1. **(1 pt) Library/Rochester/setLinearAlgebra16DeterminantOfTransf-/ur_la_16_2.pg**
Find the determinant of the linear transformation
$T(f) = 5f + 3f'$ from P_2 to P_2.
det = ____

Correct Answers:

- 125

2. **(1 pt) Library/Rochester/setLinearAlgebra16DeterminantOfTransf-/ur_la_16_7.pg**
Find the determinant of the linear transformation
$T(f) = -7f - 4f' - 3f''$ from the space V spanned by $\cos(x)$ and $\sin(x)$ to V.
det = ____

Correct Answers:

- 32

3. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_9.pg**
Find the matrix A of the linear transformation $T(f(t)) = f(6t+5)$ from P_2 to P_2 with respect to the standard basis for P_2, $\{1,t,t^2\}$.

$$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Correct Answers:

- 1
- 5
- 25
- 0
- 6
- 60
- 0
- 0
- 36

4. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_5.pg**
Find the matrix A of the linear transformation

$$T(M) = \begin{bmatrix} 4 & 6 \\ 0 & 1 \end{bmatrix} M$$

from $U^{2\times 2}$ to $U^{2\times 2}$ (upper triangular matrices), with respect to the basis

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}.$$

$$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Correct Answers:

- 4
- 0
- -6
- 0
- 4
- 6
- 0
- 0
- 1

5. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_8.pg**
Find the matrix A of the linear transformation $T(f(t)) = f(8)$ from P_2 to P_2 with respect to the standard basis for P_2, $\{1,t,t^2\}$.

$$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Note: You should be viewing the transformation as mapping to constant polynomials rather than real numbers,

e.g. $T(2+t-t^2) = -4+0t+0t^2$.

Correct Answers:

- 1
- 8
- 64
- 0
- 0
- 0
- 0
- 0
- 0

6. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_15.pg**
Let V be the plane with equation $x_1 + 4x_2 - 3x_3 = 0$ in $\mathbb{R}^3$. Find the matrix A of the linear transformation $T(x) = \begin{bmatrix} 10 & 1 & 11 \\ -1 & -1 & -2 \\ 2 & -1 & 1 \end{bmatrix} x$ with respect to the basis

$$\left\{ \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

$$A = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$$

Correct Answers:

- 3
- -5
- -9
- 7

7. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_13.pg**

Let V be the space spanned by the two functions $\cos(t)$ and $\sin(t)$. Find the matrix A of the linear transformation $T(f(t)) = f''(t)+2f'(t)+9f(t)$ from V into itself with respect to the basis $\{\cos(t), \sin(t)\}$.

$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Correct Answers:

- 8
- 2
- -2
- 8

8. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_10.pg**

Find the matrix A of the linear transformation

$$T(f(t)) = \int_{-2}^{4} f(t)dt$$

from P_3 to $\mathbb{R}$ with respect to the standard bases for P_3 and $\mathbb{R}$.

$A = [\ \_\_\_\_ \quad \_\_\_\_ \quad \_\_\_\_ \quad \_\_\_\_\ ]$

Correct Answers:

- 6
- 6
- 24
- 60

9. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_3.pg**

If $T : P_1 \to P_1$ is a linear transformation such that $T(1+4x) = -3-2x$ and $T(4+15x) = 4+2x$, then $T(4-5x) =$________.

Correct Answers:

- 324 + 202*x

10. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_16.pg**

Let $T : P_3 \to P_3$ be the linear transformation satisfying $T(1) = 4x^2-2$, $T(x) = -4x+8$, $T(x^2) = 4x^2-x-8$. Find the image of an arbitrary cubic polynomial ax^2+bx+c.

$T(ax^2+bx+c) =$ ____________.

Correct Answers:

- a * (4*x^2 + -1*x + -8) + b * (-4*x + 8) + c * (4*x*x + -2)

11. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_6.pg**

Find the matrix A of the linear transformation

$T(M) = \begin{bmatrix} 7 & 3 \\ 0 & 1 \end{bmatrix} M \begin{bmatrix} 7 & 3 \\ 0 & 1 \end{bmatrix}^{-1}$

from $U^{2\times2}$ to $U^{2\times2}$ (upper triangular matrices), with respect to the standard basis for $U^{2\times2}$:

$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

Correct Answers:

- 1
- 0
- 0
- -3
- 7
- 3
- 0
- 0
- 1

12. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_20.pg**

Let V be a vector space, $v,u \in V$, and let $T_1 : V \to V$ and $T_2 : V \to V$ be linear transformations such that $T_1(v) = 3v-7u$, $T_1(u) = -5v+7u$, $T_2(v) = -2v+3u$, and $T_2(u) = 7v-3u$. Find the images of v and u under the composite of T_1 and T_2.

$(T_2T_1)(v) =$ ________,
$(T_2T_1)(u) =$ ________.

Correct Answers:

- -55*v + 30*u
- 59*v + -36*u

13. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_7.pg**

Find the matrix A of the linear transformation $T(f(t)) = 8f'(t)+9f(t)$ from P_2 to P_2 with respect to the standard basis for P_2, $\{1,t,t^2\}$.

$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

Correct Answers:

- 9
- 8
- 0
- 0
- 9
- 16
- 0
- 0
- 9

14. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_4.pg**

The matrices $A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, and $A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ form a basis for the linear space $V = \mathbb{R}^{2\times2}$. Write the matrix of the linear transformation $T : \mathbb{R}^{2\times2} \to \mathbb{R}^{2\times2}$ such that $T(A) = 11A+3A^T$ relative to this basis:

$\begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$

Correct Answers:

- 14
- 0
- 0
- 0
- 0
- 11
- 3
- 0
- 0
- 3
- 11
- 0
- 0
- 0
- 0
- 14

15. **(1 pt) Library/Rochester/setLinearAlgebra15TransfOfLinSpaces-/ur_la_15_17.pg**

Let $T : P_3 \to P_3$ be the linear transformation such that $T(-2x^2) = -3x^2 + 2x$, $T(-0.5x - 2) = 2x^2 - 4x - 4$, and $T(4x^2 - 1) = 4x - 2$.

Find $T(1)$, $T(x)$, $T(x^2)$, and $T(ax^2 + bx + c)$, where a, b, and c are arbitrary real numbers.

$T(1) =$ ____________________,
$T(x) =$ ____________________,
$T(x^2) =$ ____________________,
$T(ax^2 + bx + c) =$ ____________________.

Correct Answers:

- `6*x^2 + -8*x + 2`
- `-28*x^2 + 40*x + 0`
- `1.5*x^2 + -1*x + 0`
- `a*(1.5*x^2 + -1*x + 0) + b*(-28*x^2 + 40*x + 0) + c*(6*x^2 + -8*x + 2)`

16. **(1 pt) Library/Rochester/setLinearAlgebra9Dependence-/ur_la_9_11.pg**

Find a linearly independent set of vectors that spans the same subspace of $\mathbb{R}^3$ as that spanned by the vectors

$$\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

Linearly independent set: $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{2} \cr
  \mbox{1} \cr
  \mbox{-1} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{0} \cr
  \mbox{1} \cr
  \mbox{2} \cr
  \end{array}\right.\)
  ```

17. **(1 pt) Library/Rochester/setLinearAlgebra9Dependence-/ur_la_9_1.pg**

Let $A = \begin{bmatrix} -1 \\ 1 \\ -6 \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ -1 \\ -5 \end{bmatrix}$, and $C = \begin{bmatrix} 3 \\ 1 \\ 18 \end{bmatrix}$.

$\boxed{?}$ 1. Determine whether or not the three vectors listed above are linearly independent or linearly dependent.

If they are linearly dependent, determine a non-trivial linear relation - (a non-trivial relation is three numbers which are not all three zero.) Otherwise, if the vectors are linearly independent, enter 0's for the coefficients, since that relationship **always** holds.

____ $A +$ ____ $B +$ ____ $C = 0$.

You can use this **row reduction tool** to help with the calculations.

Correct Answers:

- `Linearly_Independent`
- `a multiple of ( 0, 0, 0 )`

18. **(1 pt) Library/Rochester/setLinearAlgebra9Dependence-/ur_la_9_13.pg**

Find a linearly independent set of vectors that spans the same subspace of $\mathbb{R}^4$ as that spanned by the vectors

$$\begin{bmatrix} -2 \\ 2 \\ -7 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \\ 3 \end{bmatrix}.$$

Linearly independent set: $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{1} \cr
  \mbox{-1} \cr
  \mbox{2} \cr
  \mbox{3} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{0} \cr
  \mbox{1} \cr
  \mbox{-3} \cr
  \mbox{3} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{0} \cr
  \mbox{1} \cr
  \mbox{0} \cr
  \mbox{-2} \cr
  \end{array}\right.\)
  ```

19. **(1 pt) Library/Rochester/setLinearAlgebra9Dependence-/ur_la_9_10.pg**

Express the vector $v = \begin{bmatrix} -45 \\ -15 \end{bmatrix}$ as a linear combination of

$x = \begin{bmatrix} -6 \\ -3 \end{bmatrix}$ and $y = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$.

$v =$ ___ $x+$ ___ y.

Correct Answers:

- 5
- 5

---

20. **(1 pt) Library/Rochester/setLinearAlgebra9Dependence-/ur_la_9_3.pg**

Let $A = \begin{bmatrix} -2 \\ -1 \\ 4 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 7 \\ -3 \\ -5 \end{bmatrix}$, $C = \begin{bmatrix} -6 \\ -9 \\ 8 \\ 5 \end{bmatrix}$, and $D = \begin{bmatrix} -2 \\ -4 \\ 2 \\ 3 \end{bmatrix}$.

? 1. Determine whether or not the four vectors listed above are linearly independent or linearly dependent.

If they are linearly dependent, determine a non-trivial linear relation - (a non-trivial relation is three numbers which are not all three zero.) Otherwise, if the vectors are linearly independent, enter 0's for the coefficients, since that relationship **always** holds.

_____ $A+$ _____ $B+$ _____ $C+$ _____ $D = 0$.

You can use this **row reduction tool** to help with the calculations.

Correct Answers:

- Independent
- a multiple of (0, 0, 0, 0)

---

21. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_4.pg**

If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that
$T \begin{bmatrix} 1 \\ 6 \end{bmatrix} = \begin{bmatrix} 12 \\ 29 \end{bmatrix}$ and $T \begin{bmatrix} 6 \\ -5 \end{bmatrix} = \begin{bmatrix} 31 \\ -31 \end{bmatrix}$,

then the standard matrix of T is $A = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$.

