5 7 9 12 16 20 24 27 29 33 35 39 50 54 67 70 77 0 15 26 28 36 37 46 60 69 80 1 2 6 10 19 21 22 25 32 42 48 53 55 59 61 62 68 72 73 74 75 76 78 79 81 82 83 84 85 86 88 89 90];

(c) ϕ_T=[3 4 8 11 13 14 17 18 23 30 31 34 38 40 41 43 44 45 47 49 51 52 56 57 58 63 64 65 66 71 87 5 7 9 12 16 20 24 27 29 33 35 39 50 54 67 70 77 0 15 26 28 36 37 46 60 69 80 1 19 55 59 68 73 2 6 10 21 22 25 32 42 48 53 61 62 72 74 75 76 78 79 81 82 83 84 85 86 88 89 90];

(d) ϕ_T=[3 4 8 11 13 14 17 18 23 30 31 34 38 40 41 43 44 45 47 49 51 52 56 57 58 63 64 65 66 71 87 5 7 9 12 16 20 24 27 29 33 35 39 50 54 67 70 77 0 15 26 28 36 37 46 60 69 80 1 19 55 59 68 73 32 53 61 84 2 6 10 21 22 25 42 48 62 72 74 75 76 78 79 81 82 83 85 86 88];

4. For low-index bit mapper for K_{max}=72

(a) ϕ_T=[0 5 6 7 813 14 15 19 20 22 24 26 27 28 30 31 33 37 40 41 48 53 54 57 58 60 63 67 68 87 1 4 17 21 32 36 38 42 44 47 51 55 59 62 64 66 77 2 11 25 34 35 43 45 56 71 80 3 12 16 52 70 73 10 18 39 84 9 61 72 50 81 23 83 49 76 29 88 69 74 75 46 82 85 65 78 79 86 89 90];

(b) ϕ_T=[68 67 63 60 58 57 54 53 48 41 40 37 33 31 30 28 27 26 24 22 20 19 15 14 13 8 7 6 5 0 87 66 64 62 59 55 51 47 44 42 38 36 32 21 17 4 1 77 71 56 45 43 35 34 25 11 2 80 70 52 16 12 3 73 39 18 10 84 61 9 72 50 81 23 83 49 76 29 88 69 74 75 46 82 85 65 86 79 78 89 90];

(c) ϕ_T=[0 5 6 7 8 13 14 15 19 20 22 24 26 27 28 30 31 33 37 40 41 48 53 54 57 58 60 63 67 68 87 1 4 17 21 32 36 38 42 44 47 51 55 59 62 64 66 77 2 11 25 34 35 43 45 56 71 80 3 9 10 12 16 18 23 29 39 46 49 50 52 61 65 69 70 72 73 74 75 76 78 79 81 82 83 84 85 86 88 89 90];

(d) ϕ_T=[68 67 63 60 58 57 54 53 48 41 40 37 33 31 30 28 27 26 24 22 20 19 15 14 13 8 7 6 5 0 87 66 64 62 59 55 51 47 44 42 38 36 32 21 17 4 1 77 71 56 45 43 35 34 25 11 2 80 39 10 12 16 18 23 29 39 46 49 50 52 61 65 69 70 72 73 74 75 76 78 79 81 82 83 84 85 86 88 89 90];

(e) ϕ_T=[0 5 6 7 8 13 14 15 19 20 22 24 26 27 28 30 31 33 37 404 1 48 53 54 57 58 60 63 67 87 1 4 17 21 32 36 38 42 44 47 51 55 59 62 64 66 77 2 11 25 34 35 43 45 56 71 80 3 12 16 52 70 73 9 10 18 23 29 39 46 49 50 61 65 69 72 74 75 76 78 79 81 82 83 84 85 86 88 8990];

(f) ϕ_T=[68 67 63 60 58 57 54 53 48 41 40 37 33 31 30 28 27 26 24 22 20 19 15 14 13 8 7 6 5 0 87 66 64 62 59 55 51 47 44 42 38 36 32 21 17 4 1 77 71 56 45 43 35 34 25 11 2 80 70 52 16 12 3 73 9 10 18 23 29 39 46 49 50 61 65 69 72 74 75 76 78 79 81 82 83 84 85 86 88 89 90];

(g) ϕ_T=[0 5 6 7 8 13 14 15 19 20 22 24 26 27 28 30 31 33 37 404 1 48 53 54 57 58 60 63 67 68 87 1 4 17 21 32 36 38 42 44 47 51 55 59 62 64 66 77 2 11 25 34 35 43 45 56 71 80 3 12 16 52 70 73 10 18 39 84 9 23 29 46 49 50 61 65 69 72 74 75 76 78 79 81 82 83 85 86 88 89 90];

(h) ϕ_T=[68 67 63 60 58 57 54 53 48 41 40 37 33 31 30 28 27 26 24 22 20 19 15 14 13 8 7 6 5 0 87 66 64 62 59 55 51 47 44 42 38 36 32 21 17 4 1 77 71 56 45 43 35 34 25 11 2 80 70 52 16 12 3 73 39 18 10 84 9 23 29 46 49 50 61 65 69 72 74 75 76 78 79 81 82 83 85 86 88 89 90];

5. For high-index bit mapper for K_{max}=140

(e) ϕ_T=[0 3 6 9 11 12 13 14 15 17 18 19 21 22 28 29 30 37 38 42 43 48 53 57 59 64 65 66 69 71 72 73 78 79 91 92 97 106 111 114 117 119 122 125 126 127 128 130 131 134 136 137 144 1 4 7 10 16 20 23 31 39 44 49 54 58 60 67 70 74 80 93 98 107 107 112 115 118 120 123 129 132 135 138 145 2 5 8 24 32 40 45 50 55 61 68 75 81 94 99 108 113 116 121 124 133 139 146 26 33 34 41 46 77 83 87 88 89 100 101 104 109 110 147 25 51 56 62 76 82 95 140 27 84 86 96 102 161 36 103 105 150 90 159 52 163 63 153 47 143 85 151 158 160 162 35 141 142 148 149 152 154 155 156 157];

(f) ϕ_T=[0 3 6 9 11 12 13 14 15 17 18 19 21 22 28 29 30 37 38 42 43 48 53 57 59 64 65 66 69 71 72 73 78 79 91 92 97 106 111 114 117 119 122 125 126 127 128 130 131 134 136 137 144 1 4 7 10 16 20 23 31 39 44 49 54 58 60 67 70 74 80 93 98 107 112 115 118 120 123 129 132 135 138 145 2 5 8 24 32 40 45 50 55 61 68 75 81 94 99 108 113 116 121 124 133 139 146 25 26 27 33 34 35 36 41 46 47 51 52 56 62 63 76 77 82 83 84 85 86 87 88 89 90 95 96 100 101 102 103 104 105 109 110 140 141 142 143 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

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(i) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 17 23 37 48 75 80 86 137 143 13 18 38 144 39 145 40 146 41 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(j) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 17 18 23 37 38 39 40 41 48 75 80 86 137 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(k) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 17 23 37 48 75 80 86 137 143 13 18 38 39 40 41 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

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(q) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 18 23 38 39 80 137 145 17 40 75 146 37 41 48 86 143 144 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(r) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 18 23 38 39 80 137 145 17 40 75 146 48 149 37 41 86 143 144 147 148 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

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57 60 61 63 64 77 84 87 90 96 103 104 107 114 116 118 122 129 130 136 137 138 144 8 27 31 34 49 50 65 74 93 94 95 97 100 101 105 121 127 147 13 26 36 54 73 75 89 92 135 146 14 32 80 88 145 1 6 20 22 45 68 69 70 78 83 110 123 133 138 140 141 142 143 148 149 150 151 152 153 154 156 157 158];

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(j) ϕ_T=[139 136 133 130 128 127 126 125 124 122 121 120 118 117 111 110 109 102 101 97 96 91 86 82 80 75 74 73 70 68 67 66 61 60 48 47 42 33 28 25 22 20 17 14 13 12 11 9 8 5 3 2 144 138 135 132 129 123 119 116 108 100 95 90 85 81 79 72 69 65 59 46 41 32 27 24 21 19 16 10 7 4 1 145 137 134 131 115 107 99 94 89 84 78 71 64 58 45 40 31 26 23 18 15 6 0 146 113 106 105 98 93 62 56 52 51 50 39 38 35 30 29 147 114 88 83 77 63 57 44 140 112 55 53 43 37 161 103 36 34 150 49 159 87 163 76 153 92 143 54 151 158 160 162 104 141 142 148 149 152 154 155 156 157];

(k) [2 3 5 8 9 11 12 13 14 17 20 22 25 28 33 42 47 48 60 61 66 67 68 70 73 74 75 80 82 86 91 96 97 101 102 109 110 111 117 118 120 121 122 124 125 126 127 128 130 133 136 139 144 1 4 7 10 16 19 21 24 27 32 41 46 59 65 69 72 79 81 85 90 95 100 108 116 119 123 129 132 135 138 145 0 6 15 18 23 26 31 40 45 58 64 71 78 84 89 94 99 107 115 131 134 137 146 29 30 34 35 36 37 38 39 43 44 49 50 51 52 53 54 55 56 57 62 63 76 77 83 87 88 92 93 98 103 104 105 106 112 113 114 140 141 142 143 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

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(u) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 108 111 113 114 115 119 120 125 130 132 135 137 139 140 4 6 9 11 15 18 23 25 27 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 118 124 129 131 134 136 138 141 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 142 2 53 59 64 91 102 116 122 127 143 98 99 100 101 121 126 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

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(x) ϕ_T=[139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 140 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 141 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 142 127 122 116 102 91 64 59 53 2 143 126 121 101 144 98 99 100 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(y) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 142 2 53 59 64 91 102 116 122 127 143 101 121 126 144 100 145 98 99 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(z) ϕ_T=[139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 140 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 141 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 142 127 122 116 102 91 64 59 53 2 143 126 121 101 144 100 145 98 99 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

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(ii) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 142 2 5 9 100 101 116 121 126 127 145 64 99 122 146 91 149 53 98 102 143 144 147 148 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(jj) ϕ_T=[139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 140 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 141 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 142 127 126 121 116 101 100 59 2 145 122 99 64 146 91 149 53 98 102 143 144 147 148 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(kk) ϕ_T=[0 5 6 7 8 13 14 15 19 20 22 24 26 27 28 30 31 33 37 40 41 48 53 54 57 58 60 63 67 68 72 73 77 80 81 83 84 86 87 88 91 92 97 98 100 101 102 104 106 111 114 115 116 120 121 123 127 128 129 134 135 137 139 155 12 3 9 10 17 21 23 25 32 35 36 43 49 52 55 62 75 76 78 79 82 93 95 96 99 109 110 118 122 124 130 132 136 144 12 18 34 38 39 42 44 45 46 65 74 89 90 105 108 112 131 147 4 47 50 64 66 85 103 113 126 146 51 59 107 125 145 11 70 71 152 61 119 133 138 151 56 117 148 149 16 141 94 140 69 153 154 29 142 143 150 156 157 158];

(ll) ϕ_T=[139 137 135 134 129 128 127 123 121 120 116 115 114 111 106 104 102 101 100 98 97 92 91 88 87 86 84 83 81 80 77 73 72 68 67 63 60 58 57 54 53 4841 40 37 33 31 30 28 27 26 24 22 20 19 15 14 13 8 7 6 5 0 155 136 132 130 124 122 118 110 109 99 96 95 93 82 79 78 76 75 62 55 52 49 43 36 35 32 25 23 21 17 10 9 3 2 1 144 131 112 108 105 90 89 74 65 46 45 44 42 39 38 34 18 12 147 126 113 103 85 66 64 50 47 4 146 125 107 59 51 145 71 70 11 152 138 133 119 61 151 117 56 148 149 16 141 94 140 69 153 154 29 143 142 150 156 157 158];

(mm) ϕ_T=[0 5 6 7 8 13 14 15 19 20 22 24 26 27 28 30 31 33 37 40 41 48 53 54 57 58 60 63 67 68 72 73 77 80 81 83 84 86 87 88 91 92 97 98 100 101 102 104 106 111 114 115 116 120 121 123 127 128 129 134 135 137 139 155 12 3 9 10 17 21 23 25 32 35 36 43 49 52 55 62 75 76 78 79 82 93 95 96 99 109 110 118 122 124 130 132 136 144 12 18 34 38 39 42 44 45 46 65 74 89 90 105 108 112 131 147 4 11 16 29 47 50 51 56 59 61 64 66 69 70 71 85 94 103 107 113 117 119 125 126 133 138 140 141 141 142 143 145 146 148 149 150 151 152 153 154 156 157 158];

(nn) ϕ_T=[139 137 135 134 129 128 127 123 121 120 116 115 114 111 106 104 102 101 100 98 97 92 91 88 87 86 84 83 81 80 77 73 72 68 67 63 60 58 57 54 53 48 41 40 37 33 31 30 28 27 26 24 22 20 19 15 14 13 8 7 6 5 0 155 136 132 130 124 122 118 110 109 99 96 95 93 82 79 78 76 75 62 55 52 49 43 36 35 32 25 23 21 17 10 9 3 2 1 144 131 112 108 105 90 89 74 65 46 45 44 42 39 38 34 18 12 147 4 11 16 29 47 50 51 56 59 61 64 66 69 70 71 85 94 103 107 113 117 119 125 126 133 138 140 141 142 143 145 146 148 149 150 151 152 153 154 156 157 158];

(oo) ϕ_T=[0 5 6 7 8 13 14 15 19 20 22 24 26 27 28 30 31 33 37 40 41 48 53 54 57 58 60 63 67 68 72 73 77 80 81 83 84 86 87 88 91 92 97 98 100 101 102 104 106 111 114 115 116 120 121 123 127 128 129 134 135 137 139 155 1 2 3 9 10 17 21 23 25 32 35 36 43 49 52 55 62 75 76 78 79 82 93 95 96 99 109 110 118 122 124 130 132 136 144 12 18 34 38 39 42 44 45 46 65 74 89 90 105 108 112 131 147 4 47 50 64 66 85 103 113 126 146 11 16 29 51 56 59 61 69 70 71 94 107 117 119 125 133 138 138 140 141 142 143 145 148 149 150 151 152 153 154 156 157 158];

