Homework 1

[S21] Advanced Robotics

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Link to github: https://github.com/TechToker/Tripteron-MSA

## 1. Robot design

The 3-DOF translational parallel robot Tripteron consists of 3 serial kinematic legs and the end-effector platform. It is able to move in all 3 directions and each leg controls only one degree of freedom independently from the others. The visualization is given on fig. 1. The kinematic scheme is given on in the fig. 2.

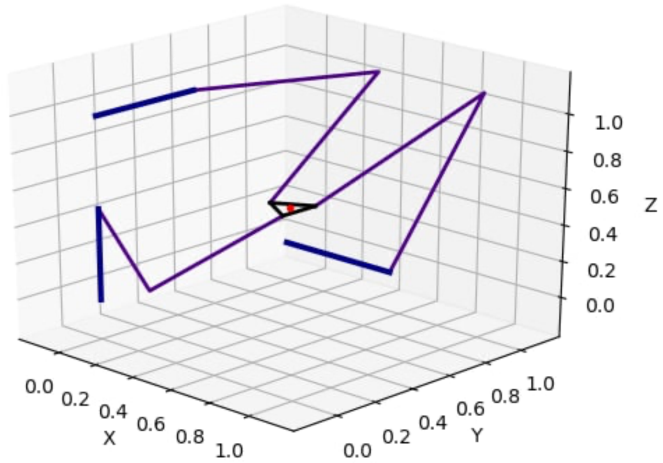



Fig. 1: Tripteron visualization

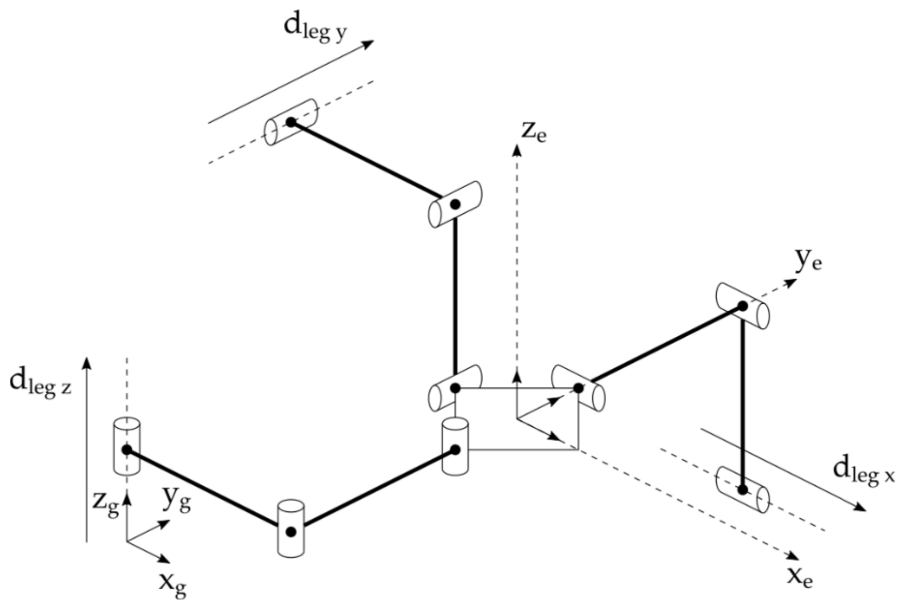



Fig. 2: Tripteron kinematics model

## 2. MSA model

First of all, we need to describe the stiffness matrixes and transform them to the global coordinates. The basic expression for that could be represented as following:

$$K_{global} = Q * K_{local} * Q'$$

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \qquad Q = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}$$

where

$$K_{11} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\ 0 & 0 & \frac{12EI_y}{L^3} & 0 & -\frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GI_p}{L} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 \\ 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} \end{bmatrix}$$

$$K_{12} = \begin{bmatrix} -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} \\ 0 & 0 & -\frac{12EI_y}{L^3} & 0 & -\frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & -\frac{GI_p}{L} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 \\ 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} \end{bmatrix}$$

$$K_{21} = K_{12}^T$$

$$K_{22} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} \\ 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GI_p}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 \\ 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} \end{bmatrix}$$

where L is the beam length, $A = pi*d^2/4$ is the area of cross section, $I_y = I_z = pi*d^4/64$ are the principal moment of inertia, $I_p = I_y + I_z$ is the torsional moment of inertia, $E = 6.9*10^{10}$ and $G = 2.55*10^{10}$ are Young's and Coulomb's modules of the aluminum.

