Determining the Hubble Constant from Observations of Distance Modulus and Redshift for Type Ia Supernovae

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Background & Purpose

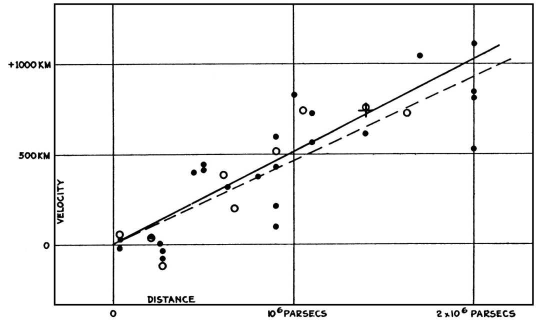
- Universe is expanding, modeled by equation

$$v = H_0 d$$

where v = recessional velocity of galaxy

H_0 = Hubble's constant

d = distance of galaxy



Plot of recessional velocity v vs distance d for several galaxies. The slope of the best-fit line is the Hubble constant H_0.

- Disagreement on H_0: current accepted value is $H_0 = 72 \pm 2$ km s^{-1} Mpc^{-1}.
- Prior research: Scolnic 2018[1] - Pantheon Dataset
  - Observed **distance modulus** and **redshift** for 1048 **Type Ia supernovae**
  - distance modulus ~ distance d, redshift ~ velocity v.

Purpose of experiment: To determine a precise value of H_0 by fitting a model to the Pantheon dataset and using the model parameters to calculate H_0.

Procedure

- Definitions of parameters:
  - Ω_m = mass density of universe
  - $w = p/\rho$ = pressure / energy density = equation of state of universe
  - Δdm = distance modulus offset between the model and data

1. Plot distance modulus vs redshift values for Pantheon dataset of 1048 points.
2. Initialize Markov Chain with $(\Omega_{m0}, w_0, \Delta dm_0) = (0.0,-1.0,0.0)$.
3. Iterate through Markov Chain. Each Markov-Chain Monte Carlo (MCMC) iteration:
   a. Draw the latest value of chain θ, as original parameters. Draw proposal θ' from normal distribution centered on θ.
   b. Calculate log likelihood function of original parameters $\ln f(\Omega_m, w, \Delta dm)$ and proposed parameters $\ln f(\Omega_m', w', \Delta dm')$.

$$\ln f(\Omega_m, w, \Delta_{dm}) = \sum_{i=1}^{N} \left[ln\left(\frac{1}{\sigma_{dm_i}\sqrt{2\pi}}\right) - \frac{1}{2}\frac{(dm_{obs} - dm_{predicted}(\Omega_m, w, \Delta_{dm}))^2}{(\sigma_{dm_i})^2} \right]$$

   c. Calculate $\ln(r)$ where $0<r<1$.

   d. If $\ln f(\Omega_m', w', \Delta dm') - \ln f(\Omega_m, w, \Delta dm) > \ln(r)$,
   then add proposed parameters $\theta' = (\Omega_m', w', \Delta dm')$ to chain. Otherwise add current parameters $\theta = (\Omega_m, w, \Delta dm)$ again.
4. After desired number of iterations, for each parameter:
   - mean → experimental value
   - standard deviation → uncertainty in experimental value

