PHY 102
Introductory Mechanics and Properties of Matter
(3 units)

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Recommended Texts:

Fundamentals of Physics:	Resnick and Halliday
Advanced Level Physics:	Nelkon and Parker
College Physics:	Any Good Author
College Physics:	Schaum Series (Worked Examples)
Mechanics and Properties of Matter	Idowu Farai

Course content

- Useful Mathematics.
- Quantities, Units and Their Dimensions.
- Introduction to Vectors.
- Kinematics.
- Newton's laws of motion.
- Gravitational Force of Attraction.
- Work and Energy.
- Dynamics of Rotating Rigid Bodies.
- Simple Harmonic Motion.
- Elastic Properties.
- Some Properties of Liquids.

Useful Mathematics

- There is so much order in the physical world.

- The orderliness in the physical world has allowed for accurate or near accurate prediction of the behaviours of many physical systems using laws or theorems.

- Laws and theorems are often expressed in the form of mathematical functions.

- Mathematical functions can be represented in a variety of ways: Table, Formula and Graph.

a) Common functions include **Periodic and Sinusoidal functions.**

- A periodic function repeats its value at regular intervals. Examples of periodic functions are trigonometric functions, inverse trigonometric functions and hyperbolic functions.
- A periodic wave repeats its shape at regular intervals.

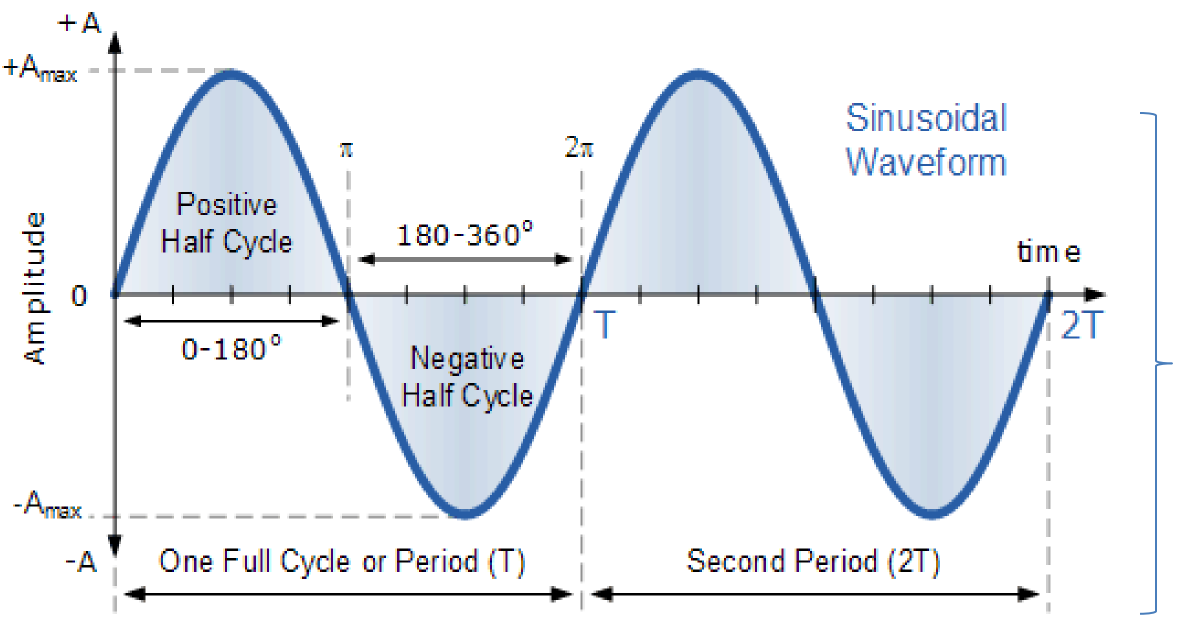
- A function that is based on the sine function which oscillates between a low (minimum) value and a high (maximum) value at regular intervals is known as Sinusoidal function.

- Since the cosine function ($y_2 = \cos B$) is a shifted sine function ($y_1 = \sin A$), cosine function is an example of sinusoidal function. Thus, both sine and cosine functions are examples of Sinusoidal functions.

