

- The row space of  $A$  is the same as the column space of  $A^T$ . TRUE The rows become the columns of  $A^T$  so this makes sense.
- If  $B$  is an echelon form of  $A$ , and if  $B$  has three nonzero rows, then the first three rows of  $A$  form a basis of Row  $A$ . FALSE The nonzero rows of  $B$  form a basis. The first three rows of  $A$  may be linear dependent.
- The dimensions of the row space and the column space of  $A$  are the same, even if  $A$  is not square. TRUE by the Rank Theorem. Also since dimension of row space = number of nonzero rows in echelon form = number pivot columns = dimension of column space.

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- The sum of the dimensions of the row space and the null space of  $A$  equals the number of rows in  $A$ . FALSE Equals number of columns by rank theorem. Also dimension of row space = number pivot columns, dimension of null space = number of non-pivot columns (free variables) so these add to total number of columns.
- On a computer, row operations can change the apparent rank of a matrix. TRUE Due to rounding error.

- If  $B$  is any echelon form of  $A$ , the the pivot columns of  $B$  form a basis for the column space of  $A$ . FALSE It's the corresponding columns in  $A$ .
- Row operations preserve the linear dependence relations among the rows of  $A$ . FALSE For example, Row interchanges mess things up.
- The dimension of null space of  $A$  is the number of columns of  $A$  that are not pivot columns. TRUE These correspond with the free variables.
- The row space of  $A^T$  is the same as the column space of  $A$ . TRUE Columns of  $A$  go to rows of  $A^T$ .
- If  $A$  and  $B$  are row equivalent, then their row spaces are the same. TRUE. This allows us to find row space of  $A$  by finding the row space of its echelon form..

- If  $A\mathbf{x} = \lambda\mathbf{x}$  for some vector  $\mathbf{x}$ , then  $\lambda$  is an eigenvalue of  $A$ .  
FALSE This is true as long as the vector is not the zero vector.
- A matrix  $A$  is not invertible if and only if 0 is an eigenvalue of  $A$ . TRUE
- A number  $c$  is an eigenvalue of  $A$  if and only if the equation  $(A - cI)\mathbf{x} = \mathbf{0}$  has a nontrivial solution. TRUE This is a rearrangement of the equation  $A\mathbf{x} = \lambda\mathbf{x}$ .
- Finding an eigenvector of  $A$  may be difficult, but checking whether a given vector is in fact an eigenvector is easy. TRUE  
Just see if  $A\mathbf{x}$  is a scalar multiple of  $\mathbf{x}$ .
- To find the eigenvalues of  $A$ , reduce  $A$  to echelon form.  
FALSE Row reducing changes the eigenvectors and eigenvalues.

- If  $A\mathbf{x} = \lambda\mathbf{x}$  for some scalar  $\lambda$ , then  $\mathbf{x}$  is an eigenvector of  $A$ . FALSE The vector must be nonzero.'
- If  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are linearly independent eigenvectors, then they correspond to different eigenvalues. FALSE The converse is true, however.
- A steady-state vector for a stochastic matrix is actually an eigenvector. TRUE A steady state vector has the property that  $A\mathbf{x} = \mathbf{x}$ . In this case  $\lambda$  is 1.
- The eigenvalues of a matrix are on its main diagonal. FALSE This is only true for triangular matrices.
- An eigenspace of  $A$  is a null space of a certain matrix. TRUE The eigenspace is the nullspace of  $A - \lambda I$ .