IIT-JEE Mathematics Questions (Hard)

1. Find the number of non-negative integral solutions to the equation x1 + x2 + x3 + x4 + x5 = 25, where xi "e 0 for i = 1, 2, 3, 4, 5.

2. Let f(x) be a twice differentiable function such that f(0) = 0 and f(1) = 2. If the area bounded by the curve y = f(x), the x-axis, and the line x = 1 is 4, find the value of the integral "+[0,1] (f'(x))^2 dx.

3. Find all real roots of the equation (z - 1)^5 + (z + 1)^5 = 32.

4. Let A be a 3x3 matrix with distinct eigenvalues. If the trace of A is 0 and the product of its eigenvalues is -8, find the determinant of A.

5. In a triangle ABC, let AD be the altitude from A to BC and BE be the altitude from B to AC. If AD = 12 and BE = 15, find the area of the triangle ABC.

6. Let f(x) be a continuous function on the interval [0,1] such [0,1]. If "+[0,1] f(x) dx = 2, find the value of the integral "+[0,1]

7. Find the number of ways to distribute 10 identical balls into 3 distinct boxes such that each box contains at least one ball.

8. A regular tetrahedron has a side length of 2a. Find the distance from the centroid of one of its faces to the centroid of the tetrahedron.

9. Let f(x,y) be a twice differentiable function such that f(0,0) = 0 and f(1,1) = 1. If the directional derivative of f at (0,0) in the direction (1,2) is 3, find the value of the integral "+[0,1] "+[0,1] (" ^2f/" x" y) dx dy.

10. Let S be the set of all complex numbers z such that |z - 1| area of the region enclosed by the set S.