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1. (1 pt) The matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix}$
has one real eigenvalue. Find this eigenvalue and a basis of the eigenspace.
eigenvalue = ____,
Basis: $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$, $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- `2`
- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{-1} \cr
  \mbox{1} \cr
  \mbox{1} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{-1} \cr
  \mbox{0} \cr
  \mbox{1} \cr
  \end{array}\right.\)
  ```

---

2. (1 pt) Given that $v_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ are eigenvectors of the matrix $A = \begin{bmatrix} -3 & -2 \\ 12 & 7 \end{bmatrix}$, determine the corresponding eigenvalues.
$\lambda_1 =$ ___.
$\lambda_2 =$ ___.

Correct Answers:

- `1`
- `3`

---

3. (1 pt) Let $v_1 = \begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$
be eigenvectors of the matrix A which correspond to the eigenvalues $\lambda_1 = -3$, $\lambda_2 = 0$, and $\lambda_3 = 2$, respectively, and let $v = \begin{bmatrix} -5 \\ 1 \\ -1 \end{bmatrix}$.
Express v as a linear combination of v_1, v_2, and v_3, and find Av.
$v =$ ______ $v_1 +$ ______ $v_2 +$ ______ v_3.
$Av = \begin{bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{bmatrix}$.

Correct Answers:

- `1`
- `1`
- `-1`
- `-6`
- `6`
- `8`

---

4. (1 pt) Let W be the subspace of $\mathbb{R}^3$ spanned by the vectors $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}$. Find the matrix A of the orthogonal projection onto W.
$A = \begin{bmatrix} \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \end{bmatrix}$.

Correct Answers:

- `0.666666666666667`
- `-0.333333333333333`
- `-0.333333333333333`
- `-0.333333333333333`
- `0.666666666666667`
- `-0.333333333333333`
- `-0.333333333333333`
- `-0.333333333333333`
- `0.666666666666667`

---

5. (1 pt) The matrix
$A = \begin{bmatrix} 1 & -2 \\ 13 & -9 \end{bmatrix}$
has complex eigenvalues, $\lambda_{1,2} = a \pm bi$, where $a =$ ___ and $b =$ ___.

The corresponding eigenvectors are $v_{1,2} = c \pm di$, where $c =$ (___, ___) and $d =$ (___, ___).

Correct Answers:

- `-4; 1; -1; -2; 1; 3`

---

6. (1 pt) Let $A = \begin{bmatrix} -1 & 3 \\ -6 & 8 \end{bmatrix}$.
Find two different diagonal matrices D and the corresponding matrix Ssuch that $A = SDS^{-1}$.

$D_1 = \begin{bmatrix} \_\_ & 0 \\ 0 & \_\_ \end{bmatrix}$ $S_1 = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$.
$D_2 = \begin{bmatrix} \_\_ & 0 \\ 0 & \_\_ \end{bmatrix}$ $S_2 = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$.

Correct Answers:

- `2; 5; 1; -1; 1; -2; 2; 5; 1; -1; 1; -2`
  ```

7. (1 pt) Given that the matrix A has eigenvalues $\lambda_1 = 4$ with corresponding eigenvector $v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\lambda_2 = -7$ with corresponding eigenvector $v_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, find A.

$A = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$

Correct Answers:

- `-18`
- `-22`
- `11`
- `15`

---

8. (1 pt) Find the projection of $v = \begin{bmatrix} -13 \\ 16 \\ -16 \end{bmatrix}$ onto the subspace V of $\mathbb{R}^3$ spanned by $\begin{bmatrix} -6 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 7 \\ 1 \end{bmatrix}$.

$\text{proj}_V(v) = \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix}$.

Correct Answers:

- `-8.36170212765957`
- `9.04255319148936`
- `4.87234042553191`

---

9. (1 pt) Given the vector $x = (2, 1, -1)$, find

$||x||_1 =$ _____

$||x||_\infty =$ _____

$||x||_2 =$ _____

Correct Answers:

- `4`
- `2`
- `2.44948974278318`

---

10. (1 pt) Given $A = \begin{bmatrix} 4 & -1 \\ -2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -4 \\ 3 & 4 \end{bmatrix}$.
Determine the value of each of the following:

$< A, B > =$ _____,

$||A||_F =$ _____,

$||B||_F =$ _____,

$\theta_{A,B} =$ _____.