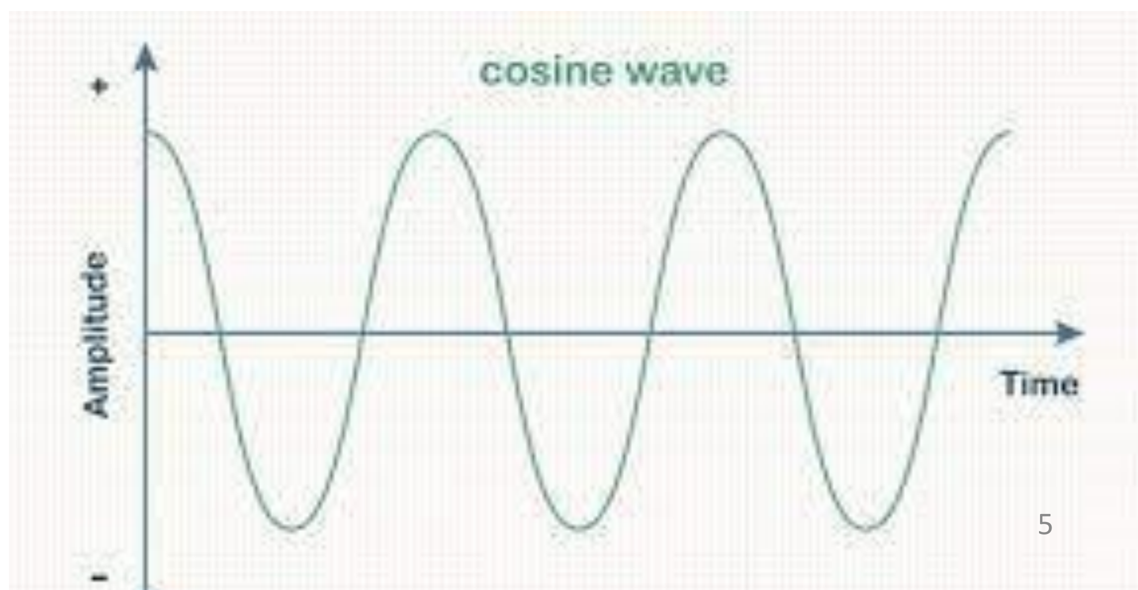
- Sinusoidal functions are necessarily periodic functions but not all periodic functions are sinusoidal functions.

Note that A and B as used above may contain more than one term.

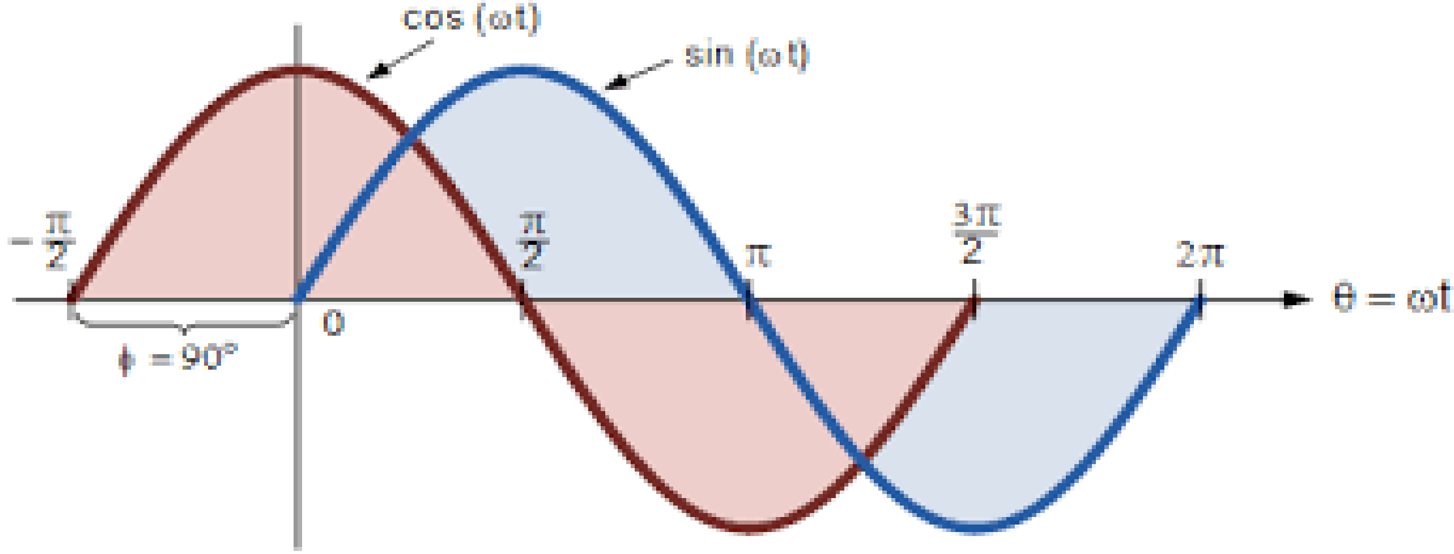
❖ The graph of either Sine or Cosine function is known as Sinusoidal wave.



