Homework 1

1) Problem 1 : i = iteration, S = Array after i^{th} iteration

Ans : i = 1 S = 32 41 58 24 40 47

i = 2 S = 32 41 58 24 40 47

i = 3 S = 24 32 41 58 40 47

i = 4 S = 24 32 40 41 58 47.

i = 5 S = 24 32 40 41 47 58

2) Problem 3.

Ans: i) $f(n) = n^2/31 + 2n^3 - 5$

$f(n) = \theta(n^3)$

Theorem Reason = $f(n)$ is a polynomial of degree 3

2) $f(n) = n \lg n + n^2/100$

$f(n) = \theta(n^2)$

Theorem $f(n)$ is a polynomial of degree 2.

* Problem → 3

1) For each i, line 4 of Algo1 was executed i times.

2) In total, line 4 was executed $\frac{n(n+1)}{2}$ times

3) $f = \theta(n^2)$

because $f(n) = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$ is of degree 2.

* Problem - 4

Ans ⇒

① To Prove $3^{n+2} = O(3^n)$

$\Rightarrow 3^{n+2} \leq c3^n$

$3^n \cdot 3^2 \leq c3^n$

$\Rightarrow 9 \leq c$ for all n.

② To Prove $3^{2n} = O(3^n)$

$\Rightarrow 3^{2n} \leq c3^n$

$3^n \leq c$

$c \geq 3^n$

$\Rightarrow c = 3$ for $n = 1$

Q5 $g_1 = \Omega(g_2) \Rightarrow g_1 \geq g_2$

$(n+1)! \geq n! \geq 2^{2^n} \geq n \cdot 2^n \geq e^n \geq 2^n \geq (3/2)^n \geq 4^{\lg n} \geq$

$2^{\lg n} \geq (\sqrt{2})^{\lg n} \geq n^3 \geq n^2 \geq n \lg n \geq$

$\lg(n!) \geq n \geq \lg^2 n \geq \lg \lg n \geq \ln n \geq 0$

2) $\Rightarrow \quad \Theta(n \lg n) = \lg(n!)$

$\Rightarrow$

Q6 $(0,4), (1,4)$

a) 1) Pairs $(2,3)$ and $(3,4)$ are the inversions of A.

b) 2) The array $\langle n, n-1, \ldots, 2, 1 \rangle$ from set $\{1, 2, 3 \ldots, n\}$ will have the most inversions

2) $\Rightarrow$ It will have $\frac{(n-1)n}{2}$ inversions. $\Rightarrow O(n^2)$

c) 3) $t = 5, \quad I = 4$

4) Both the running time of Insertion Sort and no. of inversions are $O(n^2)$ from b(2)