(pp) ϕ_T=[139 137 135 134 129 128 127 123 121 120 116 115 114 111 106 104 102 101 100 98 97 92 91 88 87 86 84 83 81 80 77 73 72 68 67 63 60 58 57 54 53 4841 40 37 33 31 3028 27 26 24 22 20 19 15 14

13 8 7 6 5 0 155 136 132 130 124 122 118 110 109 99 96 95 93 82 79 78 76 75 62 55 52 49 43 36 35 32 25 23 21 17 10 9 3 2 1 144 131 112 108 105 90 89 74 65 46 45 44 42 39 38 34 18 12 147 126 113 103 85 66 64 50 47 4 146 11 16 29 51 56 59 61 69 70 71 94 107 117 119 125 133 138 140 141 142 143 145 148 149 150 151 152 153 154 156 157 158];

(qq) ϕ_T=[0 5 6 78 13 14 15 19 20 22 24 26 27 28 30 31 33 37 40 41 48 53 54 57 58 60 63 67 68 72 73 77 80 81 83 84 86 87 88 91 92 97 98 100 101 102 104 106 111 114 115 116 120 121 123 127 128 129 134 135 137 139 155 12 3 9 10 17 21 23 25 32 35 36 43 49 52 55 62 75 76 78 79 82 93 95 96 99 109 110 118 122 124 130 132 136 144 12 18 34 38 39 42 44 45 46 65 74 89 90 105 108 112 131 147 4 47 50 64 66 85 103 113 126 146 51 59 107 125 145 11 16 29 56 61 69 70 71 94 133 138 140 141 142 143 148 149 150 151 152 153 154 156 157 158];

(rr) ϕ_T=[139 137 135 134 129 128 127 123 121 120 116 115 114 111 106 104 102 101 100 98 97 92 91 88 87 86 84 83 81 80 77 73 72 68 67 63 60 58 57 54 53 4841 40 37 33 31 30 28 27 26 24 22 20 19 15 14 13 8 7 6 5 0 155 136 132 130 124 122 118 110 109 99 96 95 93 82 79 78 76 75 62 55 52 49 43 36 35 32 25 23 21 17 10 9 3 2 1 144 131 112 108 105 90 89 74 65 46 45 44 42 39 38 34 18 12 147 126 113 103 85 66 64 50 47 4 146 125 107 59 51 145 11 16 29 56 61 69 70 71 94 117 119 133 138 140 141 142 143 148 149 150 151 152 153 154 156 157 158];

7. For high-index bit mapper for K_{max}=160

(a) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 164 59 165 60 166 61 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(b) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 103 107 108 109 120 121 124 129 130 167 10 15 45 71 76 82 96 102 115 160 11 47 104 106 116 122 181 56 123 125 170 110 179 72 183 83 173 67 163 105 171 178 180 182 55 161 162 168 169 172 174 175 176 177];

(c) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 37 38 43 57 58 59 60 61 68 95 100 106 157 163 164 165 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(d) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 10 11 15 45 46 47 53 54 55 56 61 66 67 71 72 76 82 83 96 97 102 103 104 105 106 107 108 109 110 115 116 120 121 122 123 124 125 129 130 160 161 162 163 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(e) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 59 60 61 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(f) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 1031 071 08 109 120 121 124 129 130 167 10 11 15 45 47 55 56 67 71 72 76 82 83 96 102 104 105 106 110 115 116 122 123 125 160 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(g) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 164 59 60 61 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(h) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 103 107 108 109 120 121 124 129 130 167 10 15 45 71 76 82 96 102 115 160 11 47 55 56 67 72 83 104 105 106 110 116 122 123 125 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(i) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 164 59 165 60 61 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(j) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 1031 071 08 109 120 121 124 129 130 167 10 15 45 71 76 82 96 102 115 160 11 47 104 106 116 122 181 55 56 67 72 83 105 110 123 125 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 182 183];

(k) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58 59 100 157 165 37 60 95 166 68 169 57 106 163 164 61 167 168 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(l) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 37 38 43 57 58 59 60 61 68 95 100 106 157 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(m) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58 59 100 157 165 37 57 60 61 68 95 106 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(n) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58 59 100 157 165 37 60 95 166 57 61 68 106 163 164 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(o) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58 59 100 157 165 37 60 95 166 68 169 57 61 106 163 164 167 168 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

8. For low-index bit mapper for K_{max}=160

(a) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 161 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 162 2 53 59 64 91 102 116 122 127 146 163 101 121 126 164 100 165 99 166 98 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(b) ϕ_T=[2 3 5 8 9 11 12 13 14 17 20 22 25 28 33 42 47 48 60 61 66 67 68 70 73 74 75 80 82 86 91 96 97 101 102 109 110 111 117 118 120 121 122 124 125 126 127 128 130 133 136 139 141 143 147 152 154 155 156 159 164 1 4 7 10 16 19 21 24 27 32 41 46 59 65 69 72 79 81 85 90 95 100 108 116 119 123 129 132 135 138 140 142 146 151 153 158 165 0 6 15 18 23 26 31 40 45 58 64 71 78 84 89 94 99 107 115 131 134 137 145 150 157 166 29 30 35 38 39 50 51 52 56 62 93 98 105 106 113 167 44 57 63 77 83 88 114 144 149 160 37 43 53 55 112 148 181 34 36 103 170 49 179 87 183 76 173 92 163 54 171 178 180 182 104 161 162 168 169 172 174 175 176 177];

(c) ϕ_T=[159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 160 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 161 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 162 146 127 122 116 102 91 64 59 53 2 163 126 121 101 164 100 165 99 166 98 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(d) ϕ_T=[159 156 155 154 152 147 143 141 139 136 133 130 128 127 126 125 124 122 121 120 118 117 111 110 109 102 101 97 96 91 86 82 80 75 74 73 70 68

67 66 61 60 48 47 42 33 28 25 22 20 17 14 13 12 11 9 8 5 3 2 164 158 153 151 146 142 140 138 135 132 129 123 119 116 108 100 95 90 85 81 79 72 69 65 59 46 41 32 27 24 21 19 16 107 4 1 165 157 150 145 137 134 131 115 107 99 94 89 84 78 71 64 58 58 45 40 31 26 23 18 15 6 0 166 113 106 105 98 93 62 56 52 51 50 39 38 35 30 29 167 149 144 114 88 83 77 63 57 44 160 148 112 55 53 43 37 181 103 36 34 170 49 179 87 183 76 173 92 163 54 171 178 180 182 104 161 162 168 169 172 174 175 176 177];

(e) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 161 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 162 2 53 59 64 91 98 99 100 101 102 116 121 122 126 127 146 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(f) ϕ_T=[2 3 5 8 9 11 12 13 14 17 20 22 25 28 33 42 47 48 60 61 66 67 68 70 73 74 75 80 82 86 91 96 97 101 102 109 110 111 117 118 120 121 122 124 125 126 127 128 130 133 136 139 141 143 147 152 154 155 156 159 164 1 4 7 10 16 19 21 24 27 32 41 46 59 65 69 72 79 81 85 90 95 100 108 116 119 123 129 132 135 138 140 142 146 151 153 158 165 0 6 15 18 23 26 31 40 45 58 64 71 78 84 89 94 99 107 115 131 134 137 145 150 157 166 29 30 34 35 36 37 38 39 43 44 49 50 51 52 53 54 55 56 57 62 63 76 77 83 87 88 92 93 98 103 104 105 106 112 113 114 144 148 149 160 161 162 163 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(g) ϕ_T=[159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 160 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 161 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 162 2 53 59 64 91 98 99 100 101 102 116 121 122 126 127 146 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(h) ϕ_T=[159 156 155 154 152 147 143 141 139 136 133 130 128 127 126 125 124 122 121 120 118 117 111 110 109 102 101 97 96 91 86 82 80 75 74 73 70 68 67 66 61 60 48 47 42 33 28 25 22 20 17 14 13 12 11 9 8 5 3 2 164 158 153 151 146 142 140 138 135 132 129 123 119 116 108 100 95 90 85 81 79 72 69 65 59 46 41 32 27 24 21 19 16 107 4 1 165 157 150 145 137 134 131 115 107 99 94 89 84 78 71 64 58 45 40 31 26 23 18 15 6 0 166 29 30 34 35 36 37 38 39 43 44 49 50 51 52 53 54 55 56 57 62 63 7677 83 87 88 92 93 98 103 104 105 106 112 113 114 144 148 149 160 161 162 163 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(i) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 161 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 162 2 53 59 64 91 102 116 122 127 146 163 98 99 100 101 121 126 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(j) ϕ_T=[2 3 5 8 9 11 12 13 14 17 20 22 25 28 33 42 47 48 60 61 66 67 68 70 73 74 75 80 82 86 91 96 97 101 102 109 110 111 117 118 120 121 122 124 125 126 127 128 130 133 136 139 141 143 147 152 154 155 156 159 164 1 4 7 10 16 19 21 24 27 32 41 46 59 65 69 72 79 81 85 90 95 100 108 116 119 123 129 132 135 138 140 142 146 151 153 158 165 0 6 15 18 23 26 31 40 45 58 64 71 78 84 89 94 99 107 115 131 134 137 145 150 157 166 29 30 35 38 39 50 51 52 56 62 93 98 105 106 113 167 34 36 37 43 44 49 53 54 55 57 63 76 77 83 87 88 92 103 104 112 114 144 148 149 160 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(k) ϕ_T=[159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 160 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 161 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 162 146 127 122 116 102 91 64 59 53 2 163 98 99 100 101 121 126 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(l) ϕ_T=[159 156 155 154 152 147 143 141 139 136 133 130 128 127 126 125 124 122 121 120 118 117 111 110 109 102 101 97 96 91 86 82 80 75 74 73 70 68 67 66 61 60 48 47 42 33 28 25 22 20 17 14 13 12 11 9 8 5 3 2 164 158 153 151 146 142 140 138 135 132 129 123 119 116 108 100 95 90 85 81 79 72 69 65 59 46 41 32 27 24 21 19 16 107 4 1 165 157 150 145 137 134 131 115 107 99 94 89 84 78 71 64 58 45 40 31 26 23 18 15 6 0 166 113 106 105 98 93 62 56 52 51 50 39 38 35 30 29 167 34 36 37 43 44 49 53 54 55 57 63 76 77 83 87 88 92 103 104 112 114 144 148 149 160 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(m) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 161 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 162 2 53 59 64 91 102 116 122 127 146 163 101 121 126 164 98 99 100 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(n) ϕ_T=[2 3 5 8 9 11 12 13 14 17 20 22 25 28 33 42 47 48 60 61 66 67 68 70 73 74 75 80 82 86 91 96 97 101 102 109 110 111 117 118 120 121 122 124 125 126 127 128 130 133 136 139 141 143 147 152 154 155 156 159 164 1 4 7 10 16 19 21 24 27 32 41 46 59 65 69 72 79 81 85 90 95 100 108 116 119 123 129 132 135 138 140 142 146 151 153 158 165 0 6 15 18 23

26 31 40 45 58 64 71 78 84 89 94 99 107 115 131 134 137 145 150 157 166 29 30 35 38 39 50 51 52 56 62 93 98 105 106 113 167 44 57 63 77 83 88 114 144 149 160 34 36 37 43 49 53 54 55 76 87 92 103 104 112 148 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(o) ϕ_T=[159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 160 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 161 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 162 146 127 122 116 102 91 64 59 53 2 163 126 121 101 164 98 99 100 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(p) ϕ_T=[159 156 155 154 152 147 143 141 139 136 133 130 128 127 126 125 124 122 121 120 118 117 111 110 109 102 101 97 96 91 86 82 80 75 74 73 70 68 67 66 61 60 48 47 42 33 28 25 22 20 17 14 13 12 11 9 8 5 3 2 164 158 153 151 146 142 140 138 135 132 129 123 119 116 108 100 95 90 85 81 79 72 69 65 59 46 41 32 27 24 21 19 16 107 4 1 165 157 150 145 137 134 131 115 107 99 94 89 84 78 71 64 58 45 40 31 26 23 18 15 6 0 166 113 106 105 98 93 62 56 52 51 50 39 38 35 30 29 167 149 144 114 88 83 77 63 57 44 160 34 36 37 43 49 53 54 55 76 87 92 103 104 112 148 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(q) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 161 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 162 2 53 59 64 91 102 116 122 127 146 163 101 121 126 164 100 165 98 99 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(r) ϕ_T=[2 3 5 8 9 11 12 13 14 17 20 22 25 28 33 42 47 48 60 61 66 67 68 70 73 74 75 80 82 86 91 96 97 101 102 109 110 111 117 118 120 121 122 124 125 126 127 128 130 133 136 139 141 143 147 152 154 155 156 159 164 1 4 7 10 16 19 21 24 27 32 41 46 59 65 69 72 79 81 85 90 95 100 108 116 119 123 129 132 135 138 140 142 146 151 153 158 165 0 6 15 18 23 26 31 40 45 58 64 71 78 84 89 94 99 107 115 131 134 137 145 150 157 166 29 30 35 38 39 50 51 52 56 62 93 98 105 106 113 167 44 57 63 77 83 88 114 144 149 160 37 43 53 55 112 148 181 34 36 49 54 76 87 87 92 103 104 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 181 182 183];