Correct Answers:

- 6
- 1
- -1
- 5

---

22. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_6.pg**

A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ whose matrix is
$\begin{bmatrix} -1 & 4 & 1 \\ 2 & -8 & -5.5+k \end{bmatrix}$
is onto if and only if $k \neq$ ___.

Correct Answers:

- 3.5

---

23. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_13.pg**

Match each linear transformation with its matrix.

___1. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

___2. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

___3. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

___4. $\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$

___5. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

___6. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

A. Contraction by a factor of 2
B. Rotation through an angle of 90° in the clockwise direction
C. Rotation through an angle of 90° in the counterclockwise direction
D. Reflection in the origin
E. Reflection in the x-axis
F. Projection onto the x-axis

Correct Answers:

- E
- B
- C
- A
- F
- D

---

24. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_1.pg**

If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation such that
$T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$, $T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$,

and $T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix}$,

then $T \begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$.

Correct Answers:

- -1
- -8
- -32

---

25. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_7.pg**

The matrix
$A = \begin{bmatrix} -2 & -3 & 2 & 1 \\ -5 & -6 & 5 & 2.5 \\ -9 & -12 & 9 & 8.5 \end{bmatrix}$
is a matrix of a linear transformation $T : \mathbb{R}^k \to \mathbb{R}^n$ where
$k =$ ___, $n =$ ___,
dim(Ker(T)) = ___, dim(Range(T)) = ___.
Is T onto? (enter YES or NO) ___.
Is T one-to-one? (enter YES or NO) ___.

Correct Answers:

- 4
- 3
- 1
- 3
- yes
- no

---

26. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_27.pg**

Which of the following linear transformations from $\mathbb{R}^3$ to $\mathbb{R}^3$ are invertible?

- A. Reflection in the yz -plane
- B. Projection onto the xy -plane
- C. Dilation by a factor of 4
- D. Identity transformation (i.e. $T(v) = v$ for all v)
- E. Trivial transformation (i.e. $T(v) = 0$ for all v)
- F. Rotation about the z -axis

Correct Answers:

- ACDF

---

27. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_10.pg**

Consider a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^2$ for which $T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}9\\6\end{bmatrix}$, $T\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}8\\3\end{bmatrix}$,

and $T\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}4\\5\end{bmatrix}$.

Find the matrix A of T.

$A = \begin{bmatrix}\_\_ & \_\_ & \_\_\\ \_\_ & \_\_ & \_\_\end{bmatrix}$.

Correct Answers:

- 9
- 8
- 4
- 6
- 3
- 5

---

28. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_16.pg**

The dot product of two vectors in $\mathbb{R}^3$ is defined by

$\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix} \cdot \begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix} = a_1b_1 + a_2b_2 + a_3b_3.$

Let $v = \begin{bmatrix}-1\\5\\1\end{bmatrix}$. Find the matrix A of the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}$ given by $T(x) = v \cdot x$.

$A = \begin{bmatrix}\_\_ & \_\_ & \_\_\end{bmatrix}$.

Correct Answers:

- -1
- 5
- 1

---

29. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_3.pg**

Let $b_1 = \begin{bmatrix}1\\3\end{bmatrix}$ and $b_2 = \begin{bmatrix}-2\\-7\end{bmatrix}$.

The set $B = \{b_1, b_2\}$ is a basis for $\mathbb{R}^2$.

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $T(b_1) = 3b_1 + 2b_2$ and $T(b_2) = 6b_1 + 5b_2$.

Then the matrix of T relative to the basis B is

$[T]_B = \begin{bmatrix}\_\_ & \_\_\\ \_\_ & \_\_\end{bmatrix}$,

and the matrix of T relative to the standard basis E for $\mathbb{R}^2$ is

$[T]_E = \begin{bmatrix}\_\_ & \_\_\\ \_\_ & \_\_\end{bmatrix}$.

Correct Answers:

- 3
- 6
- 2
- 5
- -19
- 6
- -86
- 27

---

30. **(1 pt) Library/Rochester/setLinearAlgebra4InverseMatrix-/ur_Ch2_1_3.pg**

Given:

$T(\begin{bmatrix}2\\-2\end{bmatrix}) = \begin{bmatrix}-10\\-14\end{bmatrix}$

$T(\begin{bmatrix}-6\\-5\end{bmatrix}) = \begin{bmatrix}41\\64\end{bmatrix}$

Find a matrix such that:

$T(\vec{v}) = \begin{bmatrix}\_\_ & \_\_\\ \_\_ & \_\_\end{bmatrix}(\vec{v})$

Correct Answers:

- \(\displaystyle\left.\begin{array}{cc} \mbox{-6} &\mbox{-1} \cr \mbox{-9} &\mbox{-2} \cr \end{array}\right.\)

---

31. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_27.pg**

Which of the following linear transformations from $\mathbb{R}^3$ to $\mathbb{R}^3$ are invertible?

- A. Reflection in the xy -plane
- B. Identity transformation (i.e. $T(v) = v$ for all v)
- C. Rotation about the y -axis
- D. Trivial transformation (i.e. $T(v) = 0$ for all v)
- E. Projection onto the xz -plane
- F. Dilation by a factor of 3

Correct Answers:

- ABCF

32. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_10.pg**
Consider a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^2$ for which
$T\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}5\\8\end{bmatrix}$, $T\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}2\\0\end{bmatrix}$,
and $T\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}9\\1\end{bmatrix}$.
Find the matrix A of T.
$A=\begin{bmatrix}\_\_ & \_\_ & \_\_\\ \_\_ & \_\_ & \_\_\end{bmatrix}$.

Correct Answers:

- 5
- 2
- 9
- 8
- 0
- 1

---

33. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_15.pg**
Find the matrix A of the linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ that rotates any vector through an angle of 45° in the counter-clockwise direction.
$A=\begin{bmatrix}\_\_ & \_\_\\ \_\_ & \_\_\end{bmatrix}$.

Correct Answers:

- `cos(45*pi/180)`
- `(-1)*sin(45*pi/180)`
- `sin(45*pi/180)`
- `cos(45*pi/180)`

---

34. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_28.pg**
Find the matrix M of the linear transformation $T:\mathbb{R}^2\to\mathbb{R}^2$ given by $T\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}-6x_1-2x_2\\-9x_1+x_2\end{bmatrix}$.
$M=\begin{bmatrix}\_\_ & \_\_\\ \_\_ & \_\_\end{bmatrix}$.

Correct Answers:

- -6
- -2
- -9
- 1

---

35. **(1 pt) Library/Rochester/setLinearAlgebra23QuadraticForms-/ur_la_23_3.pg**
The matrix
$A=\begin{bmatrix}2.2 & 0 & 0.6\\0 & 5 & 0\\0.6 & 0 & 3.8\end{bmatrix}$
has three distinct eigenvalues, $\lambda_1<\lambda_2<\lambda_3$,
$\lambda_1=$___,
$\lambda_2=$___,
$\lambda_3=$___.

Classify the quadratic form $Q(x)=x^TAx$:

- A. $Q(x)$ is positive definite
- B. $Q(x)$ is positive semidefinite
- C. $Q(x)$ is indefinite
- D. $Q(x)$ is negative definite
- E. $Q(x)$ is negative semidefinite

Correct Answers:

- 2
- 4
- 5
- A

---

36. **(1 pt) Library/Rochester/setLinearAlgebra23QuadraticForms-/ur_la_23_2.pg**
Find the eigenvalues of the matrix

$$M=\begin{bmatrix}-90 & 20\\20 & -60\end{bmatrix}.$$

Enter the two eigenvalues, separated by a comma:

____

Classify the quadratic form $Q(x)=x^TAx$:

- A. $Q(x)$ is indefinite
- B. $Q(x)$ is positive definite
- C. $Q(x)$ is positive semidefinite
- D. $Q(x)$ is negative semidefinite
- E. $Q(x)$ is negative definite

Correct Answers:

- -50, -100
- E

---

37. **(1 pt) Library/Rochester/setLinearAlgebra18OrthogonalBases-/ur_la_18_2.pg**
Let $x=\begin{bmatrix}3\\-2\\-3\end{bmatrix}$ and $y=\begin{bmatrix}-5\\-1\\3\end{bmatrix}$.
Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^3$ spanned by x and y.
$\begin{bmatrix}\_\_\\ \_\_\\ \_\_\end{bmatrix}$, $\begin{bmatrix}\_\_\\ \_\_\\ \_\_\end{bmatrix}$.

Correct Answers:

- 0.639602149066831
- -0.426401432711221
- -0.639602149066831
- -0.554700196225229
- -0.832050294337844
- 0

38. **(1 pt) Library/Rochester/setLinearAlgebra18OrthogonalBases-/ur_la_18_6.pg**

Find an orthonormal basis of the plane $x_1 + 6x_2 - x_3 = 0$.

$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$

Correct Answers:

- `\(\displaystyle\left.\begin{array}{c} \mbox{0.707106781186547} \cr \mbox{0} \cr \mbox{0.707106781186547} \cr \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c} \mbox{-0.688247201611685} \cr \mbox{0.229415733870562} \cr \mbox{0.688247201611685} \cr \end{array}\right.\)`

39. **(1 pt) Library/Rochester/setLinearAlgebra8VectorSpaces-/ur_la_8_2.pg**

Which of the following subsets of $\mathbb{R}^{3\times3}$ are subspaces of $\mathbb{R}^{3\times3}$?

- A. The 3×3 matrices of rank 2
- B. The 3×3 matrices with determinant 0
- C. The 3×3 matrices in reduced row-echelon form
- D. The 3×3 matrices with all zeros in the third row
- E. The diagonal 3×3 matrices
- F. The 3×3 matrices with trace 0 (the trace of a matrix is the sum of its diagonal entries)

Correct Answers:

- `DEF`

40. **(1 pt) Library/Rochester/setLinearAlgebra8VectorSpaces-/ur_la_8_3.pg**

Determine whether the given set S is a subspace of the vector space V.

- A. V is the vector space of all real-valued functions defined on the interval $[a,b]$, and S is the subset of V consisting of those functions satisfying $f(a) = 3$.
- B. $V = P_3$, and S is the subset of P_3 consisting of all polynomials of the form $p(x) = ax^3 + bx$.
- C. $V = M_n(\mathbb{R})$, and S is the subset of all upper triangular matrices.
- D. V is the vector space of all real-valued functions defined on the interval $(-\infty,\infty)$, and S is the subset of V consisting of those functions satisfying $f(0) = 0$.
- E. $V = M_n(\mathbb{R})$, and S is the subset of all $n\times n$ matrices with $\det(A) = 0$.
- F. $V = \mathbb{R}^2$, and S is the set of all vectors (x_1,x_2) in V satisfying $3x_1 + 4x_2 = 0$.
- G. $V = P_5$, and S is the subset of P_5 consisting of those polynomials satisfying $p(1) > p(0)$.