The MSA model of the Tripteron is presented in fig. 3. Links are flexible cylindrical beams; active joints are flexible, and the platform is rigid.

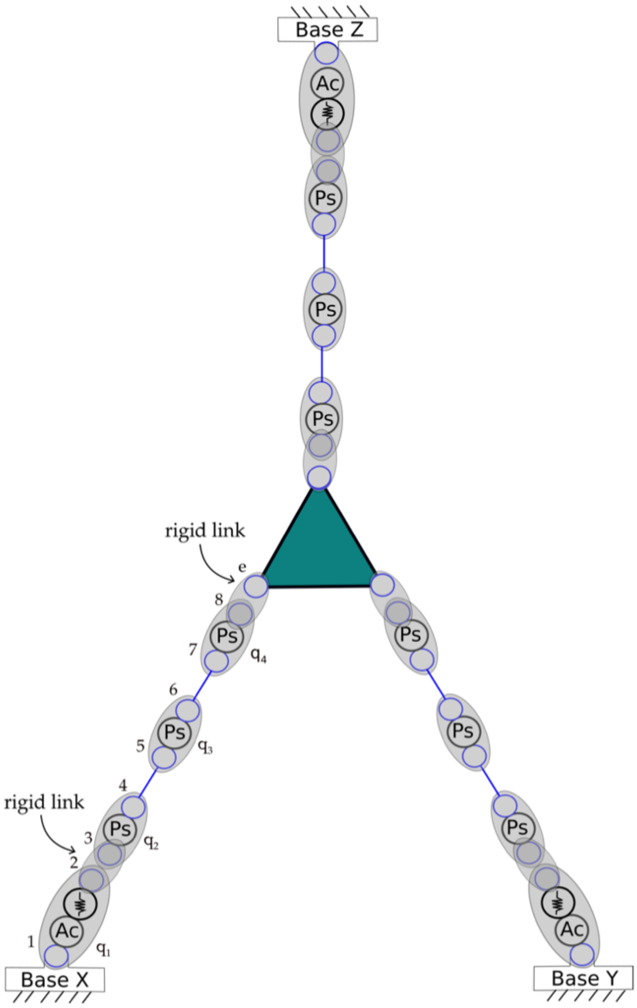



Fig. 3: MSA model

The node 1 is connected to the rigid base and could be described as:

$$[0_{6\times6} \quad I_{6\times6}]\begin{bmatrix} W_1 \\ \Delta\, t_1 \end{bmatrix} = 0_{6\times6}$$

Links 4-5 and 6-7 are flexible and have constraints on deflection and loading that are described as:

$$\begin{bmatrix} -I_{6\times6} & 0_{6\times6} & K_{i,j}^{11} & K_{i,j}^{12} \\ 0_{6\times6} & -I_{6\times6} & K_{i,j}^{21} & K_{i,j}^{22} \end{bmatrix}\begin{bmatrix} W_i \\ W_j \\ \Delta\, t_i \\ \Delta\, t_j \end{bmatrix} = \begin{bmatrix} 0_{6\times1} \\ 0_{6\times1} \end{bmatrix}$$

where *i* and *j* are the node numbers.

Rigid platform presented as a rigid link 8-e:

$$\begin{bmatrix} 0_{6\times6} & 0_{6\times6} & D_{8,e} & -I_{6\times6} \\ I_{6\times6} & D_{8,e}^{T} & 0_{6\times6} & 0_{6\times6} \end{bmatrix}\begin{bmatrix} W_8 \\ W_e \\ \Delta\, t_8 \\ \Delta\, t_e \end{bmatrix} = \begin{bmatrix} 0_{6\times1} \\ 0_{6\times1} \end{bmatrix}$$

where

$$D_{8,e} = \begin{bmatrix} I_{3\times3} & [d_{8,e}\times]^{T} \\ 0_{3\times3} & I_{3\times3} \end{bmatrix}$$

where $[d_{8,e}\times]$ is the 3×3 skew-symmetric matrix derived from the vector $d_{8,e}$ which describes the link geometry from 8$^{th}$ to e$^{th}$ node.

