# Studio integrali

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1) $\int_0^2 \frac{x^2+8}{x^2+4} = \frac{x^2+4-12}{x^2+4} = \int 1 - \frac{12}{x^2+4} = x - \int \frac{12}{x^2+4} = x - \frac{12}{2}\int \frac{2}{x^2+4} = x - 6\arctan\left(\frac{x}{2}\right)$
$\left[x - 6\arctan\frac{x}{2}\right]_0^2 = (2 - 6\arctan 1) - (0 - 6\arctan 0) = 2 - 6\arctan 1$

2) $f(x) = xe^x, media\ integrale\ [-1,1]$
$\frac{1}{b-a}\int_a^b f(x)$
$\frac{1}{2}\int xe^x$
$x \to 1$
$e^x \to e^x$
$\frac{1}{2}\left(x\,e^x - \int e^x\right) = \frac{1}{2}(xe^x - e^x) = \frac{1}{2}[\mathrm{x e^x}]_{-1}^1$
$\frac{1}{2}((-1e^1 + e^1) - (e^{-1} + e^{-1}))$
$\frac{1}{2} * 2e^{-1} = e^{-1}$

3) $f(x) = \begin{Bmatrix} 2x \to x < 0 \\ 3x^2 \to x \geq 0 \end{Bmatrix}, \int_{-1}^2 f(x)$
$\int_{-1}^0 2x = x^2 = -1$
$\int_0^2 3x^2 = \left[\frac{3x^3}{3}\right]_0^2 = x^3 \to 2^3 = 8$
$8 - 1 = 7$

4) $\int_1^3 \frac{|x-2|}{x}$
$x - 2 > 0 \to x > 2$
$\int_2^3 \frac{x-2}{x} = \int 1 - \frac{2}{x} = [x - 2\ln x]_2^3 = 3 - 2\ln 3 - 2 - 2\ln 2$
$x - 2 < 0 \to x < 2$
$\int_1^2 \frac{-(x-2)}{x} = \int -1 + \int \frac{2}{x} = [-x + 2\ln x]_1^2 = -2 + 2\ln 2 - 1$

*Somma*
$3 - 2\ln 3 - 2 - 2\ln 2 - 2 + 2\ln 2 - 1 = 4\ln 2 - 2\ln 3$

5) $\int_1^e \ln x = \int lnx + 1$
$\ln x \to \frac{1}{x}$
$1 \to x$
$\mathrm{x}\ln x - \int \frac{1}{x} * x$
$[x\ln x - x]_1^e$
$(e * \ln e - e) - (1 * \ln 1 - 1)$
$0 - (1 * 0 - 1) = 1$

6) Primitive:
$f(x) = x\cos x^2$
$\int x\cos x^2$
$x^2 = t \to help$
$\int \frac{\cos t}{2} = \frac{1}{2}\int \cos t = \frac{1}{2}\sin x^2 + c$

$\alpha(0) = 0$
$\frac{\sin 0}{2} + c = 0 \to c = 0$
$\frac{1}{2}\sin x^2 + 0 \to \frac{1}{2}\sin x^2$
*Calcolare:*

$$\int_0^{\frac{\sqrt{\pi}}{2}} x\cos x^2$$

$$\left[\frac{1}{2}\sin x^2\right]_0^{\sqrt{\frac{\pi}{2}}} = \left(\frac{1}{2}\sin\sqrt{\frac{\pi}{2}}\right) \sim \frac{1}{2} * \sin 1 = \frac{1}{2}$$

7) $\int_0^1 xe^x$

$x \to 1$

$e^x \to e^x$

$xe^x - \int e^x$

$[xe^x - e^x]_0^1 = (e^1 - e^1) - (-1) = 1$

8) $\int_1^{e^2} \frac{\ln^2 x}{x} = \frac{(\ln x)^2}{x} = \frac{3\ln^2 x}{x} = \frac{1}{3}\ln(x)^3$