Correct Answers:

- `BCDF`

41. **(1 pt) Library/Rochester/setLinearAlgebra8VectorSpaces-/ur_la_8_6.pg**

Which of the following sets are subspaces of $\mathbb{R}^3$?

- A. $\{(x,y,z) \mid x,y,z > 0\}$
- B. $\{(x,y,z) \mid -5x - 7y - 8z = 0\}$
- C. $\{(x, x-5, x-7) \mid x \text{ arbitrary number }\}$
- D. $\{(x,y,z) \mid 5x + 7y = 0, 8x + 4z = 0\}$
- E. $\{(x,y,z) \mid -9x - 2y - 3z = -4\}$
- F. $\{(4x, 9x, 2x) \mid x \text{ arbitrary number }\}$

Correct Answers:

- `BDF`

42. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_4.pg**

Let W be the subspace of $\mathbb{R}^3$ spanned by the vectors $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -2 \\ -1 \end{bmatrix}$. Find the matrix A of the orthogonal projection onto W.

$A = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}.$

Correct Answers:

- `0.833333333333333`
- `-0.166666666666667`
- `-0.333333333333333`
- `-0.166666666666667`
- `0.833333333333333`
- `-0.333333333333333`
- `-0.333333333333333`
- `-0.333333333333333`
- `0.333333333333333`

43. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_2.pg**

Let $x = \begin{bmatrix} -2 \\ 2 \\ 1 \\ -5 \end{bmatrix}$.

Find the norm of x and the unit vector in the direction of x.

$||x|| = \_\_\_\_\_$,

$u = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}.$

Correct Answers:

- `5.8309518948453`
- `-0.342997170285018`
- `0.342997170285018`
- `0.171498585142509`
- `-0.857492925712544`

44. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_16.pg**

Find the orthogonal projection of $v = \begin{bmatrix} 9 \\ -13 \\ -13 \\ -3 \end{bmatrix}$ onto the subspace V of $\mathbb{R}^3$ spanned by $\begin{bmatrix} -5 \\ -2 \\ -1 \\ -4 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -4 \\ 8 \\ 0 \end{bmatrix}$.

$\text{proj}_V(v) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.

Correct Answers:

- `-0.652173913043478`
- `2.33913043478261`
- `-5.3304347826087`
- `-0.521739130434783`

45. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_3.pg**

Find the angle α between the vectors $\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix}$.

$\alpha =$ ________.

Correct Answers:

- `0.451026811796262`

46. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_17.pg**

Find the orthogonal projection of $v = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 0 \end{bmatrix}$

onto the subspace V of $\mathbb{R}^3$ spanned by

$\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$.

$\text{proj}_V(v) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.

Correct Answers:

- `-1.25`
- `3.75`
- `-1.25`
- `1.25`

47. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_11.pg**

Let $\{e_1, e_2, e_3, e_4, e_5, e_6\}$ be the standard basis in $\mathbb{R}^6$. Find the length of the vector $x = 2e_1 - 4e_2 + 5e_3 - 3e_4 - 3e_5 - 5e_6$.

$||x|| =$ ________.

Correct Answers:

- `9.38083151964686`

48. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_14.pg**

Find two linearly independent vectors perpendicular to the vector $v = \begin{bmatrix} 8 \\ 3 \\ -1 \end{bmatrix}$.

$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$, $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{1} \cr
  \mbox{0} \cr
  \mbox{8} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{-3} \cr
  \mbox{8} \cr
  \mbox{0} \cr
  \end{array}\right.\)
  ```

49. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_15.pg**

Find the orthogonal projection of $v = \begin{bmatrix} 3 \\ -2 \\ 14 \end{bmatrix}$ onto the subspace V of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ -5 \\ -14 \end{bmatrix}$.

$\text{proj}_V(v) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.

Correct Answers:

- `-0.96`
- `1.6`
- `13.28`

50. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_1.pg**

Let $x = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} 5 \\ 5 \\ -1 \end{bmatrix}$.

Find the dot product of x and y.

$x \cdot y =$ ________.

Correct Answers:

- `-5`

51. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_21.pg**

Let $v_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$, $v_2 = \begin{bmatrix} -0.5 \\ -0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$, and $v_3 = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{bmatrix}$.

Find a vector v_4 in $\mathbb{R}^4$ such that the vectors v_1, v_2, v_3, and v_4 are
  ```

orthonormal.

$v_4 = \left[\begin{array}{c} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{array}\right]$.

Correct Answers:

- `\(\displaystyle\left.\begin{array}{c}`
  `\mbox{-0.5} \cr`
  `\mbox{0.5} \cr`
  `\mbox{-0.5} \cr`
  `\mbox{0.5} \cr`
  `\end{array}\right.\)`

---

52. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_19.pg**

Let $v = \left[\begin{array}{c} -3 \\ 7 \\ -3 \\ 1 \end{array}\right]$.

Find a basis of the subspace of $\mathbb{R}^4$ consisting of all vectors perpendicular to v.

$\left[\begin{array}{c} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{array}\right], \left[\begin{array}{c} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{array}\right], \left[\begin{array}{c} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{array}\right]$.

Correct Answers:

- `\(\displaystyle\left.\begin{array}{c}`
  `\mbox{7} \cr`
  `\mbox{3} \cr`
  `\mbox{0} \cr`
  `\mbox{0} \cr`
  `\end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}`
  `\mbox{-3} \cr`
  `\mbox{0} \cr`
  `\mbox{3} \cr`
  `\mbox{0} \cr`
  `\end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}`
  `\mbox{1} \cr`
  `\mbox{0} \cr`
  `\mbox{0} \cr`
  `\mbox{3} \cr`
  `\end{array}\right.\)`

---

53. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_5.pg**

Let W be the subspace of $\mathbb{R}^3$ spanned by the vectors $\left[\begin{array}{c} -1 \\ 1 \\ -1 \\ 1 \end{array}\right]$ and $\left[\begin{array}{c} -8 \\ 9 \\ -1 \\ 2 \end{array}\right]$. Find the matrix A of the orthogonal projection onto W.

$A = \left[\begin{array}{cccc} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{array}\right]$.

Correct Answers:

- `0.43`
- `-0.49`
- `0.01`
- `-0.07`
- `-0.49`
- `0.57`
- `0.07`
- `0.01`
- `0.01`
- `0.07`
- `0.57`
- `-0.49`
- `-0.07`
- `0.01`
- `-0.49`
- `0.43`

---

54. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_10.pg**

Find the length of the vector $x = \left[\begin{array}{c} -6 \\ -3 \\ 4 \end{array}\right]$.

$||x|| =$ __________.

Correct Answers:

- `7.81024967590665`

---

55. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_20.pg**

Let $v = \left[\begin{array}{c} 1 \\ -2 \\ 4 \\ 1 \end{array}\right]$, $u = \left[\begin{array}{c} -3 \\ 7 \\ -4 \\ 4 \end{array}\right]$, and let W the subspace of $\mathbb{R}^4$ spanned by v and u. Find a basis of $W^{\perp}$.

$\left[\begin{array}{c} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{array}\right], \left[\begin{array}{c} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{array}\right]$.

Correct Answers:

- `\(\displaystyle\left.\begin{array}{c}`
  `\mbox{-20} \cr`
  `\mbox{-8} \cr`
  `\mbox{1} \cr`
  `\mbox{0} \cr`
  `\end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c`
  `\mbox{-15} \cr`
  `\mbox{-7} \cr`
  `\mbox{0} \cr`
  `\mbox{1} \cr`
  `\end{array}\right.\)`

56. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_6.pg**

Find a vector v perpendicular to the vector $u = \left[\begin{array}{c} 5 \\ 3 \end{array}\right]$.

$v = \left[\begin{array}{c} \text{—} \\ \text{—} \end{array}\right]$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{3} \cr
  \mbox{-5} \cr
  \end{array}\right.\)
  ```

57. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_27.pg**

Find a basis of the subspace of $\mathbb{R}^4$ that consists of all vectors perpendicular to both

$\left[\begin{array}{c} 1 \\ 0 \\ 1 \\ -4 \end{array}\right]$ and $\left[\begin{array}{c} 0 \\ 1 \\ 8 \\ -1 \end{array}\right]$.

$\left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right], \left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right]$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{1} \cr
  \mbox{8} \cr
  \mbox{-1} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{-4} \cr
  \mbox{-1} \cr
  \mbox{0} \cr
  \mbox{-1} \cr
  \end{array}\right.\)
  ```

58. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_4.pg**

Find the coordinate vector of $x = \left[\begin{array}{c} -4 \\ 3 \\ -5 \end{array}\right]$ with respect to the basis $B = \left\{ \left[\begin{array}{c} 1 \\ 7 \\ 8 \end{array}\right], \left[\begin{array}{c} 0 \\ 1 \\ -4 \end{array}\right], \left[\begin{array}{c} 0 \\ 0 \\ 1 \end{array}\right] \right\}$ or $\mathbb{R}^3$.

$[x]_B = \left[\begin{array}{c} \text{\_\_\_\_} \\ \text{\_\_\_\_} \\ \text{\_\_\_\_} \end{array}\right]$

Correct Answers:

- `-4`
- `31`
- `151`

59. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_24.pg**

Find a basis of the subspace of $\mathbb{R}^4$ spanned by the following vectors:

$\left[\begin{array}{c} 3 \\ 3 \\ -12 \\ 3 \\ -3 \end{array}\right], \left[\begin{array}{c} -3 \\ -3 \\ 12 \\ -2 \\ 3 \end{array}\right], \left[\begin{array}{c} -3 \\ 0 \\ 6 \\ -6 \\ -3 \end{array}\right], \left[\begin{array}{c} 9 \\ 3 \\ -23 \\ 15 \\ 3 \end{array}\right], \left[\begin{array}{c} -9 \\ -3 \\ 23 \\ -15 \\ -3 \end{array}\right]$.

