Fundamentals of Computing Autumn 2023 Coursework

For return by 20 November; late submission by 4 December 2023

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1. **(6%)** Given the machine 32-bit word

$$1100\ 0001\ 1011\ 0000\ 0000\ 0000\ 0000\ 0000$$

find the decimal number represented by this word assuming that it is

(a) a two's complement integer;

(b) a single precision IEEE 754 floating-point number;

(c) a ones' complement integer.

2. **(6%)** Find computer representations of the following numbers:

(a) -107 as a two's complement 32-bit binary number;

(b) 17.7 as an IEEE 754 32-bit floating-point number.

3. **(8%)** Using the notation from page 28 in FoC-I, consider the floating point system with $\beta = 10$ and $p = 3$. Suppose we need to compute the value of $b^2 - 4ac$ for $b = 3.34$, $a = 1.22$, $c = 2.28$. Show all of the steps of the computation and rounding, find the relative error and compare it with the machine epsilon for the given β and p.

4. **(9%)** Consider the Boolean formulas

$$F_1 = B \wedge C, \quad F_2 = \neg A \to \neg C, \quad \text{and} \quad F_3 = \neg(B \wedge A).$$

(a) Are these formulas *consistent* in the sense that all of them are true under some assignment of the truth-values to the propositional variables?

(b) Is $C \wedge \neg A$ a logical consequence of F_1, F_2, F_3. Explain your answer.

(c) Is $C \wedge \neg A$ a logical consequence of F_1, F_2. Explain your answer.

5. **(10%)** Describe (without constructing) the truth-table for the Boolean function that returns 1 on any 32 input bits $a_{31}a_{30}\ldots a_1a_0$ if and only if the value of the input word interpreted as a 32-bit IEEE 754 (normalised or denormalised) number is strictly between 0 and 2. Explain your answer. Show a Boolean formula that computes this Boolean function. (Hint: have a look at pages 18 and 24 in FoC I.)

6. **(6%)** Let

$$A = \{1, 2, 3\}, \qquad B = \{\{1\}, 2, \{1, 3\}\}, \qquad C = \{1, 3\}.$$

Describe each of the following sets by listing its elements.

(a) $(A \cup B) - C$.

(b) $\big((A \cap B) \times C\big) \cup \big(C \times (A \cap B)\big)$.

(c) $\big(2^A \cap 2^C\big) - B$.

7. **(9%)** Which of the following relations on $\{a, b, c\}$ are partial orders? Which of them are equivalence relations? Explain your answers.

(a) $\{(a, a), (b, b), (c, c)\}$

(b) $\{(b, b), (b, c), (c, b), (a, c), (c, a)\}$

(c) $\{(a, a), (a, c), (a, b), (c, c), (c, b), (b, b)\}$

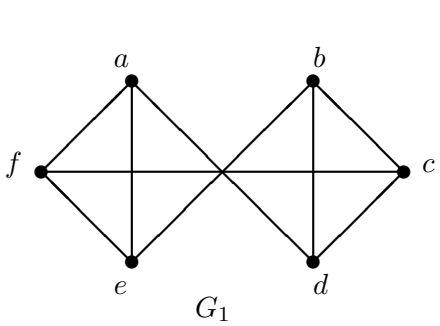
8. **(6%)** Which of the following $\mathbb{R} \to \mathbb{R}$ functions are bijections? Explain your answer:

(a) $f(x) = 2x + 1$

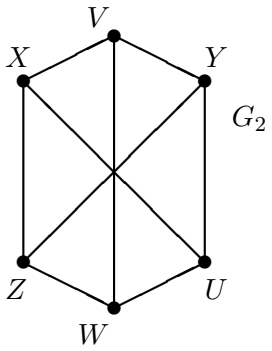
(b) $f(x) = x^2 + 1$

(c) $f(x) = x^3$

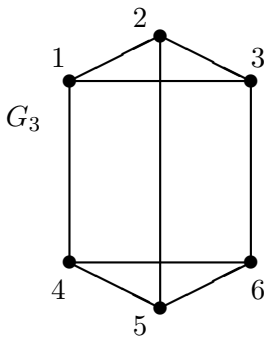
9. **(10%)** Determine which pairs of the following graphs G_1, G_2 and G_3 are isomorphic and which are not, and explain your answer:



G_1



G_2



G_3

10. **(9%)** Let $\Sigma = \{x, y\}$. For each of the following languages over Σ, find a regular expression representing it:

(i) All strings that contain either xy or $yyxx$ as a substring.

(ii) All strings that contain exactly two x's.

(ii) All strings in which the number of y's is odd.

11. **(9%)** (a) Consider the following DFA A: it has four states s, p, q and r, its initial state is s, its accepting state is r, and its transition function is given by the table

	0	1
s	p	s
p	q	s
q	q	r
r	r	r

Draw a state transition diagram for the DFA A and describe the language $L(A)$ in English.

(b) Select all regular expressions below that represent the language of A in part (a).

(i) $1^*(01)^*000^*1(0 \cup 1)^*$

(ii) $(0 \cup 1)^*001(0 \cup 1)^*$

(iii) $(1 \cup 01)^*000^*1(0 \cup 1)^*$

(iv) $(1 \cup 01)^*000^*1$

Explain your answer.

12. **(12%)** Transform, using the subset construction, the following nondeterministic finite automaton into an equivalent deterministic finite automaton. Define the language of this automaton by means of a regular expression.