(s) ϕ_T=[159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 160 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 161 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 162 146 127 122 116 102 91 64 59 53 2 163 126 121 101 164 100 165 98 99 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(t) ϕ_T=[159 156 155 154 152 147 143 141 139 136 133 130 128 127 126 125 124 122 121 120 118 117 111 110 109 102 101 97 96 91 86 82 80 75 74 73 70 68 67 66 61 60 48 47 42 33 28 25 22 20 17 14 13 12 11 9 8 5 3 2 164 158 153 151 146 142 140 138 135 132 129 123 119 116 108 100 95 90 85 81 79 72 69 65 59 46 41 32 27 24 21 19 16 107 4 1 165 157 150 145 137 134 131 115 107 99 94 89 84 78 71 64 58 45 40 31 26 23 18 15 6 0 166 113 106 105 98 93 62 56 52 51 50 39 38 35 30 29 167 149 144 114 88 83 77 63 57 44 160 148 112 55 53 43 37 181 34 36 49 54 76 87 92 103 104 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 182 183];

(u) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 161 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 162 2 59 100 101 116 121 126 127 146 165 64 99 122 166 91 169 53 102 163 164 98 167 168 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(v) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 161 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 162 2 59 100 101 116 121 126 127 146 165 64 99 122 166 91 169 53 102 163 164 98 167 168 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(w) ϕ_T=[159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 160 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 161 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 162 146 127 126 121 116 101 100 59 2 165 122 99 64 166 91 169 102 53 163 164 98 167 168 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(x) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 161 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 162 2 53 59 64 91 98 99 100 101 102 116 121 122 126

127 146 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(y) ϕ_T=[159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 160 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 161 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 162 2 53 59 64 91 98 99 100 101 102 116 121 122 126 127 146 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(z) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 161 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 162 2 59 100 101 116 121 126 127 146 165 53 64 91 98 99 102 122 163 164 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(aa) ϕ_T=[159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 160 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 161 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 162 146 127 126 121 116 101 100 59 2 165 53 64 91 98 99 102 122 163 164 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(bb) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 4 6 9 11 15 18 23 25 27 30 32 34 37 4043 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 161 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 162 2 59 100 101 116 121 126 127 146 165 64 99 122 166 53 91 98 102 163 164 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(cc) ϕ_T=[159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 160 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 161 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 162 146 127 126 121 116 101 100 59 2 165 122 99 64 166 53 91 98 102 163 164 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(dd) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 4 6 9 11 15 18 23 25 27 30 32 34 37 4043 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 161 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 162 2 59 100 101 116 121 126 127 146 165 64 99 122 166 91 169 53 98 102 163 164 167 168 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(ee) ϕ_T=[159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 160 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 161 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 162 146 127 126 121 116 101 100 59 2 165 122 99 64 166 91 169 53 98 102 163 164 167 168 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

9. For high-index bit mapper for K_{max}=200

(a) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 7475 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83 91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 8 15 86 93 94 101 106 137 143 147 148 149 160 161 164 169 170 207 6 19 50 55 85 111 116 122 136 142 155 200 87 95 107 144 150 162 165 208 51 156 214 7 146 221 163 219 96 210 145 202 212 112 211 216 223 123 201 203 209 213 215 217 218 220 222];

(b) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 74 75 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83 91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 6 7 8 15 19 50 51 55 85 86 87 93 94 95 96 101 106 107 111 112 116 122 123 136 137 142 143 144 145 146 147 148 149 150 155 156 160 161 162 163 164 165 169 170 200 201 202 203 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(c) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 74 75 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83

91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 8 15 86 93 94 101 106 137 143 147 148 149 160 161 164 169 170 207 6 7 19 50 51 55 85 87 92 96 107 111 112 116 122 123 136 142 144 145 146 150 155 156 162 163 165 200 201 202 203 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(d) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 7475 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83 91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 8 15 86 93 94 101 106 137 143 147 148 149 160 161 164 169 170 207 6 19 50 55 85 111 116 122 136 142 155 200 7 51 87 95 96 107 112 123 144 145 146 150 156 162 163 165 201 202 203 208 209 210 211 212 213 214 215 216 217 218 219 120 221 222 223];

(e) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 49 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 204 99 205 100 206 101 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(f) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 73 77 78 83 97 98 99 100 101 108 135 140 146 197 197 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(g) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 99 100 101 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(h) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 204 99 100 101 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(i) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 7176 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 204 99 205 100 101 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(j) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 100 135 206 108 209 97 146 203 204 101 207 208 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(k) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200

1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 73 77 78 83 97 98 99 100 101 108 135 140 146 197 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(l) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 97 100 101 108 135 146 203 204 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(m) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 100 135 206 97 101 108 146 203 204 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(n) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 100 135 206 108 209 97 101 146 203 204 207 208 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(o) ϕ_T=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173 175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 33 44 87 91 94 134 153 154 155 157 165 207 53 61 66 80 138 195 211 82 96 105 140 143 170 218 6 39 86 92 208 15 22 183 200 49 130 210 149 203 48 202 59 201 209 214 133 205 206 213 216 217];

(p) ϕ_T=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173 175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 6 15 22 33 39 44 48 49 53 59 61 66 80 82 86 87 91 92 94 96 105 130 133 134 138 140 143 149 153 154 155 157 165 170 183 195 200 201 202 203 205 206 207 208 209 210 211 213 214 216 217 218];

(q) ϕ_T=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173 175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 33 44 87 91 94 134 153 154 155 157 165 207 6 15 22 39 48 49 53 59 61 66 80 82 86 92 96 105 130 133 138 140 143 149 170 183 195 200 201 202 203 205 206 208 209 210 211 213 214 216 217];

(r) ϕ_T=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173 175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 33 44 87 91 94 134 153 154 155 157 165 207 53 61 66 80 138 195 211 6 15 22 39 48 49 59 82 86 92 96 105 130 133 140 143 149 170 183 200 201 202 203 205 206 208 209 210 213 214 216 217];

10. For low-index bit mapper for K_{max}=200

(a) ϕ_T=[2 3 5 8 9 11 12 13 14 17 20 22 25 28 33 42 47 48 60 61 66 67 68 70 73 74 75 80 82 86 91 96 97 101 102 109 110 111 117 118 120 121 122 124 125 126 127 128 130 133 136 139 141 143 147 152 154 155 156 159 160 161 163 165 168 170 172 173 176 178 179 183 187 189 190 196 197 198 199 204 1 47 10 16 1921 24 27 32 41 46 59 65 69 72 79 81 85 90 95 100 108 116 119 123 129 132 135 138 140 142 146 151 153 158 162 164 167 169 171 175 177 182 186 188 195 205 0 6 15 18 23 26 31 40 45 58 64 71 78 84 89 94 99 107 115 131 134 137 145 150 157 166 174 181 185 194 206 29 30 35 38 39 50 51 52 56 62

93 98 105 106 113 184 191 207 44 57 63 77 83 88 114 144 149 180 193 200 34 37 49 55 92 104 112 208 43 148 214 53 192 221 36 219 103 210 54 202 212 87 211 216 223 76 201 203 209 213 215 217 218 220 222];

(b) ϕ_T=[199 198 197 196 190 189 187 183 179 178 176 173 172 170 168 165 163 161 160 159 156 155 154 152 147 143 141 139 136 133 130 128 127 126 125 124 122 121 120 118 117 111 110 109 102 101 97 96 91 86 82 80 75 74 73 70 68 67 66 61 60 48 47 42 33 28 25 22 20 17 14 13 12 11 9 8 5 3 2 204 195 188 186 182 177 175 171 169 167 164 162 158 153 151 146 142 140 138 135 132 129 123 119 116 108 100 95 90 85 81 79 72 69 65 59 46 41 32 27 24 21 19 16 10 7 4 1 205 194 185 181 174 166 157 150 145 137 134 131 115 107 99 94 89 84 78 71 64 58 45 40 31 26 23 18 15 6 0 206 191 184 113 106 105 98 93 62 56 52 51 50 39 38 35 30 29 207 193 180 149 144 114 88 83 77 63 57 44 200 112 104 92 55 49 37 34 208 148 43 214 192 53 221 36 219 103 210 54 212 202 87 211 216 223 76 215 213 217 218 201 220 203 222 209];

(c) ϕ_T=[2 3 5 8 9 11 12 13 14 17 20 22 25 28 33 42 47 48 60 61 66 67 68 70 73 74 75 80 82 86 91 96 97 101 102 109 110 111 117 118 120 121 122 124 125 126 127 128 130 133 136 139 141 143 147 152 154 155 156 159 160 161 163 165 168 170 172 173 176 178 179 183 187 189 190 196 197 198 199 204 1 47 10 16 1921 24 27 32 41 46 59 65 69 72 79 81 85 90 95 100 108 116 119 123 129 132 135 138 140 142 146 151 153 158 162 164 167 169 171 175 177 182 186 188 195 205 0 6 15 18 23 26 31 40 45 58 64 71 78 84 89 94 99 107 115 131 134 137 145 150 157 166 174 181 185 194 206 29 30 34 35 36 37 38 39 43 44 49 50 51 52 53 54 55 56 57 62 63 76 77 83 87 88 92 93 98 103 104 105 106 112 113 114 144 148 149 180 184 191 192 193 200 201 202 203 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(d) ϕ_T=[199 198 197 196 190 189 187 183 179 178 176 173 172 170 168 165 163 161 160 159 156 155 154 152 147 143 141 139 136 133 130 128 127 126 125 124 122 121 120 118 117 111 110 109 102 101 97 96 91 86 82 80 75 74 73 70 68 67 66 61 60 48 47 42 33 28 25 22 20 17 14 13 12 11 9 8 5 3 2 204 195 188 186 182 177 175 171 169 167 164 162 158 153 151 146 142 140 138 135 132 129 123 119 116 108 100 95 90 85 81 79 72 69 65 59 46 41 32 27 24 21 19 16 10 7 4 1 205 194 185 181 174 166 157 150 145 137 134 131 115 107 99 94 89 84 78 71 64 58 45 40 31 26 23 18 15 6 0 206 29 30 34 35 36 37 38 39 43 44 49 50 51 52 53 54 55 56 57 62 63 76 77 83 87 88 92 93 98 103 104 105 106 112 113 114 144 148 149 180 184 191 192 193 200 201 202 203 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(e) ϕ_T=[2 3 5 8 9 11 12 13 14 17 20 22 25 28 33 42 47 48 60 61 66 67 68 70 73 74 75 80 82 86 91 96 97 101 102 109 110 111 117 118 120 121 122 124 125 126 127 128 130 133 136 139 141 143 147 152 154 155 156 159 160 161 163 165 168 170 172 173 176 178 179 183 187 189 190 196 197 198 199 204 1 47 10 16 1921 24 27 32 41 46 59 65 69 72 79 81 85 90 95 100 108 116 119 123 129 132 135 138 140 142 146 151 153 158 162 164 167 169 171 175 177 182 186 188 195 205 0 6 15 18 23 26 31 40 45 58 64 71 78 84 89 94 99 107 115 131 134 137 145 150 157 166 174 181 185 194 206 29 30 35 38 39 50 51 52 56 62 93 98 105 106 113 184 191 207 34 36 37 43 44 49 53 54 55 57 63 76 77 83 87 88 92 103 104 112 114 144 148 149 180 192 193 200 201 202 203 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(f) ϕ_T=[199 198 197 196 190 189 187 183 179 178 176 173 172 170 168 165 163 161 160 159 156 155 154 152 147 143 141 139 136 133 130 128 127 126 125 124 122 121 120 118 117 111 110 109 102 101 97 96 91 86 82 80 75 74 73 70 68 67 66 61 60 48 47 42 33 28 25 22 20 17 14 13 12 11 9 8 5 3 2 204 195 188 186 182 177 175 171 169 167 164 162 158 153 151 146 142 140 138 135 132 129 123 119 116 108 100 95 90 85 81 79 72 69 65 59 46 41 32 27 24 21 19 16 10 7 4 1 205 194 185 181 174 166 157 150 145 137 134 131 115 107 99 94 89 84 78 71 64 58 45 40 31 26 23 18 15 6 0 206 191 184 113 106 105 98 93 62 56 52 51 50 39 38 35 30 29 207 34 36 37 43 44 4953 54 55 57 63 76 77 83 87 88 92 103 104 112 114 144 148 149 180 192 193 200 201 202 203 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(g) ϕ_T=[2 3 5 8 9 11 12 13 14 17 20 22 25 28 33 42 47 48 60 61 66 67 68 70 73 74 75 80 82 86 91 96 97 101 102 109 110 111 117 118 120 121 122 124 125 126 127 128 130 133 136 139 141 143 147 152 154 155 156 159 160 161 163 165 168 170 172 173 176 178 179 183 187 189 190 196 197 198 199 204 1 47 10 16 1921 24 27 32 41 46 59 65 69 72 79 81 85 90 95 100 108 116 119 123 129 132 135 138 140 142 146 151 153 158 162 164 167 169 171 175 177 182 186 188 195 205 0 6 15 18 23 26 31 40 45 58 64 71 78 84 89 94 99 107 115 131 134 137 145 150 157 166 174 181 185 194 206 29 30 35 38 39 50 51 52 56 62 93 98 105 106 113 184 191 207 44 57 63 77 83 88 114 144 149 180 193 200 34 36 37 43 49 53 54 55 76 87 92 103 104 112 148 192 201 202 203 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(h) ϕ_T=[199 198 197 196 190 189 187 183 179 178 176 173 172 170 168 165 163 161 160 159 156 155 154 152 147 143 141 139 136 133 130 128 127 126 125 124 122 121 120 118 117 111 110 109 102 101 97 96 91 86 82 80 75 74 73 70 68 67 66 61 60 48 47 42 33 28 25 22 20 17 14 13 12 11 9 8 5 3 2 204 195 188 186 182 177 175 171 169 167 164 162 158 153 151 146 142 140 138 135 132 129 123 119 116 108 100 95 90 85 81 79 72 69 65 59 46 41 32 27 24 21 19 16 10 7 4 1 205 194 185 181 174 166 157 150 145 137 134 131 115 107 99 94 89 84 78 71 64 58 45 40 31 26 23 18 15 6 0 206 191 184 113 106 105 98 93 62 56 52 51 50 39 38 35 30 29 207 193 180 149 144 114 88 83 77 63 57 44 200 34 36 37 43 49 53 54 55 76 87 92 103 104 112 148 192 201 202 203 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(i) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 161 162 164 166 167 171 175 177 179 180 183 186 187 188 191 193 194 196 197 199 200 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151