$\left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right], \left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right], \left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right], \left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right]$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{3} \cr
  \mbox{3} \cr
  \mbox{-12} \cr
  \mbox{3} \cr
  \mbox{-3} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{-3} \cr
  \mbox{-3} \cr
  \mbox{12} \cr
  \mbox{-2} \cr
  \mbox{3} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{-3} \cr
  \mbox{0} \cr
  \mbox{6} \cr
  \mbox{-6} \cr
  \mbox{-3} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{9} \cr
  \mbox{3} \cr
  \mbox{-23} \cr
  \mbox{15} \cr
  \mbox{3} \cr
  \end{array}\right.\)
  ```

60. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_3.pg**

The set $B = \left\{ \left[\begin{array}{c} 1 \\ -10 \end{array}\right], \left[\begin{array}{c} 2 \\ -10 \end{array}\right] \right\}$ is a basis for $\mathbb{R}^2$.

Find the coordinates of the vector $x = \left[\begin{array}{c} 6 \\ -40 \end{array}\right]$ relative to the basis B:

$[x]_B = \left[\begin{array}{c} \text{\_\_\_\_} \\ \text{\_\_\_\_} \end{array}\right]$

Correct Answers:

- `2`
- `2`

61. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_25.pg**

Find a basis of the subspace of $\mathbb{R}^3$ defined by the equation $5x_1 + 3x_2 + 2x_3 = 0$.

$\left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \end{array}\right], \left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \end{array}\right]$.

Correct Answers:
  ```

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{2} \cr
  \mbox{0} \cr
  \mbox{-5} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{3} \cr
  \mbox{-5} \cr
  \mbox{0} \cr
  \end{array}\right.\)
  ```

---

62. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_18.pg**
Find bases of the kernel and image of the orthogonal projection onto the plane $4x+3y+z=0$ in $\mathbb{R}^3$.

Kernel: $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Image: $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$, $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{4} \cr
  \mbox{3} \cr
  \mbox{1} \cr
  \end{array}\right.\)
  ```
- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{1} \cr
  \mbox{0} \cr
  \mbox{-4} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{3} \cr
  \mbox{-4} \cr
  \mbox{0} \cr
  \end{array}\right.\)
  ```

---

63. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_26.pg**
Find a basis of the subspace of $\mathbb{R}^4$ defined by the equation $4x_1-7x_2+2x_3+7x_4=0$.

$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$, $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$, $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{-7} \cr
  \mbox{-4} \cr
  \mbox{0} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{2} \cr
  \mbox{0} \cr
  \mbox{-4} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{7} \cr
  \mbox{0} \cr
  \mbox{0} \cr
  \mbox{-4} \cr
  \end{array}\right.\)
  ```

---

64. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_14.pg**
Find two linearly independent vectors perpendicular to the vector $v=\begin{bmatrix} -8 \\ -3 \\ 5 \end{bmatrix}$.

$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$, $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{5} \cr
  \mbox{0} \cr
  \mbox{8} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{-3} \cr
  \mbox{8} \cr
  \mbox{0} \cr
  \end{array}\right.\)
  ```

---

65. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_15.pg**
Find the orthogonal projection of $v=\begin{bmatrix} 4 \\ 16 \\ -9 \end{bmatrix}$ onto the subspace V of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} -6 \\ 5 \\ 4 \end{bmatrix}$.

$\mathrm{proj}_V(v)=\begin{bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{bmatrix}$.

Correct Answers:

- 5.94155844155844
- 16.2987012987013
- -6.46103896103896

---

66. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_1.pg**
Let $x=\begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$ and $y=\begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}$.
Find the dot product of x and y.
$x \cdot y =$ ______.

Correct Answers:

- -9

---

67. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_21.pg**
Let $v_1=\begin{bmatrix} 0.5 \\ -0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$, $v_2=\begin{bmatrix} 0.5 \\ -0.5 \\ -0.5 \\ -0.5 \end{bmatrix}$, and $v_3=\begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{bmatrix}$.
Find a vector v_4 in $\mathbb{R}^4$ such that the vectors v_1, v_2, v_3, and v_4 are
  ```

orthonormal.

$$v_4 = \left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right].$$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{0.5} \cr
  \mbox{0.5} \cr
  \mbox{-0.5} \cr
  \mbox{0.5} \cr
  \end{array}\right.\)
  ```

---

68. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_19.pg**

Let $v = \left[\begin{array}{c} -7 \\ 9 \\ 9 \\ 3 \end{array}\right]$.

Find a basis of the subspace of $\mathbb{R}^4$ consisting of all vectors perpendicular to v.

$$\left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right], \left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right], \left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right].$$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{9} \cr
  \mbox{7} \cr
  \mbox{0} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{9} \cr
  \mbox{0} \cr
  \mbox{7} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{3} \cr
  \mbox{0} \cr
  \mbox{0} \cr
  \mbox{7} \cr
  \end{array}\right.\)
  ```

---

69. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_12.pg**

Find a vector x perpendicular to the vectors $v = \left[\begin{array}{c} 1 \\ 3 \\ -1 \end{array}\right]$ and $u = \left[\begin{array}{c} -1 \\ -2 \\ -2 \end{array}\right]$.

$$x = \left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \end{array}\right].$$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{-8} \cr
  \mbox{3} \cr
  \mbox{1} \cr
  \end{array}\right.\)
  ```

---

70. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_5.pg**

Let W be the subspace of $\mathbb{R}^3$ spanned by the vectors $\left[\begin{array}{c} -1 \\ 1 \\ 1 \\ 1 \end{array}\right]$ and $\left[\begin{array}{c} 0 \\ -5 \\ 1 \\ -4 \end{array}\right]$. Find the matrix A of the orthogonal projection onto W.

$$A = \left[\begin{array}{cccc} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{array}\right].$$

Correct Answers:

- 0.403846153846154
- -0.0192307692307692
- -0.480769230769231
- -0.0961538461538461
- -0.0192307692307692
- 0.596153846153846
- -0.0961538461538461
- 0.480769230769231
- -0.480769230769231
- -0.0961538461538461
- 0.596153846153846
- 0.0192307692307692
- -0.0961538461538461
- 0.480769230769231
- 0.0192307692307692
- 0.403846153846154

---

71. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_20.pg**

Let $v = \left[\begin{array}{c} 1 \\ 3 \\ -3 \\ -4 \end{array}\right]$, $u = \left[\begin{array}{c} 0 \\ -1 \\ 1 \\ 3 \end{array}\right]$, and let W the subspace of $\mathbb{R}^4$ spanned by v and u. Find a basis of $W^{\perp}$.

$$\left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right], \left[\begin{array}{c} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{array}\right].$$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{0} \cr
  \mbox{1} \cr
  \mbox{1} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{-5} \cr
  \mbox{3} \cr
  \mbox{0} \cr
  ```

```
\mbox{1} \cr
\end{array}\right.\)
```

72. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_4.pg**

Find the coordinate vector of $x = \begin{bmatrix} -4 \\ -2 \\ -4 \end{bmatrix}$ with respect to the basis $B = \left\{ \begin{bmatrix} 1 \\ 6 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ or $\mathbb{R}^3$.

$[x]_B = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

Correct Answers:

- `-4`
- `22`
- `182`

73. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_3.pg**

The set $B = \left\{ \begin{bmatrix} -1 \\ -10 \end{bmatrix}, \begin{bmatrix} 1 \\ 12 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$.

Find the coordinates of the vector $x = \begin{bmatrix} -1 \\ -16 \end{bmatrix}$ relative to the basis B:

$[x]_B = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

Correct Answers:

- `-2`
- `-3`

74. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_18.pg**

Find bases of the kernel and image of the orthogonal projection onto the plane $-2x+5y+z=0$ in $\mathbb{R}^3$.

Kernel: $\begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$.

Image: $\begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}, \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{-2} \cr
  \mbox{5} \cr
  \mbox{1} \cr
  \end{array}\right.\)
  ```
- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{1} \cr
  \mbox{0} \cr
  \mbox{2} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{5} \cr
  \mbox{2} \cr
  \mbox{0} \cr
  \end{array}\right.\)
  ```

75. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_14.pg**

Find two linearly independent vectors perpendicular to the vector $v = \begin{bmatrix} -4 \\ -7 \\ 7 \end{bmatrix}$.

$\begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}, \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{7} \cr
  \mbox{0} \cr
  \mbox{4} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{-7} \cr
  \mbox{4} \cr
  \mbox{0} \cr
  \end{array}\right.\)
  ```

76. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_4.pg**

Find the coordinate vector of $x = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$ with respect to the basis $B = \left\{ \begin{bmatrix} 1 \\ 8 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ or $\mathbb{R}^3$.

$[x]_B = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

Correct Answers:

- `1`
- `-9`
- `-55`

77. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_5.pg**

Let B be the basis of $\mathbb{R}^2$ consisting of the vectors

$\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 5 \end{bmatrix}$,

and let R be the basis consisting of

$\begin{bmatrix} -2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ -2 \end{bmatrix}$.

Find a matrix P such that $[x]_R = P[x]_B$ for all x in $\mathbb{R}^2$.

$P = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$

Correct Answers:

- `-14`
- `17`
- `8`
- `-11`

78. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_35.pg**

Find a basis for the space of 2x2 diagonal matrices: $\left\{ \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}, \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \right\}$

Correct Answers:
  ```

- ```
  \(\displaystyle\left.\begin{array}{cc}
  \mbox{1} &\mbox{0} \cr
  \mbox{0} &\mbox{0} \cr
  \end{array}\right.\) \(\displaystyle\left.\begin{array}{cc}
  \mbox{0} &\mbox{0} \cr
  \mbox{0} &\mbox{1} \cr
  \end{array}\right.\)
  ```

---

79. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_32.pg**
Find a basis of the subspace of $\mathbb{R}^4$ consisiting of all vectors of the form $\begin{bmatrix} x_1 \\ -4x_1+x_2 \\ 8x_1+8x_2 \\ -6x_1+3x_2 \end{bmatrix}$.

$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{1} \cr
  \mbox{-4} \cr
  \mbox{8} \cr
  \mbox{-6} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{0} \cr
  \mbox{1} \cr
  \mbox{8} \cr
  \mbox{3} \cr
  \end{array}\right.\)
  ```

---

80. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_6.pg**
The set $B = \{2+2x^2,\ 10-4x+10x^2,\ 16-8x+14x^2\}$ is a basis for P_2. Find the coordinates of $p(x) = -8+4x-10x^2$ relative to this basis:

$[p(x)]_B = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

Correct Answers:

- 3
- -3
- 1

---

81. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_19.pg**

Let L be the line spanned by $\begin{bmatrix} -4 \\ -8 \\ 7 \end{bmatrix}$ in $\mathbb{R}^3$.

Find a basis of the orthogonal complement $L^\perp$ of L.

$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{7} \cr
  \mbox{0} \cr
  \mbox{4} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{-8} \cr
  \mbox{4} \cr
  \mbox{0} \cr
  \end{array}\right.\)
  ```

---

82. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_20.pg**

Let L be the line spanned by $\begin{bmatrix} -9 \\ -4 \\ 6 \\ 7 \end{bmatrix}$ in $\mathbb{R}^4$.