Correct Answers:

- `2`
- `5.47722557505166`
- `7.54983443527075`
- `1.52241236779538`

---

11. (1 pt) If $p(x)$ and $q(x)$ are arbitrary polynomials of degree at most 2, then the mapping
$< p, q > = p(-1)q(-1) + p(0)q(0) + p(3)q(3)$
defines an inner product in P_3.
Use this inner product to find $< p, q >$, $||p||$, $||q||$, and the angle θ between $p(x)$ and $q(x)$ for
$p(x) = 3x^2 + 3x + 5$ and $q(x) = 3x^2 - 3x - 6$.

$< p, q > =$ _____,
$||p|| =$ _____,
$||q|| =$ _____,
$\theta =$ _____.

Correct Answers:

- `462`
- `41.6052881254294`
- `13.4164078649987`
- `0.595853233383319`

---

12. (1 pt)
(a) Find the least-squares solution $\hat{x}$ of the system
$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} x = \begin{bmatrix} -4 \\ -9 \\ 7 \end{bmatrix}$.

$\hat{x} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$.

(b) Determine the projection $p = A\hat{x}$.

$p = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Calculate the residual $r(\hat{x}) = b - A\hat{x}$.

$r(\hat{x}) = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- `-4`
- `-9`
- `-4`
- `-9`
- `0`
- `0`
- `0`
- `7`

---

13. (1 pt) Use Theorem 5.5.2 to write the vector $v = \begin{bmatrix} -3 \\ 0 \\ -5 \end{bmatrix}$, as linear combination of $u_1 = \begin{bmatrix} -3/\sqrt{29} \\ 2/\sqrt{29} \\ 4/\sqrt{29} \end{bmatrix}$, $u_2 = \begin{bmatrix} 4/\sqrt{25} \\ 0/\sqrt{25} \\ 3/\sqrt{25} \end{bmatrix}$ and $u_3 = \begin{bmatrix} -6/\sqrt{725} \\ -25/\sqrt{725} \\ 8/\sqrt{725} \end{bmatrix}$. Note that u_1, u_2

and u_3 are orthonormal.

$v =$ _____ $u_1 +$ _____ $u_2 +$ _____ u_3

Use Parseval's formula to compute $||v||^2$.
$||v||^2 =$ __________

Correct Answers:

- `-2.04264871994757`
- `-5.4`
- `-0.817059487979028`
- `34`

---

14. (1 pt) Let $A = \begin{bmatrix} -9.5 & -4 \\ 6.5 & 8 \\ -6.5 & -8 \\ 9.5 & 4 \end{bmatrix}$.

A singular value decomposition of A is as follows:

$$A = \begin{bmatrix} -0.5 & -0.5 & 0.5 & 0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 20 & 0 \\ 0 & 5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{bmatrix}.$$

(A) Find the closest (with respect to the Frobenius norm) matrix of rank 1 to A.

$$A1 = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}.$$

(B) Find the Frobenius norm of $A - A1$.
$||A - A1||_F =$ _____

Correct Answers:

- `-8`
- `-6`
- `8`
- `6`
- `-8`
- `-6`
- `8`
- `6`
- `5`

---

15. (1 pt) Supppose A is an invertible $n \times n$ matrix and v is an eigenvector of A with associated eigenvalue -7. Convince yourself that v is an eigenvector of the following matrices, and find the associated eigenvalues:

1. A^5, eigenvalue = _____,
2. A^{-1}, eigenvalue = _____,
3. $A - 4I_n$, eigenvalue = _____,
4. $5A$, eigenvalue = _____.