Results

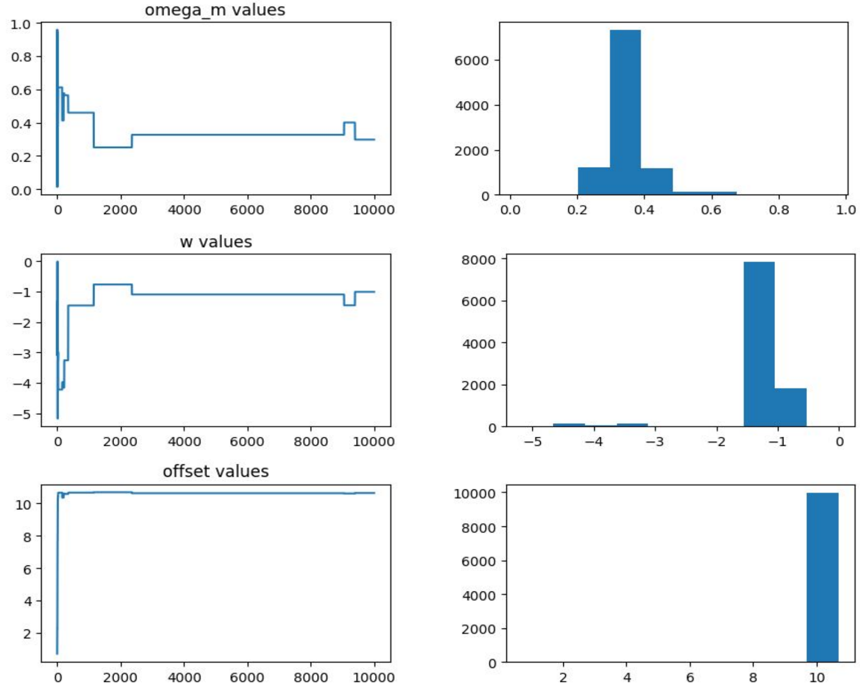
- Experimental values: $\mathbf{\Omega_m = 0.337 \pm 0.066}$, $\mathbf{w = -1.182 \pm 0.521}$, $\mathbf{\Delta dm = 10.634 \pm 0.261}$

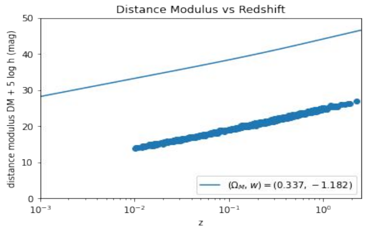
Traces | Histograms

Ω_m

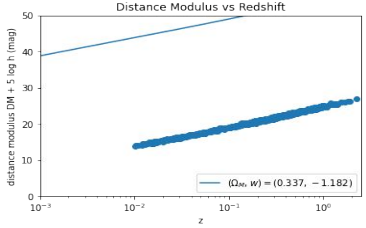
w

Δdm

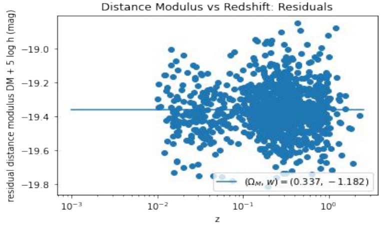




Plot of Pantheon dataset and model **before** adding Δdm



Plot of Pantheon dataset and model **after** adding Δdm



Residual plot

Discussion & Conclusion

	Scolnic (2018)	Shi and Farr	z-score	p-value	Significant? (α=0.05)
Ω_m	0.307 ± 0.012	0.337 ± 0.066	0.447	0.655	no
w	-1.026 ± 0.041	-1.182 ± 0.521	-0.299	0.765	no

- Our Ω_m and w values are not significantly different from Scolnic (2018)
- Adding Δdm decreased quality of fit → **sampling process may be faulty!**
- Unable to calculate H_0 from parameters → **results are inconclusive**

Future goals:

- Minimize systematic error in model fit → improve precision and accuracy for Ω_m, w, Δ_{dm}
- Determine the value of the Hubble constant H_0 based on Ω_m, w, Δ_{dm}

References & Acknowledgments

- Papers consulted

    - Scolnic, D.M. The Astrophysical Journal, 859:101 (2018).
    - Kirshner, Robert. PNAS, 101:8-13 (2004).
    - Hogg, David W. "Data Analysis Recipes: Using Markov Chain Monte Carlo," (2017).
    - Hogg, David W. "Distance Measures in Cosmology," (2000).
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- MCMC program written in Python and its libraries SciPy, NumPy, Matplotlib, and Jupyter Notebook, courtesy of Python Software Foundation <www.python.org>
- Log likelihood function uses cosmological distance functions from Hogg (2000)
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