Find a basis of the orthogonal complement $L^\perp$ of L.

$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{-4} \cr
  \mbox{9} \cr
  \mbox{0} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{6} \cr
  \mbox{0} \cr
  \mbox{9} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{7} \cr
  \mbox{0} \cr
  \mbox{0} \cr
  \mbox{9} \cr
  \end{array}\right.\)
  ```

---

83. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_2.pg**
Consider the basis B of $\mathbb{R}^2$ consisting of vectors
$\begin{bmatrix} -6 \\ -5 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ -2 \end{bmatrix}$.
Find x in $\mathbb{R}^2$ whose coordinate vector relative to the basis B is $[x]_B = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$.

$x = \begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$

Correct Answers:

- 22
- 6

---

84. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_36.pg**
Find a basis for the space of 2x2 lower triangular matrices:
$\{\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}\}$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{cc}
  \mbox{1} &\mbox{0} \cr
  \mbox{0} &\mbox{0} \cr
  \end{array}\right.\) \(\displaystyle\left.\begin{array}{cc
  \mbox{0} &\mbox{0} \cr
  \mbox{1} &\mbox{0} \cr
  \end{array}\right.\) \(\displaystyle\left.\begin{array}{cc
  \mbox{0} &\mbox{0} \cr
  ```
  ```

```
\mbox{0} &\mbox{1} \cr
\end{array}\right.\)
```

85. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_33.pg**

The set $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ is called the standard basis of the space of 2×2 matrices.

Find the coordinates of $M = \begin{bmatrix} -8 & 6 \\ -7 & 2 \end{bmatrix}$ with respect to this basis.

$[M]_B = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- -8
- 6
- -7
- 2

86. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_34.pg**

The set $B = \left\{ \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \right\}$ is a basis of the space of upper-triangular 2×2 matrices.

Find the coordinates of $M = \begin{bmatrix} -6 & 5 \\ 0 & 9 \end{bmatrix}$ with respect to this basis.

$[M]_B = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- 6
- -7
- -15

87. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_1.pg**

The vectors

$v_1 = \begin{bmatrix} 1 \\ -6 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} -3 \\ 7 \\ -3 \end{bmatrix}$, and $v_3 = \begin{bmatrix} -4 \\ 2 \\ k \end{bmatrix}$

form a basis for $\mathbb{R}^3$ if and only if $k \neq$ __.

Correct Answers:

- -6

88. **(1 pt) Library/TCNJ/TCNJ_IntroLinearTransformations-/problem2.pg**

Let $A = \begin{bmatrix} 3 & -1 \\ 3 & 8 \\ 9 & -6 \end{bmatrix}$.

Define the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ as $T(x) = Ax$.

Find the images of $u = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$ and $v = \begin{bmatrix} a \\ b \end{bmatrix}$ under T.

$T(u) = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$

$T(v) = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$

Correct Answers:

- -2
- -38
- 6
- (a*3+-1*b)
- (a*3+8*b)
- (a*9+-6*b)

89. **(1 pt) Library/TCNJ/TCNJ_IntroLinearTransformations-/problem23.pg**

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_2 - x_3, x_3 - x_1)$.

Find a vector $w \in \mathbb{R}^3$ that is not in the image of T.

$w =$ ______

Correct Answers:

- 23

90. **(1 pt) Library/TCNJ/TCNJ_IntroLinearTransformations-/problem18.pg**

Let $e_1 = (1,0)$, $e_2 = (0,1)$, $x_1 = (-3,-6)$, and $x_2 = (2,-7)$.

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that sends e_1 to x_1 and e_2 to x_2.

If T maps $(1,3)$ to the vector y, then $y = ($ __ , __ $)$.

Correct Answers:

- 3
- -27

91. **(1 pt) Library/TCNJ/TCNJ_IntroLinearTransformations-/problem22.pg**

Let $A = \begin{bmatrix} 1 & 5 & 5 & -5 \\ 0 & 1 & 2 & -2 \\ -1 & -3 & -1 & 1 \end{bmatrix}$

Find a vector w in $\mathbb{R}^3$ that is not in the image of the transformation $x \mapsto Ax$.

$w =$ ______

Correct Answers:

- 23

92. **(1 pt) Library/TCNJ/TCNJ_IntroLinearTransformations-/problem1.pg**

Let $A = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$.

Define the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ as $T(x) = Ax$.

Find the images of $u = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ and $v = \begin{bmatrix} a \\ b \end{bmatrix}$ under T.

$T(u) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

$T(v) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

Correct Answers:

- `21`
- `4`
- `(a*1+4*b)`
- `(a*-1+1*b)`

---

93. **(1 pt) Library/TCNJ/TCNJ_IntroLinearTransformations-/problem13.pg**

Let A be a 9×8 matrix. What must a and b be if we define the linear transformation by $T : \mathbb{R}^a \to \mathbb{R}^b$ as $T(x) = Ax$?

$a =$ _____
$b =$ _____

Correct Answers:

- `8`
- `9`

---

94. **(1 pt) Library/TCNJ/TCNJ_IntroLinearTransformations-/problem17.pg**

Hello

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that sends the vector $u = (5,2)$ into $(2,1)$ and maps $v = (1,3)$ into $(-1,3)$. Use properties of a linear transformation to calculate

$T(-5u) = (\_\_\_, \_\_\_)$, $T(9v) = (\_\_\_, \_\_\_)$

$T(-5u+9v) = (\_\_\_, \_\_\_)$

Correct Answers:

- `-10`
- `-5`
- `-9`
- `27`
- `-19`
- `22`

---

95. **(1 pt) Library/TCNJ/TCNJ_VectorSpaces/problem2.pg**

Let H be the set of all vectors of the form: $\begin{bmatrix} 5t \\ 0 \\ 4t \end{bmatrix}$. Find a vector v in $\mathbb{R}^3$ such that $H = Span\{v\}$.

$v = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$.

Correct Answers:

- `a multiple of ( 5, 0, 4 )`

---

96. **(1 pt) Library/TCNJ/TCNJ_VectorSpaces/problem4.pg**

Let W be the set of all vectors of the form: $\begin{bmatrix} 5s-2t \\ -2s-4t \\ 3s-5t \\ 2s-4t \end{bmatrix}$. Find vectors u and v such that $W = Span\{u,v\}$.

$u = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$, $v = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{5} \cr
  \mbox{-2} \cr
  \mbox{3} \cr
  \mbox{2} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{-2} \cr
  \mbox{-4} \cr
  \mbox{-5} \cr
  \mbox{-4} \cr
  \end{array}\right.\)
  ```

---

97. **(1 pt) Library/TCNJ/TCNJ_VectorSpaces/problem3.pg**

Let W be the set of all vectors of the form: $\begin{bmatrix} -3b+2c \\ b \\ c \end{bmatrix}$. Find vectors u and v such that $W = Span\{u,v\}$.