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(j) ϕ_T=[199 197 196 194 193 191 188 187 186 183 180 179 177 175 171 167 166 164 162 161 160 159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 200 198 195 192 190 185 182 178 176 174 170 165 163 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 201 189 184 181 173 169 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 202 172 168 146 127 122 116 102 91 64 59 53 2 203 126 121 126 204 100 205 99 206 98 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(k) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 161 162 164 166 167 171 175 177 179 180 183 186 187 188 191 193 194 196 197 199 200 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 163 165 170 174 176 178 182 185 190 192 195 198 201 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 169 173 181 184 189 202 2 53 59 64 91 98 99 100 101 102 116 121 122 126 127 146 168 172 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(l) ϕ_T=[199 197 196 194 193 191 188 187 186 183 180 179 177 175 171 167 166 164 162 161 160 159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 200 198 195 192 190 185 182 178 176 174 170 165 163 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 201 189 184 181 173 169 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 202 2 53 59 64 91 98 99 100 101 102 116 121 122 126 127 146 168 172 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(m) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 161 162 164 166 167 171 175 177 179 180 183 186 187 188 191 193 194 196 197 199 200 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 163 165 170 174 176 178 182 185 190 192 195 198 201 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 169 173 181 184 189 202 2 53 59 64 91 102 116 122 127 146 168 172 203 98 99 100 101 121 126 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(n) ϕ_T=[199 197 196 194 193 191 188 187 186 183 180 179 177 175 171 167 166 164 162 161 160 159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 200 198 195 192 190 185 182 178 176 174 170 165 163 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 201 189 184 181 173 169 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 202 172 168 146 127 122 116 102 91 64 59 53 2 203 98 99 100 101 121 126 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(o) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 161 162 164 166 167 171 175 177 179 180 183 186 187 188 191 193 194 196 197 199 200 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 163 165 170 174 176 178 182 185 190 192 195 198 201 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 169 173 181 184 189 202 2 53 59 64 91 102 116 122 127 146 168 172 203 101 121 126 204 98 99 100 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(p) ϕ_T=[199 197 196 194 193 191 188 187 186 183 180 179 177 175 171 167 166 164 162 161 160 159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 200 198 195 192 190 185 182 178 176 174 170 165 163 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 201 189 184 181 173 169 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 202 172 168 146 127 122 116 102 91 64 59 53 2 203 126 121 126 204 98 99 100 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(q) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 161 162 164 166 167 171 175 177 179 180 183 186 187 188 191 193 194 196 197 199 200 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45

47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 163 165 170 174 176 178 182 185 190 192 195 198 201 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 169 173 181 184 189 202 2 53 59 64 91 102 116 122 127 146 168 172 203 101 121 126 204 100 205 98 99 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(r) ϕ_T=[199 197 196 194 193 191 188 187 186 183 180 179 177 175 171 167 166 164 162 161 160 159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 200 198 195 192 190 185 182 178 176 174 170 165 163 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 201 189 184 181 173 169 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 202 172 168 146 127 122 116 102 91 64 59 53 2 203 126 121 126 204 100 205 98 99 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(s) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 161 162 164 166 167 171 175 177 179 180 183 186 187 188 191 193 194 196 197 199 200 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 163 165 170 174 176 178 182 185 190 192 195 198 201 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 169 173 181 184 189 202 2 59 100 101 116 121 126 127 146 172 205 64 99 122 168 206 91 209 53 102 203 204 98 207 208 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(t) ϕ_T=[199 197 196 194 193 191 188 187 186 183 180 179 177 175 171 167 166 164 162 161 160 159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 200 198 195 192 190 185 182 178 176 174 170 165 163 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 201 189 184 181 173 169 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 202 172 146 127 126 121 116 101 100 59 2 205 168 122 99 64 206 91 209 102 53 203 204 98 207 208 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(u) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 161 162 164 166 167 171 175 177 179 180 183 186 187 188 191 193 194 196 197 199 200 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 163 165 170 174 176 178 182 185 190 192 195 198 201 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 169 173 181 184 189 202 2 53 59 64 91 98 99 100 101 102 116 121 122 126 127 146 168 172 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(v) ϕ_T=[199 197 196 194 193 191 188 187 186 183 180 179 177 175 171 167 166 164 162 161 160 159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 200 198 195 192 190 185 182 178 176 174 170 165 163 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 201 189 184 181 173 169 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 202 2 53 59 64 91 98 99 100 101 102 116 121 122 126 127 146 168 172 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(w) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 161 162 164 166 167 171 175 177 179 180 183 186 187 188 191 193 194 196 197 199 200 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 163 165 170 174 176 178 182 185 190 192 195 198 201 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 169 173 181 184 189 202 2 59 100 101 116 121 126 127 146 172 205 53 64 91 98 99 102 122 168 203 204 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(x) ϕ_T=[199 197 196 194 193 191 188 187 186 183 180 179 177 175 171 167 166 164 162 161 160 159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 200 198 195 192 190 185 182 178 176 174 170 165 163 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 201 189 184 181 173 169 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 202 172 146 127 126 121 116 101 100 592 205 53 64 91 98 99 102 122 168 203 204 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(y) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 161 162 164 166 167 171 175 177

179 180 183 186 187 188 191 193 194 196 197 199 200 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 163 165 170 174 176 178 182 185 190 192 195 198 201 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 169 173 181 184 189 202 2 59 100 101 116 121 126 127 146 172 205 64 99 122 168 206 53 91 98 102 203 204 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(z) ϕ_T=[199 197 196 194 193 191 188 187 186 183 180 179 177 175 171 167 166 164 162 161 160 159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 200 198 195 192 190 185 182 178 176 174 170 165 163 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 201 189 184 181 173 169 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 202 172 146 127 126 121 116 101 100 59 2 205 168 122 99 64 206 53 91 98 102 203 204 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(aa) ϕ_T=[0 1 5 7 10 12 13 16 17 19 20 21 24 26 28 29 31 33 35 38 41 44 46 48 50 51 52 56 57 58 62 63 67 68 69 70 72 73 74 77 78 80 81 83 85 86 88 89 90 94 97 105 108 111 113 114 115 119 120 125 130 132 135 137 139 140 142 144 145 149 150 152 153 155 157 158 159 160 161 162 164 166 167 171 175 177 179 180 183 186 187 188 191 193 194 196 197 199 200 4 6 9 11 15 18 23 25 27 30 32 34 37 40 43 45 47 49 55 61 66 71 76 79 82 84 87 93 96 104 107 110 112 118 124 129 131 134 136 138 141 143 148 151 154 156 163 165 170 174 176 178 182 185 190 192 195 198 201 3 8 14 22 36 39 42 54 60 65 75 92 95 103 106 109 117 123 128 133 147 169 173 181 184 189 202 2 59 100 101 116 121 126 127 146 172 205 64 99 122 168 206 91 209 53 98 102 203 204 207 208 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(bb) ϕ_T=[199 197 196 194 193 191 188 187 186 183 180 179 177 175 171 167 166 164 162 161 160 159 158 157 155 153 152 150 149 145 144 142 140 139 137 135 132 130 125 120 119 115 114 113 111 108 105 97 94 90 89 88 86 85 83 81 80 78 77 74 73 72 70 69 68 67 63 62 58 57 56 52 51 50 48 46 44 41 38 35 33 31 29 28 26 24 21 20 19 17 16 13 12 10 7 5 1 0 200 198 195 192 190 185 182 178 176 174 170 165 163 156 154 151 148 143 141 138 136 134 131 129 124 118 112 110 107 104 96 93 87 84 82 79 76 71 66 61 55 49 47 45 43 40 37 34 32 30 27 25 23 18 15 11 9 6 4 201 189 184 181 173 169 147 133 128 123 117 109 106 103 95 92 75 65 60 54 42 39 36 22 14 8 3 202 172 146 127 126 121 116 101 100 59 2 205 168 122 99 64 206 91 209 53 98 102 203 204 207 208 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(cc) ϕ_T=[2 3 5 7 8 10 11 12 18 19 20 21 22 25 27 28 33 35 36 37 38 39 41 43 47 51 55 55 57 58 62 64 67 70 71 74 76 77 78 79 81 83 85 89 90 91 93 95 96 97 100 101 102 109 110 114 116 120 121 123 125 126 127 128 129 130 131 135 136 142 145 147 152 157 158 165 168 171 173 175 178 180 181 182 187 188 190 194 195 198 199 212 0 6 13 14 15 24 26 30 31 40 48 53 54 60 63 68 72 73 80 84 86 87 88 92 98 104 106 111 115 134 137 139 141 148 149 153 154 161 163 164 167 172 174 179 186 191 192 197 215 1 9 17 23 32 49 52 75 82 99 118 122 124 132 143 144 156 159 162 169 170 176 183 185 189 196 204 34 42 44 45 46 65 105 108 112 155 166 207 4 61 119 133 138 146 211 29 56 59 94 103 117 218 107 113 160 193 208 16 177 184 200 69 150 210 50 203 151 202 140 201 209 214 66 205 206 213 216 217];

(dd) ϕ_T=[199 198 195 194 190 188 187 182 181 180 178 175 173 171 168 165 158 157 152 147 145 142 136 135 131 130 129 128 127 126 125 123 121 120 116 114 110 109 102 101 100 97 96 95 93 91 90 89 85 83 81 79 78 77 76 74 71 70 67 64 62 58 57 55 51 47 43 41 39 38 37 36 35 33 28 27 25 22 21 20 19 18 12 11 10 8 7 5 3 2 212 197 192 191 186 179 174 172 167 164 163 161 154 153 149 148 141 139 137 134 115 111 106 104 98 92 88 87 86 84 80 73 72 68 63 60 54 53 48 40 31 30 26 24 15 14 13 6 0 215 196 189 185 183 176 170 169 162 159 156 144 143 132 124 122 118 99 82 75 52 49 32 23 17 9 1 204 166 155 112 108 105 65 46 45 44 42 34 207 146 138 133 119 61 4 211 117 103 94 59 56 29 218 193 160 113 107 208 184 177 16 200 150 69 210 50 203 151 202 140 209 214 201 66 206 213 216 217 205];

(ee) ϕ_T=[2 3 5 7 8 10 11 12 18 19 20 21 22 25 27 28 33 35 36 37 38 39 41 434 7 51 55 55 57 58 62 64 67 70 71 74 76 77 78 79 81 83 85 89 90 91 93 95 96 97 100 101 102 109 110 114 116 120 121 123 125 126 127 128 129 130 131 135 136 142 145 147 152 157 158 165 168 171 173 175 178 180 181 182 187 188 190 194 195 198 199 212 0 6 13 14 15 24 26 30 31 40 48 53 54 60 63 68 72 73 80 84 86 87 88 92 98 104 106 111 115 134 137 139 141 148 149 153 154 161 163 164 167 172 174 179 186 191 192 197 215 1 9 17 23 32 49 52 75 82 99 118 122 124 132 143 144 156 159 162 169 170 176 183 185 189 196 204 4 16 29 34 42 44 45 46 50 56 59 61 65 66 69 97 103 105 107 108 112 113 117 119 133 138 140 146 150 151 155 160 166 177 184 193 200 201 202 203 205 206 207 208 209 210 211 213 214 216 217 218];

(ff) ϕ_T=[199 198 195 194 190 188 187 182 181 180 178 175 173 171 168 165 158 157 152 147 145 142 136 135 131 130 129 128 127 126 125 123 121 120 116 114 110 109 102 101 100 97 96 95 93 91 90 89 85 83 81 79 78 77 76 74 71 70 67 64 62 58 57 55 51 47 43 41 39 38 37 36 35 33 28 27 25 22 21 20 19 18 12 11 10 8 7 5 3 2 212 197 192 191 186 179 174 172 167 164 163 161 154 153 149 148 141 139 137 134 115 111 106 104 98 92 88 87 86 84 80 73 72 68 63 60 54 53 48 40 31 30 26 24 15 14 13 6 0 215 196 189 185 183 176 170 169 162 159 156 144 143 132 124 122 118 99 82 75 52 49 32 23 17 9 1 204 4 16 29 34 42 44 45 46 50 56 59 61 65 66 69 94 103 105 107 108 112 113 117 119 133 138 140 146 150 151 155 160 166 177 184 193 200 201 202 203 205 206 207 208 209 210 211 213 214 216 217 218];

(gg) ϕ_T=[2 3 5 7 8 10 11 12 18 19 20 21 22 25 27 28 33 35 36 37 38 39 41 43 47 51 55 55 57 58 62 64 67 70 71 74 76 77 78 79 81 83 85 89 90 91 93 95 96 97 100 101 102 109 110 114 116 120 121 123 125 126 127 128 129 130 131 135 136 142 145 147 152 157 158 165 168 171 173 175 178 180 181 182 187 188 190 194 195 198 199 212 0 6 13 14 15 24 26 30 31 40 48 53 54 60 63 68 72 73 80 84 86 87 88 92 98 104

106 111 115 134 137 139 141 148 149 153 154 161 163 164 167 172 174 179 186 191 192 197 215 1 9 17 23 32 49 52 75 82 99 118 122 124 132 143 144 156 159 162 169 170 176 183 185 189 196 204 34 42 44 45 46 65 105 108 112 155 166 207 4 16 29 50 56 59 61 66 69 94 103 107 113 117 119 133 138 140 146 150 151 151 160 177 184 193 200 201 202 203 205 206 208 209 210 211 213 214 216 217 218];

