**PyMISES Aerodynamic Expert Guide**

**1. Mathematical Foundation**

**1.1 Euler Equations**

The core of PyMISES is based on the steady, two-dimensional Euler equations formulated in integral form:

**Mass Conservation:**

∮ ρq·n ds = 0

**Momentum Conservation:**

∮(ρ(q·n)q + pn) ds = 0

**Energy Conservation:**

∮(ρ(q·n)h) ds = 0

Where:

* ρ is density
* q is velocity vector
* n is unit normal vector
* p is pressure
* h is total enthalpy

**1.2 Streamline-Based Discretization**

The unique aspect of PyMISES is the use of an intrinsic streamline grid where:

1. One set of coordinate lines corresponds to streamlines
2. There is no convection across the corresponding faces of conservation cells
3. Mass flux and stagnation enthalpy are constant along streamtubes

This leads to several advantages:

* Reduction from 4 to 2 unknowns per grid node
* No numerical diffusion of entropy or enthalpy across streamlines
* Faithful reproduction of analytic information propagation mechanisms
* Natural adaptation to the flow field

**1.3 Artificial Dissipation**

For transonic flows, artificial dissipation is implemented as a bulk viscosity-like term:

λ = { 0, M < Mc

{ Cd(1-(Mc/M)²)/(2M²), M > Mc

Where:

* Mc is threshold Mach number (typically 0.9-0.95)
* Cd is dissipation coefficient (typically 0.5-1.0)

This approach provides:

* Stable shock capturing
* Prevention of expansion shocks
* Minimal numerical diffusion in subsonic regions

**1.4 Integral Boundary Layer Equations**

The viscous effects are modeled using integral boundary layer equations:

**Momentum Integral Equation:**

θ(dθ/dx) = (Cf/2) - (H+2-M²ₑ)(θ/Uₑ)(dUₑ/dx)

**Shape Parameter Equation:**

θ(dH\*/dx) = 2CD/H\* - Cf/2·(2H\*\*/H\* + 1 - H) - θ/Uₑ·(dUₑ/dx)

Where:

* θ is momentum thickness
* δ\* is displacement thickness
* H = δ\*/θ is shape parameter
* H\* is kinetic energy shape parameter
* H\*\* is density thickness shape parameter
* Cf is skin friction coefficient
* CD is dissipation coefficient
* Ue is edge velocity
* Me is edge Mach number

**1.5 Closure Relations**

**1.5.1 Laminar Closure**

For laminar flow, closure relations are based on the Falkner-Skan profile family:

H\* = 1.515 + 0.076(Hk-4)²/Hk, Hk < 4

H\* = 1.515 + 0.040(Hk-4)²/Hk, Hk > 4

Cf·Reθ = -0.067 + 0.01977(7.4-Hk)²/(Hk-1)

2CD/H\* = 0.207 + 0.00205(4-Hk)⁵·⁵, Hk < 4

2CD/H\* = 0.207 - 0.003(Hk-4)², Hk > 4

Where Hk is the kinematic shape parameter.

**1.5.2 Turbulent Closure**

For turbulent flow:

Cf = 0.3e⁻¹·³³Hk/(log₁₀(Reθ))¹·⁷⁴⁺⁰·³¹Hk · Fc

Where Fc is a compressibility correction.

The energy shape parameter is given by:

H\* = (1.505 + 400/Reθ) / Hk·(1 + (H₀-H)²(0.04 + 0.007ln(Reθ)/Hk))

The dissipation coefficient uses a lag-entrainment approach:

CD = Cf·Us² + CT(1-Us)

Where CT is a maximum shear stress coefficient that follows a lag equation to account for non-equilibrium effects:

δ dCT/dx = KC(CT\_EQ^(1/2) - CT^(1/2))

With KC = 5.6 as the lag constant and δ as the boundary layer thickness.

**1.6 Transition Prediction**

Based on Drela's modified Abu-Ghannam/Shaw transition criterion, transition is predicted using a hybrid approach combining the eⁿ method with a bypass transition model:

**1.6.1 Amplification Factor Method**

The amplification factor n is governed by:

θ(dn/dx) = f(H,Reθ) + g(H,Reθ)

Where:

* f(H,Reθ) is the Orr-Sommerfeld-based TS wave growth rate
* g(H,Reθ) is a bypass transition term that activates as Reθ approaches a critical value

The g term is defined as:

g(H,Reθ) = { 0 r < 0

{ A(3r² - 2r³) 0 < r < 1

{ A r > 1

r = (1/B)((Reθ/Reθs) - 1) + 1/2

Where:

* A = 0.10 (amplification rate constant)
* B = 0.30 (ramp width parameter)
* Reθs is the critical Reynolds number

**1.6.2 Critical Reynolds Number**

The critical Reynolds number is modified from the original Abu-Ghannam/Shaw formulation:

Reθs(H,ñcrit) = 155 + 89.0[0.25tanh(10/(H-1) - 5.5) + 1](ñcrit)^1.25

Where ñcrit is related to the turbulence level τ by:

τ' = 2.7tanh(τ/2.7)

ñcrit(τ) = -8.43 - 2.4ln(τ'/100)

This formulation addresses the ill-posedness of the original AGS criterion by:

1. Using H instead of λ to parameterize boundary layer instability
2. Adding a smooth transition from stability to instability
3. Accounting for viscous-inviscid interaction effects

**1.6.3 Initial Turbulence Level**

For transition modeling in varying edge velocity fields (e.g., turbine cascades), the amplification equation is modified to:

(dn̄/dx) + (dlnue/dx) = (1/θ)[f(H,Reθ) + g(H,Reθ)]

This accounts for the acceleration or deceleration of the external flow.

