Studio serie

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1) $\sum_{n=1}^{\infty}\left(n^{\frac{1}{n}}-1\right)^{n}$

Utilizzo radice

$\lim_{x\to\infty}\sqrt[n]{\left(n^{\frac{1}{n}}-1\right)^{n}}=n^{\frac{1}{n}}-1=\infty^{0}-1=1-1=0\rightarrow converge$

2) $\sum_{n=1}^{\infty}\frac{n^{2}}{n\ln n+2n^{a+1}}$

$\sim\frac{n^{2}}{n^{a+1}}=\frac{n}{n^{a}}$

$a>2\rightarrow converge$

3) $\sum_{n=1}^{+\infty}\cos(\pi x)*\sin\frac{1}{n}$

$\cos(\pi x)=(-1)^{n}$

$\rightarrow(-1)^{n}*\sin\frac{1}{n}$

Criterio Si usa libnitz

- $\sin\frac{1}{n}\geq 0\rightarrow vero$
- $\lim_{n\to\infty}\sin\frac{1}{n}\sim\frac{1}{n}\rightarrow 0$
- $a_{n+1}\leq a_{n}$
  $\frac{1}{n+1}\leq\frac{1}{n}$

-> *converge*

4) Studiare la convergenza assoluta e normale di:

$\sum(-1)^{n}\left(e^{\frac{2}{n}}-e^{\frac{1}{n}}\right)$

Normale

- $e^{\frac{2}{n}}-e^{\frac{1}{n}}\geq 0\rightarrow e^{\frac{1}{n}}\geq 0$
- $\lim_{n\to+\infty}e^{\frac{2}{n}}-e^{\frac{1}{n}}=e^{\frac{1}{n}}\rightarrow 0$
- $e^{\frac{1}{n+1}}\leq e^{\frac{1}{n}}\rightarrow vero$

Assolutamente:

$\left|(-1)^{n}\left(e^{\frac{2}{n}}-e^{\frac{1}{n}}\right)\right|=e^{\frac{2}{n}}-e^{\frac{1}{n}}=e^{\frac{1}{n}}$

$\frac{e^{\frac{1}{n+1}}}{e^{\frac{1}{n}}}=e^{\frac{1}{n+1}}*e^{-\frac{1}{n}}=e^{\frac{2n+1}{n(n+1)}}>1\rightarrow diverge$

5) $\sum_{n=1}^{\infty}\frac{n}{(n+1)!}=\frac{n}{(n+1)*n*(n-1)!}=\frac{1}{(n+1)(n-1)!}>\frac{1}{n^{a}},a=1\rightarrow a>1\rightarrow converge$

Calcoliamo la somma

$\frac{n+1-1}{(n+1)!}=\frac{n+1}{(n+1)*n!}-\frac{1}{(n+1)!}=\frac{1}{n!}-\frac{1}{(n+1)!}$

Serie telescopica:

$\left[1-\frac{1}{2}\right]_{1}+\left[\frac{1}{2}-\frac{1}{6}\right]_{2}+\left[\frac{1}{6}+\frac{1}{24}\right]_{3}$

$1-\frac{1}{(n+1)!}$

$\lim_{x\to\infty}1-\frac{1}{(n+1)!}=1$