INTRODUCTION TO CALCULUS

CHAPTER 11

We revisit an idea introduced in a Blitzer Bonus in Chapter 1. Take a rapid sequence of still photographs of a moving scene and project them onto a screen at thirty shots a second or faster. Our eyes see the result as continuous motion. The small difference between one frame and the next cannot be detected by the human visual system. The idea of calculus likewise regards continuous motion as made up of a sequence of still configurations. In this chapter, you will see how calculus uses a revolutionary concept called *limits* to master the mystery of movement by "freezing the frame" instant by instant.

HERE'S WHERE YOU'LL FIND THESE APPLICATIONS:

Using limits to describe instantaneous rates of change is introduced in the Section 11.4 opener and developed throughout the section.

SECTION 11.1 Finding Limits Using Tables and Graphs

Objectives

1. Understand limit notation.
2. Find limits using tables.
3. Find limits using graphs.
4. Find one-sided limits and use them to determine if a limit exists.

Motion and change are the very essence of life. Moving air brushes against our faces, rain falls on our heads, birds fly past us, plants spring from the earth, grow, and then die, and rocks thrown upward reach a maximum height before falling to the ground.

The tools of algebra and trigonometry are essentially static; numbers, points, lines, equations, functions, and graphs do not incorporate motion. The development of calculus in the middle of the seventeenth century provided a way to use these static tools to analyze motion and change. It took nearly two thousand years of effort for humankind to achieve this feat, made possible by a revolutionary concept called *limits*. The invention of limits marked a turning point in human history, having as dramatic an impact on our lives as the invention of the wheel and the printing press. In this section, we introduce this bold and dramatic style of thinking about mathematics.

1 Understand limit notation.

An Introduction to Limits

Suppose that you and a friend are walking along the graph of the function

$$f(x) = \frac{x^2 - 4}{x - 2}.$$

FIGURE 11.1 Walking along the graph of f, very close to 2

Figure 11.1 illustrates that you are walking uphill and your friend is walking downhill. Because 2 is not in the domain of the function, there is a hole in the graph at $x = 2$. Warning signs along the graph might be appropriate: Caution: $f(2)$ is undefined! If you or your friend reach 2, you will fall through the hole and splatter onto the x-axis.

Obviously, there is a problem at $x = 2$. But what happens along the graph of $f(x) = \dfrac{x^2 - 4}{x - 2}$ as you and your friend walk very, very close to $x = 2$? What function value, $f(x)$, will each of you approach? One way to answer this question is to construct a table of function values to analyze numerically the behavior of f as x gets closer and closer to 2. Remember that you are walking uphill, approaching 2 from the left side of 2. Your friend is walking downhill, approaching 2 from the right side of 2. Thus, we must include values of x that are less than 2 and values of x that are greater than 2.

In **Table 11.1** at the top of the next page, we choose values of x close to 2. As x approaches 2 from the left, we arbitrarily start with $x = 1.99$. Then we select two additional values of x that are closer to 2, but still less than 2. We choose 1.999 and 1.9999. As x approaches 2 from the right, we arbitrarily start with $x = 2.01$. Then we select two additional values of x that are closer to 2, but still greater than 2. We choose 2.001 and 2.0001. Finally, evaluate f at each chosen value of x to obtain **Table 11.1**.

TECHNOLOGY

A graphing utility with a TABLE feature can be used to generate the entries in **Table 11.1**. In TBLSET, change Auto to Ask for Indpnt, the independent variable. Here is a typical screen that verifies **Table 11.1**.

Table 11.1

	x approaches 2 from the left.				x approaches 2 from the right.		
x	1.99	1.999	1.9999	2	2.0001	2.001	2.01
$f(x) = \dfrac{x^2 - 4}{x - 2}$	3.99	3.999	3.9999	Undefined	4.0001	4.001	4.01
	$f(x)$ gets closer to 4.				$f(x)$ gets closer to 4.		

From **Table 11.1**, it appears that as x gets closer to 2, the values of $f(x) = \dfrac{x^2 - 4}{x - 2}$ get closer to 4. We say that

"The limit of $\dfrac{x^2 - 4}{x - 2}$ as x approaches 2 equals the number 4."

We can express this sentence in a mathematical notation called **limit notation**. We use an arrow for the word *approaches*. Likewise, we use *lim* as shorthand for the word *limit*. Thus, the limit notation for the English sentence in quotations is

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4.$$

The limit of $\dfrac{x^2 - 4}{x - 2}$ as x approaches 2 equals the number 4.

Calculus is the study of limits and their applications. Limits are the foundation of the concepts that you will encounter in calculus.

Limit Notation and Its Description

Suppose that f is a function defined on some open interval containing the number a. The function f may or may not be defined at a. The **limit notation**

$$\lim_{x \to a} f(x) = L$$

is read "the limit of $f(x)$ as x approaches a equals the number L." This means that as x gets closer to a, but remains unequal to a, the corresponding values of $f(x)$ get closer to L.

2 Find limits using tables.

Finding Limits Using Tables

To find $\lim_{x \to a} f(x)$, use a graphing utility with a TABLE feature or create a table by hand. Approach a from the left, choosing values of x that are close to a but still less than a. Then approach a from the right, choosing values of x that are close to a but still greater than a. Evaluate f at each chosen value of x to obtain the desired table.

Choose values of x so that the table makes it obvious what the corresponding values of $f(x)$ are getting close to. If the values of $f(x)$ are getting close to the number L, we infer that

$$\lim_{x \to a} f(x) = L.$$

EXAMPLE 1 Finding a Limit Using a Table

Find: $\lim_{x \to 4} 3x^2$.

SOLUTION

As x gets closer to 4, but remains unequal to 4, we must find the number that the corresponding values of $3x^2$ get closer to. The voice balloons shown below indicate that in this limit problem, $f(x) = 3x^2$ and $a = 4$.

$$\lim_{x \to 4} 3x^2$$

This is a in $\lim_{x \to a} f(x)$.

This is $f(x)$ in $\lim_{x \to a} f(x)$.

In making a table, we choose values of x close to 4. As x approaches 4 from the left, we arbitrarily start with $x = 3.99$. Then we select two additional values of x that are closer to 4, but still less than 4. We choose 3.999 and 3.9999. As x approaches 4 from the right, we arbitrarily start with $x = 4.01$. Then we select two additional numbers that are closer to 4 but still greater than 4. We choose 4.001 and 4.0001. Finally, we evaluate f at each chosen value of x to obtain **Table 11.2**. The values of $f(x)$ in the table are rounded to four decimal places.

x approaches 4 from the left. **x approaches 4 from the right.**

Table 11.2

x	3.99	3.999	3.9999	4.0001	4.001	4.01
$f(x) = 3x^2$	47.7603	47.9760	47.9976	48.0024	48.0240	48.2403

$f(x)$ gets closer to 48. **$f(x)$ gets closer to 48.**

From **Table 11.2**, it appears that as x gets closer to 4, the values of $3x^2$ get closer to 48. We infer that

$$\lim_{x \to 4} 3x^2 = 48.$$

Check Point 1 Find: $\lim_{x \to 3} 4x^2$.

EXAMPLE 2 Finding a Limit Using a Table

Find: $\lim_{x \to 0} \frac{\sin x}{x}$.

SOLUTION

As x gets closer to 0, but remains unequal to 0, we must find the number that the corresponding values of $\frac{\sin x}{x}$ get closer to. The voice balloons shown below indicate that in this limit problem $f(x) = \frac{\sin x}{x}$ and $a = 0$.

$$\lim_{x \to 0} \frac{\sin x}{x}$$

This is a in $\lim_{x \to a} f(x)$.

This is $f(x)$ in $\lim_{x \to a} f(x)$.

Because division by 0 is undefined, the domain of $f(x) = \dfrac{\sin x}{x}$ is $\{x \mid x \neq 0\}$. Thus, f is not defined at 0. However, in this limit problem, we do not care what is happening at $x = 0$. We are interested in the behavior of the function as x gets close to 0. **Table 11.3** shows the values of $f(x)$, rounded to five decimal places, as x approaches 0 from the left and from the right. Values of x in the table are measured in radians.

TECHNOLOGY

Graphic Connections

The graph of $f(x) = \dfrac{\sin x}{x}$ illustrates that as x gets closer to 0, the values of $f(x)$ are approaching 1. This supports our inference that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

[−10, 10, 1] by [−2, 2, 1]

Table 11.3

	x approaches 0 from the left.				x approaches 0 from the right.		
x	−0.03	−0.02	−0.01	→ ←	0.01	0.02	0.03
$f(x) = \dfrac{\sin x}{x}$	0.99985	0.99993	0.99998	→ ←	0.99998	0.99993	0.99985
	$f(x)$ gets closer to 1.				$f(x)$ gets closer to 1.		

From **Table 11.3**, it appears that as x gets closer to 0, the values of $\dfrac{\sin x}{x}$ get closer to 1. We infer that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

Check Point 2 Find: $\displaystyle\lim_{x \to 0} \frac{\cos x - 1}{x}$.

3 Find limits using graphs.

Finding Limits Using Graphs

The limit statement

$$\lim_{x \to a} f(x) = L$$

is illustrated in **Figure 11.2**. In the three graphs, the number that x is approaching, a, is shown on the x-axis. The limit, L, is shown on the y-axis. Take a few minutes to examine the graphs. Can you see that as x approaches a along the x-axis, $f(x)$ approaches L along the y-axis? In each graph, as x gets closer to a, the values of $f(x)$ get closer to L.

FIGURE 11.2 In each graph, as x gets closer to a, the values of f get closer to L: $\lim_{x \to a} f(x) = L$.

In **Figure 11.2(a)**, as x approaches a, $f(x)$ approaches L. At a, the value of the function is L: $f(a) = L$. In **Figure 11.2(b)**, as x approaches a, $f(x)$ approaches L. This is true although f is not defined at a, shown by the hole in the graph. In **Figure 11.2(c)**, we again see that as x approaches a, $f(x)$ approaches L. Notice, however, that the value of the function at a, $f(a)$, shown by the blue dot, is not equal to the limit: $f(a) \neq L$. What you get as you approach a is not the same as what you get at a.

Example 3 illustrates that the graph of a function can sometimes be helpful in finding limits.

FIGURE 11.3

EXAMPLE 3 Finding a Limit Using a Graph

Use the graph in **Figure 11.3** to find each of the following:

a. $\lim_{x \to 4} f(x)$ **b.** $f(4)$.

SOLUTION

a. To find $\lim_{x \to 4} f(x)$, examine the graph of f near $x = 4$. As x gets closer to 4, the values of $f(x)$ get closer to the y-coordinate of the point shown by the open dot on the right. The y-coordinate of this point is 7. Thus, as x gets closer to 4, the values of $f(x)$ get closer to 7. We conclude from the graph that

$$\lim_{x \to 4} f(x) = 7.$$

b. To find $f(4)$, examine the graph of f at $x = 4$. At $x = 4$, the open dot is not included in the graph of f. The graph of f at 4 is shown by the closed dot with coordinates $(4, 2)$. Thus, $f(4) = 2$.

In Example 3, notice that the value of f at 4 has nothing to do with the conclusion that $\lim_{x \to 4} f(x) = 7$. Regardless of how f is defined at 4, it is still true that $\lim_{x \to 4} f(x) = 7$. Furthermore, if f were undefined at 4, the limit of $f(x)$ as $x \to 4$ would still equal 7.

Check Point 3 Use the graph in **Figure 11.3** to find each of the following:

a. $\lim_{x \to -2} f(x)$ **b.** $f(-2)$.

EXAMPLE 4 Finding a Limit by Graphing a Function

Graph the function

$$f(x) = \begin{cases} 2x - 4 & \text{if } x \neq 3 \\ -5 & \text{if } x = 3. \end{cases}$$

Use the graph to find $\lim_{x \to 3} f(x)$.

SOLUTION

This piecewise function is defined by two equations. Graph the piece defined by the linear function, $f(x) = 2x - 4$, using the y-intercept, -4, and the slope, 2. Because $x = 3$ is not included, show an open dot on the line corresponding to $x = 3$. This open dot, with coordinates $(3, 2)$, is shown in **Figure 11.4**.

Now we complete the graph using $f(x) = -5$ if $x = 3$. This part of the function is graphed as the point $(3, -5)$, shown as a closed blue dot in **Figure 11.4**.

To find $\lim_{x \to 3} f(x)$, examine the graph of f near $x = 3$. As x gets closer to 3, the values of $f(x)$ get closer to the y-coordinate of the point shown by the open dot. The y-coordinate of this point is 2. We conclude from the graph that

$$\lim_{x \to 3} f(x) = 2.$$

FIGURE 11.4 As x gets closer to 3, what number are the function values getting closer to?

Check Point 4 Graph the function

$$f(x) = \begin{cases} 3x - 2 & \text{if } x \neq 2 \\ 1 & \text{if } x = 2. \end{cases}$$

Use the graph to find $\lim_{x \to 2} f(x)$.

4 Find one-sided limits and use them to determine if a limit exists.

FIGURE 11.5 As x approaches 2 from the left (red arrow) or from the right (blue arrow), values of $f(x)$ get closer to 4.

One-Sided Limits

The graph in **Figure 11.5** shows a portion of the graph of the function $f(x) = x^2$. The graph illustrates that

$$\lim_{x \to 2} x^2 = 4.$$

As x gets closer to 2, but remains unequal to 2, the corresponding values of $f(x)$ get closer to 4. The values of x near 2 fall into two categories: those that lie to the left of 2, shown by the red arrow on the x-axis, and those that lie to the right of 2, shown by the blue arrow on the x-axis.

The values of x can get closer to 2 in two ways. The values of x can approach 2 from the left, through numbers that are less than 2. **Table 11.4** shows some values of x and the corresponding values of $f(x)$ rounded to four decimal places. The red portion of the graph in **Figure 11.5** shows that as x approaches 2 from the left of 2, $f(x)$ approaches 4.

Table 11.4

x	1.99	1.999	1.9999 →
$f(x) = x^2$	3.9601	3.9960	3.9996 →

We say that "the limit of x^2 as x approaches 2 from the left equals 4." The mathematical notation for this English sentence is

$$\lim_{x \to 2^-} x^2 = 4.$$

The notation $x \to 2^-$ indicates that x is less than 2 and is approaching 2 from the left.

The values of x can also approach 2 from the right, through numbers that are greater than 2. **Table 11.5** shows some values of x and the corresponding values of $f(x)$ rounded to four decimal places. The blue portion of the graph in **Figure 11.5** shows that as x approaches 2 from the right of 2, $f(x)$ approaches 4.

Table 11.5

x	← 2.0001	2.001	2.01
$f(x) = x^2$	← 4.0004	4.0040	4.0401

We say that "the limit of x^2 as x approaches 2 from the right equals 4." The mathematical notation for this English sentence is

$$\lim_{x \to 2^+} x^2 = 4.$$

The notation $x \to 2^+$ indicates that x is greater than 2 and is approaching 2 from the right.

In general, if x approaches a from one side, we have a **one-sided limit**.

One-Sided Limits

Left-Hand Limit The limit notation

$$\lim_{x \to a^-} f(x) = L$$

is read "the limit of $f(x)$ as x approaches a from the left equals L" and is called the **left-hand limit**. This means that as x gets closer to a, but remains less than a, the corresponding values of $f(x)$ get closer to L.

Right-Hand Limit The limit notation

$$\lim_{x \to a^+} f(x) = L$$

is read "the limit of $f(x)$ as x approaches a from the right equals L" and is called the **right-hand limit**. This means that as x gets closer to a, but remains greater than a, the corresponding values of $f(x)$ get closer to L.

FIGURE 11.6 As x approaches 1 from the left (red arrow) and from the right (blue arrow), values of $f(x)$ do not get closer to a single number.

A function's graph can be helpful in finding one-sided limits. For example, **Figure 11.6** shows the graph of the piecewise function

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ x + 3 & \text{if } x \geq 1. \end{cases}$$

The red portion of the graph, part of a parabola, illustrates that as x approaches 1 from the left, the corresponding values of $f(x)$ get closer to 2. The left-hand limit is 2:

$$\lim_{x \to 1^-} f(x) = 2.$$

The blue portion of the graph, part of a line, illustrates that as x approaches 1 from the right, the corresponding values of $f(x)$ get closer to 4. The right-hand limit is 4:

$$\lim_{x \to 1^+} f(x) = 4.$$

Because $\lim_{x \to 1^-} f(x) = 2$ and $\lim_{x \to 1^+} f(x) = 4$, there is no single number that the values of $f(x)$ are close to when x is close to 1. In this case, we say that **f has no limit as x approaches 1** or that **$\lim_{x \to 1} f(x)$ does not exist**.

In general, a function f has a limit as x approaches a if and only if the left-hand limit equals the right-hand limit.

Equal and Unequal One-Sided Limits

- One-sided limits can be used to show that a function has a limit as x approaches a.

$$\lim_{x \to a} f(x) = L \text{ if and only if both}$$

$$\lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = L.$$

- One-sided limits can be used to show that a function has no limit as x approaches a.

$$\text{If } \lim_{x \to a^-} f(x) = L \text{ and } \lim_{x \to a^+} f(x) = M, \text{ where } L \neq M,$$

$$\lim_{x \to a} f(x) \text{ does not exist.}$$

GREAT QUESTION!

Can you clarify the use of the word *from* when describing one-sided limits?

The word *from* is helpful in distinguishing left- and right-hand limits. A left-hand limit means you approach the given x-value *from* the left. It does not mean that you approach toward the left on the graph. A right-hand limit means you approach the given x-value *from* the right. It does not mean you approach toward the right on the graph.

FIGURE 11.7

EXAMPLE 5 Finding One-Sided Limits Using a Graph

Use the graph of the piecewise function f in **Figure 11.7** to find each of the following, or state that the limit or function value does not exist:

a. $\lim_{x \to -2^-} f(x)$ **b.** $\lim_{x \to -2^+} f(x)$ **c.** $\lim_{x \to -2} f(x)$ **d.** $f(-2)$.

SOLUTION

a. To find $\lim_{x \to -2^-} f(x)$, examine the portion of the graph shown in red that is near, but to the left of $x = -2$. As x approaches -2 from the left, the values of $f(x)$ get close to the y-coordinate of the point shown by the red open dot. This point, $(-2, 0)$, has a y-coordinate of 0. Thus,

$$\lim_{x \to -2^-} f(x) = 0.$$

b. To find $\lim_{x \to -2^+} f(x)$, examine the portion of the graph shown in blue that is near but to the right of $x = -2$. As x approaches -2 from the right, the

values of $f(x)$ get close to the y-coordinate of the point shown by the blue open dot. This point, $(-2, -2)$, has a y-coordinate of -2. Thus,

$$\lim_{x \to -2^+} f(x) = -2.$$

c. We found that

$$\lim_{x \to -2^-} f(x) = 0 \quad \text{and} \quad \lim_{x \to -2^+} f(x) = -2.$$

The limit as x approaches -2 from the left equals 0.

The limit as x approaches -2 from the right equals -2.

Because the left- and right-hand limits are unequal, $\lim_{x \to -2} f(x)$ does not exist.

d. To find $f(-2)$, examine the graph of f *at* $x = -2$. The graph of f at -2 is shown by the blue closed dot with coordinates $(-2, -1)$. Thus, $f(-2) = -1$.

Check Point 5 Use the graph of the piecewise function f in **Figure 11.7** to find each of the following, or state that the limit or function value does not exist:

a. $\lim_{x \to 0^-} f(x)$ **b.** $\lim_{x \to 0^+} f(x)$ **c.** $\lim_{x \to 0} f(x)$ **d.** $f(0)$.

CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

1. The notation $\lim_{x \to a} f(x) = L$ is read "the _____ of _____ as _____ approaches _____ equals the number _____."
2. The notation $\lim_{x \to a} f(x) = L$ means that as _____ gets closer to _____, but remains unequal to _____, the corresponding values of _____ get closer to _____.
3. True or false: If $\lim_{x \to a} f(x) = L$, then the value of f at a is equal to L. _____
4. The notation $\lim_{x \to a^-} f(x) = L$ is called the _____ limit. The notation means that as _____ gets closer to _____, but remains less than _____, the corresponding values of _____ get closer to _____.
5. The notation $\lim_{x \to a^+} f(x) = L$ is called the _____ limit. The notation means that as _____ gets closer to _____, but remains greater than _____, the corresponding values of _____ get closer to _____.
6. If $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$, then $\lim_{x \to a} f(x)$_____.
7. If $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = M$, where $L \neq M$, then $\lim_{x \to a} f(x)$ _____.

EXERCISE SET 11.1

Practice Exercises

In Exercises 1–4, use each table to find the indicated limit.

1. $\lim_{x \to 2} 2x^2$

x	1.99	1.999	1.9999 → ← 2.0001	2.001	2.01
$f(x) = 2x^2$	7.9202	7.9920	7.9992 → ← 8.0008	8.0080	8.0802

2. $\lim_{x \to 3} 5x^2$

x	2.99	2.999	2.9999 → ← 3.0001	3.001	3.01
$f(x) = 5x^2$	44.701	44.970	44.997 → ← 45.003	45.030	45.301

3. $\lim_{x \to 0} \frac{\sin 3x}{x}$

x	−0.03	−0.02	−0.01 → ← 0.01	0.02	0.03
$f(x) = \frac{\sin 3x}{x}$	2.9960	2.9982	2.9996 → ← 2.9996	2.9982	2.9960

4. $\lim_{x \to 0} \frac{\sin 4x}{\sin 2x}$

x	−0.03	−0.02	−0.01 → ← 0.01	0.02	0.03
$f(x) = \frac{\sin 4x}{\sin 2x}$	1.9964	1.9984	1.9996 → ← 1.9996	1.9984	1.9964

In Exercises 5–18, construct a table to find the indicated limit.

5. $\lim_{x \to 2} 5x^2$

6. $\lim_{x \to 2} (x^2 - 1)$

7. $\lim_{x \to 3} \frac{1}{x - 2}$

8. $\lim_{x \to 4} \frac{1}{x - 3}$

9. $\lim_{x \to 0} \frac{x}{x^2 + 1}$

10. $\lim_{x \to 0} \frac{x + 1}{x^2 + 1}$

11. $\lim_{x \to -2} \frac{x^3 + 8}{x + 2}$

12. $\lim_{x \to -5} \frac{x^2 - 25}{x + 5}$

13. $\lim_{x \to 0} \frac{2x^2 + x}{\sin x}$

14. $\lim_{x \to 0} \frac{\sin x^2}{x}$

15. $\lim_{x \to 0} \frac{\tan x}{x}$

16. $\lim_{x \to 0} \frac{x^2}{\sec x - 1}$

17. $\lim_{x \to 0} f(x)$, where $f(x) = \begin{cases} x + 1 & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases}$

18. $\lim_{x \to 0} f(x)$, where $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 3x + 2 & \text{if } x \geq 0 \end{cases}$

In Exercises 19–22, use the graph of f to find the indicated limit and function value.

19.

a. $\lim_{x \to 3} f(x)$ **b.** $f(3)$

20.

a. $\lim_{x \to 2} f(x)$ **b.** $f(2)$

21.

a. $\lim_{x \to 2} f(x)$ **b.** $f(2)$

22.

a. $\lim_{x \to 1} f(x)$ **b.** $f(1)$

In Exercises 23–26, use the graph and the viewing rectangle shown below the graph to find the indicated limit.

23. $\lim_{x \to 2} (1 - x^2)$

$[-4, 4, 1]$ by $[-4, 4, 1]$

24. $\lim_{x \to -2} |2x|$

$[-4, 4, 1]$ by $[-1, 7, 1]$

25. $\lim_{x \to -\frac{\pi}{2}} \sin x$

$\left[-\pi, \pi, \frac{\pi}{2}\right]$ by $[-2, 2, 1]$

26. $\lim_{x \to -\frac{\pi}{2}} \cos x$

$\left[-\pi, \pi, \frac{\pi}{2}\right]$ by $[-2, 2, 1]$

In Exercises 27–32, the graph of a function is given. Use the graph to find the indicated limits and function values, or state that the limit or function value does not exist.

27.

a. $\lim_{x\to 2^-} f(x)$ **b.** $\lim_{x\to 2^+} f(x)$ **c.** $\lim_{x\to 2} f(x)$ **d.** $f(2)$

28.

a. $\lim_{x\to -2^-} f(x)$ **b.** $\lim_{x\to -2^+} f(x)$ **c.** $\lim_{x\to -2} f(x)$ **d.** $f(-2)$

29.

a. $\lim_{x\to -3^-} f(x)$ **b.** $\lim_{x\to -3^+} f(x)$ **c.** $\lim_{x\to -3} f(x)$ **d.** $f(-3)$

e. $\lim_{x\to -1^-} f(x)$ **f.** $\lim_{x\to -1^+} f(x)$ **g.** $\lim_{x\to -1} f(x)$ **h.** $f(-1)$

i. $\lim_{x\to 3^-} f(x)$ **j.** $\lim_{x\to 3^+} f(x)$ **k.** $\lim_{x\to 3} f(x)$ **l.** $f(3)$

30.

a. $\lim_{x\to -3^-} f(x)$ **b.** $\lim_{x\to -3^+} f(x)$ **c.** $\lim_{x\to -3} f(x)$ **d.** $f(-3)$

e. $\lim_{x\to 0^-} f(x)$ **f.** $\lim_{x\to 0^+} f(x)$ **g.** $\lim_{x\to 0} f(x)$ **h.** $f(0)$

i. $\lim_{x\to 2^-} f(x)$ **j.** $\lim_{x\to 2^+} f(x)$ **k.** $\lim_{x\to 2} f(x)$ **l.** $f(2)$

31.

a. $\lim_{x\to 2^-} f(x)$ **b.** $\lim_{x\to 2^+} f(x)$ **c.** $\lim_{x\to 2} f(x)$ **d.** $f(2)$

e. $\lim_{x\to 2.5^-} f(x)$ **f.** $\lim_{x\to 2.5^+} f(x)$ **g.** $\lim_{x\to 2.5} f(x)$ **h.** $f(2.5)$

32.

a. $\lim_{x\to 3^-} f(x)$ **b.** $\lim_{x\to 3^+} f(x)$ **c.** $\lim_{x\to 3} f(x)$ **d.** $f(3)$

e. $\lim_{x\to 3.5^-} f(x)$ **f.** $\lim_{x\to 3.5^+} f(x)$ **g.** $\lim_{x\to 3.5} f(x)$ **h.** $f(3.5)$

In Exercises 33–54, graph each function. Then use your graph to find the indicated limit, or state that the limit does not exist.

33. $f(x) = 2x + 1$, $\lim_{x\to 3} f(x)$

34. $f(x) = 2x - 1$, $\lim_{x\to 3} f(x)$

35. $f(x) = 4 - x^2$, $\lim_{x\to -3} f(x)$

36. $f(x) = 9 - x^2$, $\lim_{x\to -2} f(x)$

37. $f(x) = |x + 1|$, $\lim_{x\to -1} f(x)$

38. $f(x) = |x + 2|$, $\lim_{x\to -2} f(x)$

39. $f(x) = \dfrac{1}{x}$, $\lim_{x\to -1} f(x)$

40. $f(x) = \dfrac{1}{x^2}$, $\lim_{x\to -1} f(x)$

41. $f(x) = \dfrac{x^2 - 1}{x - 1}$, $\lim_{x\to 1} f(x)$

42. $f(x) = \dfrac{x^2 - 100}{x - 10}$, $\lim_{x\to 10} f(x)$

43. $f(x) = e^x$, $\lim_{x\to 0} f(x)$

44. $f(x) = \ln x$, $\lim_{x\to 1} f(x)$

45. $f(x) = \sin x$, $\lim_{x\to \pi} f(x)$

46. $f(x) = \cos x$, $\lim_{x\to \pi} f(x)$

47. $f(x) = \begin{cases} x + 1 & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases}$, $\lim_{x\to 2} f(x)$

48. $f(x) = \begin{cases} x - 1 & \text{if } x \neq 3 \\ 4 & \text{if } x = 3 \end{cases}$, $\lim_{x\to 3} f(x)$

49. $f(x) = \begin{cases} x + 3 & \text{if } x < 0 \\ 4 & \text{if } x \geq 0 \end{cases}$, $\lim_{x\to 0} f(x)$

50. $f(x) = \begin{cases} x + 4 & \text{if } x < 0 \\ 5 & \text{if } x \geq 0 \end{cases}$, $\lim_{x\to 0} f(x)$

51. $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ x + 1 & \text{if } x \geq 1 \end{cases}$, $\lim_{x\to 1} f(x)$

52. $f(x) = \begin{cases} 3x & \text{if } x < 1 \\ x + 2 & \text{if } x \geq 1 \end{cases}$, $\lim_{x\to 1} f(x)$

53. $f(x) = \begin{cases} x + 1 & \text{if } x < 0 \\ \sin x & \text{if } x \geq 0 \end{cases}$, $\lim_{x\to 0} f(x)$

54. $f(x) = \begin{cases} x & \text{if } x < 0 \\ \cos x & \text{if } x \geq 0, \end{cases}$ $\lim\limits_{x \to 0} f(x)$

Practice Plus

In Exercises 55–56, use the equations for the functions f and g to graph the function $y = (f \circ g)(x)$. Then use the graph of $f \circ g$ to find the indicated limit.

55. $f(x) = x^2 - 5, g(x) = \sqrt{x}; \lim\limits_{x \to 2} (f \circ g)(x)$

56. $f(x) = x^2 + 3, g(x) = \sqrt{x}; \lim\limits_{x \to 1} (f \circ g)(x)$

In Exercises 57–58, use the equation for the function f to find and graph the function $y = f^{-1}(x)$. Then use the graph of f^{-1} to find the indicated limit.

57. $f(x) = x^3 - 2; \lim\limits_{x \to 6} f^{-1}(x)$ **58.** $f(x) = x^3 - 4; \lim\limits_{x \to 4} f^{-1}(x)$

In Exercises 59–66, use the graph of $y = f(x)$ to graph each function g. Then use the graph of g to find the indicated limit.

59. $g(x) = f(x) + 2; \lim\limits_{x \to 3} g(x)$

60. $g(x) = f(x) - 2; \lim\limits_{x \to 3} g(x)$

61. $g(x) = f(x + 3); \lim\limits_{x \to 1^-} g(x)$

62. $g(x) = f(x + 2); \lim\limits_{x \to 2^-} g(x)$

63. $g(x) = -f(x); \lim\limits_{x \to -3^+} g(x)$

64. $g(x) = -2f(x); \lim\limits_{x \to -3^+} g(x)$

65. $g(x) = f(2x); \lim\limits_{x \to 1} g(x)$

66. $g(x) = f\left(\frac{1}{2}x\right); \lim\limits_{x \to 1} g(x)$

Application Exercises

67. You are approaching a fan located at 3 on the x-axis.

The function f describes the breeze that you feel, $f(x)$, in miles per hour, when your nose is at position x on the x-axis. Use the values in the table to solve this exercise.

x	2.9	2.99	2.999 → ← 3.001	3.01	3.1
$f(x)$	7.7	7.92	7.991 → ← 7.991	7.92	7.7

a. Find $\lim\limits_{x \to 3} f(x)$. Describe what this means in terms of the location of your nose and the breeze that you feel.

b. Would it be a good idea to move closer so that you actually reach $x = 3$? Describe the difference between what you feel for $\lim\limits_{x \to 3} f(x)$ and $f(3)$.

68. You are riding along an expressway traveling x miles per hour. The function $f(x) = 0.015x^2 + x + 10$ describes the recommended safe distance, $f(x)$, in feet, between your car and other cars on the expressway. Use the values in the table below to find $\lim\limits_{x \to 60} f(x)$. Describe what this means in terms of your car's speed and the recommended safe distance.

x	59.9	59.99	59.999 → ← 60.001	60.01	60.1
$f(x) = 0.015x^2 + x + 10$	123.72	123.972	123.997 → ← 124.003	124.028	124.28

Functions can be used to model changes in intellectual abilities over one's life span. The graphs of f and g show mean scores on standardized tests measuring spatial orientation and verbal ability, respectively, as a function of age. Use the graphs of f and g to solve Exercises 69–70.

Source: Wade and Tavris, *Psychology Sixth Edition,* Prentice Hall, 2000

69. What mean score in spatial orientation is associated with a person whose age is close to 67? Use limit notation to express the answer.

70. What mean score in verbal ability is associated with a person whose age is close to 60? Use limit notation to express the answer.

71. You rent a car from a company that charges \$20 per day plus \$0.10 per mile. The car is driven 200 miles in the first day. The figure shows the graph of the cost, $f(x)$, in dollars, as a function of the miles, x, that you drive the car.

a. Find $\lim_{x \to 100} f(x)$. Interpret the limit, referring to miles driven and cost.

b. For the first day only, what is the rental cost approaching as the mileage gets closer to 200?

c. What is the cost to rent the car at the start of the second day?

72. You are building a greenhouse next to your house, as shown in the figure. Because the house will be used for one side of the enclosure, only three sides will need to be enclosed. You have 60 feet of fiberglass to enclose the three walls.

The function $f(x) = x(60 - 2x)$ describes the area of the greenhouse that you can enclose, $f(x)$, in square feet, if the width of the greenhouse is x feet.

a. Use the table shown to find $\lim_{x \to 15} f(x)$.

X	Y1
14.7	449.82
14.8	449.92
14.9	449.98
15	
15.1	449.98
15.2	449.92
15.3	449.82

Y1=X*(60-2X)

b. Use the graph shown to find $\lim_{x \to 15} f(x)$. Do you get the same limit as you did in part (a)? What information about the limit is shown by the graph that might not be obvious from the table?

[−10, 40, 10] by [0, 500, 50]

Writing in Mathematics

73. Explain how to read $\lim_{x \to a} f(x) = L$.

74. What does the limit notation $\lim_{x \to a} f(x) = L$ mean?

75. Without showing the details, explain how to use a table to find $\lim_{x \to 4} x^2$.

76. Explain how a graph can be used to find a limit.

77. When we find $\lim_{x \to a} f(x)$, we do not care about the value of the function at $x = a$. Explain why this is so.

78. Explain how to read $\lim_{x \to a^-} f(x) = L$.

79. What does the limit notation $\lim_{x \to a^-} f(x) = L$ mean?

80. Explain how to read $\lim_{x \to a^+} f(x) = L$.

81. What does the limit notation $\lim_{x \to a^+} f(x) = L$ mean?

82. What does it mean if the limits in Exercises 79 and 81 are not both equal to the same number L?

Technology Exercises

83. Use the $\boxed{\text{TABLE}}$ feature of your graphing utility to verify any five of the limits that you found in Exercises 5–16.

84. Use the $\boxed{\text{ZOOM IN}}$ feature of your graphing utility to verify any five of the limits that you found in Exercises 33–46. Zoom in on the graph of the given function, f, near $x = a$ to verify each limit.

In Exercises 85–88, estimate $\lim_{x \to a} f(x)$ by using the $\boxed{\text{TABLE}}$ *feature of your graphing utility to create a table of values. Then use the* $\boxed{\text{ZOOM IN}}$ *feature to zoom in on the graph of f near $x = a$ to justify or improve your estimate.*

85. $\lim_{x \to 0} \dfrac{2^x - 1}{x}$

86. $\lim_{x \to 4} \dfrac{\ln x - \ln 4}{x - 4}$

87. $\lim_{x \to 1} \dfrac{x^{3/2} - 1}{x - 1}$

88. $\lim_{x \to 0} \dfrac{x^2}{1 - \cos 2x}$

Critical Thinking Exercises

Make Sense? *In Exercises 89–92, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

89. Limits indicate that a graph can get really close to values without actually reaching them.

90. I'm working with a function that is undefined at 5, so $\lim_{x \to 5} f(x)$ does not exist.

91. I'm working with a function that is undefined at 3, but defined at 2.99, 2.999, 2.9999, as well as at 3.01, 3.001, and 3.0001.

92. I'm working with a function for which $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, so I cannot draw the graph of the function near a without lifting my pencil off the paper.

93. Give an example of a function that is not defined at 2 for which $\lim_{x \to 2} f(x) = 5$.

94. Consider the function $f(x) = 3x + 2$. As x approaches 1, $f(x)$ approaches 5: $\lim_{x \to 1} f(x) = 5$. Find the values of x such that $f(x)$ is within 0.1 of 5 by solving

$$|f(x) - 5| < 0.1.$$

Then find the values of x such that $f(x)$ is within 0.01 of 5.

95. Find an estimate of 3^{π} ($\pi \approx 3.14159265$) by taking a sequence of rational numbers, $x_1, x_2, x_3, \ldots$ that approaches π. Obtain your estimate by evaluating $3^{x_1}, 3^{x_2}, 3^{x_3}, \ldots$.

Preview Exercises

Exercises 96–98 will help you prepare for the material covered in the next section.

96. a. Graph the piecewise function:

$$f(x) = \begin{cases} x^2 + 5 & \text{if } x < 2 \\ 3x + 1 & \text{if } x \geq 2. \end{cases}$$

b. Use your graph from part (a) to find each of the following limits, or indicate that the limit does not exist: $\lim_{x \to 2^-} f(x)$; $\lim_{x \to 2^+} f(x)$; $\lim_{x \to 2} f(x)$.

97. Simplify: $\dfrac{x^2 - x - 6}{x - 3}$.

98. Rationalize the numerator: $\dfrac{\sqrt{4 + x} - 2}{x}$.

SECTION 11.2 Finding Limits Using Properties of Limits

Objectives

1. Find limits of constant functions and the identity function.
2. Find limits using properties of limits.
3. Find one-sided limits using properties of limits.
4. Find limits of fractional expressions in which the limit of the denominator is zero.

Isaac Newton

Gottfried Leibniz

Calculus was invented independently by British mathematician Isaac Newton (1642–1727) and German mathematician Gottfried Leibniz (1646–1716). Although Newton stated that limits were the basic concept in calculus, neither he nor Leibniz was able to express the idea of a limit in a precise mathematical fashion. In essence, Newton and Leibniz developed calculus into a powerful tool even though they could not fully understand why the tool worked.

A great triumph of calculus came with the work of German mathematician Karl Weierstrass (1815–1897). Weierstrass provided a precise definition of $\lim_{x \to a} f(x) = L$, placing calculus on a sound footing almost two hundred years after its invention. The properties of limits presented in this section are theorems that you will prove in calculus using this definition. In this section, you will learn to apply these properties to find limits.

FIGURE 11.8 The graph of the constant function $f(x) = c$. No matter how close x is to a, the corresponding value of $f(x)$ is c.

Limits Involving Constant Functions and the Identity Function

We frequently encounter the constant function, $f(x) = c$, and the identity function, $f(x) = x$. **Figure 11.8** shows the graph of the constant function. The graph is a horizontal line. What does this mean about the limit as x approaches a? Regardless of how close x is to a, the corresponding value of $f(x)$ is c. Thus, if $f(x) = c$, then $\lim_{x \to a} f(x) = c$.

1 Find limits of constant functions and the identity function.

Limit of a Constant Function

For the constant function $f(x) = c$,

$$\lim_{x \to a} f(x) = \lim_{x \to a} c = c,$$

where a is any number. In words, regardless of what number x is approaching, the limit of any constant is that constant.

EXAMPLE 1 Finding Limits of Constant Functions

Find the following limits:

a. $\lim_{x \to 4} 7$ **b.** $\lim_{x \to 0} (-5)$.

SOLUTION

Regardless of what number x is approaching, the limit of any constant is that constant: $\lim_{x \to a} c = c$. Using this formula, we find the given limits.

a. $\lim_{x \to 4} 7 = 7$ **b.** $\lim_{x \to 0} (-5) = -5$

Check Point 1 Find the following limits:

a. $\lim_{x \to 8} 11$ **b.** $\lim_{x \to 0} (-9)$.

The graph of the identity function, $f(x) = x$, is shown in **Figure 11.9**. Each input for this function is associated with an identical output. What does this mean about the limit as x approaches a? For any value of a, as x gets closer to a, the corresponding value of $f(x)$ is just as close to a. Thus, if $f(x) = x$, then $\lim_{x \to a} f(x) = a$.

FIGURE 11.9 The graph of the identity function $f(x) = x$. No matter how close x is to a, the corresponding value of $f(x)$ is just as close to a.

Limit of the Identity Function

For the identity function $f(x) = x$,

$$\lim_{x \to a} f(x) = \lim_{x \to a} x = a,$$

where a is any number. In words, the limit of x as x approaches any number is that number.

EXAMPLE 2 Finding Limits of the Identity Function

Find the following limits:

a. $\lim_{x \to 7} x$ **b.** $\lim_{x \to -\pi} x$.

SOLUTION

We use the formula $\lim_{x \to a} x = a$. The number that x is approaching is also the limit.

a. $\lim_{x \to 7} x = 7$ **b.** $\lim_{x \to -\pi} x = -\pi$

Check Point 2 Find the following limits:

a. $\lim_{x \to 19} x$ **b.** $\lim_{x \to -\sqrt{2}} x$.

2 Find limits using properties of limits.

Properties of Limits

How do we find the limit of a sum, such as

$$\lim_{x \to 5} (x + 7)?$$

We find the limit of each function in the sum:

$$\lim_{x \to 5} x = 5 \quad \text{and} \quad \lim_{x \to 5} 7 = 7.$$

Use the formula $\lim_{x \to a} x = a$.

Use the formula $\lim_{x \to a} c = c$.

Blitzer Bonus
The Feud Over Who Invented Calculus

"How dare Leibniz publish a book on calculus!" fumed Isaac Newton. "He stole my ideas when he was in England. This will not pass easily."

Although Newton and Leibniz invented calculus independently, Newton's overwhelming fear of criticism kept him from publishing. Leibniz published his work on calculus in 1684; Newton waited over 20 years after he invented calculus and did not publish until 1687. Newton was dismayed that he did not publish his version first. Leibniz was accused of plagiarism in a report written by members of the Royal Society in England. The author of the report was not a matter of public record, for Newton himself had written most of it. Newton's accusations against Leibniz set off a bitter dispute between British mathematicians and mathematicians on Continental Europe that lasted over 100 years.

Then we add each of these limits. Thus,

$$\lim_{x\to 5}(x+7) = \lim_{x\to 5} x + \lim_{x\to 5} 7 = 5 + 7 = 12.$$

This is an application of a limit property involving the limit of a sum.

The Limit of a Sum

If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then

$$\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = L + M.$$

In words, the limit of the sum of two functions equals the sum of their limits.

EXAMPLE 3 Finding the Limit of a Sum

Find: $\lim_{x\to -4}(x+9)$.

SOLUTION

The two functions in this limit problem are $f(x) = x$ and $g(x) = 9$. We seek the limit of the sum of these functions.

$$\begin{aligned}\lim_{x\to -4}(x+9) &= \lim_{x\to -4} x + \lim_{x\to -4} 9 && \text{The limit of a sum is the sum of the limits.}\\ &= -4 + 9 && \lim_{x\to a} x = a \text{ and } \lim_{x\to a} c = c.\\ &= 5\end{aligned}$$

Check Point 3 Find: $\lim_{x\to -3}(x+16)$.

In calculus, you will prove the following property involving the limit of the difference of two functions:

The Limit of a Difference

If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then

$$\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = L - M.$$

In words, the limit of the difference of two functions equals the difference of their limits.

EXAMPLE 4 Finding the Limit of a Difference

Find: $\lim_{x\to 5}(12-x)$.

SOLUTION

The two functions in this limit problem are $f(x) = 12$ and $g(x) = x$. We seek the limit of the difference of these functions.

$$\begin{aligned}\lim_{x\to 5}(12-x) &= \lim_{x\to 5} 12 - \lim_{x\to 5} x && \text{The limit of a difference is the difference of the limits.}\\ &= 12 - 5 && \lim_{x\to a} c = c \text{ and } \lim_{x\to a} x = a.\\ &= 7\end{aligned}$$

Check Point 4 Find: $\lim_{x\to 14}(19-x)$.

Now we consider a property that will enable you to find the limit of the product of two functions.

The Limit of a Product

If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, then

$$\lim_{x \to a}[f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = LM.$$

In words, the limit of the product of two functions equals the product of their limits.

EXAMPLE 5 Finding the Limit of a Product

Find: $\lim_{x \to 5}(-6x)$.

SOLUTION

The two functions in this limit problem are $f(x) = -6$ and $g(x) = x$. We seek the limit of the product of these functions.

$$\begin{aligned}
\lim_{x \to 5}(-6x) &= \lim_{x \to 5}(-6) \cdot \lim_{x \to 5} x && \text{The limit of a product is the product of the limits.} \\
&= -6 \cdot 5 && \lim_{x \to a} c = c \text{ and } \lim_{x \to a} x = a. \\
&= -30
\end{aligned}$$

Check Point 5 Find: $\lim_{x \to 7}(-10x)$.

EXAMPLE 6 Finding Limits Using Properties of Limits

Find the following limits:

a. $\lim_{x \to -3}(7x - 4)$ **b.** $\lim_{x \to 5} 6x^2$.

SOLUTION

a.

$$\begin{aligned}
\lim_{x \to -3}(7x - 4) &= \lim_{x \to -3}(7x) - \lim_{x \to -3} 4 && \text{The limit of a difference is the difference of the limits.} \\
&= \lim_{x \to -3} 7 \cdot \lim_{x \to -3} x - \lim_{x \to -3} 4 && \text{The limit of a product is the product of the limits.} \\
&= 7(-3) - 4 && \lim_{x \to a} c = c \text{ and } \lim_{x \to a} x = a. \\
&= -21 - 4 \\
&= -25
\end{aligned}$$

b.

$$\begin{aligned}
\lim_{x \to 5} 6x^2 &= \lim_{x \to 5} 6 \cdot \lim_{x \to 5} x^2 && \text{The limit of a product is the product of the limits.} \\
&= 6 \cdot \lim_{x \to 5}(x \cdot x) && \lim_{x \to a} c = c \\
&= 6 \cdot \lim_{x \to 5} x \cdot \lim_{x \to 5} x && \text{The limit of a product is the product of the limits.} \\
&= 6 \cdot 5 \cdot 5 && \lim_{x \to a} x = a \\
&= 150
\end{aligned}$$

Check Point 6 Find the following limits:

a. $\lim_{x \to -5}(3x - 7)$ **b.** $\lim_{x \to 3} 8x^2$.

The procedure used to find $\lim_{x \to 5} 6x^2$ in Example 6(b) can be used to determine the limit of any monomial function in the form $f(x) = b_n x^n$, where n is a positive integer and b_n is a constant.

$$\lim_{x \to a} b_n x^n = \lim_{x \to a} b_n \cdot \lim_{x \to a} x^n$$

The limit of a product is the product of the limits.

$$= b_n \cdot \lim_{x \to a} \underbrace{(x \cdot x \cdot x \cdot \cdots \cdot x)}_{\text{By definition, } x^n \text{ contains } n \text{ factors of } x.}$$

$\lim_{x \to a} c = c$

$$= b_n \cdot \underbrace{\lim_{x \to a} x \cdot \lim_{x \to a} x \cdot \lim_{x \to a} x \cdot \cdots \cdot \lim_{x \to a} x}_{\text{There are } n \text{ factors of } \lim_{x \to a} x.}$$

The limit of the product containing n factors is the product of the limits.

$$= b_n \cdot \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{\text{There are } n \text{ factors of } a.}$$

$\lim_{x \to a} x = a$

$$= b_n a^n$$

This is the monomial function $f(x) = b_n x^n$ evaluated at a.

The Limit of a Monomial

If n is a positive integer and b_n is a constant, then

$$\lim_{x \to a} b_n x^n = b_n a^n$$

for any number a. In words, the limit of a monomial as x approaches a is the monomial evaluated at a.

EXAMPLE 7 Finding the Limit of a Monomial

Find: $\lim_{x \to 2} (-6x^4)$.

SOLUTION

The limit of the monomial $-6x^4$ as x approaches 2 is the monomial evaluated at 2. Thus, we find the limit by substituting 2 for x.

$$\lim_{x \to 2} (-6x^4) = -6 \cdot 2^4 = -6 \cdot 16 = -96$$

Check Point 7 Find: $\lim_{x \to 2} (-7x^3)$.

How do we find the limit of a polynomial function

$$f(x) = b_n x^n + b_{n-1} x^{n-1} + \cdots + b_1 x + b_0$$

as x approaches a? A polynomial is a sum of monomials. Thus, the limit of a polynomial is the sum of the limits of its monomials.

$$\lim_{x \to a} f(x) = \lim_{x \to a} (b_n x^n + b_{n-1} x^{n-1} + \cdots + b_1 x + b_0)$$

f is a polynomial function.

$$= \lim_{x \to a} b_n x^n + \lim_{x \to a} b_{n-1} x^{n-1} + \cdots + \lim_{x \to a} b_1 x + \lim_{x \to a} b_0$$

The limit of a sum is the sum of the limits.

$$= b_n a^n + b_{n-1} a^{n-1} + \cdots + b_1 a + b_0$$

Find limits by evaluating monomials at a. Find the last limit in the sum using $\lim_{x \to a} c = c$ with $c = b_0$.

This is the polynomial function $f(x) = b_n x^n + b_{n-1} x^{n-1} + \cdots + b_1 x + b_0$ evaluated at a.

$$= f(a)$$

The Limit of a Polynomial

If f is a polynomial function, then

$$\lim_{x \to a} f(x) = f(a)$$

for any number a. In words, the limit of a polynomial as x approaches a is the polynomial evaluated at a.

EXAMPLE 8 Finding the Limit of a Polynomial

Find: $\lim_{x \to 3}(4x^3 + 2x^2 - 6x + 5)$.

SOLUTION

The limit of the polynomial $4x^3 + 2x^2 - 6x + 5$ as x approaches 3 is the polynomial evaluated at 3. Thus, we find the limit by substituting 3 for x.

$$\begin{aligned}
&\lim_{x \to 3}(4x^3 + 2x^2 - 6x + 5) \\
&\quad = 4 \cdot 3^3 + 2 \cdot 3^2 - 6 \cdot 3 + 5 \\
&\quad = 4 \cdot 27 + 2 \cdot 9 - 6 \cdot 3 + 5 \\
&\quad = 108 + 18 - 18 + 5 \\
&\quad = 113
\end{aligned}$$

Check Point 8 Find: $\lim_{x \to 2}(7x^3 + 3x^2 - 5x + 3)$.

A linear function, $f(x) = mx + b$, is a polynomial function of degree one. This means that the limit of a linear function as x approaches a is the linear function evaluated at a:

$$\lim_{x \to a}(mx + b) = ma + b.$$

For example,

$$\lim_{x \to 4}(3x - 7) = 3 \cdot 4 - 7 = 12 - 7 = 5.$$

The next limit property involves the limit of a function to a power, such as

$$\lim_{x \to 2}(x^2 + 2x - 3)^4.$$

To find such a limit, first find $\lim_{x \to 2}(x^2 + 2x - 3)$:

$$\lim_{x \to 2}(x^2 + 2x - 3) = 2^2 + 2 \cdot 2 - 3 = 4 + 4 - 3 = 5.$$

The limit that we seek is found by taking this limit, 5, and raising it to the fourth power. Thus,

$$\lim_{x \to 2}(x^2 + 2x - 3)^4 = \left[\lim_{x \to 2}(x^2 + 2x - 3)\right]^4 = 5^4 = 625.$$

The Limit of a Power

If $\lim_{x \to a} f(x) = L$ and n is a positive integer, then

$$\lim_{x \to a}[f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n = L^n.$$

In words, the limit of a function to a power is found by taking the limit of the function and then raising this limit to the power.

Blitzer Bonus
Calculus in Japan

Mathematics was developed simultaneously in various cultures in all parts of the world. Seki Kowa, a 17th-century Japanese mathematician, is credited with the invention of calculus in Japan. The illustration shown below was drawn in 1670 by a pupil of Seki Kowa. It measures the circle's area with a series of rectangles.

EXAMPLE 9 Finding the Limit of a Power

Find: $\lim_{x \to 5}(2x - 7)^3$.

SOLUTION

The limit of the linear function $f(x) = 2x - 7$ as x approaches 5 is the linear function evaluated at 5. Because this function is raised to the third power, the limit that we seek is the limit of the linear function raised to the third power.

$$\lim_{x \to 5}(2x - 7)^3 = \left[\lim_{x \to 5}(2x - 7)\right]^3 = (2 \cdot 5 - 7)^3 = 3^3 = 27$$

Check Point 9 Find: $\lim_{x \to 4}(3x - 5)^3$.

How do we find the limit of a root? Recall that if $\sqrt[n]{L}$ represents a real number and $n \geq 2$, then

$$\sqrt[n]{L} = L^{1/n}.$$

Because a root is a power, we find the limit of a root using a similar procedure for finding the limit of a power.

The Limit of a Root

If $\lim_{x \to a} f(x) = L$ and n is a positive integer greater than or equal to 2, then

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L}$$

provided that all roots represent real numbers. In words, the limit of the nth root of a function is found by taking the limit of the function and then taking the nth root of this limit.

EXAMPLE 10 Finding the Limit of a Root

Find: $\lim_{x \to -2} \sqrt{4x^2 + 5}$.

SOLUTION

The limit of the quadratic (polynomial) function $f(x) = 4x^2 + 5$ as x approaches -2 is the function evaluated at -2. Because we have the square root of this function, the limit that we seek is the square root of the limit of the quadratic function.

$$\lim_{x \to -2} \sqrt{4x^2 + 5} = \sqrt{\lim_{x \to -2}(4x^2 + 5)} = \sqrt{4(-2)^2 + 5} = \sqrt{16 + 5} = \sqrt{21}$$

Check Point 10 Find: $\lim_{x \to -1} \sqrt{6x^2 - 4}$.

We have considered limits of sums, differences, products, and roots. We conclude with a property that deals with the limit of a quotient.

The Limit of a Quotient

If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, $M \neq 0$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M}, M \neq 0.$$

In words, the limit of the quotient of two functions equals the quotient of their limits, **provided that the limit of the denominator is not zero**.

Before possibly applying the quotient property, begin by finding the limit of the denominator. If this limit is not zero, you can apply the quotient property. If this limit is zero, the quotient property cannot be used.

EXAMPLE 11 Finding the Limit of a Quotient

Find: $\lim_{x \to 1} \frac{x^3 - 3x^2 + 7}{2x - 5}$.

SOLUTION

The two functions in this limit problem are $f(x) = x^3 - 3x^2 + 7$ and $g(x) = 2x - 5$. We seek the limit of the quotient of these functions. Can we use the quotient property for limits? We answer the question by finding the limit of the denominator, $g(x)$.

$$\lim_{x \to 1}(2x - 5) = 2 \cdot 1 - 5 = -3$$

Because the limit of the denominator is not zero, we can apply the quotient property for limits. The limit of the quotient is the quotient of the limits.

$$\lim_{x \to 1} \frac{x^3 - 3x^2 + 7}{2x - 5} = \frac{\lim_{x \to 1}(x^3 - 3x^2 + 7)}{\lim_{x \to 1}(2x - 5)} = \frac{1^3 - 3 \cdot 1^2 + 7}{2 \cdot 1 - 5} = \frac{5}{-3} = -\frac{5}{3}$$

Check Point 11 Find: $\lim_{x \to 2} \frac{x^2 - 4x + 1}{3x - 5}$.

We've considered a number of limit properties. Let's take a moment to summarize these properties.

Properties of Limits

Formulas for Finding Limits

1. $\lim_{x \to a} c = c$
2. $\lim_{x \to a} x = a$
3. If f is a polynomial (linear, quadratic, cubic, etc.) function, $\lim_{x \to a} f(x) = f(a)$.

Limits of Sums, Differences, Products, Powers, Roots, and Quotients

If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, then

4. $\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = L + M$.
5. $\lim_{x \to a}[f(x) - (x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = L - M$.
6. $\lim_{x \to a}[f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = LM$.
7. $\lim_{x \to a}[f(x)]^n = [\lim_{x \to a} f(x)]^n = L^n$, where $n \geq 2$ is an integer.
8. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L}$, where $n \geq 2$ is an integer and all roots represent real numbers.
9. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M}$, $M \neq 0$.

3 Find one-sided limits using properties of limits.

Properties of Limits and Piecewise Functions

In Section 11.1, we used graphs of piecewise functions to find one-sided limits. We can now find such limits by applying properties of limits to the appropriate part of a piecewise function's equation.

EXAMPLE 12 Using Limit Properties to Find One-Sided Limits

Consider the piecewise function defined by

$$f(x) = \begin{cases} x^2 + 5 & \text{if} \quad x < 2 \\ 3x + 1 & \text{if} \quad x \geq 2. \end{cases}$$

Find each of the following limits, or state that the limit does not exist:

a. $\lim\limits_{x \to 2^-} f(x)$ **b.** $\lim\limits_{x \to 2^+} f(x)$ **c.** $\lim\limits_{x \to 2} f(x)$.

SOLUTION

a. To find $\lim\limits_{x \to 2^-} f(x)$, we look at values of $f(x)$ when x is close to 2 but less than 2. Because x is less than 2, we use the first line of the piecewise function's equation, $f(x) = x^2 + 5$.

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x^2 + 5) = 2^2 + 5 = 9$$

b. To find $\lim\limits_{x \to 2^+} f(x)$, we look at values of $f(x)$ when x is close to 2 but greater than 2. Because x is greater than 2, we use the second line of the piecewise function's equation, $f(x) = 3x + 1$.

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3x + 1) = 3 \cdot 2 + 1 = 7$$

c. We found that

$$\lim_{x \to 2^-} f(x) = 9 \quad \text{and} \quad \lim_{x \to 2^+} f(x) = 7.$$

The limit as x approaches 2 from the left equals 9.

The limit as x approaches 2 from the right equals 7.

These one-sided limits are illustrated in **Figure 11.10**. Because the left- and right-hand limits are unequal, $\lim\limits_{x \to 2} f(x)$ does not exist.