**1.7 Viscous-Inviscid Coupling**

Coupling is achieved through the displacement body concept:

* Inviscid flow sees a body displaced from the physical body by δ\*
* Boundary layer equations are integrated into the global Newton system
* No separate iteration between solvers

This provides robust convergence even with separation bubbles present.

**1.8 Inverse Design Framework**

The inverse design capability includes:

**Full Inverse:**

* Pressure specified on entire airfoil
* Leading and trailing edge closure constraints
* Free parameters for well-posed problem

**Mixed Inverse:**

* Pressure specified on part of airfoil
* Geometry fixed elsewhere
* Continuity enforced at segment boundaries
* Additional free parameters for segment continuity

**2. Numerical Solution Procedure**

**2.1 Newton Method**

The coupled system of equations is solved using Newton's method:

[dF/dU]·ΔU = -F(U)

U^(v+1) = U^(v) + ΔU

Where:

* F(U) is the residual vector
* U is the solution vector (ρ, δn at each node)
* [dF/dU] is the Jacobian matrix
* v is the iteration index

**2.2 Jacobian Structure**

The Jacobian matrix has a block structure with:

* Four block diagonals (A, B, C, Z) for local variables
* Additional rows/columns for global variables and constraints
* Typical size ~4000-10000 equations

**2.3 Linear System Solution**

The linear system is solved using block Gaussian elimination:

* Forward sweep to eliminate Z and B blocks
* Backward sweep to eliminate modified C blocks
* Separate solution for global variables
* Back-substitution for local variables

**2.4 Convergence Acceleration**

Convergence is accelerated through:

* Proper under-relaxation in early iterations (r < 1)
* Full Newton steps (r = 1) once near solution
* Grid redistribution when needed
* Careful initialization strategies

**3. Implementation Guidelines**

**3.1 Grid Generation**

1. Start with elliptic grid generation (Thompson method)
2. Initialize streamlines for incompressible flow
3. Allow grid adaptation during Newton iterations
4. Redistribute grid periodically to maintain quality
5. Special treatment for leading edge stagnation point

**3.2 Boundary Conditions**

1. **Wall Boundary Conditions:**
   * No-flow-through (streamline coincides with wall)
   * Displacement thickness offset in viscous mode
2. **Far-Field Conditions:**
   * Pressure specification based on potential flow (vortex + source + doublet)
   * Consistent inflow/outflow angle specification
   * Treatment of wake streamline
3. **Cascade Conditions:**
   * Periodicity in tangential direction
   * Specified inlet flow angle
   * Kutta condition at trailing edge

**3.3 Stability Considerations**

1. Enhanced artificial dissipation near shocks
2. Pressure correction to eliminate grid sawtooth modes
3. Careful treatment of stagnation points
4. Edge velocity correction in boundary layer coupling
5. Special attention to transition region to avoid numerical instabilities

**3.4 Performance Metrics**

1. **Lift and Moment:**
   * Integrated from surface pressure distribution
   * Consistent with far-field circulation
2. **Drag Components:**
   * Profile drag from momentum deficit in wake
   * Wave drag from entropy increase across shock
   * Induced drag from far-field analysis
3. **Boundary Layer Parameters:**
   * Displacement thickness (δ\*)
   * Momentum thickness (θ)
   * Shape factor (H)
   * Separation locations

**4. Validation Test Cases**

The following test cases are recommended for validation:

1. **Joukowski airfoil:** Compare with exact incompressible solution
2. **RAE 2822 (Case 6):** Attached transonic flow with shock
3. **RAE 2822 (Case 10):** Transonic flow with shock-induced separation
4. **NACA 4412 at high angle:** Trailing edge separation
5. **LA203A at low Reynolds number:** Transitional separation bubbles
6. **Supercritical cascade:** For periodicity and shock capturing

**5. Common Issues and Remedies**

1. **Convergence Difficulties:**
   * Increase artificial dissipation (Cd)
   * Lower threshold Mach number (Mc)
   * Use stronger under-relaxation
   * Improve initial grid quality
2. **Shock Instabilities:**
   * Refine grid near shock
   * Increase artificial dissipation locally
   * Smooth pressure distribution in early iterations
3. **Separation Problems:**
   * Carefully tune boundary layer closure parameters
   * Improve grid resolution in separation region
   * Modify transition criteria if needed
4. **Transition Prediction Issues:**
   * Use the modified transition model based on H instead of λ
   * Verify proper coupling between boundary layer and inviscid flow
   * Consider appropriate transition ramp parameters (A and B)
5. **Inverse Design Issues:**
   * Smooth target pressure distribution
   * Add more degrees of freedom
   * Enforce reasonable geometric constraints
   * Start from well-converged direct solution

**6. Advanced Features**

**6.1 Multi-Point Design**

For multi-point design:

1. Perform inverse design at primary design point
2. Check performance at off-design conditions
3. Modify target pressure to improve off-design performance
4. Repeat until satisfactory compromise is achieved

**6.2 Performance Optimization**

For direct optimization:

1. Parameterize geometry (e.g., control points)
2. Define objective function (e.g., L/D, Cd at fixed Cl)
3. Use gradient-based methods leveraging sensitivity information from Newton solver
4. Apply constraints on geometry and aerodynamic parameters

**6.3 Sensitivity Analysis**

Newton method automatically provides sensitivity information:

* dCl/dα: Lift curve slope
* dCd/dM: Drag-divergence characteristics
* dCm/dCl: Stability derivatives

These can be used for design refinement without additional solutions.