Sine wave



Sine and Cosine waves: The cosine wave leads the sine wave by 90^0 or $\frac{\pi}{2}$ $radians$ as seen below.



Examples of Sinusoidal wave or Sinusoidal function include AC Voltage, AC Current, Simple harmonic motion (SHM) and Displacement in a wave motion.

Sometimes we may need to add (or subtract) two or more sinusoidal functions to get the functions that will describe the periodic behaviour of some other systems. There are standard ways of adding such functions:

$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\cos^2 A + \sin^2 A = 1$$

$$\cos^2 A - \sin^2 A = \cos 2A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

b.) Logarithm Functions and Indices

We can recall that if x is related to *y* by: $\boldsymbol{y = \log_b x}$, then, y is related to x by: $\boldsymbol{x = b^{y}}$.

e.g. $\boldsymbol{2^3 = 8}$ ***or*** $\boldsymbol{\log_2 8 = 3}$

The letter b is called the base of the logarithm and it is usually omitted when it is 10.

e.g. $\boldsymbol{1000 = 10^3}$ ***or*** $\boldsymbol{\log 1000 = 3}$

For the exponential function, $e^x = N$, then $x = \log_e N$, where $e \approx 2.7183$ and $\log_e e = 1$.

This logarithm is called a Naperian or natural log because many natural systems are described by exponential functions.

Properties of log functions:

$$\log MN = \log M + \log N, \qquad a^p \times a^q = a^{p+q}$$

$$\log\left(\frac{M}{N}\right) = \log M - \log N, \qquad a^0 = 1$$

$$\log M^p = p \log M, \qquad a^{-p} = \frac{1}{a^p}$$

Example

For a mass of a gas at pressure P and volume V, $\mathbf{PV^{\gamma} = C}$, where γ and C are constants. From the values of *P* and *V* obtained in an experiment, determine the values of γ and C and estimate P when V = 100 cm^3

Take log of both sides: $\log P + \gamma \log V = \log C$

Rearrange: $\log P = -\gamma \log V + \log C$

Compare with straight line graph: $y = mx + c$

Plot a graph of ***log P*** on y-axis against ***log V*** on x-axis

1. ***Slope = - γ and intercept on y-axis = log C***
2. ***Read log P corresponding to log 100, hence find P***

Derived Functions

For linear equations like: $\boldsymbol{y = 4x + 2}$, the rate of change of *y* with *x* is constant and it is equal to the slope of the graph of *y* against *x*

For functions like: $\boldsymbol{y = 2x^2}$ ***or*** $\boldsymbol{y = 5x^3 + 2x}$, the slope is different at different values of *x*.

A new equation, called ***the derivative*** of the original equation is required to get the slope at any value of x

Differentiation Operation:

It can be shown that if: $\boldsymbol{y = kx^n}$, then its first derivative is

$$\frac{dy}{dx} = nkx^{n-1}.$$

That is, if $y = 8x^5 + 4x^3 + 2x + 7$.