FIGURE 11.10

Check Point 12 Consider the piecewise function defined by

$$f(x) = \begin{cases} -1 & \text{if} \quad x < 1 \\ \sqrt[3]{2x - 1} & \text{if} \quad x \geq 1. \end{cases}$$

Find each of the following limits, or state that the limit does not exist:

a. $\lim\limits_{x \to 1^-} f(x)$ **b.** $\lim\limits_{x \to 1^+} f(x)$ **c.** $\lim\limits_{x \to 1} f(x)$.

4 Find limits of fractional expressions in which the limit of the denominator is zero.

Strategies for Finding Limits When the Limit of the Denominator Is Zero

When taking the limit of a fractional expression in which the limit of the denominator is zero, the quotient property for limits cannot be used. In such cases, it is necessary to rewrite the expression before the limit can be found. Factoring is one technique that can be used to rewrite an expression.

EXAMPLE 13 Using Factoring to Find a Limit

Find: $\lim\limits_{x \to 3} \dfrac{x^2 - x - 6}{x - 3}$.

SOLUTION

The limit of the denominator is zero:

$$\lim_{x \to 3} (x - 3) = 3 - 3 = 0.$$

GREAT QUESTION!

Can I apply the quotient property for limits when the limit of the denominator is zero?

No. Avoid this common error:

INCORRECT!

$$\cancel{\lim_{x\to 3}\frac{x^2-x-6}{x-3} = \frac{\lim_{x\to 3}(x^2-x-6)}{\lim_{x\to 3}(x-3)} = \frac{3^2-3-6}{3-3} = \frac{0}{0} = 1.}$$

Do not use the quotient property if the limit of the denominator is zero. Furthermore, division by zero is not permitted: $\frac{0}{0}$ is *not* equal to 1.

Thus, the quotient property for limits cannot be used. Instead, try simplifying the expression using factoring:

$$\frac{x^2-x-6}{x-3} = \frac{(x-3)(x+2)}{x-3}.$$

We seek the limit of this expression as x approaches 3. Because x is close to 3 but not equal to 3, the common factor in the numerator and denominator, $x-3$, is not equal to zero. With $x-3 \neq 0$, we can divide the numerator and denominator by $x-3$. Cancel the common factor, $x-3$, and then take the limit.

$$\lim_{x\to 3}\frac{x^2-x-6}{x-3} = \lim_{x\to 3}\frac{\cancel{(x-3)}(x+2)}{\cancel{x-3}} = \lim_{x\to 3}(x+2) = 3+2 = 5$$

The graph of $f(x) = \dfrac{x^2-x-6}{x-3}$ is shown in **Figure 11.11**. The hole in the graph at $x = 3$ shows that $f(3)$ is undefined. However, as x approaches 3, the graph shows that the values of f get closer to 5. This verifies the limit that we found in Example 13:

$$\lim_{x\to 3}\frac{x^2-x-6}{x-3} = 5.$$

Check Point 13 Find: $\displaystyle\lim_{x\to 1}\frac{x^2+2x-3}{x-1}$.

FIGURE 11.11 As x approaches 3, values of f get closer to 5:

$$\lim_{x\to 3}\frac{x^2-x-6}{x-3} = 5.$$

Rationalizing the numerator or denominator of a fractional expression is another technique that can be used to find a limit when the limit of the denominator is zero.

EXAMPLE 14 Rationalizing a Numerator to Find a Limit

Find: $\displaystyle\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}$.

SOLUTION

As x approaches 0, the denominator of the expression approaches zero. Thus, the quotient property for limits cannot be used. Instead, try rewriting the expression by rationalizing the numerator. If we multiply the numerator and denominator by $\sqrt{4+x}+2$, the numerator will not contain radicals.

$$\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}$$

$$= \lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}\cdot\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$$ Rationalize the numerator.

$$= \lim_{x\to 0}\frac{(\sqrt{4+x})^2-2^2}{x(\sqrt{4+x}+2)}$$ $(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b}) = (\sqrt{a})^2-(\sqrt{b})^2$

$$= \lim_{x\to 0}\frac{4+x-4}{x(\sqrt{4+x}+2)}$$ $(\sqrt{4+x})^2 = 4+x$

$$= \lim_{x\to 0}\frac{x}{x(\sqrt{4+x}+2)}$$ Simplify: $4+x-4 = x$.

$$= \lim_{x\to 0}\frac{1}{\sqrt{4+x}+2}$$ Divide both the numerator and denominator by x. This is permitted because x approaches 0 but is not equal to 0.

$$= \frac{\lim_{x\to 0}1}{\sqrt{\lim_{x\to 0}(4+x)}+\lim_{x\to 0}2}$$ Use limit properties.

$$= \frac{1}{\sqrt{4+0}+2}$$ Take the limits.

$$= \frac{1}{2+2} = \frac{1}{4}$$ Simplify.

Check Point 14 Find: $\lim_{x \to 0} \frac{\sqrt{9 + x} - 3}{x}$.

CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

1. $\lim_{x \to a} c =$ _____
2. $\lim_{x \to a} x =$ _____
3. If f is a polynomial function, then $\lim_{x \to a} f(x) =$ _____.

If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, use L, M, or both to complete the statements in Exercises 4–9.

4. $\lim_{x \to a}[f(x) + g(x)] =$ _____
5. $\lim_{x \to a}[f(x) - g(x)] =$ _____
6. $\lim_{x \to a}[f(x) \cdot g(x)] =$ _____
7. $\lim_{x \to a}[f(x)]^n =$ _____, where $n \geq 2$ is an integer
8. $\lim_{x \to a} \sqrt[n]{f(x)} =$ _____, where $n \geq 2$ is an integer and all roots represent real numbers
9. $\lim_{x \to a} \frac{f(x)}{g(x)} =$ _____, $M \neq 0$
10. For the piecewise function defined by
$$f(x) = \begin{cases} x^2 + 5 & \text{if } x < 2 \\ x^3 + 1 & \text{if } x \geq 2 \end{cases},$$
we find $\lim_{x \to 2^-} f(x)$ using $f(x) =$ _____ and we find $\lim_{x \to 2^+} f(x)$ using $f(x) =$ _____.
11. True or false: $\lim_{x \to 6} \frac{x^2 - 25}{x - 5} = \frac{\lim_{x \to 6}(x^2 - 25)}{\lim_{x \to 6}(x - 5)}$ _____
12. True or false: $\lim_{x \to 5} \frac{x^2 - 25}{x - 5} = \frac{\lim_{x \to 5}(x^2 - 25)}{\lim_{x \to 5}(x - 5)}$ _____

EXERCISE SET 11.2

Practice Exercises

In Exercises 1–42, use properties of limits to find the indicated limit. It may be necessary to rewrite an expression before limit properties can be applied.

1. $\lim_{x \to 2} 8$
2. $\lim_{x \to 3}(-6)$
3. $\lim_{x \to 2} x$
4. $\lim_{x \to 3} x$
5. $\lim_{x \to 6}(3x - 4)$
6. $\lim_{x \to 7}(4x - 3)$
7. $\lim_{x \to -2} 7x^2$
8. $\lim_{x \to -3} 5x^2$
9. $\lim_{x \to 5}(x^2 - 3x - 4)$
10. $\lim_{x \to 6}(x^2 - 4x - 7)$
11. $\lim_{x \to 2}(5x - 8)^3$
12. $\lim_{x \to 4}(6x - 21)^3$
13. $\lim_{x \to 1}(2x^2 - 3x + 5)^2$
14. $\lim_{x \to 2}(2x^2 + 3x - 1)^2$
15. $\lim_{x \to -4} \sqrt{x^2 + 9}$
16. $\lim_{x \to -1} \sqrt{5x^2 + 4}$
17. $\lim_{x \to 5} \frac{x}{x + 1}$
18. $\lim_{x \to 2} \frac{3x}{x - 4}$
19. $\lim_{x \to 2} \frac{x^2 - 1}{x - 1}$
20. $\lim_{x \to 3} \frac{x^2 - 4}{x - 2}$
21. $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$
22. $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$
23. $\lim_{x \to 2} \frac{2x - 4}{x - 2}$
24. $\lim_{x \to 3} \frac{4x - 12}{x - 3}$
25. $\lim_{x \to 1} \frac{x^2 + 2x - 3}{x^2 - 1}$
26. $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 9}$
27. $\lim_{x \to 2} \frac{x^3 - 2x^2 + 4x - 8}{x^4 - 2x^3 + x - 2}$
28. $\lim_{x \to -1} \frac{x^3 + 2x^2 + x}{x^4 + x^3 + 2x + 2}$
29. $\lim_{x \to 0} \frac{\sqrt{1 + x} - 1}{x}$
30. $\lim_{x \to 0} \frac{\sqrt{16 + x} - 4}{x}$
31. $\lim_{x \to 2} [(x + 1)^2(3x - 1)^3]$
32. $\lim_{x \to -1} [(x + 2)^3(3x + 2)]$
33. $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$
34. $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$
35. $\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$
36. $\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$
37. $\lim_{x \to 4} \frac{\sqrt{x} + 5}{x - 5}$
38. $\lim_{x \to 9} \frac{\sqrt{x} + 10}{x - 10}$
39. $\lim_{x \to 0} \frac{\frac{1}{x + 3} - \frac{1}{3}}{x}$
40. $\lim_{x \to 0} \frac{\frac{1}{x + 4} - \frac{1}{4}}{x}$
41. $\lim_{x \to 2} \frac{x^2 - 4}{x^3 - 8}$
42. $\lim_{x \to 1} \frac{x^2 - 1}{x^3 - 1}$

In Exercises 43–50, a piecewise function is given. Use properties of limits to find the indicated limit, or state that the limit does not exist.

43. $f(x) = \begin{cases} x + 5 & \text{if } x < 1 \\ x + 7 & \text{if } x \geq 1 \end{cases}$
    a. $\lim_{x \to 1^-} f(x)$ b. $\lim_{x \to 1^+} f(x)$ c. $\lim_{x \to 1} f(x)$
44. $f(x) = \begin{cases} x + 6 & \text{if } x < 1 \\ x + 9 & \text{if } x \geq 1 \end{cases}$
    a. $\lim_{x \to 1^-} f(x)$ b. $\lim_{x \to 1^+} f(x)$ c. $\lim_{x \to 1} f(x)$

45. $f(x) = \begin{cases} x^2 + 5 & \text{if } x < 2 \\ x^3 + 1 & \text{if } x \geq 2 \end{cases}$

a. $\lim\limits_{x \to 2^-} f(x)$ **b.** $\lim\limits_{x \to 2^+} f(x)$ **c.** $\lim\limits_{x \to 2} f(x)$

46. $f(x) = \begin{cases} x^2 + 6 & \text{if } x < 2 \\ x^3 + 2 & \text{if } x \geq 2 \end{cases}$

a. $\lim\limits_{x \to 2^-} f(x)$ **b.** $\lim\limits_{x \to 2^+} f(x)$ **c.** $\lim\limits_{x \to 2} f(x)$

47. $f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 5 & \text{if } x = 3 \end{cases}$

a. $\lim\limits_{x \to 3^-} f(x)$ **b.** $\lim\limits_{x \to 3^+} f(x)$ **c.** $\lim\limits_{x \to 3} f(x)$

48. $f(x) = \begin{cases} \dfrac{x^2 - 16}{x - 4} & \text{if } x \neq 4 \\ 7 & \text{if } x = 4 \end{cases}$

a. $\lim\limits_{x \to 4^-} f(x)$ **b.** $\lim\limits_{x \to 4^+} f(x)$ **c.** $\lim\limits_{x \to 4} f(x)$

49. $f(x) = \begin{cases} 1 - x & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ x^2 - 1 & \text{if } x > 1 \end{cases}$

a. $\lim\limits_{x \to 1^-} f(x)$ **b.** $\lim\limits_{x \to 1^+} f(x)$ **c.** $\lim\limits_{x \to 1} f(x)$

50. $f(x) = \begin{cases} 4 - x & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ x^2 + 2 & \text{if } x > 1 \end{cases}$

a. $\lim\limits_{x \to 1^-} f(x)$ **b.** $\lim\limits_{x \to 1^+} f(x)$ **c.** $\lim\limits_{x \to 1} f(x)$

Practice Plus

51. Let $f(x) = x^3 - x^2 + 5x - 1$ and $g(x) = 2$. Find $\lim\limits_{x \to 3}(f \circ g)(x)$ and $\lim\limits_{x \to 3}(g \circ f)(x)$.

52. Let $f(x) = x^3 + x^2 - 6x - 1$ and $g(x) = 3$. Find $\lim\limits_{x \to 4}(f \circ g)(x)$ and $\lim\limits_{x \to 4}(g \circ f)(x)$.

53. Let $f(x) = \dfrac{2}{x}$ and $g(x) = \dfrac{3}{x - 1}$.

Find $\lim\limits_{x \to 1}(f \circ g)(x)$ and $\lim\limits_{x \to 1}(g \circ f)(x)$.

54. Let $f(x) = \dfrac{4}{x - 1}$ and $g(x) = \dfrac{1}{x + 2}$.

Find $\lim\limits_{x \to 1}(f \circ g)(x)$ and $\lim\limits_{x \to 1}(g \circ f)(x)$.

55. Let $f(x) = x^2 + 4, x \geq 0$. Find $\lim\limits_{x \to 8} f^{-1}(x)$.

56. Let $f(x) = x^2 + 9, x \geq 0$. Find $\lim\limits_{x \to 25} f^{-1}(x)$.

57. Let $f(x) = \dfrac{2x + 1}{x - 1}$. Find $\lim\limits_{x \to 4} f^{-1}(x)$.

58. Let $f(x) = \dfrac{2x + 3}{x + 4}$. Find $\lim\limits_{x \to 5} f^{-1}(x)$.

Application Exercises

In Albert Einstein's special theory of relativity, time slows down and length in the direction of motion decreases from the point of view of an observer watching an object moving at velocities approaching the speed of light. (The speed of light is approximately 186,000 miles per second. At this speed, a beam of light can travel around the world about seven times in a single second.) Einstein's theory, verified with experiments in atomic physics, forms the basis of Exercises 59–60.

Starship at rest

Change in starship's length when moving at 99% of light's speed when viewed by an observer

59. The formula

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

expresses the length, L, of a starship moving at velocity v with respect to an observer on Earth, where L_0 is the length of the starship at rest and c is the speed of light.

a. Find $\lim\limits_{v \to c^-} L$.

b. If a starship is traveling at velocities approaching the speed of light, what does the limit in part (a) indicate about its length from the perspective of a stationary viewer on Earth?

c. Explain why a left-hand limit is used in part (a).

60. The formula

$$R_a = R_f\sqrt{1 - \frac{v^2}{c^2}}$$

expresses the aging rate of an astronaut, R_a, relative to the aging rate of a friend on Earth, R_f, where v is the astronaut's velocity and c is the speed of light.

a. Find $\lim\limits_{v \to c^-} R_a$.

b. If you are traveling in a starship at velocities approaching the speed of light, what does the limit in part (a) indicate about your aging rate relative to a friend on Earth?

c. Explain why a left-hand limit is used in part (a).

Writing in Mathematics

61. Explain how to find the limit of a constant. Then express your written explanation using limit notation.

62. Explain how to find the limit of the identity function $f(x) = x$. Then express your written explanation using limit notation.

63. Explain how to find the limit of a sum. Then express your written explanation using limit notation.

64. Explain how to find the limit of a difference. Then express your written explanation using limit notation.

65. Explain how to find the limit of a product. Then express your written explanation using limit notation.

66. Describe how to find the limit of a polynomial function. Provide an example with your description.

67. Explain how to find the following limit: $\lim\limits_{x \to 2}(3x^2 - 10)^3$. Then use limit notation to write the limit property that supports your explanation.

68. Explain how to find the following limit: $\lim_{x \to 2} \sqrt{5x - 6}$. Then use limit notation to write the limit property that supports your explanation.

69. Explain how to find the limit of a quotient if the limit of the denominator is not zero. Then express your written explanation using limit notation.

70. Write an example involving the limit of a quotient in which the quotient property for limits cannot be applied. Explain why the property cannot be applied to your limit problem.

71. Explain why

$$\lim_{x \to 4} \frac{(x + 4)(x - 4)}{x - 4}$$

can be found by first dividing the numerator and the denominator of the expression by $x - 4$. Division by zero is undefined. How can we be sure that we are not dividing the numerator and the denominator by zero?

Technology Exercises

72. Use the $\boxed{\text{TABLE}}$ feature of your graphing utility to verify any five of the limits that you found in Exercises 1–42.

73. Use the $\boxed{\text{ZOOM IN}}$ feature of your graphing utility to verify any five of the limits that you found in Exercises 1–42. Zoom in on the graph of the given function, f, near $x = a$ to verify each limit.

Critical Thinking Exercises

Make Sense? *In Exercises 74–77, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

74. I evaluated a polynomial function f at 3 and obtained 7, so 7 must be $\lim_{x \to 3} f(x)$.

75. I'm working with functions f and g for which $\lim_{x \to 4} f(x) = 0$, $\lim_{x \to 4} g(x) = -5$, and $\lim_{x \to 4} [f(x) - g(x)] = 5$.

76. I'm working with functions f and g for which $\lim_{x \to 4} f(x) = 0$, $\lim_{x \to 4} g(x) = -5$, and $\lim_{x \to 4} \frac{f(x)}{g(x)} = 0$.

77. I'm working with functions f and g for which $\lim_{x \to 4} f(x) = 0$, $\lim_{x \to 4} g(x) = -5$, and $\lim_{x \to 4} \frac{g(x)}{f(x)} \neq 0$.

In Exercises 78–79, find the indicated limit.

78. $\lim_{x \to 0} x\left(1 - \frac{1}{x}\right)$

79. $\lim_{x \to 4} \left(\frac{1}{x} - \frac{1}{4}\right)\left(\frac{1}{x - 4}\right)$

In Exercises 80–81, find $\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$.

80. $f(x) = x^2 + 2x - 3, a = 1$

81. $f(x) = \sqrt{x}, a = 1$

In Exercises 82–83, use properties of limits and the following limits

$$\lim_{x \to 0} \frac{\sin x}{x} = 1, \qquad \lim_{x \to 0} \frac{\cos x - 1}{x} = 0,$$
$$\lim_{x \to 0} \sin x = 0, \qquad \lim_{x \to 0} \cos x = 1$$

to find the indicated limit.

82. $\lim_{x \to 0} \frac{\tan x}{x}$

83. $\lim_{x \to 0} \frac{2 \sin x + \cos x - 1}{3x}$

Group Exercises

84. Below is a list of ten common errors involving algebra, trigonometry, and limits that students frequently make in calculus. Group members should examine each error and describe the mistake. Where possible, correct each error. Finally, group members should offer suggestions for avoiding each error.

a. $(x + h)^3 - x^3 = x^3 + h^3 - x^3 = h^3$

b. $\frac{1}{a + b} = \frac{1}{a} + \frac{1}{b}$

c. $\frac{1}{a + b} = \frac{1}{a} + b$

d. $\sqrt{x + h} - \sqrt{x} = \sqrt{x} + \sqrt{h} - \sqrt{x} = \sqrt{h}$

e. $\frac{\sin 2x}{x} = \sin 2$

f. $\frac{a + bx}{a} = 1 + bx$

g. $\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \frac{1^3 - 1}{1 - 1} = \frac{0}{0} = 1$

h. $\sin(x + h) - \sin x = \sin x + \sin h - \sin x = \sin h$

i. $ax = bx$, so $a = b$

j. To find $\lim_{x \to 4} \frac{x^2 - 9}{x - 3}$, it is necessary to rewrite $\frac{x^2 - 9}{x - 3}$ by factoring $x^2 - 9$.

85. Research and present a group report about the history of the feud between Newton and Leibniz over who invented calculus. What other interests did these men have in addition to mathematics? What practical problems led them to the invention of calculus? What were their personalities like? Whose version established the notation and rules of calculus that we use today?

Preview Exercises

Exercises 86–88 will help you prepare for the material covered in the next section. In each exercise, use what occurs near 3 and at 3 to graph the function in an open interval about 3. (Graphs will vary.) Is it necessary to lift your pencil off the paper to obtain each graph? Explain your answer.

86. $\lim_{x \to 3} f(x) = 5; f(3) = 5$

87. $\lim_{x \to 3} f(x) = 5; f(3) = 6$

88. $\lim_{x \to 3^-} f(x) = 5; \lim_{x \to 3^+} f(x) = 6; f(3) = 5$

SECTION 11.3 Limits and Continuity

Objectives

1. Determine whether a function is continuous at a number.
2. Determine for what numbers a function is discontinuous.

Why you should not ski down discontinuous slopes

In everyday speech, a continuous process is one that goes on without interruption and without abrupt changes. In mathematics, a continuous function has much the same meaning. The graph of a continuous function does not have interrupting breaks, such as holes, gaps, or jumps. Thus, the graph of a continuous function can be drawn without lifting a pencil off the paper. In this section, you will learn how limits can be used to describe continuity.

1 Determine whether a function is continuous at a number.

Limits and Continuity

Figure 11.12 shows three graphs that cannot be drawn without lifting a pencil from the paper. In each case, there appears to be an interruption of the graph of f at $x = a$.

FIGURE 11.12 Each graph has an interruption at $x = a$.

Examine **Figure 11.12(a)**. The interruption occurs because the open dot indicates there is no point on the graph corresponding to $x = a$. This shows that $f(a)$ is not defined.

Now, examine **Figure 11.12(b)**. The closed blue dot at $x = a$ shows that $f(a)$ is defined. However, there is a jump at a. As x approaches a from the left, the values of f get closer to the y-coordinate of the point shown by the open dot. By contrast, as x approaches a from the right, the values of f get closer to the y-coordinate of the point shown by the closed dot. There is no single limit as x approaches a. The jump in the graph reflects the fact that $\lim_{x \to a} f(x)$ does not exist.

Finally, examine **Figure 11.12(c)**. The closed blue dot at $x = a$ shows that $f(a)$ is defined. Furthermore, as x approaches a from the left or from the right, the values of f get closer to the y-coordinate of the point shown by the open dot. Thus, $\lim_{x \to a} f(x)$ exists. However, there is still an interruption at a. Do you see why? The limit as x approaches a, $\lim_{x \to a} f(x)$, is the y-coordinate of the open dot. By contrast, the value of the function at a, $f(a)$, is the y-coordinate of the closed dot. The interruption in the graph reflects the fact that $\lim_{x \to a} f(x)$ and $f(a)$ are not equal.

We now provide a precise definition of what it means for a function to be continuous at a number. Notice how each part of this definition avoids the interruptions that occurred in **Figure 11.12**.

Definition of a Function Continuous at a Number

A function f **is continuous at** a when three conditions are satisfied.

1. f is defined at a; that is, a is in the domain of f, so that $f(a)$ is a real number.
2. $\lim_{x \to a} f(x)$ exists.
3. $\lim_{x \to a} f(x) = f(a)$

If f is not continuous at a, we say that f is **discontinuous at** a. Each of the functions whose graph is shown in **Figure 11.12** on the previous page is discontinuous at a.

EXAMPLE 1 Determining Whether a Function Is Continuous at a Number

Determine whether the function

$$f(x) = \frac{2x + 1}{2x^2 - x - 1}$$

is continuous: **a.** at 2; **b.** at 1.

SOLUTION

According to the definition, three conditions must be satisfied to have continuity at a.

a. To determine whether $f(x) = \dfrac{2x + 1}{2x^2 - x - 1}$ is continuous at 2, we check the conditions for continuity with $a = 2$.

Condition 1 f is defined at a. Is $f(2)$ defined?

$$f(2) = \frac{2 \cdot 2 + 1}{2 \cdot 2^2 - 2 - 1} = \frac{4 + 1}{8 - 2 - 1} = \frac{5}{5} = 1$$

Because $f(2)$ is a real number, 1, $f(2)$ is defined.

Condition 2 $\lim_{x \to a} f(x)$ exists. Does $\lim_{x \to 2} f(x)$ exist?

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{2x + 1}{2x^2 - x - 1} = \frac{\lim_{x \to 2}(2x + 1)}{\lim_{x \to 2}(2x^2 - x - 1)}$$

$$= \frac{2 \cdot 2 + 1}{2 \cdot 2^2 - 2 - 1} = \frac{4 + 1}{8 - 2 - 1} = \frac{5}{5} = 1$$

Using properties of limits, we see that $\lim_{x \to 2} f(x)$ exists.

Condition 3 $\lim_{x \to a} f(x) = f(a)$ Does $\lim_{x \to 2} f(x) = f(2)$? We found that $\lim_{x \to 2} f(x) = 1$ and $f(2) = 1$. Thus, as x gets closer to 2, the corresponding values of $f(x)$ get closer to the function value at 2: $\lim_{x \to 2} f(x) = f(2)$.

Because the three conditions are satisfied, we conclude that f is continuous at 2.

b. To determine whether $f(x) = \dfrac{2x + 1}{2x^2 - x - 1}$ is continuous at 1, we check the conditions for continuity with $a = 1$.

Condition 1 f is defined at a. Is $f(1)$ defined? Factor the denominator of the function's equation:

$$f(x) = \frac{2x + 1}{(x - 1)(2x + 1)}$$

Denominator is zero at $x = 1$.

Denominator is zero at $x = -\frac{1}{2}$.

Because division by zero is undefined, the domain of f is $\{x \mid x \neq 1, x \neq -\frac{1}{2}\}$. Thus, f is not defined at 1.

FIGURE 11.13 f is continuous at 2. It is not continuous at 1 or at $-\frac{1}{2}$.

Because one of the three conditions is not satisfied, we conclude that f is not continuous at 1. Equivalently, we can say that f is discontinuous at 1.

The graph of $f(x) = \dfrac{2x+1}{2x^2-x-1}$ is shown in **Figure 11.13**. The graph verifies our work in Example 1. Can you see that f is continuous at 2? By contrast, it is not continuous at 1, where the graph has a vertical asymptote.

The graph in **Figure 11.13** also reveals a discontinuity at $-\frac{1}{2}$. The open dot indicates that there is no point on the graph corresponding to $x = -\frac{1}{2}$. Can you see what is happening as x approaches $-\frac{1}{2}$?

$$\lim_{x\to-\frac{1}{2}} \frac{2x+1}{2x^2-x-1} = \lim_{x\to-\frac{1}{2}} \frac{\cancel{2x+1}}{(x-1)\cancel{(2x+1)}} = \lim_{x\to-\frac{1}{2}} \frac{1}{x-1} = \frac{1}{-\frac{1}{2}-1} = -\frac{2}{3}$$

As x gets closer to $-\frac{1}{2}$, the graph of f gets closer to $-\frac{2}{3}$. Because f is not defined at $-\frac{1}{2}$, the graph has a hole at $\left(-\frac{1}{2}, -\frac{2}{3}\right)$. This is shown by the open dot in **Figure 11.13**.

Check Point 1 Determine whether the function

$$f(x) = \frac{x-2}{x^2-4}$$

is continuous: **a.** at 1; **b.** at 2.

2 Determine for what numbers a function is discontinuous.

Determining Where Functions Are Discontinuous

We have seen that the limit of a polynomial function as x approaches a is the polynomial function evaluated at a. Thus, if f is a polynomial function, then $\lim_{x\to a} f(x) = f(a)$ for any number a. This means that **a polynomial function is continuous at every number**.

Many of the functions discussed throughout this book are continuous at every number in their domain. For example, rational functions are continuous at every number, except any at which they are not defined. At numbers that are not in the domain of a rational function, a hole in the graph or an asymptote appears. Exponential, logarithmic, sine, and cosine functions are continuous at every number in their domain. Like rational functions, the tangent, cotangent, secant, and cosecant functions are continuous at every number, except any at which they are not defined. At numbers that are not in the domain of these trigonometric functions, an asymptote occurs.

GREAT QUESTION!

What's the bottom line on when functions are continuous and when they're not?

Most functions are always continuous at every number in their domain, including polynomial, rational, radical, exponential, logarithmic, and trigonometric functions. Most of the discontinuities you will encounter in calculus will be due to jumps in piecewise functions.

Example 2 illustrates how to determine where a piecewise function is discontinuous.

EXAMPLE 2 Determining Where a Piecewise Function Is Discontinuous

Determine for what numbers x, if any, the following function is discontinuous:

$$f(x) = \begin{cases} x+2 & \text{if } x \le 0 \\ 2 & \text{if } 0 < x \le 1. \\ x^2+2 & \text{if } x > 1 \end{cases}$$

$$f(x) = \begin{cases} x + 2 & \text{if } x \le 0 \\ 2 & \text{if } 0 < x \le 1 \\ x^2 + 2 & \text{if } x > 1 \end{cases}$$

The given piecewise function (repeated)

SOLUTION

First, let's determine whether each of the three pieces of f is continuous. The first piece, $f(x) = x + 2$, is a linear function; it is continuous at every number x. The second piece, $f(x) = 2$, a constant function, is continuous at every number x. And the third piece, $f(x) = x^2 + 2$, a polynomial function, is also continuous at every number x. Thus, these three functions, a linear function, a constant function, and a polynomial function, can be graphed without lifting a pencil from the paper. However, the pieces change at $x = 0$ and at $x = 1$. Is it necessary to lift a pencil from the paper when graphing f at these values? It appears that we must investigate continuity at 0 and at 1.

To determine whether the function is continuous at 0, we check the conditions for continuity with $a = 0$.

Condition 1 f is defined at a. Is $f(0)$ defined? Because $a = 0$, we use the first line of the piecewise function, where $x \le 0$.

$$\begin{aligned} f(x) &= x + 2 && \text{This is the function's equation for } x \le 0\text{, which includes } x = 0. \\ f(0) &= 0 + 2 && \text{Replace } x \text{ with } 0. \\ &= 2 \end{aligned}$$

Because $f(0)$ is a real number, 2, $f(0)$ is defined.

Condition 2 $\lim_{x \to a} f(x)$ exists. Does $\lim_{x \to 0} f(x)$ exist? To answer this question, we look at the values of $f(x)$ when x is close to 0. Let us investigate the left- and right-hand limits. If these limits are equal, then $\lim_{x \to 0} f(x)$ exists. To find $\lim_{x \to 0^-} f(x)$, the left-hand limit, we look at the values of $f(x)$ when x is close to 0 but less than 0. Because x is less than 0, we use the first line of the piecewise function, $f(x) = x + 2$ if $x \le 0$. Thus,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x + 2) = 0 + 2 = 2.$$

To find $\lim_{x \to 0^+} f(x)$, the right-hand limit, we look at the values of $f(x)$ when x is close to 0 but greater than 0. Because x is greater than 0, we use the second line of the piecewise function, $f(x) = 2$ if $0 < x \le 1$. Thus,

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 2 = 2.$$

Because the left- and right-hand limits are both equal to 2, $\lim_{x \to 0} f(x) = 2$. Thus, we see that $\lim_{x \to 0} f(x)$ exists.

Condition 3 $\lim_{x \to a} f(x) = f(a)$ Does $\lim_{x \to 0} f(x) = f(0)$? We found that $\lim_{x \to 0} f(x) = 2$ and $f(0) = 2$. This means that as x gets closer to 0, the corresponding values of $f(x)$ get closer to the function value at 0: $\lim_{x \to 0} f(x) = f(0)$.

Because the three conditions are satisfied, we conclude that f is continuous at 0.

Now we must determine whether the function is continuous at 1, the other value of x where the pieces change. We check the conditions for continuity with $a = 1$.

Condition 1 f is defined at a. Is $f(1)$ defined? Because $a = 1$, we use the second line of the piecewise function, where $0 < x \le 1$.

$$\begin{aligned} f(x) &= 2 && \text{This is the function's equation for } 0 < x \le 1\text{, which includes } x = 1. \\ f(1) &= 2 && \text{Replace } x \text{ with } 1. \end{aligned}$$

Because $f(1)$ is a real number, 2, $f(1)$ is defined.

Condition 2 $\lim_{x \to a} f(x)$ exists. Does $\lim_{x \to 1} f(x)$ exist? We investigate left- and right-hand limits as x approaches 1. To find $\lim_{x \to 1^-} f(x)$, the left-hand limit, we look at values of $f(x)$ when x is close to 1 but less than 1. Thus, we use the second line of the piecewise function, $f(x) = 2$ if $0 < x \le 1$. The left-hand limit is

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 2 = 2.$$

FIGURE 11.14 This piecewise function is continuous at 0, where pieces change, and discontinuous at 1, where pieces change.

To find $\lim_{x \to 1^+} f(x)$, the right-hand limit, we look at values of $f(x)$ when x is close to 1 but greater than 1. Thus, we use the third line of the piecewise function, $f(x) = x^2 + 2$ if $x > 1$. The right-hand limit is

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^2 + 2) = 1^2 + 2 = 3.$$

The left- and right-hand limits are not equal: $\lim_{x \to 1^-} f(x) = 2$ and $\lim_{x \to 1^+} f(x) = 3$. This means that $\lim_{x \to 1} f(x)$ does not exist.

Because one of the three conditions is not satisfied, we conclude that f is not continuous at 1.

In summary, the given function is discontinuous at 1 only. The graph of f, shown in **Figure 11.14**, illustrates this conclusion.

Check Point 2 Determine for what numbers x, if any, the following function is discontinuous:

$$f(x) = \begin{cases} 2x & \text{if } x \le 0 \\ x^2 + 1 & \text{if } 0 < x \le 2. \\ 7 - x & \text{if } x > 2 \end{cases}$$

CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

1. A function f is continuous at a when three conditions are satisfied.
   - f is defined at _____, so that _____ is a real number.
   - $\lim$_____ exists.
   - $\lim$_____ = _____
2. True or false: For the function $f(x) = \dfrac{1}{x - 3}$, f is not defined at 3, so f is discontinuous at 3. _____
3. True or false: For the piecewise function defined by
$$f(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5} & \text{if } x \ne 5 \\ 12 & \text{if } x = 5, \end{cases}$$
$\lim_{x \to 5} f(x)$ exists and is equal to 10, so f is continuous at 5. _____
4. Consider the piecewise function defined by
$$f(x) = \begin{cases} 1 - x & \text{if } x < 1 \\ 0 & \text{if } x = 1. \\ x^2 - 1 & \text{if } x > 1 \end{cases}$$
We find $\lim_{x \to 1^-} f(x)$ using $f(x) =$ _____. We find $\lim_{x \to 1^+} f(x)$ using $f(x) =$ _____. The function's definition indicates that $f(1) =$ _____. If we determine that $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$, we can conclude that _____.
5. True or false: A polynomial function is continuous at every number. _____
6. True or false: A piecewise function is always discontinuous at each of its jumps. _____

EXERCISE SET 11.3

Practice Exercises

In Exercises 1–18, use the definition of continuity to determine whether f is continuous at a.

1. $f(x) = 2x + 5$, $a = 1$
2. $f(x) = 3x + 4$, $a = 1$
3. $f(x) = x^2 - 3x + 7$, $a = 4$
4. $f(x) = x^2 - 5x + 6$, $a = 4$
5. $f(x) = \dfrac{x^2 + 4}{x - 2}$, $a = 3$
6. $f(x) = \dfrac{x^2 + 6}{x - 5}$, $a = 6$
7. $f(x) = \dfrac{x + 5}{x - 5}$, $a = 5$
8. $f(x) = \dfrac{x + 7}{x - 7}$, $a = 7$
9. $f(x) = \dfrac{x - 5}{x + 5}$, $a = 5$
10. $f(x) = \dfrac{x - 7}{x + 7}$, $a = 7$
11. $f(x) = \dfrac{x^2 + 5x}{x^2 - 5x}$, $a = 0$
12. $f(x) = \dfrac{x^2 + 8x}{x^2 - 8x}$, $a = 0$
13. $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \ne 2 \\ 5 & \text{if } x = 2 \end{cases}$, $a = 2$
14. $f(x) = \begin{cases} \dfrac{x^2 - 36}{x - 6} & \text{if } x \ne 6 \\ 13 & \text{if } x = 6 \end{cases}$, $a = 6$

15. $f(x) = \begin{cases} x - 5 & \text{if } x \le 0 \\ x^2 + x - 5 & \text{if } x > 0 \end{cases}$

$a = 0$

16. $f(x) = \begin{cases} x - 4 & \text{if } x \le 0 \\ x^2 + x - 4 & \text{if } x > 0 \end{cases}$

$a = 0$

17. $f(x) = \begin{cases} 1 - x & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ x^2 - 1 & \text{if } x > 1 \end{cases}$

$a = 1$

18. $f(x) = \begin{cases} 2 - x & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$

$a = 1$

In Exercises 19–34, determine for what numbers, if any, the given function is discontinuous.

19. $f(x) = x^2 + 4x - 6$

20. $f(x) = x^2 + 8x - 10$

21. $f(x) = \dfrac{x + 1}{(x + 1)(x - 4)}$

22. $f(x) = \dfrac{x + 2}{(x + 2)(x - 5)}$

23. $f(x) = \dfrac{\sin x}{x}$

24. $f(x) = \dfrac{1 - \cos x}{x}$

25. $f(x) = \pi$

26. $f(x) = c$

27. $f(x) = \begin{cases} x - 1 & \text{if } x \le 1 \\ x^2 & \text{if } x > 1 \end{cases}$

28. $f(x) = \begin{cases} x - 2 & \text{if } x \le 2 \\ x^2 - 1 & \text{if } x > 2 \end{cases}$

29. $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1} & \text{if } x \ne 1 \\ 2 & \text{if } x = 1 \end{cases}$

30. $f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \ne 3 \\ 6 & \text{if } x = 3 \end{cases}$

31. $f(x) = \begin{cases} x + 6 & \text{if } x \le 0 \\ 6 & \text{if } 0 < x \le 2 \\ x^2 + 1 & \text{if } x > 2 \end{cases}$

32. $f(x) = \begin{cases} x + 7 & \text{if } x \le 0 \\ 7 & \text{if } 0 < x \le 3 \\ x^2 - 1 & \text{if } x > 3 \end{cases}$

33. $f(x) = \begin{cases} 5x & \text{if } x < 4 \\ 21 & \text{if } x = 4 \\ x^2 + 4 & \text{if } x > 4 \end{cases}$

34. $f(x) = \begin{cases} 7x & \text{if } x < 6 \\ 41 & \text{if } x = 6 \\ x^2 + 6 & \text{if } x > 6 \end{cases}$

Practice Plus

In Exercises 35–38, graph each function. Then determine for what numbers, if any, the function is discontinuous.

35. $f(x) = \begin{cases} \sin x & \text{if } -\pi \le x < 0 \\ -\sin x & \text{if } 0 \le x < \pi \\ \cos x & \text{if } \pi \le x \le 2\pi \end{cases}$

36. $f(x) = \begin{cases} -\cos x & \text{if } -\pi \le x < 0 \\ -\sin x & \text{if } 0 \le x < \pi \\ \sin x & \text{if } \pi \le x \le 2\pi \end{cases}$

37. $f(x) = \begin{cases} -1 & \text{if } x \text{ is an integer.} \\ 1 & \text{if } x \text{ is not an integer.} \end{cases}$

38. $f(x) = \begin{cases} 2 & \text{if } x \text{ is an odd integer.} \\ -2 & \text{if } x \text{ is not an odd integer.} \end{cases}$

In Exercises 39–42, determine for what numbers, if any, the function is discontinuous. Construct a table to find any required limits.

39. $f(x) = \begin{cases} \dfrac{\sin 2x}{x} & \text{if } x \ne 0 \\ 2 & \text{if } x = 0 \end{cases}$

40. $f(x) = \begin{cases} \dfrac{\sin 3x}{x} & \text{if } x \ne 0 \\ 3 & \text{if } x = 0 \end{cases}$

41. $f(x) = \begin{cases} \dfrac{\cos x}{x - \dfrac{\pi}{2}} & \text{if } x \ne \dfrac{\pi}{2} \\ 1 & \text{if } x = \dfrac{\pi}{2} \end{cases}$

42. $f(x) = \begin{cases} \dfrac{\sin x}{x - \pi} & \text{if } x \ne \pi \\ 1 & \text{if } x = \pi \end{cases}$

Application Exercises

43. The graph represents the percentage of required topics in a precalculus course, $p(t)$, that a student learned at various times, t, throughout the course. At time t_2, the student panicked for an instant during an exam. At time t_3, after working on a set of cumulative review exercises, a big jump in understanding suddenly took place.

a. Find $\lim_{t \to t_1} p(t)$ and $p(t_1)$.

b. Is p continuous at t_1? Use the definition of continuity to explain your answer.

c. Find $\lim_{t \to t_2} p(t)$ and $p(t_2)$.

d. Is p continuous at t_2? Use the definition of continuity to explain your answer.

e. Find $\lim_{t \to t_3} p(t)$ and $p(t_3)$.

f. Is p continuous at t_3? Use the definition of continuity to explain your answer.

g. Find $\lim_{t \to t_4} p(t)$ and $p(t_4)$.

h. Explain the meaning of both the limit and the function value in part (g) in terms of the time in the course and the percentage of topics learned.

44. The figure shows the cost of mailing a first-class letter, $f(x)$, as a function of its weight, x, in ounces.

Source: Lynn E. Baring, Postmaster, Inverness, CA

a. Find $\lim_{x \to 3^-} f(x)$.

b. Find $\lim_{x \to 3^+} f(x)$.

c. What can you conclude about $\lim_{x \to 3} f(x)$? How is this shown by the graph?

d. What aspect of costs for mailing a letter causes the graph to jump vertically by the same amount at its discontinuities?

45. The following piecewise function gives the tax owed, $T(x)$, by a single taxpayer in 2011 on a taxable income of x dollars.

$$T(x) = \begin{cases} 0.10x & \text{if } 0 < x \le 8500 \\ 850.00 + 0.15(x - 8500) & \text{if } 8500 < x \le 34{,}500 \\ 4750.00 + 0.25(x - 34{,}500) & \text{if } 34{,}500 < x \le 83{,}600 \\ 17{,}025.00 + 0.28(x - 83{,}600) & \text{if } 83{,}600 < x \le 174{,}400 \\ 42{,}449.00 + 0.33(x - 174{,}400) & \text{if } 174{,}400 < x \le 379{,}150 \\ 110{,}016.50 + 0.35(x - 379{,}150) & \text{if } x > 379{,}150. \end{cases}$$

a. Determine whether T is continuous at 8500.

b. Determine whether T is continuous at 34,500.

c. If T had discontinuities, use one of these discontinuities to describe a situation where it might be advantageous to earn less money in taxable income.

Writing in Mathematics

46. Explain how to determine whether a function is continuous at a number.

47. If a function is not defined at a, how is this shown on the function's graph?

48. If a function is defined at a, but $\lim_{x \to a} f(x)$ does not exist, how is this shown on the function's graph?

49. If a function is defined at a, $\lim_{x \to a} f(x)$ exists, but $\lim_{x \to a} f(x) \neq f(a)$, how is this shown on the function's graph?

50. In Exercises 43–44, functions that modeled learning in a precalculus course and the cost of mailing a letter had jumps in their graphs. Describe another situation that can be modeled by a function with discontinuities. What aspect of this situation causes the discontinuities?

51. Give two examples of the use of the word *continuous* in everyday English. Compare its use in your examples to its meaning in mathematics.

Technology Exercises

52. Use your graphing utility to graph any five of the functions in Exercises 1–18 and verify whether f is continuous at a.

53. Estimate $\lim_{x \to 0^+} (1 + x)^{1/x}$ by using the $\boxed{\text{TABLE}}$ feature of your graphing utility to create a table of values. Then use the $\boxed{\text{ZOOM IN}}$ feature to zoom in on the graph of f near and to the right of $x = 0$ to justify or improve your estimate.

Critical Thinking Exercises

Make Sense? *In Exercises 54–57, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

54. If $\lim_{x \to a} f(x) \neq f(a)$ and $\lim_{x \to a} f(x)$ exists, I can redefine $f(a)$ to make f continuous at a.

55. If $\lim_{x \to a^-} f(x) \neq f(a)$ and $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, I can redefine $f(a)$ to make f continuous at a.

56. f and g are both continuous at a, although $f + g$ is not.

57. f and g are both continuous at a, although $\dfrac{f}{g}$ is not.

58. Define $f(x) = \dfrac{x^2 - 81}{x - 9}$ at $x = 9$ so that the function becomes continuous at 9.

59. Is it possible to define $f(x) = \dfrac{1}{x - 9}$ at $x = 9$ so that the function becomes continuous at 9? How does this discontinuity differ from the discontinuity in Exercise 58?

60. For the function

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ Ax - 3 & \text{if } x \ge 1 \end{cases}$$

find A so that the function is continuous at 1.

Group Exercise

61. In this exercise, the group will define three piecewise functions. Each function should have three pieces and two values of x at which the pieces change.

a. Define and graph a piecewise function that is continuous at both values of x where the pieces change.

b. Define and graph a piecewise function that is continuous at one value of x where the pieces change and discontinuous at the other value of x where the pieces change.

c. Define and graph a piecewise function that is discontinuous at both values of x where the pieces change.

At the end of the activity, group members should turn in the functions and their graphs. Do not use any of the piecewise functions or graphs that appear anywhere in this book.

Preview Exercises

Exercises 62–64 will help you prepare for the material covered in the next section. In each exercise, find the indicated difference quotient and simplify.

62. If $f(x) = x^2 + x$, find $\dfrac{f(2 + h) - f(2)}{h}$.

63. If $f(x) = x^3$, find $\dfrac{f(x + h) - f(x)}{h}$.

64. If $s(t) = -16t^2 + 48t + 160$, find $\dfrac{s(a + h) - s(a)}{h}$.

CHAPTER 11 Mid-Chapter Check Point

WHAT YOU KNOW: We learned that $\lim_{x \to a} f(x) = L$ means that as x gets closer to a, but remains unequal to a, the corresponding values of $f(x)$ get closer to L. We found limits using tables, graphs, and properties of limits. The quotient property for limits did not apply to fractional expressions in which the limit of the denominator is zero. In these cases, rewriting the expression using factoring or rationalizing the numerator or denominator was helpful before finding the limit. We saw that if the left-hand limit, $\lim_{x \to a^-} f(x)$, (x approaches a from the left) is not equal to the right-hand limit, $\lim_{x \to a^+} f(x)$, (x approaches a from the right), then $\lim_{x \to a} f(x)$ does not exist. Finally, we defined continuity in terms of limits. A function f is continuous at a when f is defined at a, $\lim_{x \to a} f(x)$ exists, and $\lim_{x \to a} f(x) = f(a)$. If f is not continuous at a, we say that f is discontinuous at a.

In Exercises 1–7, use the graphs of f and g to find the indicated limit or function value, or state that the limit or function value does not exist.

1. $\lim_{x \to -1^-} f(x)$

2. $\lim_{x \to -1^+} f(x)$

3. $\lim_{x \to -1} f(x)$

4. $\lim_{x \to 1} [f(x) + g(x)]$

5. $\lim_{x \to 0} [f(x) - g(x)]$

6. $(f - g)(0)$

7. $\lim_{x \to 1} \sqrt{10 + f(x)}$

8. Use the graph of f shown above to determine for what numbers the function is discontinuous. Then use the definition of continuity to verify each discontinuity.

In Exercises 9–11, use the table to find the indicated limit.

x	−0.03	−0.02	−0.01	−0.007	0.007	0.01	0.02	0.03
$f(x) = \dfrac{\sin x}{2x^2 - x}$	−0.9433	−0.9615	−0.9804	−0.9862	−1.014	−1.02	−1.042	−1.064
$g(x) = \dfrac{e^x - \tan x}{\cos^2 x}$	1.0014	1.0006	1.0002	1.0001	1.0001	1.0001	1.0006	1.0013

9. $\lim_{x \to 0} f(x)$

10. $\lim_{x \to 0} g(x)$

11. $\lim_{x \to 0} \dfrac{4g(x)}{[f(x)]^2}$

In Exercises 12–17, find the limits.

12. $\lim_{x \to -2} (x^3 - x + 5)$

13. $\lim_{x \to 3} \sqrt{x^2 - 3x + 4}$

14. $\lim_{x \to 5} \dfrac{2x^2 - x + 4}{x - 1}$

15. $\lim_{x \to 5} \dfrac{2x^2 - 7x - 15}{x - 5}$

16. $\lim_{x \to 0} \dfrac{\sqrt{x^2 + 9} - 3}{x^2}$

17. $\lim_{x \to 0} \dfrac{\dfrac{1}{x + 10} - \dfrac{1}{x}}{x}$

In Exercises 18–19, a piecewise function is given. Use the function to find the indicated limit, or state that the limit does not exist.

18. $f(x) = \begin{cases} 9 - 2x & \text{if } x < 4 \\ \sqrt{x - 4} & \text{if } x \geq 4 \end{cases}$

a. $\lim_{x \to 4^-} f(x)$ **b.** $\lim_{x \to 4^+} f(x)$ **c.** $\lim_{x \to 4} f(x)$

19. $f(x) = \begin{cases} \dfrac{x^4 - 16}{x - 2} & \text{if } x \neq 2 \\ 32 & \text{if } x = 2 \end{cases}$

a. $\lim_{x \to 2^-} f(x)$ **b.** $\lim_{x \to 2^+} f(x)$ **c.** $\lim_{x \to 2} f(x)$

In Exercises 20–21, use the definition of continuity to determine whether f is continuous at a.

20. $f(x) = \begin{cases} \sqrt{3 - x} & \text{if } x \leq 3 \\ x^2 - 3x & \text{if } x > 3 \end{cases}$

$a = 3$

21. $f(x) = \begin{cases} \dfrac{(x + 3)^2 - 9}{x} & \text{if } x \neq 0 \\ 6 & \text{if } x = 0 \end{cases}$

$a = 0$

22. Determine for what numbers, if any, the following function is discontinuous:

$$f(x) = \begin{cases} \dfrac{x^2 - 1}{x + 1} & \text{if } x < -1 \\ 2x & \text{if } -1 \leq x \leq 5. \\ 3x - 4 & \text{if } x > 5 \end{cases}$$

SECTION 11.4 Introduction to Derivatives

Objectives

1. Find slopes and equations of tangent lines.
2. Find the derivative of a function.
3. Find average and instantaneous rates of change.
4. Find instantaneous velocity.

Things change over time and most changes occur at uneven rates. This is illustrated in the chapter opener (page 1091) with a sequence of photos of a young boy transforming into an adult. What does calculus have to say about this radical transformation?

In this section, we will see how calculus allows motion and change to be analyzed by "freezing the frame" of a continuously changing process, instant by instant. For example, **Figure 11.15** shows a male's changing height over intervals of time. Over the period of time from P to D, his average rate of growth is his change in height—that is, his height at time D minus his height at time P—divided by the change in time from P to D.

The lines PD, PC, PB, and PA shown in **Figure 11.15** have slopes that show the man's average growth rates for successively shorter periods of time. Calculus makes these time frames so small that their limit approaches a single point—that is, a single instant in time. This point is shown as point P in **Figure 11.15**. The slope of the line that touches the graph at P gives the man's growth rate at one instant in time, P.

Keep this informal discussion of this man and his growth rate in mind as you read this section. We begin with the calculus that describes the slope of the line that touches the graph in **Figure 11.15** at P.

FIGURE 11.15

Slopes and Equations of Tangent Lines

1 Find slopes and equations of tangent lines.

In Chapter 1, we saw that if the graph of a function is not a straight line, the **average rate of change** between any two points is the slope of the line containing the two points. We called this line a **secant line**.

Figure 11.16 shows the graph of a male's height, in inches, as a function of his age, in years. Two points on the graph are labeled: (13, 57) and (18, 76). At age 13, this person was 57 inches tall, and at age 18, he was 76 inches tall. The slope of the secant line containing these two points is

$$\frac{76 - 57}{18 - 13} = \frac{19}{5} = 3\frac{4}{5}.$$

Slope is the change in the y-coordinates divided by the change in the x-coordinates.

FIGURE 11.16 Height as a function of age

The man's average rate of change, or average growth rate, from 13 to 18 was $3\frac{4}{5}$ inches per year.

How can we find this person's growth rate at the instant when he was 13? We can find this *instantaneous rate of change* by repeating the computation of slope from 13 to 17, then from 13 to 16, then from 13 to 15, again from 13 to 14, again from 13 to $13\frac{1}{2}$, and once again from 13 to 13.01. What limit is approached by these computations as the shrinking interval of time gets closer and closer to the instant when the man was 13?

We answer these questions by considering the graph of any function f, shown in **Figure 11.17**. We need to find the slope, or steepness, of this curve at the point $P = (a, f(a))$. This slope will reveal the function's instantaneous rate of change at a. We begin by choosing a second point, Q, whose x-coordinate is $a + h$, where $h \neq 0$. The point $Q = (a + h, f(a + h))$ is shown in **Figure 11.17**.

How do we find the average rate of change of f between points P and Q? We find the slope of the secant line, the line containing P and Q.

FIGURE 11.17 Finding the average rate of change, or the slope of the secant line

Slope of secant line

$$= \frac{f(a + h) - f(a)}{a + h - a}$$ Slope is the change in y-coordinates, $f(a + h) - f(a)$, divided by the change in x-coordinates, $(a + h) - a$.

$$= \frac{f(a + h) - f(a)}{h}$$ Simplify.

Do you recognize this expression as the difference quotient presented in Chapter 1? We will make use of this expression and our understanding of limits to find the slope of a graph at a specific point.

What happens if the distance labeled h in **Figure 11.17** approaches 0? The value of the x-coordinate of point Q, $a + h$, will get closer and closer to a. Can you see that a is the x-coordinate of point P? Thus, as h approaches 0, point Q approaches point P. Examine **Figure 11.18** to see how we visualize the changing position of point Q.

FIGURE 11.18 As Q approaches P, the succession of secant lines approaches the tangent line.

Figure 11.18 also shows how the secant line between points P and Q changes as h approaches 0. Note how the position of the secant line changes as the position of Q changes. The secant line between point P and point Q approaches the red line that touches the graph of f at point P. This limiting position of the secant line is called the **tangent line** to the graph of f at the point $P = (a, f(a))$.

According to our earlier derivation, the slope of each secant line in **Figure 11.18** is

$$\frac{f(a+h) - f(a)}{h}.$$

This difference quotient is also the average rate of change of f from $x_1 = a$ to $x_2 = a + h$.

As h approaches 0, this slope approaches the slope of the tangent line to the curve at $(a, f(a))$. Thus, the slope of the tangent line to the curve at $(a, f(a))$ is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

This limit also represents the **instantaneous rate of change of f with respect to x at a**.

Slope of the Tangent Line to a Curve at a Point

The **slope of the tangent line** to the graph of a function $y = f(x)$ at $(a, f(a))$ is given by

$$m_{\tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided that this limit exists. This limit also describes

- the **slope of the graph of f at $(a, f(a))$**.
- the **instantaneous rate of change of f with respect to x at a**.

EXAMPLE 1 Finding the Slope of a Tangent Line

Find the slope of the tangent line to the graph of $f(x) = x^2 + x$ at $(2, 6)$.

SOLUTION

The slope of the tangent line at $(a, f(a))$ is

$$m_{\tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

We use this formula to find the slope of the tangent line at the given point. Because we are finding the slope of the tangent line at $(2, 6)$, we know that $a = 2$.

$$m_{\tan} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

Because $a = 2$, substitute 2 into the formula for each occurrence of a.

$$= \lim_{h \to 0} \frac{[(2+h)^2 + (2+h)] - [2^2 + 2]}{h}$$

To find $f(2 + h)$, replace x in $f(x) = x^2 + x$ with $2 + h$. To find $f(2)$ replace x with 2.

$$= \lim_{h \to 0} \frac{[4 + 4h + h^2 + 2 + h] - 6}{h}$$

Square $2 + h$ using $(A + B)^2 = A^2 + 2AB + B^2$.

$$= \lim_{h \to 0} \frac{h^2 + 5h}{h}$$

Combine like terms in the numerator.

$$= \lim_{h \to 0} \frac{h(h+5)}{h}$$

Factor the numerator.

$$= \lim_{h \to 0} (h + 5)$$

Divide both the numerator and denominator by h. This is permitted because h approaches 0, but $h \neq 0$.

$$= 0 + 5$$

Use limit properties.

$$= 5$$

FIGURE 11.19

Thus, the slope of the tangent line to the graph of $f(x) = x^2 + x$ at $(2, 6)$ is 5. This is shown in **Figure 11.19**. We also say that the slope of the graph of $f(x) = x^2 + x$ at $(2, 6)$ is 5.

Check Point 1 Find the slope of the tangent line to the graph of $f(x) = x^2 - x$ at $(4, 12)$.

In Example 1, we found the *slope* of the tangent line shown in **Figure 11.19**. We can find an *equation* of this line using the point-slope form of the equation of a line

$$y - y_1 = m(x - x_1).$$

The tangent line passes through $(2, 6)$: $x_1 = 2$ and $y_1 = 6$. The slope of the tangent line is 5: $m = 5$. The point-slope equation of the tangent line is

$$y - 6 = 5(x - 2).$$

We can solve for y and express the equation of the tangent line in slope-intercept form: $y = mx + b$. The slope-intercept equation of the tangent line is

$$y - 6 = 5x - 10 \quad \text{Apply the distributive property.}$$
$$y = 5x - 4. \quad \text{Add 6 to both sides and write in slope-intercept form.}$$

TECHNOLOGY

Graphing utilities with a DRAW TANGENT feature will draw tangent lines to curves and display their slope-intercept equations. **Figure 11.20** shows the tangent line to the graph of $y = x^2 + x$ at the point whose x-coordinate is 2. Also displayed is the slope-intercept equation of the tangent line, $y = 5x - 4$.

FIGURE 11.20

EXAMPLE 2 Finding the Slope-Intercept Equation of a Tangent Line

Find the slope-intercept equation of the tangent line to the graph of $f(x) = \sqrt{x}$ at $(4, 2)$.

