Studio integrali

martedì 1 febbraio 2022 13:36

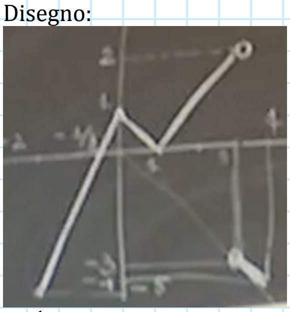
- Scrivere primitive di $x^2 + \frac{1}{x} - \sin x$

$$\int x^2 + \int \frac{1}{x} - \int \sin x$$

$$\frac{x^3}{3} + \ln x + \cos x + c, \qquad c\ e\ R$$

- *Calcolare* $\int_{-2}^{4} f$ dove

$3x + 1 \rightarrow x < 0$
$|x - 1| \rightarrow 0 \leq x < 3$
$-x \rightarrow x \geq 3$
Disegno:



$$\int_{-2}^{-\frac{1}{3}} f + \int_{-\frac{1}{3}}^{1} f + \int_{1}^{3} f + \int_{3}^{4} f \qquad trapazio = \frac{1}{2} * (b + B) * h$$

$$-\left(\frac{1}{2} * \left(2 - \frac{1}{3}\right) * 5\right) + \frac{1}{2} * \left(\frac{4}{5}\right) + \frac{1}{2} * (4) - \frac{1}{2} * (4 + 3) * 1$$

2) Det se esistono le primitive q di f

$f(x) = \frac{x-1}{x+2}$ e $2q(1) + q(2) = 4$

$D = (-inf, -2), (-2, +\inf)$

Primitive

$$\frac{x - 1}{x + 2} = \frac{x + 2 - 3}{x + 2} = 1 - \frac{3}{x + 2}$$

$$\int 1 - \frac{3}{x + 2} = \int 1 - \int \frac{3}{x + 2} = x - 3\ln(x + 2) + c$$

Calcoliamo

$2q(1) + q(2) = 4$

$2(1 - 3\ln(1 + 2) + c) + (2 - 3\ln(2 + 2) + c) = 4$

$3c = 6\ln 3 + 3\ln 4$

$$c = \frac{6\ln 3 + 3\ln 4}{3}$$

3) $f(x) = x \operatorname{arctg} x\,, q(1) = 2q(\sqrt{3})$

$$\int x * \operatorname{arctg} x = \frac{x^2}{2} * \operatorname{arctg} x - \int \frac{x^2}{2} * \frac{1}{1 + x^2}$$

$$\frac{x^2}{2} * \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2 + 1 - 1}{x^2 + 1} \rightarrow 1 - \frac{1}{x^2 + 1}$$

$$\frac{x^2}{2} * \operatorname{arctg} x - \frac{1}{2}(x - \operatorname{arctg} x) + c$$

$$= \frac{x^2}{2} * \operatorname{arctg} x - \frac{x}{2} + \frac{\operatorname{arctg} x}{2} + c$$

$q(1) = 2q(\sqrt{3})$

$$\frac{1}{2} * \frac{n}{4} - \frac{1}{2} + \frac{1}{2} * \frac{n}{4} + c = 2\left(\frac{3}{2} * \frac{n}{3} - \frac{\sqrt{3}}{2} + \frac{1}{2} * \frac{n}{3} + c\right)$$

(si risolve)

3) $si\ f(x) = \int_{2}^{x} \frac{e^{x^2}}{x^2}$

Scrivere l'eq della rette tangente nel punto $x_0 = 2$
$y_0 = 0$

$$y - y_0 = m(x - x_0) \rightarrow y - 0 = \frac{e^4}{4}(x - 2)$$

4) $\int_{0}^{2} \frac{x}{x^2 + 1} = \frac{1}{2} \int \frac{2x}{x^2 + 1} = \frac{1}{2} * \ln x^2 + 1$

$\frac{1}{2} * \ln(2^2 + 1) - \frac{1}{2} * \ln 1$

$\frac{\ln 5}{2}$

5) $\int \frac{x}{x^4 + 1} = \frac{1}{2} \int \frac{2x}{1 + (x^2)^2} = \frac{1}{2} * \tan x^2$

6) $\int \frac{1}{x * \ln^2 x} = \int \ln^{-2} x * \frac{1}{x} = -\frac{1}{\ln x} + c$

7) $f(x) = -x * \sin x^2$

Primitive:

$\int -x * \sin x^2 = \frac{1}{2} \int -2x * \sin x^2 = \frac{1}{2} * -\cos x^2 + c$

$q\left(\sqrt{\frac{3\pi}{2}}\right) = 0$

$\frac{1}{2} * \cos\left(\sqrt{\frac{3\pi}{2}}\right)^2 + c = 0$

$c = \frac{1}{2} * \cos\left(\frac{3\pi}{2}\right)$

$2c = \cos\left(\frac{3}{2}\pi\right) \rightarrow c = 0$

$\int_{\sqrt{\frac{3\pi}{2}}}^{\sqrt{2\pi}} f(x) = \left(\frac{1}{2} - \cos(2\pi)\right) - \left(\frac{1}{2} - \cos\left(\frac{3\pi}{2}\right)\right)$