Correct Answers:

- `-16807`
- `-0.142857142857143`
- `-11`
- `-35`

---

16. (1 pt) Find a 2×2 matrix A such that
$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$
are eigenvectors of A, with eigenvalues 2 and -7 respectively.

$$A = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}.$$

Correct Answers:

- `-4.92307692307692`
- `10.3846153846154`
- `1.38461538461538`
- `-0.0769230769230769`

---

17. (1 pt) The matrix $A = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 4 & -2 \\ 0 & 0 & 4 \end{bmatrix}$ has two real eigenvalues, $\lambda_1 = 4$ of multiplicity 2, and $\lambda_2 = 3$ of multiplicity 1. Find an orthonormal basis for the eigenspace corresponding to λ_1.

$$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{0.894427190999916} \cr
  \mbox{0} \cr
  \mbox{0.447213595499958} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{0} \cr
  \mbox{1} \cr
  \mbox{0} \cr
  \end{array}\right.\)
  ```

---

18. (1 pt) Find the orthogonal projection of $v = \begin{bmatrix} 14 \\ 14 \\ -8 \\ -3 \end{bmatrix}$ onto the subspace V of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 2 \\ 2 \\ -1 \\ -4 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ 2 \\ -4 \\ 0 \end{bmatrix}$.

$$\text{proj}_V(v) = \begin{bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{bmatrix}.$$

Correct Answers:

- `5.63555555555556`
- `6.30222222222222`
- `-3.48444444444444`
- `-12.16`
  ```

- 3
- 3
- -9
- 0
- 6
- -6
- 0
- 0
- 6

---

19. (1 pt) Let $v_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0.5 \\ -0.5 \\ -0.5 \\ -0.5 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 0.5 \\ -0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$.

Find a vector v_4 in $\mathbb{R}^4$ such that the vectors v_1, v_2, v_3, and v_4 are orthonormal.

$v_4 = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- \(\displaystyle\left.\begin{array}{c}
  \mbox{0.5} \cr
  \mbox{0.5} \cr
  \mbox{-0.5} \cr
  \mbox{0.5} \cr
  \end{array}\right.\)

---

20. (1 pt) Find the QR factorization of $M = \begin{bmatrix} 12 & -12 \\ 6 & 1 \\ 4 & 10 \end{bmatrix}$.

$M = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix} \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$.

Correct Answers:

- 0.857142857142857
- -0.428571428571429
- 0.428571428571429
- 0.285714285714286
- 0.285714285714286
- 0.857142857142857
- 14
- -7
- 0
- 14

---

21. (1 pt) Find the QR factorization of $M = \begin{bmatrix} -2 & -6 & 8 \\ -2 & 0 & 8 \\ -1 & 3 & -5 \end{bmatrix}$.

$M = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix} \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$.

Correct Answers:

- -0.666666666666667
- -0.666666666666667
- -0.333333333333333
- -0.666666666666667
- 0.333333333333333
- 0.666666666666667
- -0.333333333333333
- 0.666666666666667
- -0.666666666666667

---

22. (1 pt) Fit a linear function of the form $f(t) = c_0 + c_1 t$ to the data points $(-2,-8)$, $(0,-7)$, $(2,0)$, using least squares.

$c_0 =$ ____,

$c_1 =$ ____.

Correct Answers:

- -5
- 2

---

23. (1 pt) Find the singular values $\sigma_1 \geq \sigma_2$ of

$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$.

$\sigma_1 =$ ______,

$\sigma_2 =$ ______.

Correct Answers:

- 3.43961507945952
- 0.411154600065108

---

24. (1 pt) Find the singular values $\sigma_1 \geq \sigma_2 \geq \sigma_3$ of

$A = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 0 & -2 \end{bmatrix}$.

$\sigma_1 =$ _____,

$\sigma_2 =$ _____,

$\sigma_3 =$ _____.

Correct Answers:

- 2.23606797749979
- 2.23606797749979
- 0

---

25. (1 pt) Let $A = \begin{bmatrix} -9.5 & -4 \\ 9.5 & 4 \\ 6.5 & 8 \\ -6.5 & -8 \end{bmatrix}$.

A singular value decomposition of A is as follows:

$$A = \begin{bmatrix} -0.5 & -0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \end{bmatrix} \begin{bmatrix} 20 & 0 \\ 0 & 5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{bmatrix}.$$

Find the least-squares solution of the linear system

$Ax = b$, where $b = \begin{bmatrix} 3 \\ -4 \\ 5 \\ 5 \end{bmatrix}$.

$x_1^* =$ ______,

$x_2^* =$ ______.

Correct Answers:

- -0.56
- 0.455