The connection between the nodes 2 and 3 is rigid link with the length 0:

$$\begin{bmatrix} 0_{6\times6} & 0_{6\times6} & I_{6\times6} & -I_{6\times6} \\ I_{6\times6} & I_{6\times6} & 0_{6\times6} & 0_{6\times6} \end{bmatrix}\begin{bmatrix} W_2 \\ W_3 \\ \Delta\, t_2 \\ \Delta\, t_3 \end{bmatrix} = \begin{bmatrix} 0_{6\times1} \\ 0_{6\times1} \end{bmatrix}$$

Active elastic joint 1-2 is described by the following equation:

$$\begin{bmatrix} 0_{5\times6} & 0_{5\times6} & \lambda_{1,2}^{r} & -\lambda_{1,2}^{r} \\ I_{6\times6} & I_{6\times6} & 0_{6\times6} & 0_{6\times6} \\ \lambda_{1,2}^{e} & 0_{1\times6} & K_a\lambda_{1,2}^{e} & -K_a\lambda_{1,2}^{e} \end{bmatrix}\begin{bmatrix} W_1 \\ W_2 \\ \Delta\, t_1 \\ \Delta\, t_2 \end{bmatrix} = \begin{bmatrix} 0_{6\times1} \\ 0_{6\times1} \\ 0_{6\times1} \end{bmatrix}$$

where

$$\lambda_{1,2}^{e,x} = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$\lambda_{1,2}^{e,y} = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$\lambda_{1,2}^{e,z} = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]$$

$$\lambda_{1,2}^{r,x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\lambda_{1,2}^{r,y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\lambda_{1,2}^{r,z} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$K_a$ represents the stiffness of the active joint = $10^6$ H/m.

The passive joints 3-4, 5-6 and 7-8:

$$\begin{bmatrix} 0_{5\times6} & 0_{5\times6} & \lambda_{i,j}^{r} & -\lambda_{i,j}^{r} \\ \lambda_{i,j}^{r} & \lambda_{i,j}^{r} & 0_{5\times6} & 0_{5\times6} \\ \lambda_{i,j}^{p} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} \\ 0_{1\times6} & \lambda_{i,j}^{p} & 0_{1\times6} & 0_{1\times6} \end{bmatrix} \begin{bmatrix} W_i \\ W_j \\ \Delta t_i \\ \Delta t_j \end{bmatrix} = \begin{bmatrix} 0_{6\times1} \\ 0_{6\times1} \\ 0_{6\times1} \\ 0_{6\times1} \end{bmatrix}$$

where

$$\lambda_{3,4}^{p,x} = \lambda_{5,6}^{p,x} = \lambda_{7,8}^{p,x} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\lambda_{3,4}^{p,y} = \lambda_{5,6}^{p,y} = \lambda_{7,8}^{p,y} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\lambda_{3,4}^{p,z} = \lambda_{5,6}^{p,z} = \lambda_{7,8}^{p,z} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\lambda_{3,4}^{r,x} = \lambda_{5,6}^{r,x} = \lambda_{7,8}^{r,x} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\lambda_{3,4}^{r,y} = \lambda_{5,6}^{r,y} = \lambda_{7,8}^{r,y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
\lambda_{3,4}^{r,z} = \lambda_{5,6}^{r,z} = \lambda_{7,8}^{r,z} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}
$$

The external loading denotes by:

$$
\begin{bmatrix} -I_{6\times6} & 0_{6\times6} \end{bmatrix} \begin{bmatrix} W_e \\ \Delta\, t_e \end{bmatrix} = W_{ext}
$$

Using all above-mentioned equations, we can create aggregated equation to the following type:

$$
\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} W_{ag} \\ \Delta\, t_{ag} \\ \Delta\, t_e \end{bmatrix} = \begin{bmatrix} 0 \\ W_{ext} \end{bmatrix}
$$