$u = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$, $v = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{-3} \cr
  \mbox{1} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{2} \cr
  \mbox{0} \cr
  \mbox{1} \cr
  \end{array}\right.\)
  ```

---

98. **(1 pt) Library/TCNJ/TCNJ_VectorSpaces/problem5.pg**

Let $v_1 = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 12 \\ 3 \\ 16 \end{bmatrix}$, $v_3 = \begin{bmatrix} 46 \\ 12 \\ 60 \end{bmatrix}$ and $w = \begin{bmatrix} 8 \\ 4 \\ 10 \end{bmatrix}$.

1. Is w in $\{v_1, v_2, v_3\}$? Type "yes" or "no". _____

2. How many vectors are in $\{v_1, v_2, v_3\}$? Enter "inf" if the answer is infinitely many. _____

3. How many vectors are in $Span\{v_1, v_2, v_3\}$? Enter "inf" if the answer is infinitely many. _____

4. Is w in the subspace spanned by $\{v_1, v_2, v_3\}$? Type "yes" or "no". _____

Correct Answers:

- `no`
- `3`
- `inf`
- `yes`

---

99. **(1 pt) Library/TCNJ/TCNJ_VectorSpaces/problem6.pg**
Determine if each of the following sets is a subspace of $\mathbb{P}_n$, for an appropriate value of n. Type "yes" or "no" for each answer.

Let W_1 be the set of all polynomials of the form $p(t) = at^2$, where a is in $\mathbb{R}$. _____

Let W_2 be the set of all polynomials of the form $p(t) = t^2 + a$, where a is in $\mathbb{R}$. _____

Let W_3 be the set of all polynomials of the form $p(t) = at^2 + at$, where a is in $\mathbb{R}$. _____

Correct Answers:

- `yes`
- `no`
- `yes`

---

100. **(1 pt) Library/TCNJ/TCNJ_OrthogonalSets/problem4.pg**
Let $y = \begin{bmatrix} -10 \\ 5 \\ -8 \end{bmatrix}$ and $u = \begin{bmatrix} -3 \\ -4 \\ 7 \end{bmatrix}$. Describe y as the sum of two orthogonal vectors, x_1 in $Span\{u\}$ and x_2 orthogonal to u.

$x_1 = \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix}$, $x_2 = \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix}$.

Correct Answers:

- `1.86486486486486`
- `2.48648648648649`
- `-4.35135135135135`
- `-11.8648648648649`
- `2.51351351351351`
- `-3.64864864864865`

---

101. **(1 pt) Library/TCNJ/TCNJ_OrthogonalSets/problem15.pg**
Given $v = \begin{bmatrix} -1 \\ -2 \\ -7 \\ 6 \\ -10 \end{bmatrix}$, find the coordinates for v in the subspace W spanned by $u_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} -7 \\ 3 \\ 5 \\ 16 \\ 5 \end{bmatrix}$, $u_4 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ and $u_5 = \begin{bmatrix} -4 \\ -2 \\ 1 \\ -2 \\ 1 \end{bmatrix}$. Note that u_1, u_2, u_3, u_4 and u_5 are orthogonal.

$v =$ _____ u_1+ _____ u_2+ _____ u_3+ _____ u_4+ _____ u_5

Correct Answers:

- `-4`
- `-3.85714285714286`
- `0.032967032967033`
- `-1.5`
- `-0.807692307692308`

---

102. **(1 pt) Library/TCNJ/TCNJ_OrthogonalSets/problem14.pg**
Given $v = \begin{bmatrix} -10 \\ 5 \\ -6 \\ 8 \end{bmatrix}$, find the coordinates for v in the subspace W spanned by $u_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 4 \end{bmatrix}$, $u_3 = \begin{bmatrix} 11 \\ -1 \\ 4 \\ -7 \end{bmatrix}$ and $u_4 = \begin{bmatrix} 0 \\ -3 \\ 1 \\ 1 \end{bmatrix}$. Note that u_1, u_2, u_3 and u_4 are orthogonal.

$v =$ _____ u_1+ _____ u_2+ _____ u_3+ _____ u_4

Correct Answers:

- `1.33333333333333`
- `0.137254901960784`
- `-1.0427807486631`
- `-1.18181818181818`

---

103. **(1 pt) Library/TCNJ/TCNJ_OrthogonalSets/problem8.pg**
All vectors are in $\mathbb{R}^n$.

Check the true statements below:

- A. If the columns of an $m \times n$ matrix are orthonormal, then the linear mapping $x \to Ax$ preserves lengths.
- B. The orthogonal projection of y onto v is the same as the orthogonal projection of y onto cv whenever $c \neq 0$.
- C. If a set $S = \{u_1, ..., u_p\}$ has the property that $u_i \cdot u_j = 0$ whenever $i \neq j$, then S is an orthonormal set.
- D. Not every orthogonal set in $\mathbb{R}^n$ is a linearly independent set.
- E. An orthogonal matrix is invertible.

Correct Answers:

- `ABDE`

---

104. **(1 pt) Library/TCNJ/TCNJ_OrthogonalSets/problem16.pg**
Suppose v_1, v_2, v_3 is an orthogonal set of vectors in $\mathbb{R}^5$. Let w be a vector in $\text{Span}(v_1, v_2, v_3)$ such that
$v_1 \cdot v_1 = 45, v_2 \cdot v_2 = 145.25, v_3 \cdot v_3 = 25,$
$w \cdot v_1 = 135, w \cdot v_2 = -290.5, w \cdot v_3 = -75,$
then $w =$_____v_1+_____v_2+_____v_3.

Correct Answers:

- `3`
- `-2`
- `-3`

105. **(1 pt) Library/TCNJ/TCNJ_OrthogonalSets/problem3.pg**
Let $y = \begin{bmatrix} -6 \\ 9 \end{bmatrix}$ and $u = \begin{bmatrix} -7 \\ -4 \end{bmatrix}$. Describe y as the sum of two orthogonal vectors, x_1 in $Span\{u\}$ and x_2 orthogonal to u.
$x_1 = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$, $x_2 = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.
Correct Answers:

- `-0.646153846153846`
- `-0.369230769230769`
- `-5.35384615384615`
- `9.36923076923077`

106. **(1 pt) Library/TCNJ/TCNJ_OrthogonalSets/problem11.pg**
Given $v = \begin{bmatrix} -9 \\ 9 \\ 0 \\ 8 \end{bmatrix}$, find the coordinates for v in the subspace W spanned by $u_1 = \begin{bmatrix} 3 \\ -3 \\ 3 \\ 0 \end{bmatrix}$, $u_2 = \begin{bmatrix} 3 \\ 0 \\ -3 \\ 1 \end{bmatrix}$ and $u_3 = \begin{bmatrix} -6 \\ -3 \\ 3 \\ 27 \end{bmatrix}$. Note that u_1, u_2 and u_3 are orthogonal.

$v = \_\_\_\_ u_1 + \_\_\_\_ u_2 + \_\_\_\_ u_3$
Correct Answers:

- `-2`
- `-1`
- `0.310344827586207`

107. **(1 pt) Library/TCNJ/TCNJ_OrthogonalSets/problem12.pg**
Given $v = \begin{bmatrix} -5 \\ 3 \\ -9 \end{bmatrix}$, find the coordinates for v in the subspace W spanned by $u_1 = \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ and $u_3 = \begin{bmatrix} 3 \\ -5 \\ -6 \end{bmatrix}$. Note that u_1, u_2 and u_3 are orthogonal.

$v = \_\_\_\_ u_1 + \_\_\_\_ u_2 + \_\_\_\_ u_3$
Correct Answers:

- `-1.57142857142857`
- `-3.8`
- `0.342857142857143`

108. **(1 pt) Library/TCNJ/TCNJ_MatrixEquations/problem1.pg**
Show that the vectors $\langle 1,2,1\rangle, \langle 1,3,1\rangle, \langle 1,4,1\rangle$ do not span R^3 by giving a vector not in their span: ________
Correct Answers:

- `23`

109. **(1 pt) Library/TCNJ/TCNJ_MatrixEquations/problem13.pg**
Do the following sets of vectors span $\mathbb{R}^3$?

[?] 1. $\begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -8 \\ 8 \\ -3 \end{bmatrix}$

[?] 2. $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -7 \\ 7 \end{bmatrix}, \begin{bmatrix} 9 \\ -12 \\ 12 \end{bmatrix}$

[?] 3. $\begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 3 \end{bmatrix}, \begin{bmatrix} -12 \\ 18 \\ -10 \end{bmatrix}$

[?] 4. $\begin{bmatrix} -3 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 9 \\ 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 12 \\ 8 \\ 9 \end{bmatrix}, \begin{bmatrix} 33 \\ 22 \\ 25 \end{bmatrix}$

Correct Answers:

- `No`
- `Yes`
- `No`
- `No`

110. **(1 pt) Library/TCNJ/TCNJ_MatrixEquations/problem3.pg**
Do the columns of the matrix span $\mathbb{R}^3$?

[?] 1. $A = \begin{bmatrix} 5 & 25 & 29 \\ 5 & 24 & 28 \\ 2 & 10 & 12 \end{bmatrix}$

[?] 2. $A = \begin{bmatrix} -4 & -8 & 0 & 48 \\ -4 & -9 & 1 & 52 \\ 2 & 4 & 0 & -24 \end{bmatrix}$

[?] 3. $A = \begin{bmatrix} 9 & 9 & -9 \\ -4 & -4 & 4 \\ 1 & 1 & -1 \end{bmatrix}$

[?] 4. $A = \begin{bmatrix} -3 & 5 \\ -2 & -6 \\ -6 & 8 \end{bmatrix}$

Correct Answers:

- `Yes`
- `No`
- `No`
- `No`

111. **(1 pt) Library/TCNJ/TCNJ_OrthogonalProjections/problem2.pg**
Let $y = \begin{bmatrix} 8 \\ 9 \\ -2 \end{bmatrix}$, $u_1 = \begin{bmatrix} -3 \\ -2 \\ 2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$. Compute the distance d from y to the plane in $\mathbb{R}^3$ spanned by u_1 and u_2.

$d = \_\_\_\_$
Correct Answers:

- `0.171498585142508`

112. **(1 pt) Library/TCNJ/TCNJ_OrthogonalProjections/problem7.pg**
Find the projection of $v = \begin{bmatrix} 5 \\ -5 \\ -2 \end{bmatrix}$ onto the line l of $\mathbb{R}^3$ given by the parametric equation $l = tu$, where $u = \begin{bmatrix} -1 \\ 3 \\ -4 \end{bmatrix}$

$\text{proj}_l(v) = \begin{bmatrix} \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \end{bmatrix}$.

Correct Answers:

- `0.461538461538462`
- `-1.38461538461538`
- `1.84615384615385`

---

113. **(1 pt) Library/TCNJ/TCNJ_OrthogonalProjections/problem5.pg**
All vectors and subspaces are in $\mathbb{R}^n$.

Check the true statements below:

- A. If W is a subspace of $\mathbb{R}^n$ and if v is in both W and $W^\perp$, then v must be the zero vector.
- B. If an $n \times p$ matrix U has orthonormal columns, then $UU^T x = x$ for all x in $\mathbb{R}^n$.
- C. If $y = z_1 + z_2$, where z_1 is in a subspace W and z_2 is in $W^\perp$, then z_1 must be the orthogonal projection of y onto W.
- D. The best approximation to y by elements of a subspace W is given by the vector $y - \text{proj}_W(y)$.
- E. In the Orthogonal Decomposition Theorem, each term $\hat{y} = \frac{y \cdot u_1}{u_1 \cdot u_1} u_1 + ... + \frac{y \cdot u_p}{u_p \cdot u_p} u_p$ is itself an orthogonal projection of y onto a subspace of W.

Correct Answers:

- `ACE`

---

114. **(1 pt) Library/TCNJ/TCNJ_OrthogonalProjections/problem9.pg**
Find the projection of $v = \begin{bmatrix} -2 \\ 13 \\ -17 \end{bmatrix}$ onto the subspace V of $\mathbb{R}^3$ spanned by $\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -6 \\ 6 \\ -2 \end{bmatrix}$.

$\text{proj}_V(v) = \begin{bmatrix} \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \end{bmatrix}$.

Correct Answers:

- `-10.2622950819672`
- `5.77049180327869`
- `-13.9016393442623`

---

115. **(1 pt) Library/TCNJ/TCNJ_OrthogonalProjections/problem6.pg**
Find the shortest distance from the point $P = (-5, 4, -2)$ to a point on the line given by $l : (x, y, z) = (-7t, 5t, -1t)$.
The distance is ____________________.

Correct Answers:

- `1.29614813968157`

---

