Integrali per parte

giovedì 9 giugno 2022 11:52

Siccome a febbraio ho sbagliato completamente l'esercizio sull'integrale per parte
Mo mi spacco di integrali fino al punto di sognarli la notte

1. $\int xe^x$
   $x \to 1$
   $e^x \to e^x$
   $xe^x - \int e^x = xe^x - e^x + c$

2. $\int x \sin x$
   $x \to 1$
   $\sin x \to -\cos x$
   $-x * \cos x - \int -\cos x$
   $-x * \cos x + \sin x + c$

3. $\int x^2 \ln x$
   $\ln x \to \frac{1}{x}$
   $x^2 \to \frac{1}{3}x^3$
   $\ln x * \frac{x^3}{3} - \int \frac{1}{x} * \frac{x^3}{3}$
   $\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2$
   $\frac{1}{3}x^3 \ln x - \frac{1}{3} * \frac{x^3}{3} + c$

4. $\int \ln x$
   $\ln x \to \frac{1}{x}$
   $1 \to x$
   $x * \ln x - \int \frac{1}{x} * x = x \ln x - x + c$

5. $\int x^2 * \sin x$
   $x^2 \to 2x$
   $\sin x \to -\cos x$
   $x^2 * -\cos x - \int -\cos x * 2x$
   $-x^2 \cos x + 2\int x \cos x$
   $x \to 1$
   $\cos x \to \sin x$
   $-x^2 \cos x + 2\left(x \cos x - \int \sin x\right)$
   $\sin x \to -\cos x$
   $-x^2 \cos x + 2(x \cos x + \cos x) + c$

6. $\int x^2 e^x$
   $x^2 \to 2x$
   $e^x \to e^x$
   $x^2 e^x - 2\int x * e^x$
   $x \to 1$
   $e^x \to e^x$
   $x^2 e^x - 2\left(xe^x - \int e^x\right)$

$x^2e^x - 2(xe^x - e^x) + c$

7. $\int \ln^2 x$

$\ln^2 x = (\ln x)^2 \rightarrow 2 * \ln x$

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$\ln^2 x \rightarrow 2 \ln x * \frac{1}{x}$

$1 \rightarrow x$

$x * \ln^2 x - 2 \int x \ln x * \frac{1}{x}$

$x * \ln^2 x - 2 \int \ln x$

$\ln x \rightarrow \frac{1}{x}$

$1 \rightarrow x$

$x \ln^2 x - 2\left(x \ln x - \int 1\right)$

$x \ln^2 x - 2(x \ln x - x) + c$

8. $\int e^x * \sin x$

$\sin x \rightarrow \cos x$

$e^x \rightarrow e^x$

$\sin x * e^x - \int \cos x * e^x$

$\cos x = -\sin x$

$e^x = e^x$

$\sin x * e^x - \left(\cos x * e^x - \int -\sin x * e^x\right)$

$\int e^x \sin x = e^x \sin x - e^x \cos x - \int e^x \sin x$

$2 \int e^x \sin x = e^x \sin x - e^x \cos x$

$\int e^x \sin x = \frac{e^x \sin x - e^x \cos x}{2} + c$

9. $\int \frac{\ln^2 x}{x} = \int \ln^2 x * \frac{1}{x}$

$\ln^2 x \rightarrow 2 \ln x * \frac{1}{x}$

$\frac{1}{x} \rightarrow \ln x$

$\ln^2 x * \ln x - 2 \int \ln x * \frac{1}{x} * \ln x$

$\ln^2 x * \ln x - 2 \int \ln^2 x * \frac{1}{x}$

Stesso ragionamento del punto 8

$\int \frac{\ln^2 x}{x} = \ln^3 x - 2 \int \frac{\ln^2 x}{x}$

$3 \int \frac{\ln^2 x}{x} = \ln^3 x$

$\int \frac{\ln^2 x}{x} = \frac{\ln^3 x}{3} + c$

10. $\int e^{3x} * \cos 4x$

$\cos 4x \rightarrow -4 \sin 4x$

$e^{3x} \rightarrow \frac{1}{3} e^{3x}$

$\frac{1}{3} e^{3x} * \cos 4x - \int -4 \sin 4x * \frac{1}{3} e^{3x}$

$\frac{1}{3} e^{3x} * \cos 4x + \frac{4}{3} \int \sin 4x * e^{3x}$

$\sin 4x = 4 \cos 4x$

$e^{3x} = \frac{1}{3} e^{3x}$

$$\frac{1}{3}e^{3x} * \cos 4x + \frac{4}{3}\left(\sin 4x * \frac{1}{3}e^{3x} - \int 4\cos x * \frac{1}{3}e^{3x}\right)$$

$$\frac{1}{3}e^{3x} * \cos 4x + \frac{4}{9}e^{3x}\sin 4x - \frac{16}{9}\int e^{3x}\cos 4x$$

$$\int e^{3x}\cos 4x = \frac{1}{3}e^{3x} * \cos 4x + \frac{4}{9}e^{3x}\sin 4x - \frac{16}{9}\int e^{3x}\cos 4x$$

$$\frac{25}{9}\int e^{x}\cos 4x = \frac{1}{3}e^{3x}\cos 4x + \frac{4}{9}e^{3x}\sin 4x$$

$$\int e^{x}\cos 4x = \frac{9}{25}\left(\frac{1}{3}e^{3x}\cos 4x + \frac{4}{9}e^{3x}\sin 4x\right) + c$$