The first derivative is $\frac{dy}{dx} = 40x^4 + 12x^2 + 2$

The second derivative is: $\frac{d^2y}{dx^2} = 160x^3 + 24x$ and so on

Exercise: Find the first and the second derivatives of $x = ut + \frac{1}{2}at^2$, if *u* and *a* are constants.

Some common derivatives of sinusoidal functions are:

$$y = \cos x, \quad \frac{dy}{dx} = -\sin x \qquad y = \sin x, \quad \frac{dy}{dx} = \cos x$$

$$y = B \sin kx, \quad \frac{dy}{dx} = B k \cos kx$$

$$y = B \cos ax, \quad \frac{dy}{dx} = -B a \sin ax$$

Integration Operation

Integration is a mathematical operation denoted by the sign $\int$ and it implies the reverse of the differentiation operation. It can be shown that

$$\int kx^n dx = \frac{k}{n+1} x^{n+1} + c$$

For example,

$$\int 15x^2 dx = \frac{15}{3}x^3 + c = 5x^3 + c$$

A special integration is that of $\int \frac{1}{x} dx = \log_e x + c$

From Summation to Integration

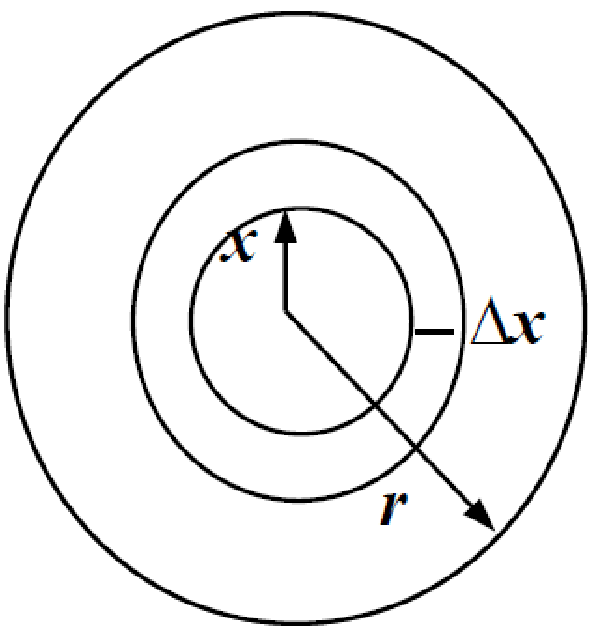
The summation of all integer numbers between 1 and 10 is

$$\sum_{1}^{10} x = \sum (1 + 2 + 3 + 4 + \ldots\ldots + 10) = 55$$

When the quantity *x* can assume any value (e.g. 3.6434..) and not only integers, we say it is continuous. The summation operation is replaced by integration.

Application

If we divide a circle into an infinite number of tiny rings such that the radius of a ring of thickness Δx varies continuously between point *0(i.e. O is the centre of the circle)* and point *r (i.e. r is the distance between point O and the circumference of the circle) then* , the area of the circle is the summation of the areas of all the tiny rings.



Area of each ring,

$$\Delta A = 2\pi x \Delta x,$$

where Δx is the thickness of the ring.

Total area, $\mathbf{A} = \int_{\mathbf{0}}^{\boldsymbol{r}} \Delta A$

Therefore,

$$\boldsymbol{A} = \int_{\mathbf{0}}^{\boldsymbol{r}} \mathbf{2}\boldsymbol{\pi x d x} = \mathbf{2}\boldsymbol{\pi}\frac{\boldsymbol{x}^{\mathbf{2}}}{\mathbf{2}} - \mathbf{0} = \boldsymbol{\pi r}^{\mathbf{2}}$$

We are all familiar with the formula $\boldsymbol{A = \pi r^2}$ for a circle.

Homework

The surface area of a sphere is $\boldsymbol{4\pi r^2}$. By dividing the sphere into an infinite number of spheres, each of very small thickness Δx when radius is x, show that the volume of the whole sphere of radius r is

$$\boldsymbol{A} = \frac{\mathbf{4}}{\mathbf{3}}\boldsymbol{\pi r}^{\mathbf{3}}$$