SOLUTION

We begin by finding the slope of the tangent line to the graph of $f(x) = \sqrt{x}$ at $(4, 2)$.

$$m_{\tan} = \lim_{h \to 0} \frac{f(4 + h) - f(4)}{h}$$

The slope of the tangent line at $(a, f(a))$ is $\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$.

$$= \lim_{h \to 0} \frac{\sqrt{4 + h} - 2}{h}$$

To find $f(4 + h)$, replace x in $f(x) = \sqrt{x}$ with $4 + h$. $f(4) = \sqrt{4} = 2$

$$= \lim_{h \to 0} \left[\frac{\sqrt{4 + h} - 2}{h} \cdot \frac{\sqrt{4 + h} + 2}{\sqrt{4 + h} + 2} \right]$$

Rationalize the numerator.

$$= \lim_{h \to 0} \frac{4 + h - 4}{h\left(\sqrt{4 + h} + 2\right)}$$

Multiply the numerators.

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{4 + h} + 2\right)}$$

Simplify the numerator.

$$= \lim_{h \to 0} \frac{1}{\sqrt{4 + h} + 2}$$

Divide both the numerator and the denominator by h. This is permitted because h approaches 0, but $h \neq 0$.

$$= \frac{1}{\sqrt{4 + 0} + 2}$$

Use limit properties.

$$= \frac{1}{4}$$

Now that we have the slope of the tangent line, we can write the slope-intercept equation. Begin with the point-slope form

$$y - y_1 = m(x - x_1).$$

The tangent line is given to pass through $(4, 2)$: $x_1 = 4$ and $y_1 = 2$. We found the slope of the tangent line to be $\frac{1}{4}$: $m = \frac{1}{4}$. The point-slope equation of the tangent line is

$$y - 2 = \tfrac{1}{4}(x - 4).$$

Solving for y, we obtain the slope-intercept equation of the tangent line.

$$y - 2 = \tfrac{1}{4}x - 1$$ Apply the distributive property.

$$y = \tfrac{1}{4}x + 1$$ Add 2 to both sides. This is the slope-intercept form, $y = mx + b$, of the equation.

The slope-intercept equation of the tangent line to the graph of $f(x) = \sqrt{x}$ at $(4, 2)$ is $y = \frac{1}{4}x + 1$. **Figure 11.21** shows the graph of f and the tangent line.

FIGURE 11.21

Check Point 2 Find the slope-intercept equation of the tangent line to the graph of $f(x) = \sqrt{x}$ at $(1, 1)$.

2 Find the derivative of a function.

The Derivative of a Function

In Examples 1 and 2, we found the slope of a tangent line to the graph of f at $(a, f(a))$, where a was a specific number. We can also find the slope of a tangent line at $(x, f(x))$, where x can represent any number in the domain of f for which the slope of the tangent line is defined. The resulting function is called the *derivative of f at x*.

Definition of the Derivative of a Function

Let $y = f(x)$ denote a function f. The **derivative of f at x**, denoted by $f'(x)$, read "f prime of x," is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

provided that this limit exists. The derivative of a function f gives the slope of f for any value of x in the domain of f'.

By evaluating the derivative, you can compute the slopes of various tangent lines to the graph of a function. Thus, the derivative gives you a way to analyze your moving world by revealing a function's instantaneous rate of change at any moment.

Blitzer Bonus || The Seeds of Change

Every shape that's born bears in its womb the seeds of change.

—Ovid (Roman poet)

Figure 11.22 shows a graph involving change, namely, a male's height as a function of his age. The derivative of this function provides a formula for the slope of the tangent line to the function's graph at any point. The figure shows four tangent lines. The derivative of this function would reveal that the tangent line with the greatest slope touches the curve somewhere between $x = 3$ and $x = 4$. Thus, the instantaneous rate of change in the boy's growth is greatest at some moment in time between the ages of 3 and 4.

FIGURE 11.22 Analyzing continuous change at an instant

EXAMPLE 3 Finding the Derivative of a Function

a. Find the derivative of $f(x) = x^2 + 3x$ at x. That is, find $f'(x)$.

b. Find the slope of the tangent line to the graph of $f(x) = x^2 + 3x$ at $x = -2$ and at $x = -\frac{3}{2}$.

SOLUTION

a. We use the definition of the derivative of f at x to find the derivative of the given function.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Use the definition of the derivative.

$$= \lim_{h \to 0} \frac{[(x+h)^2 + 3(x+h)] - (x^2 + 3x)}{h}$$

To find $f(x + h)$, replace x in $f(x) = x^2 + 3x$ with $x + h$.

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h}$$

Perform the indicated operations in the numerator.

$$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$$

Simplify the numerator: $x^2 - x^2 = 0$ and $3x - 3x = 0$.

$$= \lim_{h \to 0} \frac{h(2x + h + 3)}{h}$$

Factor the numerator.

$$= \lim_{h \to 0} (2x + h + 3)$$

Divide the numerator and the denominator by h.

$$= 2x + 0 + 3$$

Use limit properties. As h approaches 0, only the term containing h is affected.

$$= 2x + 3$$

The derivative of $f(x) = x^2 + 3x$ is

$$f'(x) = 2x + 3.$$

b. The derivative gives the slope of the tangent line at any point. Thus, to find the slope of the tangent line to the graph of $f(x) = x^2 + 3x$ at $x = -2$, evaluate the derivative at -2. Similarly, to find the slope of the tangent line at $x = -\frac{3}{2}$, evaluate the derivative at $-\frac{3}{2}$.

$$\begin{aligned} f'(x) &= 2x + 3 \\ f'(-2) &= 2(-2) + 3 = -4 + 3 = -1 \\ f'\left(-\tfrac{3}{2}\right) &= 2\left(-\tfrac{3}{2}\right) + 3 = -3 + 3 = 0 \end{aligned}$$

Figure 11.23 shows the graph of $f(x) = x^2 + 3x$ and tangent lines at $x = -2$ and $x = -\frac{3}{2}$. The slope of the decreasing green tangent line at $x = -2$ is -1. The slope of the horizontal red tangent line at $x = -\frac{3}{2}$ is 0.

FIGURE 11.23 Two tangent lines to the graph of $f(x) = x^2 + 3x$ and their slopes

Check Point 3

a. Find the derivative of $f(x) = x^2 - 5x$ at x. That is, find $f'(x)$.

b. Find the slope of the tangent line to the graph of $f(x) = x^2 - 5x$ at $x = -1$ and at $x = 3$.

3 Find average and instantaneous rates of change.

Applications of the Derivative

Many applications of the derivative involve analyzing change by determining a function's instantaneous rate of change at any moment. How do we use the derivative of a function to reveal such changes? We know that the derivative of f at x is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.$$

Thus, the derivative of f at a is

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}.$$

Do you recognize this limit? It describes the instantaneous rate of change of f with respect to x at a.

Average and Instantaneous Rates of Change

Average Rate of Change The **average rate of change of f from $x = a$ to $x = a + h$** is given by the difference quotient

$$\frac{f(a + h) - f(a)}{h}.$$

Instantaneous Rate of Change The **instantaneous rate of change of f with respect to x at a** is the derivative of f at a:

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}.$$

EXAMPLE 4 Finding Average and Instantaneous Rates of Change

The function $f(x) = x^3$ describes the volume of a cube, $f(x)$, in cubic inches, whose length, width, and height each measure x inches. If x is changing,

a. Find the average rate of change of the volume with respect to x as x changes from 5 inches to 5.1 inches and from 5 inches to 5.01 inches.

b. Find the instantaneous rate of change of the volume with respect to x at the moment when $x = 5$ inches.

SOLUTION

a. As x changes from 5 to 5.1, $a = 5$ and $h = 0.1$. The average rate of change of the volume with respect to x as x changes from 5 to 5.1 is determined as follows.

$$\frac{f(a+h)-f(a)}{h}$$ The difference quotient gives the average rate of change from a to $a + h$.

$$= \frac{f(5+0.1)-f(5)}{0.1}$$ This is the average rate of change from 5 to 5.1.

$$= \frac{f(5.1)-f(5)}{0.1}$$ Simplify.

$$= \frac{5.1^3-5^3}{0.1}$$ Use $f(x) = x^3$ and substitute 5.1 and 5, respectively, for x.

$$= 76.51$$

The average rate of change in the volume is 76.51 cubic inches per inch as x changes from 5 to 5.1 inches.

As x changes from 5 to 5.01, $a = 5$ and $h = 0.01$. The average rate of change of the volume with respect to x as x changes from 5 to 5.01 is determined as follows.

$$\frac{f(a+h)-f(a)}{h}$$ The difference quotient gives the average rate of change from a to $a + h$.

$$= \frac{f(5+0.01)-f(5)}{0.01}$$ This is the average rate of change from 5 to 5.01.

$$= \frac{f(5.01)-f(5)}{0.01}$$ Simplify.

$$= \frac{5.01^3-5^3}{0.01}$$ Use $f(x) = x^3$ and substitute 5.01 and 5, respectively, for x.

$$= 75.1501$$

The average rate of change in the volume is 75.1501 cubic inches per inch as x changes from 5 to 5.01 inches.

b. Instantaneous rates of change are given by the derivative. The derivative of f at a, $f'(a)$, is the instantaneous rate of change of f with respect to x at a. We must find the instantaneous rate of change of the volume with respect to x at the moment when $x = 5$ inches. This means that we must find $f'(5)$. We find $f'(5)$ by first finding $f'(x)$, the derivative, and then evaluating f' at 5.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ Use the definition of the derivative.

$$= \lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$$ To find $f(x + h)$, replace x in $f(x) = x^3$ with $x + h$.

$$= \lim_{h\to 0}\frac{(x^3+3x^2h+3xh^2+h^3)-x^3}{h}$$ Use the Binomial Theorem to cube $x + h$.

$$= \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}$$ Simplify the numerator: $x^3 - x^3 = 0$.

$$= \lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}$$ Factor the numerator.

$$= \lim_{h\to 0}(3x^2+3xh+h^2)$$ Divide the numerator and the denominator by h.

$$= 3x^2 + 3x\cdot 0 + 0^2$$ Use limit properties. As h approaches 0, only terms containing h are affected.

$$= 3x^2$$

TECHNOLOGY

Graphing utilities have a feature that gives (or approximates) the derivative of a function evaluated at any number. Consult your manual for details. The screen below verifies that if $f(x) = x^3$, then $f'(5) = 75$.

The derivative of $f(x) = x^3$ is $f'(x) = 3x^2$. To find the instantaneous change of f at 5, evaluate the derivative at 5.

$$f'(x) = 3x^2$$
$$f'(5) = 3 \cdot 5^2 = 75$$

The instantaneous rate of change of the volume with respect to x at the moment when $x = 5$ inches is 75 cubic inches per inch. Notice how the average rates of change that we computed in part (a), 76.51 and 75.1501, are approaching the instantaneous rate of change, 75.

Check Point 4 Use the function in Example 4, $f(x) = x^3$, to find each of the following:

a. the average rate of change of the volume with respect to x as x changes from 4 inches to 4.1 inches and from 4 inches to 4.01 inches.

b. the instantaneous rate of change of the volume with respect to x at the moment when $x = 4$ inches.

---

4 Find instantaneous velocity.

The ideas of calculus are frequently applied to position functions that express an object's position, $s(t)$, in terms of time, t. In the time interval from $t = a$ to $t = a + h$, the change in the object's position is

$$s(a + h) - s(a).$$

The **average velocity** over this time interval is

$$\frac{s(a + h) - s(a)}{h}.$$

The numerator is the change in position.

The denominator is the change in time from $t = a$ to $t = a + h$.

Now suppose that we compute the average velocities over shorter and shorter time intervals $[a, a + h]$. This means that we let h approach 0. As in our previous discussion, we define the *instantaneous velocity* at time $t = a$ to be the limit of these average velocities. This limit is the derivative of s at a.

Instantaneous Velocity

Suppose that a function expresses an object's position, $s(t)$, in terms of time, t. The **instantaneous velocity** of the object at time $t = a$ is

$$s'(a) = \lim_{h \to 0} \frac{s(a + h) - s(a)}{h}.$$

Instantaneous velocity at time a is also called **velocity** at time a.

EXAMPLE 5 Finding Instantaneous Velocity

A ball is thrown straight up from a rooftop 160 feet high with an initial velocity of 48 feet per second. The function

$$s(t) = -16t^2 + 48t + 160$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is thrown. The ball misses the rooftop on its way down and eventually strikes the ground.

a. What is the instantaneous velocity of the ball 2 seconds after it is thrown?

b. What is the instantaneous velocity of the ball when it hits the ground?

Blitzer Bonus

Roller Coasters and Derivatives

Roller coaster rides give you the opportunity to spend a few hair-raising minutes plunging hundreds of feet, accelerating to 80 miles an hour in seven seconds, and enduring vertical loops that turn you upside-down. By finding a function that models your distance above the ground at every moment of the ride and taking its derivative, you can determine when the instantaneous velocity is the greatest. As you experience the glorious agony of the roller coaster, this is your moment of peak terror.

SOLUTION

Instantaneous velocity is given by the derivative of a function that expresses an object's position, $s(t)$, in terms of time, t. The instantaneous velocity of the ball at a seconds is $s'(a)$.

$$s'(a) = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$$ This derivative describes instantaneous velocity at time a.

To find $s(a+h)$, replace t in $s(t) = -16t^2 + 48t + 160$ with $a + h$. To find $s(a)$, replace t with a. Thus,

$$s'(a) = \lim_{h \to 0} \frac{-16(a+h)^2 + 48(a+h) + 160 - (-16a^2 + 48a + 160)}{h}$$

Take a few minutes to simplify the numerator of the difference quotient and factor out h. You should obtain

$$s'(a) = \lim_{h \to 0} \frac{\cancel{h}(-32a - 16h + 48)}{\cancel{h}} = -32a - 16 \cdot 0 + 48 = -32a + 48.$$

The instantaneous velocity of the ball at a seconds is

$$s'(a) = -32a + 48.$$

a. The instantaneous velocity of the ball at 2 seconds is found by replacing a with 2.

$$s'(2) = -32 \cdot 2 + 48 = -64 + 48 = -16$$

Two seconds after the ball is thrown, its instantaneous velocity is -16 feet per second. The negative sign indicates that the ball is moving downward when $t = 2$ seconds.

b. To find the instantaneous velocity of the ball when it hits the ground, we need to know how many seconds elapse between the time the ball is thrown from the rooftop and the time it hits the ground. The ball hits the ground when $s(t)$, its height above the ground, is 0. Thus, we set $s(t)$ equal to 0.

$$-16t^2 + 48t + 160 = 0 \quad \text{Set } s(t) = 0.$$
$$-16(t^2 - 3t - 10) = 0 \quad \text{Factor out } -16.$$
$$-16(t-5)(t+2) = 0 \quad \text{Factor completely.}$$
$$t - 5 = 0 \qquad t + 2 = 0 \quad \text{Set each variable factor equal to 0.}$$
$$t = 5 \qquad t = -2 \quad \text{Solve for } t.$$

Because we are describing the ball's position for $t \geq 0$, we discard the solution $t = -2$. The ball hits the ground at 5 seconds. Its instantaneous velocity at 5 seconds is found by replacing a with 5 in $s'(a)$.

$$s'(a) = -32a + 48 \quad \text{This is the ball's instantaneous velocity after } a \text{ seconds.}$$
$$s'(5) = -32 \cdot 5 + 48 = -160 + 48 = -112$$

The instantaneous velocity of the ball when it hits the ground is -112 feet per second. The negative sign indicates that the ball is moving downward at the instant that it strikes the ground. •••

Check Point 5 A ball is thrown straight up from ground level with an initial velocity of 96 feet per second. The function

$$s(t) = -16t^2 + 96t$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is thrown.

a. What is the instantaneous velocity of the ball after 4 seconds?

b. What is the instantaneous velocity of the ball when it hits the ground?

CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

1. The slope of the tangent line to the graph of a function $y = f(x)$ at $(a, f(a))$ is given by

$$\lim_{h \to 0} _____$$

provided that this limit exists. This limit also describes the _____ rate of change of _____ with respect to _____ at a.

2. Using $f(x) = 3x^2 + x$, we can determine that

$$\lim_{h \to 0} \frac{f(1 + h) - f(1)}{h} = 7.$$

This means that the point-slope equation of the tangent line to the graph of $f(x) = 3x^2 + x$ at $(1, 4)$ is $y - ___ = ___(x - ___)$.

3. The derivative of f at x, denoted by _____, is defined by

$$\lim_{h \to 0} _____$$

provided that this limit exists.

4. True or false: The derivative of a function f gives the slope of f for any value of x in the domain of f'.

_____

5. Using $f(x) = x^2 - 3x + 5$, we can determine that $f'(x) = 2x - 3$. This means that the point-slope equation of the tangent line to the graph of $f(x) = x^2 - 3x + 5$ at $(6, 23)$ is $y - ___ = ___(x - ___)$.

6. A foul tip of a baseball is hit straight upward from a height of 4 feet with an initial velocity of 64 feet per second. The function $s(t) = -16t^2 + 64t + 4$ describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is hit. Using this function, we can determine that $s'(a) = -32a + 64$. This tells us that one second after the ball is hit, its instantaneous velocity is _____ feet per second. The positive sign indicates that the ball is moving _____ when $t = 1$ second.

EXERCISE SET 11.4

Practice Exercises

In Exercises 1–14,

a. Find the slope of the tangent line to the graph of f at the given point.

b. Find the slope-intercept equation of the tangent line to the graph of f at the given point.

1. $f(x) = 2x + 3$ at $(1, 5)$
2. $f(x) = 4x + 2$ at $(1, 6)$
3. $f(x) = x^2 + 4$ at $(-1, 5)$
4. $f(x) = x^2 + 7$ at $(-1, 8)$
5. $f(x) = 5x^2$ at $(-2, 20)$
6. $f(x) = 4x^2$ at $(-2, 16)$
7. $f(x) = 2x^2 - x$ at $(2, 6)$
8. $f(x) = 3x^2 + x$ at $(1, 4)$
9. $f(x) = 2x^2 + x - 3$ at $(0, -3)$
10. $f(x) = 2x^2 - x + 5$ at $(0, 5)$
11. $f(x) = \sqrt{x}$ at $(9, 3)$
12. $f(x) = \sqrt{x}$ at $(16, 4)$
13. $f(x) = \dfrac{1}{x}$ at $(1, 1)$
14. $f(x) = \dfrac{2}{x}$ at $(1, 2)$

In Exercises 15–28,

a. Find the derivative of f at x. That is, find $f'(x)$.

b. Find the slope of the tangent line to the graph of f at each of the two values of x given to the right of the function.

15. $f(x) = -3x + 7; x = 1, x = 4$
16. $f(x) = -5x + 3; x = 1, x = 4$
17. $f(x) = x^2 - 6; x = -1, x = 3$
18. $f(x) = x^2 - 8; x = -1, x = 3$
19. $f(x) = x^2 - 3x + 5; x = \frac{3}{2}, x = 2$
20. $f(x) = x^2 - 4x + 7; x = \frac{3}{2}, x = 2$
21. $f(x) = x^3 + 2; x = -1, x = 1$
22. $f(x) = x^3 - 2; x = -1, x = 1$
23. $f(x) = \sqrt{x}; x = 1, x = 4$
24. $f(x) = \sqrt{x}; x = 25, x = 100$
25. $f(x) = \dfrac{4}{x}; x = -2, x = 1$
26. $f(x) = \dfrac{8}{x}; x = -2, x = 1$
27. $f(x) = 3.2x^2 + 2.1x; x = 0, x = 4$
28. $f(x) = 1.3x^2 - 1.4x; x = 0, x = 4$

Practice Plus

In Exercises 29–36,

a. Use transformations of a common graph to obtain the graph of f.

b. Find the slope-intercept equation of the tangent line to the graph of f at the point whose x-coordinate is given.

c. Use the y-intercept and the slope to graph the tangent line in the same rectangular coordinate system as the graph of f.

29. $f(x) = (x - 2)^2$; tangent line at 3
30. $f(x) = (x + 2)^2$; tangent line at -1
31. $f(x) = \sqrt{x + 1} - 2$; tangent line at 0
32. $f(x) = \sqrt{x - 1} + 2$; tangent line at 2
33. $f(x) = x^3 + 2$; tangent line at -1
34. $f(x) = x^3 - 2$; tangent line at 1
35. $f(x) = -\dfrac{1}{x + 3}$; tangent line at -2
36. $f(x) = -\dfrac{1}{x - 2}$; tangent line at 3

Application Exercises

37. The function $f(x) = x^2$ describes the area of a square, $f(x)$, in square inches, whose sides each measure x inches. If x is changing,

a. Find the average rate of change of the area with respect to x as x changes from 6 inches to 6.1 inches and from 6 inches to 6.01 inches.

b. Find the instantaneous rate of change of the area with respect to x at the moment when $x = 6$ inches.

38. The function $f(x) = x^2$ describes the area of a square, $f(x)$, in square inches, whose sides each measure x inches. If x is changing,

a. Find the average rate of change of the area with respect to x as x changes from 10 inches to 10.1 inches and from 10 inches to 10.01 inches.

b. Find the instantaneous rate of change of the area with respect to x at the moment when $x = 10$ inches.

In Exercises 39–42, express all answers in terms of π.

39. The function $f(x) = \pi x^2$ describes the area of a circle, $f(x)$, in square inches, whose radius measures x inches. If the radius is changing,

a. Find the average rate of change of the area with respect to the radius as the radius changes from 2 inches to 2.1 inches and from 2 inches to 2.01 inches.

b. Find the instantaneous rate of change of the area with respect to the radius when the radius is 2 inches.

40. The function $f(x) = \pi x^2$ describes the area of a circle, $f(x)$, in square inches, whose radius measures x inches. If the radius is changing,

a. Find the average rate of change of the area with respect to the radius as the radius changes from 4 inches to 4.1 inches and from 4 inches to 4.01 inches.

b. Find the instantaneous rate of change of the area with respect to the radius when the radius is 4 inches.

41. The function $f(x) = 4\pi x^2$ describes the surface area, $f(x)$, of a sphere of radius x inches. If the radius is changing, find the instantaneous rate of change of the surface area with respect to the radius when the radius is 6 inches.

42. The function $f(x) = 5\pi x^2$ describes the volume, $f(x)$, of a right circular cylinder of height 5 feet and radius x feet. If the radius is changing, find the instantaneous rate of change of the volume with respect to the radius when the radius is 8 feet.

43. An explosion causes debris to rise vertically with an initial velocity of 64 feet per second. The function

$$s(t) = -16t^2 + 64t$$

describes the height of the debris above the ground, $s(t)$, in feet, t seconds after the explosion.

a. What is the instantaneous velocity of the debris 1 second after the explosion? 3 seconds after the explosion?

b. What is the instantaneous velocity of the debris when it hits the ground?

44. An explosion causes debris to rise vertically with an initial velocity of 72 feet per second. The function

$$s(t) = -16t^2 + 72t$$

describes the height of the debris above the ground, $s(t)$, in feet, t seconds after the explosion.

a. What is the instantaneous velocity of the debris $\frac{1}{2}$ second after the explosion? 4 seconds after the explosion?

b. What is the instantaneous velocity of the debris when it hits the ground?

45. A foul tip of a baseball is hit straight upward from a height of 4 feet with an initial velocity of 96 feet per second. The function

$$s(t) = -16t^2 + 96t + 4$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is hit.

a. What is the instantaneous velocity of the ball 2 seconds after it is hit? 4 seconds after it is hit?

b. The ball reaches its maximum height above the ground when the instantaneous velocity is zero. After how many seconds does the ball reach its maximum height? What is its maximum height?

46. A foul tip of a baseball is hit straight upward from a height of 4 feet with an initial velocity of 64 feet per second. The function

$$s(t) = -16t^2 + 64t + 4$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is hit.

a. What is the instantaneous velocity of the ball 1 second after it is hit? 3 seconds after it is hit?

b. The ball reaches its maximum height above the ground when the instantaneous velocity is zero. After how many seconds does the ball reach its maximum height? What is its maximum height?

Writing in Mathematics

47. Explain how the tangent line to the graph of a function at point P is related to the secant lines between points P and Q on the function's graph.

48. Explain what we mean by the slope of the graph of a function at a point.

49. Explain how to find the slope of $f(x) = x^2$ at $(2, 4)$.

50. Explain how to write an equation of the tangent line to the graph of $f(x) = x^2$ at $(2, 4)$.

51. If you are given $y = f(x)$, the equation of function f, describe how to find $f'(x)$.

52. Explain how to use the derivative to compute the slopes of various tangent lines to the graph of a function.

53. Explain how the instantaneous rate of change of a function at a point is related to its average rates of change.

54. If a function expresses an object's position in terms of time, how do you find the instantaneous velocity of the object at any time during its motion?

55. Use the concept of an interval of time to describe how calculus views a particular instant of time.

56. You are about to take a great picture of fog rolling into San Francisco from the middle of the Golden Gate Bridge, 400 feet above the water. Whoops! You accidently lean too far over the safety rail and drop your camera. Your friend quips, "Well at least you know calculus; you can figure out the velocity with which the camera is going to hit the water." If the camera's height, $s(t)$, in feet, over the water after t seconds is $s(t) = 400 - 16t^2$, describe how to determine the camera's velocity at the instant of its demise.

57. A calculus professor introduced the derivative by saying that it could be summed up in one word: *slope*. Explain what this means.

58. For an unusual introduction to calculus using graphics and humor, read the first 100 pages (up through and including Chapter 2) of *The Cartoon Guide to Calculus* (Harpercollins, 2012) by cartoonist and Harvard-educated mathematician Larry Gonick. Write a report describing five new things you learned from the book that you did not know about calculus after completing this chapter. Did you find Gonick's attempt to teach calculus, using cartoon characters talking, commenting, and joking, helpful or distracting? (If you enjoy the use of comic art to humanize the world of functions, limits, and derivatives, and you're planning to take a calculus course in the future, read *The Cartoon Guide to Calculus* in its entirety, where you'll be taught the essentials of first-semester calculus.)

Technology Exercises

59. Use the $\boxed{\text{DRAW TANGENT}}$ feature of a graphing utility to graph the functions and tangent lines for any five exercises from Exercises 1–14. Use the equation that is displayed on the screen to verify the slope-intercept equation of the tangent line that you found in each exercise.

60. Without using the $\boxed{\text{DRAW TANGENT}}$ feature of a graphing utility, graph the function and the tangent line whose equation you found for any five exercises from Exercises 1–14. Does the line appear to be tangent to the graph of f at the point on f that is given in the exercise?

61. Use the feature on a graphing utility that gives the derivative of a function evaluated at any number to verify part (b) for any five of your answers in Exercises 15–28.

In Exercises 62–65, find, or approximate to two decimal places, the derivative of each function at the given number using a graphing utility.

62. $f(x) = x^4 - x^3 + x^2 - x + 1$ at 1

63. $f(x) = \dfrac{x}{x-3}$ at 6

64. $f(x) = x^2 \cos x$ at $\dfrac{\pi}{4}$

65. $f(x) = e^x \sin x$ at 2

Critical Thinking Exercises

Make Sense? *In Exercises 66–69, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

66. Because I have two points to work with, I use the formula for slope, $\dfrac{y_2 - y_1}{x_2 - x_1}$, to find the slope of the tangent line to the graph of a function $y = f(x)$ at $(a, f(a))$.

67. I can find the slope of the tangent line to the graph of $f(x)$ at $(3, f(3))$ using $\lim\limits_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$ or finding $f'(x)$ and then replacing x with 3.

68. I obtained $f'(x)$ by finding $\lim\limits_{h \to 0}[f(x+h) - f(x)]$ and $\lim\limits_{h \to 0} h$, and then using the quotient rule for limits.

69. If $f(x) = \pi x^2$ describes the area of a circle, $f(x)$, with radius x, $f'(5) > f'(2)$ because the area increases more rapidly as the radius increases.

In Exercises 70–75, graphs of functions are shown in $[-5, 5, 1]$ *by* $[-5, 5, 1]$ *viewing rectangles. Match each function with the graph of its derivative. Graphs of derivatives are labeled (a)–(f) and are shown in* $[-5, 5, 1]$ *by* $[-5, 5, 1]$ *viewing rectangles.*

70.

71.

72.

73.

74.

75.

a.

b.

c.

d.

e.

f.

76. A ball is thrown straight up from a rooftop 96 feet high with an initial velocity of 80 feet per second. The function

$$s(t) = -16t^2 + 80t + 96$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is thrown. The ball misses the rooftop on its way down and eventually strikes the ground. What is its instantaneous velocity as it passes the rooftop on the way down?

77. Show that the rate of change of the area of a circle with respect to its radius is equal to the circumference of the circle.

78. Show that the x-coordinate of the vertex of the parabola whose equation is $y = ax^2 + bx + c$ occurs when the derivative of the function is zero.

79. For any positive integer n, prove that if $f(x) = x^n$, then $f'(x) = nx^{n-1}$.

CHAPTER 11 Summary, Review, and Test

SUMMARY

DEFINITIONS AND CONCEPTS	EXAMPLES
11.1 Finding Limits Using Tables and Graphs	
a. Limit Notation and Its Description $\lim_{x \to a} f(x) = L$ is read "the limit of $f(x)$ as x approaches a equals the number L." This means that as x gets closer to a, but remains unequal to a, the corresponding values of $f(x)$ get closer to L.	
b. Limits can be found using tables.	Ex. 1, p. 1094; Ex. 2, p. 1094
c. Limits can be found using graphs.	Ex. 3, p. 1096; Ex. 4, p. 1096
d. Left-Hand Limit $\lim_{x \to a^-} f(x) = L$ is read "the limit of $f(x)$ as x approaches a from the left equals L." This means that as x gets closer to a, but remains less than a, the corresponding values of $f(x)$ get closer to L.	
e. Right-Hand Limit $\lim_{x \to a^+} f(x) = L$ is read "the limit of $f(x)$ as x approaches a from the right equals L." This means that as x gets closer to a, but remains greater than a, the corresponding values of $f(x)$ get closer to L.	
f. If $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, then $\lim_{x \to a} f(x)$ does not exist.	Ex. 5, p. 1098
11.2 Finding Limits Using Properties of Limits	
a. Properties of limits are given in the box on page 1111.	Ex. 1–Ex. 11, pp. 1105–1111
b. Properties of limits can be used to find one-sided limits.	Ex. 12, p. 1112
c. When taking the limit of a fractional expression in which the limit of the denominator is zero, the quotient property for limits cannot be used. Rewriting the expression using factoring or rationalizing the numerator or denominator may be helpful before the limit is found.	Ex. 13, p. 1112; Ex. 14, p. 1113
11.3 Limits and Continuity	
a. A function f is continuous at a when f is defined at a, $\lim_{x \to a} f(x)$ exists, and $\lim_{x \to a} f(x) = f(a)$. If f is not continuous at a, we say that f is discontinuous at a.	Ex. 1, p. 1118; Ex. 2, p. 1119
11.4 Introduction to Derivatives	
a. The slope of the tangent line to the graph of a function $y = f(x)$ at $(a, f(a))$ is given by $$m_{\tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ provided that this limit exists. The limit also describes the slope of the graph of f at $(a, f(a))$.	Ex, 1, p. 1127; Ex. 2, p. 1128
b. The Derivative of a Function The derivative of f at x is given by $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ provided that this limit exists. The derivative gives the slope of f for any value at x in the domain of f'.	Ex. 3, p. 1130
c. The derivative of f at a, $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$, gives the instantaneous rate of change of f with respect to x at a. Expressions for average and instantaneous rates of change are given in the box on page 1131.	Ex. 4, p. 1131
d. If a function expresses an object's position, $s(t)$, in terms of time, t, the instantaneous velocity of the object at time $t = a$ is $$s'(a) = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}.$$	Ex. 5, p. 1133

REVIEW EXERCISES

11.1

In Exercises 1–3, construct a table to find the indicated limit.

1. $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$ **2.** $\lim_{x \to 0} \frac{\sqrt{x + 1} - 1}{x}$

3. $\lim_{x \to 0} \frac{\sin 2x}{x}$

In Exercises 4–8, use the graph of f to find the indicated limit or function value.

4. $\lim_{x \to -4} f(x)$ **5.** $\lim_{x \to -1} f(x)$ **6.** $\lim_{x \to 3} f(x)$

7. $f(-4)$ **8.** $f(3)$

In Exercises 9–23, use the graph of function f to find the indicated limit or function value, or state that the limit or function value does not exist.

9. $\lim_{x \to -6^+} f(x)$ **10.** $\lim_{x \to -4^-} f(x)$ **11.** $\lim_{x \to -4^+} f(x)$

12. $\lim_{x \to -4} f(x)$ **13.** $f(-4)$ **14.** $\lim_{x \to -1^+} f(x)$

15. $\lim_{x \to -1^-} f(x)$ **16.** $\lim_{x \to -1} f(x)$ **17.** $f(-1)$

18. $f(2)$ **19.** $\lim_{x \to 2^-} f(x)$ **20.** $\lim_{x \to 2^+} f(x)$

21. $\lim_{x \to 2} f(x)$ **22.** $\lim_{x \to 5} f(x)$ **23.** $f(5)$

In Exercises 24–26, graph each function. Then use your graph to find the indicated limit.

24. $f(x) = \frac{x^2 - 9}{x - 3}$, $\lim_{x \to 3} f(x)$

25. $f(x) = \sin x$, $\lim_{x \to \frac{3\pi}{2}} f(x)$

26. $f(x) = \begin{cases} 1 - x & \text{if } x < 0 \\ \cos x & \text{if } x \geq 0, \end{cases}$ $\lim_{x \to 0} f(x)$

11.2

In Exercises 27–37, find the limit.

27. $\lim_{x \to 4} (2x^2 - 5x + 3)$ **28.** $\lim_{x \to -1} (-2x^3 - x + 5)$

29. $\lim_{x \to -3} (x^2 + 1)^3$ **30.** $\lim_{x \to 4} \sqrt{x^2 + 9}$

31. $\lim_{x \to 5} \frac{11x - 3}{x^2 + 1}$ **32.** $\lim_{x \to -4} \frac{x^2 - 16}{x + 4}$

33. $\lim_{x \to 7} \frac{5x - 35}{x - 7}$ **34.** $\lim_{x \to 0} \frac{\sqrt{x + 100} - 10}{x}$

35. $\lim_{x \to -1} \frac{x^2 - 1}{x^2 + x}$ **36.** $\lim_{x \to 100} \frac{\sqrt{x} - 10}{x - 100}$

37. $\lim_{x \to 0} \frac{\frac{1}{x + 5} - \frac{1}{5}}{x}$

In Exercises 38–40, a piecewise function is given. Use properties of limits to find the indicated limit, or state that the limit does not exist.

38. $f(x) = \begin{cases} x^2 + 1 & \text{if } x < 2 \\ 3x + 1 & \text{if } x \geq 2 \end{cases}$

a. $\lim_{x \to 2^-} f(x)$ **b.** $\lim_{x \to 2^+} f(x)$ **c.** $\lim_{x \to 2} f(x)$

39. $f(x) = \begin{cases} \sqrt[3]{x^2 + 7} & \text{if } x < 1 \\ 4x & \text{if } x \geq 1 \end{cases}$

a. $\lim_{x \to 1^-} f(x)$ **b.** $\lim_{x \to 1^+} f(x)$ **c.** $\lim_{x \to 1} f(x)$

40. $f(x) = \begin{cases} \frac{x^2 - 25}{x + 5} & \text{if } x \neq -5 \\ 13 & \text{if } x = -5 \end{cases}$

a. $\lim_{x \to -5^-} f(x)$ **b.** $\lim_{x \to -5^+} f(x)$ **c.** $\lim_{x \to -5} f(x)$

11.3

In Exercises 41–45, use the definition of continuity to determine whether f is continuous at a.

41. $f(x) = 3x^2 - 2x + 1$, $a = 4$ **42.** $f(x) = \frac{x^2 - 9}{x + 3}$, $a = -3$

43. $f(x) = \begin{cases} \frac{x^2 + 5x}{x^2 - 5x} & \text{if } x \neq 0 \\ -2 & \text{if } x = 0 \end{cases}$

$a = 0$

44. $f(x) = \begin{cases} \frac{x^2 + x}{x^2 - 3x - 4} & \text{if } x \neq -1 \\ \frac{1}{5} & \text{if } x = -1 \end{cases}$

$a = -1$

45. $f(x) = \begin{cases} 3x & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ x + 4 & \text{if } x > 2 \end{cases}$

$a = 2$

In Exercises 46–51, determine for what numbers, if any, the given function is discontinuous.

46. $f(x) = x^3 + 5x^2 - 1$ **47.** $f(x) = \frac{x - 1}{(x - 1)(x + 3)}$

48. $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ **49.** $f(x) = \begin{cases} 4x & \text{if } x < 5 \\ x^2 - 5 & \text{if } x \geq 5 \end{cases}$

50. $f(x) = \begin{cases} \dfrac{x^2 - 4}{x + 2} & \text{if } x \neq -2 \\ 4 & \text{if } x = -2 \end{cases}$

51. $f(x) = \begin{cases} \dfrac{x^2 - 121}{x - 11} & \text{if } x \neq 11 \\ 20 & \text{if } x = 11 \end{cases}$

11.4

In Exercises 52–53,

a. *Find the slope of the tangent line to the graph of f at the given point.*

b. *Find the slope-intercept equation of the tangent line to the graph of f at the given point.*

52. $f(x) = 2x^2 + 5x$ at $(1, 7)$

53. $f(x) = x^2 - 7x - 4$ at $(-1, 4)$

In Exercises 54–57,

a. *Find $f'(x)$.*

b. *Find the slope of the tangent line to the graph of f at each of the two values of x given to the right of the function.*

54. $f(x) = 3x^2 + 12x - 1; x = -2, x = 1$

55. $f(x) = 2x^3 - x; x = -1, x = 1$

56. $f(x) = \dfrac{1}{x}; x = -2, x = 2$ **57.** $f(x) = \sqrt{x}; x = 36, x = 81$

58. The function $f(x) = 5x^2$ describes the volume of a rectangular box, $f(x)$, in cubic inches, whose square base has sides that each measure x inches and whose height is 5 inches. If x is changing,

a. Find the average rate of change of the volume with respect to x as x changes from 2 inches to 2.1 inches and from 2 inches to 2.01 inches.

b. Find the instantaneous rate of change of the volume with respect to x at the moment when $x = 2$ inches.

59. The function $f(x) = \frac{4}{3}\pi x^3$ describes the volume, $f(x)$, of a sphere of radius x inches. If the radius is changing, find the instantaneous rate of change of the volume with respect to the radius when the radius is 5 inches. Express the answer in terms of π.

60. A baseball is thrown straight upward from a height of 5 feet with an initial velocity of 80 feet per second. The function

$$s(t) = -16t^2 + 80t + 5$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is thrown.

a. What is the instantaneous velocity of the ball 2 seconds after it is thrown? 4 seconds after it is thrown?

b. The ball reaches its maximum height above the ground when the instantaneous velocity is zero. After how many seconds does the ball reach its maximum height? What is the maximum height?

CHAPTER 11 TEST

1. Construct a table to find $\lim\limits_{x \to 9} \dfrac{9 - x}{3 - \sqrt{x}}$.

In Exercises 2–7, use the graph of function f to find the indicated limit or function value, or state that the limit or function value does not exist.

2. $\lim\limits_{x \to -2} f(x)$

3. $f(-2)$

4. $\lim\limits_{x \to 2^-} f(x)$

5. $\lim\limits_{x \to 2^+} f(x)$

6. $\lim\limits_{x \to 2} f(x)$

7. $\lim\limits_{x \to 4} f(x)$

In Exercises 8–10, find the limit.

8. $\lim\limits_{x \to -2} (x^2 + x + 1)^4$ **9.** $\lim\limits_{x \to -1} \dfrac{x^2 - x - 2}{x + 1}$

10. $\lim\limits_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$

In Exercises 11–12, determine whether f is continuous at a.

11. $f(x) = \begin{cases} \dfrac{x^2 - 1}{x + 1} & \text{if } x \neq -1 \\ 6 & \text{if } x = -1 \end{cases}$

$a = -1$

12. $f(x) = \begin{cases} 2 - x & \text{if } x \leq 2 \\ x^2 - 2x & \text{if } x > 2 \end{cases}$

$a = 2$

In Exercises 13–14, find $f'(x)$.

13. $f(x) = x^2 - 5x + 1$ **14.** $f(x) = \dfrac{10}{x}$

15. Find the slope-intercept equation of the tangent line to the graph of $f(x) = x^2$ at $(-3, 9)$.

16. A ball is thrown straight upward. The function

$$s(t) = -16t^2 + 72t$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is thrown. What is the instantaneous velocity of the ball 3 seconds after it is thrown?

CUMULATIVE REVIEW EXERCISES (CHAPTERS P–11)

Solve each equation or inequality in Exercises 1–5.

1. $\dfrac{1}{x+2} > \dfrac{3}{x+1}$

2. $2x^3 + 11x^2 - 7x - 6 = 0$

3. $|2x + 4| > 3$

4. $\cos^2 x + \sin x + 1 = 0, 0 \le x < 2\pi$

5. $\log_4(x^2 - 9) - \log_4(x + 3) = 3$

In Exercises 6–15, graph each equation, function, or system in a rectangular coordinate system.

6. $f(x) = x^3 + x^2 - 12x$

7. $f(x) = \dfrac{2x^2 - 5x + 2}{x^2 - 4}$

8. $f(x) = \begin{cases} -x + 1 & \text{if } -1 \le x < 1 \\ 2 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$

9. $y = 2\sin\left(2x + \dfrac{\pi}{2}\right)$ (Graph one period.)

10. $y = \frac{1}{2}\sec 2\pi x, \quad 0 \le x \le 2$

11. $\begin{cases} x - 2y \le 4 \\ x \ge 2 \end{cases}$

12. $x^2 - 4y^2 - 4x - 24y - 48 = 0$

13. $f(x) = \sqrt{x}, g(x) = \sqrt{x - 2} + 1$ (Graph f and g in the same rectangular coordinate system.)

14. $x = 3\sin t, y = 4\cos t + 2; 0 \le t \le 2\pi$

15. $2x^2 + 5xy + 2y^2 - \frac{9}{2} = 0$

16. Find $f'(x)$ if $f(x) = -2x^2 + 7x - 1$.

17. Find $f^{-1}(x)$ if $f(x) = 7x - 1$.

18. Find the limit: $\lim\limits_{x \to -3} \dfrac{x^2 + x - 6}{x^2 + 2x - 3}$.

19. Expand and simplify: $(x^2 - 3y)^4$.

20. Write the slope-intercept form of the equation of the line passing through the point $(2, -3)$ and parallel to the line whose equation is $2x + y - 6 = 0$.

21. Find the dot product $\mathbf{v} \cdot \mathbf{w}$ and the angle between $\mathbf{v}$ and $\mathbf{w}$:

$$\mathbf{v} = -2\mathbf{i} + \mathbf{j}, \quad \mathbf{w} = 4\mathbf{i} - 3\mathbf{j}.$$

22. Find the partial fraction decomposition for

$$\frac{1}{x(x^2 + x + 1)}.$$

Verify each identity in Exercises 23–24.

23. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

24. $\tan(\theta + \pi) = \tan\theta$

25. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 2 & 1 \end{bmatrix}$, find BA.

26. Graph the polar equation: $r = 4\sin\theta$.

27. Express $h(x) = (x^2 - 3x + 7)^9$ as a composition of two functions f and g such that $h(x) = (f \circ g)(x)$.

28. Solve using matrices:

$$\begin{cases} 2x - y - 2z = -1 \\ x - 2y - z = 1 \\ x + y + z = 1. \end{cases}$$

29. Use the formula for the sum of the first n terms of a geometric sequence to find $\sum\limits_{i=1}^{6} 4(-2)^i$.

30. Use DeMoivre's Theorem to find

$$\left[\sqrt{2}(\cos 15° + i\sin 15°)\right]^4.$$

Write the answer in rectangular form.

31. A bank loaned out \$120,000, part of it at 8% per year and the rest at 18% per year.

a. Express the interest, I, on the two loans as a function of the amount loaned at 8%, x.

b. If the interest received totaled \$10,000, how much was loaned at each rate?

32. A machine produces open boxes using square sheets of metal. The machine cuts equal-sized squares measuring 9 centimeters on a side from each corner. Then the machine shapes the metal into an open box by turning up the sides. If each box must have a volume of 225 cubic centimeters, what should be the dimensions of the piece of sheet metal?

33. You have 200 feet of fencing to enclose a small rectangular garden with one side against a barn. If you do not fence the side along the barn, find the length and width of the garden that will maximize its area. What is the largest area that can be enclosed?

34. Use Newton's Law of Cooling, $T = C + (T_0 - C)e^{kt}$, to solve this exercise. You remove a pie that has a temperature of 375°F from the oven. You leave the pie in a room whose temperature is 72°F. After 60 minutes, the temperature of the pie is 75°F.

a. Write a model for the temperature of the pie, T, after t minutes.

b. When will the temperature of the pie be 250°F?

35. You just purchased a rectangular waterfront lot along a river's edge. The area of the lot is 60,000 square feet. To create a sense of privacy, you decide to fence along three sides, excluding the side that fronts the river. An expensive fencing along the lot's front length costs \$25 per foot. An inexpensive fencing along the two side widths costs only \$5 per foot. Express the total cost, C, of fencing along the three sides as a function of the lot's length, x.

36. Two ships leave a harbor at the same time. One ship travels at a bearing of N42°E for 23 miles. The other ship travels at a bearing of N38°W for 72 miles. After both ships are anchored, how far apart are they? Round to the nearest tenth of a mile.

37. At a fixed temperature, the volume of a given mass of gas varies inversely as the pressure applied to the gas. A certain mass of gas has a volume of 40 cubic inches when the pressure is 22 pounds. What is the volume of the gas when the pressure is 30 pounds?

38. A ball is thrown straight upward. The function

$$s(t) = -16t^2 + 40t$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is thrown. What is the instantaneous velocity of the ball 2 seconds after it is thrown?

39. The figure shows an open box with a square base. The box is to have a volume of 4 cubic feet. Express the surface area of the box, A, as a function of the length of a side of its square base, x.

40. The function $f(x) = -2.32x^2 + 76.58x - 559.87$ models the percentage of U.S. students, $f(x)$, who are x years old who say their school is not drug free, where $12 \le x \le 17$. At what age do 70% of U.S. students say that their school is not drug free? Round to the nearest tenth of a year.

APPENDIX A

Where Did That Come From?
Selected Proofs

SECTION 3.3 Properties of Logarithms

The Product Rule

Let b, M, and N be positive real numbers with $b \neq 1$.

$$\log_b(MN) = \log_b M + \log_b N$$

Proof

We begin by letting $\log_b M = R$ and $\log_b N = S$.
Now we write each logarithm in exponential form.

$$\log_b M = R \quad \text{means} \quad b^R = M.$$

$$\log_b N = S \quad \text{means} \quad b^S = N.$$

By substituting and using a property of exponents, we see that

$$MN = b^R b^S = b^{R+S}.$$

Now we change $MN = b^{R+S}$ to logarithmic form.

$$MN = b^{R+S} \quad \text{means} \quad \log_b(MN) = R + S.$$

Finally, substituting $\log_b M$ for R and $\log_b N$ for S gives us

$$\log_b(MN) = \log_b M + \log_b N,$$

the property that we wanted to prove.

The quotient and power rules for logarithms are proved using similar procedures.

The Change-of-Base Property

For any logarithmic bases a and b, and any positive number M,

$$\log_b M = \frac{\log_a M}{\log_a b}.$$

Proof

To prove the change-of-base property, we let x equal the logarithm on the left side:

$$\log_b M = x.$$

Now we rewrite this logarithm in exponential form.

$$\log_b M = x \quad \text{means} \quad b^x = M.$$

Because b^x and M are equal, the logarithms with base a for each of these expressions must be equal. This means that

$$\log_a b^x = \log_a M$$

$$x \log_a b = \log_a M \quad \text{Apply the power rule for logarithms on the left side.}$$

$$x = \frac{\log_a M}{\log_a b} \quad \text{Solve for } x \text{ by dividing both sides by } \log_a b.$$

In our first step we let x equal $\log_b M$. Replacing x on the left side by $\log_b M$ gives us

$$\log_b M = \frac{\log_a M}{\log_a b},$$

which is the change-of-base property.

SECTION 6.2 The Law of Cosines

Heron's Formula for the Area of a Triangle

The area of a triangle with sides a, b, and c is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is one-half its perimeter: $s = \frac{1}{2}(a + b + c)$.

Proof

The proof of Heron's formula begins with a half-angle formula and the Law of Cosines.

$$\cos\frac{C}{2} = \sqrt{\frac{1+\cos C}{2}} = \sqrt{\frac{1 + \dfrac{a^2+b^2-c^2}{2ab}}{2}}$$

This is the Law of Cosines $c^2 = a^2 + b^2 - 2ab\cos C$ solved for $\cos C$.

$$= \sqrt{\frac{a^2+2ab+b^2-c^2}{4ab}} = \sqrt{\frac{(a+b)^2-c^2}{4ab}} = \sqrt{\frac{(a+b+c)(a+b-c)}{4ab}}$$

Multiply the numerator and denominator of the radicand by $2ab$.

Factor $a^2 + 2ab + b^2$.

Factor the numerator as the difference of two squares.

We now introduce the expression for one-half the perimeter: $s = \frac{1}{2}(a + b + c)$. We replace $a + b + c$ in the numerator by $2s$. We also find an expression for $a + b - c$ as follows:

$$a + b - c = a + b + c - 2c = 2s - 2c = 2(s - c).$$

Thus,

$$\cos\frac{C}{2} = \sqrt{\frac{(a+b+c)(a+b-c)}{4ab}} = \sqrt{\frac{2s\cdot 2(s-c)}{4ab}} = \sqrt{\frac{s(s-c)}{ab}}.$$

In a similar manner, we obtain

$$\sin\frac{C}{2} = \sqrt{\frac{1-\cos C}{2}} = \sqrt{\frac{(s-a)(s-b)}{ab}}.$$

From our work in Section 6.1, we know that the area of a triangle is one-half the product of the length of two sides times the sine of their included angle.

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}ab \cdot 2\sin\frac{C}{2}\cos\frac{C}{2}$$

$\sin C = \sin 2\frac{C}{2} = 2\sin\frac{C}{2}\cos\frac{C}{2}$

$$= ab\sqrt{\frac{(s-a)(s-b)}{ab}}\sqrt{\frac{s(s-c)}{ab}}$$

Use the expressions for $\sin\frac{C}{2}$ and $\cos\frac{C}{2}$ on page 00.

$$= ab\frac{\sqrt{s(s-a)(s-b)(s-c)}}{\sqrt{a^2b^2}}$$

Multiply the radicands.

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

Simplify: $\frac{ab}{\sqrt{a^2b^2}} = \frac{ab}{ab} = 1.$

SECTION 6.5 Complex Numbers in Polar Form; DeMoivre's Theorem

The Quotient of Two Complex Numbers in Polar Form

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be two complex numbers in polar form. Their quotient, $\frac{z_1}{z_2}$, is

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)].$$

Proof

We begin by multiplying the numerator and denominator of the quotient, $\frac{z_1}{z_2}$, by the conjugate of the expression in parentheses in the denominator. Then we simplify the quotient using the difference formulas for sine and cosine.

$$\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)}$$

This is the given quotient.

$$= \frac{r_1(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 - i\sin\theta_2)}{r_2(\cos\theta_2 + i\sin\theta_2)(\cos\theta_2 - i\sin\theta_2)}$$

Multiply the numerator and denominator by the conjugate of the expression in parentheses in the denominator. Recall that the conjugate of $a + bi$ is $a - bi$.

$$= \frac{r_1(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 - i\sin\theta_2)}{r_2(\cos^2\theta_2 + \sin^2\theta_2)}$$

Multiply the conjugates in the denominator.

$$= \frac{r_1(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 - i\sin\theta_2)}{r_2}$$

Use a Pythagorean identity: $\cos^2\theta_2 + \sin^2\theta_2 = 1.$

$$= \frac{r_1}{r_2}(\cos\theta_1\cos\theta_2 - i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 - i^2\sin\theta_1\sin\theta_2)$$

Use the FOIL method.

$$= \frac{r_1}{r_2}[\cos\theta_1\cos\theta_2 + i(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2) - i^2\sin\theta_1\sin\theta_2]$$

Factor i from the second and third terms.

$$= \frac{r_1}{r_2}[\cos\theta_1\cos\theta_2 + i(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2) - (-1)\sin\theta_1\sin\theta_2]$$

$i^2 = -1$

$$= \frac{r_1}{r_2}[(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)]$$

Rearrange terms.

This is $\cos(\theta_1 - \theta_2)$. This is $\sin(\theta_1 - \theta_2)$.

$$= \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

SECTION 6.7 The Dot Product

Properties of the Dot Product

If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors, and c is a scalar, then

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
3. $\mathbf{0} \cdot \mathbf{v} = 0$
4. $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$
5. $(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot (c\mathbf{v})$

Proof

To prove the second property, let

$$\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}, \quad \mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}, \quad \text{and} \quad \mathbf{w} = w_1\mathbf{i} + w_2\mathbf{j}.$$

Then

$$\begin{aligned}
\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) &= (u_1\mathbf{i} + u_2\mathbf{j}) \cdot [(v_1\mathbf{i} + v_2\mathbf{j}) + (w_1\mathbf{i} + w_2\mathbf{j})] && \text{These are the given vectors.} \\
&= (u_1\mathbf{i} + u_2\mathbf{j}) \cdot [(v_1 + w_1)\mathbf{i} + (v_2 + w_2)\mathbf{j}] && \text{Add horizontal components and add vertical components.} \\
&= u_1(v_1 + w_1) + u_2(v_2 + w_2) && \text{Multiply horizontal components and multiply vertical components.} \\
&= u_1v_1 + u_1w_1 + u_2v_2 + u_2w_2 && \text{Use the distributive property.} \\
&= \underbrace{u_1v_1 + u_2v_2}_{\text{This is the dot product of } \mathbf{u} \text{ and } \mathbf{v}.} + \underbrace{u_1w_1 + u_2w_2}_{\text{This is the dot product of } \mathbf{u} \text{ and } \mathbf{w}.} && \text{Rearrange terms.} \\
&= \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}.
\end{aligned}$$

To prove the third property, let

$$\mathbf{0} = 0\mathbf{i} + 0\mathbf{j} \quad \text{and} \quad \mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}.$$

Then

$$\begin{aligned}
\mathbf{0} \cdot \mathbf{v} &= (0\mathbf{i} + 0\mathbf{j}) \cdot (v_1\mathbf{i} + v_2\mathbf{j}) && \text{These are the given vectors.} \\
&= 0 \cdot v_1 + 0 \cdot v_2 && \text{Multiply horizontal components and multiply vertical components.} \\
&= 0 + 0 \\
&= 0.
\end{aligned}$$

To prove the first part of the fifth property, let

$$\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} \quad \text{and} \quad \mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}.$$

Then

$$\begin{aligned}
(c\mathbf{u}) \cdot \mathbf{v} &= [c(u_1\mathbf{i} + u_2\mathbf{j})] \cdot (v_1\mathbf{i} + v_2\mathbf{j}) && \text{These are the given vectors.} \\
&= (cu_1\mathbf{i} + cu_2\mathbf{j}) \cdot (v_1\mathbf{i} + v_2\mathbf{j}) && \text{Multiply each component of } u_1\mathbf{i} + u_2\mathbf{j} \text{ by } c. \\
&= cu_1v_1 + cu_2v_2 && \text{Multiply horizontal components and multiply vertical components.} \\
&= c\underbrace{(u_1v_1 + u_2v_2)}_{\text{This is the dot product of } \mathbf{u} \text{ and } \mathbf{v}.} && \text{Factor out } c \text{ from both terms.} \\
&= c(\mathbf{u} \cdot \mathbf{v})
\end{aligned}$$

SECTION 9.1 The Ellipse

The Standard Form of the Equation of an Ellipse with a Horizontal Major Axis Centered at the Origin

Proof

Refer to **Figure A.1**.

FIGURE A.1

$$d_1 + d_2 = 2a$$ The sum of the distances from P to the foci equals a constant, 2a.

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$ Use the distance formula.

$$\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$$ Isolate a radical.

$$(x+c)^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$$ Square both sides.

$$x^2 + 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + x^2 - 2cx + c^2 + y^2$$ Square $x + c$ and $x - c$.

$$4cx - 4a^2 = -4a\sqrt{(x-c)^2 + y^2}$$ Simplify and isolate the radical.

$$cx - a^2 = -a\sqrt{(x-c)^2 + y^2}$$ Divide both sides by 4.

$$(cx - a^2)^2 = a^2[(x-c)^2 + y^2]$$ Square both sides.

$$c^2x^2 - 2a^2cx + a^4 = a^2(x^2 - 2cx + c^2 + y^2)$$ Square $cx - a^2$ and $x - c$.

$$c^2x^2 - 2a^2cx + a^4 = a^2x^2 - 2a^2cx + a^2c^2 + a^2y^2$$ Use the distributive property.

$$c^2x^2 + a^4 = a^2x^2 + a^2c^2 + a^2y^2$$ Add $2a^2cx$ to both sides.

$$c^2x^2 - a^2x^2 - a^2y^2 = a^2c^2 - a^4$$ Rearrange the terms.

$$(c^2 - a^2)x^2 - a^2y^2 = a^2(c^2 - a^2)$$ Factor out x^2 and a^2, respectively.

$$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$ Multiply both sides by -1.

Refer to the discussion on page 00 and let $b^2 = a^2 - c^2$ in the preceding equation.