(hh) ϕ_T=[199 198 195 194 190 188 187 182 181 180 178 175 173 171 168 165 158 157 152 147 145 142 136 135 131 130 129 128 127 126 125 123 121 120 116 114 110 109 102 101 100 97 96 95 93 91 90 89 85 83 81 79 78 77 76 74 71 70 67 64 62 58 57 55 51 47 43 41 39 38 37 36 35 33 28 27 25 22 21 20 19 18 12 11 10 8 7 5 3 2 212 197 192 191 186 179 174 172 167 164 163 161 154 153 149 148 141 139 137 134 115 111 106 104 98 92 88 87 86 84 80 73 72 68 63 60 54 53 48 40 31 30 26 24 15 14 13 6 0 215 196 189 185 183 176 170 169 162 159 156 144 143 132 124 122 118 99 82 75 52 49 32 23 17 9 1 204 166 155 112 108 105 65 46 45 44 42 34 207 4 16 29 50 56 59 61 66 69 94 103 107 113 117 119 133 138 140 146 150 151 160 177 184 193 200 201 202 203 205 206 208 209 210 211 213 214 216 217 218];

(ii) ϕ_T=[2 3 5 7 8 10 11 12 18 19 20 21 22 25 27 28 33 35 36 37 38 39 41 43 47 51 55 57 58 62 64 67 70 71 74 76 77 78 79 81 83 85 89 90 91 93 95 96 97 100 101 102 109 110 114 116 120 121 123 125 126 127 128 129 130 131 135 136 142 145 147 152 157 158 165 168 171 173 175 178 180 181 182 187 188 190 194 195 198 199 212 0 6 13 14 15 24 26 30 31 40 48 53 54 60 63 68 72 73 80 84 86 87 88 92 98 104 106 111 115 134 137 139 141 148 149 153 154 161 163 164 167 172 174 179 186 191 192 197 215 1 9 17 23 32 49 52 75 82 99 118 122 124 132 143 144 156 159 162 169 170 176 183 185 189 196 204 34 42 44 45 46 65 105 108 112 155 166 207 4 61 119 133 138 146 211 16 29 50 56 59 66 69 94 103 107 113 117 140 150 151 160 177 184 193 200 201 202 203 205 206 208 209 210 213 214 216 217 218];

(jj) ϕ_T=[199 198 195 194 190 188 187 182 181 180 178 175 173 171 168 165 158 157 152 147 145 142 136 135 131 130 129 128 127 126 125 123 121 120 116 114 110 109 102 101 100 97 96 95 93 91 90 89 85 83 81 79 78 77 76 74 71 70 67 64 62 58 57 55 51 47 43 41 39 38 37 36 35 33 28 27 25 22 21 20 19 18 12 11 10 8 7 5 3 2 212 197 192 191 186 179 174 172 167 164 163 161 154 153 149 148 141 139 137 134 115 111 106 104 98 92 88 87 86 84 80 73 72 68 63 60 54 53 48 40 31 30 26 24 15 14 13 6 0 215 196 189 185 183 176 170 169 162 159 156 144 143 132 124 122 118 99 82 75 52 49 32 23 17 9 1 204 166 155 112 108 105 65 46 45 44 42 34 207 146 138 133 119 61 4 211 16 29 50 56 59 66 69 94 103 107 113 117 140 150 151 160 177 184 193 200 201 202 203 205 206 208 209 210 213 214 216 217 218];

(kk) ϕ_T=[2, 3, 5, 7, 8, 10, 11, 12, 18, 19, 20, 21, 22, 25, 27, 28, 33, 35, 36, 37, 38, 39, 41, 43, 47, 51, 55, 57, 58, 62, 64, 67, 70, 71, 74, 76, 77, 78, 79, 81, 83, 85, 89, 90, 91, 93, 95, 96, 97, 100, 101, 102, 109, 110, 114, 116, 120, 121, 123, 125, 126, 127, 128, 129, 130, 131, 135, 136, 142, 145, 147, 152, 157, 158, 165, 168, 171, 173, 175, 178, 180, 181, 182, 187, 188, 190, 194, 195, 198, 199, 212, 0, 6, 13, 14, 15, 24, 26, 30, 31, 40, 48, 53, 54, 60, 63, 68, 72, 73, 80, 84, 86, 87, 88, 92, 98, 104, 106, 111, 115, 134, 137, 139, 141, 148, 149, 153, 154, 161, 163, 164, 167, 172, 174, 179, 186, 191, 192, 197, 215, 1, 9, 17, 23, 32, 49, 52, 75, 82, 99, 118, 122, 124, 132, 143, 144, 156, 159, 162, 169, 170, 176, 183, 185, 189, 196, 204, 34, 42, 44, 45, 46, 65, 105, 108, 112, 155, 166, 207, 4, 61, 119, 133, 138, 146, 211, 29, 56, 59, 94, 103, 117, 218, 107, 113, 160, 193, 208, 16, 177, 184, 200, 140, 151, 201, 66, 205, 50, 209, 150, 206, 69, 202, 203, 210, 213, 214, 216, 217];

Particular embodiments include some of the following features and benefits. In particular embodiments, CRC checking can be done bit-by-bit for each individual CRC bit. This is in contrast to existing methods where the entire length-K_{CRC} vector of CRC bits are used in CRC checking. Bit-by-bit CRC checking enables the decoder to terminate the decoding process earlier should the decoded value of any of the CRC bits be found to be inconsistent with the decoded values of the information bits upon which the CRC bits depend, for all candidate paths in the list. The CRC checking can be performed during the SCL decoding. This is in contrast to existing methods which performs CRC checking only after the end of SCL decoding.

In particular embodiments, the decoder at the receiver can be run with the early termination feature, using the distributed CRC bits. In some embodiments, the distributed CRC bits do not carry a mask. When the decoder reaches a distributed CRC bit p_i, the decoder performs the following to decide if the decoding process should terminate early.

Step 1: The decoder calculates L estimated values, $p_i(\ell)$, of the distributed CRC bit p_i, one for each list ℓ, ℓ=0, 1, . . . , L−1.

Step 2: For each $p_i(\ell)$, the decoder checks if the info bits associated with $p_i(\ell)$ results in a successful parity check.

Step 3: if no parity check for each $p_i(\ell)$ are successful, the decoding process can terminate and deliver a 'decoding failure' message as the decoder output. If parity check(s) for one or more of $p_i(\ell)$ are successful, then the decoding process continues normally.

In another embodiment, the distributed CRC bits carry a mask (i.e., the distributed CRC bit q_i, and becomes: $w_i=(p_i+q_i) \bmod 2$. When the decoder reaches the bit location of a distributed CRC bit p_i, the decoder performs the following to decide if the decoding process should terminate early.

Step 1: The decoder calculates L estimated values, $w_i(\ell)$, of the distributed CRC bit p_i, one for each list ℓ, ℓ=0, 1, . . . , L−1.

Step 2: For each $w_i(\ell)$, the decoder removes the mask, $p_i=(w_i+q_i) \bmod 2$.

Step 3: For each $p_i(\ell)$, the decoder checks if the info bits associated with $p_i(t)$ results in a successful parity check.

Step 4: If no parity check for each $p_i(\ell)$ are successful, the decoding process can terminate and deliver the 'decoding failure' message as the decoder output. If parity check(s) for one or more of $p_i(\ell)$ are successful, then the decoding process continues normally.

FIG. **7** is a flow diagram illustrating an example method in a wireless transmitter, according to particular embodiments. In particular embodiments, one or more steps of FIG. **7** may be performed by network node **120** or wireless device **110** of network **100** described with respect to FIG. **2**.

The method begins at step **712**, where a wireless transmitter encodes a set of information carrying data bits u of length K with a linear outer code to generate a set of outer parity bits p along with the data bits u. For example, network

node **120** may encode a set of outer parity bits (e.g., using a CRC) and data bits according to any of the embodiments and examples described above with respect to FIGS. **3-6**.

At step **714**, the wireless transmitter interleaves the set of outer parity bits p and the data bits u using a predetermined interleaving mapping function that depends on the number of data bits K and is operable to distribute some bits of the set of parity bits p in front of some data bits u. For example, network node **120** may interleave the set of outer parity bits p and the data bits u using any of the embodiments and examples described above with respect to FIGS. **3-6**.

At step **716**, the wireless transmitter encodes the interleaved bits using a polar encoder to generate a set of encoded bits x. For example, network node **120** may encode the interleaved bits according to any of the embodiments and examples described above with respect to FIGS. **3-6**.

At step **718**, the wireless transmitter may transmit the set of encoded bits x to a wireless receiver. For example, network node **120** transmits the set of encoded bits x to wireless device **110**.

Modifications, additions, or omissions may be made to method **700** of FIG. **7**. Additionally, one or more steps in the method of FIG. **7** may be performed in parallel or in any suitable order. The steps may be repeated over time as necessary.

FIG. **8** is a flow diagram illustrating an example method in a wireless receiver, according to particular embodiments. In particular embodiments, one or more steps of FIG. **8** may be performed by network node **120** or wireless device **110** of network **100** described with respect to FIG. **2**.

The method begins at step **812**, where a wireless receiver determines that a decoder reaches a distributed CRC bit p_i when decoding a received set of polar encoded bits. For example, wireless device **110** may decode a set of polar encoded bits according to any of the deinterleaving embodiments or examples described above, such as successive cancellation list (SCL) decoding.

At step **814**, the wireless receiver calculates L estimated values, $p_i(\ell)$, of the distributed CRC bit p_i, one for each list ℓ, $\ell=0, 1, \ldots, L-1$. For example, wireless device **110** may estimate a list of values for a parity bit.

At step **814**, for each $p_i(\ell)$, the wireless receiver determines whether the info bits associated with $p_i(\ell)$ leads to a successful parity check. For example, wireless device **110** determines if a data bit was correctly received using the list of values for the parity bit. If the decoding is not successful, then the method continues to step **816** where the wireless receiver terminates the decoding. If the decoding is successful, then the method continues to step **818** where the wireless receiver continues the decoding.

Modifications, additions, or omissions may be made to method **800** of FIG. **8**. Additionally, one or more steps in the method of FIG. **8** may be performed in parallel or in any suitable order. The steps may be repeated over time as necessary.

FIG. **9**A is a block diagram illustrating an example embodiment of a wireless device. The wireless device is an example of the wireless devices **110** illustrated in FIG. **2**. In particular embodiments, the wireless device is capable of encoding and decoding a transmission using a CRC interleaving pattern for polar codes.

Particular examples of a wireless device include a mobile phone, a smart phone, a PDA (Personal Digital Assistant), a portable computer (e.g., laptop, tablet), a sensor, a modem, a machine type (MTC) device/machine to machine (M2M) device, laptop embedded equipment (LEE), laptop mounted equipment (LME), USB dongles, a device-to-device capable device, a vehicle-to-vehicle device, or any other device that can provide wireless communication. The wireless device includes transceiver **1310**, processing circuitry **1320**, memory **1330**, and power source **1340**. In some embodiments, transceiver **1310** facilitates transmitting wireless signals to and receiving wireless signals from wireless network node **120** (e.g., via an antenna), processing circuitry **1320** executes instructions to provide some or all of the functionality described herein as provided by the wireless device, and memory **1330** stores the instructions executed by processing circuitry **1320**. Power source **1340** supplies electrical power to one or more of the components of wireless device **110**, such as transceiver **1310**, processing circuitry **1320**, and/or memory **1330**.

Processing circuitry **1320** includes any suitable combination of hardware and software implemented in one or more integrated circuits or modules to execute instructions and manipulate data to perform some or all of the described functions of the wireless device. In some embodiments, processing circuitry **1320** may include, for example, one or more computers, one more programmable logic devices, one or more central processing units (CPUs), one or more microprocessors, one or more applications, and/or other logic, and/or any suitable combination of the preceding. Processing circuitry **1320** may include analog and/or digital circuitry configured to perform some or all of the described functions of wireless device **110**. For example, processing circuitry **1320** may include resistors, capacitors, inductors, transistors, diodes, and/or any other suitable circuit components.

Memory **1330** is generally operable to store computer executable code and data. Examples of memory **1330** include computer memory (e.g., Random Access Memory (RAM) or Read Only Memory (ROM)), mass storage media (e.g., a hard disk), removable storage media (e.g., a Compact Disk (CD) or a Digital Video Disk (DVD)), and/or or any other volatile or non-volatile, non-transitory computer-readable and/or computer-executable memory devices that store information.

Power source **1340** is generally operable to supply electrical power to the components of wireless device **110**. Power source **1340** may include any suitable type of battery, such as lithium-ion, lithium-air, lithium polymer, nickel cadmium, nickel metal hydride, or any other suitable type of battery for supplying power to a wireless device.

Other embodiments of the wireless device may include additional components (beyond those shown in FIG. **9**A) responsible for providing certain aspects of the wireless device's functionality, including any of the functionality described above and/or any additional functionality (including any functionality necessary to support the solution described above).

FIG. **9**B is a block diagram illustrating example components of a wireless device **110**. The components may include encoding/decoding module **1350**, transmitting module **1352** and receiving module **1354**.

Encoding/decoding module **1350** may perform the encoding and decoding functions of wireless device **110**. For example, encoding/decoding module **1350** may encode and decode a set of bits according to any of the CRC interleaving examples and embodiments described above. In some embodiments, encoding/decoding module **1350** may perform only encoding, may perform only decoding, or may perform both encoding and decoding. In certain embodiments, encoding/decoding module **1350** may include or be

included in processing circuitry **1320**. In particular embodiments, encoding/decoding module **1350** may communicate with transmitting module **1352** and receiving module **1354**.

Transmitting module **1352** may perform the transmitting functions of wireless device **110**. For example, transmitting module **1352** may transmit an encoded set of bits to network node **120**. In certain embodiments, transmitting module **1352** may include or be included in processing circuitry **1320**. In particular embodiments, transmitting module **1352** may communicate with scheduling module **1350** and receiving module **1354**.