116. **(1 pt) Library/TCNJ/TCNJ_MatrixLinearTransformation-/problem25.pg**
Let T be an onto linear transformation from $\mathbb{R}^r$ to $\mathbb{R}^s$.

___1. What can one say about the relationship between r and s.

A. $r < s$
B. $r > s$
C. $r \geq s$
D. $r \leq s$
E. There is not enough information to tell

Correct Answers:

- `C`

---

117. **(1 pt) Library/TCNJ/TCNJ_MatrixLinearTransformation-/problem18.pg**
Let T be an injective linear transformation from $\mathbb{R}^r$ to $\mathbb{R}^s$. Let A be the matrix associated to T and let B be the row-echelon reduction of A.

___1. Determine which of the following conditions can hold:

A. $r = 5, s = 7$ and B has 5 pivots.
B. $r = 5, s = 7$ and B has 4 pivots.
C. $r = 7, s = 5$ and B has 5 pivots.
D. $r = 7, s = 5$ and B has 4 pivots.
E. None of the above.

Correct Answers:

- `A`

---

118. **(1 pt) Library/TCNJ/TCNJ_MatrixLinearTransformation-/problem22.pg**
Match the following concepts with the correct definitions:

___1. f is a function from $\mathbb{R}^3$ to $\mathbb{R}^3$
___2. f is an onto function from $\mathbb{R}^3$ to $\mathbb{R}^3$
___3. f is a one-to-one function from $\mathbb{R}^3$ to $\mathbb{R}^3$

A. For every $y \in \mathbb{R}^3$, there is a $x \in \mathbb{R}^3$ such that $f(x) = y$.
B. For every $x \in \mathbb{R}^3$, there is a $y \in \mathbb{R}^3$ such that $f(x) = y$.
C. For every $y \in \mathbb{R}^3$, there is a unique $x \in \mathbb{R}^3$ such that $f(x) = y$.
D. For every $y \in \mathbb{R}^3$, there is at most one $x \in \mathbb{R}^3$ such that $f(x) = y$.

Correct Answers:

- `B`
- `A`
- `D`

119. **(1 pt) Library/TCNJ/TCNJ_MatrixLinearTransformation-/problem20.pg**

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(x,y) = (12x - 12y, 6x - 6y)$$

Find a vector w that is not in the image of T.

$w =$ ____

Correct Answers:

- `23`

---

120. **(1 pt) Library/TCNJ/TCNJ_MatrixLinearTransformation-/problem4.pg**

Let T be the linear transformation defined by

$T(x_1,x_2,x_3) = (5x_1+4x_3, 9x_1-3x_2+x_3, 7x_1+8x_2, 2x_1-x_2-6x_3)$.

Its associated matrix A is an $n \times m$ matrix,

where $n =$ ___, and $m =$ ___.

Correct Answers:

- `4`
- `3`

---

121. **(1 pt) Library/TCNJ/TCNJ_MatrixLinearTransformation-/problem3.pg**

Let T be the linear transformation defined by

$T(x_1,x_2,x_3,x_4,x_5) = -9x_1+6x_2+7x_3+5x_4-8x_5$.

Its associated matrix A is a ___ $\times$ ___ matrix.

Correct Answers:

- `1`
- `5`

---

122. **(1 pt) Library/TCNJ/TCNJ_MatrixLinearTransformation-/problem9.pg**

Consider a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^2$ for which
$T\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}1\\2\\6\end{bmatrix}, \quad T\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}7\\4\\3\end{bmatrix}, \quad T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}5\\9\\0\end{bmatrix}$

Find the matrix A of T.

$$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}.$$

Correct Answers:

- `5`
- `1`
- `7`
- `9`
- `2`
- `4`
- `0`
- `6`
- `3`

---

123. **(1 pt) Library/TCNJ/TCNJ_MatrixLinearTransformation-/problem10.pg**

To every linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$, there is an associated 2×2 matrix. Match the following linear transformations with their associated matrix.

___1. The projection onto the x-axis given by T(x,y)=(x,0)
___2. Reflection about the line y=x
___3. Clockwise rotation by $\pi/2$ radians
___4. Reflection about the x-axis
___5. Reflection about the y-axis
___6. Counter-clockwise rotation by $\pi/2$ radians

A. $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$
B. $\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$
C. $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$
D. $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
E. $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$
F. $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$
G. None of the above

Correct Answers:

- `F`
- `D`
- `E`
- `A`
- `B`
- `C`

---

124. **(1 pt) Library/TCNJ/TCNJ_MatrixLinearTransformation-/problem6.pg**

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first rotates points clockwise through 30° and then reflects points through the line $y = x$.

Find the standard matrix A for T.

$$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}.$$

Correct Answers:

- `-0.5`
- `0.866025403784439`
- `0.866025403784439`
- `0.5`

125. **(1 pt) Library/TCNJ/TCNJ_LinearIndependence/problem2.pg**

Determine whether or not the following sets S of 2×2 matrices are linearly independent.

$\boxed{?}$1. $S=\left\{\begin{pmatrix}1 & 4\\ -4 & 3\end{pmatrix},\begin{pmatrix}4 & 1\\ 1 & -4\end{pmatrix},\begin{pmatrix}4 & -4\\ 1 & 0\end{pmatrix}\right\}$

$\boxed{?}$2. $S=\left\{\begin{pmatrix}-1 & 6\\ 5 & 0\end{pmatrix},\begin{pmatrix}5 & -30\\ -25 & 0\end{pmatrix}\right\}$

$\boxed{?}$3. $S=\left\{\begin{pmatrix}-1 & 6\\ 5 & 0\end{pmatrix},\begin{pmatrix}5 & -55\\ 5 & 0\end{pmatrix}\right\}$

$\boxed{?}$4. $S=\left\{\begin{pmatrix}-1 & 6\\ 5 & 0\end{pmatrix},\begin{pmatrix}5 & -55\\ 5 & 0\end{pmatrix},\begin{pmatrix}1 & -3\\ 9 & 10\end{pmatrix},\begin{pmatrix}6 & -1\\ -30 & -5\end{pmatrix},\begin{pmatrix}17 & -31\\ \pi & e^2\end{pmatrix}\right\}$

Correct Answers:

- `Linearly_Independent`
- `Linearly_Dependent`
- `Linearly_Independent`
- `Linearly_Dependent`

126. **(1 pt) Library/TCNJ/TCNJ_LengthOrthogonality/problem4.pg**

Let W be the set of all vectors $\begin{bmatrix}\text{x}\\ \text{y}\\ \text{x + y}\end{bmatrix}$ with x and y real.

Determine whether each of the following vectors is in $W^{\perp}$.

$\boxed{?}$1. $v=\begin{bmatrix}7\\ -8\\ 8\end{bmatrix}$

$\boxed{?}$2. $v=\begin{bmatrix}2\\ 9\\ -7\end{bmatrix}$

$\boxed{?}$3. $v=\begin{bmatrix}3\\ 3\\ -3\end{bmatrix}$

Correct Answers:

- `No`
- `No`
- `Yes`

127. **(1 pt) Library/TCNJ/TCNJ_LengthOrthogonality/problem7.pg**

Let W be the set of all vectors $\begin{bmatrix}\text{x}\\ \text{y}\\ \text{x + y}\end{bmatrix}$ with x and y real.

Determine whether each of the following vectors is in $W^{\perp}$.

$\boxed{?}$1. $v=\begin{bmatrix}9\\ -8\\ -1\end{bmatrix}$

$\boxed{?}$2. $v=\begin{bmatrix}6\\ -9\\ 7\end{bmatrix}$

$\boxed{?}$3. $v=\begin{bmatrix}5\\ 5\\ -5\end{bmatrix}$

Correct Answers:

- `No`
- `No`
- `Yes`

128. **(1 pt) Library/TCNJ/TCNJ_LengthOrthogonality/problem3.pg**

Let W be the set of all vectors $\begin{bmatrix}\text{x}\\ \text{y}\\ \text{x + y}\end{bmatrix}$ with x and y real. Find a basis of $W^{\perp}$.

$\begin{bmatrix}\_\_\\ \_\_\\ \_\_\end{bmatrix}$

Correct Answers:

- `a multiple of ( -1, -1, 1 )`

129. **(1 pt) Library/TCNJ/TCNJ_LengthOrthogonality/problem6.pg**

Find the angle α between the vectors $\begin{bmatrix}1\\ -4\end{bmatrix}$ and $\begin{bmatrix}5\\ 4\end{bmatrix}$.

$\alpha=$ __________.

Correct Answers:

- `2.00055860589159`

130. **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem2.pg**

Let A = $\begin{bmatrix}2 & -3 & 7\\ -5 & -3 & -7\\ -4 & -5 & -3\\ 2 & 4 & 0\\ -5 & -9 & -1\end{bmatrix}$

Give a basis for the column space of A.

$u=\begin{bmatrix}\_\_\\ \_\_\\ \_\_\\ \_\_\\ \_\_\end{bmatrix}, v=\begin{bmatrix}\_\_\\ \_\_\\ \_\_\\ \_\_\\ \_\_\end{bmatrix}$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{2} \cr
  \mbox{-5} \cr
  \mbox{-4} \cr
  \mbox{2} \cr
  \mbox{-5} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{-3} \cr
  \mbox{-3} \cr
  \mbox{-5} \cr
  \mbox{4} \cr
  \mbox{-9} \cr
  \end{array}\right.\)
  ```
  ```

131. **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem4.pg**

Determine which of the following pairs of functions are linearly independent.

$\boxed{?}$ 1. $f(t) = 3t^2 + 21t \quad , \quad g(t) = 3t^2 - 21t$
$\boxed{?}$ 2. $f(\theta) = \cos(3\theta) \quad , \quad g(\theta) = 3\cos^3(\theta) - 6\cos(\theta)$
$\boxed{?}$ 3. $f(t) = 17t^3 \quad , \quad g(t) = e^x$
$\boxed{?}$ 4. $f(x) = x^3 \quad , \quad g(x) = |x|^3$
$\boxed{?}$ 5. $f(x,y) = 2x - 4y - 12, \quad g(x,y) = -3x + 6y + 18$
$\boxed{?}$ 6. $f(t) = e^{\lambda t}\cos(\mu t) \quad , \quad g(t) = e^{\lambda t}\sin(\mu t) \quad , \mu \neq 0$
$\boxed{?}$ 7. $f(x) = x^2 \quad , \quad g(x) = 4|x|^2$
$\boxed{?}$ 8. $f(x) = e^{3x} \quad , \quad g(x) = e^{3(x-3)}$
$\boxed{?}$ 9. $f(t) = 3t \quad , \quad g(t) = |t|$

Correct Answers:

- `Linearly independent`
- `Linearly dependent`
- `Linearly independent`
- `Linearly independent`
- `Linearly independent`
- `Linearly independent`
- `Linearly dependent`
- `Linearly dependent`
- `Linearly independent`

132. **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem7.pg**

Let W_1 be the set: $\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 5 \end{bmatrix}.$

Determine if W_1 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_1 is a basis.
- B. W_1 is not a basis because it is linearly dependent.
- C. W_1 is not a basis because it does not span $\mathbb{R}^3$.

Let W_2 be the set: $\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix}.$

Determine if W_2 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_2 is not a basis because it is linearly dependent.
- B. W_2 is not a basis because it does not span $\mathbb{R}^3$.
- C. W_2 is a basis.

Correct Answers:

- `B`
- `B`

133. **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem1.pg**

Let A = $\begin{bmatrix} -5 & 4 & -1 & -2 & 1 \\ 4 & 4 & 2 & -4 & 3 \\ -14 & 4 & -4 & 0 & -1 \end{bmatrix}$

Give a basis for the row space of A.

$u = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, v = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{-5} \cr
  \mbox{4} \cr
  \mbox{-1} \cr
  \mbox{-2} \cr
  \mbox{1} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{4} \cr
  \mbox{4} \cr
  \mbox{2} \cr
  \mbox{-4} \cr
  \mbox{3} \cr
  \end{array}\right.\)
  ```