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ Divide both sides by a^2b^2.

SECTION 9.2 The Hyperbola

The Asymptotes of a Hyperbola Centered at the Origin

The hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

with a horizontal transverse axis has the two asymptotes

$$y = \frac{b}{a}x \quad \text{and} \quad y = -\frac{b}{a}x.$$

Proof

Begin by solving the hyperbola's equation for y.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

This is the standard form of the equation of a hyperbola.

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$

We isolate the term involving y^2 to solve for y.

$$y^2 = \frac{b^2x^2}{a^2} - b^2$$

Multiply both sides by b^2.

$$y^2 = \frac{b^2x^2}{a^2}\left(1 - \frac{a^2}{x^2}\right)$$

Factor out $\frac{b^2x^2}{a^2}$ on the right. Verify that this result is correct by multiplying using the distributive property and obtaining the previous step.

$$y = \pm\sqrt{\frac{b^2x^2}{a^2}\left(1 - \frac{a^2}{x^2}\right)}$$

Solve for y using the square root property: If $u^2 = d$, then $u = \pm\sqrt{d}$.

$$y = \pm\frac{b}{a}x\sqrt{1 - \frac{a^2}{x^2}}$$

Simplify.

As $|x| \to \infty$, the value of $\frac{a^2}{x^2}$ approaches 0. Consequently, the value of y can be approximated by

$$y = \pm\frac{b}{a}x.$$

This means that the lines whose equations are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$ are asymptotes for the graph of the hyperbola.

APPENDIX B

The Transition from Precalculus to Calculus

Calculus is the study of limits and their applications. (Limits are introduced in Sections 11.1 and 11.2.)

Precalculus	Calculus
Substitution: The value of function f at a $f(a)$	Limits: What occurs to the value of a function f as x gets closer to a, but remains unequal to a $\lim_{x \to a} f(x)$

Calculus uses limits to extend ideas and topics from precalculus to more general situations.

Precalculus Topic	Calculus Generalization
Slope of a line	Slope of a curve
Average rate of change; average velocity	Instantaneous rate of change; instantaneous velocity
Location of a quadratic function's $[f(x) = ax^2 + bx + c]$ maximum or minimum value	Location of any function's relative maxima and/or minima
Area of a rectangle	Area of a plane region bounded by curves
Length of a line segment	Length of a curve
Center of a circle	Center of gravity (the balance point) of a plane region
Work done by a constant force moving an object along a linear path	Work done by a variable force moving an object along a curved path

The vast array of generalizations made possible by the invention of limits marked a turning point in human history, having as dramatic an impact on our lives as the invention of the wheel and the printing press.

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ANSWERS TO SELECTED EXERCISES

CHAPTER P

Exercise Set P.1

19. 60 ft **20.** 40 ft **24.** $\{r, e, a, l\}$ **30.** $\{1, 2, 3, 7, 8\}$ **31.** $\{1, 2, 3, 4, 5, 6, 7, 8, 10\}$ **32.** $\{0, 1, 2, 3, 4, 5, 6\}$ **34.** $\{e, m, p, t, y\}$

35. a. $\sqrt{100}$ **b.** $0, \sqrt{100}$ **c.** $-9, 0, \sqrt{100}$ **d.** $-9, -\frac{4}{5}, 0, 0.25, 9.2, \sqrt{100}$ **e.** $\sqrt{3}$ **f.** $-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}$

36. a. $\sqrt{49}$ **b.** $0, \sqrt{49}$ **c.** $-7, 0, \sqrt{49}$ **d.** $-7, -0.\overline{6}, 0, \sqrt{49}$ **e.** $\sqrt{50}$ **f.** $-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}$

37. a. $\sqrt{64}$ **b.** $0, \sqrt{64}$ **c.** $-11, 0, \sqrt{64}$ **d.** $-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}$ **e.** $\sqrt{5}, \pi$ **f.** $-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}$ **38. a.** $\sqrt{4}$

b. $0, \sqrt{4}$ **c.** $-5, 0, \sqrt{4}$ **d.** $-5, -0.\overline{3}, 0, \sqrt{4}$ **e.** $\sqrt{2}$ **f.** $-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}$ **40.** Answers may vary; an example is $\frac{1}{2}$.

72. $|-3 - (-26)|; 23$ **74.** $|-1.2 - (-5.4)|; 4.2$ **79.** commutative property of addition **88.** $24x - 2$ **94.** $10y - 54$

100. $-5x + 13y + 1$ **121.** $x - (x + 4); -4$ **123.** $6(-5x); -30x$ **124.** $10(-4x); -40x$ **129. a.** 140 beats per minute
b. 160 beats per minute **130. a.** 95 beats per minute **b.** 114 beats per minute **145.** does not make sense

Exercise Set P.2

48. $30x^{11}y^{12}$ **75.** $-60{,}000{,}100{,}000$ **76.** $-70{,}000{,}100{,}000$ **79.** 6.38×10^{17} **80.** 5.79×10^{17} **87.** 6.3×10^{7} **88.** 8.2×10^{7}
89. 6.4×10^{4} **90.** 4.2×10^{4} **91.** 1.22×10^{-11} **92.** 1.53×10^{-11} **93.** 2.67×10^{13} **94.** 3.77×10^{13} **117.** $\$1.0586 \times 10^{10}$
118. $\$8.832 \times 10^{9}$ **122.** 3.1536×10^{7} **132.** does not make sense **134.** does not make sense **146.** about 2.94×10^{9} times

Exercise Set P.3

116. b. 15.8; underestimates by 2.7 million **117.** $\frac{\sqrt{5} + 1}{2}$; 1.62 to 1

Exercise Set P.4

4. yes; $x^4 - x^3 + x^2 - 5$ **9.** $11x^3 + 7x^2 - 12x - 4; 3$ **10.** $12x^3 - 5x^2 - 4x - 4; 3$ **11.** $12x^3 + 4x^2 + 12x - 14; 3$
12. $9x^4 + 4x^3 - 2x + 1; 4$ **13.** $6x^2 - 6x + 2; 2$ **14.** $6x^3 + 10x^2 + 11x - 8; 3$ **16.** $x^3 + 125$ **17.** $2x^3 - 9x^2 + 19x - 15$
18. $2x^3 - 9x^2 + 10x - 3$ **19.** $x^2 + 10x + 21$ **20.** $x^2 + 13x + 40$ **21.** $x^2 - 2x - 15$ **22.** $x^2 + x - 2$ **23.** $6x^2 + 13x + 5$
24. $21x^2 + 19x + 4$ **25.** $10x^2 - 9x - 9$ **26.** $14x^2 - 31x - 10$ **27.** $15x^4 - 47x^2 + 28$ **28.** $21x^4 - 41x^2 + 10$
29. $8x^5 - 40x^3 + 3x^2 - 15$ **30.** $7x^5 - 14x^3 + 5x^2 - 10$ **37.** $16x^4 - 25x^2$ **38.** $9x^4 - 16x^2$ **52.** $x^3 + 6x^2 + 12x + 8$
53. $8x^3 + 36x^2 + 54x + 27$ **54.** $27x^3 + 108x^2 + 144x + 64$ **57.** $27x^3 - 108x^2 + 144x - 64$ **58.** $8x^3 - 36x^2 + 54x - 27$
59. $7x^2 + 38xy + 15y^2$ **60.** $6x^2 + 61xy + 63y^2$ **61.** $2x^2 + xy - 21y^2$ **62.** $6x^2 + 13xy - 5y^2$ **63.** $15x^2y^2 + xy - 2$
64. $14x^4y^2 - 19x^2y - 3$ **69.** $x^3 - y^3$ **70.** $x^3 + y^3$ **71.** $9x^2 - 25y^2$ **72.** $49x^2 - 9y^2$ **91. a.** \$54,647; overestimates by \$556
b. $M - W = 1.8x^3 - 82x^2 + 2644x - 11{,}449$ **92. b.** $M - W = 1.8x^3 - 82x^2 + 2644x - 11{,}449$ **106.** makes sense, although answers may vary

Exercise Set P.5

5. $9x^2(x^2 - 2x + 3)$ **6.** $6x^2(x^2 - 3x + 2)$ **7.** $(x + 5)(x + 3)$ **8.** $(2x + 1)(x + 4)$ **9.** $(x - 3)(x^2 + 12)$ **10.** $(2x + 5)(x^2 + 17)$
11. $(x - 2)(x^2 + 5)$ **12.** $(x - 3)(x^2 + 4)$ **13.** $(x - 1)(x^2 + 2)$ **14.** $(x + 6)(x^2 - 2)$ **15.** $(3x - 2)(x^2 - 2)$ **16.** $(x - 1)(x^2 - 5)$
25. $(3x - 28)(x + 1)$ **27.** $(2x - 1)(3x - 4)$ **28.** $(2x - 3)(3x - 4)$ **29.** $(2x + 3)(2x + 5)$ **30.** $(8x + 1)(x + 4)$ **33.** $(5x + 8)(4x - 1)$
34. $(5x - 3)(3x - 2)$ **35.** $(2x + y)(x + y)$ **36.** $(3x + y)(x + y)$ **37.** $(3x + 2y)(2x - 3y)$ **38.** $(3x - 5y)(2x + y)$ **43.** $(3x + 5y)(3x - 5y)$
44. $(6x + 7y)(6x - 7y)$ **45.** $(x^2 + 4)(x + 2)(x - 2)$ **46.** $(x^2 + 1)(x + 1)(x - 1)$ **47.** $(4x^2 + 9)(2x + 3)(2x - 3)$
48. $(9x^2 + 1)(3x + 1)(3x - 1)$ **57.** $(x + 3)(x^2 - 3x + 9)$ **58.** $(x + 4)(x^2 - 4x + 16)$ **59.** $(x - 4)(x^2 + 4x + 16)$
60. $(x - 3)(x^2 + 3x + 9)$ **61.** $(2x - 1)(4x^2 + 2x + 1)$ **62.** $(3x - 1)(9x^2 + 3x + 1)$ **63.** $(4x + 3)(16x^2 - 12x + 9)$
64. $(2x + 5)(4x^2 - 10x + 25)$ **67.** $4(x + 2)(x - 3)$ **68.** $6(x + 2)(x - 5)$ **69.** $2(x^2 + 9)(x + 3)(x - 3)$
70. $7(x^2 + 1)(x + 1)(x - 1)$ **71.** $(x - 3)(x + 3)(x + 2)$ **72.** $(x - 5)(x + 5)(x + 3)$ **73.** $2(x - 8)(x + 7)$ **74.** $6(x - 2)(x + 1)$
79. $(x - 2)(x + 2)^2$ **80.** $(x - 1)(x + 1)(x + 2)$ **81.** $y(y^2 + 9)(y + 3)(y - 3)$ **82.** $y(y^2 + 4)(y + 2)(y - 2)$ **83.** $5y^2(2y + 3)(2y - 3)$

84. $3y^2(4y+1)(4y-1)$ **85.** $(x-6+7y)(x-6-7y)$ **86.** $(x-5+6y)(x-5-6y)$ **89.** $(y-2)(x+4)(x-4)$
90. $(3y-1)(2x+3)(2x-3)$ **95.** $\dfrac{4(1+2x)}{x^{2/3}}$ **96.** $\dfrac{6(2+x)}{x^{3/4}}$ **97.** $-(x+3)^{1/2}(x+2)$ **98.** $(x^2+4)^{3/2}(x^4+8x^2+17)$
102. $\dfrac{2(100x^2+95x+11)}{(4x+3)^2}$ **103.** $(x+1)(5x-6)(2x+1)$ **104.** $(x-1)(6x-5)(2x+1)$ **105.** $(x^2+6)(6x^2-1)$ **106.** $(x^2+5)(7x^2-1)$
107. $y(y^2+1)(y^4-y^2+1)$ **108.** $(y+2)(y^2+y+1)$ **109.** $(x+2y)(x-2y)(x+y)(x-y)$ **110.** $(x+3y)(x-3y)(x+y)(x-y)$
111. $(x-y)^2(x-y+2)(x-y-2)$ **112.** $(x+y)^2(x+y+10)(x+y-10)$ **113.** $(2x-y^2)(x-3y^2)$ **114.** $(3x+2y^2)(x+y^2)$
117. a. $9x^2-16$ **b.** $(3x+4)(3x-4)$ **118. a.** $49x^2-36$ **b.** $(7x+6)(7x-6)$ **119. a.** $x(x+y)-y(x+y)$ **b.** $(x+y)(x-y)$
131. makes sense **138.** $(x^n+4)(x^n+2)$ **139.** $-(x+5)(x-1)$ **141.** $-\dfrac{10}{(x-5)^{3/2}(x+5)^{1/2}}$ **143.** $b=0,3,4,-c(c+4)$, where $c>0$ is an integer
144. $\dfrac{(x+5)(x+1)}{(x+5)(x-5)}=\dfrac{x+1}{x-5}$

Mid-Chapter P Check Point

1. $12x^2-x-35$ **5.** $x+45$ **10.** $2x^3-11x^2+17x-5$ **12.** $18a^2-11ab-10b^2$ **13.** $\{a,c,d,e,f,h\}$ **15.** $5x^2y^3+2xy-y^2$
28. $(7x-1)(x-3)$ **30.** $(x^2+3)(x+5)$ **31.** $(3x-7y)(x+y)$ **32.** $y(4-y)(16+4y+y^2)$ **33.** $2x(5x+1)^2$
35. $\dfrac{(1-x)^2}{x^{3/2}}$ **36.** $\dfrac{(x-3)(x+3)}{(x^2+1)^{1/2}}$

Exercise Set P.6

8. $\dfrac{4}{x-2}, x\neq 2$ **10.** $\dfrac{x-4}{3}, x\neq 4$ **12.** $\dfrac{y-5}{y+4}, y\neq -4,-1$ **14.** $\dfrac{x-7}{x+7}, x\neq 7,-7$ **16.** $\dfrac{1}{2}, x\neq 5,-\dfrac{3}{2}$ **17.** $\dfrac{(x-3)(x+3)}{x(x+4)}, x\neq 0,-4,3$
18. $2, x\neq 2,-2$ **21.** $\dfrac{x^2+2x+4}{3x}, x\neq -2,0,2$ **22.** $\dfrac{1}{x^2-3x+9}, x\neq -3$ **24.** $\dfrac{9}{28}, x\neq -5$ **25.** $\dfrac{(x-2)^2}{x}, x\neq 0,-2,2$
26. $4(x-2), x\neq 2,-2$ **31.** $\dfrac{(x+2)(x+4)}{x-5}, x\neq -6,-3,-1,3,5$ **32.** $\dfrac{7(x+1)^2}{2x^2}, x\neq -5,-1,0,1,5$ **34.** $2, x\neq -\dfrac{4}{3}$ **38.** $\dfrac{x}{x-2}, x\neq 2$
41. $\dfrac{9x+39}{(x+4)(x+5)}, x\neq -4,-5$ **42.** $\dfrac{10x-28}{(x-2)(x-3)}, x\neq 2,3$ **44.** $\dfrac{x+12}{x(x+3)}, x\neq -3,0$ **45.** $\dfrac{3x^2+4}{(x+2)(x-2)}, x\neq -2,2$
46. $\dfrac{2x^2+5x+12}{(x-3)(x+2)}, x\neq 3,-2$ **47.** $\dfrac{2x^2+50}{(x-5)(x+5)}, x\neq -5,5$ **48.** $\dfrac{2x^2+18}{(x-3)(x+3)}, x\neq -3,3$ **49.** $\dfrac{13}{6(x+2)}, x\neq -2$
50. $\dfrac{29}{6(x+4)}, x\neq -4$ **51.** $\dfrac{4x+16}{(x+3)^2}, x\neq -3$ **52.** $\dfrac{20x-6}{(5x-2)(5x+2)}, x\neq -\dfrac{2}{5},\dfrac{2}{5}$ **53.** $\dfrac{x^2-x}{(x+5)(x-2)(x+3)}, x\neq -5,2,-3$
54. $-\dfrac{5x}{(x-6)(x-1)(x+4)}, x\neq 6,1,-4$ **55.** $\dfrac{-x^2-2x+1}{(x+1)(x-1)}, x\neq -1,1$ **56.** $\dfrac{-x^2-2x+3}{(x+2)(x-2)}, x\neq -2,2$ **58.** $\dfrac{x^2+40x-25}{(x+5)(x-4)}, x\neq -5,4$
66. $\dfrac{x-2}{x+1}, x\neq 2,3,-1$ **68.** $\dfrac{2(x+2)}{x+1}, x\neq -1,2,-2,1$ **71.** $-\dfrac{2x+h}{x^2(x+h)^2}, x\neq 0, h\neq 0, x\neq -h$
72. $\dfrac{1}{(x+1)(x+h+1)}, x\neq -1, h\neq 0, x\neq -h-1$ **75.** $-\dfrac{2}{x^2\sqrt{x^2+2}}$ **76.** $\dfrac{5}{\sqrt{(5-x^2)^3}}$ **77.** $\dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$ **78.** $\dfrac{\sqrt{x}-\sqrt{x+3}}{3\sqrt{x}\sqrt{x+3}}$
89. $\dfrac{2d}{a^2+ab+b^2}$ **90.** $\dfrac{a^2+b^2}{a^2+ab+b^2}$ **91. a.** 86.67, 520, 1170; It costs \$86,670,000 to inoculate 40% of the population against this strain of flu, \$520,000,000 to inoculate 80% of the population, and \$1,170,000,000 to inoculate 90% of the population. **c.** The cost increases rapidly; it is impossible to inoculate 100% of the population.
92. $\dfrac{2r_1r_2}{r_1+r_2}$; $34\dfrac{2}{7}$ mph **93. c.** $\dfrac{-33x^2+263x+515}{-60x^2+499x+295}$ **109.** does not make sense **112.** makes sense **119.** $\dfrac{x-y+1}{(x-y)(x-y)}$

Exercise Set P.7

4. $\left\{\dfrac{25}{3}\right\}$ **8.** $\{-19\}$ **16.** $\left\{\dfrac{25}{7}\right\}$ **17. a.** 1 **b.** $\{3\}$ **18. a.** -4 **b.** $\{-3\}$ **19. a.** -1 **b.** $\varnothing$ **20. a.** 2 **b.** $\varnothing$ **29.** $p=\dfrac{T-D}{m}$
30. $M=\dfrac{P-C}{C}$ **31.** $a=\dfrac{2A}{h}-b$ **32.** $b=\dfrac{2A}{h}-a$ **33.** $r=\dfrac{S-P}{Pt}$ **34.** $t=\dfrac{S-P}{Pr}$ **35.** $S=\dfrac{F}{B}+V$ **37.** $I=\dfrac{E}{R+r}$
38. $h=\dfrac{A-2lw}{2l+2w}$ **39.** $f=\dfrac{pq}{p+q}$ **40.** $R_1=\dfrac{RR_2}{R_2-R}$ **41.** $f_1=-\dfrac{ff_2}{f-f_2}$ or $f_1=\dfrac{ff_2}{f_2-f}$ **42.** $f_2=-\dfrac{ff_1}{f-f_1}$ or $f_2=\dfrac{ff_1}{f_1-f}$
49. $\left\{-\dfrac{4}{5},4\right\}$ **50.** $\left\{\dfrac{1}{3},\dfrac{7}{3}\right\}$ **71.** $\{3+2\sqrt{5},3-2\sqrt{5}\}$ **73.** $\{-2+\sqrt{3},-2-\sqrt{3}\}$ **75.** $\{-5,-3\}$ **77.** $\left\{\dfrac{-5+\sqrt{13}}{2},\dfrac{-5-\sqrt{13}}{2}\right\}$
78. $\left\{\dfrac{-5+\sqrt{17}}{2},\dfrac{-5-\sqrt{17}}{2}\right\}$ **79.** $\left\{\dfrac{3+\sqrt{57}}{6},\dfrac{3-\sqrt{57}}{6}\right\}$ **80.** $\left\{\dfrac{-1+\sqrt{41}}{10},\dfrac{-1-\sqrt{41}}{10}\right\}$ **81.** $\left\{\dfrac{1+\sqrt{29}}{4},\dfrac{1-\sqrt{29}}{4}\right\}$
82. $\left\{\dfrac{3+\sqrt{6}}{3},\dfrac{3-\sqrt{6}}{3}\right\}$ **83.** 36; 2 unequal real solutions **84.** -44; no real solutions **85.** 97; 2 unequal real solutions

86. 169; 2 unequal real solutions **87.** 0; 1 real solution **88.** -8; no real solutions **89.** 37; 2 unequal real solutions

90. 40; 2 unequal real solutions **98.** $\left\{\frac{-3+\sqrt{17}}{4}, \frac{-3-\sqrt{17}}{4}\right\}$ **99.** $\left\{\frac{-11+\sqrt{33}}{4}, \frac{-11-\sqrt{33}}{4}\right\}$ **100.** $\left\{\frac{3+\sqrt{65}}{4}, \frac{3-\sqrt{65}}{4}\right\}$

135. $\frac{-2-\sqrt{22}}{2}$ and $\frac{-2+\sqrt{22}}{2}$ **136.** $\frac{4-\sqrt{6}}{2}$ and $\frac{4+\sqrt{6}}{2}$ **140. a.** $C(x) = \frac{x + 0.35(200)}{x + 200}$ **163.** false

Exercise Set P.8

27. 5 m **31.** 13.2 ft **41.** 6 miles per hour **43.** 11 hr **44.** 17 hr **55.** does not make sense **57.** 3 miles, 4 miles, 5 miles
61. \$4000 for the mother; \$8000 for the boy; \$2000 for the girl

Section P.9

Check Point Exercises

1. a. $\{x|-2 \le x < 5\}$

b. $\{x|1 \le x \le 3.5\}$

c. $\{x|x < -1\}$

3. $[-1, \infty)$ or $\{x|x \ge -1\}$

4. $\{x|x < 4\}$ or $(-\infty, 4)$

5. $\{x|x \ge 13\}$ or $[13, \infty)$

6. $[-1, 4)$ or $\{x|-1 \le x < 4\}$

7. $(-3, 7)$ or $\{x|-3 < x < 7\}$

8. $\left\{x \middle| -\frac{11}{5} \le x \le 3\right\}$ or $\left[-\frac{11}{5}, 3\right]$

9. $\{x|x < -4 \text{ or } x > 8\}$ or $(-\infty, -4) \cup (8, \infty)$

Exercise Set P.9

1. $\{x|1 < x \le 6\}$

2. $\{x|-2 < x \le 4\}$

3. $\{x|-5 \le x < 2\}$

4. $\{x|-4 \le x < 3\}$

5. $\{x|-3 \le x \le 1\}$

6. $\{x|-2 \le x \le 5\}$

7. $\{x|x > 2\}$

8. $\{x|x > 3\}$

9. $\{x|x \ge -3\}$

10. $\{x|x \ge -5\}$

11. $\{x|x < 3\}$

12. $\{x|x < 2\}$

13. $\{x|x < 5.5\}$

14. $\{x|x \le 3.5\}$

27. $(-\infty, 3)$

28. $(-\infty, 6)$

29. $\left[\frac{20}{3}, \infty\right)$

30. $[2, \infty)$

31. $(-\infty, -4]$

32. $[-6, \infty)$

33. $\left(-\infty, -\frac{2}{5}\right]$

34. $\left(-\infty, -\frac{53}{6}\right]$

35. $[0, \infty)$

36. $(5, \infty)$

37. $(-\infty, 1)$

38. $(-\infty, -4)$

39. $[6, \infty)$

40. $[2, \infty)$

41. $[-10, \infty)$

42. $[-2, \infty)$

43. $(-\infty, -6)$

44. $(8, \infty)$

45. $[13, \infty)$

46. $\left[-\frac{19}{6}, \infty\right)$

47. $(-\infty, -2)$

48. $\left(-\frac{3}{4}, \infty\right)$

53. $(-5, -2]$ **54.** $\left[\frac{3}{2}, \frac{11}{2}\right)$ **56.** $[-4, 2)$ **64.** $\left[-\frac{19}{3}, 7\right]$ **69.** $(-\infty, -1] \cup [3, \infty)$ **70.** $(-\infty, -7] \cup [1, \infty)$ **71.** $\left(-\infty, \frac{1}{3}\right) \cup (5, \infty)$
72. $\left(-\infty, -\frac{11}{5}\right) \cup (3, \infty)$ **73.** $(-\infty, -5] \cup [3, \infty)$ **74.** $(-\infty, -2] \cup [4, \infty)$ **75.** $(-\infty, -3) \cup (12, \infty)$ **76.** $(-\infty, -8) \cup (16, \infty)$
77. $(-\infty, -1] \cup [3, \infty)$ **78.** $\left(-\infty, -\frac{17}{10}\right] \cup \left[\frac{7}{10}, \infty\right)$ **81.** $(-\infty, -3) \cup (5, \infty)$ **82.** $(-\infty, 2) \cup (8, \infty)$ **83.** $(-\infty, -1] \cup [2, \infty)$
84. $(-\infty, -4] \cup \left[\frac{1}{2}, \infty\right)$ **87.** $\left(-\infty, \frac{1}{3}\right) \cup (1, \infty)$ **88.** $(-\infty, -2) \cup (6, \infty)$ **89.** $\left(-\infty, -\frac{75}{14}\right) \cup \left(\frac{87}{14}, \infty\right)$ **90.** $(-\infty, \infty)$ **91.** $(-\infty, -6] \cup [24, \infty)$
97. $\left(-\infty, -\frac{1}{3}\right] \cup [3, \infty)$ **98.** $\left[-2, \frac{9}{2}\right]$ **101.** intimacy $\geq$ passion or passion $\leq$ intimacy **102.** commitment $\geq$ intimacy or intimacy $\leq$ commitment
103. commitment $>$ passion or passion $<$ commitment **104.** commitment $>$ passion or passion $<$ commitment
109. between 80 and 110 minutes, inclusive **112.** $50 + 0.20x < 20 + 0.50x$; more than 100 mi **113.** $15 + 0.08x < 3 + 0.12x$; more than 300 texts
114. $1800 + 0.03x < 200 + 0.08x$; greater than \$32,000 **115.** $2 + 0.08x < 8 + 0.05x$; 199 checks or fewer **116.** $2x > 10{,}000 + 0.40x$; more than 6250 tapes **117.** $5.50x > 3000 + 3x$; more than 1200 packages **118.** $265 + 65x \leq 2800$; at most 39 bags **119.** $245 + 95x \leq 3000$; at most 29 bags **120.** $\frac{86 + 88 + 92 + 84 + x + x}{6} \geq 90$; at least 95% **121. a.** $\frac{86 + 88 + x}{3} \geq 90$; at least a 96 **b.** $\frac{86 + 88 + x}{3} < 80$; a grade less than 66 **122.** 1.5 to 3.5 hours, inclusive **140.** Because $x > y$, $y - x$ represents a negative number, so when both sides are multiplied by $(y - x)$, the inequality must be reversed. **144.** -5; 0; 3; 4; 3; 0; -5

Chapter P Review Exercises

3. 124 ft **8. a.** $\sqrt{81}$ **b.** $0, \sqrt{81}$ **c.** $-17, 0, \sqrt{81}$ **d.** $-17, -\frac{9}{13}, 0, 0.75, \sqrt{81}$ **e.** $\sqrt{2}, \pi$ **f.** $-17, -\frac{9}{13}, 0, 0.75, \sqrt{2}, \pi, \sqrt{81}$
11. $\sqrt{17} - 3$ **12.** $|4 - (-17)|; 21$ **13.** commutative property of addition **14.** associative property of multiplication **17.** commutative property of multiplication **18.** commutative property of addition **36.** 390,000 **46.** $4x\sqrt{3}$ **73.** $8x^3 + 10x^2 - 20x - 4$; degree 3
74. $8x^4 - 5x^3 + 6$; degree 4 **75.** $12x^3 + x^2 - 21x + 10$ **76.** $6x^2 - 7x - 5$ **80.** $8x^3 + 12x^2 + 6x + 1$ **82.** $-x^2 - 17xy - 3y^2$; degree 2
83. $24x^3y^2 + x^2y - 12x^2 + 4$; degree 5 **84.** $3x^2 + 16xy - 35y^2$ **85.** $9x^2 - 30xy + 25y^2$ **86.** $9x^4 + 12x^2y + 4y^2$ **87.** $49x^2 - 16y^2$
90. $(x - 4)(x - 7)$ **91.** $(3x + 1)(5x - 2)$ **94.** $3x^2(x - 5)(x + 2)$ **96.** $(x + 3)(x - 3)^2$ **98.** $(x^2 + 4)(x + 2)(x - 2)$
101. $3x^2(x - 2)(x + 2)$ **102.** $(3x - 5)(9x^2 + 15x + 25)$ **103.** $x(x - 1)(x + 1)(x^2 + 1)$ **104.** $(x^2 - 2)(x + 5)$
107. $(x + 2)(x - 2)(x^2 + 3)^{1/2}(-x^4 + x^2 + 13)$ **109.** $x^2, x \neq -2$ **110.** $\frac{x - 3}{x - 6}, x \neq -6, 6$ **111.** $\frac{x}{x + 2}, x \neq -2$
112. $\frac{(x + 3)^3}{(x - 2)^2(x + 2)}, x \neq 2, -2$ **113.** $\frac{2}{x(x + 1)}, x \neq 0, 1, -1, -\frac{1}{3}$ **114.** $\frac{x + 3}{x - 4}, x \neq -3, 4, 2, 8$ **115.** $\frac{1}{x - 3}, x \neq 3, -3$
116. $\frac{4x(x - 1)}{(x + 2)(x - 2)}, x \neq 2, -2$ **117.** $\frac{2x^2 - 3}{(x - 3)(x + 3)(x - 2)}, x \neq 3, -3, 2$ **118.** $\frac{11x^2 - x - 11}{(2x - 1)(x + 3)(3x + 2)}, x \neq \frac{1}{2}, -3, -\frac{2}{3}$ **119.** $\frac{3}{x}, x \neq 0, 2$
120. $\frac{3x}{x - 4}, x \neq 0, 4, -4$ **121.** $\frac{3x + 8}{3x + 10}, x \neq -3, -\frac{10}{3}$ **128.** $\{-2, 1\}$ **129.** $\left\{\frac{1}{2}, 5\right\}$ **130.** $\left\{-2, \frac{10}{3}\right\}$ **131.** $\left\{\frac{7 + \sqrt{37}}{6}, \frac{7 - \sqrt{37}}{6}\right\}$
133. $\{3 \pm 2\sqrt{6}\}$ **137.** $g = \frac{s - vt}{t^2}$ **138.** $P = \frac{A}{1 + rT}$ **139.** no real solutions **140.** one repeated real solution
144. \$10,000 in sales

151. $\{x | -3 \leq x < 5\}$

152. $\{x | x > -2\}$

153. $\{x | x \leq 0\}$

158. $[-2, \infty)$

159. $\left[\frac{3}{5}, \infty\right)$

160. $\left(-\infty, -\frac{21}{2}\right)$

161. $(-3, \infty)$

162. $(-\infty, -2]$

163. $(2, 3]$

164. $[-9, 6]$

165. $(-\infty, -6) \cup (0, \infty)$

166. $(-\infty, -3] \cup [-2, \infty)$

167. $(-\infty, -5] \cup [1, \infty)$

168. no more than 80 miles per day

Chapter P Test

10. $\frac{x + 3}{x - 2}, x \neq 2, 1$ **11.** 2.5×10^1 **12.** $2x^3 - 13x^2 + 26x - 15$ **13.** $25x^2 + 30xy + 9y^2$ **14.** $\frac{2(x + 3)}{x + 1}, x \neq 3, -1, -4, -3$
15. $\frac{x^2 + 2x + 15}{(x + 3)(x - 3)}, x \neq 3, -3$ **16.** $\frac{11}{(x - 3)(x - 4)}, x \neq 3, 4$ **20.** $(x^2 + 3)(x + 2)$ **24.** $(x + 5 + 3y)(x + 5 - 3y)$
26. $-7, -\frac{4}{5}, 0, 0.25, \sqrt{4}, \frac{22}{7}$ **31.** 1.32×10^{10} **37.** $\left\{\frac{1 - 5\sqrt{3}}{3}, \frac{1 + 5\sqrt{3}}{3}\right\}$

44. $(-\infty, 12]$

45. $\left[\frac{21}{8}, \infty\right)$

46. $\left[-7, \frac{13}{2}\right)$

47. $\left(-\infty, -\frac{5}{3}\right] \cup \left[\frac{1}{3}, \infty\right)$

CHAPTER 1

Section 1.1

Check Point Exercises

1.

2.

$y = 4 - x$

3.

$y = |x + 1|$

4. minimum x-value: -100; maximum x-value: 100; distance between tick marks on x-axis: 50; minimum y-value: -100; maximum y-value: 100; distance between tick marks on y-axis: 10

Exercise Set 1.1

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.

13.

$y = x^2 - 2$

14.

$y = x^2 + 2$

15.

$y = x - 2$

16.

$y = x + 2$

17.

$y = 2x + 1$

18.

$y = 2x - 4$

19.

$y = -\frac{1}{2}x$

20.

$y = -\frac{1}{2}x + 2$

21.

$y = 2|x|$

22.

$y = -2|x|$

23.

$y = |x| + 1$

24.

$y = |x| - 1$

25.

$y = 9 - x^2$

26.

$y = -x^2$

27.

$y = x^3$

28.

$y = x^3 - 1$

39. $(-2, 4)$ and $(1, 1)$

47. $y = 2x + 4$

$y = 2x + 4$

48. $y = 4 - 2x$

$y = 4 - 2x$

49. $y = 3 - x^2$

$y = 3 - x^2$

50. $y = x^2 + 2$

$y = x^2 + 2$

51.

$y = 5$

52.

$y = -1$

53.

54.

55. b. 19.5%; underestimates by 0.5 **d.** 46.5%; It's less than the estimate, although answers may vary. **e.** 1990; 14%
56. b. 51%; overestimates by 1 **d.** 21%; It's less than the estimate, although answers may vary. **e.** 1980; 72% **67.** makes sense
68. does not make sense

88.

89. c. all real numbers **d.** all real numbers greater than or equal to 1

Section 1.2

Check Point Exercises

5.

Exercise Set 1.2

1. function; $\{1, 3, 5\}$; $\{2, 4, 5\}$ **2.** function; $\{4, 6, 8\}$; $\{5, 7, 8\}$ **7.** function; $\{-3, -2, -1, 0\}$; $\{-3, -2, -1, 0\}$
8. function; $\{-7, -5, -3, 0\}$; $\{-7, -5, -3, 0\}$ **27. c.** $-4x + 5$ **28. c.** $-3x + 7$ **29. b.** $x^2 + 12x + 38$ **c.** $x^2 - 2x + 3$
30. b. $x^2 - 6x - 19$ **c.** $x^2 + 10x - 3$ **33. c.** $\sqrt{x} + 3$ **34. c.** $\sqrt{2x} - 6$ **36. c.** $\frac{-4x^3 + 1}{-x^3}$ or $\frac{4x^3 - 1}{x^3}$ **38. c.** $\begin{cases} 1, & \text{if } x < -6 \\ -1, & \text{if } x > -6 \end{cases}$

39.

The graph of g is the graph of f shifted up by 3 units.

40.

The graph of g is the graph of f shifted down by 4 units.

41.

The graph of g is the graph of f shifted down by 1 unit.

42.

The graph of g is the graph of f shifted up by 3 units.

43.

The graph of g is the graph of f shifted up by 1 unit.

44.

The graph of g is the graph of f shifted down by 2 units.

45.

The graph of g is the graph of f shifted down by 2 units.

46.

The graph of g is the graph of f shifted up by 1 unit.

47.

The graph of g is the graph of f shifted up by 2 units.

48.

The graph of g is the graph of f shifted down by 1 unit.

49.

The graph of g is the graph of f shifted up by 2 units.

50.

The graph of g is the graph of f shifted up by 5 units.

51.

The graph of g is the graph of f shifted down by 1 unit.

52.

The graph of g is the graph of f shifted up by 2 units.

53.

The graph of g is the graph of f shifted to the right by 1 unit.

54.

The graph of g is the graph of f shifted to the left by 2 units.

77. a. $(-\infty, \infty)$ **b.** $[-4, \infty)$ **c.** -3 and 1 **d.** -3 **e.** $f(-2) = -3$ and $f(2) = 5$ **78. a.** $(-\infty, \infty)$ **b.** $(-\infty, 4]$ **c.** -3 and 1 **d.** 3 **e.** $f(-2) = 3$ and $f(2) = -5$ **79. a.** $(-\infty, \infty)$ **b.** $[1, \infty)$ **c.** none **d.** 1 **e.** $f(-1) = 2$ and $f(3) = 4$ **80. a.** $(-\infty, \infty)$ **b.** $[0, \infty)$ **c.** -1 **d.** 1 **e.** $f(-4) = 3$ and $f(3) = 4$ **81. a.** $[0, 5)$ **b.** $[-1, 5)$ **c.** 2 **d.** -1 **e.** $f(3) = 1$ **82. a.** $(-6, 0]$ **b.** $[-3, 4)$ **c.** -4 **d.** -3 **e.** $f(-5) = 2$ **83. a.** $[0, \infty)$ **b.** $[1, \infty)$ **c.** none **d.** 1 **e.** $f(4) = 3$ **84. a.** $[-1, \infty)$ **b.** $[0, \infty)$ **c.** -1 **d.** 1 **e.** $f(3) = 2$ **85. a.** $[-2, 6]$ **b.** $[-2, 6]$ **c.** 4 **d.** 4 **e.** $f(-1) = 5$ **86. a.** $[-3, 2]$ **b.** $[-5, 5]$ **c.** $-\frac{1}{2}$ **d.** 1 **e.** $f(-2) = -3$ **87. a.** $(-\infty, \infty)$ **b.** $(-\infty, -2]$ **c.** none **d.** -2 **e.** $f(-4) = -5$ and $f(4) = -2$ **88. a.** $(-\infty, \infty)$ **b.** $[0, \infty)$ **c.** $\{x|x \le 0\}$ **d.** 0 **e.** $f(-2) = 0$ and $f(2) = 4$ **91. a.** $\{-5, -2, 0, 1, 3\}$ **b.** $\{2\}$ **c.** none **d.** 2 **e.** $f(-5) + f(3) = 4$ **92. a.** $\{-5, -2, 0, 1, 4\}$ **b.** $\{-2\}$ **c.** none **d.** -2 **e.** $f(-5) + f(4) = -4$ **93.** -2; 10 **94.** -8; 76

99. a. {(Philippines, 12), (Spain, 13), (Italy, 14), (Germany, 14), (Russia, 16)} **c.** {(12, Philippines), (13, Spain), (14, Italy), (14, Germany), (16, Russia)} **d.** no; 14 in the domain corresponds to two members of the range, Italy and Germany. **100. a.** {(Philippines, 18), (Spain, 18), (Italy, 16), (Germany, 16), (Russia, 16)} **c.** {(18, Philippines), (18, Spain), (16, Italy), (16, Germany), (16, Russia)} **d.** no; 18 in the domain corresponds to two members of the range, Philippines and Spain, and 16 in the domain corresponds to three members of the range, Italy, Germany, and Russia.
101. a. 83; The chance that a 60-year-old will survive to age 70 is 83%. **b.** 76; The chance that a 60-year-old will survive to age 70 is 76%.
102. a. 25; The chance that a 60-year-old will survive to age 90 is 25%. **b.** 10; The chance that a 60-year-old will survive to age 90 is 10%.
103. a. 81; In 2010, the wage gap was 81%.; (30, 81) **104. a.** 69; In 1990, the wage gap was 69%.; (10, 69) **105.** $C = 100{,}000 + 100x$, where x is the number of bicycles produced; $C(90) = 109{,}000$; It costs \$109,000 to produce 90 bicycles. **106.** $V = 22{,}500 - 3200x$; $V(3) = 12{,}900$; The value of the car after 3 years is \$12,900. **107.** $T = \frac{40}{x} + \frac{40}{x + 30}$, where x is the rate on the outgoing trip; $T(30) = 2$; It takes 2 hours, traveling 30 mph outgoing and 60 mph returning. **108.** $S = 0.1x + 0.6(50 - x)$, where x is the number of milliliters of the 10% solution; $S(30) = 15$; There are 15 milliliters of sodium iodine in the mixture when 30 milliliters of the 10% solution are used. **118.** makes sense **120.** does not make sense **121.** does not make sense **127.** Answers will vary; an example is $\{(1, 1), (2, 1)\}$. **128.** $f(2) = 6$; $f(3) = 9$; $f(4) = 12$; no
129. 36; For 100 calling minutes, the monthly cost is \$36. **130.** $f(x) = x + 2,\ x \le 1$

131. $4xh + 2h^2 + 3h$

Section 1.3

Check Point Exercises

4.

$f(x) = \begin{cases} 3 & \text{if } x \le -1 \\ x - 2 & \text{if } x > -1 \end{cases}$

Exercise Set 1.3

1. a. $(-1, \infty)$ **b.** $(-\infty, -1)$ **c.** none **2. a.** $(-\infty, -1)$ **b.** $(-1, \infty)$ **c.** none **3. a.** $(0, \infty)$ **b.** none **c.** none **4. a.** $(-1, \infty)$ **b.** none **c.** none **5. a.** none **b.** $(-2, 6)$ **c.** none **6. a.** $(-3, 2)$ **b.** none **c.** none **7. a.** $(-\infty, -1)$ **b.** none **c.** $(-1, \infty)$ **8. a.** $(0, \infty)$ **b.** none **c.** $(-\infty, 0)$ **9. a.** $(-\infty, 0)$ or $(1.5, 3)$ **b.** $(0, 1.5)$ or $(3, \infty)$ **c.** none **10. a.** $(-5, -4)$ or $(-2, 0)$ or $(2, 4)$ **b.** $(-4, -2)$ or $(0, 2)$ or $(4, 5)$ **c.** none **11. a.** $(-2, 4)$ **b.** none **c.** $(-\infty, -2)$ or $(4, \infty)$ **12. a.** none **b.** $(-4, 2)$ **c.** $(-\infty, -4)$ or $(2, \infty)$

43. a.

$f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$

44. a.

$f(x) = \begin{cases} x & \text{if } x < 0 \\ -x & \text{if } x \ge 0 \end{cases}$

45. a.

$f(x) = \begin{cases} 2x & \text{if } x \le 0 \\ 2 & \text{if } x > 0 \end{cases}$

46. a.

$f(x) = \begin{cases} \frac{1}{2}x & \text{if } x \le 0 \\ 3 & \text{if } x > 0 \end{cases}$

47. a.

$f(x) = \begin{cases} x + 3 \text{ if } x < -2 \\ x - 3 \text{ if } x \geq -2 \end{cases}$

48. a.

$f(x) = \begin{cases} x + 2 \text{ if } x < -3 \\ x - 2 \text{ if } x \geq -3 \end{cases}$

49. a.

$f(x) = \begin{cases} 3 \text{ if } x \leq -1 \\ -3 \text{ if } x > -1 \end{cases}$

50. a.

$f(x) = \begin{cases} 4 \text{ if } x \leq -1 \\ -4 \text{ if } x > -1 \end{cases}$

51. a.

$f(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x < 1 \\ 2x - 1 \text{ if } x \geq 1 \end{cases}$

52. a.

$f(x) = \begin{cases} -\frac{1}{2}x^2 \text{ if } x < 1 \\ 2x + 1 \text{ if } x \geq 1 \end{cases}$

53. a.

$f(x) = \begin{cases} 0 \text{ if } x < -4 \\ -x \text{ if } -4 \leq x < 0 \\ x^2 \text{ if } x \geq 0 \end{cases}$

54. a.

$f(x) = \begin{cases} 0 & \text{if } x < -3 \\ -x & \text{if } -3 \leq x < 0 \\ x^2 - 1 \text{ if } x \geq 0 \end{cases}$

61. $2x + h - 4, h \neq 0$ **62.** $2x + h - 5, h \neq 0$ **63.** $4x + 2h + 1, h \neq 0$ **64.** $6x + 3h + 1, h \neq 0$ **65.** $-2x - h + 2, h \neq 0$
66. $-2x - h - 3, h \neq 0$ **67.** $-4x - 2h + 5, h \neq 0$ **68.** $-6x - 3h + 2, h \neq 0$ **69.** $-4x - 2h - 1, h \neq 0$ **70.** $-6x - 3h + 1, h \neq 0$

75. $\dfrac{1}{\sqrt{x + h} + \sqrt{x}}, h \neq 0$ **76.** $\dfrac{1}{\sqrt{x + h - 1} + \sqrt{x - 1}}, h \neq 0$

79.

80.

81. $C(t) = \begin{cases} 50 & \text{if } 0 \leq t \leq 400 \\ 50 + 0.30(t - 400) & \text{if } t > 400 \end{cases}$

82. $C(t) = \begin{cases} 60 & \text{if } 0 \leq t \leq 450 \\ 60 + 0.35(t - 450) & \text{if } t > 450 \end{cases}$

83. increasing: (25, 55); decreasing: (55, 75) **84.** increasing: (25, 65); decreasing: (65, 75)
85. 55 years old; 38% **86.** 65 years old; 26% **87.** domain: [25, 75]; range: [34, 38]
88. domain: [25, 75]; range: [23, 26] **89.** men **91.** 2575; A single taxpayer with taxable income of \$20,000 owes \$2575. **92.** 8625; A single taxpayer with taxable income of \$50,000 owes \$8625.
93. $42{,}449 + 0.33(x - 174{,}400)$ **94.** $110{,}016.50 + 0.35(x - 379{,}150)$ **95.** 0.85; It costs \$0.85 to mail a 3-ounce first-class letter. **96.** 1.05; It costs \$1.05 to mail a 3.5-ounce first-class letter.
97. \$0.65 **98.** \$0.65

99.

106.

The number of doctor visits decreases during childhood and then increases as you get older. The minimum is (20.29, 3.99), which means that the minimum number of annual doctor visits, about 4, occurs at around age 20.

107.

increasing: $(-\infty, 1)$ or $(3, \infty)$
decreasing: $(1, 3)$

108.

increasing: $(-2, 0)$ or $(2, \infty)$
decreasing: $(-\infty, -2)$ or $(0, 2)$

109.

increasing: $(2, \infty)$
decreasing: $(-\infty, -2)$
constant: $(-2, 2)$

110.

increasing: $(1, \infty)$
decreasing: $(-\infty, 1)$

111.

increasing: $(0, \infty)$
decreasing: $(-\infty, 0)$

112.

increasing: $(-\infty, 0)$
decreasing: $(0, \infty)$

113. a.

b.

c. increasing: $(0, \infty)$; decreasing: $(-\infty, 0)$

d. $f(x) = x^n$ is increasing for $(-\infty, \infty)$ when n is positive and odd.

e.

As n increases the steepness increases.

120. a. h is even if both f and g are even or if both f and g are odd. **b.** h is odd if f is odd and g is even or if f is even and g is odd. **122.** 3

Section 1.4

Check Point Exercises

4.

$f(x) = \frac{3}{5}x + 1$

5.

6.

7.

$3x + 6y - 12 = 0$

8.

$3x - 2y - 6 = 0$

Exercise Set 1.4

5. 0; horizontal **6.** 0; horizontal **9.** undefined; vertical **10.** undefined; vertical **11.** $y - 5 = 2(x - 3); y = 2x - 1$

12. $y - 3 = 4(x - 1); y = 4x - 1$ **13.** $y - 5 = 6(x + 2); y = 6x + 17$ **14.** $y + 1 = 8(x - 4); y = 8x - 33$

15. $y + 3 = -3(x + 2); y = -3x - 9$ **16.** $y + 2 = -5(x + 4); y = -5x - 22$ **17.** $y - 0 = -4(x + 4); y = -4x - 16$

18. $y + 3 = -2(x - 0); y = -2x - 3$ **19.** $y + 2 = -1\left(x + \frac{1}{2}\right); y = -x - \frac{5}{2}$ **20.** $y + \frac{1}{4} = -1(x + 4); y = -x - \frac{17}{4}$

21. $y - 0 = \frac{1}{2}(x - 0); y = \frac{1}{2}x$ **22.** $y - 0 = \frac{1}{3}(x - 0); y = \frac{1}{3}x$ **23.** $y + 2 = -\frac{2}{3}(x - 6); y = -\frac{2}{3}x + 2$

24. $y + 4 = -\frac{3}{5}(x - 10); y = -\frac{3}{5}x + 2$ **25.** using $(1, 2)$, $y - 2 = 2(x - 1); y = 2x$ **26.** using $(3, 5)$, $y - 5 = 2(x - 3); y = 2x - 1$

27. using $(-3, 0)$, $y - 0 = 1(x + 3); y = x + 3$ **28.** using $(-2, 0)$, $y - 0 = 1(x + 2); y = x + 2$ **29.** using $(-3, -1)$, $y + 1 = 1(x + 3); y = x + 2$

30. using $(-2, -4)$, $y + 4 = 1(x + 2); y = x - 2$ **31.** using $(-3, -2)$, $y + 2 = \frac{4}{3}(x + 3); y = \frac{4}{3}x + 2$

32. using $(-3, 6)$, $y - 6 = -\frac{4}{3}(x + 3); y = -\frac{4}{3}x + 2$ **33.** using $(-3, -1)$, $y + 1 = 0(x + 3); y = -1$

34. using $(-2, -5)$, $y + 5 = 0(x + 2); y = -5$ **35.** using $(2, 4)$, $y - 4 = 1(x - 2); y = x + 2$

36. using $(1, -3)$, $y + 3 = -\frac{3}{2}(x - 1); y = -\frac{3}{2}x - \frac{3}{2}$ **37.** using $(0, 4)$, $y - 4 = 8(x - 0); y = 8x + 4$

38. using $(4, 0)$, $y - 0 = \frac{1}{2}(x - 4); y = \frac{1}{2}x - 2$

39. $m = 2; b = 1$

$y = 2x + 1$

40. $m = 3; b = 2$

$y = 3x + 2$

41. $m = -2; b = 1$

$f(x) = -2x + 1$

42. $m = -3; b = 2$

$f(x) = -3x + 2$

43. $m = \frac{3}{4}; b = -2$

$f(x) = \frac{3}{4}x - 2$

44. $m = \frac{3}{4}; b = -3$

$f(x) = \frac{3}{4}x - 3$

45. $m = -\frac{3}{5}; b = 7$

$y = -\frac{3}{5}x + 7$

46. $m = -\frac{2}{5}; b = 6$

$y = -\frac{2}{5}x + 6$

47. $m = -\frac{1}{2}; b = 0$

$g(x) = -\frac{1}{2}x$

48. $m = -\frac{1}{3}; b = 0$

$g(x) = -\frac{1}{3}x$

49. $y = -2$

50. $y = 4$

51. $x = -3$

52. $x = 5$

53. $y = 0$

54. $x = 0$

55. $f(x) = 1$

56. $f(x) = 3$

57. $3x - 18 = 0$

58. $3x + 12 = 0$

59. a. $y = -3x + 5$

b. $m = -3; b = 5$

c.

$3x + y - 5 = 0$

60. a. $y = -4x + 6$

b. $m = -4; b = 6$

c.

$4x + y - 6 = 0$

61. a. $y = -\frac{2}{3}x + 6$

b. $m = -\frac{2}{3}; b = 6$

c.

$2x + 3y - 18 = 0$

62. a. $y = -\frac{2}{3}x - 2$

b. $m = -\frac{2}{3}; b = -2$

c.

$4x + 6y + 12 = 0$

63. a. $y = 2x - 3$

b. $m = 2; b = -3$

c.

$8x - 4y - 12 = 0$

64. a. $y = \frac{6}{5}x - 4$

b. $m = \frac{6}{5}; b = -4$

c.

$6x - 5y - 20 = 0$

65. a. $y = 3$

b. $m = 0; b = 3$

c.

66. a. $y = -7$

b. $m = 0; b = -7$

c.

$4y + 28 = 0$

67.

$6x - 2y - 12 = 0$

68.

$6x - 9y - 18 = 0$

69.

70.

71.

$8x - 2y + 12 = 0$

72.

$6x - 3y + 15 = 0$

74. $m = -\frac{b}{a}$; falls

75. undefined slope; vertical

76. $m = \frac{a}{b}$; rises

77. $m = -\frac{A}{B}; b = \frac{C}{B}$

78. $m = \frac{A}{B}; b = \frac{C}{B}$

81. $3x - 4f(x) - 6 = 0$

82.

$6x - 5f(x) - 20 = 0$

87. a. $y - 31.1 = 0.78(x - 10)$ or $y - 38.9 = 0.78(x - 20)$

c. 54.5%

88. a. $y - 45.2 = 0.65(x - 10)$ or $y - 51.7 = 0.65(x - 20)$

89. a & b. **Life Expectancy for United States Males, by Year of Birth**; $E(x) = 0.215x + 65.7$

90. Answers will vary due to rounding.

a & b. **Life Expectancy for United States Females, by Year of Birth**; $E(x) = 0.17x + 72.9$ or $E(x) = 0.17x + 73$

c. 83.1 yr or 83.2 yr

91. Answers will vary; an example is $y = -2.3x + 255$, where x is the percentage of adult females who are literate and y is under-five mortality per thousand.; Predictions will vary.

100.

101.

102.

103.

104. a.

x	y
2.5	211
3.9	167
⋮	⋮
2.7	172

b.

c. $a = -22.96876741$
$b = 260.5633751$
$r = -0.8428126855$

d.

106. does not make sense **108.** makes sense **110.** false **113.** coefficient of x: -6; coefficient of y: 3 **114.** coefficient of x: 1; coefficient of y: -2

Exercise Set 1.5

9. $y - 2 = \frac{2}{3}(x + 2); 2x - 3y + 10 = 0$ **10.** $y - 3 = \frac{3}{2}(x + 1); 3x - 2y + 9 = 0$ **11.** $y + 7 = -2(x - 4); 2x + y - 1 = 0$

12. $y + 9 = 7(x - 5); 7x - y - 44 = 0$ **19. a.** 70 ft/sec **b.** 65 ft/sec **c.** 60.1 ft/sec **d.** 60.01 ft/sec **20. a.** 84 ft/sec **b.** 78 ft/sec **c.** 72.12 ft/sec **d.** 72.012 ft/sec **29.** 137; There was an average increase of approximately 137 discharges per year. **30.** -130; There was an average decrease of approximately 130 discharges per year. **31. b.** overestimates by 5 discharges per year **32. b.** underestimates by 34 discharges per year

39. b.

; no **c.**

; The lines now appear to be perpendicular. **44.** $\frac{B}{A}$

46. a.

b.

c. The graph in part (b) is the graph in part (a) shifted down 4 units.

47. a.

b.

c. The graph in part (b) is the graph in part (a) shifted to the right 2 units.

48. a.

b.

c. The graph in part (b) is the graph in part (a) reflected across the y-axis.

Mid-Chapter 1 Check Point

1. not a function; domain: $\{1, 2\}$; range: $\{-6, 4, 6\}$ **2.** function; domain: $\{0, 2, 3\}$; range: $\{1, 4\}$ **3.** function; domain: $\{x|-2 \le x < 2\}$ or $[-2, 2)$; range: $\{y|0 \le y \le 3\}$ or $[0, 3]$ **4.** not a function; domain: $\{x|-3 < x \le 4\}$ or $(-3, 4]$; range: $\{y|-1 \le y \le 2\}$ or $[-1, 2]$ **5.** not a function; domain: $\{-2, -1, 0, 1, 2\}$; range: $\{-2, -1, 1, 3\}$ **6.** function; domain: $\{x|x \le 1\}$ or $(-\infty, 1]$; range: $\{y|y \ge -1\}$ or $[-1, \infty)$ **9.** No vertical line intersects the graph in more than one point.

25. $y = -2x$

26. $y = -2$

27. $x + y = -2$

28. $y = \frac{1}{3}x - 2$

29. $x = 3.5$

30.

$4x - 2y = 8$

31.

$f(x) = x^2 - 4$

32.

$f(x) = x - 4$

33.

$f(x) = |x| - 4$

34.

$5y = -3x$

35.

$5y = 20$

36.

$f(x) = \begin{cases} -1 \text{ if } x \le 0 \\ 2x + 1 \text{ if } x > 0 \end{cases}$

37. a. $f(-x) = -2x^2 - x - 5$; neither

Section 1.6

Check Point Exercises

1.

2.

3.

4.

5.

6.

7. a.

$g(x) = f(2x)$

b.

$h(x) = f\left(\frac{1}{2}x\right)$

8.

$y = -\frac{1}{3}f(x + 1) - 2$

9.

$g(x) = 2(x - 1)^2 + 3$

Exercise Set 1.6

1.

$g(x) = f(x) + 1$

2.

$g(x) = f(x) - 1$

3.

$g(x) = f(x + 1)$

4.

$g(x) = f(x - 1)$

5.

$g(x) = f(x - 1) - 2$

6.

$g(x) = f(x + 1) + 2$

7.

$g(x) = f(-x)$

8.

$g(x) = -f(x)$

9.

$g(x) = -f(x) + 3$

10.

$g(x) = f(-x) + 3$

11.

$g(x) = \frac{1}{2}f(x)$

12.

$g(x) = 2f(x)$

13. $g(x) = f\left(\frac{1}{2}x\right)$

14. $g(x) = f(2x)$

15.

$g(x) = -f\left(\frac{1}{2}x\right) + 1$

16.

$g(x) = -f(2x) - 1$

17.

$g(x) = f(x) - 1$

18.

$g(x) = f(x) + 1$

19.

$g(x) = f(x - 1)$

20.

$g(x) = f(x + 1)$

21.

$g(x) = f(x - 1) + 2$

22.

$g(x) = f(x + 1) - 2$

23.

$g(x) = -f(x)$

24.

$g(x) = f(-x)$

25.

$g(x) = f(-x) + 1$

26.