Receiving module **1354** may perform the receiving functions of wireless device **110**. For example, receiving module **1354** may receive an encoded set of bits from network node **120**. In certain embodiments, receiving module **1354** may include or be included in processing circuitry **1320**. In particular embodiments, transmitting module **1352** may communicate with scheduling module **1350** and transmitting module **1352**.

FIG. **10**A is a block diagram illustrating an example embodiment of a network node. The network node is an example of the network node **120** illustrated in FIG. **2**. In particular embodiments, the network node is capable of encoding and decoding a transmission using a CRC interleaving pattern for polar codes.

Network node **120** can be an eNodeB, a nodeB, gNB, a base station, a wireless access point (e.g., a Wi-Fi access point), a low power node, a base transceiver station (BTS), a transmission point or node, a remote RF unit (RRU), a remote radio head (RRH), or other radio access node. The network node includes at least one transceiver **1410**, at least one processing circuitry **1420**, at least one memory **1430**, and at least one network interface **1440**. Transceiver facilitates transmitting wireless signals to and receiving wireless signals from a wireless device, such as wireless devices **110** (e.g., via an antenna); processing circuitry **1420** executes instructions to provide some or all of the functionality described above as being provided by a network node **120**; memory **1430** stores the instructions executed by processing circuitry **1420**; and network interface **1440** communicates signals to backend network components, such as a gateway, switch, router, Internet, Public Switched Telephone Network (PSTN), controller, and/or other network nodes **120**. Processing circuitry **1420** and memory **1430** can be of the same types as described with respect to processing circuitry **1320** and memory **1330** of FIG. **9**A above.

In some embodiments, network interface **1440** is communicatively coupled to processing circuitry **1420** and refers to any suitable device operable to receive input for network node **120**, send output from network node **120**, perform suitable processing of the input or output or both, communicate to other devices, or any combination of the preceding. Network interface **1440** includes appropriate hardware (e.g., port, modem, network interface card, etc.) and software, including protocol conversion and data processing capabilities, to communicate through a network.

FIG. **10**B is a block diagram illustrating example components of a network node **120**. The components may include encoding/decoding module **1450**, transmitting module **1452** and receiving module **1454**.

Encoding/decoding module **1450** may perform the encoding and decoding functions of network node **120**. For example, encoding/decoding module **1450** may encode and decode a set of bits according to any of the CRC interleaving examples and embodiments described above. In some embodiments, encoding/decoding module **1450** may perform only encoding, may perform only decoding, or may perform both encoding and decoding. In certain embodiments, encoding/decoding module **1450** may include or be included in processing circuitry **1420**. In particular embodiments, encoding/decoding module **1450** may communicate with transmitting module **1452** and receiving module **1454**.

Transmitting module **1452** may perform the transmitting functions of network node **120**. For example, transmitting module **1452** may transmit an encoded set of bits to wireless device **110**. In certain embodiments, transmitting module **1452** may include or be included in processing circuitry **1420**. In particular embodiments, transmitting module **1452** may communicate with encoding/decoding module **1450** and receiving module **1454**.

Receiving module **1454** may perform the receiving functions of network node **120**. For example, receiving module **1454** may receive an encoded set of bits from wireless device **110**. In certain embodiments, receiving module **1454** may include or be included in processing circuitry **1420**. In particular embodiments, transmitting module **1452** may communicate with encoding/decoding module **1450** and transmitting module **1452**.

Modifications, additions, or omissions may be made to the systems and apparatuses disclosed herein without departing from the scope of the invention. The components of the systems and apparatuses may be integrated or separated. Moreover, the operations of the systems and apparatuses may be performed by more, fewer, or other components. Additionally, operations of the systems and apparatuses may be performed using any suitable logic comprising software, hardware, and/or other logic. As used in this document, "each" refers to each member of a set or each member of a subset of a set.

Modifications, additions, or omissions may be made to the methods disclosed herein without departing from the scope of the invention. The methods may include more, fewer, or other steps. Additionally, steps may be performed in any suitable order.

Although this disclosure has been described in terms of certain embodiments, alterations and permutations of the embodiments will be apparent to those skilled in the art. Accordingly, the above description of the embodiments does not constrain this disclosure. Other changes, substitutions, and alterations are possible without departing from the spirit and scope of this disclosure, as defined by the claims below.

Abbreviations used in the preceding description include

3GPP Third Generation Partnership Project
BBU Baseband Unit
BTS Base Transceiver Station
CC Component Carrier
CRC Cyclic Redundancy Check
CQI Channel Quality Information
CSI Channel State Information
D2D Device to Device
DCI Downlink Control Information
DFT Discrete Fourier Transform
DMRS Demodulation Reference Signal
eNB eNodeB
FDD Frequency Division Duplex
FFT Fast Fourier Transform
gNB Next-generation NodeB
LAA Licensed-Assisted Access
LBT Listen-before-talk
LDPC Low-Density Parity Check
LTE Long Term Evolution
LTE-U LTE in Unlicensed Spectrum
M2M Machine to Machine
MCS Modulation and Coding Scheme

MIB Master Information Block
MIMO Multi-Input Multi-Output
MTC Machine Type Communication
NR New Radio
OFDM Orthogonal Frequency Division Multiplexing
PCM Parity Check Matrix
PRB Physical Resource Block
RAN Radio Access Network
RAT Radio Access Technology
RBS Radio Base Station
RNC Radio Network Controller
RRC Radio Resource Control
RRH Remote Radio Head
RRU Remote Radio Unit
SCell Secondary Cell
SI System Information
SIB System Information Block
TB Transport Block
TBS Transport Block Size
TDD Time Division Duplex
TTI Transmission Time Interval
UE User Equipment
UL Uplink
UTRAN Universal Terrestrial Radio Access Network
WAN Wireless Access Network

The invention claimed is:

1. A method of operation of a wireless transmitter in a wireless communication network, the method comprising:

encoding a set of information carrying a number K of data bits u to generate a set of cyclic redundancy check (CRC) parity bits p along with the data bits u;

interleaving the set of CRC parity bits p and the data bits u to generate interleaved bits, the interleaving using a predetermined interleaving mapping function that depends on the number K of the data bits u and is operable to distribute some bits of the set of CRC parity bits p in front of some of the data bits u;

encoding the interleaved bits using polar coding to generate a set of encoded bits x; and

transmitting the set of encoded bits x to a wireless receiver.

2. The method of claim **1**, wherein the wireless receiver comprises user equipment (UE).

3. The method of claim **1**, wherein the predetermined interleaving mapping function comprises a template interleaver for a largest value of the number K, referred to as K_{max}, and the template interleaver comprises a high-index bit mapper wherein:

the number K of the data bits u are loaded at the high-index positions of the input of the template interleaver, where $u=[u_0, u_1, \ldots, u_{K-1}]$ and the input of the template interleaver, denoted by $v=[v_0, v_1, \ldots, v_{K_{max}-1}]$, is given by the following bit mapping:

$$v_i = \begin{cases} u_{i-K_{max}+K} & K_{max} - K \le i < K_{max} \\ p_{i-K_{max}} & i \ge K_{max} \\ \text{NULL} & \text{otherwise} \end{cases}.$$

4. The method of claim **3**, wherein the K_{max} is 140 and the template interleaver uses an interleaving pattern comprising any one of the following interleaving patterns, wherein indices corresponding to the set of CRC parity bits p are underlined:

(a) ϕ_T=[0 3 6 9 11 12 13 14 15 17 18 19 21 22 28 29 30 37 38 42 43 48 53 57 59 64 65 66 69 71 72 73 78 79 91 92 97 106 111 114 117 119 122 125 126 127 128 130 131 134 136 137 144 1 4 7 10 16 20 23 31 39 44 49 54 58 60 67 70 74 80 93 98 107 107 112 115 118 120 123 129 132 135 138 145 2 5 8 24 32 40 45 50 55 61 68 75 81 94 99 108 113 116 121 124 133 139 146 26 33 34 41 46 77 83 87 88 89 100 101 104 109 110 147 25 51 56 62 76 82 95 140 27 84 86 96 102 161 36 103 105 150 90 159 52 163 63 153 47 143 85 151 158 160 162 35 141 142 148 149 152 154 155 156 157];

(b) ϕ_T=[0 3 6 9 11 12 13 14 15 17 18 19 21 22 28 29 30 37 38 42 43 48 53 57 59 64 65 66 69 71 72 73 78 79 91 92 97 106 111 114 117 119 122 125 126 127 128 130 131 134 136 137 144 1 4 7 10 16 20 23 31 39 44 49 54 58 60 67 70 74 80 93 98 107 112 115 118 120 123 129 132 135 138 145 2 5 8 24 32 40 45 50 55 61 68 75 81 94 99 108 113 116 121 124 133 139 146 25 26 27 33 34 35 36 41 46 47 51 52 56 62 63 76 77 82 83 84 85 86 87 88 89 90 95 96 100 101 102 103 104 105 109 110 140 141 142 143 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(c) ϕ_T=[0 3 6 9 11 12 13 14 15 17 18 19 21 22 28 29 30 37 38 42 43 48 53 57 59 64 65 66 69 71 72 73 78 79 91 92 97 106 111 114 117 119 122 125 126 127 128 130 131 134 136 137 144 1 4 7 10 16 20 23 31 39 44 49 54 58 60 67 70 74 80 93 98 107 112 115 118 120 123 129 132 135 138 145 2 5 8 24 32 40 45 50 55 61 68 75 81 94 99 108 113 116 121 124 133 139 146 26 33 34 41 46 77 83 87 88 89 100 101 109 110 147 25 27 35 36 47 51 52 56 62 63 76 82 84 85 86 90 95 96 102 103 105 140 141 142 143 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(d) ϕ_T=[0 3 6 9 11 12 13 14 15 17 18 19 21 22 28 29 30 37 38 42 43 48 53 57 59 64 65 66 69 71 72 73 78 79 91 92 97 106 111 114 117 119 122 125 126 127 128 130 131 134 136 137 144 1 4 7 10 16 20 23 31 39 44 49 54 58 60 67 70 74 80 93 98 107 112 115 118 120 123 129 132 135 138 145 2 5 8 24 32 40 45 50 55 61 68 75 81 94 99 108 113 116 121 124 133 139 146 26 33 34 41 46 77 83 87 88 89 100 101 104 109 110 147 25 51 56 62 76 82 95 140 27 35 36 47 52 63 84 85 86 90 96 102 103 105 141 142 143 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(e) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 17 23 37 48 75 80 86 137 143 13 18 38 144 39 145 40 146 41 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(f) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 17 18 23 37 38 39 40 41 48 75 80 86 137 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(g) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110

111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 17 23 37 48 75 80 86 137 143 13 18 38 39 40 41 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(h) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 17 23 37 48 75 80 86 137 143 13 18 38 144 39 41 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(i) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 17 23 37 48 75 80 86 137 143 13 18 38 144 39 145 40 41 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(j) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 18 23 38 39 80 137 145 17 40 75 146 48 149 37 86 143 144 41 147 148 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(k) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 17 18 23 37 38 39 40 41 48 75 80 86 137 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(l) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 18 23 38 39 80 137 145 17 37 40 41 48 75 86 143 144 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(m) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 18 23 38 39 80 137 145 17 40 75 146 37 41 48 86 143 144 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(n) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 18 23 38 39 80 137 145 17 40 75 146 48 149 37 41 86 143 144 147 148 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(o) ϕ_T=[0 2 4 5 10 11 12 16 18 19 23 24 25 28 33 35 37 38 39 41 42 47 48 51 52 53 55 56 58 59 62 66 67 71 72 76 79 81 82 85 86 91 98 99 102 106 108 109 111 112 113 115 117 119 120 124 125 126 131 132 133 134 139 155 3 7 9 15 17 21 29 30 40 43 44 46 57 60 61 63 64 77 84 87 90 96 103 104 107 114 116 118 122 129 130 136 137 138 144 8 27 31 34 49 50 65 74 93 94 95 97 100 101 105 121 127 147 13 26 36 54 73 75 89 92 135 146 14 32 80 88 145 68 69 128 152 1 6 20 78 151 22 83 148 149 123 141 45 140 70 153 154 110 142 143 150 156 157 158];

(p) ϕ_T=[0 2 4 5 10 11 12 16 18 19 23 24 25 28 33 35 37 38 39 41 42 47 48 51 52 53 55 56 58 59 62 66 67 71 72 76 79 81 82 85 86 91 98 99 102 106 108 109 111 112 113 115 117 119 120 124 125 126 131 132 133 134 139 155 3 7 9 15 17 21 29 30 40 43 44 46 57 60 61 63 64 77 84 87 90 96 103 104 107 114 116 118 122 129 130 136 137 138 144 8 27 31 34 49 50 65 74 93 94 95 97 100 101 105 121 127 147 16 13 14 20 22 26 32 36 45 54 68 69 70 73 75 78 80 83 88 89 92 110 123 128 135 140 141 142 143 145 146 148 149 150 151 152 153 154 156 157 158];

(q) ϕ_T=[0 2 4 5 10 11 12 16 18 19 23 24 25 28 33 35 37 38 39 41 42 47 48 51 52 53 55 56 58 59 62 66 67 71 72 76 79 81 82 85 86 91 98 99 102 106 108 109 111 112 113 115 117 119 120 124 125 126 131 132 133 134 139 155 3 7 9 15 17 21 29 30 40 43 44 46 57 60 61 63 64 77 84 87 90 96 103 104 107 114 116 118 122 129 130 136 137 138 144 8 27 31 34 49 50 65 74 93 94 95 97 100 101 105 121 127 147 13 26 36 54 73 75 89 92 135 146 1 6 14 20 22 32 45 68 69 70 78 80 83 88 110 123 138 140 141 142 143 145 148 149 150 151 152 153 154 156 157 158];

(r) ϕ_T=[0 2 4 5 10 11 12 16 18 19 23 24 25 28 33 35 37 38 39 41 42 47 48 51 52 53 55 56 58 59 62 66 67 71 72 76 79 81 82 85 86 91 98 99 102 106 108 109 111 112 113 115 117 119 120 124 125 126 131 132 133 134 139 155 3 7 9 15 17 21 29 30 40 43 44 46 57 60 61 63 64 77 84 87 90 96 103 104 107 114 116 118 122 129 130 136 137 138 144 8 27 31 34 49 50 65 74 93 94 95 97 100 101 105 121 127 147 13 26 36 54 73 75 89 92 135 146 14 32 80 88 145 1 6 20 22 45 68 69 70 78 83 110 123 133 138 140 141 142 143 148 149 150 151 152 153 154 156 157 158].

