# Homework 1

**Problem 1.** Illustrate the operation of *InsertionSort* on the array $\langle 32, 41, 58, 24, 40, 47\rangle$ (show the array after each iteration of the algorithm).

**Problem 2.** Express the functions $n^2/31 + 2n^3 - 5$ and $n \lg n + n^2/100$ in terms of $\Theta$-notation.

**Problem 3.** How many times line 4 of ALG1 is executed for each $i$? In total? Give an asymptotic analysis of the running time using big-Oh (or big-Theta which would be technically more precise).

```
ALG1(n)
1   s = 0
2   for i = 1 to n
3       for j = 1 to i
4           s += j * j
```

**Problem 4.** Prove or disprove
(a) $3^{n+2} = O(3^n)$,
(b) $3^{2n} = O(3^n)$.

**Problem 5.** ***a***. Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, \ldots$ of the functions satisfying $g_1 = \Omega(g_2), g_2 = \Omega(g_3), \ldots$ Partition your list into equivalence classes such that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n)$.

| | | | | | |
|---|---|---|---|---|---|
| $(\sqrt{2})^{\lg n}$ | $n^2$ | $n!$ | $\ln n$ | $(\frac{3}{2})^n$ | $n^3$ |
| $\lg^2 n$ | $\lg(n!)$ | $2^{2^n}$ | $n \lg n$ | $\lg \lg n$ | $n \cdot 2^n$ |
| $4^{\lg n}$ | $(n+1)!$ | $n$ | $2^n$ | $2^{\lg n}$ | $e^n$ |

***b***. Give an example of a single non-negative function $f(n)$ such that for all functions $g_i(n)$ in part (a), $f(n)$ is neither $O(g_i(n))$ nor $\Omega(g_i(n))$.

**Problem 6.** Let $S[..]$ be an array of $n$ distinct numbers. If $i < j$ and $S[i] > S[j]$, then the pair $(i, j)$ is called an inversion of $A$.

***a***. Find all inversions of the array $\langle 3, 4, 9, 7, 1\rangle$.

***b***. What array with elements from the set $\{1, 2, \ldots, n\}$ has the most inversions? How many inversions does it have?

***c***. Let $t$ be the number of shifts (by one) done by *InsertionSort* on a given sequence and let $I$ be the number of its inversions. Find $t$ and $I$ for a sequence $\langle 3, 4, 9, 7, 1\rangle$. What is the relationship between the running time of insertion sort and the number of inversions in the array? Justify your answer.