$g(x) = -f(x) + 1$

27.

$g(x) = 2f(x)$

28.

$g(x) = \frac{1}{2}f(x)$

29.

$g(x) = f(2x)$

30.

$g(x) = f\left(\frac{1}{2}x\right)$

31.

$g(x) = 2f(x + 2) + 1$

32.

$g(x) = 2f(x + 2) - 1$

33.

$g(x) = f(x) + 2$

34.

$g(x) = f(x) - 2$

35.

$g(x) = f(x + 2)$

36.

$g(x) = f(x - 2)$

37.

$g(x) = -f(x + 2)$

38.

$g(x) = -f(x - 2)$

39.

$g(x) = -\frac{1}{2}f(x + 2)$

40.

$g(x) = -\frac{1}{2}f(x - 2)$

41.

$g(x) = -\frac{1}{2}f(x + 2) - 2$

42.

$g(x) = -\frac{1}{2}f(x - 2) + 2$

43.

$g(x) = \frac{1}{2}f(2x)$

44.

$g(x) = 2f\left(\frac{1}{2}x\right)$

45.

$g(x) = f(x - 1) - 1$

46.

$g(x) = f(x + 1) + 1$

47.

$g(x) = -f(x - 1) + 1$

48.

$g(x) = -f(x + 1) - 1$

49.

$g(x) = 2f\left(\frac{1}{2}x\right)$

50.

$g(x) = \frac{1}{2}f(2x)$

51.

$g(x) = \frac{1}{2}f(x + 1)$

52.

$g(x) = 2f(x - 1)$

53.

54.

55.

56.

57.

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59.

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61.

62.

63.

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100.

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111.

112.

113.

114.

115.

116.

117.

118.

119.

$g(x) = 2 \text{ int } (x + 1)$

120.

$g(x) = 3 \text{ int } (x - 1)$

121.

$h(x) = \text{int } (-x) + 1$

122.

$h(x) = \text{int } (-x) - 1$

123. $y = \sqrt{x - 2}$ **124.** $y = -x^3 + 2$ **125.** $y = (x + 1)^2 - 4$
126. $y = \sqrt{x - 2} + 1$ **127. a.** First, vertically stretch the graph of $f(x) = \sqrt{x}$ by the factor 2.9; then, shift the result up 20.1 units. **d.** 0.2 in. per month; This is a much smaller rate of change; The graph is not as steep between 50 and 60 as it is between 0 and 10.
128. a. First, vertically stretch the graph of $f(x) = \sqrt{x}$ by the factor 3.1; then, shift the result up 19 units. **d.** 0.2 in. per month; This is a much smaller rate of change; The graph is not as steep between 50 and 60 as it is between 0 and 10.

135. a.

b.

136. a.

b.

137. makes sense **138.** makes sense

Exercise Set 1.7

3. $(-\infty, 4) \cup (4, \infty)$ **4.** $(-\infty, -5) \cup (-5, \infty)$ **5.** $(-\infty, \infty)$ **6.** $(-\infty, \infty)$ **7.** $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$ **8.** $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$
9. $(-\infty, -7) \cup (-7, 9) \cup (9, \infty)$ **10.** $(-\infty, -8) \cup (-8, 10) \cup (10, \infty)$ **13.** $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$ **14.** $(-\infty, 0) \cup (0, 4) \cup (4, \infty)$
15. $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$ **16.** $(-\infty, 2) \cup \left(2, \frac{10}{3}\right) \cup \left(\frac{10}{3}, \infty\right)$ **27.** $[2, 5) \cup (5, \infty)$ **28.** $[3, 6) \cup (6, \infty)$

31. $(f + g)(x) = 3x + 2$; domain: $(-\infty, \infty)$; $(f - g)(x) = x + 4$; domain: $(-\infty, \infty)$; $(fg)(x) = 2x^2 + x - 3$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{2x + 3}{x - 1}$; domain: $(-\infty, 1) \cup (1, \infty)$ **32.** $(f + g)(x) = 4x - 2$; domain: $(-\infty, \infty)$; $(f - g)(x) = 2x - 6$; domain: $(-\infty, \infty)$; $(fg)(x) = 3x^2 + 2x - 8$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{3x - 4}{x + 2}$; domain: $(-\infty, -2) \cup (-2, \infty)$ **33.** $(f + g)(x) = 3x^2 + x - 5$; domain: $(-\infty, \infty)$; $(f - g)(x) = -3x^2 + x - 5$; domain: $(-\infty, \infty)$; $(fg)(x) = 3x^3 - 15x^2$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{x - 5}{3x^2}$; domain: $(-\infty, 0) \cup (0, \infty)$ **34.** $(f + g)(x) = 5x^2 + x - 6$; domain: $(-\infty, \infty)$; $(f - g)(x) = -5x^2 + x - 6$; domain: $(-\infty, \infty)$; $(fg)(x) = 5x^3 - 30x^2$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{x - 6}{5x^2}$; domain: $(-\infty, 0) \cup (0, \infty)$

35. $(f + g)(x) = 2x^2 - 2$; domain: $(-\infty, \infty)$; $(f - g)(x) = 2x^2 - 2x - 4$; domain: $(-\infty, \infty)$; $(fg)(x) = 2x^3 + x^2 - 4x - 3$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = 2x - 3$; domain: $(-\infty, -1) \cup (-1, \infty)$ **36.** $(f + g)(x) = 6x^2 - 2$; domain: $(-\infty, \infty)$; $(f - g)(x) = 6x^2 - 2x$; domain: $(-\infty, \infty)$; $(fg)(x) = 6x^3 - 7x^2 + 1$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{6x^2 - x - 1}{x - 1}$; domain: $(-\infty, 1) \cup (1, \infty)$ **37.** $(f + g)(x) = 2x - 12$; domain: $(-\infty, \infty)$; $(f - g)(x) = -2x^2 - 2x + 18$; domain: $(-\infty, \infty)$; $(fg)(x) = -x^4 - 2x^3 + 18x^2 + 6x - 45$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{3 - x^2}{x^2 + 2x - 15}$; domain: $(-\infty, -5) \cup (-5, 3) \cup (3, \infty)$ **38.** $(f + g)(x) = 4x - 7$; domain: $(-\infty, \infty)$; $(f - g)(x) = -2x^2 - 4x + 17$; domain: $(-\infty, \infty)$; $(fg)(x) = -x^4 - 4x^3 + 17x^2 + 20x - 60$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{5 - x^2}{x^2 + 4x - 12}$; domain: $(-\infty, -6) \cup (-6, 2) \cup (2, \infty)$

39. $(f + g)(x) = \sqrt{x} + x - 4$; domain: $[0, \infty)$; $(f - g)(x) = \sqrt{x} - x + 4$; domain: $[0, \infty)$; $(fg)(x) = \sqrt{x}(x - 4)$; domain: $[0, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x - 4}$; domain: $[0, 4) \cup (4, \infty)$ **40.** $(f + g)(x) = \sqrt{x} + x - 5$; domain: $[0, \infty)$; $(f - g)(x) = \sqrt{x} - x + 5$; domain: $[0, \infty)$; $(fg)(x) = \sqrt{x}(x - 5)$;

domain: $[0, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x-5}$; domain: $[0, 5) \cup (5, \infty)$ **41.** $(f+g)(x) = \frac{2x+2}{x}$; domain: $(-\infty, 0) \cup (0, \infty)$; $(f-g)(x) = 2$; domain: $(-\infty, 0) \cup (0, \infty)$; $(fg)(x) = \frac{2x+1}{x^2}$; domain: $(-\infty, 0) \cup (0, \infty)$; $\left(\frac{f}{g}\right)(x) = 2x + 1$; domain: $(-\infty, 0) \cup (0, \infty)$ **42.** $(f+g)(x) = 6$; domain: $(-\infty, 0) \cup (0, \infty)$; $(f-g)(x) = \frac{6x-2}{x}$; domain: $(-\infty, 0) \cup (0, \infty)$; $(fg)(x) = \frac{6x-1}{x^2}$; domain: $(-\infty, 0) \cup (0, \infty)$; $\left(\frac{f}{g}\right)(x) = 6x - 1$; domain: $(-\infty, 0) \cup (0, \infty)$ **43.** $(f+g)(x) = \frac{9x-1}{x^2-9}$; domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$; $(f-g)(x) = \frac{x+3}{x^2-9} = \frac{1}{x-3}$; domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$; $(fg)(x) = \frac{20x^2-6x-2}{(x^2-9)^2}$; domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{5x+1}{4x-2}$; domain: $(-\infty, -3) \cup \left(-3, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 3\right) \cup (3, \infty)$ **44.** $(f+g)(x) = \frac{5x-3}{x^2-25}$; domain: $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$; $(f-g)(x) = \frac{x+5}{x^2-25} = \frac{1}{x-5}$; domain: $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$; $(fg)(x) = \frac{6x^2-10x-4}{(x^2-25)^2}$; domain: $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{3x+1}{2x-4}$; domain: $(-\infty, -5) \cup (-5, 2) \cup (2, 5) \cup (5, \infty)$ **45.** $(f+g)(x) = \frac{8x^2+30x-12}{(x-2)(x+3)}$; domain: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$; $(f-g)(x) = \frac{8x^2+18x+12}{(x-2)(x+3)}$; domain: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$; $(fg)(x) = \frac{48x}{(x-2)(x+3)}$; domain: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{4x(x+3)}{3(x-2)}$; domain: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$ **46.** $(f+g)(x) = \frac{9x^2+79x-28}{(x-4)(x+8)}$; domain: $(-\infty, -8) \cup (-8, 4) \cup (4, \infty)$; $(f-g)(x) = \frac{9x^2+65x+28}{(x-4)(x+8)}$; domain: $(-\infty, -8) \cup (-8, 4) \cup (4, \infty)$; $(fg)(x) = \frac{63x}{(x-4)(x+8)}$; domain: $(-\infty, -8) \cup (-8, 4) \cup (4, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{9x(x+8)}{7(x-4)}$; domain: $(-\infty, -8) \cup (-8, 4) \cup (4, \infty)$ **47.** $(f+g)(x) = \sqrt{x+4} + \sqrt{x-1}$; domain: $[1, \infty)$; $(f-g)(x) = \sqrt{x+4} - \sqrt{x-1}$; domain: $[1, \infty)$; $(fg)(x) = \sqrt{x^2+3x-4}$; domain: $[1, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+4}}{\sqrt{x-1}}$; domain: $(1, \infty)$ **48.** $(f+g)(x) = \sqrt{x+6} + \sqrt{x-3}$; domain: $[3, \infty)$; $(f-g)(x) = \sqrt{x+6} - \sqrt{x-3}$; domain: $[3, \infty)$; $(fg)(x) = \sqrt{x^2+3x-18}$; domain: $[3, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+6}}{\sqrt{x-3}}$; domain: $(3, \infty)$

49. $(f+g)(x) = \sqrt{x-2} + \sqrt{2-x}$; domain: $\{2\}$; $(f-g)(x) = \sqrt{x-2} - \sqrt{2-x}$; domain: $\{2\}$; $(fg)(x) = \sqrt{x-2} \cdot \sqrt{2-x}$; domain: $\{2\}$; $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-2}}{\sqrt{2-x}}$; domain: $\varnothing$ **50.** $(f+g)(x) = \sqrt{x-5} + \sqrt{5-x}$; domain: $\{5\}$; $(f-g)(x) = \sqrt{x-5} - \sqrt{5-x}$; domain: $\{5\}$; $(fg)(x) = \sqrt{x-5} \cdot \sqrt{5-x}$; $\{5\}$; $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-5}}{\sqrt{5-x}}$; domain: $\varnothing$ **55. a.** $(f \circ g)(x) = 20x^2 - 11$ **b.** $(g \circ f)(x) = 80x^2 - 120x + 43$

56. a. $(f \circ g)(x) = 14x^2 - 62$ **b.** $(g \circ f)(x) = 98x^2 + 28x - 7$ **57. a.** $(f \circ g)(x) = x^4 - 4x^2 + 6$ **b.** $(g \circ f)(x) = x^4 + 4x^2 + 2$

58. a. $(f \circ g)(x) = x^4 - 6x^2 + 10$ **b.** $(g \circ f)(x) = x^4 + 2x^2 - 2$ **59. a.** $(f \circ g)(x) = -2x^2 - x - 1$ **b.** $(g \circ f)(x) = 2x^2 - 17x + 41$

60. a. $(f \circ g)(x) = -5x^2 + 20x - 7$ **b.** $(g \circ f)(x) = -25x^2 + 40x - 13$ **61. a.** $(f \circ g)(x) = \sqrt{x-1}$ **b.** $(g \circ f)(x) = \sqrt{x} - 1$

62. a. $(f \circ g)(x) = \sqrt{x+2}$ **b.** $(g \circ f)(x) = \sqrt{x} + 2$ **67. b.** $\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, 0\right) \cup (0, \infty)$ **68. b.** $\left(-\infty, -\frac{1}{4}\right) \cup \left(-\frac{1}{4}, 0\right) \cup (0, \infty)$

69. b. $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$ **70. b.** $\left(-\infty, -\frac{6}{5}\right) \cup \left(-\frac{6}{5}, 0\right) \cup (0, \infty)$ **73. b.** $(-\infty, 1]$ **74. b.** $(-\infty, 2]$

75. $f(x) = x^4, g(x) = 3x - 1$ **76.** $f(x) = x^3, g(x) = 2x - 5$ **77.** $f(x) = \sqrt[3]{x}, g(x) = x^2 - 9$ **78.** $f(x) = \sqrt{x}, g(x) = 5x^2 + 3$

79. $f(x) = |x|, g(x) = 2x - 5$ **80.** $f(x) = |x|, g(x) = 3x - 4$ **81.** $f(x) = \frac{1}{x}, g(x) = 2x - 3$ **82.** $f(x) = \frac{1}{x}, g(x) = 4x + 5$

87. $[-4, 3]$ **88.** $(-4, 3)$

89.

90.

95. 1 and 2 **96.** $-\frac{4}{3}$ and 1

99. $(R - C)(20{,}000) = -200{,}000$; The company loses \$200,000 when 20,000 radios are sold.; $(R - C)(30{,}000) = 0$; The company breaks even when 30,000 radios are sold.; $(R - C)(40{,}000) = 200{,}000$; The company makes a profit of \$200,000 when 40,000 radios are sold.
100. a. $m = -0.44$; Profit is decreasing. **b.** $m = 0.51$; Profit is increasing. **c.** $0.07x + 24.76$; $m = 0.07$; Profit is increasing.
101. a. f gives the price of the computer after a \$400 discount. g gives the price of the computer after a 25% discount. **b.** $(f \circ g)(x) = 0.75x - 400$; This models the price of a computer after first a 25% discount and then a \$400 discount. **c.** $(g \circ f)(x) = 0.75(x - 400)$; This models the price of a computer after first a \$400 discount and then a 25% discount. **d.** $f \circ g$ because $0.75x - 400 < 0.75(x - 400)$ **102. a.** f gives the cost of a pair of jeans after a \$5 discount. g gives the cost of a pair of jeans that has been discounted 40%. **b.** $(f \circ g)(x) = 0.6x - 5$; The cost of a pair of jeans is 60% of the regular price minus \$5 rebate. **c.** $(g \circ f)(x) = 0.6(x - 5) = 0.6x - 3$; The cost of a pair of jeans is 60% of the price that has been reduced by \$5. **d.** $f \circ g$ because $0.6x - 5 < 0.6x - 3$

108.

At $x = 0$, there is no y-value since the function is not defined there.

109.

Domain: $[0, 4]$

117. true **118.** Assume f and g are even; then $f(-x) = f(x)$ and $g(-x) = g(x)$. $(fg)(-x) = f(-x)g(-x) = f(x)g(x) = (fg)(x)$, so fg is even.
119. Answers will vary; One possible answer is $f(x) = x + 1$ and $g(x) = x - 1$. **120.** $\{(4, -2), (1, -1), (1, 1), (4, 2)\}$; no

Section 1.8

Check Point Exercises

6.

7. $f^{-1}(x) = \sqrt{x - 1}$

Exercise Set 1.8

3. $f(g(x)) = x$; $g(f(x)) = x$; f and g are inverses. **4.** $f(g(x)) = x$; $g(f(x)) = x$; f and g are inverses.
5. $f(g(x)) = \dfrac{5x - 56}{9}$; $g(f(x)) = \dfrac{5x - 4}{9}$; f and g are not inverses. **6.** $f(g(x)) = \dfrac{3x - 40}{7}$; $g(f(x)) = \dfrac{3x - 4}{7}$; f and g are not inverses.
7. $f(g(x)) = x$; $g(f(x)) = x$; f and g are inverses. **8.** $f(g(x)) = x$; $g(f(x)) = x$; f and g are inverses. **10.** $f(g(x)) = x$; $g(f(x)) = x$; f and g are inverses.
27. $f^{-1}(x) = \dfrac{3x + 1}{x - 2}$; $x \neq 2$ **28.** $f^{-1}(x) = \dfrac{-x - 3}{x - 2}$; $x \neq 2$
29. The function is not one-to-one, so it does not have an inverse function. **30.** The function is one-to-one, so it does have an inverse function.
31. The function is not one-to-one, so it does not have an inverse function. **32.** The function is not one-to-one, so it does not have an inverse function.
33. The function is one-to-one, so it does have an inverse function. **34.** The function is one-to-one, so it does have an inverse function.

35.

36.

37.

38.

39. a. $f^{-1}(x) = \dfrac{x + 1}{2}$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$;
range of f = domain of $f^{-1} = (-\infty, \infty)$

40. a. $f^{-1}(x) = \dfrac{x + 3}{2}$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$;
range of f = domain of $f^{-1} = (-\infty, \infty)$

41. a. $f^{-1}(x) = \sqrt{x + 4}$

b.

c. domain of f = range of $f^{-1} = [0, \infty)$;
range of f = domain of $f^{-1} = [-4, \infty)$

42. a. $f^{-1}(x) = -\sqrt{x + 1}$

b.

c. domain of f = range of $f^{-1} = (-\infty, 0]$;
range of f = domain of $f^{-1} = [-1, \infty)$

43. a. $f^{-1}(x) = -\sqrt{x} + 1$

b.

c. domain of f = range of $f^{-1} = (-\infty, 1]$;
range of f = domain of $f^{-1} = [0, \infty)$

44. a. $f^{-1}(x) = \sqrt{x} + 1$

b.

c. domain of f = range of $f^{-1} = [1, \infty)$;
range of f = domain of $f^{-1} = [0, \infty)$

45. a. $f^{-1}(x) = \sqrt[3]{x + 1}$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$;
range of f = domain of $f^{-1} = (-\infty, \infty)$

46. a. $f^{-1}(x) = \sqrt[3]{x - 1}$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$;
range of f = domain of $f^{-1} = (-\infty, \infty)$

47. a. $f^{-1}(x) = \sqrt[3]{x} - 2$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$;
range of f = domain of $f^{-1} = (-\infty, \infty)$

48. a. $f^{-1}(x) = \sqrt[3]{x} + 2$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$;
range of f = domain of $f^{-1} = (-\infty, \infty)$

49. a. $f^{-1}(x) = x^2 + 1, x \geq 0$

b.

c. domain of f = range of $f^{-1} = [1, \infty)$;
range of f = domain of $f^{-1} = [0, \infty)$

50. a. $f^{-1}(x) = (x - 2)^2, x \geq 2$

b.

c. domain of f = range of $f^{-1} = [0, \infty)$;
range of f = domain of $f^{-1} = [2, \infty)$

51. a. $f^{-1}(x) = (x - 1)^3$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$;
range of f = domain of $f^{-1} = (-\infty, \infty)$

52. a. $f^{-1}(x) = x^3 + 1$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$;
range of f = domain of $f^{-1} = (-\infty, \infty)$

65. a. {(Zambia, 4.2), (Colombia, 4.5), (Poland, 3.3), (Italy, 3.3), (United States, 2.5)}
b. {(4.2, Zambia), (4.5, Colombia), (3.3, Poland), (3.3, Italy), (2.5, United States)}; no; The inverse of f is not a function.
66. a. {(Zambia, −7.3), (Colombia, −4.5), (Poland, −2.8), (Italy, −2.8), (United States, −1.9)}
b. {(−7.3, Zambia), (−4.5, Colombia), (−2.8, Poland), (−2.8, Italy), (−1.9, United States)}; no; The inverse of g is not a function.
67. a. f is a one-to-one function. **b.** $f^{-1}(0.25)$ is the number of people in a room for a 25% probability of two people sharing a birthday. $f^{-1}(0.5)$ is the number of people in a room for a 50% probability of two people sharing a birthday. $f^{-1}(0.7)$ is the number of people in a room for a 70% probability of two people sharing a birthday. **68. a.** No; the horizontal line $y = 3$ intersects the graph in more than one point.
c. No, the graph does not represent a one-to-one function; the points (12, 3) and (19, 3) have the same second coordinate but different first coordinates.
69. $f(g(x)) = \frac{9}{5}\left[\frac{5}{9}(x - 32)\right] + 32 = x$ and $g(f(x)) = \frac{5}{9}\left[\left(\frac{9}{5}x + 32\right) - 32\right] = x$

76.

not one-to-one

77.

one-to-one

78.

one-to-one

79.

not one-to-one

80.

not one-to-one

81.

not one-to-one

82.

one-to-one

83.

not one-to-one

84.

f and g are inverses.

85.

f and g are inverses.

86.

f and g are inverses.

95. $(f \circ g)^{-1}(x) = \dfrac{x - 15}{3}$; $(g^{-1} \circ f^{-1})(x) = \dfrac{x}{3} - 5 = \dfrac{x - 15}{3}$

97. No; The spacecraft was at the same height, $s(t)$, for two different values of t-once during the ascent and once again during the descent.

101.

Section 1.9

Check Point Exercises

5. b.

$(x + 3)^2 + (y - 1)^2 = 4$

6.

$x^2 + y^2 + 4x - 4y - 1 = 0$

Exercise Set 1.9

3. $2\sqrt{29} \approx 10.77$ **4.** $\sqrt{73} \approx 8.54$ **7.** $\sqrt{29} \approx 5.39$ **8.** $2\sqrt{10} \approx 6.32$ **10.** $\sqrt{41} \approx 6.40$ **11.** $2\sqrt{5} \approx 4.47$

12. $5\sqrt{2} \approx 7.07$ **18.** $\sqrt{2} \approx 1.41$ **21.** $(-4, -5)$ **22.** $\left(-\dfrac{5}{2}, -5\right)$ **31.** $x^2 + y^2 = 49$ **32.** $x^2 + y^2 = 64$

33. $(x - 3)^2 + (y - 2)^2 = 25$ **34.** $(x - 2)^2 + (y + 1)^2 = 16$ **35.** $(x + 1)^2 + (y - 4)^2 = 4$ **36.** $(x + 3)^2 + (y - 5)^2 = 9$

37. $(x + 3)^2 + (y + 1)^2 = 3$ **38.** $(x + 5)^2 + (y + 3)^2 = 5$ **39.** $(x + 4)^2 + y^2 = 100$ **40.** $(x + 2)^2 + y^2 = 36$

41. center: $(0, 0)$
radius: 4
domain: $[-4, 4]$
range: $[-4, 4]$

$x^2 + y^2 = 16$

42. center: $(0, 0)$
radius: 7
domain: $[-7, 7]$
range: $[-7, 7]$

$x^2 + y^2 = 49$

43. center: $(3, 1)$
radius: 6
domain: $[-3, 9]$
range: $[-5, 7]$

$(x - 3)^2 + (y - 1)^2 = 36$

44. center: $(2, 3)$
radius: 4
domain: $[-2, 6]$
range: $[-1, 7]$

$(x - 2)^2 + (y - 3)^2 = 16$

45. center: $(-3, 2)$
radius: 2
domain: $[-5, -1]$
range: $[0, 4]$

$(x + 3)^2 + (y - 2)^2 = 4$

46. center: $(-1, 4)$
radius: 5
domain: $[-6, 4]$
range: $[-1, 9]$

$(x + 1)^2 + (y - 4)^2 = 25$

47. center: $(-2, -2)$
radius: 2
domain: $[-4, 0]$
range: $[-4, 0]$

$(x + 2)^2 + (y + 2)^2 = 4$

48. center: $(-4, -5)$
radius: 6
domain: $[-10, 2]$
range: $[-11, 1]$

$(x + 4)^2 + (y + 5)^2 = 36$

49. center: $(0, 1)$
radius: 1
domain: $[-1, 1]$
range: $[0, 2]$

$x^2 + (y - 1)^2 = 1$

50. center: $(0, 2)$
radius: 2
domain: $[-2, 2]$
range: $[0, 4]$

$x^2 + (y - 2)^2 = 4$

51. center: $(-1, 0)$
radius: 5
domain: $[-6, 4]$
range: $[-5, 5]$

$(x + 1)^2 + y^2 = 25$

52. center: $(-2, 0)$
radius: 4
domain: $[-6, 2]$
range: $[-4, 4]$

$(x + 2)^2 + y^2 = 16$

53. $(x + 3)^2 + (y + 1)^2 = 4$
center: $(-3, -1)$
radius: 2

$x^2 + y^2 + 6x + 2y + 6 = 0$

54. $(x + 4)^2 + (y + 2)^2 = 4$
center: $(-4, -2)$
radius: 2

$x^2 + y^2 + 8x + 4y + 16 = 0$

55. $(x - 5)^2 + (y - 3)^2 = 64$
center: $(5, 3)$
radius: 8

$x^2 + y^2 - 10x - 6y - 30 = 0$

56. $(x - 2)^2 + (y - 6)^2 = 49$
center: $(2, 6)$
radius: 7

$x^2 + y^2 - 4x - 12y - 9 = 0$

57. $(x + 4)^2 + (y - 1)^2 = 25$
center: $(-4, 1)$
radius: 5

$x^2 + y^2 + 8x - 2y - 8 = 0$

58. $(x + 6)^2 + (y - 3)^2 = 49$
center: $(-6, 3)$
radius: 7

$x^2 + y^2 + 12x - 6y - 4 = 0$

59. $(x - 1)^2 + y^2 = 16$

center: $(1, 0)$

radius: 4

$x^2 - 2x + y^2 - 15 = 0$

60. $x^2 + (y - 3)^2 = 16$

center: $(0, 3)$

radius: 4

$x^2 + y^2 - 6y - 7 = 0$

61. $\left(x - \frac{1}{2}\right)^2 + (y + 1)^2 = \frac{1}{4}$

center: $\left(\frac{1}{2}, -1\right)$

radius: $\frac{1}{2}$

$x^2 + y^2 - x + 2y + 1 = 0$

62. $\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 1$

center: $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

radius: 1

$x^2 + y^2 + x + y - \frac{1}{2} = 0$

63. $\left(x + \frac{3}{2}\right)^2 + (y - 1)^2 = \frac{17}{4}$

center: $\left(-\frac{3}{2}, 1\right)$

radius: $\frac{\sqrt{17}}{2}$

$x^2 + y^2 + 3x - 2y - 1 = 0$

64. $\left(x + \frac{3}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{25}{4}$

center: $\left(-\frac{3}{2}, -\frac{5}{2}\right)$

radius: $\frac{5}{2}$

$x^2 + y^2 + 3x + 5y + \frac{9}{4} = 0$

65. a. $(5, 10)$ **b.** $\sqrt{5}$ **c.** $(x - 5)^2 + (y - 10)^2 = 5$

67. $x^2 + y^2 = 16$
$x - y = 4$

68. $x^2 + y^2 = 9$
$x - y = 3$

69. $(x - 2)^2 + (y + 3)^2 = 4$
$y = x - 3$

70. $(x - 3)^2 + (y + 1)^2 = 9$
$y = x - 1$

83.

84.

85.

89. makes sense

95. a Distance between (x_1, y_1) and $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$
$$= \sqrt{\left(\frac{x_1 + x_2 - 2x_1}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_1}{2}\right)^2}$$
$$= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$
$$= \sqrt{\frac{x_2^2 - 2x_1x_2 + x_1^2}{4} + \frac{y_2^2 - 2y_1y_2 + y_1^2}{4}}$$
$$= \sqrt{\frac{x_1^2 - 2x_1x_2 + x_2^2}{4} + \frac{y_1^2 - 2y_1y_2 + y_2^2}{4}}$$
$$= \sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2}$$
$$= \sqrt{\left(\frac{x_1 + x_2 - 2x_2}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_2}{2}\right)^2}$$
$$= \sqrt{\left(\frac{x_1 + x_2}{2} - x_2\right)^2 + \left(\frac{y_1 + y_2}{2} - y_2\right)^2}$$

= Distance between (x_2, y_2) and $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

b. $$\sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2} + \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$
$$= 2\sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$
$$= 2\sqrt{\frac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{4}}$$
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

= Distance from (x_1, y_1) to (x_2, y_2)

99. a. perimeter: 140 yd; area: 1200 yd^2
b. perimeter: 140 yd; area: 1000 yd^2

Exercise Set 1.10

3. a. $M(x) = 239.4 - 0.3x$ **13. b.** $T(x) = -4x^2 + 520x$ **15. b.** $V(2) = 800$; If a 2-in. square is cut from each corner, the volume is 800 cubic in.; $V(3) = 972$; if a 3-in. square is cut from each corner, the volume is 972 cubic in.; $V(4) = 1024$; if a 4-in. square is cut from each corner, the volume is 1024 cubic in.; $V(5) = 980$; if a 5-in. square is cut from each corner, the volume is 980 cubic in.; $V(6) = 864$; if a 6-in. square is cut from each corner, the volume is 864 cubic in. Initially, as x increases, V increases. When $x = 4$, V is a maximum. As x increases beyond 4, V decreases. **16. b.** $V(3) = 1728$; if a 3-in. square is cut from each corner, the volume is 1728 cubic in.; $V(4) = 1936$; if a 4-in. square is cut from each corner, the volume is 1936 cubic in.; $V(5) = 2000$; if a 5-in. square is cut from each corner, the volume is 2000 cubic in.; $V(6) = 1944$; if a 6-in. square is cut from each corner, the volume is 1944 cubic in.; $V(7) = 1792$; if a 7-in. square is cut from each corner, the volume is 1792 cubic in. Initially, as x increases, V increases. When $x = 5$, V is a maximum. As x increases beyond 5, V decreases. **19.** $P(x) = x(66 - x)$ **20.** $P(x) = x(50 - x)$ **23.** Let x be the length of the side perpendicular to the canal. $A(x) = x(800 - 2x)$ **24.** Let x be the length of the side perpendicular to the canal. $A(x) = x(600 - 2x)$ **29.** Let x be the length of the interior wall. $C(x) = 475x + \frac{1{,}400{,}000}{x}$ **30.** Let x be the length of the fence. $C(x) = 29x + \frac{5000}{x}$ **32.** $A(x) = \frac{2000}{x} + x^2$
35. a. $I(x) = 0.15x + 0.07(50{,}000 - x)$ **b.** \$31,250 at 15%, \$18,750 at 7% **36. a.** $I(x) = 0.10x + 0.12(18{,}750 - x)$
b. \$6650 at 10%, \$12,100 at 12% **37.** $I(x) = 0.12x - 0.05(8000 - x)$ **38.** $I(x) = 0.14x - 0.06(12{,}000 - x)$ **63.** does not make sense
65. does not make sense

Chapter 1 Review Exercises

1.

2.

3.

4.

5.

9. a. (20, 8); 8% of college students anticipated a starting salary of \$20 thousand. **15.** function; domain: {2, 3, 5}; range: {7}
16. function; domain: {1, 2, 13}; range: {10, 500, π} **34. b.** $-2, 3$; $f(-2) = -3$, $f(3) = -5$ **36.** odd; symmetric with respect to the origin
37. even; symmetric with respect to the y-axis **38.** odd; symmetric with respect to the origin

39.

40.

42. $-4x - 2h + 1$
43. a. yes; The graph passes the vertical line test. **b.** (3, 12); The eagle descended. **c.** (0, 3) and (12, 17); The eagle's height held steady during the first 3 seconds and the eagle was on the ground for 5 seconds. **d.** (17, 30); The eagle was ascending.

44.

46. 1; rises **47.** 0; horizontal **48.** undefined; vertical **49.** $y - 2 = -6(x + 3)$; $y = -6x - 16$
50. using (1, 6), $y - 6 = 2(x - 1)$; $y = 2x + 4$

54. Slope: $\frac{2}{5}$; y-intercept: -1

$y = \frac{2}{5}x - 1$

55. Slope: -4; y-intercept: 5

$f(x) = -4x + 5$

56. Slope: $-\frac{2}{3}$; y-intercept: -2

$2x + 3y + 6 = 0$

57. Slope: 0; y-intercept: 4

$2y - 8 = 0$

58.

$2x - 5y - 10 = 0$

59. $2x - 10 = 0$

60. a. $y - 2.3 = 0.116(x - 15)$ or $y - 11 = 0.116(x - 90)$ **d.** 4.3 deaths per 100,000 persons; underestimates by 0.7 death per 100,000 persons; The line passes below the point for France. **63. c.** The positive sign means the ball is moving up on (0, 2). The negative sign means the ball is moving down on (2, 4).

64.

65.

66.

67.

68.

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70.

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72.

73.

74.

75.

76.

77.

78.

79.

80.

81.

82.

83.

84.

86. $(-\infty, 7) \cup (7, \infty)$ **88.** $(-\infty, -7) \cup (-7, 3) \cup (3, \infty)$ **91.** $(f + g)(x) = 4x - 6$; domain: $(-\infty, \infty)$; $(f - g)(x) = 2x + 4$; domain: $(-\infty, \infty)$; $(fg)(x) = 3x^2 - 16x + 5$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{3x - 1}{x - 5}$; domain: $(-\infty, 5) \cup (5, \infty)$ **92.** $(f + g)(x) = 2x^2 + x$; domain: $(-\infty, \infty)$; $(f - g)(x) = x + 2$; domain: $(-\infty, \infty)$; $(fg)(x) = x^4 + x^3 - x - 1$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{x^2 + x + 1}{x^2 - 1}$; domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

93. $(f + g)(x) = \sqrt{x + 7} + \sqrt{x - 2}$; domain: $[2, \infty)$; $(f - g)(x) = \sqrt{x + 7} - \sqrt{x - 2}$; domain: $[2, \infty)$; $(fg)(x) = \sqrt{x^2 + 5x - 14}$; domain: $[2, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x + 7}}{\sqrt{x - 2}}$; domain: $(2, \infty)$ **94. a.** $(f \circ g)(x) = 16x^2 - 8x + 4$ **b.** $(g \circ f)(x) = 4x^2 + 11$

95. a. $(f \circ g)(x) = \sqrt{x + 1}$ **b.** $(g \circ f)(x) = \sqrt{x} + 1$ **96. b.** $(-\infty, 0) \cup \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$ **97. b.** $[-2, \infty)$

98. $f(x) = x^4, g(x) = x^2 + 2x - 1$ **99.** $f(x) = \sqrt[3]{x}, g(x) = 7x + 4$ **100.** $f(g(x)) = x - \frac{7}{10}$; $g(f(x)) = x - \frac{7}{6}$; f and g are not inverses of each other.

101. $f(g(x)) = x$; $g(f(x)) = x$; f and g are inverses of each other.

109.

110. $f^{-1}(x) = \sqrt{1 - x}$

111. $f^{-1}(x) = (x - 1)^2, x \geq 1$

116. $x^2 + y^2 = 9$

117. $(x + 2)^2 + (y - 4)^2 = 36$

118. center: (0, 0); radius: 1
domain: [−1, 1]; range: [−1, 1]

$x^2 + y^2 = 1$

119. center: (−2, 3); radius: 3
domain: [−5, 1]; range: [0, 6]

$(x + 2)^2 + (y - 3)^2 = 9$

120. center: (2, −1); radius: 3
domain: [−1, 5]; range: [−4, 2]

$x^2 + y^2 - 4x + 2y - 4 = 0$

125. $A(x) = x\left(\dfrac{400 - 3x}{2}\right)$

Chapter 1 Test

2. e. (−5, −1) or (2, 6)

3. g.

h.

i.

4.

domain: $(-\infty, \infty)$;
range: $(-\infty, \infty)$

5.

domain: [−2, 2];
range: [−2, 2]

6.

domain: $(-\infty, \infty)$;
range: {4}

7.

$f(x) = -\frac{1}{3}x + 2$

domain: $(-\infty, \infty)$;
range: $(-\infty, \infty)$

8.

$(x + 2)^2 + (y - 1)^2 = 9$

domain: [−5, 1];
range: [−2, 4]

9.

$f(x) = \begin{cases} 2 \text{ if } x \leq 0 \\ -1 \text{ if } x > 0 \end{cases}$

domain: $(-\infty, \infty)$;
range: {−1, 2}

10.

$x^2 + y^2 + 4x - 6y - 3 = 0$

domain: [−6, 2];
range: [−1, 7]

11.

domain of f = domain of g = $(-\infty, \infty)$;
range of f = $[0, \infty)$; range of g = $[-2, \infty)$

12.

domain of f = domain of g = $(-\infty, \infty)$;
range of f = $[0, \infty)$; range of g = $(-\infty, 4]$

13.

domain of f = range of f^{-1} = $(-\infty, \infty)$;
range of f = domain of f^{-1} = $(-\infty, \infty)$

14.

domain of f = range of f^{-1} = $(-\infty, \infty)$;
range of f = domain of f^{-1} = $(-\infty, \infty)$

15.

domain of f = range of f^{-1} = $[0, \infty)$;
range of f = domain of f^{-1} = $[-1, \infty)$

16. $f(x - 1) = x^2 - 3x - 2$

17. $2x + h - 1$

19. $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 - x - 4}{2x - 6}$; $(-\infty, 3) \cup (3, \infty)$

20. $(f \circ g)(x) = 4x^2 - 26x + 38$

21. $(g \circ f)(x) = 2x^2 - 2x - 14$

23. $f(-x) = x^2 + x - 4$; neither

24. using (2, 1), $y - 1 = 3(x - 2)$; $y = 3x - 5$

25. $y - 6 = 4(x + 4)$; $y = 4x + 22$

CHAPTER 2

Exercise Set 2.1

4. $2 + 16i$ **63.** $(5 + i\sqrt{15}) + (5 - i\sqrt{15}) = 10; (5 + i\sqrt{15})(5 - i\sqrt{15}) = 25 - 15i^2 = 25 + 15 = 40$ **86.**

$f(x) = x^2$
$g(x) = (x + 3)^2 + 1$

Section 2.2

Check Point Exercises

1.

$f(x) = -(x - 1)^2 + 4$

2.

$f(x) = (x - 2)^2 + 1$

3.

$f(x) = -x^2 + 4x + 1$

5. c.

Exercise Set 2.2

10. $(2, 12)$ **11.** $(-1, 5)$ **12.** $(-4, -8)$ **14.** $(2, -11)$ **16.** $(2, 7)$

17. domain: $(-\infty, \infty)$
range: $[-1, \infty)$
axis of symmetry: $x = 4$

$f(x) = (x - 4)^2 - 1$

18. domain: $(-\infty, \infty)$
range: $[-2, \infty)$
axis of symmetry: $x = 1$

$f(x) = (x - 1)^2 - 2$

19. domain: $(-\infty, \infty)$
range: $[2, \infty)$
axis of symmetry: $x = 1$

$f(x) = (x - 1)^2 + 2$

20. domain: $(-\infty, \infty)$
range: $[2, \infty)$
axis of symmetry: $x = 3$

$f(x) = (x - 3)^2 + 2$

21. domain: $(-\infty, \infty)$
range: $[1, \infty)$
axis of symmetry: $x = 3$

$y - 1 = (x - 3)^2$

22. domain: $(-\infty, \infty)$
range: $[3, \infty)$
axis of symmetry: $x = 1$

$y - 3 = (x - 1)^2$

23. domain: $(-\infty, \infty)$
range: $[-1, \infty)$
axis of symmetry: $x = -2$

$f(x) = 2(x + 2)^2 - 1$

24. domain: $(-\infty, \infty)$
range: $\left(-\infty, \frac{5}{4}\right]$
axis of symmetry: $x = \frac{1}{2}$

$f(x) = \frac{5}{4} - \left(x - \frac{1}{2}\right)^2$

25. domain: $(-\infty, \infty)$
range: $(-\infty, 4]$
axis of symmetry: $x = 1$

$f(x) = 4 - (x - 1)^2$

26. domain: $(-\infty, \infty)$
range: $(-\infty, 1]$
axis of symmetry: $x = 3$

$f(x) = 1 - (x - 3)^2$

27. domain: $(-\infty, \infty)$
range: $[-4, \infty)$
axis of symmetry: $x = 1$

$f(x) = x^2 - 2x - 3$

28. domain: $(-\infty, \infty)$
range: $[-16, \infty)$
axis of symmetry: $x = 1$

$f(x) = x^2 - 2x - 15$

29. domain: $(-\infty, \infty)$
range: $\left[-\frac{49}{4}, \infty\right)$
axis of symmetry: $x = -\frac{3}{2}$

$f(x) = x^2 + 3x - 10$

30. domain: $(-\infty, \infty)$
range: $\left[-\frac{81}{8}, \infty\right)$
axis of symmetry: $x = \frac{7}{4}$

$f(x) = 2x^2 - 7x - 4$

31. domain: $(-\infty, \infty)$
range: $(-\infty, 4]$
axis of symmetry: $x = 1$

$f(x) = 2x - x^2 + 3$

32. domain: $(-\infty, \infty)$
range: $(-\infty, 9]$
axis of symmetry: $x = -2$

$f(x) = 5 - 4x - x^2$

33. domain: $(-\infty, \infty)$
range: $[-6, \infty)$
axis of symmetry: $x = -3$

$f(x) = x^2 + 6x + 3$

34. domain: $(-\infty, \infty)$
range: $[-5, \infty)$
axis of symmetry: $x = -2$

$f(x) = x^2 + 4x - 1$

35. domain: $(-\infty, \infty)$
range: $[-5, \infty)$
axis of symmetry: $x = -1$

$f(x) = 2x^2 + 4x - 3$

36. domain: $(-\infty, \infty)$
range: $\left[-\frac{13}{3}, \infty\right)$
axis of symmetry: $x = \frac{1}{3}$

$f(x) = 3x^2 - 2x - 4$

37. domain: $(-\infty, \infty)$
range: $(-\infty, -1]$
axis of symmetry: $x = 1$

$f(x) = 2x - x^2 - 2$

38. domain: $(-\infty, \infty)$
range: $[2, \infty)$
axis of symmetry: $x = 2$

$f(x) = 6 - 4x + x^2$

39. **a.** minimum **b.** Minimum is -13 at $x = 2$.
c. domain: $(-\infty, \infty)$; range: $[-13, \infty)$

40. **a.** minimum **b.** Minimum is -11 at $x = 2$.
c. domain: $(-\infty, \infty)$; range: $[-11, \infty)$

41. **a.** maximum **b.** Maximum is 1 at $x = 1$.
c. domain: $(-\infty, \infty)$; range: $(-\infty, 1]$

42. **a.** maximum **b.** Maximum is 21 at $x = -3$.
c. domain: $(-\infty, \infty)$; range: $(-\infty, 21]$

43. **a.** minimum **b.** Minimum is $-\frac{5}{4}$ at $x = \frac{1}{2}$. **c.** domain: $(-\infty, \infty)$; range: $\left[-\frac{5}{4}, \infty\right)$

44. **a.** minimum **b.** Minimum is $-\frac{3}{2}$ at $x = \frac{1}{2}$. **c.** domain: $(-\infty, \infty)$; range: $\left[-\frac{3}{2}, \infty\right)$

45. domain: $(-\infty, \infty)$; range: $[-2, \infty)$ **46.** domain: $(-\infty, \infty)$; range: $(-\infty, -4]$ **51.** $f(x) = 2(x + 10)^2 - 5$ **53.** $f(x) = -3(x + 2)^2 + 4$

54. $f(x) = -3(x - 5)^2 - 7$ **55.** $f(x) = 3(x - 11)^2$ **56.** $f(x) = 3(x - 9)^2$

59. **c.**

60. **c.**

61. 8 and 8; 64 **62.** 10 and 10; 100 **67.** 12.5 yd by 12.5 yd; 156.25 sq yd
68. 20 yd by 20 yd; 400 sq yd **72.** 3 in.; 18 sq in. **74.** \$35; \$6125

84. **a.**

You can only see a little of the parabola.

d. You can choose Xmin and Xmax so the x-value of the vertex is in the center of the graph. Choose Ymin to include the y-value of the vertex.

85. (80, 1600)

86. (2.5, 185)

87. (−4, 520)

88. (−30, 91)

89. **a & e.**

d. The greatest number of viewers actually occurred in Season 5, not Season 6, and the model underestimates the greatest number by 0.5 million.

100. $f(x) = \frac{1}{2}(x + 3)^2 - 4$ **102.** $\left(\frac{4}{5}, \frac{2}{5}\right)$ **103.** \$95; \$21,675

107. $f(2) = -1$; $f(3) = 16$; The graph passes through $(2, -1)$, which is below the x-axis, and $(3, 16)$, which is above the x-axis. Since the graph of f is continuous, it must cross the x-axis somewhere between 2 and 3 to get from one of these points to the other.

Section 2.3

Check Point Exercises

9.

$f(x) = x^3 - 3x^2$

10.

$f(x) = 2(x + 2)^2(x - 3)$

Exercise Set 2.3

3. polynomial function; degree: 5 **4.** polynomial function; degree: 7

25. $x = 5$ has multiplicity 1; The graph crosses the x-axis; $x = -4$ has multiplicity 2; The graph touches the x-axis and turns around.

26. $x = -5$ has multiplicity 1; The graph crosses the x-axis; $x = -2$ has multiplicity 2; The graph touches the x-axis and turns around.

27. $x = 3$ has multiplicity 1; The graph crosses the x-axis; $x = -6$ has multiplicity 3; The graph crosses the x-axis.

28. $x = -\frac{1}{2}$ has multiplicity 1; The graph crosses the x-axis; $x = 4$ has multiplicity 3; The graph crosses the x-axis.

29. $x = 0$ has multiplicity 1; The graph crosses the x-axis; $x = 1$ has multiplicity 2; The graph touches the x-axis and turns around.

30. $x = 0$ has multiplicity 1; The graph crosses the x-axis; $x = -2$ has multiplicity 2; The graph touches the x-axis and turns around.

31. $x = 2, x = -2$ and $x = -7$ have multiplicity 1; The graph crosses the x-axis.

32. $x = 3, x = -3$, and $x = -5$ have multiplicity 1; The graph crosses the x-axis.

37. $f(-3) = -11; f(-2) = 1$ **39.** $f(-3) = -42; f(-2) = 5$

41. a. $f(x)$ rises to the right and falls to the left.
b. $x = -2, x = 1, x = -1$;
$f(x)$ crosses the x-axis at each.
c. The y-intercept is -2.
d. neither
e.

$f(x) = x^3 + 2x^2 - x - 2$

42. a. $f(x)$ rises to the right and falls to the left.
b. $x = -1, x = 2, x = -2$;
$f(x)$ crosses the x-axis at each.
c. The y-intercept is -4.
d. neither
e.

$f(x) = x^3 + x^2 - 4x - 4$

43. a. $f(x)$ rises to the left and the right.
b. $x = 0, x = 3, x = -3$;
$f(x)$ crosses the x-axis at -3 and 3;
$f(x)$ touches the x-axis at 0.
c. The y-intercept is 0.
d. y-axis symmetry
e.

$f(x) = x^4 - 9x^2$

44. a. $f(x)$ rises to the left and the right.
b. $x = 0, x = 1, x = -1$;
$f(x)$ touches but does not cross the x-axis at 0;
$f(x)$ crosses the x-axis at -1 and 1.
c. The y-intercept is 0.
d. y-axis symmetry
e.

$f(x) = x^4 - x^2$

45. a. $f(x)$ falls to the left and the right.
b. $x = 0, x = 4, x = -4$;
$f(x)$ crosses the x-axis at -4 and 4;
$f(x)$ touches the x-axis at 0.
c. The y-intercept is 0.
d. y-axis symmetry
e.

$f(x) = -x^4 + 16x^2$

46. a. $f(x)$ falls to the left and the right.
b. $x = 0, x = 2, x = -2$;
$f(x)$ touches but does not cross the x-axis at 0;
$f(x)$ crosses the x-axis at -2 and 2.
c. The y-intercept is 0.
d. y-axis symmetry
e.

$f(x) = -x^4 + 4x^2$

47. a. $f(x)$ rises to the left and the right.
b. $x = 0, x = 1$;
$f(x)$ touches the x-axis at 0 and 1.
c. The y-intercept is 0.
d. neither
e.

$f(x) = x^4 - 2x^3 + x^2$

48. a. $f(x)$ rises to the left and the right.
b. $x = 0, x = 3$;
$f(x)$ touches the x-axis at 3 and 0.
c. The y-intercept is 0.
d. neither
e.

$f(x) = x^4 - 6x^3 + 9x^2$

49. a. $f(x)$ falls to the left and the right.
b. $x = 0, x = 2$;
$f(x)$ crosses the x-axis at 0 and 2.
c. The y-intercept is 0.
d. neither
e.

$f(x) = -2x^4 + 4x^3$

50. a. $f(x)$ falls to the left and the right.
b. $x = 0, x = 1$;
$f(x)$ crosses the x-axis at 0 and 1.
c. The y-intercept is 0.
d. neither
e.

$f(x) = -2x^4 + 2x^3$

51. a. $f(x)$ rises to the left and falls to the right.
b. $x = 0, x = \pm\sqrt{3}$;
$f(x)$ crosses the x-axis at 0;
$f(x)$ touches the x-axis at $\sqrt{3}$ and $-\sqrt{3}$.
c. The y-intercept is 0.
d. origin symmetry
e.

$f(x) = 6x^3 - 9x - x^5$

52. a. $f(x)$ rises to the left and falls to the right.
b. $x = 0, x = \pm\sqrt{2}$;
$f(x)$ crosses the x-axis at $-\sqrt{2}$, 0, and $\sqrt{2}$.
c. The y-intercept is 0.
d. origin symmetry
e.

$f(x) = 6x - x^3 - x^5$

53. a. $f(x)$ rises to the left and falls to the right.
b. $x = 0, x = 3$;
$f(x)$ crosses the x-axis at 3;
$f(x)$ touches the x-axis at 0.
c. The y-intercept is 0.
d. neither
e.

$f(x) = 3x^2 - x^3$

54. a. $f(x)$ falls to the left and the right.
b. $x = \pm 1$;
$f(x)$ crosses the x-axis at -1 and 1.
c. The y-intercept is $\frac{1}{2}$.
d. y-axis symmetry
e.

$f(x) = \frac{1}{2} - \frac{1}{2}x^4$

55. a. $f(x)$ falls to the left and the right.
b. $x = 1, x = -2, x = 2$;
$f(x)$ crosses the x-axis at -2 and 2;
$f(x)$ touches the x-axis at 1.
c. The y-intercept is 12.
d. neither
e.

$f(x) = -3(x - 1)^2(x^2 - 4)$

56. a. $f(x)$ falls to the left and the right.
b. $x = 4, x = -5, x = 5$;
$f(x)$ crosses the x-axis at -5 and 5;
$f(x)$ touches the x-axis at 4.
c. The y-intercept is 800.
d. neither
e.

$f(x) = -2(x - 4)^2(x^2 - 25)$

57. a. $f(x)$ rises to the left and the right.
b. $x = -2, x = 0, x = 1$;
$f(x)$ crosses the x-axis at -2 and 1;
$f(x)$ touches the x-axis at 0.
c. The y-intercept is 0.
d. neither
e.

$f(x) = x^2(x-1)^3(x+2)$

58. a. $f(x)$ rises to the left and the right.
b. $x = -2, x = -1, x = 0$;
$f(x)$ crosses the x-axis at -1 and 0;
$f(x)$ touches the x-axis at -2.
c. The y-intercept is 0.
d. neither
e.

$f(x) = x^3(x+2)^2(x+1)$

59. a. $f(x)$ falls to the left and the right.
b. $x = -3, x = 0, x = 1$;
$f(x)$ crosses the x-axis at -3 and 1;
$f(x)$ touches the x-axis at 0.
c. The y-intercept is 0.
d. neither
e.

$f(x) = -x^2(x-1)(x+3)$

60. a. $f(x)$ falls to the left and the right.
b. $x = -2, x = 0, x = 2$;
$f(x)$ crosses the x-axis at -2 and 2;
$f(x)$ touches the x-axis at 0.
c. The y-intercept is 0.
d. y-axis symmetry
e.

$f(x) = -x^2(x+2)(x-2)$

61. a. $f(x)$ falls to the left and the right.
b. $x = -5, x = 0, x = 1$;
$f(x)$ crosses the x-axis at -5 and 0;
$f(x)$ touches the x-axis at 1.
c. The y-intercept is 0.
d. neither
e.

$f(x) = -2x^3(x-1)^2(x+5)$

62. a. $f(x)$ falls to the left and the right.
b. $x = -3, x = 0, x = 1$;
$f(x)$ crosses the x-axis at -3 and 0;
$f(x)$ touches the x-axis at 1.
c. The y-intercept is 0.
d. neither
e.

$f(x) = -3x^3(x-1)^2(x+3)$

63. a. $f(x)$ rises to the left and the right.
b. $x = -4, x = 1, x = 2$;
$f(x)$ crosses the x-axis at -4 and 1;
$f(x)$ touches the x-axis at 2.
c. The y-intercept is -16.
d. neither
e.

$f(x) = (x-2)^2(x+4)(x-1)$

64. a. $f(x)$ rises to the right and falls to the left.
b. $x = -4, x = -3, x = -1$;
$f(x)$ crosses the x-axis at each.
c. The y-intercept is 12.
d. neither
e.

$f(x) = (x+3)(x+1)^3(x+4)$

65. a. -2, odd; 1, odd; 4, odd **b.** $f(x) = (x+2)(x-1)(x-4)$ **c.** 8 **66. a.** -3, odd; 2, odd; 5, odd **b.** $f(x) = (x+3)(x-2)(x-5)$ **c.** 30 **67. a.** -1, odd; 3, even **b.** $f(x) = (x+1)(x-3)^2$ **c.** 9 **68. a.** -2, odd; 1, even **b.** $f(x) = (x+2)(x-1)^2$ **c.** 2 **69. a.** -3, even; 2, even **b.** $f(x) = -(x+3)^2(x-2)^2$ **c.** -36 **70. a.** -1, even; 4, even **b.** $f(x) = -(x+1)^2(x-4)^2$ **c.** -16 **71. a.** -2, even; -1, odd; 1, odd **b.** $f(x) = (x+2)^2(x+1)(x-1)^3$ **c.** -4 **72. a.** -2, odd; -1, even; 1, odd **b.** $f(x) = (x+2)(x+1)^2(x-1)^3$ **c.** -2 **73. a.** 3167; The world tiger population in 2010 was approximately 3167.; (40, 3167) **b.** underestimates by 33 **c.** rises to the right; no; The model indicates an increasing world tiger population that will actually decrease without conservation efforts. **74. a.** 26,747; The world tiger population in 1980 was approximately 26,747.; (10, 26,747) **b.** underestimates by 1253 **c.** rises to the right; yes; The model indicates an increasing world tiger population that might actually increase with conservation efforts. **75. a.** from 1 through 4 min and from 8 through 10 min **b.** from 4 through 8 min and from 10 through 12 min **e.** negative; The graph falls to the left and falls to the right. **f.** 116 ± 1 beats per min; 10 min **g.** 64 ± 1 beats per min; 8 min **76. e.** positive; The graph falls to the left and rises to the right. **f.** $\$3.15 \pm \$0.05$; 2011 **g.** $\$1.85 \pm \$0.05$; 2009

94.

95.

96.

97.

98.

99.

111. $6x^3 - x^2 - 5x + 4$

Exercise Set 2.4

9. $2x^2 + x + 6 - \dfrac{38}{x+3}$ **10.** $3x + 7 + \dfrac{26}{x-3}$ **11.** $4x^3 + 16x^2 + 60x + 246 + \dfrac{984}{x-4}$ **13.** $2x + 5$ **14.** $x^2 + x - 3 - \dfrac{12}{x^2+x-2}$

15. $6x^2 + 3x - 1 - \dfrac{3x-1}{3x^2+1}$ **16.** $x^2 - 4x + 1 + \dfrac{4x-1}{2x^3+1}$ **17.** $2x + 5$ **18.** $x + 2$ **23.** $6x^4 + 12x^3 + 22x^2 + 48x + 93 + \dfrac{187}{x-2}$

24. $x^4 + 7x^3 + 21x^2 + 60x + 182 + \dfrac{549}{x-3}$ **27.** $x^4 + x^3 + 2x^2 + 2x + 2$ **28.** $x^6 - 2x^5 + 5x^4 - 10x^3 + 10x^2 - 20x + 40 - \dfrac{68}{x+2}$

29. $x^3 + 4x^2 + 16x + 64$ **30.** $x^6 + 2x^5 + 4x^4 + 8x^3 + 16x^2 + 32x + 64$ **31.** $2x^4 - 7x^3 + 15x^2 - 31x + 64 - \dfrac{129}{x+2}$

55. a. 70; When the tax rate is 30%, tax revenue is \$700 billion.; (30, 70) **b.** $80 + \dfrac{800}{x-110}$; $f(30) = 70$; yes
c. No, f is a rational function because it is a quotient of two polynomials.

56. a. $68\frac{4}{7}$; When the tax rate is 40%, tax revenue is $68\frac{4}{7}$ tens of billions of dollars, or approximately \$685.7 billion.; $\left(40, 68\frac{4}{7}\right)$
b. $80 + \dfrac{800}{x-110}$; $f(40) \approx 68.57$; yes **c.** No; f is a rational function because it is a quotient of two polynomials.