5. The method of claim **3**, wherein the K_{max} is 160 and the template interleaver uses an interleaving pattern comprising any one of the following interleaving patterns, wherein indices corresponding to the set of CRC parity bits p are underlined:

(a) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 164 59 165 60 166 61 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(b) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 103 107 108 109 120 121 124 129 130 167 10 15 45 71 76 82 96 102 115 160 11 47 104 106 116 122 181 56 123 125 170 110 179 72 183 83 173 67 163 105 171 178 180 182 55 161 162 168 169 172 174 175 176 177];

(c) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 37 38 43 57 58 59 60 61 68 95 100 106 157 163 164 165 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(d) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 10 11 15 45 46 47 53 54 55 56 61 66 67 71 72 76 82 83 96 97 102 103 104 105 106 107 108 109 110 115 116 120 121 122 123 124 125 129 130 160 161 162 163 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(e) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 59 60 61 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(f) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 1031 071 08 109 120 121 124 129 130 167 10 11 15 45 47 55 56 67 71 72 76 82 83 96 102 104 105 106 110 115 116 122 123 125 160 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(g) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 164 59 60 61 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(h) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 103 107 108 109 120 121 124 129 130 167 10 15 45 71 76 82 96 102 115 160 11 47 55 56 67 72 83 104 105 106 110 116 122 123 125 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(i) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 164 59 165 60 61 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(j) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 1031 071 08 109 120 121 124 129 130 167 10 15 45 71 76 82 96 102 115 160 11 47 104 106 116 122 181 55 56 67 72 83 105 110 123 125 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 182 183];

(k) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133

135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58 59 100 157 165 37 60 95 166 68 169 57 106 163 164 61 167 168 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(l) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 37 38 43 57 58 59 60 61 68 95 100 106 157 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(m) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58 59 100 157 165 37 57 60 61 68 95 106 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(n) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58 59 100 157 165 37 60 95 166 57 61 68 106 163 164 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(o) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58 59 100 157 165 37 60 95 166 68 169 57 61 106 163 164 167 168 170 171 172 173 174 175 176 177 178 179 180 181 182 183].

6. The method of claim **3**, wherein the K_{max} is 200 and the template interleaver uses an interleaving pattern comprising any one of the following interleaving patterns, wherein indices corresponding to the set of CRC parity bits p are underlined:

(a) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 7475 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83 91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 8 15 86 93 94 101 106 137 143 147 148 149 160 161 164 169 170 207 6 19 50 55 85 111 116 122 136 142 155 200 87 95 107 144 150 162 165 208 51 156 214 7 146 221 163 219 96 210 145 202 212 112 211 216 223 123 201 203 209 213 215 217 218 220 222];

(b) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 74 75 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83 91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 6 7 8 15 19 50 51 55 85 86 87 93 94 95 96 101 106 107 111 112 116 122 123 136 137 142 143 144 145 146 147 148 149 150 155 156 160 161 162 163 164 165 169 170 200 201 202 203 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(c) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 74 75 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83 91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 8 15 86 93 94 101 106 137 143 147 148 149 160 161 164 169 170 207 6 7 19 50 51 55 85 87 92 96 107 111 112 116 122 123 136 142 144 145 146 150 155 156 162 163 165 200 201 202 203 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(d) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 7475 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83 91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 8 15 86 93 94 101 106 137 143 147 148 149 160 161 164 169 170 207 6 19 50 55 85 111 116 122 136 142 155 200 7 51 87 95 96 107 112 123 144 145 146 150 156 162 163 165 201 202 203 208 209 210 211 212 213 214 215 216 217 218 219 120 221 222 223];

(e) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 49 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110

111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 204 99 205 100 206 101 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(f) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 73 77 78 83 97 98 99 100 101 108 135 140 146 197 197 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(g) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 99 100 101 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(h) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 204 99 100 101 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(i) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 204 99 205 100 101 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(j) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 100 135 206 108 209 97 146 203 204 101 207 208 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(k) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 73 77 78 83 97 98 99 100 101 108 135 140 146 197 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(l) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 97 100 101 108 135 146 203 204 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(m) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150

152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 100 135 206 97 101 108 146 203 204 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(n) ϕ_7=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 100 135 206 108 209 97 101 146 203 204 207 208 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(o) ϕ_7=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173 175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 33 44 87 91 94 134 153 154 155 157 165 207 53 61 66 80 138 195 211 82 96 105 140 143 170 218 6 39 86 92 208 15 22 183 200 49 130 210 149 203 48 202 59 201 209 214 133 205 206 213 216 217];

(p) ϕ_7=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173 175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 6 15 22 33 39 44 48 49 53 59 61 66 80 82 86 87 91 92 94 96 105 130 133 134 138 140 143 149 153 154 155 157 165 170 183 195 200 201 202 203 205 206 207 208 209 210 211 213 214 216 217 218];

(q) ϕ_7=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173 175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 33 44 87 91 94 134 153 154 155 157 165 207 6 15 22 39 48 49 53 59 61 66 80 82 86 92 96 105 130 133 138 140 143 149 170 183 195 200 201 202 203 205 206 208 209 210 211 213 214 216 217];

(r) ϕ_7=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173 175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 33 44 87 91 94 134 153 154 155 157 165 207 53 61 66 80 138 195 211 6 15 22 39 48 49 59 82 86 92 96 105 130 133 140 143 149 170 183 200 201 202 203 205 206 208 209 210 213 214 216 217 218].

7. The method of claim **1**, wherein the wireless transmitter comprises a network node.

8. A wireless transmitter comprising processing circuitry, the processing circuitry operable to:

encode a set of information carrying a number K of data bits u to generate a set of cyclic redundancy check (CRC) parity bits p along with the data bits u;

interleave the set of CRC parity bits p and the data bits u to generate interleaved bits, the interleaving using a predetermined interleaving mapping function that depends on the number K of data bits u and is operable to distribute some bits of the set CRC of parity bits p in front of some of the data bits u;

encode the interleaved bits using polar coding to generate a set of encoded bits x; and

transmit the set of encoded bits x to a wireless receiver.

9. The wireless transmitter of claim **8**, wherein the wireless transmitter comprises a network node and the wireless receiver comprises user equipment (UE).

10. The wireless transmitter of claim **8**, wherein the predetermined interleaving mapping function comprises a template interleaver for a largest value of the number K, referred to as K_{max}, and the template interleaver comprises a high-index bit mapper wherein:

the number K of the data bits u are loaded at the high-index positions of the input of the template interleaver, where $u=[u_0, u_1, \ldots, u_{K-1}]$ and the input of the template interleaver, denoted by $v=[v_0, v_1, \ldots, v_{K_{max}-1}]$, is given by the following bit mapping:

$$v_i = \begin{cases} u_{K-1-i} & 0 \le i < K \\ p_{i-K_{max}} & i \ge K_{max} \\ \text{NULL} & \text{otherwise} \end{cases}.$$

11. A method of operation of a wireless receiver in a wireless communication network, the method comprising:

receiving a set of polar encoded bits x from a wireless transmitter, the set of polar encoded bits x generated by encoding a set of information carrying a number K of data bits u to generate a set of cyclic redundancy check (CRC) parity bits p along with the data bits u, interleaving the set of CRC parity bits p and the data bits u to generate interleaved bits, the interleaving using a predetermined interleaving mapping function that depends on the number K of the data bits u and is operable to distribute some bits of the set of CRC parity bits p in front of some of the data bits u; and encoding

the interleaved bits using polar coding to generate the set of polar encoded bits x; and

performing a parity check based on one or more CRC parity bits of the set of CRC parity bits p, the one or more CRC parity bits obtained from decoding one or more polar encoded bits of the set of polar encoded bits x.

12. The method of claim **11**, wherein the predetermined interleaving mapping function comprises a template interleaver for a largest value of the number K, referred to as K_{max}, and the template interleaver comprises a high-index bit mapper wherein:

the number K of the data bits u are loaded at the high-index positions of the input of the template interleaver, where $u=[u_0, u_1, \ldots, u_{K-1}]$ and the input of the template interleaver, denoted by $v=[v_0, v_1, \ldots, v_{K_{max}-1}]$, is given by the following bit mapping:

$$v_i = \begin{cases} u_{i-K_{max}+K} & K_{max} - K \le i < K_{max} \\ p_{i-K_{max}} & i \ge K_{max} \\ \text{NULL} & \text{otherwise} \end{cases}.$$

13. The method of claim **12**, wherein the K_{max} is 140 and the template interleaver uses an interleaving pattern comprising any one of the following interleaving patterns, wherein indices corresponding to the set of CRC parity bits p are underlined:

(a) ϕ_7=[0 3 6 9 11 12 13 14 15 17 18 19 21 22 28 29 30 37 38 42 43 48 53 57 59 64 65 66 69 71 72 73 78 79 91 92 97 106 111 114 117 119 122 125 126 127 128 130 131 134 136 137 144 1 4 7 10 16 20 23 31 39 44 49 54 58 60 67 70 74 80 93 98 107 107 112 115 118 120 123 129 132 135 138 145 2 5 8 24 32 40 45 50 55 61 68 75 81 94 99 108 113 116 121 124 133 139 146 26 33 34 41 46 77 83 87 88 89 100 101 104 109 110 147 25 51 56 62 76 82 95 140 27 84 86 96 102 161 36 103 105 150 90 159 52 163 63 153 47 143 85 151 158 160 162 35 141 142 148 149 152 154 155 156 157];

(b) ϕ_7=[0 3 6 9 11 12 13 14 15 17 18 19 21 22 28 29 30 37 38 42 43 48 53 57 59 64 65 66 69 71 72 73 78 79 91 92 97 106 111 114 117 119 122 125 126 127 128 130 131 134 136 137 144 1 4 7 10 16 20 23 31 39 44 49 54 58 60 67 70 74 80 93 98 107 112 115 118 120 123 129 132 135 138 145 2 5 8 24 32 40 45 50 55 61 68 75 81 94 99 108 113 116 121 124 133 139 146 25 26 27 33 34 35 36 41 46 47 51 52 56 62 63 76 77 82 83 84 85 86 87 88 89 90 95 96 100 101 102 103 104 105 109 110 140 141 142 143 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(c) ϕ_7=[0 3 6 9 11 12 13 14 15 17 18 19 21 22 28 29 30 37 38 42 43 48 53 57 59 64 65 66 69 71 72 73 78 79 91 92 97 106 111 114 117 119 122 125 126 127 128 130 131 134 136 137 144 1 4 7 10 16 20 23 31 39 44 49 54 58 60 67 70 74 80 93 98 107 112 115 118 120 123 129 132 135 138 145 2 5 8 24 32 40 45 50 55 61 68 75 81 94 99 108 113 116 121 124 133 139 146 26 33 34 41 46 77 83 87 88 89 100 101 109 110 147 25 27 35 36 47 51 52 56 62 63 76 82 84 85 86 90 95 96 102 103 105 140 141 142 143 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(d) ϕ_7=[0 3 6 9 11 12 13 14 15 17 18 19 21 22 28 29 30 37 38 42 43 48 53 57 59 64 65 66 69 71 72 73 78 79 91 92 97 106 111 114 117 119 122 125 126 127 128 130 131 134 136 137 144 1 4 7 10 16 20 23 31 39 44 49 54 58 60 67 70 74 80 93 98 107 112 115 118 120 123 129 132 135 138 145 2 5 8 24 32 40 45 50 55 61 68 75 81 94 99 108 113 116 121 124 133 139 146 26 33 34 41 46 77 83 87 88 89 100 101 104 109 110 147 25 51 56 62 76 82 95 140 27 35 36 47 52 63 84 85 86 90 96 102 103 105 141 142 143 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(e) ϕ_7=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 17 23 37 48 75 80 86 137 143 13 18 38 144 39 145 40 146 41 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(f) ϕ_7=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 17 18 23 37 38 39 40 41 48 75 80 86 137 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(g) ϕ_7=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 17 23 37 48 75 80 86 137 143 13 18 38 39 40 41 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(h) ϕ_7=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 17 23 37 48 75 80 86 137 143 13 18 38 144 39 41 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(i) ϕ_7=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 17 23 37 48 75 80 86 137 143 13 18 38 144 39 145 40 41 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(j) ϕ_7=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109

112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 18 23 38 39 80 137 145 17 40 75 146 48 149 37 86 143 144 41 147 148 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(k) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 17 18 23 37 38 39 40 41 48 75 80 86 137 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(l) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 18 23 38 39 80 137 145 17 37 40 41 48 75 86 143 144 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(m) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 18 23 38 39 80 137 145 17 40 75 146 37 41 48 86 143 144 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(n) ϕ_T=[0 2 4 7 9 14 19 20 24 25 26 28 31 34 42 45 49 50 51 53 54 56 58 59 61 62 65 66 67 69 70 71 72 76 77 81 82 83 87 88 89 91 93 95 98 101 104 106 108 110 111 113 115 118 119 120 122 123 126 127 129 132 134 138 139 140 1 3 5 8 10 15 21 27 29 32 35 43 46 52 55 57 60 63 68 73 78 84 90 92 94 96 99 102 105 107 109 112 114 116 121 124 128 130 133 135 141 6 11 16 22 30 33 36 44 47 64 74 79 85 97 100 103 117 125 131 136 142 12 13 18 23 38 39 80 137 145 17 40 75 146 48 149 37 41 86 143 144 147 148 150 151 152 153 154 155 156 157 158 159 160 161 162 163];