where

$$
B = \begin{bmatrix} 0_{30x6} \\ -I_{6x6} \\ 0_{36x6} \end{bmatrix}
$$

$$
D = [0_{6x6}]
$$

$$
C = [0_{48x6} \quad -I_{6x6} \quad 0_{48x6}]
$$

$$
A= \begin{bmatrix}
0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & I_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} \\
0_{6x6} & 0_{6x6} & 0_{6x6} & -I_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & K_{4,5}^{11} & K_{12}^{12} & 0_{6x6} & 0_{6x6} & 0_{6x6} \\
0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & -I_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & K_{4,5}^{21} & K_{4,5}^{22} & 0_{6x6} & 0_{6x6} & 0_{6x6} \\
0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & -I_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & K_{6,7}^{11} & K_{6,7}^{12} & 0_{6x6} \\
0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & -I_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & K_{6,7}^{21} & K_{6,7}^{22} & 0_{6x6} \\
0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & D_{2,3} \\
0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & I_{6x6} & D_{2,3}^T & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} \\
0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & I_{6x6} & -I_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} \\
0_{6x6} & I_{6x6} & I_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} \\
0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & \lambda_{1,2}^{r} & -\lambda_{1,2}^{r} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} \\
I_{6x6} & I_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} & 0_{6x6} \\
\lambda_{1,2}^{e} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & K_a \lambda_{1,2}^{e} & -K_a \lambda_{1,2}^{e} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} \\
0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & \lambda_{3,4}^{r} & -\lambda_{3,4}^{r} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} \\
0_{5x6} & 0_{5x6} & \lambda_{3,4}^{r} & \lambda_{3,4}^{r} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} \\
0_{1x6} & 0_{1x6} & \lambda_{3,4}^{p} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} \\
0_{1x6} & 0_{1x6} & 0_{1x6} & \lambda_{3,4}^{p} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} \\
0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & \lambda_{5,6}^{r} & -\lambda_{5,6}^{r} & 0_{5x6} & 0_{5x6} \\
0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & \lambda_{5,6}^{r} & \lambda_{5,6}^{r} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} \\
0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & \lambda_{5,6}^{r} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} \\
0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & \lambda_{5,6}^{p} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} \\
0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & \lambda_{7,8}^{r} & \lambda_{7,8}^{r} \\
0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & \lambda_{7,8}^{r} & \lambda_{7,8}^{r} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} & 0_{5x6} \\
0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & \lambda_{7,8}^{p} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} \\
0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & \lambda_{7,8}^{p} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6} & 0_{1x6}
\end{bmatrix}
$$

We find the stiffness matrix using the following formula:

$$
K_{ci} = D - CA^{-1}B
$$

Then we find the resulting stiffness matrix of the whole manipulator:

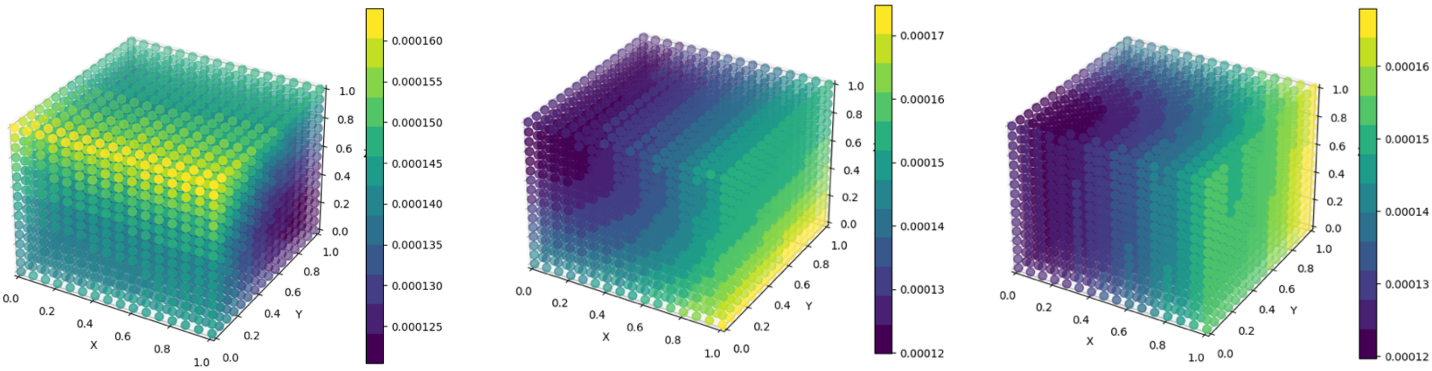
$$K_c = \sum_{i=1}^{3} K_{ci}$$

To find the displacement we use the Hook's law:

$$W = K_c \, \Delta \, t$$

## 3. Deflection map

The deflection maps are represented on fig. 4.




(a) (b) (c)

Fig. 4: Deflection maps 3D

(a) Deflection 3D map for the wrench applied on x axis

(b) Deflection 3D map for the wrench applied on y axis

(c) Deflection 3D map for the wrench applied on z axis

## 4. Obtained results

We implemented and applied MSA in order to obtain Cartesian stiffness matrix of the whole robot. Then we found the deflection in different points of the robot workspace to get the deflection maps. They show us that, if we apply the force along the *i* axis, then the deflection only depends on the distance from the joint along that axis. Also, if we compare our results to the given article, we can see that the difference is barely noticeable. That is because the added platform is rigid and doesn't add much deflection.