Exercise Set 2.5

4. $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$ **5.** $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{3}{2}, \pm\frac{3}{4}$ **6.** $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

7. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ **9. b.** $-2, -1$, or 2 **10. a.** $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ **b.** $-3, 1$, or 4

11. a. $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$ **b.** $-2, \frac{1}{2}$, or 3 **12. b.** $-\frac{1}{2}, 1$, or 2 **13. c.** $-1, \dfrac{-3-\sqrt{33}}{2}$, and $\dfrac{-3+\sqrt{33}}{2}$

14. c. $\frac{1}{2}, \dfrac{-1-\sqrt{5}}{2}$, and $\dfrac{-1+\sqrt{5}}{2}$ **15. c.** $-2, \dfrac{-1+i}{2}$, and $\dfrac{-1-i}{2}$ **16. c.** $1, \dfrac{3+i\sqrt{11}}{2}$, and $\dfrac{3-i\sqrt{11}}{2}$ **17. a.** $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b. $-3, 1$, or 4 **18. b.** -1 or 4 **19. a.** $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ **21. a.** $\pm 1, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$ **b.** $-5, \frac{1}{3}$, or $\frac{1}{2}$

22. a. $\pm 1, \pm 2, \pm 4, \pm\frac{1}{2}$ **23. a.** $\pm 1, \pm 2, \pm 4$ **b.** -2 or 2 **24. a.** $\pm 1, \pm 3, \pm 5, \pm 15$ **b.** -1 or 3 **25.** $f(x) = 2x^3 - 2x^2 + 50x - 50$

26. $f(x) = 2x^3 - 8x^2 + 8x - 32$ **27.** $f(x) = x^3 - 3x^2 - 15x + 125$ **28.** $f(x) = 3x^3 + 12x^2 - 93x - 522$ **30.** $f(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$

31. $f(x) = x^4 - 9x^3 + 21x^2 + 21x - 130$ **32.** $f(x) = 3x^4 - x^3 - 9x^2 + 159x - 52$

53. b.

$f(x) = -x^3 + x^2 + 16x - 16$

54. b.

$f(x) = -x^3 + 3x^2 - 4$

55. b.

$f(x) = 4x^3 - 8x^2 - 3x + 9$

56. b.

$f(x) = 3x^3 + 2x^2 + 2x - 1$

57. b.

$f(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$

58. b.

$f(x) = 2x^4 + 2x^3 - 22x^2 - 18x + 36$

59. b.

$f(x) = 3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$

60. b.

$f(x) = -5x^4 + 4x^3 - 19x^2 + 16x + 4$

72. $\left\{-\frac{1}{2}, 3, 5\right\}$ **77.** 5, 3, or 1 positive real zeros; no negative real zeros

79.

1 real zero, 2 nonreal complex zeros

80.

3 real zeros, 2 nonreal complex zeros

81.

2 real zeros, 2 nonreal complex zeros

82.

2 real zeros, 4 nonreal complex zeros

83. makes sense
85. makes sense
90. false

Mid-Chapter 2 Check Point

7. $\frac{3}{4} \pm i\frac{\sqrt{23}}{4}$

8.

$f(x) = (x - 3)^2 - 4$

domain: $(-\infty, \infty)$
range: $[-4, \infty)$

9.

$f(x) = 5 - (x + 2)^2$

domain: $(-\infty, \infty)$
range: $(-\infty, 5]$

10.

$f(x) = -x^2 - 4x + 5$

domain: $(-\infty, \infty)$
range: $(-\infty, 9]$

11.

$f(x) = 3x^2 - 6x + 1$

domain: $(-\infty, \infty)$
range: $[-2, \infty)$

12.

$f(x) = (x - 2)^2 (x + 1)^3$

13.

$f(x) = -(x - 2)^2 (x + 1)^2$

14.

$f(x) = x^3 - x^2 - 4x + 4$

15.

$f(x) = x^4 - 5x^2 + 4$

16.

$f(x) = -(x + 1)^6$

17.

$f(x) = -6x^3 + 7x^2 - 1$

18.

$f(x) = 2x^3 - 2x$

19.

$f(x) = x^3 - 2x^2 + 26x$

20.

$f(x) = -x^3 + 5x^2 - 5x - 3$

30. $2x^2 - x - 3 + \frac{x + 1}{3x^2 - 1}$ **31.** $2x^3 - 5x^2 - 3x + 6$
32. $f(x) = -2x^3 + 2x^2 - 2x + 2$ **33.** $f(x) = x^4 - 4x^3 + 13x^2 - 36x + 36$
34. yes

Section 2.6

Check Point Exercises

4.

$g(x) = \frac{1}{x + 2} - 1$

5.

$f(x) = \frac{3x - 3}{x - 2}$

6.

$f(x) = \frac{2x^2}{x^2 - 9}$

7.

$f(x) = \frac{x^4}{x^2 + 2}$

Exercise Set 2.6

3. $\{x \mid x \neq 5, x \neq -4\}$ **4.** $\{x \mid x \neq 2, x \neq -6\}$ **5.** $\{x \mid x \neq 7, x \neq -7\}$ **6.** $\{x \mid x \neq 8, x \neq -8\}$ **7.** all real numbers
8. all real numbers **21.** vertical asymptote: $x = -4$; no holes **22.** vertical asymptote: $x = 3$; no holes **23.** vertical asymptotes: $x = -4, x = 0$; no holes **24.** vertical asymptotes: $x = 3, x = 0$; no holes **25.** vertical asymptote: $x = -4$; hole at $x = 0$ **26.** vertical asymptote: $x = 3$; hole at $x = 0$

27. no vertical asymptotes; no holes **28.** no vertical asymptotes; no holes **29.** no vertical asymptotes: hole at $x = 3$ **30.** no vertical asymptotes: hole at $x = 5$ **31.** vertical asymptote: $x = -3$; hole at $x = 3$ **32.** vertical asymptote: $x = -5$; hole at $x = 5$ **33.** vertical asymptote: $x = 3$; hole at $x = -7$ **34.** vertical asymptote: $x = 4$; hole at $x = -6$ **35.** no vertical asymptotes; hole at $x = -7$ **36.** no vertical asymptotes; hole at $x = -6$

45.

$g(x) = \frac{1}{x-1}$

46.

$g(x) = \frac{1}{x-2}$

47.

$h(x) = \frac{1}{x} + 2$

48.

$h(x) = \frac{1}{x} + 1$

49.

$g(x) = \frac{1}{x+1} - 2$

50.

$g(x) = \frac{1}{x+2} - 2$

51.

$g(x) = \frac{1}{(x+2)^2}$

52.

$g(x) = \frac{1}{(x+1)^2}$

53.

$h(x) = \frac{1}{x^2} - 4$

54.

$h(x) = \frac{1}{x^2} - 3$

55.

$h(x) = \frac{1}{(x-3)^2} + 1$

56.

$h(x) = \frac{1}{(x-3)^2} + 2$

57.

$f(x) = \frac{4x}{x-2}$

58.

$f(x) = \frac{3x}{x-1}$

59.

$f(x) = \frac{2x}{x^2-4}$

60.

$f(x) = \frac{4x}{x^2-1}$

61.

$f(x) = \frac{2x^2}{x^2-1}$

62.

$f(x) = \frac{4x^2}{x^2-9}$

63.

$f(x) = \frac{-x}{x+1}$

64.

$f(x) = \frac{-3x}{x+2}$

65.

$f(x) = -\frac{1}{x^2-4}$

66.

$f(x) = -\frac{2}{x^2-1}$

67.

$f(x) = \frac{2}{x^2+x-2}$

68.

$f(x) = \frac{-2}{x^2-x-2}$

69.

$f(x) = \frac{2x^2}{x^2+4}$

70.

$f(x) = \frac{4x^2}{x^2+1}$

71.

$f(x) = \frac{x+2}{x^2+x-6}$

72.

$f(x) = \frac{x-4}{x^2-x-6}$

73.

$f(x) = \dfrac{x - 2}{x^2 - 4}$

74.

$f(x) = \dfrac{x - 3}{x^2 - 9}$

75.

$f(x) = \dfrac{x^4}{x^2 + 2}$

76.

$f(x) = \dfrac{2x^4}{x^2 + 1}$

77.

$f(x) = \dfrac{x^2 + x - 12}{x^2 - 4}$

78.

$f(x) = \dfrac{x^2}{x^2 + x - 6}$

79.

$f(x) = \dfrac{3x^2 + x - 4}{2x^2 - 5x}$

80.

$f(x) = \dfrac{x^2 - 4x + 3}{(x + 1)^2}$

81. b.

$f(x) = \dfrac{x^2 - 1}{x}$

82. b.

$f(x) = \dfrac{x^2 - 4}{x}$

83. b.

$f(x) = \dfrac{x^2 + 1}{x}$

84. b.

$f(x) = \dfrac{x^2 + 4}{x}$

85. a. Slant asymptote: $y = x + 4$

b.

$f(x) = \dfrac{x^2 + x - 6}{x - 3}$

86. a. Slant asymptote: $y = x$

b.

$f(x) = \dfrac{x^2 - x + 1}{x - 1}$

87. b.

$f(x) = \dfrac{x^3 + 1}{x^2 + 2x}$

88. b.

$f(x) = \dfrac{x^3 - 1}{x^2 - 9}$

89.

$f(x) = \dfrac{x + 2}{2x(x - 2)}$

90.

$f(x) = \dfrac{5x + 1}{2(x - 5)}$

91.

$f(x) = \dfrac{x - 6}{2(x - 3)}$

92.

$f(x) = \dfrac{-2}{(x + 2)(x + 3)}$

93.

$f(x) = \dfrac{x - 2}{x + 2}$

94.

$f(x) = \dfrac{(x - 1)(x + 1)}{x^2 + 1}$

95. $g(x) = \dfrac{1}{x + 3} + 2$

$g(x) = \dfrac{1}{x + 3} + 2$

96. $g(x) = \dfrac{1}{x + 2} + 3$

$g(x) = \dfrac{1}{x + 2} + 3$

97. $g(x) = \dfrac{-1}{x - 2} + 3$

$g(x) = \dfrac{-1}{x - 2} + 3$

98. $g(x) = \dfrac{-1}{x - 4} + 2$

$g(x) = \dfrac{-1}{x - 4} + 2$

99. b. $\overline{C}(x) = \dfrac{100x + 100{,}000}{x}$

c. $\overline{C}(500) = 300$, when 500 bicycles are produced, it costs \$300 to produce each bicycle; $\overline{C}(1000) = 200$, when 1000 bicycles are produced, it costs \$200 to produce each bicycle; $\overline{C}(2000) = 150$, when 2000 bicycles are produced, it costs \$150 to produce each bicycle; $\overline{C}(4000) = 125$, when 4000 bicycles are produced, it costs \$125 to produce each bicycle.

d. $y = 100$; The cost per bicycle approaches \$100 as more bicycles are produced.

100. b. $\overline{C}(x) = \dfrac{30x + 300{,}000}{x}$

c. $\overline{C}(1000) = 330$, when 1000 pairs of shoes are produced, it costs \$330 to produce each pair; $\overline{C}(10{,}000) = 60$, when 10,000 pairs of shoes are produced, its costs \$60 to produce each pair; $\overline{C}(100{,}000) = 33$, when 100,000 pairs of shoes are produced, it costs \$33 to produce each pair.

d. $y = 30$; The cost per pair of shoes approaches \$30 as more shoes are produced.

101. d. $y = 6.5$; Over time, the pH level rises back to normal. **e.** It quickly drops below normal and then slowly begins to approach the normal level. **102. c.** $y = 0$; Over time, the drug's concentration will approach 0 mg per liter. **103.** 90; An incidence ratio of 10 means 90% of the deaths are smoking related. **104.** 89; An incidence ratio of 9 means about 89% of the deaths are smoking related. **105.** $y = 100$; The percentage of deaths cannot exceed 100% as the incidence ratios increase. **106.** No; according to the model and its graph, there is no incidence ratio that will produce 100 percentage of deaths. **107. a.** $f(x) = \dfrac{1.75x^2 - 15.9x + 160}{2.1x^2 - 3.5x + 296}$ **c.** 66%; underestimates by 1%

122.

The graph approaches the horizontal asymptote faster and the vertical asymptote more slowly as n increases.

123.

The graph approaches the horizontal asymptote faster and the vertical asymptote more slowly as n increases.

124.

$g(x)$ is the graph of a line whereas $f(x)$ is the graph of a rational function with a slant asymptote; In $g(x)$, $x - 2$ is a factor of $x^2 - 5x + 6$.

125. a.

b. The graph increases and reaches a maximum of about 356 arrests per 100,000 drivers at age 25.

126. does not make sense

Section 2.7

Check Point Exercises

1.

2.

3.

4.

Exercise Set 2.7

1. $(-\infty, -2) \cup (4, \infty)$

2. $(-\infty, -3) \cup (5, \infty)$

3. $[-3, 7]$

4. $[-1, 7]$

5. $(-\infty, 1) \cup (4, \infty)$

6. $(1, 3)$

7. $(-\infty, -4) \cup (-1, \infty)$

8. $(-\infty, -3) \cup (2, \infty)$

9. $\varnothing$

10. $(-\infty, 1) \cup (1, \infty)$

11. $\left[-4, \frac{2}{3}\right]$

12. $\left(-\infty, -\frac{2}{3}\right] \cup \left[\frac{1}{3}, \infty\right)$

13. $\left(-3, \frac{5}{2}\right)$

14. $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{3}, \infty\right)$

15. $\left(-1, -\frac{3}{4}\right)$

16. $\left(-5, -\frac{1}{3}\right)$

17. $\left[-2, \frac{1}{3}\right]$

18. $(-\infty, \infty)$

19. $(-\infty, 0] \cup [4, \infty)$

20. $(-2, 0)$

21. $\left(-\infty, -\frac{3}{2}\right) \cup (0, \infty)$

22. $\left[0, \frac{5}{3}\right]$

23. $[0, 1]$

24. $[0, 2]$

25. $[2 - \sqrt{2}, 2 + \sqrt{2}]$

26. $[1 - \sqrt{3}, 1 + \sqrt{3}]$

27. $\varnothing$

28. $(-\infty, \infty)$

29. $[1, 2] \cup [3, \infty)$

30. $[-3, -2] \cup [-1, \infty)$

31. $[0, 3] \cup [5, \infty)$

32. $[0, 4] \cup [6, \infty)$

33. $(-\infty, 2) \cup \left(2, \frac{7}{2}\right)$

34. $(-\infty, 5) \cup \left(5, \frac{13}{2}\right)$

35. $[-2, -1] \cup [1, \infty)$

36. $\{-2\} \cup [2, \infty)$

37. $(-\infty, -3)$

38. $(-\infty, -7) \cup (-1, 1)$

39. $(-1, \infty)$

40. $(1, \infty)$

41. $\{0\} \cup [9, \infty)$

42. $(-\infty, 4]$

43. $(-\infty, -3) \cup (4, \infty)$

44. $(-\infty, -5) \cup (2, \infty)$

45. $(-4, -3)$

46. $(-5, -2)$

47. $[2, 4)$

48. $(-\infty, -3] \cup (-2, \infty)$

49. $\left(-\infty, -\frac{4}{3}\right) \cup [2, \infty)$

50. $\left[-\frac{5}{3}, 3\right)$

51. $(-\infty, 0) \cup (3, \infty)$

52. $(-\infty, -4) \cup (0, \infty)$

53. $(-\infty, -4] \cup (-2, 1]$

54. $(-\infty, -3] \cup (-1, 2]$

55. $(-\infty, -5) \cup (-3, \infty)$

56. $(1, 2)$

57. $\left(-\infty, \frac{1}{2}\right) \cup \left[\frac{7}{5}, \infty\right)$

58. $(-\infty, 3) \cup (4, \infty)$

59. $(-\infty, -6] \cup (-2, \infty)$

60. $[-4, -2)$

61. $\left(-\infty, \frac{1}{2}\right] \cup [2, \infty)$ **62.** $\left(-\infty, \frac{1}{4}\right) \cup (2, \infty)$ **63.** $(-\infty, -1) \cup [1, \infty)$ **64.** $\left(\frac{1}{2}, 1\right]$

65. $(-\infty, -8) \cup (-6, 4) \cup (6, \infty)$

66. $(-\infty, -7) \cup (1, \infty)$

67. $(-3, 2)$

68. $(-\infty, -3) \cup (-1, 1)$

69. $(-\infty, -1) \cup (1, 2) \cup (3, \infty)$

70. $(-\infty, -1) \cup (1, 2) \cup (3, \infty)$

71. $\left[-6, -\frac{1}{2}\right] \cup [1, \infty)$ **72.** $(-\infty, -6) \cup \left(-\frac{1}{2}, 1\right)$
73. $(-\infty, -2) \cup [-1, 2)$ **74.** $(-2, -1) \cup (2, \infty)$

75. between 0 and $\frac{1}{2}$ second **76.** between 0 and 3 seconds **77. c.** extremely well; Function values and data are identical. **d.** speeds exceeding 76 miles per hour; points on (b) to the right of (76, 540) **78. c.** extremely well **d.** speeds exceeding 68 miles per hour; points on (a) to the right of (68, 540) **86.** $(-\infty, -5) \cup (2, \infty)$ **87.** $\left[-3, \frac{1}{2}\right]$ **88.** $(-2, -1) \cup (2, \infty)$ **91.** $(-4, -1) \cup [2, \infty)$ **92. b.** speeds exceeding 60 miles per hour **93. b.** speeds exceeding 52 miles per hour **95.** does not make sense **96.** makes sense **97.** does not make sense **104.** $(-\infty, 2) \cup (2, \infty)$
107. $(-\infty, 2) \cup (2, \infty)$
109.
$$\begin{aligned} 27 - 3x^2 &\geq 0 \\ 3x^2 &\leq 27 \\ x^2 &\leq 9 \\ -3 \leq x &\leq 3 \end{aligned}$$
110. a. 16 **c.** 400 **111. a.** 96 **c.** 32

Exercise Set 2.8

13. $x = \frac{kz^3}{y}; y = \frac{kz^3}{x}$ **14.** $x = \frac{k\sqrt[3]{z}}{y}; y = \frac{k\sqrt[3]{z}}{x}$ **15.** $x = \frac{kyz}{\sqrt{w}}; y = \frac{x\sqrt{w}}{kz}$ **16.** $x = \frac{kyz}{w^2}; y = \frac{xw^2}{kz}$ **17.** $x = kz(y + w); y = \frac{x - kzw}{kz}$

18. $x = kz(y - w); y = \frac{x + kzw}{kz}$ **19.** $x = \frac{kz}{y - w}; y = \frac{xw + kz}{x}$ **20.** $x = \frac{kz}{y + w}; y = \frac{kz - xw}{x}$ **36.** $\frac{1}{3}$ as long as it seems now

37. a. $C = \dfrac{kP_1P_2}{d^2}$ **38.** Yes, the wind will exert a force of 360 pounds.

39. a.

b. Current varies inversely as resistance.

51. makes sense **55.** Reduce the resistance by a factor of $\frac{1}{3}$.

58.

59.

60.

Chapter 2 Review Exercises

12. $\left\{\frac{3}{2} \pm \frac{1}{2}i\right\}$

13.

axis of symmetry; $x = -1$
domain: $(-\infty, \infty)$; range: $(-\infty, 4]$

14.

axis of symmetry: $x = -4$
domain: $(-\infty, \infty)$; range: $[-2, \infty)$

15.

axis of symmetry: $x = 1$
domain: $(-\infty, \infty)$; range: $(-\infty, 4]$

16.

axis of symmetry: $x = 1$
domain: $(-\infty, \infty)$; range: $[-8, \infty)$

17. a. maximum is -57 at $x = 7$ **b.** domain: $(-\infty, \infty)$; range: $(-\infty, -57]$

18. a. minimum is 685 at $x = -3$
b. domain: $(-\infty, \infty)$; range: $[685, \infty)$

19. b. 6 ft **d.**

20. 250 yd by 500 yd; 125,000 sq yard

28. b. no; The model indicates increasing deforestation despite a declining rate in which the forest is being cut down. **d.** no; The model indicates the amount of forest cleared, in square kilometers, will eventually be negative, which is not possible. **29.** No; the graph falls to the right, so eventually there would be a negative number of thefts, which is not possible. **30.** $x = 1$, multiplicity 1, crosses; $x = -2$, multiplicity 2, touches; $x = -5$, multiplicity 3, crosses **31.** $x = -5$, multiplicity 1, crosses; $x = 5$, multiplicity 2, touches **32.** $f(1)$ is negative and $f(2)$ is positive, so by the Intermediate Value Theorem, f has a real zero between 1 and 2.

33. a. The graph falls to the left and rises to the right.
b. no symmetry
c.

34. a. The graph rises to the left and falls to the right.
b. origin symmetry
c.

35. a. The graph falls to the left and rises to the right.
b. no symmetry
c.

36. a. The graph falls to the left and to the right.
b. y-axis symmetry
c.

37. a. The graph falls to the left and to the right.
b. no symmetry
c.

38. a. The graph rises to the left and to the right.
b. no symmetry
c.

39.

$f(x) = 2x^2(x-1)^3(x+2)$

40.

$f(x) = -x^3(x+4)^2(x-1)$

42. $2x^2 - 4x + 1 - \dfrac{10}{5x-3}$ **48.** $\{4, -2 \pm \sqrt{5}\}$

50. $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{8}{3}, \pm\frac{4}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}$

52. 3 or 1 positive real zeros; 2 or 0 negative real zeros

53. No sign variations exist for either $f(x)$ or $f(-x)$, so no real roots exist.

54. b. 1 positive real zero; 2 or no negative real zeros **c.** -2 or 1

55. b. 2 or 0 positive real zeros; 1 negative real zero **c.** $-1, \frac{1}{3}$, or $\frac{1}{2}$

56. a. $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{8}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{3}{8}, \pm\frac{5}{2}, \pm\frac{5}{4}, \pm\frac{5}{8}, \pm\frac{15}{2}, \pm\frac{15}{4}, \pm\frac{15}{8}$ **b.** 3 or 1 positive real solutions; no negative real solutions **c.** $\frac{1}{2}, \frac{3}{2}$, or $\frac{5}{2}$ **57. b.** 2 or 0 positive real solutions; 1 negative solution **d.** $\left\{\frac{1}{2}, \frac{-5-\sqrt{29}}{2}, \frac{-5+\sqrt{29}}{2}\right\}$

58. b. 2 or zero positive real solutions; 2 or zero negative real solutions **c.** $-2, -1, 1$, or 3

59. b. 1 positive real root; 1 negative real root **c.** $-\frac{1}{2}$ or $\frac{1}{2}$ **60. b.** 2 or no positive zeros; 2 or no negative zeros **c.** $-2, -1, \frac{1}{2}$, or 2

61. $f(x) = x^3 - 6x^2 + 21x - 26$ **62.** $f(x) = 2x^4 + 12x^3 + 20x^2 + 12x + 18$ **63.** $f(x) = (x-i)(x+i)(x+2)(2x-1)$

64. $g(x) = (x+1)^2(x-4)^2$ **65.** 4 real zeros, one with multiplicity two **66.** 3 real zeros; 2 nonreal complex zeros

67. 2 real zeros, one with multiplicity two; 2 nonreal complex zeros **68.** 1 real zero; 4 nonreal complex zeros

69.

$g(x) = \dfrac{1}{(x+2)^2} - 1$

70.

$h(x) = \dfrac{1}{x-1} + 3$

71. Vertical asymptotes: $x = 3$ and $x = -3$
horizontal asymptote: $y = 0$

$f(x) = \dfrac{2x}{x^2 - 9}$

72. Vertical asymptote: $x = -3$
horizontal asymptote: $y = 2$

$f(x) = \dfrac{2x-4}{x+3}$

73. Vertical asymptotes: $x = 3$ and $x = -2$
horizontal asymptote: $y = 1$

$h(x) = \dfrac{x^2 - 3x - 4}{x^2 - x - 6}$

74. Vertical asymptote: $x = -2$
horizontal asymptote: $y = 1$

$r(x) = \dfrac{x^2 + 4x + 3}{(x+2)^2}$

75. Vertical asymptote: $x = -1$
no horizontal asymptote
slant asymptote: $y = x - 1$

$y = \dfrac{x^2}{x+1}$

76. Vertical asymptote: $x = 3$
no horizontal asymptote
slant asymptote: $y = x + 5$

$y = \dfrac{x^2 + 2x - 3}{x - 3}$

77. No vertical asymptote
no horizontal asymptote
slant asymptote: $y = -2x$

$f(x) = \dfrac{-2x^3}{x^2 + 1}$

78. Vertical asymptote: $x = \dfrac{3}{2}$
no horizontal asymptote
slant asymptote: $y = 2x - 5$

$g(x) = \dfrac{4x^2 - 16x + 16}{2x - 3}$

79. b. $\overline{C}(x) = \dfrac{25x + 50{,}000}{x}$

c. $\overline{C}(50) = 1025$, when 50 calculators are manufactured, it costs \$1025 to manufacture each; $\overline{C}(100) = 525$, when 100 calculators are manufactured, it costs \$525 to manufacture each; $\overline{C}(1000) = 75$, when 1000 calculators are manufactured, it costs \$75 to manufacture each; $\overline{C}(100{,}000) = 25.5$, when 100,000 calculators are manufactured, it costs \$25.50 to manufacture each.

d. $y = 25$; costs will approach \$25.

80. $y = 3000$; The number of fish in the pond approaches 3000.

81. $y = 0$; As the number of years of education increases the percentage rate of unemployment approaches zero.

82. b. $R(x) = \dfrac{1.53x + 114.8}{2.99x + 235.5}$ **c.** $y = 0.51$; Over time the percentage of men in the U.S. population will approach 51%.

85. $\left(-3, \frac{1}{2}\right)$

86. $(-\infty, -4] \cup \left[-\frac{1}{2}, \infty\right)$

87. $(-3, 0) \cup (1, \infty)$

88. $(-\infty, -2) \cup (6, \infty)$

89. $[-1, 1) \cup [2, \infty)$

90. $(-\infty, 4) \cup \left[\frac{23}{4}, \infty\right)$

91. a. 261 ft; overestimates by 1 ft **b.** speeds exceeding 40 miles per hour

Chapter 2 Test

4. $\{2 \pm i\}$

5.

$f(x) = (x + 1)^2 + 4$

axis of symmetry: $x = -1$
domain: $(-\infty, \infty)$; range: $[4, \infty)$

6.

$f(x) = x^2 - 2x - 3$

axis of symmetry: $x = 1$
domain: $(-\infty, \infty)$; range: $[-4, \infty)$

7. a. maximum of 2 at $x = 3$;
b. domain: $(-\infty, \infty)$; range: $(-\infty, 2]$

10. b.

$f(x) = x^3 - 5x^2 - 4x + 20$

11. Since the degree of the polynomial is odd and the leading coefficient is positive, the graph of f should fall to the left and rise to the right. The x-intercepts should be -1, 0, and 1.

14. 3 or 1 positive real zeros; no negative real zeros

16. a. $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

19.

$f(x) = -3x^3 - 4x^2 + x + 2$

20. domain: $\{x | x \neq -3\}$

$f(x) = \dfrac{1}{(x + 3)^2}$

21. domain: $\{x | x \neq 1\}$

$f(x) = \dfrac{1}{x - 1} + 2$

22. domain: $\{x | x \neq 4, x \neq -4\}$

$f(x) = \dfrac{x}{x^2 - 16}$

23. domain: $\{x | x \neq 2\}$

$f(x) = \dfrac{x^2 - 9}{x - 2}$

24. domain: $\{x | x \neq -3, x \neq 1\}$

$f(x) = \dfrac{x + 1}{x^2 + 2x - 3}$

25. domain: all real numbers

$f(x) = \dfrac{4x^2}{x^2 + 3}$

26. a. $\overline{C}(x) = \dfrac{300{,}000 + 10x}{x}$

b. $y = 10$; As the number of satellite radio players increases, the average cost approaches \$10.

27. $(-3, 4)$

28. $(-\infty, 3) \cup [10, \infty)$

Cumulative Review Exercises (Chapters P–2)

1. domain: $(-2, 2)$; range: $[0, \infty)$ **2.** -1 and 1, both of multiplicity 2

6.

$g(x) = f(x + 2) + 1$

8. $\left\{\dfrac{5 + \sqrt{13}}{6}, \dfrac{5 - \sqrt{13}}{6}\right\}$ **10.** $\{-3, -1, 2\}$ **11.** $(-\infty, 1) \cup (4, \infty)$ **12.** $(-\infty, -1) \cup \left(\frac{5}{3}, \infty\right)$

13.

$f(x) = x^3 - 4x^2 - x + 4$

14.

$f(x) = x^2 + 2x - 8$

15.

$f(x) = x^2(x - 3)$

16.

$f(x) = \dfrac{x - 1}{x - 2}$

17.

18.

$x^2 + y^2 - 2x + 4y - 4 = 0$

19. $(f \circ g)(x) = 32x^2 - 20x + 2$ **20.** $4x + 2h - 1$

CHAPTER 3

Section 3.1

Check Point Exercises

2.

$f(x) = 3^x$

3.

$f(x) = \left(\dfrac{1}{3}\right)^x$

4.

$f(x) = 3^x$
$g(x) = 3^{x-1}$

5.

$f(x) = 2^x$
$g(x) = 2^x + 1$

Exercise Set 3.1

1. 10.556 **2.** 13.967 **3.** 11.665 **4.** 16.242 **5.** 0.125 **6.** 0.116 **7.** 9.974 **8.** 29.964 **9.** 0.387 **10.** 0.472

11.

$f(x) = 4^x$

12.

$f(x) = 5^x$

13.

$g(x) = \left(\dfrac{3}{2}\right)^x$

14.

$g(x) = \left(\dfrac{4}{3}\right)^x$

15.

$h(x) = \left(\dfrac{1}{2}\right)^x$

16.

$h(x) = \left(\dfrac{1}{3}\right)^x$

17.

$f(x) = (0.6)^x$

18.

$f(x) = (0.8)^x$

19. $H(x) = -3^{-x}$
20. $g(x) = 3^{x-1}$
21. $F(x) = -3^x$
22. $f(x) = 3^x$
23. $h(x) = 3^x - 1$
24. $G(x) = 3^{-x}$

25.

$f(x) = 2^x$
$g(x) = 2^{x+1}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

26.

$f(x) = 2^x$
$g(x) = 2^{x+2}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

27.

$f(x) = 2^x$
$g(x) = 2^x - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$
range: $(-1, \infty)$

28.

$f(x) = 2^x$
$g(x) = 2^x + 2$

asymptote: $y = 2$
domain: $(-\infty, \infty)$
range: $(2, \infty)$

29.

$f(x) = 2^x$
$h(x) = 2^{x+1} - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$
range: $(-1, \infty)$

30.

$f(x) = 2^x$
$h(x) = 2^{x+2} - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$
range: $(-1, \infty)$

31.

$f(x) = 2^x$
$g(x) = -2^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(-\infty, 0)$

32.

$f(x) = 2^x$
$g(x) = 2^{-x}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

33.

$f(x) = 2^x$
$g(x) = 2 \cdot 2^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

34.

$f(x) = 2^x$
$g(x) = \frac{1}{2} \cdot 2^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

35.

$g(x) = e^{x-1}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

36.

$g(x) = e^{x+1}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

37.

$g(x) = e^x + 2$

asymptote: $y = 2$
domain: $(-\infty, \infty)$
range: $(2, \infty)$

38.

$g(x) = e^x - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$
range: $(-1, \infty)$

39.

$h(x) = e^{x-1} + 2$

asymptote: $y = 2$
domain: $(-\infty, \infty)$
range: $(2, \infty)$

40.

$h(x) = e^{x+1} - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$
range: $(-1, \infty)$

41.

$h(x) = e^{-x}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

42.

$h(x) = -e^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(-\infty, 0)$

43.

$g(x) = 2e^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

44.

$g(x) = \frac{1}{2} e^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

45.

$h(x) = e^{2x} + 1$

asymptote: $y = 1$
domain: $(-\infty, \infty)$
range: $(1, \infty)$

46.

$h(x) = e^{x/2} + 2$

asymptote: $y = 2$
domain: $(-\infty, \infty)$
range: $(2, \infty)$

47.

$f(x) = 3^x$
$g(x) = 3^{-x}$

asymptote of f: $y = 0$
asymptote of g: $y = 0$

48.

$f(x) = 3^x$
$g(x) = -3^x$

asymptote of f: $y = 0$
asymptote of g: $y = 0$

49.

$f(x) = 3^x$
$g(x) = \frac{1}{3} \cdot 3^x$

asymptote of f: $y = 0$
asymptote of g: $y = 0$

50.

$f(x) = 3^x$
$g(x) = 3 \cdot 3^x$

asymptote of f: $y = 0$
asymptote of g: $y = 0$

51.

$f(x) = \left(\frac{1}{2}\right)^x$
$g(x) = \left(\frac{1}{2}\right)^{x-1} + 1$

asymptote of f: $y = 0$
asymptote of g: $y = 1$

52.

$f(x) = \left(\frac{1}{2}\right)^x$
$g(x) = \left(\frac{1}{2}\right)^{x-1} + 2$

asymptote of f: $y = 0$
asymptote of g: $y = 2$

53. **a.** \$13,116.51
b. \$13,140.67
c. \$13,157.04
d. \$13,165.31

54. **a.** \$9479.19
b. \$9527.79
c. \$9560.92
d. \$9577.70

57.

58.

59.

60.

69. 3.249009585; 3.317278183; 3.321880096; 3.321995226; 3.321997068; $2^{\sqrt{3}} \approx 3.321997085$; The closer the exponent is to $\sqrt{3}$, the closer the value is to $2^{\sqrt{3}}$.
70. 8; 8.5741877; 8.815240927; 8.821353305; 8.824411082; 8.824961595; 8.824979946; $2^{\pi} \approx 8.824977827$; The closer the exponent gets to π, the closer the value is to 2^{π}. **74. a.** approximately \$5,027,378,919 **b.** approximately \$5,231,970,592

81. a. $A = 10{,}000\left(1 + \frac{0.05}{4}\right)^{4t}$; $A = 10{,}000\left(1 + \frac{0.045}{12}\right)^{12t}$

b.

5% interest compounded quarterly

82. a.

b.

c.

84. makes sense

92.

93. a. $\cosh(-x) = \frac{e^{-x} + e^{-(-x)}}{2} = \frac{e^{-x} + e^{x}}{2} = \frac{e^{x} + e^{-x}}{2} = \cosh x$

b. $\sinh(-x) = \frac{e^{-x} - e^{-(-x)}}{2} = \frac{e^{-x} - e^{x}}{2} = -\frac{e^{x} - e^{-x}}{2} = -\sinh x$

c. $\left(\frac{e^{x} + e^{-x}}{2}\right)^2 - \left(\frac{e^{x} - e^{-x}}{2}\right)^2 \stackrel{?}{=} 1$

$\frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} \stackrel{?}{=} 1$

$\frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4} \stackrel{?}{=} 1$

$\frac{4}{4} \stackrel{?}{=} 1$

$1 = 1$

94. We don't know how to solve $x = 2^y$ for y.

Section 3.2

Check Point Exercises

6.

Exercise Set 3.2

9. $\log_2 8 = 3$ **10.** $\log_5 625 = 4$ **11.** $\log_2 \frac{1}{16} = -4$ **12.** $\log_5 \frac{1}{125} = -3$ **13.** $\log_8 2 = \frac{1}{3}$ **14.** $\log_{64} 4 = \frac{1}{3}$ **15.** $\log_{13} x = 2$

16. $\log_{15} x = 2$ **17.** $\log_b 1000 = 3$ **18.** $\log_b 343 = 3$ **19.** $\log_7 200 = y$ **20.** $\log_8 300 = y$

43.

44.

45.

46.

47. $H(x) = 1 - \log_3 x$ **48.** $G(x) = \log_3(-x)$ **49.** $h(x) = \log_3 x - 1$ **50.** $F(x) = -\log_3 x$ **51.** $g(x) = \log_3(x - 1)$ **52.** $f(x) = \log_3 x$

53.

vertical asymptote: $x = -1$
domain: $(-1, \infty)$
range: $(-\infty, \infty)$

54.

vertical asymptote: $x = -2$
domain: $(-2, \infty)$
range: $(-\infty, \infty)$

55.

vertical asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

56.

vertical asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

57.

$f(x) = \log_2 x$

$g(x) = \frac{1}{2}\log_2 x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

58.

$f(x) = \log_2 x$

$g(x) = -2\log_2 x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

59.

$g(x) = \log(x - 1)$

asymptote: $x = 1$
domain: $(1, \infty)$
range: $(-\infty, \infty)$

60.

$g(x) = \log(x - 2)$

asymptote: $x = 2$
domain: $(2, \infty)$
range: $(-\infty, \infty)$

61.

$h(x) = \log x - 1$

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

62.

$h(x) = \log x - 2$

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

63.

$g(x) = 1 - \log x$

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

64.

$g(x) = 2 - \log x$

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

65.

$g(x) = \ln(x + 2)$

asymptote: $x = -2$
domain: $(-2, \infty)$
range: $(-\infty, \infty)$

66.

$x = -1$
$g(x) = \ln(x + 1)$

asymptote: $x = -1$
domain: $(-1, \infty)$
range: $(-\infty, \infty)$

67.

$h(x) = \ln(2x)$

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

68.

$h(x) = \ln\left(\frac{1}{2}x\right)$

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

69.

$g(x) = 2\ln x$

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

70.

$g(x) = \frac{1}{2}\ln x$

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

71.

$h(x) = -\ln x$

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

72.

$h(x) = \ln(-x)$

asymptote: $x = 0$
domain: $(-\infty, 0)$
range: $(-\infty, \infty)$

73.

$g(x) = 2 - \ln x$

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

74.

$g(x) = 1 - \ln x$

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

75. $(-4, \infty)$
79. $(-\infty, 2) \cup (2, \infty)$ **80.** $(-\infty, 7) \cup (7, \infty)$
101. $3^2 = x - 1; \{10\}$ **102.** $5^2 = x + 4; \{21\}$
109. $(-\infty, -1) \cup (2, \infty)$
110. $(-\infty, -2) \cup (6, \infty)$
111. $(-\infty, -1) \cup (5, \infty)$
112. $(-\infty, -5) \cup (2, \infty)$
117. ≈ 188 db; yes

119. b. 71.5; 63.9; 58.8; 55; 52; 49.5

c.

Material retention decreases as time passes.

128.

$g(x)$ is $f(x)$ shifted left 3 units.

129.

$g(x)$ is $f(x)$ shifted upward 3 units.

130.

$g(x)$ is $f(x)$ reflected about the x-axis.

131.

$g(x)$ is $f(x)$ shifted right 2 units and upward 1 unit.

132.

The score falls below 65 after 9 months.

133. a.

b.

c.

d. They are the same. $\log_b M + \log_b N$

134. $y = \ln x, y = \sqrt{x}, y = x, y = x^2, y = e^x, y = x^x$

135. makes sense **138.** does not make sense **142.** true **147. c.** $\log_2(8 \cdot 4) = \log_2 8 + \log_2 4$

Exercise Set 3.3

1. $\log_5 7 + \log_5 3$ **2.** $\log_8 13 + \log_8 7$ **3.** $1 + \log_7 x$ **4.** $1 + \log_9 x$ **5.** $3 + \log x$ **6.** $4 + \log x$ **7.** $1 - \log_7 x$ **8.** $1 - \log_9 x$
9. $\log x - 2$ **10.** $\log x - 3$ **11.** $3 - \log_4 y$ **12.** $3 - \log_5 y$ **13.** $2 - \ln 5$ **14.** $4 - \ln 8$ **21.** $2\log_b x + \log_b y$ **22.** $\log_b x + 3\log_b y$
23. $\frac{1}{2}\log_4 x - 3$ **24.** $\frac{1}{2}\log_5 x - 2$ **25.** $2 - \frac{1}{2}\log_6(x + 1)$ **26.** $2 - \frac{1}{2}\log_8(x + 1)$ **27.** $2\log_b x + \log_b y - 2\log_b z$
28. $3\log_b x + \log_b y - 2\log_b z$ **29.** $1 + \frac{1}{2}\log x$ **30.** $\frac{1}{2} + \frac{1}{2}\ln x$ **31.** $\frac{1}{3}\log x - \frac{1}{3}\log y$ **32.** $\frac{1}{5}\log x - \frac{1}{5}\log y$
33. $\frac{1}{2}\log_b x + 3\log_b y - 3\log_b z$ **34.** $\frac{1}{3}\log_b x + 4\log_b y - 5\log_b z$ **35.** $\frac{2}{3}\log_5 x + \frac{1}{3}\log_5 y - \frac{2}{3}$ **36.** $\frac{1}{5}\log_2 x + \frac{4}{5}\log_2 y - \frac{4}{5}$
37. $3\ln x + \frac{1}{2}\ln(x^2 + 1) - 4\ln(x + 1)$ **38.** $4\ln x + \frac{1}{2}\ln(x^2 + 3) - 5\ln(x + 3)$ **39.** $1 + 2\log x + \frac{1}{3}\log(1 - x) - \log 7 - 2\log(x + 1)$
40. $2 + 3\log x + \frac{1}{3}\log(5 - x) - \log 3 - 2\log(x + 7)$ **47.** $\log\left(\frac{2x + 5}{x}\right)$ **48.** $\log\left(\frac{3x + 7}{x}\right)$ **51.** $\ln(x^{1/2}y)$ or $\ln(y\sqrt{x})$
52. $\ln(x^{1/3}y)$ or $\ln(y\sqrt[3]{x})$ **55.** $\ln\left(\frac{x^5}{y^2}\right)$ **56.** $\ln\left(\frac{x^7}{y^3}\right)$ **57.** $\ln\left(\frac{x^3}{y^{1/3}}\right)$ or $\ln\left(\frac{x^3}{\sqrt[3]{y}}\right)$ **58.** $\ln\left(\frac{x^2}{y^{1/2}}\right)$ or $\ln\left(\frac{x^2}{\sqrt{y}}\right)$ **59.** $\ln\frac{(x + 6)^4}{x^3}$
60. $\ln\frac{(x + 9)^8}{x^4}$ **61.** $\ln\left(\frac{x^3y^5}{z^6}\right)$ **62.** $\ln\left(\frac{x^4y^7}{z^3}\right)$ **65.** $\log_5\left(\frac{\sqrt{xy}}{(x + 1)^2}\right)$ **66.** $\log_4\left((x + 1)^2\sqrt[3]{\frac{x}{y}}\right)$ **67.** $\ln\sqrt[3]{\frac{(x + 5)^2}{x(x^2 - 4)}}$
68. $\ln\sqrt[3]{\frac{(x + 6)^5}{x(x^2 - 25)}}$ **69.** $\log\frac{x(x^2 - 1)}{7(x + 1)} = \log\frac{x(x - 1)}{7}$ **70.** $\log\frac{x(x^2 - 4)}{15(x + 2)} = \log\frac{x(x - 2)}{15}$

79.

80.

81.

82.

87. $\frac{1}{2}A - \frac{3}{2}C$ **88.** $\frac{1}{2}C - 2A$ **90.** false; ln 0 is undefined. **91.** false; $\log_4(2x)^3 = 3\log_4(2x)$ **94.** false; $\ln(x \cdot 1) = \ln x + \ln 1$
95. true **96.** false; $\ln x + \ln(2x) = \ln(2x^2)$ **97.** false; $\log(x + 3) - \log(2x) = \log\left(\frac{x + 3}{2x}\right)$ **98.** false; $\log\left(\frac{x + 2}{x - 1}\right) = \log(x + 2) - \log(x - 1)$
102. false; $e^x = \ln e^{e^x}$

113. a.

b.

$y = 2 + \log_3 x$ shifts the graph of $y = \log_3 x$ two units upward; $y = \log_3(x + 2)$ shifts the graph of $y = \log_3 x$ two units left; $y = -\log_3 x$ reflects the graph of $y = \log_3 x$ about the x-axis.

114.

$y = \log(10x) = 1 + \log x$
shifts $y = \log x$ one unit upward;
$y = \log(0.1x) = -1 + \log x$
shifts $y = \log x$ one unit downward;
the product rule

115.

c. The graph of the equation with the largest b will be on the top in the interval $(0, 1)$ and on the bottom in the interval $(1, \infty)$.

121. makes sense **123.** makes sense **129.** $\log e = \log_{10} e = \dfrac{\ln e}{\ln 10} = \dfrac{1}{\ln 10}$ **130.** $\dfrac{2A}{B}$

132.
$$\frac{f(x+h) - f(x)}{h} = \frac{\log_b(x+h) - \log_b x}{h} = \frac{\log_b\left(\dfrac{x+h}{x}\right)}{h} = \frac{1}{h}\log_b\left(1 + \frac{h}{x}\right) = \log_b\left(1 + \frac{h}{x}\right)^{\frac{1}{h}}$$

133. Let $\log_b M = R$ and $\log_b N = S$.
Then $\log_b M = R$ means $b^R = M$ and $\log_b N = S$ means $b^S = N$.
$$\frac{M}{N} = \frac{b^R}{b^S} = b^{R-S}$$
$$\log_b \frac{M}{N} = \log_b b^{R-S} = R - S = \log_b M - \log_b N$$

Mid-Chapter 3 Check Point

1.

$f(x) = 2^x$
$g(x) = 2^x - 3$

asymptote of f: $y = 0$
asymptote of g: $y = -3$
domain of f = domain of $g = (-\infty, \infty)$
range of $f = (0, \infty)$; range of $g = (-3, \infty)$

2.

$f(x) = \left(\dfrac{1}{2}\right)^x$
$g(x) = \left(\dfrac{1}{2}\right)^{x-1}$

asymptote of f: $y = 0$
asymptote of g: $y = 0$
domain of f = domain of $g = (-\infty, \infty)$
range of f = range of $g = (0, \infty)$

3.

$f(x) = e^x$
$g(x) = \ln x$

asymptote of f: $y = 0$
asymptote of g: $x = 0$
domain of f = range of $g = (-\infty, \infty)$
range of f = domain of $g = (0, \infty)$

4.

$f(x) = \log_2 x$
$g(x) = \log_2 (x - 1) + 1$

asymptote of f: $x = 0$
asymptote of g: $x = 1$
domain of $f = (0, \infty)$; domain of $g = (1, \infty)$
range of f = range of $g = (-\infty, \infty)$

5.

$f(x) = \log_{1/2} x$
$g(x) = -2 \log_{1/2} x$

asymptote of f: $x = 0$
asymptote of g: $x = 0$
domain of f = domain of $g = (0, \infty)$
range of f = range of $g = (-\infty, \infty)$

8. $(-\infty, -6) \cup (-6, \infty)$ **14.** 2

15. Evaluation not possible; $\log_2 \dfrac{1}{8} = -3$ and $\log_3(-3)$ is undefined.

Exercise Set 3.4

23. $\left\{\dfrac{\ln 3.91}{\ln 10}\right\}$; ≈ 0.59 **24.** $\left\{\dfrac{\ln 8.07}{\ln 10}\right\}$; ≈ 0.91 **27.** $\left\{\dfrac{\ln 17}{\ln 5}\right\}$; ≈ 1.76 **28.** $\left\{\dfrac{\ln 143}{\ln 19}\right\}$; ≈ 1.69 **31.** $\left\{\dfrac{\ln 659}{5}\right\}$; ≈ 1.30

32. $\left\{\dfrac{\ln 2568.25}{7}\right\}$; ≈ 1.12 **33.** $\left\{\dfrac{\ln 793 - 1}{-5}\right\}$; ≈ -1.14 **34.** $\left\{\dfrac{\ln 7957 - 1}{-8}\right\}$; ≈ -1.00 **35.** $\left\{\dfrac{\ln 10{,}478 + 3}{5}\right\}$; ≈ 2.45

36. $\left\{\dfrac{\ln 11{,}250 + 5}{4}\right\}$; ≈ 3.58 **37.** $\left\{\dfrac{\ln 410}{\ln 7} - 2\right\}$; ≈ 1.09 **38.** $\left\{\dfrac{\ln 137}{\ln 5} + 3\right\}$; ≈ 6.06 **39.** $\left\{\dfrac{\ln 813}{0.3 \ln 7}\right\}$; ≈ 11.48

40. $\left\{\dfrac{7 \ln 0.2}{\ln 3}\right\}$; ≈ -10.25 **41.** $\left\{\dfrac{3 \ln 5 + \ln 3}{\ln 3 - 2 \ln 5}\right\}$; ≈ -2.80 **42.** $\left\{\dfrac{2 \ln 3 - \ln 7}{2 \ln 7 - \ln 3}\right\}$; ≈ 0.09 **43.** $\{0, \ln 2\}$; $\ln 2 \approx 0.69$ **44.** $\{\ln 3\}$; ≈ 1.10

45. $\left\{\dfrac{\ln 3}{2}\right\}$; ≈ 0.55 **46.** $\left\{\dfrac{\ln 6}{2}\right\}$; ≈ 0.90 **48.** $\left\{\dfrac{\ln 3}{\ln 2}\right\}$; ≈ 1.58 **61.** $\left\{\dfrac{e^4}{2}\right\}$; ≈ 27.30 **62.** $\left\{\dfrac{e^5}{2}\right\}$; ≈ 74.21 **63.** $\{e^{-1/2}\}$; ≈ 0.61

64. $\{e^{-1/3}\}$; ≈ 0.72 **65.** $\{e^2 - 3\}$; ≈ 4.39 **66.** $\{e^2 - 4\}$; ≈ 3.39 **97.** $\left\{\pm\sqrt{\dfrac{\ln 45}{\ln 3}}\right\}$ **98.** $\left\{\pm\sqrt{\dfrac{\ln 50}{\ln 5}}\right\}$

103. a. 37.3 million **104. b.** 2016 **119. b.** $10^{-2.4}$ mole per liter **130.** $\{20\}$

131. $\{-1.391606, 1.6855579\}$ **132.** $\{-1.291641, 1.2793139\}$

133.

As distance from eye increases, barometric air pressure increases.

135.

about 7.9 min

136.

about 20 years old

152.

Section 3.5

Check Point Exercises

5.

6.

Exercise Set 3.5

15. approximately 8 grams **17.** 8 grams after 10 seconds; 4 grams after 20 seconds; 2 grams after 30 seconds; 1 gram after 40 seconds; 0.5 gram after 50 seconds **18.** 8 grams after 25,000 years; 4 grams after 50,000 years; 2 grams after 75,000 years; 1 gram after 100,000 years; 0.5 gram after 125,000 years

27. a. $\frac{1}{2} = e^{1.31k}$ yields $k = \frac{\ln\left(\frac{1}{2}\right)}{1.31} \approx -0.52912.$ **32.** about 39 years after the year when the population was 1400

33. $2A_0 = A_0e^{kt}$; $2 = e^{kt}$; $\ln 2 = \ln e^{kt}$; $\ln 2 = kt$; $\frac{\ln 2}{k} = t$ **34.** $3A_0 = A_0e^{kt}$; $3 = e^{kt}$; $\ln 3 = \ln e^{kt}$; $\ln 3 = kt$; $\frac{\ln 3}{k} = t$

36. b. about 58 yr **38.** quite well **39.** quite well **47. a.** $T = 45 + 25e^{-0.0916t}$ **b.** 51°F **48. a.** $T = 70 + 380e^{-0.1004t}$ **b.** 121°F

51. a.

52. a.

b. exponential function

53. a.

b. logarithmic function

54. a.

b. logarithmic function

55. a.

56. a.

57. $y = 100e^{(\ln 4.6)x}$; $y = 100e^{1.526x}$ **58.** $y = 1000e^{(\ln 7.3)x}$; $y = 1000e^{1.988x}$ **59.** $y = 2.5e^{(\ln 0.7)x}$; $y = 2.5e^{-0.357x}$ **60.** $y = 4.5e^{(\ln 0.6)x}$; $y = 4.5e^{-0.511x}$
70. a. $r \approx 0.999$; Since r is close to 1, the model fits the data well. **71.** $r \approx 0.870$; Fit is ok, but not great.
74. $y = 201.2(1.011)^x$; $y = 2.657x + 197.923$; using exponential; by 2016; using linear: by 2021; Answers will vary.
75. b. $y = 3.46e^{(\ln 1.02)x}$; $y = 3.46e^{0.02x}$; by approximately 2% **76.** Models will vary. Examples are given. Predictions will vary. For Exercise 47: $y = 1.402(1.078)^x$; For Exercise 48: $y = 2896.7(1.056)^x$; For Exercise 49: $y = 120 + 4.343 \ln x$; For Exercise 50: $y = -11.629 + 13.424 \ln x$; For Exercise 51: $y = 0.063x - 124.16$; For Exercise 52: $y = 0.742x - 1449.669$
77. does not make sense **79.** makes sense

Chapter 3 Review Exercises

1. $g(x) = 4^{-x}$ **2.** $h(x) = -4^{-x}$ **3.** $r(x) = -4^{-x} + 3$ **4.** $f(x) = 4^x$

5.

$f(x) = 2^x$
$g(x) = 2^{x-1}$

asymptote of f: $y = 0$
asymptote of g: $y = 0$
domain of f = domain of g = $(-\infty, \infty)$
range of f = range of g = $(0, \infty)$

6.

$f(x) = 3^x$
$g(x) = 3^x - 1$

asymptote of f: $y = 0$
asymptote of g: $y = -1$
domain of f = domain of g = $(-\infty, \infty)$
range of f = $(0, \infty)$; range of g = $(-1, \infty)$

7.

$f(x) = 3^x$
$g(x) = -3^x$

asymptote of f: $y = 0$
asymptote of g: $y = 0$
domain of f = domain of g = $(-\infty, \infty)$
range of f = $(0, \infty)$; range of g = $(-\infty, 0)$

8.

$f(x) = \left(\frac{1}{2}\right)^x$
$g(x) = \left(\frac{1}{2}\right)^{-x}$

asymptote of f: $y = 0$
asymptote of g: $y = 0$
domain of f = domain of g = $(-\infty, \infty)$
range of f = range of g = $(0, \infty)$

9.

$f(x) = e^x$
$g(x) = 2e^{x/2}$

asymptote of f: $y = 0$
asymptote of g: $y = 0$
domain of f = domain of g = $(-\infty, \infty)$
range of f = range of g = $(0, \infty)$

13. $49^{1/2} = 7$ **14.** $4^3 = x$ **15.** $3^y = 81$
16. $\log_6 216 = 3$ **17.** $\log_b 625 = 4$
18. $\log_{13} 874 = y$
21. undefined; $\log_b x$ is defined only for $x > 0$.

30.

$f(x) = 2^x$
$g(x) = \log_2 x$

domain of f = range of g = $(-\infty, \infty)$
range of f = domain of g = $(0, \infty)$

31.

$f(x) = \left(\frac{1}{3}\right)^x$
$g(x) = \log_{1/3} x$

domain of f = range of g = $(-\infty, \infty)$
range of f = domain of g = $(0, \infty)$

32. $g(x) = \log(-x)$
33. $r(x) = 1 + \log(2 - x)$
34. $h(x) = \log(2 - x)$
35. $f(x) = \log x$

36.

$x = 2$
$f(x) = \log_2 x$
$g(x) = \log_2(x - 2)$

x-intercept: $(3, 0)$
vertical asymptote: $x = 2$
domain: $(2, \infty)$; range: $(-\infty, \infty)$

37.

$f(x) = \log_2 x$
$h(x) = -1 + \log_2 x$

x-intercept: $(2, 0)$
vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

38.

$f(x) = \log_2 x$
$r(x) = \log_2(-x)$

x-intercept: $(-1, 0)$
vertical asymptote: $x = 0$
domain: $(-\infty, 0)$; range: $(-\infty, \infty)$

39.

$f(x) = \log x$
$g(x) = -\log(x + 3)$

asymptote of f: $x = 0$
asymptote of g: $x = -3$
domain of $f = (0, \infty)$; domain of $g = (-3, \infty)$
range of f = range of $g = (-\infty, \infty)$

40.

$f(x) = \ln x$
$g(x) = -\ln(2x)$

asymptote of f: $x = 0$
asymptote of g: $x = 0$
domain of f = domain of $g = (0, \infty)$
range of f = range of $g = (-\infty, \infty)$

48. b. $\approx 67, \approx 63, \approx 61, \approx 59, \approx 56$

c.

61. false; $\log(x + 9) - \log(x + 1) = \log\left(\dfrac{x + 9}{x + 1}\right)$ **62.** false; $4 \log_2 x = \log_2 x^4$

68. $\left\{\dfrac{\ln 12,143}{\ln 8}\right\}$; ≈ 4.52 **69.** $\left\{\dfrac{1}{5}\ln 141\right\}$; ≈ 0.99

70. $\left\{\dfrac{12 - \ln 130}{5}\right\}$; ≈ 1.43 **71.** $\left\{\dfrac{\ln 37{,}500 - 2\ln 5}{4\ln 5}\right\}$; ≈ 1.14

72. $\left\{\dfrac{\ln 7 + 4\ln 3}{2\ln 7 - \ln 3}\right\}$; ≈ 2.27 **73.** $\{\ln 3\}$; ≈ 1.10 **88. b.** about 45,411 people

90. a.

91. a.

92. a.

Chapter 3 Test

1.

$f(x) = 2^x$
$g(x) = 2^{x+1}$

2.

$f(x) = \log_2 x$
$g(x) = \log_2(x - 1)$

24. 13.9 years **29. a.** 14 elk

Cumulative Review Exercises (Chapters P–3)

9.

$(x - 3)^2 + (y + 2)^2 = 4$

10.

$f(x) = (x - 2)^2 - 1$

11.

$f(x) = \dfrac{x^2 - 1}{x^2 - 4}$

12.

$f(x) = (x - 2)^2(x + 1)$

13.

