# Esercizi

sabato 11 giugno 2022 17:38

1) $f(x) = (2 - x^2)e^x$
$\lim_{x\to+\infty} f(x) = -\infty$
$\lim_{x\to-\infty} f(x) = \infty * 0 = 0$
Asintoto $y = 0$ per $x \to -\infty$

$f'(x) = -2xe^x + (2 - x^2)e^x$
$e^x(-2x + 2 - x^2) \to -x^2 - 2x + 2$
$D = 4 + 8 = 12$
$x_{12} = \dfrac{2 \pm \sqrt{12}}{-2} = \dfrac{2 \pm 2\sqrt{3}}{-2} = \cancel{-1(\pm\sqrt{3})} = -1 \pm \sqrt{3} = -1 + \sqrt{3}, -1 - \sqrt{3}$
+++++-1 $- \sqrt{3}$---- *-1 +* $\sqrt{3}$++++
$f'(x) = -2xe^x + 2e^x - e^x x^2$
$f''(x) = -2e^x - 2xe^x + 2e^x - 2xe^x - e^x x^2$
$e^x(-x^2 - 4x)$
$xe^x(-x - 4)$
$x > 0$
$-x - 4 > 0 \to -x >$ -4 $\to x < 4$
$(0, 4) \to concava$

McLaurin 2 ordine
$f(0) = 2$
$f'(0) = 2$
$f''(0) = 0$
$f_{mc}(0) = 2x + 2$

Retta tangente x = 1
$y_1 = (2 - 1)e = e$
$f'(1) = -e$
$y - e = -e(x - 1)$
$y = -ex + 2e$

2) $f(x) = x - \dfrac{\ln^2 x}{x}$
Primitive
$\int x - \int \dfrac{\ln^2 x}{x}$
$\dfrac{x^2}{2} - \dfrac{\ln x^3}{3}$
$\alpha(e) = 2\alpha(1)$
$\dfrac{e^2}{2} - \dfrac{\ln^3 e}{3} = 2\left(\dfrac{1}{2} - \dfrac{\ln^3 1}{3}\right)$
$\dfrac{e^2}{2} - \dfrac{\ln^3 e}{3} + c = 1 - \dfrac{2}{3} + 2c$
$\dfrac{e^2}{2} - \dfrac{1}{3} - 1 + \dfrac{2}{3} = c$

$\left[\dfrac{x^2}{2} - \dfrac{\ln^3 x}{3}\right]_e^{e^2} = \left(\dfrac{e^4}{2} - \dfrac{1}{3}\right) - \left(\dfrac{e^2}{2} - \dfrac{\ln^3 e^2}{3}\right)$
$= \dfrac{e^4}{2} - \dfrac{e^2}{2} - \dfrac{1}{3} + \dfrac{2^3}{3} = \dfrac{e^4}{2} - \dfrac{e^2}{2} - \dfrac{7}{3}$

3) $\sum q_n$
La serie converge solo se -1<q<1
Diverge solo se q >= 1
$\sum \left(\dfrac{2x}{x^2 + 1}\right)^n$
Per quali valori di x non converge?
$\dfrac{2x}{x^2 + 1} \leq -1 \; v \; \dfrac{2x}{x^2 + 1} \geq 1$

$$\frac{2x}{x^2+1} \leq -1$$
$$2x \leq -x^2 - 1$$
$$x^2 + 2x + 1 \leq 0$$
$$x_{12} = \frac{-2 \pm \sqrt{4-4}}{2} = -\frac{2}{2} = -1 \rightarrow indeterminata$$
$$\frac{2x}{x^2+1} \geq 1$$
$$-x^2 + 2x - 1 \geq 0$$
$$x_{12} = \frac{-2 \pm \sqrt{4-4}}{-2} = 1$$
Non converge quando
$x = \pm 1$

Somma per x=-2
$$\frac{-4}{4+1} = -\frac{4}{5}$$
$$\frac{1}{1-q} = \frac{1}{1+\frac{4}{5}} = \frac{1}{\frac{9}{5}} = \frac{5}{9}$$