(o) ϕ_T=[0 2 4 5 10 11 12 16 18 19 23 24 25 28 33 35 37 38 39 41 42 47 48 51 52 53 55 56 58 59 62 66 67 71 72 76 79 81 82 85 86 91 98 99 102 106 108 109 111 112 113 115 117 119 120 124 125 126 131 132 133 134 139 155 3 7 9 15 17 21 29 30 40 43 44 46 57 60 61 63 64 77 84 87 90 96 103 104 107 114 116 118 122 129 130 136 137 138 144 8 27 31 34 49 50 65 74 93 94 95 97 100 101 105 121 127 147 13 26 36 54 73 75 89 92 135 146 14 32 80 88 145 68 69 128 152 1 6 20 78 151 22 83 148 149 123 141 45 140 70 153 154 110 142 143 150 156 157 158];

(p) ϕ_T=[0 2 4 5 10 11 12 16 18 19 23 24 25 28 33 35 37 38 39 41 42 47 48 51 52 53 55 56 58 59 62 66 67 71 72 76 79 81 82 85 86 91 98 99 102 106 108 109 111 112 113 115 117 119 120 124 125 126 131 132 133 134 139 155 3 7 9 15 17 21 29 30 40 43 44 46 57 60 61 63 64 77 84 87 90 96 103 104 107 114 116 118 122 129 130 136 137 138 144 8 27 31 34 49 50 65 74 93 94 95 97 100 101 105 121 127 147 16 13 14 20 22 26 32 36 45 54 68 69 70 73 75 78 80 83 88 89 92 110 123 128 135 140 141 142 143 145 146 148 149 150 151 152 153 154 156 157 158];

(q) ϕ_T=[0 2 4 5 10 11 12 16 18 19 23 24 25 28 33 35 37 38 39 41 42 47 48 51 52 53 55 56 58 59 62 66 67 71 72 76 79 81 82 85 86 91 98 99 102 106 108 109 111 112 113 115 117 119 120 124 125 126 131 132 133 134 139 155 3 7 9 15 17 21 29 30 40 43 44 46 57 60 61 63 64 77 84 87 90 96 103 104 107 114 116 118 122 129 130 136 137 138 144 8 27 31 34 49 50 65 74 93 94 95 97 100 101 105 121 127 147 13 26 36 54 73 75 89 92 135 146 1 6 14 20 22 32 45 68 69 70 78 80 83 88 110 123 138 140 141 142 143 145 148 149 150 151 152 153 154 156 157 158];

(r) ϕ_T=[0 2 4 5 10 11 12 16 18 19 23 24 25 28 33 35 37 38 39 41 42 47 48 51 52 53 55 56 58 59 62 66 67 71 72 76 79 81 82 85 86 91 98 99 102 106 108 109 111 112 113 115 117 119 120 124 125 126 131 132 133 134 139 155 3 7 9 15 17 21 29 30 40 43 44 46 57 60 61 63 64 77 84 87 90 96 103 104 107 114 116 118 122 129 130 136 137 138 144 8 27 31 34 49 50 65 74 93 94 95 97 100 101 105 121 127 147 13 26 36 54 73 75 89 92 135 146 14 32 80 88 145 1 6 20 22 45 68 69 70 78 83 110 123 133 138 140 141 142 143 148 149 150 151 152 153 154 156 157 158].

14. The method of claim **12**, wherein the K_{max} is 160 and the template interleaver uses an interleaving pattern comprising any one of the following interleaving patterns, wherein indices corresponding to the set of CRC parity bits p are underlined:

(a) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 164 59 165 60 166 61 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(b) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 103 107 108 109 120 121 124 129 130 167 10 15 45 71 76 82 96 102 115 160 11 47 104 106 116 122 181 56 123 125 170 110 179 72 183 83 173 67 163 105 171 178 180 182 55 161 162 168 169 172 174 175 176 177];

(c) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 37 38 43

57 58 59 60 61 68 95 100 106 157 163 164 165 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(d) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 10 11 15 45 46 47 53 54 55 56 61 66 67 71 72 76 82 83 96 97 102 103 104 105 106 107 108 109 110 115 116 120 121 122 123 124 125 129 130 160 161 162 163 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(e) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 59 60 61 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(f) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 1031 071 08 109 120 121 124 129 130 167 10 11 15 45 47 55 56 67 71 72 76 82 83 96 102 104 105 106 110 115 116 122 123 125 160 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(g) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 164 59 60 61 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(h) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 103 107 108 109 120 121 124 129 130 167 10 15 45 71 76 82 96 102 115 160 11 47 55 56 67 72 83 104 105 106 110 116 122 123 125 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(i) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 37 43 57 68 95 100 106 157 163 33 38 58 164 59 165 60 61 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(j) ϕ_T=[0 3 4 5 7 12 16 18 20 23 26 29 31 32 33 34 35 37 38 39 41 42 48 49 50 57 58 62 63 68 73 77 79 84 85 86 89 91 92 93 98 99 111 112 117 126 131 134 137 139 142 145 146 147 148 150 151 154 156 157 164 1 68 13 17 1921 2427 3036 40 43 51 59 64 69 74 78 80 87 90 94 100 113 118 127 132 135 138 140 143 149 152 155 158 165 2 9 14 22 25 28 44 52 60 65 70 75 81 88 95 101 114 119 128 133 136 141 144 153 159 166 46 53 54 61 66 97 1031 071 08 109 120 121 124 129 130 167 10 15 45 71 76 82 96 102 115 160 11 47 104 106 116 122 181 55 56 67 72 83 105 110 123 125 161 162 163 168 169 170 171 172 173 174 175 176 177 178 179 180 182 183];

(k) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58 59 100 157 165 37 60 95 166 68 169 57 106 163 164 61 167 168 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(l) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 37 38 43 57 58 59 60 61 68 95 100 106 157 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(m) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58

59 100 157 165 37 57 60 61 68 95 106 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(n) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58 59 100 157 165 37 60 95 166 57 61 68 106 163 164 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183];

(o) ϕ_T=[0 1 2 4 6 7 9 10 14 15 17 19 20 22 24 27 29 34 39 40 44 45 46 48 51 54 62 65 69 70 71 73 74 76 78 79 81 82 85 86 87 89 90 91 92 96 97 101 102 103 107 108 109 111 113 115 118 121 124 126 128 130 131 133 135 138 139 140 142 143 146 147 149 152 154 158 159 160 3 5 8 11 16 18 21 23 25 28 30 35 41 47 49 52 55 63 66 72 75 77 80 83 88 93 98 104 110 112 114 116 119 122 125 127 129 132 134 136 141 144 148 150 153 155 161 12 26 31 36 42 50 53 56 64 67 84 94 99 105 117 120 123 137 145 151 156 162 13 32 33 38 43 58 59 100 157 165 37 60 95 166 68 169 57 61 106 163 164 167 168 170 171 172 173 174 175 176 177 178 179 180 181 182 183].

15. The method of claim **12**, wherein the K_{max} is 200 and the template interleaver uses an interleaving pattern comprising any one of the following interleaving patterns, wherein indices corresponding to the set of CRC parity bits p are underlined:

(a) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 7475 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83 91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 8 15 86 93 94 101 106 137 143 147 148 149 160 161 164 169 170 207 6 19 50 55 85 111 116 122 136 142 155 200 87 95 107 144 150 162 165 208 51 156 214 7 146 221 163 219 96 210 145 202 212 112 211 216 223 123 201 203 209 213 215 217 218 220 222];

(b) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 74 75 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83 91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 6 7 8 15 19 50 51 55 85 86 87 93 94 95 96 101 106 107 111 112 116 122 123 136 137 142 143 144 145 146 147 148 149 150 155 156 160 161 162 163 164 165 169 170 200 201 202 203 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(c) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 74 75 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83 91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 8 15 86 93 94 101 106 137 143 147 148 149 160 161 164 169 170 207 6 7 19 50 51 55 85 87 92 96 107 111 112 116 122 123 136 142 144 145 146 150 155 156 162 163 165 200 201 202 203 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(d) ϕ_T=[0 1 2 3 9 10 12 16 20 21 23 26 27 29 31 34 36 38 39 40 43 44 45 47 52 56 58 60 63 66 69 71 72 73 7475 77 78 79 81 82 88 89 90 97 98 102 103 108 113 117 119 124 125 126 129 131 132 133 138 139 151 152 157 166 171 174 177 179 182 185 186 187 188 190 191 194 196 197 204 4 11 13 17 22 24 28 30 32 35 37 41 46 48 53 57 59 61 64 67 70 76 80 83 91 99 104 109 114 118 120 127 130 134 140 153 158 167 172 175 178 180 183 189 192 195 198 205 5 14 18 25 33 42 49 54 62 65 68 84 92 100 105 110 115 121 128 135 141 154 159 168 173 176 181 184 193 199 206 8 15 86 93 94 101 106 137 143 147 148 149 160 161 164 169 170 207 6 19 50 55 85 111 116 122 136 142 155 200 7 51 87 95 96 107 112 123 144 145 146 150 156 162 163 165 201 202 203 208 209 210 211 212 213 214 215 216 217 218 219 120 221 222 223];

(e) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 49 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 204 99 205 100 206 101 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(f) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 73 77 78 83 97 98 99 100 101 108 135 140 146 197 197 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(g) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111

114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 99 100 101 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(h) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 204 99 100 101 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(i) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 7176 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 77 83 97 108 135 140 146 197 203 73 78 98 204 99 205 100 101 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(j) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 100 135 206 108 209 97 146 203 204 101 207 208 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(k) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 31 53 72 73 77 78 83 97 98 99 100 101 108 135 140 146 197 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(l) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 1 47 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 97 100 101 108 135 146 203 204 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(m) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 100 135 206 97 101 108 146 203 204 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(n) ϕ_T=[0 2 3 5 6 8 11 12 13 16 19 20 22 24 28 32 33 35 37 38 39 40 41 42 44 46 47 50 54 55 57 59 60 62 64 67 69 74 79 80 84 85 86 88 91 94 102 105 109 110 111 114 116 118 119 121 122 125 126 127 129 130 131 132 136 137 141 142 143 147 148 149 151 153 155 158 161 164 166 168 170 171 173 175 178 179 180 182 183 186 187 189 192 194 198 199 200 147 9 14 17 21 23 25 29 34 36 43 45 48 51 56 58 61 63 65 68 7075 81 87 89 92 95 103 106 112 115 117 120 123 128 133 138 144 150 152 154 156 159 162 165 167 169 172 174 176 181 184 188 190 193 195 201 10 15 18 26 30 52 66 71 76 82 90 93 96 104 107 124 134 139 145 157 160 163 177 185 191 196 202 27 53 72 73 78 83 98 99 140 197 205 31 77 100 135 206 108 209 97 101 146 203 204 207 208 210 211 212 213 214 215 216 217 218 219 220 221 222 223];

(o) ϕ_T=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173

175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 33 44 87 91 94 134 153 154 155 157 165 207 53 61 66 80 138 195 211 82 96 105 140 143 170 218 6 39 86 92 208 15 22 183 200 49 130 210 149 203 48 202 59 201 209 214 133 205 206 213 216 217];

(p) ϕ_T=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173 175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 6 15 22 33 39 44 48 49 53 59 61 66 80 82 86 87 91 92 94 96 105 130 133 134 138 140 143 149 153 154 155 157 165 170 183 195 200 201 202 203 205 206 207 208 209 210 211 213 214 216 217 218];

(q) ϕ_T=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173 175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 33 44 87 91 94 134 153 154 155 157 165 207 6 15 22 39 48 49 53 59 61 66 80 82 86 92 96 105 130 133 138 140 143 149 170 183 195 200 201 202 203 205 206 208 209 210 211 213 214 216 217];

(r) ϕ_T=[0 1 4 5 9 11 12 17 18 19 21 24 26 28 31 34 41 42 47 52 54 57 63 64 68 69 70 71 72 73 74 76 78 79 83 85 89 90 97 98 99 102 103 104 106 108 109 110 114 116 118 120 121 122 123 125 128 129 132 135 137 141 142 144 148 152 156 158 160 161 162 163 164 166 171 172 174 177 178 179 180 181 187 188 189 191 192 194 196 197 212 2 7 8 13 20 25 27 32 35 36 38 45 46 50 51 58 60 62 65 84 88 93 95 101 107 111 112 113 115 119 126 127 131 136 139 145 146 151 159 168 169 173 175 184 185 186 193 199 215 3 10 14 16 23 29 30 37 40 43 55 56 67 75 77 81 100 117 124 147 150 167 176 182 190 198 204 33 44 87 91 94 134 153 154 155 157 165 207 53 61 66 80 138 195 211 6 15 22 39 48 49 59 82 86 92 96 105 130 133 140 143 149 170 183 200 201 202 203 205 206 208 209 210 213 214 216 217 218].

16. The method of claim **11**, wherein the wireless receiver comprises a wireless device.

17. A wireless receiver comprising processing circuitry, the processing circuitry operable to:

receive a set of polar encoded bits x from a wireless transmitter, the set of polar encoded bits generated by encoding a set of information carrying a number K of data bits u to generate a set of cyclic redundancy check (CRC) parity bits p along with the data bits u, interleaving the set of CRC parity bits p and the data bits u to generate interleaved bits, the interleaving using a predetermined interleaving mapping function that depends on the number K of data bits u and is operable to distribute some bits of the set of CRC parity bits p in front of some of the data bits u; and encoding the interleaved bits using polar coding to generate the set of polar encoded bits x; and

perform a parity check based on one or more CRC parity bits of the set of CRC parity bits p, the one or more CRC parity bits obtained from decoding one or more polar encoded bits of the set of polar encoded bits x.

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