$f(x) = 2x - 4$
$f^{-1}(x) = \dfrac{x + 4}{2}$

14.

$f(x) = \ln x$
$g(x) = \ln(x - 2) + 1$

15. using $(1, 3)$, $y - 3 = -3(x - 1)$; $y = -3x + 6$

CHAPTER 4

Section 4.1

Check Point Exercises

4. a.

b.

c.

d.

Exercise Set 4.1

41.

42.

43.

44.

45.

46.

47.

48.

49.

50.

51.

52.

53.

54.

55.

56.

121. smaller than a right angle

123.

Exercise Set 4.2

25. $\tan t = \frac{8}{15}$; $\csc t = \frac{17}{8}$; $\sec t = \frac{17}{15}$; $\cot t = \frac{15}{8}$ **26.** $\tan t = \frac{3}{4}$; $\csc t = \frac{5}{3}$; $\sec t = \frac{5}{4}$; $\cot t = \frac{4}{3}$ **27.** $\tan t = \frac{\sqrt{2}}{4}$; $\csc t = 3$; $\sec t = \frac{3\sqrt{2}}{4}$; $\cot t = 2\sqrt{2}$ **28.** $\tan t = \frac{2\sqrt{5}}{5}$; $\csc t = \frac{3}{2}$; $\sec t = \frac{3\sqrt{5}}{5}$; $\cot t = \frac{\sqrt{5}}{2}$ **83. a.** 1; 0; −1; 0; 1 **b.** 28 days **84. a.** 10 ft; 14 ft; 10 ft; 10 ft; 10 ft; 6 ft **b.** low tide: 3 A.M., 3 P.M., high tide: 9 A.M., 9 P.M. **c.** 12 hr; Low tides occur every 12 hours and high tides occur every 12 hours. **98.** does not make sense **99.** does not make sense

Exercise Set 4.3

1. 15; $\sin\theta = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$; $\tan\theta = \frac{3}{4}$; $\csc\theta = \frac{5}{3}$; $\sec\theta = \frac{5}{4}$; $\cot\theta = \frac{4}{3}$ **2.** 10; $\sin\theta = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$; $\tan\theta = \frac{3}{4}$; $\csc\theta = \frac{5}{3}$; $\sec\theta = \frac{5}{4}$; $\cot\theta = \frac{4}{3}$ **3.** 20; $\sin\theta = \frac{20}{29}$; $\cos\theta = \frac{21}{29}$; $\tan\theta = \frac{20}{21}$; $\csc\theta = \frac{29}{20}$; $\sec\theta = \frac{29}{21}$; $\cot\theta = \frac{21}{20}$ **4.** 8; $\sin\theta = \frac{8}{17}$; $\cos\theta = \frac{15}{17}$; $\tan\theta = \frac{8}{15}$; $\csc\theta = \frac{17}{8}$; $\sec\theta = \frac{17}{15}$; $\cot\theta = \frac{15}{8}$ **68.** 0.3894; 0.9736; 0.2955, 0.9851; 0.1987, 0.9933; 0.0998, 0.9983; 0.0099998, 0.99998; 9.999998×10^{-4}, 0.9999998; $9.99999998 \times 10^{-5}$, 0.999999998; 1×10^{-5}, 0.99999999998; $\frac{\sin\theta}{\theta}$ approaches 1 as θ approaches 0. **69.** 0.92106, −0.19735; 0.95534, −0.148878; 0.98007, −0.099667; 0.99500, −0.04996; 0.99995, −0.005; 0.9999995, −0.0005; 0.999999995, −0.00005; 0.99999999995, −0.000005; $\frac{\cos\theta - 1}{\theta}$ approaches 0 as θ approaches 0. **75.** true **76.** false **78.** In a right triangle, the hypotenuse is greater than either other side. Therefore, both $\frac{\text{opposite}}{\text{hypotenuse}}$ and $\frac{\text{adjacent}}{\text{hypotenuse}}$ must be less than 1 for an acute angle in a right triangle. **79.** As θ approaches 90°, $\tan\theta$ increases without bound. At 90°, $\tan\theta$ is undefined.

Exercise Set 4.4

1. $\sin\theta = \frac{3}{5}$; $\cos\theta = -\frac{4}{5}$; $\tan\theta = -\frac{3}{4}$; $\csc\theta = \frac{5}{3}$; $\sec\theta = -\frac{5}{4}$; $\cot\theta = -\frac{4}{3}$ **2.** $\sin\theta = \frac{5}{13}$; $\cos\theta = -\frac{12}{13}$; $\tan\theta = -\frac{5}{12}$; $\csc\theta = \frac{13}{5}$; $\sec\theta = -\frac{13}{12}$; $\cot\theta = -\frac{12}{5}$ **3.** $\sin\theta = \frac{3\sqrt{13}}{13}$; $\cos\theta = \frac{2\sqrt{13}}{13}$; $\tan\theta = \frac{3}{2}$; $\csc\theta = \frac{\sqrt{13}}{3}$; $\sec\theta = \frac{\sqrt{13}}{2}$; $\cot\theta = \frac{2}{3}$ **4.** $\sin\theta = \frac{7\sqrt{58}}{58}$;

$\cos\theta = \frac{3\sqrt{58}}{58}$; $\tan\theta = \frac{7}{3}$; $\csc\theta = \frac{\sqrt{58}}{7}$; $\sec\theta = \frac{\sqrt{58}}{3}$; $\cot\theta = \frac{3}{7}$ **5.** $\sin\theta = -\frac{\sqrt{2}}{2}$; $\cos\theta = \frac{\sqrt{2}}{2}$; $\tan\theta = -1$; $\csc\theta = -\sqrt{2}$; $\sec\theta = \sqrt{2}$; $\cot\theta = -1$ **6.** $\sin\theta = -\frac{\sqrt{2}}{2}$; $\cos\theta = \frac{\sqrt{2}}{2}$; $\tan\theta = -1$; $\csc\theta = -\sqrt{2}$; $\sec\theta = \sqrt{2}$; $\cot\theta = -1$

7. $\sin\theta = -\frac{5\sqrt{29}}{29}$; $\cos\theta = -\frac{2\sqrt{29}}{29}$; $\tan\theta = \frac{5}{2}$; $\csc\theta = -\frac{\sqrt{29}}{5}$; $\sec\theta = -\frac{\sqrt{29}}{2}$; $\cot\theta = \frac{2}{5}$ **8.** $\sin\theta = -\frac{3\sqrt{10}}{10}$; $\cos\theta = -\frac{\sqrt{10}}{10}$; $\tan\theta = 3$; $\csc\theta = -\frac{\sqrt{10}}{3}$; $\sec\theta = -\sqrt{10}$; $\cot\theta = \frac{1}{3}$ **23.** $\sin\theta = -\frac{4}{5}$; $\tan\theta = \frac{4}{3}$; $\csc\theta = -\frac{5}{4}$; $\sec\theta = -\frac{5}{3}$; $\cot\theta = \frac{3}{4}$

24. $\cos\theta = -\frac{5}{13}$; $\tan\theta = \frac{12}{5}$; $\csc\theta = -\frac{13}{12}$; $\sec\theta = -\frac{13}{5}$; $\cot\theta = \frac{5}{12}$ **25.** $\cos\theta = -\frac{12}{13}$; $\tan\theta = -\frac{5}{12}$; $\csc\theta = \frac{13}{5}$; $\sec\theta = -\frac{13}{12}$; $\cot\theta = -\frac{12}{5}$

26. $\sin\theta = -\frac{3}{5}$; $\tan\theta = -\frac{3}{4}$; $\csc\theta = -\frac{5}{3}$; $\sec\theta = \frac{5}{4}$; $\cot\theta = -\frac{4}{3}$ **27.** $\sin\theta = -\frac{15}{17}$; $\tan\theta = -\frac{15}{8}$; $\csc\theta = -\frac{17}{15}$; $\sec\theta = \frac{17}{8}$; $\cot\theta = -\frac{8}{15}$

28. $\sin\theta = -\frac{2\sqrt{2}}{3}$; $\tan\theta = -2\sqrt{2}$; $\csc\theta = -\frac{3\sqrt{2}}{4}$; $\sec\theta = 3$; $\cot\theta = -\frac{\sqrt{2}}{4}$ **29.** $\sin\theta = \frac{2\sqrt{13}}{13}$; $\cos\theta = -\frac{3\sqrt{13}}{13}$; $\csc\theta = \frac{\sqrt{13}}{2}$; $\sec\theta = -\frac{\sqrt{13}}{3}$; $\cot\theta = -\frac{3}{2}$

30. $\sin\theta = \frac{\sqrt{10}}{10}$; $\cos\theta = -\frac{3\sqrt{10}}{10}$; $\csc\theta = \sqrt{10}$; $\sec\theta = -\frac{\sqrt{10}}{3}$; $\cot\theta = -3$ **31.** $\sin\theta = -\frac{4}{5}$; $\cos\theta = -\frac{3}{5}$; $\csc\theta = -\frac{5}{4}$; $\sec\theta = -\frac{5}{3}$; $\cot\theta = \frac{3}{4}$

32. $\sin\theta = -\frac{5}{13}$; $\cos\theta = -\frac{12}{13}$; $\csc\theta = -\frac{13}{5}$; $\sec\theta = -\frac{13}{12}$; $\cot\theta = \frac{12}{5}$ **33.** $\sin\theta = -\frac{2\sqrt{2}}{3}$; $\cos\theta = -\frac{1}{3}$; $\tan\theta = 2\sqrt{2}$; $\csc\theta = -\frac{3\sqrt{2}}{4}$; $\cot\theta = \frac{\sqrt{2}}{4}$

34. $\sin\theta = -\frac{1}{4}$; $\cos\theta = -\frac{\sqrt{15}}{4}$; $\tan\theta = \frac{\sqrt{15}}{15}$; $\sec\theta = -\frac{4\sqrt{15}}{15}$; $\cot\theta = \sqrt{15}$ **97.** $\frac{2\sqrt{2}-4}{\pi}$ **98.** $\frac{2\sqrt{2}-4}{\pi}$ **116.**

Mid-Chapter 4 Check Point

5. a. $\frac{5\pi}{3}$

b.

c. $\frac{\pi}{3}$

6. a. $\frac{5\pi}{4}$

b.

c. $\frac{\pi}{4}$

7. a. 150°

b.

c. 30°

11. $\sin\theta = \frac{3}{5}$; $\cos\theta = -\frac{4}{5}$; $\csc\theta = \frac{5}{3}$; $\sec\theta = -\frac{5}{4}$; $\cot\theta = -\frac{4}{3}$ **12.** $\sin\theta = -\frac{2\sqrt{10}}{7}$; $\tan\theta = -\frac{2\sqrt{10}}{3}$; $\csc\theta = -\frac{7\sqrt{10}}{20}$; $\sec\theta = \frac{7}{3}$; $\cot\theta = -\frac{3\sqrt{10}}{20}$

27. 8π cm $\approx$ 25.13 cm

Section 4.5

Check Point Exercises

1.

2.

3.

4.

5.

6.

7.

Exercise Set 4.5

1.

2.

3.

4.

5.

6.

7.

8.

9.

$y = 3 \sin \frac{1}{2} x$

10.

$y = 2 \sin \frac{1}{4} x$

11.

$y = 4 \sin \pi x$

12.

$y = 3 \sin 2\pi x$

13.

$y = -3 \sin 2\pi x$

14.

$y = -2 \sin \pi x$

15.

$y = -\sin \frac{2}{3} x$

16.

$y = -\sin \frac{4}{3} x$

17. $1; 2\pi; \pi$

$y = \sin (x - \pi)$

18. $1; 2\pi; \frac{\pi}{2}$

$y = \sin \left(x - \frac{\pi}{2}\right)$

19. $1; \pi; \frac{\pi}{2}$

$y = \sin (2x - \pi)$

20. $1; \pi; \frac{\pi}{4}$

$y = \sin \left(2x - \frac{\pi}{2}\right)$

21. $3; \pi; \frac{\pi}{2}$

$y = 3 \sin (2x - \pi)$

22. $3; \pi; \frac{\pi}{4}$

$y = 3 \sin \left(2x - \frac{\pi}{2}\right)$

23. $\frac{1}{2}; 2\pi; -\frac{\pi}{2}$

$y = \frac{1}{2} \sin \left(x + \frac{\pi}{2}\right)$

24. $\frac{1}{2}; 2\pi; -\pi$

$y = \frac{1}{2} \sin \left(x + \pi\right)$

25. $2;\ \pi;\ -\frac{\pi}{4}$

$y = -2 \sin\left(2x + \frac{\pi}{2}\right)$

26. $3;\ \pi;\ -\frac{\pi}{4}$

$y = -3 \sin\left(2x + \frac{\pi}{2}\right)$

27. $3;\ 2;\ -\frac{2}{\pi}$

$y = 3 \sin(\pi x + 2)$

28. $3;\ 1;\ -\frac{2}{\pi}$

$y = 3 \sin(2\pi x + 4)$

29. $2;\ 1;\ -2$

$y = -2 \sin(2\pi x + 4\pi)$

30. $3;\ 1;\ -2$

$y = -3 \sin(2\pi x + 4\pi)$

31.

32.

33.

34.

35.

$y = \cos 2x$

36.

$y = \cos 4x$

37.

$y = 4 \cos 2\pi x$

38.

$y = 5 \cos 2\pi x$

39.

$y = -4 \cos \frac{1}{2} x$

40.

$y = -3 \cos \frac{1}{3} x$

41.

$y = -\frac{1}{2} \cos \frac{\pi}{3} x$

42.

$y = -\frac{1}{2} \cos \frac{\pi}{4} x$

43. $1;\ 2\pi,\ \frac{\pi}{2}$

$y = \cos\left(x - \frac{\pi}{2}\right)$

44. $1;\ 2\pi,\ -\frac{\pi}{2}$

$y = \cos\left(x + \frac{\pi}{2}\right)$

45. $3;\ \pi;\ \frac{\pi}{2}$

$y = 3 \cos(2x - \pi)$

46. $4;\ \pi;\ \frac{\pi}{2}$

$y = 4 \cos(2x - \pi)$

47. $\frac{1}{2};\ \frac{2\pi}{3};\ -\frac{\pi}{6}$

$y = \frac{1}{2} \cos\left(3x + \frac{\pi}{2}\right)$

48. $\frac{1}{2};\ \pi;\ -\frac{\pi}{2}$

$y = \frac{1}{2} \cos(2x + \pi)$

49. $3;\ \pi;\ \frac{\pi}{4}$

$y = -3 \cos\left(2x - \frac{\pi}{2}\right)$

50. $4;\ \pi;\ \frac{\pi}{4}$

$y = -4 \cos\left(2x - \frac{\pi}{2}\right)$

51. $2;\ 1;\ -4$

$y = 2 \cos(2\pi x + 8\pi)$

52. $3;\ 1;\ -2$

$y = 3 \cos(2\pi x + 4\pi)$

53.

$y = \sin x + 2$

54.

$y = \sin x - 2$

55.

$y = \cos x - 3$

56.

$y = \cos x + 3$

57.

$y = 2 \sin \frac{1}{2} x + 1$

58.

$y = 2 \cos \frac{1}{2} x + 1$

59.

$y = -3 \cos 2\pi x + 2$

60.

$y = -3 \sin 2\pi x + 2$

67.

$y = \left|2 \cos \frac{x}{2}\right|$

68.

$y = \left|3 \cos \frac{2x}{3}\right|$

69.

$y = -\left|3 \sin \pi x\right|$

70.

$y = -\left|2 \sin \frac{\pi x}{2}\right|$

71.

$f(x) = -2 \sin x$
$g(x) = \sin 2x$

72.

$f(x) = 2 \cos x$
$g(x) = \cos 2x$

73.

$f(x) = \sin x$
$g(x) = \cos 2x$

74.

$f(x) = \cos x$
$g(x) = \sin 2x$

80. February 11

83.

84.

85. e.

86.

101.

102.

103.

104.

105.

106.

The graphs appear to be the same from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

107.

The graph is similar to $y = \sin x$, except the amplitude is greater and the curve is less smooth.

108.

The graph is very similar to $y = \sin x$, except the curve is less smooth.

109. a.

c.

112. does not make sense

115. a. range: $[-5, 1]$; $\left[-\frac{\pi}{6}, \frac{23\pi}{6}, \frac{\pi}{6}\right]$ by $[-5, 1, 1]$

b. range: $[-3, -1]$; $\left[-\frac{\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{6}\right]$ by $[-3, -1, 1]$

117.

$y = \sin^2 x$

or $y = \frac{1}{2} - \frac{1}{2}\cos 2x$

118.

$y = \cos^2 x$

or $y = \frac{1}{2} + \frac{1}{2}\cos 2x$

122. a.

$y = -3\cos\frac{x}{2}$

b. The reciprocal function is undefined.

Section 4.6

Check Point Exercises

1.

$y = 3\tan 2x$

2.

$y = \tan\left(x - \frac{\pi}{2}\right)$

3.

$y = \frac{1}{2}\cot\frac{\pi}{2}x$

4.

$y = \csc\left(x + \frac{\pi}{4}\right)$

5.

$y = 2\sec 2x$

Exercise Set 4.6

5.

$y = 3\tan\frac{x}{4}$

6.

$y = 2\tan\frac{x}{4}$

7.

$y = \frac{1}{2}\tan 2x$

8.

$y = 2\tan 2x$

9.

$y = -2\tan\frac{1}{2}x$

10.

$y = -3\tan\frac{1}{2}x$

11.

$y = \tan(x - \pi)$

12.

$y = \tan\left(x - \frac{\pi}{4}\right)$

17.

$y = 2\cot x$

18.

$y = \frac{1}{2}\cot x$

19.

$y = \frac{1}{2}\cot 2x$

20.

$y = 2\cot 2x$

21.

$y = -3\cot\frac{\pi}{2}x$

22.

$y = -2\cot\frac{\pi}{4}x$

23.

$y = 3\cot\left(x + \frac{\pi}{2}\right)$

24.

$y = 3\cot\left(x + \frac{\pi}{4}\right)$

25.

$y = -\frac{1}{2}\csc\frac{x}{2}$

26.

$y = 3\csc 4x$

27.

$y = \frac{1}{2}\sec 2\pi x$

28.

$y = -3\sec\frac{\pi}{2}x$

29.

$y = 3\csc x$

30.

$y = 2\csc x$

31.

$y = \frac{1}{2}\csc\frac{x}{2}$

32.

$y = \frac{3}{2}\csc\frac{x}{4}$

33.

$y = 2\sec x$

34.

$y = 3\sec x$

35.

$y = \sec\frac{x}{3}$

36.

$y = \sec\frac{x}{2}$

37.

$y = -2\csc \pi x$

38.

$y = -\frac{1}{2}\csc \pi x$

39.

$y = -\frac{1}{2}\sec \pi x$

40.

$y = -\frac{3}{2}\sec \pi x$

41.

$y = \csc(x - \pi)$

42.

$y = \csc\left(x - \frac{\pi}{2}\right)$

43.

$y = 2\sec(x + \pi)$

44.

$y = 2\sec\left(x + \frac{\pi}{2}\right)$

45.

$y = 2\tan\left(x - \frac{\pi}{6}\right) + 1$

46.

$y = 2\cot\left(x + \frac{\pi}{6}\right) - 1$

47.

$x = \frac{\pi}{2}$

$y = \sec\left(2x + \frac{\pi}{2}\right) - 1$

48.

$y = \csc\left(2x - \frac{\pi}{2}\right) + 1$

49.

$y = \csc|x|$

50.

$y = \sec|x|$

51.

$y = \left|\cot\frac{1}{2}x\right|$

52.

$y = \left|\tan\frac{1}{2}x\right|$

53.

$y = 2\sec\left(2x - \frac{\pi}{2}\right)$

54.

$y = -2\tan\left(2x - \frac{\pi}{2}\right)$

59. a.

b. 0.25, 0.75, 1.25, 1.75; The beam of light is shining parallel to the wall at these times.

60.

61.

62. Answers may vary.

63.

64. Answers may vary.

77.

78.

79.

80.

81.

82.

83.

84.

85.

86.

87.

88.

92. does not make sense

97. a. range: $(-\infty, -1] \cup [1, \infty)$; $\left[-\frac{\pi}{6}, \pi, \frac{7\pi}{6}\right]$ by $[-3, 3, 1]$

b. range: $(-\infty, -3] \cup [3, \infty)$; $\left[-\frac{1}{2}, \frac{7}{2}, 1\right]$ by $[-6, 6, 1]$

98. 2^{-x} decreases the amplitude as x gets larger.

99. a.

100. a.

101. a.

Exercise Set 4.7

71. $\dfrac{\sqrt{x^2 + 4}}{2}$ **72.** $\dfrac{3\sqrt{x^2 - 9}}{x^2 - 9}$

73. a.

b. No horizontal line intersects the graph of $y = \sec x$ more than once, so the function is one-to-one and has an inverse function.

c.

74. a.

b. No horizontal line intersects the graph of $y = \cot x$ more than once, so the function is one-to-one and has an inverse function.

c.

75.

domain: $[-1, 1]$;

range: $[0, \pi]$

76.

domain: $[-1, 1]$;

range: $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

77.

domain: $[-2, 0]$;

range: $[0, \pi]$

78.

domain: $[-2, 0]$;

range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

79.

domain: $(-\infty, \infty)$;

range: $(-\pi, \pi)$

80.

domain: $(-\infty, \infty)$;

range: $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

81.

domain: $[1, 3]$;

range: $[-\pi, 0]$

82.

domain: $[1, 3]$;

range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

83.

domain: $[-2, 2]$;

range: $[0, \pi]$

84.

domain: $[-2, 2]$;

range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

93. 0.408 radian; 0.602 radian; 0.654 radian; 0.645 radian; 0.613 radian **94.** about 16 ft; 0.6542 radian; 0.6553 radian **95.** 1.3157 radians or 75.4° **96.** 0.1440 radian or 8.2° **97.** 1.1071 sq units **98.** 1.8925 sq units

110.

111.

112.

113.

Shifted left 2 units and up 1 unit

114.

Observations may vary.

115.

It seems

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for $-1 \le x \le 1$.

116. does not make sense **117.** does not make sense **118.** makes sense

123. $\tan\alpha = \frac{8}{x}$, so $\tan^{-1}\frac{8}{x} = \alpha$. $\tan(\alpha + \theta) = \frac{33}{x}$, so $\tan^{-1}\frac{33}{x} = \alpha + \theta$. $\theta = \alpha + \theta - \alpha = \tan^{-1}\frac{33}{x} - \tan^{-1}\frac{8}{x}$.

Exercise Set 4.8

1. $B = 66.5°$; $a \approx 4.35$; $c \approx 10.90$ **2.** $B = 48.5°$; $a \approx 17.69$; $c \approx 26.70$ **3.** $B = 37.4°$; $a \approx 42.90$; $b \approx 32.80$
4. $B = 35.2°$; $a \approx 65.37$; $b \approx 46.11$ **5.** $A = 73.2°$; $a \approx 101.02$; $c \approx 105.52$ **6.** $A = 66.2°$; $a \approx 91.83$; $c \approx 100.36$
7. $b \approx 39.95$; $A \approx 37.3°$; $B \approx 52.7°$ **8.** $A \approx 9.8°$; $B \approx 80.2°$; $b \approx 64.84$ **9.** $c \approx 26.96$; $A \approx 23.6°$; $B \approx 66.4°$
10. $A \approx 41.0°$; $B \approx 49.0°$; $c \approx 23.32$ **11.** $a \approx 6.71$; $B \approx 16.6°$; $A \approx 73.4°$ **12.** $A \approx 63.6°$; $B \approx 26.4°$, $a \approx 8.06$

17. $d = -6\cos\frac{\pi}{2}t$ **18.** $d = -8\cos\pi t$ **19.** $d = -3\sin\frac{4\pi}{3}t$ **20.** $d = -5\sin\frac{4\pi}{5}t$ **21. a.** 5 in. **b.** $\frac{1}{4}$ in. per sec **c.** 4 sec

22. a. 10 in. **b.** 1 in. per sec **c.** 1 sec **23. a.** 6 in. **b.** 1 in. per sec **c.** 1 sec **24. a.** 8 in. **b.** $\frac{1}{4}$ in. per sec **c.** 4 sec

25. a. $\frac{1}{2}$ in. **b.** 0.32 in. per sec **c.** 3.14 sec **26. a.** $\frac{1}{3}$ in. **b.** 0.32 in. per sec **c.** 3.14 sec **27. a.** 5 in. **b.** $\frac{1}{3}$ in. per sec **c.** 3 sec
28. a. 4 in. **b.** $\frac{3}{4}$ in. per sec **c.** $\frac{4}{3}$ sec

37.

$d = 4\cos\left(\pi t - \frac{\pi}{2}\right)$

a. 4 in. **b.** $\frac{1}{2}$ in. per sec **c.** 2 sec **d.** $\frac{1}{2}$

38.

$d = 3\cos\left(\pi t + \frac{\pi}{2}\right)$

a. 3 in. **b.** $\frac{1}{2}$ in. per sec **c.** 2 sec **d.** $-\frac{1}{2}$

39.

$d = -2\sin\left(\frac{\pi}{4}t + \frac{\pi}{2}\right)$

a. 2 in. **b.** $\frac{1}{8}$ in. per sec **c.** 8 sec **d.** -2

40.

$d = -\frac{1}{2}\sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right)$

a. $\frac{1}{2}$ in. **b.** $\frac{1}{8}$ in. per sec **c.** 8 sec **d.** 2

41. 2059 ft **43.** 695 ft **51.** 90 mi north and 120 mi east **52.** 17.5 mi south and 36.0 mi east **56.** N 56° W

70.

3 complete oscillations

71.

10 complete oscillations

Chapter 4 Review Exercises

8.

9.

10.

11.

12.

20. 42,412 ft per min **27.** $\cos t = \frac{\sqrt{21}}{7}$; $\tan t = \frac{2\sqrt{3}}{3}$; $\csc t = \frac{\sqrt{7}}{2}$; $\sec t = \frac{\sqrt{21}}{3}$; $\cot t = \frac{\sqrt{3}}{2}$

45. $\sin\theta = -\frac{5\sqrt{26}}{26}$; $\cos\theta = -\frac{\sqrt{26}}{26}$; $\tan\theta = 5$; $\csc\theta = -\frac{\sqrt{26}}{5}$; $\sec\theta = -\sqrt{26}$; $\cot\theta = \frac{1}{5}$

46. $\sin\theta = -1$; $\cos\theta = 0$; $\tan\theta$ is undefined; $\csc\theta = -1$; $\sec\theta$ is undefined; $\cot\theta = 0$ **47.** quadrant I

49. $\sin\theta = -\frac{\sqrt{21}}{5}$; $\tan\theta = -\frac{\sqrt{21}}{2}$; $\csc\theta = -\frac{5\sqrt{21}}{21}$; $\sec\theta = \frac{5}{2}$; $\cot\theta = -\frac{2\sqrt{21}}{21}$

50. $\sin\theta = \frac{\sqrt{10}}{10}$; $\cos\theta = -\frac{3\sqrt{10}}{10}$; $\csc\theta = \sqrt{10}$; $\sec\theta = -\frac{\sqrt{10}}{3}$; $\cot\theta = -3$

51. $\sin\theta = -\frac{\sqrt{10}}{10}$; $\cos\theta = -\frac{3\sqrt{10}}{10}$; $\tan\theta = \frac{1}{3}$; $\csc\theta = -\sqrt{10}$; $\sec\theta = -\frac{\sqrt{10}}{3}$

68.

$y = 3\sin 4x$

69.

$y = -2\cos 2x$

70.

$y = 2\cos\frac{1}{2}x$

71.

$y = \frac{1}{2}\sin\frac{\pi}{3}x$

72.

$y = -\sin \pi x$

73.

$y = 3 \cos \frac{x}{3}$

74.

$y = 2 \sin (x - \pi)$

75.

$y = -3 \cos (x + \pi)$

76.

$y = \frac{3}{2} \cos \left(2x + \frac{\pi}{4}\right)$

77.

$y = \frac{5}{2} \sin \left(2x + \frac{\pi}{2}\right)$

78.

$y = -3 \sin \left(\frac{\pi}{3}x - 3\pi\right)$

79.

$y = \sin 2x + 1$

80.

$y = 2 \cos \frac{1}{3}x - 2$

81. e.

82. blue: $y = \sin \frac{\pi}{240}x$; red: $y = \sin \frac{\pi}{320}x$

83.

$y = 4 \tan 2x$

84.

$y = -2 \tan \frac{\pi}{4}x$

85.

$y = \tan (x + \pi)$

86.

$y = -\tan \left(x - \frac{\pi}{4}\right)$

87.

$y = 2 \cot 3x$

88.

$y = -\frac{1}{2} \cot \frac{\pi}{2}x$

89.

$y = 2 \cot \left(x + \frac{\pi}{2}\right)$

90.

$y = 3 \sec 2\pi x$

91.

$y = -2 \csc \pi x$

92.

$y = 3 \sec (x + \pi)$

93.

$y = \frac{5}{2} \csc (x - \pi)$

115. $B \approx 67.7°$; $a \approx 3.79$; $b \approx 9.25$ **116.** $A \approx 52.6°$; $a \approx 7.85$; $c \approx 9.88$ **117.** $A \approx 16.6°$; $B \approx 73.4°$; $b \approx 6.71$

118. $A \approx 21.3°$; $B \approx 68.7°$; $c \approx 3.86$ **120.** 90 yd **126. a.** 20 cm **b.** $\frac{1}{8}$ cm per sec **c.** 8 sec **127. a.** $\frac{1}{2}$ cm **b.** 0.64 cm per sec

c. 1.57 sec **128.** $d = -30 \cos \pi t$ **129.** $d = \frac{1}{4} \sin \frac{2\pi}{5}t$

Chapter 4 Test

2. $\frac{25\pi}{3}$ ft $\approx$ 26.18 ft **4.** $\sin \theta = \frac{5\sqrt{29}}{29}$; $\cos \theta = -\frac{2\sqrt{29}}{29}$; $\tan \theta = -\frac{5}{2}$; $\csc \theta = \frac{\sqrt{29}}{5}$; $\sec \theta = -\frac{\sqrt{29}}{2}$; $\cot \theta = -\frac{2}{5}$

6. $\sin \theta = -\frac{2\sqrt{2}}{3}$; $\tan \theta = -2\sqrt{2}$; $\csc \theta = -\frac{3\sqrt{2}}{4}$; $\sec \theta = 3$; $\cot \theta = -\frac{\sqrt{2}}{4}$

13. a. $-a + b$ or $b - a$ **b.** $\frac{a}{b} - \frac{1}{b}$ or $\frac{a-1}{b}$

14.

$y = 3 \sin 2x$

15.

$y = -2 \cos\left(x - \frac{\pi}{2}\right)$

16.

$y = 2 \tan \frac{x}{2}$

17.

$y = -\frac{1}{2} \csc \pi x$

19. $\dfrac{\sqrt{9 - x^2}}{3}$ **24. a.** 6 in. **b.** $\frac{1}{2}$ in. per sec **c.** 2 sec

Cumulative Review Exercises (Chapters P–4)

8. $4x^2 - \dfrac{14}{5}x - \dfrac{17}{25} + \dfrac{284}{125x + 50}$ **9.** $\log 1000 = 3$ **11.** 3 positive real roots; 1 negative real root

12.

$f(x) = \dfrac{x}{x^2 - 1}$

13.

$(x - 2)^2 + y^2 = 1$

14.

$y = (x - 1)(x + 2)^2$

15.

$y = \sin\left(2x + \frac{\pi}{2}\right)$

16.

$y = 2 \tan 3x$

18. a. $A = 110e^{0.1013t}$ where t is the number of years after 2000
b. 13 years after 2000, or 2013

CHAPTER 5

Section 5.1

Check Point Exercises

1. $\csc x \tan x = \dfrac{1}{\sin x} \cdot \dfrac{\sin x}{\cos x} = \dfrac{1}{\cos x} = \sec x$

2. $\cos x \cot x + \sin x = \cos x \cdot \dfrac{\cos x}{\sin x} + \sin x = \dfrac{\cos^2 x}{\sin x} + \sin x \cdot \dfrac{\sin x}{\sin x} = \dfrac{\cos^2 x + \sin^2 x}{\sin x} = \dfrac{1}{\sin x} = \csc x$

3. $\sin x - \sin x \cos^2 x = \sin x(1 - \cos^2 x) = \sin x \cdot \sin^2 x = \sin^3 x$ **4.** $\dfrac{1 + \cos\theta}{\sin\theta} = \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta} = \csc\theta + \cot\theta$

5. $\dfrac{\sin x}{1 + \cos x} + \dfrac{1 + \cos x}{\sin x} = \dfrac{\sin x(\sin x)}{(1 + \cos x)(\sin x)} + \dfrac{(1 + \cos x)(1 + \cos x)}{(\sin x)(1 + \cos x)} = \dfrac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{(1 + \cos x)(\sin x)}$

$= \dfrac{\sin^2 x + \cos^2 x + 1 + 2\cos x}{(1 + \cos x)(\sin x)} = \dfrac{1 + 1 + 2\cos x}{(1 + \cos x)(\sin x)} = \dfrac{2 + 2\cos x}{(1 + \cos x)(\sin x)} = \dfrac{2(1 + \cos x)}{(1 + \cos x)(\sin x)} = \dfrac{2}{\sin x} = 2\csc x$

6. $\dfrac{\cos x}{1 + \sin x} = \dfrac{\cos x(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} = \dfrac{\cos x(1 - \sin x)}{1 - \sin^2 x} = \dfrac{\cos x(1 - \sin x)}{\cos^2 x} = \dfrac{1 - \sin x}{\cos x}$

7. $\dfrac{\sec x + \csc(-x)}{\sec x \csc x} = \dfrac{\sec x - \csc x}{\sec x \csc x} = \dfrac{\dfrac{1}{\cos x} - \dfrac{1}{\sin x}}{\dfrac{1}{\cos x} \cdot \dfrac{1}{\sin x}} = \dfrac{\dfrac{\sin x}{\cos x \cdot \sin x} - \dfrac{\cos x}{\cos x \cdot \sin x}}{\dfrac{1}{\cos x \cdot \sin x}} = \dfrac{\dfrac{\sin x - \cos x}{\cos x \cdot \sin x}}{\dfrac{1}{\cos x \cdot \sin x}} = \dfrac{\sin x - \cos x}{\cos x \cdot \sin x} \cdot \dfrac{\cos x \cdot \sin x}{1} = \sin x - \cos x$

8. Left side: $\dfrac{1}{1 + \sin\theta} + \dfrac{1}{1 - \sin\theta} = \dfrac{1(1 - \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)} + \dfrac{1(1 + \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)} = \dfrac{1 - \sin\theta + 1 + \sin\theta}{(1 + \sin\theta)(1 - \sin\theta)} = \dfrac{2}{1 - \sin^2\theta}$;

Right side: $2 + 2\tan^2\theta = 2 + 2\left(\dfrac{\sin^2\theta}{\cos^2\theta}\right) = \dfrac{2\cos^2\theta}{\cos^2\theta} + \dfrac{2\sin^2\theta}{\cos^2\theta} = \dfrac{2\cos^2\theta + 2\sin^2\theta}{\cos^2\theta} = \dfrac{2(\cos^2\theta + \sin^2\theta)}{\cos^2\theta} = \dfrac{2}{\cos^2\theta} = \dfrac{2}{1 - \sin^2\theta}$

Exercise Set 5.1

69. $\dfrac{1}{\cos x}$ **70.** $\dfrac{1}{\cot x}$

79.

Proofs may vary.

80.

Proofs may vary.

81.

Values for x may vary.

82.

Values for x may vary.

83.

Values for x may vary.

84.

Values for x may vary.

85.

Proofs may vary.

86.

Proofs may vary.

87.

Values for x may vary.

90. makes sense

Section 5.2

Check Point Exercises

3. $\dfrac{\cos(\alpha - \beta)}{\cos\alpha\cos\beta} = \dfrac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta} = \dfrac{\cos\alpha}{\cos\alpha}\cdot\dfrac{\cos\beta}{\cos\beta} + \dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\sin\beta}{\cos\beta} = 1 + \tan\alpha\tan\beta$

6. b. $\cos\left(x + \dfrac{3\pi}{2}\right) = \cos x\cos\dfrac{3\pi}{2} - \sin x\sin\dfrac{3\pi}{2} = \cos x\cdot 0 - \sin x\cdot(-1) = \sin x$

7. $\tan(x + \pi) = \dfrac{\tan x + \tan\pi}{1 - \tan x\tan\pi} = \dfrac{\tan x + 0}{1 - \tan x\cdot 0} = \dfrac{\tan x}{1} = \tan x$

Exercise Set 5.2

28. $\tan 30°; \dfrac{\sqrt{3}}{3}$ **59. a.** $-\dfrac{4 + 6\sqrt{2}}{15}$ **b.** $\dfrac{3 - 8\sqrt{2}}{15}$ **c.** $\dfrac{54 - 25\sqrt{2}}{28}$ **60. a.** $-\dfrac{6 + 4\sqrt{5}}{15}$ **b.** $\dfrac{8 - 3\sqrt{5}}{15}$ **c.** $\dfrac{54 - 25\sqrt{5}}{22}$

61. a. $-\dfrac{8\sqrt{3} + 15}{34}$ **b.** $\dfrac{15\sqrt{3} - 8}{34}$ **c.** $\dfrac{480 - 289\sqrt{3}}{33}$ **62. a.** $-\dfrac{2\sqrt{2} + \sqrt{3}}{6}$ **b.** $\dfrac{2\sqrt{6} - 1}{6}$ **c.** $\dfrac{8\sqrt{2} - 9\sqrt{3}}{5}$

63. a. $-\dfrac{4 + 3\sqrt{15}}{20}$ **b.** $\dfrac{-3 + 4\sqrt{15}}{20}$ **c.** $\dfrac{3 - 4\sqrt{15}}{4 + 3\sqrt{15}}$ **64. a.** $-\dfrac{(15 + 7\sqrt{11})\sqrt{58}}{348}$ **b.** $\dfrac{(35 - 3\sqrt{11})\sqrt{58}}{348}$ **c.** $\dfrac{3\sqrt{11} - 35}{15 + 7\sqrt{11}}$

65. b. $\sin(\pi - x) = \sin\pi\cos x - \cos\pi\sin x = 0\cdot\cos x - (-1)\sin x = \sin x$

66. b. $\cos(x - 2\pi) = \cos x\cos 2\pi + \sin x\sin 2\pi = \cos x\cdot 1 + \sin x\cdot 0 = \cos x$

67. b. $\sin\left(x + \dfrac{\pi}{2}\right) + \sin\left(\dfrac{\pi}{2} - x\right) = \sin x\cos\dfrac{\pi}{2} + \cos x\sin\dfrac{\pi}{2} + \sin\dfrac{\pi}{2}\cos x - \cos\dfrac{\pi}{2}\sin x = \sin x\cdot 0 + \cos x\cdot 1 + 1\cdot\cos x - 0\cdot\sin x$

$= \cos x + \cos x = 2\cos x$ **68. a.** $\cos\left(x - \dfrac{\pi}{2}\right) - \cos\left(x + \dfrac{\pi}{2}\right) = \cos x\cos\dfrac{\pi}{2} + \sin x\sin\dfrac{\pi}{2} - \left(\cos x\cos\dfrac{\pi}{2} - \sin x\sin\dfrac{\pi}{2}\right)$

$= \cos x\cdot 0 + \sin x\cdot 1 - (\cos x\cdot 0 - \sin x\cdot 1) = \sin x + \sin x = 2\sin x$ **73.** $\cos\dfrac{\pi}{3} = \dfrac{1}{2}$ **74.** $\sin\dfrac{2\pi}{3} = \dfrac{\sqrt{3}}{2}$

79. Proofs may vary.; amplitude is $\sqrt{13}$; period is 2π

88.

Proofs may vary.

89.

Proofs may vary.

90.

Values for x may vary.

91.

Values for x may vary.

92.

Proofs may vary.

93.

Proofs may vary.

107. $\sin 30° = \frac{1}{2}$; $\cos 30° = \frac{\sqrt{3}}{2}$; $\sin 60° = \frac{\sqrt{3}}{2}$; $\cos 60° = \frac{1}{2}$

Section 5.3

Check Point Exercises

3. $\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos\theta + \cos 2\theta \sin\theta = 2\sin\theta\cos\theta\cos\theta + (2\cos^2\theta - 1)\sin\theta = 2\sin\theta\cos^2\theta + 2\sin\theta\cos^2\theta - \sin\theta$
$= 4\sin\theta\cos^2\theta - \sin\theta = 4\sin\theta(1 - \sin^2\theta) - \sin\theta = 4\sin\theta - 4\sin^3\theta - \sin\theta = 3\sin\theta - 4\sin^3\theta$

4. $\sin^4 x = (\sin^2 x)^2 = \left(\frac{1 - \cos 2x}{2}\right)^2 = \frac{1 - 2\cos 2x + \cos^2 2x}{4} = \frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x = \frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\left(\frac{1 + \cos 2(2x)}{2}\right)$
$= \frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{8} + \frac{1}{8}\cos 4x = \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$

6. $\frac{\sin 2\theta}{1 + \cos 2\theta} = \frac{2\sin\theta\cos\theta}{1 + (1 - 2\sin^2\theta)} = \frac{2\sin\theta\cos\theta}{2 - 2\sin^2\theta} = \frac{2\sin\theta\cos\theta}{2(1 - \sin^2\theta)} = \frac{2\sin\theta\cos\theta}{2\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

7. $\frac{\sec\alpha}{\sec\alpha\csc\alpha + \csc\alpha} = \frac{\frac{1}{\cos\alpha}}{\frac{1}{\cos\alpha}\cdot\frac{1}{\sin\alpha} + \frac{1}{\sin\alpha}} = \frac{\frac{1}{\cos\alpha}}{\frac{1}{\cos\alpha\sin\alpha} + \frac{\cos\alpha}{\cos\alpha\sin\alpha}} = \frac{\frac{1}{\cos\alpha}}{\frac{1 + \cos\alpha}{\cos\alpha\sin\alpha}} = \frac{1}{\cos\alpha}\cdot\frac{\cos\alpha\sin\alpha}{1 + \cos\alpha} = \frac{\sin\alpha}{1 + \cos\alpha} = \tan\frac{\alpha}{2}$

Exercise Set 5.3

39. $\frac{\sqrt{2 - \sqrt{3}}}{2}$ **40.** $\frac{\sqrt{2 + \sqrt{2}}}{2}$ **41.** $-\frac{\sqrt{2 + \sqrt{2}}}{2}$ **42.** $\frac{\sqrt{2 + \sqrt{3}}}{2}$

95.

Proofs may vary.

96.

Proofs may vary.

97.

Values for x may vary.

98.

Values for x may vary.

99.

a. $y = 1 + 2\sin x$
b. Proofs may vary.

100.

a. $y = \sin x$
b. Proofs may vary.

101.

a. $y = \tan\frac{x}{2}$ **b.** Proofs may vary. **103.** does not make sense

112. $\frac{5}{16} - \frac{7}{16}\cos 2x + \frac{3}{16}\cos 4x - \frac{1}{16}\cos 2x\cos 4x$ **113.** Both sides equal $\frac{\sqrt{3}}{4}$.

114. Both sides equal 0. **115.** Both sides equal 0.

Section 5.4

Check Point Exercises

3. $\frac{\cos 3x - \cos x}{\sin 3x + \sin x} = \frac{-2\sin\left(\frac{3x + x}{2}\right)\sin\left(\frac{3x - x}{2}\right)}{2\sin\left(\frac{3x + x}{2}\right)\cos\left(\frac{3x - x}{2}\right)} = \frac{-2\sin\left(\frac{4x}{2}\right)\sin\left(\frac{2x}{2}\right)}{2\sin\left(\frac{4x}{2}\right)\cos\left(\frac{2x}{2}\right)} = \frac{-2\sin 2x\sin x}{2\sin 2x\cos x} = -\frac{\sin x}{\cos x} = -\tan x$

Exercise Set 5.4

1. $\frac{1}{2}[\cos 4x - \cos 8x]$ **2.** $\frac{1}{2}[\cos 4x - \cos 12x]$ **3.** $\frac{1}{2}[\cos 4x + \cos 10x]$ **4.** $\frac{1}{2}[\cos 7x + \cos 11x]$ **5.** $\frac{1}{2}[\sin 3x - \sin x]$

6. $\frac{1}{2}[\sin 5x - \sin x]$ **7.** $\frac{1}{2}[\sin 2x - \sin x]$ **8.** $\frac{1}{2}[\sin 3x - \sin 2x]$ **15.** $2 \sin \frac{3x}{2} \cos \frac{x}{2}$ **16.** $-2 \sin \frac{x}{2} \cos \frac{3x}{2}$

45.

Values for x may vary.

46.

Values for x may vary.

47.

Proofs may vary.

48.

Proofs may vary.

49.

50.

51. a.

b.

Exercise Set 5.5

11. $x = \frac{\pi}{3} + 2n\pi$ or $x = \frac{2\pi}{3} + 2n\pi$, where n is any integer. **12.** $x = \frac{\pi}{6} + 2n\pi$ or $x = \frac{11\pi}{6} + 2n\pi$, where n is any integer.

13. $x = \frac{\pi}{4} + n\pi$, where n is any integer. **14.** $x = \frac{\pi}{3} + n\pi$, where n is any integer. **15.** $x = \frac{2\pi}{3} + 2n\pi$ or $x = \frac{4\pi}{3} + 2n\pi$, where n is any integer.

16. $x = \frac{5\pi}{4} + 2n\pi$ or $x = \frac{7\pi}{4} + 2n\pi$, where n is any integer. **17.** $x = n\pi$, where n is any integer. **18.** $x = n\pi$, where n is any integer.

19. $x = \frac{5\pi}{6} + 2n\pi$ or $x = \frac{7\pi}{6} + 2n\pi$, where n is any integer. **20.** $x = \frac{4\pi}{3} + 2n\pi$ or $x = \frac{5\pi}{3} + 2n\pi$, where n is any integer.

21. $\theta = \frac{\pi}{6} + 2n\pi$ or $\theta = \frac{5\pi}{6} + 2n\pi$, where n is any integer. **22.** $\theta = \frac{7\pi}{6} + 2n\pi$ or $\theta = \frac{11\pi}{6} + 2n\pi$, where n is any integer.

23. $\theta = \frac{3\pi}{2} + 2n\pi$, where n is any integer. **24.** $\theta = \pi + 2n\pi$, where n is any integer. **26.** $\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$

27. $\frac{5\pi}{24}, \frac{7\pi}{24}, \frac{17\pi}{24}, \frac{19\pi}{24}, \frac{29\pi}{24}, \frac{31\pi}{24}, \frac{41\pi}{24}, \frac{43\pi}{24}$ **28.** $\frac{5\pi}{16}, \frac{7\pi}{16}, \frac{13\pi}{16}, \frac{15\pi}{16}, \frac{21\pi}{16}, \frac{23\pi}{16}, \frac{29\pi}{16}, \frac{31\pi}{16}$ **29.** $\frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$

30. $\frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}, \frac{16\pi}{9}$ **36.** $\frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}$ **37.** $0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}$ **38.** $\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$ **41.** $\frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ **47.** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

48. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **49.** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **50.** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **52.** no solution **59.** $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ **60.** $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **63.** $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

64. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **68.** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **73.** $\frac{2\pi}{3}, \frac{4\pi}{3}$ **74.** $\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}$ **75.** $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$ **76.** $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

97. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **98.** $\frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$ **99.** $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ **100.** $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

103. 1.7798, 4.9214 **104.** 1.7303, 4.8719 **107.** $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **108.** $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ **109.** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **110.** $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

117. $\left(\frac{2\pi}{3}, -\frac{3}{2}\right), \left(\frac{4\pi}{3}, -\frac{3}{2}\right)$ **118.** $\left(\frac{7\pi}{6}, -\frac{3}{2}\right), \left(\frac{11\pi}{6}, -\frac{3}{2}\right)$ **119.** (3.5163, 0.7321), (5.9085, 0.7321) **120.** $(0, 1), \left(\frac{\pi}{6}, \frac{1}{2}\right), \left(\frac{5\pi}{6}, \frac{1}{2}\right), (\pi, 1), (2\pi, 1)$

$f(x) = 3\cos x$
$g(x) = \cos x - 1$

$y = 3\sin x$
$y = \sin x - 1$

$f(x) = \cos 2x$
$g(x) = -2\sin x$

$f(x) = \cos 2x$
$g(x) = 1 - \sin x$

121. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **129.** 0.4 sec and 2.1 sec **133.** $t = 2 + 6n$ or $t = 4 + 6n$ where n is any nonnegative integer.
134. $t = 1 + 6n$ or $t = 5 + 6n$ where n is any nonnegative integer.

147.

$x = 1.37$, $x = 2.30$, $x = 3.98$, or $x = 4.91$

148.

$x = 0.74$

149.

$x = 0.37$ or $x = 2.77$

150.

$x = 1.08$

151.

$x = 0$, $x = 1.57$, $x = 2.09$, $x = 3.14$, $x = 4.19$, or $x = 4.71$

156. true

Chapter 5 Review Exercises

32. b. $\sin\left(x - \frac{3\pi}{2}\right) = \sin x \cos\frac{3\pi}{2} - \cos x \sin\frac{3\pi}{2} = \sin x \cdot 0 - \cos x \cdot -1 = \cos x$

33. b. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2} = \cos x \cdot 0 - \sin x \cdot 1 = -\sin x$

34. b. $y = \dfrac{\tan x - 1}{1 - \cot x} = \dfrac{\dfrac{\sin x}{\cos x} - 1}{1 - \dfrac{\cos x}{\sin x}} = \dfrac{\dfrac{\sin x - \cos x}{\cos x}}{\dfrac{\sin x - \cos x}{\sin x}} = \dfrac{\sin x - \cos x}{\cos x} \cdot \dfrac{\sin x}{\sin x - \cos x} = \dfrac{\sin x}{\cos x} = \tan x$

35. a. $\frac{33}{65}$ **b.** $\frac{16}{65}$ **c.** $-\frac{33}{56}$ **d.** $\frac{24}{25}$ **e.** $\frac{2\sqrt{13}}{13}$ **36. a.** $-\frac{63}{65}$ **b.** $-\frac{56}{65}$ **c.** $\frac{63}{16}$ **d.** $\frac{24}{25}$ **e.** $\frac{5\sqrt{26}}{26}$

37. a. 1 **b.** $-\frac{3}{5}$ **c.** undefined **d.** $-\frac{3}{5}$ **e.** $\frac{\sqrt{10 + 3\sqrt{10}}}{2\sqrt{5}}$ **38. a.** 1 **b.** $\frac{4\sqrt{2}}{9}$ **c.** undefined **d.** $\frac{4\sqrt{2}}{9}$ **e.** $-\frac{\sqrt{3}}{3}$

43. $\frac{1}{2}[\cos 2x - \cos 10x]$ **44.** $\frac{1}{2}[\sin 10x + \sin 4x]$ **50.** $x = \frac{2\pi}{3} + 2n\pi$ or $x = \frac{4\pi}{3} + 2n\pi$, where n is any integer.

51. $x = \frac{\pi}{4} + 2n\pi$ or $x = \frac{3\pi}{4} + 2n\pi$, where n is any integer. **52.** $x = \frac{7\pi}{6} + 2n\pi$ or $x = \frac{11\pi}{6} + 2n\pi$, where n is any integer.

53. $x = \frac{\pi}{6} + n\pi$, where n is any integer. **57.** $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ **59.** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **61.** $0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$ **62.** $0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}$ **64.** 3.7890, 5.6358

65. 0.6847, 2.4569, 3.8263, 5.5985 **66.** $\frac{\pi}{4}$, 1.2490, $\frac{5\pi}{4}$, 4.3906 **67.** 0.8959, 2.2457 **68.** $t = \frac{2}{3} + 4n$ or $t = \frac{10}{3} + 4n$, where n is any integer.

Chapter 5 Test

5. $\frac{\sqrt{6} + \sqrt{2}}{4}$ **6.** $\cos x \csc x = \cos x \cdot \frac{1}{\sin x} = \frac{\cos x}{\sin x} = \cot x$ **7.** $\dfrac{\sec x}{\cot x + \tan x} = \dfrac{\dfrac{1}{\cos x}}{\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}} = \dfrac{\dfrac{1}{\cos x}}{\dfrac{\cos^2 x + \sin^2 x}{\sin x \cos x}} = \dfrac{1}{\cos x} \cdot \dfrac{\sin x \cos x}{1} = \sin x$

8. $1 - \dfrac{\cos^2 x}{1 + \sin x} = 1 - \dfrac{(1 - \sin^2 x)}{1 + \sin x} = 1 - \dfrac{(1 + \sin x)(1 - \sin x)}{1 + \sin x} = 1 - (1 - \sin x) = \sin x$

9. $\cos\left(\theta + \frac{\pi}{2}\right) = \cos\theta \cos\frac{\pi}{2} - \sin\theta \sin\frac{\pi}{2} = \cos\theta \cdot 0 - \sin\theta \cdot 1 = -\sin\theta$

10. $\dfrac{\sin(\alpha - \beta)}{\sin\alpha\cos\beta} = \dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\alpha\cos\beta} = \dfrac{\sin\alpha\cos\beta}{\sin\alpha\cos\beta} - \dfrac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta} = 1 - \cot\alpha\tan\beta$

11. $\sin t\cos t(\tan t + \cot t) = \sin t\cos t\left(\dfrac{\sin t}{\cos t} + \dfrac{\cos t}{\sin t}\right) = \sin^2 t + \cos^2 t = 1$

Cumulative Review Exercises (Chapters P–5)

6.

$y = \sqrt{x+2} - 1$

7.

$(x-1)^2 + (y+2)^2 = 9$

8.

$y + 2 = \frac{1}{3}(x - 1)$

9.

10.

$y = 2\sin\frac{x}{2} + 1$

11.

$f(x) = (x-1)^2(x-3)$

14. Proofs may vary.
19. 106 mg
20. $h \approx 15.9$ft

CHAPTER 6

Exercise Set 6.1

25. two triangles; $B_1 \approx 77°$, $C_1 \approx 43°$, $c_1 \approx 12.6$; $B_2 \approx 103°$, $C_2 \approx 17°$, $c_2 \approx 5.4$
26. two triangles; $B_1 \approx 27°$, $C_1 \approx 133°$, $c_1 \approx 64.2$; $B_2 \approx 153°$, $C_2 \approx 7°$, $c_2 \approx 10.7$
27. two triangles; $B_1 \approx 54°$, $C_1 \approx 89°$, $c_1 \approx 19.9$; $B_2 \approx 126°$, $C_2 \approx 17°$, $c_2 \approx 5.8$
28. two triangles; $B_1 \approx 56°$, $C_1 \approx 112°$, $c_1 \approx 31.2$; $B_2 \approx 124°$, $C_2 \approx 44°$, $c_2 \approx 23.4$
29. two triangles; $C_1 \approx 68°$, $B_1 \approx 54°$, $b_1 \approx 21.0$; $C_2 \approx 112°$, $B_2 \approx 10°$, $b_2 \approx 4.5$
30. two triangles; $C_1 \approx 83°$, $B_1 \approx 48°$, $b_1 \approx 93.5$; $C_2 \approx 97°$, $B_2 \approx 34°$, $b_2 \approx 70.4$
76. $\sqrt{7280} = 4\sqrt{455} \approx 85$
77.

Exercise Set 6.2

33. $A \approx 51°$, $B \approx 61°$, $C \approx 68°$, $AB = 9$, $AC = 8.5$, $BC = 7.5$
34. $A \approx 38°$, $B \approx 61°$, $C \approx 81°$, $AB = 11.8$, $AC = 10.5$, $BC = 7.3$
35. $A \approx 145°$, $B \approx 13°$, $C \approx 22°$, $a = \sqrt{61} \approx 7.8$, $b = \sqrt{10} \approx 3.2$, $c = 5$
36. $A \approx 42°$, $B \approx 71°$, $C \approx 67°$, $a = \sqrt{13} \approx 3.6$, $b = \sqrt{26} \approx 5.1$, $c = 5$
49. about 63.7 ft
50. about 42.6 ft

69.

70.

$x^2 + (y-1)^2 = 1$

71.

$x^2 + 6x + y^2 = 0$

Section 6.3

Check Point Exercises

1. a.

b.

c.

Exercise Set 6.3

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

21.

a. $\left(5, \frac{13\pi}{6}\right)$ **b.** $\left(-5, \frac{7\pi}{6}\right)$

c. $\left(5, -\frac{11\pi}{6}\right)$

22.

a. $\left(8, \frac{13\pi}{6}\right)$ **b.** $\left(-8, \frac{7\pi}{6}\right)$

c. $\left(8, -\frac{11\pi}{6}\right)$

23.

a. $\left(10, \frac{11\pi}{4}\right)$ **b.** $\left(-10, \frac{7\pi}{4}\right)$

c. $\left(10, -\frac{5\pi}{4}\right)$

24.

a. $\left(12, \frac{8\pi}{3}\right)$ **b.** $\left(-12, \frac{5\pi}{3}\right)$

c. $\left(12, -\frac{4\pi}{3}\right)$

25.

a. $\left(4, \frac{5\pi}{2}\right)$ **b.** $\left(-4, \frac{3\pi}{2}\right)$

c. $\left(4, -\frac{3\pi}{2}\right)$

26.

a. $\left(6, \frac{5\pi}{2}\right)$ **b.** $\left(-6, \frac{3\pi}{2}\right)$ **c.** $\left(6, -\frac{3\pi}{2}\right)$

39. approximately $(-5.9, 4.4)$

50. $r = \dfrac{8}{\cos\theta + 5\sin\theta}$

56. $r = -6\sin\theta$

59.