134. **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem3.pg**

Let A = $\begin{bmatrix} 2 & -3 & 5 & 1 & 1 \\ 2 & 4 & 5 & 1 & 3 \\ 2 & -10 & 5 & 1 & -1 \end{bmatrix}$

Give a basis for the row space of A.

$u = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, v = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{2} \cr
  \mbox{-3} \cr
  \mbox{5} \cr
  \mbox{1} \cr
  \mbox{1} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{2} \cr
  \mbox{4} \cr
  \mbox{5} \cr
  \mbox{1} \cr
  \mbox{3} \cr
  \end{array}\right.\)
  ```

135. **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem5.pg**

Let W_1 be the set: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Determine if W_1 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_1 is not a basis because it is linearly dependent.
- B. W_1 is not a basis because it does not span $\mathbb{R}^3$.
- C. W_1 is a basis.

Let W_2 be the set: $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

Determine if W_2 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_2 is not a basis because it is linearly dependent.
- B. W_2 is not a basis because it does not span $\mathbb{R}^3$.
- C. W_2 is a basis.

Correct Answers:

- `C`
- `AB`

136. **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem9.pg**

Check the true statements below:

- A. If $H = Span\{b_1, ..., b_p\}$, then $\{b_1, ..., b_p\}$ is a basis for H.
- B. A basis is a spanning set that is as large as possible.
- C. The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.
- D. In some cases, the linear dependence relations amoung the columns of a matrix can be affected by certain elementary row operations on the matrix.
- E. A single vector by itself is linearly dependent.

Correct Answers:

- `C`

137. **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem10.pg**

Determine whether each set $\{p_1, p_2\}$ is a linearly independent set in $\mathbb{P}_3$. Type "yes" or "no" for each answer.

The polynomials $p_1(t) = 1 + t^2$ and $p_2(t) = 1 - t^2$. ____

The polynomials $p_1(t) = 2t + t^2$ and $p_2(t) = 1 + t$. ____

The polynomials $p_1(t) = 2t - 4t^2$ and $p_2(t) = 6t^2 - 3t$. ____

Correct Answers:

- `yes`
- `yes`
- `no`

138. **(1 pt) Library/TCNJ/TCNJ_VectorEquations/problem4.pg**

Let $u = \begin{bmatrix} 2 \\ -4 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} 4 \\ -9 \\ 7 \end{bmatrix}$

Find two vectors in $span\{u, v\}$ that are not multiples of u or v and show the weights on u and v used to generate them.

___ $u +$ ___ $v =$ _______

___ $u +$ ___ $v =$ _______

Correct Answers:

-
-
-
-
-
-

139. **(1 pt) Library/TCNJ/TCNJ_VectorEquations/problem7.pg**

Let $A = \begin{bmatrix} -3 & 3 & 9 \\ 1 & 1 & -1 \\ 4 & -2 & -9 \end{bmatrix}$ and $b = \begin{bmatrix} 6 \\ -4 \\ 4 \end{bmatrix}$.

Denote the columns of A by a_1, a_2, a_3, and let $W = span\{a_1, a_2, a_3\}$.

$\boxed{?}$ 1. Determine if b is in W

$\boxed{?}$ 2. Determine if b is in $\{a_1, a_2, a_3\}$

How many vectors are in $\{a_1, a_2, a_3\}$? (For infinitely many, enter -1) ____

How many vectors are in W? (For infinitely many, enter -1) ____

Correct Answers:

- `Yes`
- `No`
- `3`
- `-1`

140. **(1 pt) Library/TCNJ/TCNJ_VectorEquations/problem1.pg**

Let x, y, z be vectors and suppose $z = -3x - 1y$ and $w = 3x + 2y + 1z$.

Mark the statements below that must be true.

- A. Span(x, y)=Span(w)
- B. Span(y, w)= Span(z)
- C. Span(y) = Span(w)
- D. Span(x, y) = Span(x, w, z)

Correct Answers:

- `CD`

141. **(1 pt) Library/TCNJ/TCNJ_LengthOrthogonality/problem2.pg**
All vectors are in $\mathbb{R}^n$.

Check the true statements below:

- A. If x is orthogonal to every vector in a subspace W, then x is in $W^\perp$.
- B. If $||u||^2 + ||v||^2 = ||u+v||^2$, then u and v are orthogonal.
- C. For an $m \times n$ matrix A, vectors in the null space of A are orthogonal to vectors in the row space of A.
- D. $u \cdot v - v \cdot u = 0$.
- E. For any scalar c, $||cv|| = c||v||$.

Correct Answers:

- `ABCD`

142. **(1 pt) Library/TCNJ/TCNJ_LengthOrthogonality/problem4.pg**
Let W be the set of all vectors $\begin{bmatrix} \text{x} \\ \text{y} \\ \text{x + y} \end{bmatrix}$ with x and y real.
Determine whether each of the following vectors is in $W^\perp$.

$\boxed{?}$ 1. $v = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$

$\boxed{?}$ 2. $v = \begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}$

$\boxed{?}$ 3. $v = \begin{bmatrix} 8 \\ 1 \\ -8 \end{bmatrix}$

Correct Answers:

- `Yes`
- `No`
- `No`

143. **(1 pt) Library/TCNJ/TCNJ_LengthOrthogonality/problem6.pg**
Find the angle α between the vectors $\begin{bmatrix} -4 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$.
$\alpha =$ __________.

Correct Answers:

- `2.89661399046293`

144. **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem2.pg**
Let A = $\begin{bmatrix} 5 & 1 & 9 \\ -3 & 4 & -10 \\ -2 & -2 & -2 \\ 1 & 2 & 0 \\ 3 & 7 & -1 \end{bmatrix}$

Give a basis for the column space of A.

$u = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$, $v = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- ```
\(\displaystyle\left.\begin{array}{c}
\mbox{5} \cr
\mbox{-3} \cr
\mbox{-2} \cr
\mbox{1} \cr
\mbox{3} \cr
\end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
\mbox{1} \cr
\mbox{4} \cr
\mbox{-2} \cr
\mbox{2} \cr
\mbox{7} \cr
\end{array}\right.\)
```

**145.** **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem7.pg**
Let $W_1$ be the set: $\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 5 \end{bmatrix}$.
Determine if $W_1$ is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. $W_1$ is a basis.
- B. $W_1$ is not a basis because it does not span $\mathbb{R}^3$.
- C. $W_1$ is not a basis because it is linearly dependent.

Let $W_2$ be the set: $\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix}$.
Determine if $W_2$ is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. $W_2$ is a basis.
- B. $W_2$ is not a basis because it does not span $\mathbb{R}^3$.
- C. $W_2$ is not a basis because it is linearly dependent.

*Correct Answers:*

- `C`
- `B`

**146.** **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem1.pg**
Let A = $\begin{bmatrix} -5 & 3 & -2 & -5 & -4 \\ 5 & -3 & 2 & -10 & -7 \\ -15 & 9 & -6 & 0 & -1 \end{bmatrix}$

Give a basis for the row space of A.
```

$$u = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, v = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

Correct Answers:

- `\(\displaystyle\left.\begin{array}{c}`
  `\mbox{-5} \cr`
  `\mbox{3} \cr`
  `\mbox{-2} \cr`
  `\mbox{-5} \cr`
  `\mbox{-4} \cr`
  `\end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}`
  `\mbox{5} \cr`
  `\mbox{-3} \cr`
  `\mbox{2} \cr`
  `\mbox{-10} \cr`
  `\mbox{-7} \cr`
  `\end{array}\right.\)`

147. **(1 pt) Library/TCNJ/TCNJ_VectorEquations/problem8.pg**

Find the value of a for which

$$v = \begin{bmatrix} 5 \\ a \\ -7 \\ -7 \end{bmatrix}$$

is in the set

$$H = span\left\{ \begin{bmatrix} 5 \\ 2 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -3 \\ -4 \end{bmatrix} \right\}.$$

$a =$ _____

Correct Answers:

- -1

148. **(1 pt) Library/TCNJ/TCNJ_VectorEquations/problem5.pg**

Let $H = span\{u, v\}$. For each of the following sets of vectors determine whether H is a line or a plane.

? 1. $u = \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}, v = \begin{bmatrix} -10 \\ 5 \\ -5 \end{bmatrix},$

? 2. $u = \begin{bmatrix} 8 \\ -2 \\ -3 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$

? 3. $u = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}, v = \begin{bmatrix} -12 \\ -12 \\ -4 \end{bmatrix},$

? 4. $u = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}, v = \begin{bmatrix} -8 \\ 6 \\ -4 \end{bmatrix},$

Correct Answers:

- Plane
- Line
- Line
- Plane

149. **(1 pt) Library/TCNJ/TCNJ_VectorEquations/problem9.pg**

Find the value of a for which

$$v = \begin{bmatrix} -2 \\ -1 \\ -4 \\ a \end{bmatrix}$$

is in the set

$$H = span\left\{ \begin{bmatrix} -1 \\ -3 \\ -5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -5 \\ -2 \end{bmatrix} \right\}.$$

$a =$ _____

Correct Answers:

- 13

150. **(1 pt) Library/TCNJ/TCNJ_VectorEquations/problem6.pg**

Let $u = \langle -1, -2, -1 \rangle$ and $v = \langle -1, 1, 2 \rangle$

Find a vector w not in $span\{u, v\}$.

$w =$ __________

Correct Answers:

- 23

151. **(1 pt) Library/TCNJ/TCNJ_LinearTransformations/problem20.pg**

Let $A = \begin{bmatrix} -1 & -8 & -9 \\ 6 & 3 & -9 \end{bmatrix}$.

Define the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ as $T(x) = Ax$.

Find the images of $u = \begin{bmatrix} -5 \\ 5 \\ 1 \end{bmatrix}$ and $v = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ under T .

$$T(u) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

$$T(v) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

Correct Answers:

- -44
- -24
- (a*-1+-8*b+-9*c)
- (a*6+3*b+-9*c)