Section 5.2 21

- The determinant of A is the product of the diagonal entries in A. FALSE in general. True is A is triangular.
- An elementary row operation on A does not change the determinant. FALSE interchanging rows and multiply a row by a constant changes the determinant.
- (det A)*(det B)=det(AB) TRUE Yay!
- If $\lambda + 5$ is a factor of the characteristic polynomial of A, then 5 is an eigenvalue of A. FALSE -5 is an eigenvalue. (The zeros are the eigenvalues.

- If A is 3×3, with columns $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ then det A equals the volume of the parallelepiped determined $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$. FALSE it's the absolute value of the determinant. We can prove this by thinking of the columns as changing under a linear transformation from the unit cube. When we apply a transformation, the volumes gets multiplied by determinant.
- det A^T=(-1)det A FALSE detA^T=det A.
- The multiplicity of a root r of a characteristic equation of A is called the algebraic multiplicity of r as an eigenvalue of A. TRUE That's the definition.
- A row replacement operation on A does not change the eigenvalues. FALSE.

- A is diagonalizable if $A = PDP^{-1}$ for some matrix D and some invertible matrix P. FALSE D must be a diagonal matrix.
- If $\mathbb{R}^n$ has a basis of eigenvectors of A, then A is diagonalizable. TRUE In this case we can construct a P which will be invertible. And a D.
- A is diagonalizable if and only if A has n eigenvalues, counting multiplicity. FALSE It always has n eigenvalues, counting multiplicity.
- If A is diagonalizable, then A is invertible. FALSE It's invertible if it doesn't have zero an eigenvector but this doesn't affect diagonalizabilty.

- A is diagonalizable if A has n eigenvectors. The eigenvectors must be linear independent.
- If A is diagonalizable, then A had n distinct eigenvalues. FALSE It could have repeated eigenvalues as long as the basis of each eigenspace is equal to the multiplicity of that eigenvalue. The converse is true however.
- If $AP = PD$, with D diagonal then the nonzero columns of P must be the eigenvectors of A. TRUE. Each column of PD is a column of P times A and is equal to the corresponding entry in D times the vector P. This satisfies the eigenvector definition as long as the column is nonzero.
- If A is invertible, then A is diagonalizable. FALSE these are not directly related.

- $\mathbf{v} \cdot \mathbf{v} = ||\mathbf{v}||^2$ TRUE by definition.
- For any scalar c, $\mathbf{u} \cdot (c\mathbf{v}) = c(\mathbf{u} \cdot \mathbf{v})$. TRUE
- If the distance from $\mathbf{u}$ to $\mathbf{v}$ equals the distance from $\mathbf{u}$ to $-\mathbf{v}$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. TRUE
- For a square matrix A, vectors in Col A are orthogonal to vectors in Nul A. FALSE Counterexample $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$
- If vectors $\mathbf{v}_1, \ldots, \mathbf{v_p}$ span a subspace W and if $\mathbf{x}$ is orthogonal to each $\mathbf{v_j}$ for $j = 1, \ldots, p$ then x is in $W^{\perp}$. TRUE since any vector in W can be written as linear combination of basis vectors and dot product splits up nicely over sums and constants.

Section 6.1 20

- $\mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} = 0$ TRUE since dot product is commutative.
- For any scalar c, $||c\mathbf{v}|| = c||v||$. FALSE need absolute value of c.
- If $\mathbf{x}$ is orthogonal to every vector in a subspace W, then $\mathbf{x}$ is in $W^{\perp}$. TRUE by definition of $W^{\perp}$
- If $||\mathbf{u}||^2 + ||\mathbf{v}||^2 = ||\mathbf{u} + \mathbf{v}||^2$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. TRUE By Pythagorean Theorem.
- For an $m \times n$ matrix A, vectors in the null space of A are orthogonal to vectors in the row space of A. TRUE by Thm 3