$x^2 + y^2 = 64$

60.

$x^2 + y^2 = 100$

61.

$x = 0$

62.

$y = \sqrt{3}x$

63.

$y = 3$

64.

$x = 7$

65.

$y = 4$

66.

$x = 6$

67.

$x^2 + y^2 = y$

68.

$x^2 + y^2 = x$

69.

$(x - 6)^2 + y^2 = 36$

70. $x^2 + (y + 2)^2 = 4$

$x^2 + (y + 2)^2 = 4$

71. $x^2 + y^2 = 6x + 4y$

$x^2 + y^2 = 6x + 4y$

72. $x^2 + y^2 = 8x + 2y$

$x^2 + y^2 = 8x + 2y$

73.

$y = \frac{1}{x}$

74.

$y = \frac{2}{x}$

75. $r = a\sec\theta$; $r\cos\theta = a$; $x = a$; $x = a$ is a vertical line a units to the right of the y-axis when $a > 0$ and $|a|$ to the left of the y-axis when $a < 0$.

76. $r = a\csc\theta$; $r\sin\theta = a$; $y = a$; $y = a$ is a horizontal line a units above the x-axis when $a > 0$ and $|a|$ below the x-axis when $a < 0$.

77. $r = a\sin\theta$; $r^2 = ar\sin\theta$; $x^2 + y^2 = ay$; $x^2 + y^2 - ay = 0$; $x^2 + \left(y - \frac{a}{2}\right)^2 = \left(\frac{a}{2}\right)^2$

78. $r = a\cos\theta$; $r^2 = ar\cos\theta$; $x^2 + y^2 = ax$; $x^2 - ax + y^2 = 0$; $\left(x - \frac{a}{2}\right)^2 + y^2 = \left(\frac{a}{2}\right)^2$ **79.** $y = x + 2\sqrt{2}$; slope: 1; y-intercept: $2\sqrt{2}$

80. $y = \sqrt{3}x - 16$; slope: $\sqrt{3}$; y-intercept: -16 **81.** $(-1, \sqrt{3})$, $(2\sqrt{3}, 2)$; $2\sqrt{5}$ **82.** $(-6, 0)$, $\left(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$; $\sqrt{61 + 30\sqrt{2}}$

85. 6.3 knots at an angle of 50° to the wind **86.** 7.4 knots at an angle of 85° to the wind

109.

110.

111.

Section 6.4

Check Point Exercises

1.

$r = 4\sin\theta$

2.

$r = 1 + \cos\theta$

3.

$r = 1 - 2\sin\theta$

4.

$r = 3\cos 2\theta$

5.

$r^2 = 4\cos 2\theta$

Exercise Set 6.4

7. a. May or may not have symmetry with respect to polar axis. **b.** Has symmetry with respect to the line $\theta = \frac{\pi}{2}$. **c.** May or may not have symmetry about the pole. **8. a.** Has symmetry with respect to the polar axis. **b.** May or may not have symmetry with respect to the line $\theta = \frac{\pi}{2}$. **c.** May or may not have symmetry about the pole. **9. a.** Has symmetry with respect to polar axis. **b.** May or may not have symmetry with respect to the line $\theta = \frac{\pi}{2}$. **c.** May or may not have symmetry about pole. **10. a.** Has symmetry with respect to the polar axis. **b.** May or may not have symmetry with respect to the line $\theta = \frac{\pi}{2}$. **c.** May or may not have symmetry about the pole. **11. a.** Has symmetry with respect to polar axis. **b.** Has symmetry with respect to the line $\theta = \frac{\pi}{2}$. **c.** Has symmetry about the pole. **12. a.** May or may not have symmetry with respect to the polar axis. **b.** May or may not have symmetry with respect to the line $\theta = \frac{\pi}{2}$. **c.** Has symmetry with respect to the pole.

13.

$r = 2\cos\theta$

14.

$r = 2\sin\theta$

15.

$r = 1 - \sin\theta$

16.

$r = 1 + \sin\theta$

17.

$r = 2 + 2\cos\theta$

18.

$r = 2 - 2\cos\theta$

19.

$r = 2 + \cos\theta$

20.

$r = 2 - \sin\theta$

21.

$r = 1 + 2\cos\theta$

22.

$r = 1 - 2\cos\theta$

23.

$r = 2 - 3\sin\theta$

24.

$r = 2 + 4\sin\theta$

25.

$r = 2\cos 2\theta$

26.

$r = 2\sin 2\theta$

27.

$r = 4\sin 3\theta$

28.

$r = 4\cos 3\theta$

29.

$r^2 = 9\cos 2\theta$

30.

$r^2 = 9\sin 2\theta$

31.

$r = 1 - 3\sin\theta$

32.

$r = 3 + \sin\theta$

33.

34.

35. $\left(\frac{\sqrt{2}}{2}, \frac{\pi}{2}\right)$ or $\left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{2}\right)$

$r = \cos\frac{\theta}{2}$

36.

$r = \sin\frac{\theta}{2}$

37.

$r = \sin\theta + \cos\theta$

38.

$r = 4\cos\theta + 4\sin\theta$

39.

$r = \dfrac{1}{1 - \cos\theta}$

40.

$r = \dfrac{2}{1 - \cos\theta}$

41.

$r = \sin\theta\cos^2\theta$

42.

$r = \dfrac{3\sin 2\theta}{\sin^3\theta + \cos^3\theta}$

43.

$r = 2 + 3\sin 2\theta$

44.

$r = 2 - 4\cos 2\theta$

58.

59.

60.

61.

62.

63.

64.

65.

66.

67.

68.

69.

70.

71.

72.

73.

74.

75.

79.

80.

81.

82.

83. If n is odd, there are n loops and $\theta\max = \pi$ traces the graph once; while if n is even, there are $2n$ loops and $\theta\max = 2\pi$ traces the graph once. In each separate case, as n increases, $\sin n\theta$ increases its number of loops.

85. There are n small petals and n large petals for each value of n. For odd values of n, the small petals are inside the large petals. For even n, they are between the large petals.

87.

88.

93.

The graph of r_2 is the graph of r_1 rotated $\frac{\pi}{4}$ or 45°.

94.

The graph of r_2 is the graph of r_1 rotated $-\frac{\pi}{6}$ or −30°.

Mid-Chapter 6 Check Point

1. $C = 107°$, $b \approx 24.8$, $c \approx 36.1$ **2.** $B \approx 37°$, $C \approx 101°$, $c \approx 92.4$ **3.** no triangle **4.** $A \approx 26°$, $C \approx 44°$, $b \approx 21.6$
5. Two triangles: $A_1 \approx 55°$, $B_1 \approx 83°$, $b_1 \approx 19.3$; $A_2 \approx 125°$, $B_2 \approx 13°$, $b_2 \approx 4.4$ **6.** $A \approx 28°$, $B \approx 42°$, $C \approx 110°$

16.

a. $\left(4, \frac{11\pi}{4}\right)$ **b.** $\left(-4, \frac{7\pi}{4}\right)$

c. $\left(4, -\frac{5\pi}{4}\right)$

17.

a. $\left(\frac{5}{2}, \frac{5\pi}{2}\right)$ **b.** $\left(-\frac{5}{2}, \frac{3\pi}{2}\right)$

c. $\left(\frac{5}{2}, -\frac{3\pi}{2}\right)$

18. $r = \dfrac{7}{5\cos\theta - \sin\theta}$

21.

22.

23.

24.

25.

26. a. Has symmetry with respect to the polar axis. **b.** May or may not have symmetry with respect to the line $\theta = \frac{\pi}{2}$. **c.** May or may not have symmetry with respect to the pole. **27. a.** Has symmetry with respect to the polar axis. **b.** Has symmetry with respect to the line $\theta = \frac{\pi}{2}$. **c.** Has symmetry with respect to the pole.

28.

29.

30.

31.

32.

Section 6.5

Check Point Exercises

1. a.

b.

c.

d.

3.

Exercise Set 6.5

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

$2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

or $2\sqrt{2}(\cos 45° + i\sin 45°)$

12.

$2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ or $2(\cos 60° + i\sin 60°)$

13.

$\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

or $\sqrt{2}(\cos 225° + i\sin 225°)$

14.

$2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$ or $2\sqrt{2}(\cos 315° + i\sin 315°)$

15.

$4\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$ or $4(\cos 270° + i\sin 270°)$

16.

$3\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$ or $3(\cos 270° + i\sin 270°)$

17.

$4\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$ or $4(\cos 330° + i\sin 330°)$

18.

$4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ or $4(\cos 120° + i\sin 120°)$

19.

$3(\cos\pi + i\sin\pi)$ or $3(\cos 180° + i\sin 180°)$

20.

$4(\cos\pi + i\sin\pi)$ or $4(\cos 180° + i\sin 180°)$

21.

$\approx 3\sqrt{5}(\cos 230.8° + i\sin 230.8°)$

22.

$6\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$ or $6(\cos 315° + i\sin 315°)$

23.

$\approx 5(\cos 126.9° + i\sin 126.9°)$

24.

$\approx \sqrt{13}(\cos 123.7° + i\sin 123.7°)$

25.

$\approx \sqrt{7}(\cos 319.1° + i\sin 319.1°)$

26.

$\approx \sqrt{6}(\cos 294.1° + i\sin 294.1°)$

27. $3\sqrt{3} + 3i$ **28.** $6 + 6i\sqrt{3}$ **29.** $-2 - 2i\sqrt{3}$ **30.** $-5\sqrt{3} - 5i$ **31.** $4\sqrt{2} - 4i\sqrt{2}$

32. $-2\sqrt{3} + 2i$ **35.** $-18.1 - 8.5i$ **36.** $-20.0 + 22.4i$ **41.** $\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}$

42. $\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}$ **44.** $4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ **65.** $3(\cos 15° + i\sin 15°)$; $3(\cos 195° + i\sin 195°)$

66. $5(\cos 105° + i\sin 105°)$; $5(\cos 285° + i\sin 285°)$

67. $2(\cos 70° + i\sin 70°)$; $2(\cos 190° + i\sin 190°)$; $2(\cos 310° + i\sin 310°)$

68. $3(\cos 102° + i\sin 102°)$; $3(\cos 222° + i\sin 222°)$; $3(\cos 342° + i\sin 342°)$ **69.** $\frac{3}{2} + \frac{3\sqrt{3}}{2}i; -\frac{3\sqrt{3}}{2} + \frac{3}{2}i; -\frac{3}{2} - \frac{3\sqrt{3}}{2}i; \frac{3\sqrt{3}}{2} - \frac{3}{2}i$

70. $1 + i\sqrt{3}$; $\approx -1.3 + 1.5i$; $\approx -1.8 - 0.8i$; $\approx 0.2 - 2.0i$; $\approx 2.0 - 0.4i$ **71.** 2; $\approx 0.6 + 1.9i$; $\approx -1.6 + 1.2i$; $\approx -1.6 - 1.2i$; $\approx 0.6 - 1.9i$

72. $2; 1 + i\sqrt{3}; -1 + i\sqrt{3}; -2; -1 - i\sqrt{3}; 1 - i\sqrt{3}$ **73.** $1; -\frac{1}{2} + \frac{\sqrt{3}}{2}i; -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ **74.** $\frac{\sqrt{3}}{2} + \frac{1}{2}i; -\frac{\sqrt{3}}{2} + \frac{1}{2}i; -i$

75. $\approx 1.1 + 0.2i$; $\approx -0.2 + 1.1i$; $\approx -1.1 - 0.2i$; $\approx 0.2 - 1.1i$ **76.** $\approx 1.0 + 0.5i$; $\approx -0.2 + 1.1i$; $\approx -1.1 + 0.2i$; $\approx -0.5 - 1.0i$; $0.8 - 0.8i$

77. $[1(\cos 90° + i\sin 90°)][2\sqrt{2}(\cos 45° + i\sin 45°)][2(\cos 150° + i\sin 150°)]$; $4\sqrt{2}(\cos 285° + i\sin 285°)$; $\approx 1.4641 - 5.4641i$

78. $[\sqrt{2}(\cos 45° + i\sin 45°)][2(\cos(300°) + i\sin(300°))][2(\cos 150° + i\sin 150°)]$; $4\sqrt{2}(\cos 135° + i\sin 135°)$; $-4 + 4i$

79. $\dfrac{[2(\cos 60° + i\sin 60°)][\sqrt{2}(\cos(315°) + i\sin(315°))]}{4(\cos(330°) + i\sin(330°))}$; $\frac{\sqrt{2}}{2}(\cos 45° + i\sin 45°)$; $\frac{1}{2} + \frac{1}{2}i$

80. $\dfrac{[2(\cos 120° + i\sin 120°)][4(\cos(300°) + i\sin(300°))]}{8(\cos(330°) + i\sin(330°))}$; $\cos 90° + i\sin 90°$; i

81. $\cos 0° + i \sin 0°, \cos 60° + i \sin 60°, \cos 120° + i \sin 120°, \cos 180° + i \sin 180°, \cos 240° + i \sin 240°, \cos 300° + i \sin 300°;$
$1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -1, -\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$

82. $\cos 30° + i \sin 30°, \cos 90° + i \sin 90°, \cos 150° + i \sin 150°, \cos 210° + i \sin 210°, \cos 270° + i \sin 270°, \cos 330° + i \sin 330°;$
$\frac{\sqrt{3}}{2} + \frac{1}{2}i, i, -\frac{\sqrt{3}}{2} + \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i, -i, \frac{\sqrt{3}}{2} - \frac{1}{2}i$

83. $2(\cos 67.5° + i \sin 67.5°), 2(\cos 157.5° + i \sin 157.5°), 2(\cos 247.5° + i \sin 247.5°), 2(\cos 337.5° + i \sin 337.5°);$
$0.7654 + 1.8478i, -1.8478 + 0.7654i, -0.7654 - 1.8478i, 1.8478 - 0.7654i$

84. $2(\cos 18° + i \sin 18°), 2(\cos 90° + i \sin 90°), 2(\cos 162° + i \sin 162°), 2(\cos 234° + i \sin 234°), 2(\cos 306° + i \sin 306°);$
$1.9021 + 0.6180i, 2i, -1.9021 + 0.6180i, -1.1756 - 1.6180i, 1.1756 - 1.6180i$

85. $\sqrt[3]{2}(\cos 20° + i \sin 20°), \sqrt[3]{2}(\cos 140° + i \sin 140°), \sqrt[3]{2}(\cos 260° + i \sin 260°); 1.1839 + 0.4309i, -0.9652 + 0.8099i, -0.2188 - 1.2408i$

86. $\sqrt[3]{2}(\cos 100° + i \sin 100°), \sqrt[3]{2}(\cos 220° + i \sin 220°), \sqrt[3]{2}(\cos 340° + i \sin 340°); -0.2188 + 1.2408i, -0.9652 - 0.8099i, 1.1839 - 0.4309i$

87.

88.

89.

90.

91. a. $i; -1 + i; -i; -1 + i; -i; -1 + i$ **b.** Complex numbers may vary. **92. a.** $-i; -1 - i; i; -1 - i; i; -1 - i$ **b.** Complex numbers may vary.

111.

113. Yes, both have length $3\sqrt{5}$. **114.** Yes, both have slope 2.

Section 6.6

Check Point Exercises

2.

Exercise Set 6.6

5.

6.

7.

8.

9.

10.

11.

12.

47. $3\sqrt{3}\mathbf{i} + 3\mathbf{j}$ **48.** $4\sqrt{2}\mathbf{i} + 4\sqrt{2}\mathbf{j}$ **49.** $-6\sqrt{2}\mathbf{i} - 6\sqrt{2}\mathbf{j}$ **50.** $5\sqrt{3}\mathbf{i} - 5\mathbf{j}$ **51.** $\approx -0.20\mathbf{i} + 0.46\mathbf{j}$ **52.** $\approx -0.23\mathbf{i} - 0.09\mathbf{j}$

80. a. $\mathbf{F} = 4\mathbf{i} - 10\mathbf{j}$ **b.** $\mathbf{F}_4 = -4\mathbf{i} + 10\mathbf{j}$ **83. a.** $\mathbf{v} = 180 \cos 40°\mathbf{i} + 180 \sin 40°\mathbf{j} \approx 137.89\mathbf{i} + 115.70\mathbf{j}$, $\mathbf{w} = 40 \cos 0°\mathbf{i} + 40 \sin 0°\mathbf{j} = 40\mathbf{i}$

105. does not make sense **107.** does not make sense **118. b.** $\|\mathbf{u}\| = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$; $\|\mathbf{u}\|^2 = (a_1 - a_2)^2 + (b_1 - b_2)^2$;
$\|\mathbf{v}\| = \sqrt{a_1^2 + b_1^2}$; $\|\mathbf{v}\|^2 = a_1^2 + b_1^2$; $\|\mathbf{w}\| = \sqrt{a_2^2 + b_2^2}$; $\|\mathbf{w}\|^2 = a_2^2 + b_2^2$

Exercise Set 6.7

1. 6; 10 **2.** 15; 18 **33.** $\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = \frac{5}{2}\mathbf{i} - \frac{5}{2}\mathbf{j}$; $\mathbf{v}_2 = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$ **34.** $\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = \frac{8}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$; $\mathbf{v}_2 = \frac{7}{5}\mathbf{i} - \frac{14}{5}\mathbf{j}$

35. $\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = -\frac{26}{29}\mathbf{i} + \frac{65}{29}\mathbf{j}$; $\mathbf{v}_2 = \frac{55}{29}\mathbf{i} + \frac{22}{29}\mathbf{j}$ **36.** $\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = -\frac{6}{5}\mathbf{i} + \frac{12}{5}\mathbf{j}$; $\mathbf{v}_2 = \frac{16}{5}\mathbf{i} + \frac{8}{5}\mathbf{j}$

51. 1617; $\mathbf{v} \cdot \mathbf{w} = 1617$ means that \$1617 in revenue is generated when 240 gallons of regular gasoline are sold at \$2.90 per gallon and 300 gallons of premium gasoline are sold at \$3.09 per gallon. **52.** 1440; $\mathbf{v} \cdot \mathbf{w} = 1440$ is the total collected by the video store when it rented 180 one-day videos at \$3 each and 450 three-day videos at \$2 each. **53.** 7600 foot-pounds **60.** 113 meter-newtons

63. c. 350; A force of 350 pounds is required to keep the boat from rolling down the ramp.

64. c. 325; A force of 325 pounds is required to keep the boat from rolling down the ramp.

79.
$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= (a_1\mathbf{i} + b_1\mathbf{j})\cdot(a_2\mathbf{i} + b_2\mathbf{j})\\ &= a_1a_2 + b_1b_2\\ &= a_2a_1 + b_2b_1\\ &= (a_2\mathbf{i} + b_2\mathbf{j})\cdot(a_1\mathbf{i} + b_1\mathbf{j})\\ &= \mathbf{v}\cdot\mathbf{u}\end{aligned}$$

80.
$$\begin{aligned}(c\mathbf{u})\cdot\mathbf{v} &= [c(a_1\mathbf{i} + b\mathbf{j})]\cdot(a_2\mathbf{i} + b_2\mathbf{j})\\ &= (ca_1\mathbf{i} + cb_1\mathbf{j})\cdot(a_2\mathbf{i} + b_2\mathbf{j})\\ &= ca_1a_2 + cb_1b_2\\ &= c(a_1a_2 + b_1b_2)\\ &= c[(a_1\mathbf{i} + b_1\mathbf{j})\cdot(a_2\mathbf{i} + b_2\mathbf{j})]\\ &= c(\mathbf{u}\cdot\mathbf{v})\end{aligned}$$

81.
$$\begin{aligned}\mathbf{u}\cdot(\mathbf{v} + \mathbf{w}) &= (a_1\mathbf{i} + b_1\mathbf{j})\cdot[(a_2\mathbf{i} + b_2\mathbf{j}) + (a_3\mathbf{i} + b_3\mathbf{j})]\\ &= (a_1\mathbf{i} + b_1\mathbf{j})\cdot[(a_2 + a_3)\mathbf{i} + (b_2 + b_3)\mathbf{j}]\\ &= a_1(a_2 + a_3) + b_1(b_2 + b_3)\\ &= a_1a_2 + a_1a_3 + b_1b_2 + b_1b_3\\ &= a_1a_2 + b_1b_2 + a_1a_3 + b_1b_3\\ &= (a_1\mathbf{i} + b_1\mathbf{j})\cdot(a_2\mathbf{i} + b_2\mathbf{j}) + (a_1\mathbf{i} + b_1\mathbf{j})\cdot(a_3\mathbf{i} + b_3\mathbf{j})\\ &= \mathbf{u}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{w}\end{aligned}$$

82. Answers may vary. One possible vector is $\mathbf{u} = 5\mathbf{i} + 2\mathbf{j}$.

84. By definition, the projection of v onto **i** is $\dfrac{\mathbf{v}\cdot\mathbf{i}}{\|\mathbf{i}\|^2}\mathbf{i}$. But $\|\mathbf{i}\| = 1$, so the projection simplifies to $(\mathbf{v}\cdot\mathbf{i})\mathbf{i}$.

88. $(4, -1)$;

Chapter 6 Review Exercises

6. two triangles; $B_1 \approx 55°$, $C_1 \approx 86°$, and $c_1 \approx 31.7$; $B_2 \approx 125°$, $C_2 \approx 16°$, and $c_2 \approx 8.8$

11. two triangles; $A_1 \approx 59°$, $C_1 \approx 84°$, $c_1 \approx 14.4$; $A_2 \approx 121°$, $C_2 \approx 22°$, $c_2 \approx 5.4$

22.

$(2, 2\sqrt{3})$

23.

$\left(-\dfrac{3\sqrt{3}}{2}, \dfrac{3}{2}\right)$

24.

$(2, 2\sqrt{3})$

25.

$(\sqrt{2}, \sqrt{2})$

26.

$(0, 4)$

27.

$(-\sqrt{2}, \sqrt{2})$

28.

a. $\left(3, \frac{13\pi}{6}\right)$ **b.** $\left(-3, \frac{7\pi}{6}\right)$

c. $\left(3, -\frac{11\pi}{6}\right)$

29.

a. $\left(2, \frac{8\pi}{3}\right)$ **b.** $\left(-2, \frac{5\pi}{3}\right)$

c. $\left(2, -\frac{4\pi}{3}\right)$

30.

a. $\left(3.5, \frac{5\pi}{2}\right)$ **b.** $\left(-3.5, \frac{3\pi}{2}\right)$

c. $\left(3.5, -\frac{3\pi}{2}\right)$

31. $\left(4\sqrt{2}, \frac{3\pi}{4}\right)$ **32.** $\left(3\sqrt{2}, \frac{7\pi}{4}\right)$ **33.** approximately $(13, 67°)$ **34.** approximately $(5, 127°)$

40.

41.

42.

43.

44. $\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$

45. $y = -4x + 8$

46.

47. a. has symmetry **b.** may or may not have symmetry **c.** may or may not have symmetry

48. a. may or may not have symmetry **b.** has symmetry **c.** may or may not have symmetry

49. a. has symmetry **b.** has symmetry **c.** has symmetry

50.

$r = 3\cos\theta$

51.

$r = 2 + 2\sin\theta$

52.

$r = \sin 2\theta$

53.

$r = 2 + \cos\theta$

54.

$r = 1 + 3\sin\theta$

55.

$r = 1 - 2\cos\theta$

56.

$r^2 = \cos 2\theta$

57.

$\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$ or

$\sqrt{2}(\cos 315° + i\sin 315°)$

58.

$4(\cos 150° + i\sin 150°)$ or

$4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

59.

$z = -3 - 4i \approx 5(\cos 233° + i\sin 233°)$

60.

$5\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$ or

$5(\cos 270° + i\sin 270°)$

61. $z = 4 + 4\sqrt{3}i$
62. $z = -2\sqrt{3} - 2i$
63. $z = -3 + 3\sqrt{3}i$
64. $z \approx -0.1 + 0.6i$

76. $7(\cos 25° + i\sin 25°)$; $7(\cos 205° + i\sin 205°)$

77. $5(\cos 55° + i\sin 55°)$; $5(\cos 175° + i\sin 175°)$; $5(\cos 295° + i\sin 295°)$

78. $\sqrt{3} + i; -1 + i\sqrt{3}; -\sqrt{3} - i; 1 - i\sqrt{3}$

81. $\frac{\sqrt[10]{2}}{2} + \frac{\sqrt[10]{2}}{2}i;\ \approx -0.49 + 0.95i;\ \approx -1.06 - 0.17i;\ \approx -0.17 - 1.06i;\ \approx 0.95 - 0.49i$

82.

83.

84.

102. $\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = \frac{50}{41}\mathbf{i} + \frac{40}{41}\mathbf{j}; \mathbf{v}_2 = -\frac{132}{41}\mathbf{i} + \frac{165}{41}\mathbf{j}$ **103.** $\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = -\frac{3}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}; \mathbf{v}_2 = \frac{1}{2}\mathbf{i} + \frac{3}{2}\mathbf{j}$

Chapter 6 Test

4.

; Ordered pairs may vary.

7.

8.

9.

10. $2(\cos 150° + i \sin 150°)$ or $2\left(\cos\frac{5\pi}{6} + i \sin\frac{5\pi}{6}\right)$

21. 323 pounds; 3.4°

Cumulative Review Exercises (Chapters P–6)

5.

6.

7. $\sin\theta \csc\theta - \cos^2\theta = \sin\theta\left(\frac{1}{\sin\theta}\right) - \cos^2\theta$
$= 1 - \cos^2\theta = \sin^2\theta$

8. $\cos\left(\theta + \frac{3\pi}{2}\right) = \cos\theta\cos\frac{3\pi}{2} - \sin\theta\sin\frac{3\pi}{2}$
$= \cos\theta(0) - \sin\theta(-1) = \sin\theta$

16. $\frac{\sqrt{\sqrt{2} + 2}}{2}$ **17. a.** $5\mathbf{i} + 23\mathbf{j}$ **b.** -12

CHAPTER 7

Exercise Set 7.1

33. $\{(x, y) | y = 3x - 5\}$ **34.** $\{(x, y) | y = 3x - 4\}$ **37.** $\{(x, y) | x + 3y = 2\}$ **38.** $\{(x, y) | 2x - y = 1\}$ **43.** $x + y = 7; x - y = -1$; 3 and 4 **44.** $x + y = 2; x - y = 8$; 5 and -3 **46.** $3x + 2y = 8; 2x - y = 3$; 2 and 1 **53.** $y = x - 4; y = -\frac{1}{3}x + 4$ **54.** $y = \frac{1}{3}x + 2, y = \frac{1}{3}x - 2$ **55.** California: 100 gal; French: 100 gal **57.** 18-karat gold: 96 g; 12-karat gold: 204 g **69.** -6000; When the company produces and sells 200 radios, the loss is \$6000. **70.** -4000; When the company produces and sells 300 radios, the loss is \$4000. **73. a.** $C(x) = 18{,}000 + 20x$ **b.** $R(x) = 80x$ **c.** (300, 24,000); When 300 canoes are produced and sold, both revenue and cost are \$24,000. **74. a.** $C(x) = 100{,}000 + 100x$ **b.** $R(x) = 300x$ **c.** (500, 150,000); When 500 bicycles are produced and sold, both cost and revenue are \$150,000. **75. a.** $C(x) = 30{,}000 + 2500x$ **b.** $R(x) = 3125x$ **c.** (48, 150,000); For 48 sold-out performances, both cost and revenue are \$150,000. **76. a.** $C(x) = 30{,}000 + 0.02x$ **b.** $R(x) = 0.5x$ **c.** (62,500, 31,250); For 62,500 cards, both cost and revenue are \$31,250. **77. a.** 4 million workers; \$4.50 per hour **81. a.** $y = 0.45x + 0.8$ **c.** week 6; 3.5 symptoms; by the intersection point (6, 3.5) **85.** Mr. Goodbar: 264 cal; Mounds: 258 cal **87.** 3 Mr. Goodbars and 2 Mounds bars **106.** does not make sense **109.** $y = \dfrac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}; x = \dfrac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$

Exercise Set 7.2

19. $y = 2x^2 - x + 3$ **20.** $y = 2x^2 - x - 3$ **21.** $y = 2x^2 + x - 5$ **22.** $y = x^2 - 6x + 8$ **29.** $\left\{\left(\frac{8}{a}, -\frac{3}{b}, -\frac{5}{c}\right)\right\}$ **30.** $\left\{\left(-\frac{9}{a}, \frac{5}{b}, \frac{5}{c}\right)\right\}$ **31. b.** $y = 0$ when $x = 5$; The ball hits the ground after 5 seconds. **32. b.** 156; When a car is in motion for 6 seconds after the brakes are applied, it travels 156 feet. **49.** does not make sense **51.** makes sense

Exercise Set 7.3

1. $\frac{A}{x-2} + \frac{B}{x-1}$ **2.** $\frac{A}{x-1} + \frac{B}{x+3}$ **3.** $\frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$ **4.** $\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$ **5.** $\frac{A}{x-1} + \frac{Bx+C}{x^2+1}$ **6.** $\frac{A}{x-4} + \frac{Bx+C}{x^2+5}$ **8.** $\frac{Ax+B}{x^2+7} + \frac{Cx+D}{(x^2+7)^2}$ **9.** $\frac{3}{x-3} - \frac{2}{x-2}$ **11.** $\frac{7}{x-9} - \frac{4}{x+2}$ **12.** $\frac{3}{x-2} + \frac{2}{x+1}$ **13.** $\frac{24}{7(x-4)} + \frac{25}{7(x+3)}$ **14.** $\frac{6}{x-3} + \frac{3}{x+5}$ **15.** $\frac{4}{7(x-3)} - \frac{8}{7(2x+1)}$ **16.** $\frac{3}{4(x+3)} + \frac{1}{4(x-1)}$ **17.** $\frac{3}{x} + \frac{2}{x-1} - \frac{1}{x+3}$ **18.** $\frac{3}{x} - \frac{1}{x+1} + \frac{2}{x-5}$ **19.** $\frac{3}{x} + \frac{4}{x+1} - \frac{3}{x-1}$ **20.** $\frac{3}{x} - \frac{5}{x-2} + \frac{4}{x+2}$ **23.** $\frac{1}{x-2} - \frac{2}{(x-2)^2} - \frac{5}{(x-2)^3}$ **24.** $\frac{2}{x+1} + \frac{4}{(x+1)^2} - \frac{3}{(x+1)^3}$ **25.** $\frac{7}{x} - \frac{6}{x-1} + \frac{10}{(x-1)^2}$ **26.** $\frac{1}{x} + \frac{2}{x+7} - \frac{28}{(x+7)^2}$ **27.** $\frac{1}{4(x+1)} + \frac{3}{4(x-1)} + \frac{1}{2(x-1)^2}$ **28.** $-\frac{1}{4(x+1)} + \frac{1}{4(x-1)} + \frac{1}{4(x+1)^2} + \frac{1}{4(x-1)^2}$ **29.** $\frac{3}{x-1} + \frac{2x-4}{x^2+1}$ **30.** $\frac{3}{x-4} + \frac{2x-1}{x^2+5}$ **31.** $\frac{2}{x+1} + \frac{3x-1}{x^2+2x+2}$ **32.** $\frac{2}{x-2} - \frac{2x-1}{x^2+2x+2}$ **33.** $\frac{1}{4x} + \frac{1}{x^2} - \frac{x+4}{4(x^2+4)}$ **34.** $\frac{14}{3(x-1)} + \frac{4}{(x-1)^2} - \frac{14x-4}{3(x^2+2)}$ **35.** $\frac{4}{x+1} + \frac{2x-3}{x^2+1}$ **36.** $\frac{3}{x+2} - \frac{2}{x^2+4}$ **37.** $\frac{x+1}{x^2+2} - \frac{2x}{(x^2+2)^2}$ **38.** $\frac{1}{x^2+4} + \frac{2x-1}{(x^2+4)^2}$ **39.** $\frac{x-2}{x^2-2x+3} + \frac{2x+1}{(x^2-2x+3)^2}$ **40.** $\frac{3x}{x^2-2x+2} + \frac{x-2}{(x^2-2x+2)^2}$ **41.** $\frac{3}{x-2} + \frac{x-1}{x^2+2x+4}$ **42.** $-\frac{2}{3(x-1)} + \frac{2x+13}{3(x^2+x+1)}$ **43.** $x^3 + x - \frac{1}{2(x+1)} + \frac{3}{2(x-1)}$ **44.** $x^3 + 4x^2 + 12x + 32 + \frac{80}{x-2} + \frac{32}{(x-2)^2}$ **45.** $x + 1 - \frac{2}{x} - \frac{2}{x^2} + \frac{2}{x-1}$ **46.** $x^2 + 3x + 1 + \frac{5}{x-2} + \frac{3}{x+1}$ **47.** $\frac{\frac{1}{2c}}{x-c} - \frac{\frac{1}{2c}}{x+c}$ **48.** $\frac{\frac{ac+b}{2c}}{x-c} + \frac{\frac{ac-b}{2c}}{x+c}$ **61.** does not make sense **62.** does not make sense

68.

; (0, −3) and (2, −1); $0 - (-3) = 3$ and $(0 - 2)^2 + (-3 + 3)^2 = 4$ are true; $2 - (-1) = 3$ and $(2 - 2)^2 + (-1 + 3)^2 = 4$ are true.

Exercise Set 7.4

5. $\{(4, -10), (-3, 11)\}$ **13.** $\{(3, 1), (-3, -1), (1, 3), (-1, -3)\}$ **19.** $\{(3, 2), (3, -2), (-3, 2), (-3, -2)\}$ **21.** $\{(3, 2), (3, -2), (-3, 2), (-3, -2)\}$ **22.** $\{(1, 2), (1, -2), (-1, 2), (-1, -2)\}$ **23.** $\{(2, 1), (2, -1), (-2, 1), (-2, -1)\}$ **24.** $\{(\sqrt{5}, \sqrt{2}), (-\sqrt{5}, \sqrt{2}), (-\sqrt{5}, -\sqrt{2}), (\sqrt{5}, -\sqrt{2})\}$ **26.** $\left\{\left(\frac{\sqrt{31}}{4}, \frac{7}{4}\right), \left(-\frac{\sqrt{31}}{4}, \frac{7}{4}\right)\right\}$ **27.** $\{(0, 2), (0, -2), (-1, \sqrt{3}), (-1, -\sqrt{3})\}$ **28.** $\{(0, -4), (2\sqrt{3}, 2), (-2\sqrt{3}, 2)\}$ **29.** $\{(2, 1), (2, -1), (-2, 1), (-2, -1)\}$ **30.** $\{(4, 0), (-3, \sqrt{7}), (-3, -\sqrt{7})\}$ **31.** $\{(-2\sqrt{2}, -\sqrt{2}), (-1, -4), (1, 4), (2\sqrt{2}, \sqrt{2})\}$ **32.** $\{(-2, -2), (2, 2), (4, 1), (-4, -1)\}$ **34.** $\left\{(1, 1), \left(\frac{19}{29}, -\frac{11}{29}\right)\right\}$ **39.** $\left\{(-4, 1), \left(-\frac{5}{2}, \frac{1}{4}\right)\right\}$ **41.** $\left\{\left(\frac{12}{5}, -\frac{29}{5}\right), (-2, 3)\right\}$

42. $\{(-5, 0), (4, 3)\}$ **45.** 2 and 1, 2 and -1, -2 and 1, or -2 and -1 **46.** 3 and 2, 3 and -2, -3 and 2, or -3 and -2 **47.** $\{(2, -1), (-2, 1)\}$

48. $\{(-3, 2), (3, -2)\}$ **49.** $\{(2, 20), (-2, 4), (-3, 0)\}$ **50.** $\{(-5, 0), (-3, 18), (3, 72)\}$ **51.** $\left\{\left(-1, -\frac{1}{2}\right), \left(-1, \frac{1}{2}\right), \left(1, -\frac{1}{2}\right), \left(1, \frac{1}{2}\right)\right\}$

52. $\left\{\left(-1, -\frac{1}{3}\right), \left(-1, \frac{1}{3}\right), \left(1, -\frac{1}{3}\right), \left(1, \frac{1}{3}\right)\right\}$

53.

$y = x^2 - 4$
$y = 2x - 4$

54.

$\begin{cases} y = 4 - x^2 \\ y = -4x + 8 \end{cases}$

63. a. between the 1940s and the 1960s
d. 1919; white collar: 27.5%; farmers: 27.4%; fairly well, although answers may vary.

72. makes sense

81.

$2x - 3y = 6$

82.

83.

Mid-Chapter 7 Check Point

4. $\{(x, y) | y = 4x - 5\}$ or $\{(x, y) | 8x - 2y = 10\}$ **7.** $\{(-1, 2, -2)\}$ **8.** $\{(4, -2, 3)\}$ **9.** $\left\{\left(-\frac{9}{5}, \frac{12}{5}\right), (3, 0)\right\}$

10. $\{(-2, -1), (-2, 1), (2, -1), (2, 1)\}$ **11.** $\{(-\sqrt{7}, 1), (-2, -2), (2, -2), (\sqrt{7}, 1)\}$ **13.** $\frac{1}{x - 2} - \frac{2}{(x - 2)^2} - \frac{5}{(x - 2)^3}$

14. $\frac{5}{x + 2} + \frac{3}{x + 1} + \frac{2}{x - 1}$ **15.** $-\frac{2}{x + 3} + \frac{3x - 5}{x^2 + 4}$ **16.** $\frac{x}{x^2 + 4} - \frac{4x}{(x^2 + 4)^2}$ **17. a.** $C(x) = 400{,}000 + 20x$ **b.** $R(x) = 100x$
d. (5000, 500,000); The company will break even when it produces and sells 5000 PDAs. At this level, both revenue and cost are \$500,000.

18. 6 roses and 14 carnations

Section 7.5

Check Point Exercises

1. $4x - 2y \geq 8$

2. $y > -\frac{3}{4}x$

3. a. $y > 1$

b. $x \leq -2$

4.

$x^2 + y^2 \geq 16$

6. $\begin{cases} x - 3y < 6 \\ 2x + 3y \geq -6 \end{cases}$

7.

$\begin{cases} y \geq x^2 - 4 \\ x + y \leq 2 \end{cases}$

8. $\begin{cases} x + y < 2 \\ -2 \leq x < 1 \\ y > -3 \end{cases}$

Exercise Set 7.5

1.

$x + 2y \leq 8$

2.

$3x - 6y \leq 12$

3.

$x - 2y > 10$

4.

$2x - y > 4$

5.

$y \leq \frac{1}{3}x$

6.

$y \leq \frac{1}{4}x$

7.

$y > 2x - 1$

8.

$y > 3x + 2$

9.

$x \leq 1$

10.

$x \leq -3$

11.

$y > 1$

12.

$y > -3$

13.

$x^2 + y^2 \le 1$

14.

$x^2 + y^2 \le 4$

15.

$x^2 + y^2 > 25$

16.

$x^2 + y^2 > 36$

17.

$(x - 2)^2 + (y + 1)^2 < 9$

18.

$(x + 2)^2 + (y - 1)^2 < 16$

19.

$y < x^2 - 1$

20.

$y < x^2 - 9$

21.

$y \ge x^2 - 9$

22.

$y \ge x^2 - 1$

23.

$y > 2^x$

24.

$y \le 3^x$

25.

$y \ge \log_2 (x + 1)$

26.

$y \ge \log_3 (x - 1)$

27. $\begin{cases} 3x + 6y \le 6 \\ 2x + y \le 8 \end{cases}$

28. $\begin{cases} x - y \ge 4 \\ x + y \le 6 \end{cases}$

29. $\begin{cases} 2x - 5y \le 10 \\ 3x - 2y > 6 \end{cases}$

30. $\begin{cases} 2x - y \le 4 \\ 3x + 2y > -6 \end{cases}$

31. $\begin{cases} y > 2x - 3 \\ y < -x + 6 \end{cases}$

32. $\begin{cases} y < -2x + 4 \\ y < x - 4 \end{cases}$

33. $\begin{cases} x + 2y \le 4 \\ y \ge x - 3 \end{cases}$

34. $\begin{cases} x + y \le 4 \\ y \ge 2x - 4 \end{cases}$

35. $\begin{cases} x \le 2 \\ y \ge -1 \end{cases}$

36. $\begin{cases} x \le 3 \\ y \le -1 \end{cases}$

37. $-2 \le x < 5$

38. $-2 < y \le 5$

39. $\begin{cases} x - y \le 1 \\ x \ge 2 \end{cases}$

40. $\begin{cases} 4x - 5y \ge -20 \\ x \ge -3 \end{cases}$

43. $\begin{cases} x + y > 4 \\ x + y > -1 \end{cases}$

44. $\begin{cases} x + y > 3 \\ x + y > -2 \end{cases}$

45.

$\begin{cases} y \ge x^2 - 1 \\ x - y \ge -1 \end{cases}$

46.

$\begin{cases} y \ge x^2 - 4 \\ x - y \ge 2 \end{cases}$

47.

$\begin{cases} x^2 + y^2 \le 16 \\ x + y > 2 \end{cases}$

48.

$\begin{cases} x^2 + y^2 \le 4 \\ x + y > 1 \end{cases}$

49.

$\begin{cases} x^2 + y^2 > 1 \\ x^2 + y^2 < 16 \end{cases}$

50.

$\begin{cases} x^2 + y^2 > 1 \\ x^2 + y^2 < 9 \end{cases}$

51.

$\begin{cases} (x - 1)^2 + (y + 1)^2 < 25 \\ (x - 1)^2 + (y + 1)^2 \ge 16 \end{cases}$

52.

$\begin{cases} (x + 1)^2 + (y - 1)^2 < 16 \\ (x + 1)^2 + (y - 1)^2 \ge 4 \end{cases}$

53.

$\begin{cases} x^2 + y^2 \le 1 \\ y - x^2 > 0 \end{cases}$

54.

$\begin{cases} x^2 + y^2 < 4 \\ y - x^2 \ge 0 \end{cases}$

55.

$\begin{cases} x^2 + y^2 < 16 \\ y \ge 2^x \end{cases}$

56.

$\begin{cases} x^2 + y^2 \le 16 \\ y < 2^x \end{cases}$

57. $\begin{cases} x - y \le 2 \\ x > -2 \\ y \le 3 \end{cases}$

58. $\begin{cases} 3x + y \le 6 \\ x > -2 \\ y \le 4 \end{cases}$

59. $\begin{cases} x \ge 0 \\ y \ge 0 \\ 2x + 5y < 10 \\ 3x + 4y \le 12 \end{cases}$

60. $\begin{cases} x \ge 0 \\ y \ge 0 \\ 2x + y < 4 \\ 2x - 3y \le 6 \end{cases}$

61. $\begin{cases} 3x + y \le 6 \\ 2x - y \le -1 \\ x > -2 \\ y < 4 \end{cases}$

62. $\begin{cases} 2x + y \le 6 \\ x + y > 2 \\ 1 \le x \le 2 \\ y < 3 \end{cases}$

63. $y \ge -2x + 4$

64. $y \ge -3x + 2$

65. $\begin{cases} x + y \le 4 \\ 3x + y \le 6 \end{cases}$

66. $\begin{cases} x + y \le 3 \\ 4x + y \le 6 \end{cases}$

67.

$\begin{cases} x + y \le 2 \\ y \ge x^2 - 4 \end{cases}$

68.

$\begin{cases} x^2 + y^2 \le 25 \\ 2y + x \ge 5 \end{cases}$

69. $\begin{cases} -2 \le x \le 2 \\ -3 \le y \le 3 \end{cases}$

70. $\begin{cases} -1 \le x \le 1 \\ -2 \le y \le 2 \end{cases}$

71.

$y > \frac{3}{2}x - 2$ or $y < 4$

72. $x - y \ge -1$ or $5x - 2y \le 10$

77. Point $A = (66, 160)$; $5.3(66) - 160 \ge 180$, or $189.8 \ge 180$, is true; $4.1(66) - 160 \le 140$, or $110.6 \le 140$, is true.

78. Point $B = (76, 220)$; $5.3(76) - 220 \ge 180$, or $182.8 \ge 180$, is true; $4.1(76) - 220 \le 140$, or $91.6 \le 140$, is true.

81. a. $50x + 150y > 2000$

b.

$50x + 150y > 2000$

c. Answers may vary. Example: (20, 20): 20 children and 20 adults will cause the elevator to be overloaded.

82. a. $165x + 110y \le 330$

b.

$165x + 110y \le 330$

c. Answers may vary. Example: (1, 1): One egg and one ounce of meat are within the patient's dietary restrictions.

83. b. $\begin{cases} y \ge 0 \\ x + y \ge 5 \\ x \ge 1 \\ 200x + 100y \le 700 \end{cases}$

84. $\begin{cases} x + y \le 15{,}000 \\ x \ge 2000 \\ y \ge 3x \\ x \ge 0 \\ y \ge 0 \end{cases}$

$\begin{cases} x + y \le 15{,}000 \\ x \ge 2000,\ y \ge 3x \\ x \ge 0,\ y \ge 0 \end{cases}$

85. a. 27.1

97.

98.

99.

100.

101.

102.

110. $\begin{cases} x \geq -2 \\ y > -1 \end{cases}$ **111.** $\begin{cases} y > x - 3 \\ y \leq x \end{cases}$ **112.** $\begin{cases} x^2 + y^2 \leq 9 \\ y < x^2 \end{cases}$ **113.** $x + 2y \leq 6$ or $2x + y \leq 6$

115. $y \geq nx + b$
$y \leq mx + b$

116. a.

$x + y \geq 6$
$x \leq 8$
$y \leq 5$

117. a.

$x \geq 0$
$y \geq 0$
$3x - 2y \leq 6$
$y \leq -x + 7$

Exercise Set 7.6

1. (1, 2): 17; (2, 10): 70; (7, 5): 65; (8, 3): 58; maximum: $z = 70$; minimum: $z = 17$
2. (3, 2): 13; (4, 10): 32; (5, 12): 39; (8, 6): 36; (7, 4): 29; maximum: $z = 39$; minimum: $z = 13$
3. (0, 0): 0; (0, 8): 400; (4, 9): 610; (8, 0): 320; maximum: $z = 610$; minimum: $z = 0$
4. (0, 0): 0; (0, 9): 405; (4, 4): 300; (3, 0): 90; maximum: $z = 405$; minimum: $z = 0$

5. a.

b. (0, 8): 16; (0, 4): 8; (4, 0): 12

6. a.

b. (0, 0): 0; (0, 4): 12; (3, 2): 12; (4, 0): 8

7. a.

b. (0, 4): 4; (0, 3): 3; (3, 0): 12; (6, 0): 24

8. a.

b. (0, 5): 30; (0, 0): 0; (2, 6): 38; (5, 0): 5

9. a.

b. (1, 2): −1; (1, 4): −5; (5, 8): −1; (5, 2): 11

10. a.

b. (0, 3): −6; (0, 2): −4; (2, 0): 10; (5, 0): 25; (5, 3): 19

11. a.

b. (0, 4): 8; (0, 2): 4; (2, 0): 8; (4, 0): 16; $\left(\frac{12}{5}, \frac{12}{5}\right)$: $\frac{72}{5}$

12. a.

b. (0, 9): 36; (0, 3): 12; (6, 0): 12; (9, 0): 18; $\left(\frac{3}{2}, \frac{3}{2}\right)$: 9

13. a.

b. (0, 6): 72; (0, 0): 0; (5, 0): 50; (3, 4): 78

14. a.

b. (0, 10): 60; (10, 0): 50; $\left(\frac{10}{3}, \frac{10}{3}\right)$: $\frac{110}{3}$

15. a. $z = 125x + 200y$ **b.** $\begin{cases} x \leq 450 \\ y \leq 200 \\ 600x + 900y \leq 360{,}000 \end{cases}$

c.

d. (0, 0): 0; (0, 200): 40,000; (300, 200): 77,500; (450, 100): 76,250; (450, 0): 56,250

16. c.

d. (3, 0): 30; (8, 0): 80; (3, 17): 149; (8, 12): 164

34. (0, 0): 0; (0, 3): $3B$; (3, 1): $3B$; (2, 0): $\frac{4}{3}B$; maximum value: $3B$ at (0, 3) and (3, 1)

39. $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -11 & -11 \end{bmatrix}$

Chapter 7 Review Exercises

6. a. $C(x) = 60{,}000 + 200x$ **b.** $R(x) = 450x$ **c.** (240, 108,000); This means the company will break even if it produces and sells 240 desks.

8. a. $y = 219x + 142$ **14.** $y = 3x^2 - 4x + 5$ **16.** $\dfrac{3}{5(x-3)} + \dfrac{2}{5(x+2)}$ **17.** $\dfrac{6}{x-4} + \dfrac{5}{x+3}$ **18.** $\dfrac{2}{x} + \dfrac{3}{x+2} - \dfrac{1}{x-1}$

20. $-\dfrac{4}{x-1} + \dfrac{4}{x-2} - \dfrac{2}{(x-2)^2}$ **21.** $\dfrac{6}{5(x-2)} + \dfrac{-6x+3}{5(x^2+1)}$ **22.** $\dfrac{5}{x-3} + \dfrac{2x-1}{x^2+4}$ **23.** $\dfrac{x}{x^2+4} - \dfrac{4x}{(x^2+4)^2}$ **26.** $\{(0, 1), (-3, 4)\}$

28. $\{(3, \sqrt{6}), (3, -\sqrt{6}), (-3, \sqrt{6}), (-3, -\sqrt{6})\}$ **33.** $\left\{\left(\dfrac{5}{2}, -\dfrac{7}{2}\right), (0, -1)\right\}$ **34.** $\{(2, -3), (-2, -3), (3, 2), (-3, 2)\}$

39.

$3x - 4y > 12$

40.

$y \le -\frac{1}{2}x + 2$

41.

$x < -2$

42.

$y \ge 3$

43.

$x^2 + y^2 > 4$

44.

$y \le x^2 - 1$

45.

$y \le 2^x$

46.

$\begin{cases} 3x + 2y \ge 6 \\ 2x + y \ge 6 \end{cases}$

47.

$\begin{cases} 2x - y \ge 4 \\ x + 2y < 2 \end{cases}$

48.

$\begin{cases} y < x \\ y \le 2 \end{cases}$

49.

$\begin{cases} x + y \le 6 \\ y \ge 2x - 3 \end{cases}$

50.

$\begin{cases} 0 \le x \le 3 \\ y > 2 \end{cases}$

52.

$\begin{cases} x^2 + y^2 \le 16 \\ x + y < 2 \end{cases}$

53.

$\begin{cases} x^2 + y^2 \le 9 \\ y < -3x + 1 \end{cases}$

54.

$\begin{cases} y > x^2 \\ x + y < 6 \\ y < x + 6 \end{cases}$

55.

$\begin{cases} y \ge 0 \\ 3x + 2y \ge 4 \\ x - y \le 3 \end{cases}$

56. (2, 2): 10; (4, 0): 8; $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$: $\dfrac{5}{2}$; (1, 0): 2; maximum value: 10; minimum value: 2

57.

$x \ge 0,\ y \ge 0$
$x + y \le 8$
$3x + 2y \ge 6$

Maximum is 24 at $x = 0, y = 8$.

58.

$0 \le x \le 5$
$0 \le y \le 7$
$x + y \ge 3$

Maximum is 33 at $x = 5, y = 7$.

59.

$x \ge 0, y \ge 0$
$y \le x$
$2x + y \le 12$
$2x + 3y \ge 6$

Maximum is 44 at $x = y = 4$.

60. b. $\begin{cases} x + y \le 200 \\ x \ge 10 \\ y \ge 80 \end{cases}$

c.

$x + y \le 200$
$x \ge 10, y \ge 80$

d. (10, 80): 33,000; (10, 190): 71,500; (120, 80): 88,000

Chapter 7 Test

6. $\dfrac{-1}{10(x+1)} + \dfrac{x+9}{10(x^2+9)}$

7.

$x - 2y < 8$

8.

$\begin{cases} x \ge 0,\ y \ge 0 \\ 3x + y \le 9 \\ 2x + 3y \ge 6 \end{cases}$

9.

$\begin{cases} x^2 + y^2 > 1 \\ x^2 + y^2 < 4 \end{cases}$

10.

$\begin{cases} y \le 1 - x^2 \\ x^2 + y^2 \le 9 \end{cases}$

13. a. $C(x) = 360{,}000 + 850x$ **c.** (1200, 1,380,000); The company will break even if it produces and sells 1200 computers.

14. 40 oz of 20%; 20 oz of 50%

Cumulative Review Exercises (Chapters P–7)

1. domain: $(-2, 2)$; range: $(-\infty, 3]$ **3.** maximum of 3 at $x = 0$

9.

$g(x) = f(x + 2) - 1$

10.

$h(x) = \frac{1}{2} f(\frac{1}{2} x)$

11. $\{3, 4\}$ **12.** $\left\{\frac{2 + i\sqrt{3}}{2}, \frac{2 - i\sqrt{3}}{2}\right\}$

15. $\left\{-3, \frac{1}{2}, 2\right\}$ **16.** $\{-2\}$

17. $\{2\}$ **20.** $\left\{\left(-\frac{1}{2}, \frac{1}{2}\right), (2, 8)\right\}$

22.

$f(x) = (x + 2)^2 - 4$

23.

$2x - 3y \le 6$

24.

$y = 3^{x-2}$

25.

$f(x) = \frac{x^2 - x - 6}{x + 1}$

26.

$f(x) = 2x - 4$

$f^{-1}(x) = \frac{x + 4}{2}$

27.

$(x - 2)^2 + (y - 4)^2 > 9$

28.

$f(x) = |x|$

$g(x) = -|x - 2|$

29. $(f \circ g)(x) = 2x^2 - 3x$;
$(g \circ f)(x) = -2x^2 + x + 2$

36. $\sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta = \frac{1 - \cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}\sin\theta = \tan\theta\sin\theta$

37. $\tan x + \tan y = \frac{\sin x}{\cos x} + \frac{\sin y}{\cos y} = \frac{\sin x\cos y + \sin y\cos x}{\cos x\cos y} = \frac{\sin(x + y)}{\cos x\cos y}$

CHAPTER 8

Section 8.1

Check Point Exercises

1. a. $\left[\begin{array}{rrr|r} 1 & 6 & -3 & 7 \\ 4 & 12 & -20 & 8 \\ -3 & -2 & 1 & -9 \end{array}\right]$ **b.** $\left[\begin{array}{rrr|r} 1 & 3 & -5 & 2 \\ 1 & 6 & -3 & 7 \\ -3 & -2 & 1 & -9 \end{array}\right]$ **c.** $\left[\begin{array}{rrr|r} 4 & 12 & -20 & 8 \\ 1 & 6 & -3 & 7 \\ 0 & 16 & -8 & 12 \end{array}\right]$

Exercise Set 8.1

1. $\left[\begin{array}{rrr|r} 2 & 1 & 2 & 2 \\ 3 & -5 & -1 & 4 \\ 1 & -2 & -3 & -6 \end{array}\right]$ **2.** $\left[\begin{array}{rrr|r} 3 & -2 & 5 & 31 \\ 1 & 3 & -3 & -12 \\ -2 & -5 & 3 & 11 \end{array}\right]$ **3.** $\left[\begin{array}{rrr|r} 1 & -1 & 1 & 8 \\ 0 & 1 & -12 & -15 \\ 0 & 0 & 1 & 1 \end{array}\right]$ **4.** $\left[\begin{array}{rrr|r} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$ **5.** $\left[\begin{array}{rrr|r} 5 & -2 & -3 & 0 \\ 1 & 1 & 0 & 5 \\ 2 & 0 & -3 & 4 \end{array}\right]$

6. $\left[\begin{array}{rrr|r} 1 & -2 & 1 & 10 \\ 3 & 1 & 0 & 5 \\ 7 & 0 & 2 & 2 \end{array}\right]$ **7.** $\left[\begin{array}{rrrr|r} 2 & 5 & -3 & 1 & 2 \\ 0 & 3 & 1 & 0 & 4 \\ 1 & -1 & 5 & 0 & 9 \\ 5 & -5 & -2 & 0 & 1 \end{array}\right]$ **8.** $\left[\begin{array}{rrrr|r} 4 & 7 & -8 & 1 & 3 \\ 0 & 5 & 1 & 0 & 5 \\ 1 & -1 & -1 & 0 & 17 \\ 2 & -2 & 11 & 0 & 4 \end{array}\right]$ **9.** $\begin{cases} 5x + 3z = -11 \\ y - 4z = 12 \\ 7x + 2y = 3 \end{cases}$ **10.** $\begin{cases} 7x + 4z = -13 \\ y - 5z = 11 \\ 2x + 7y = 6 \end{cases}$

11. $\begin{cases} w + x + 4y + z = 3 \\ -w + x - y = 7 \\ 2w + 5z = 11 \\ 12y + 4z = 5 \end{cases}$ **12.** $\begin{cases} 4w + x + 5y + z = 6 \\ w - x - z = 8 \\ 3w + 7z = 4 \\ 11y + 5z = 3 \end{cases}$ **17.** $\left[\begin{array}{rrrr|r} 1 & -1 & 1 & 1 & 3 \\ 0 & 1 & -2 & -1 & 0 \\ 0 & 2 & 1 & 2 & 5 \\ 0 & 6 & -3 & -1 & -9 \end{array}\right]$ **18.** $\left[\begin{array}{rrrr|r} 1 & -5 & 2 & -2 & 4 \\ 0 & 1 & -3 & -1 & 0 \\ 0 & 15 & -4 & 5 & -6 \\ 0 & -19 & 12 & -6 & 13 \end{array}\right]$

19. R_2: $-3, -18$; R_3: $-12, -15$; R_2: $-\frac{3}{5}, -\frac{18}{5}$; R_3: $-12, -15$ **20.** R_2: $-10, -5$; R_3: $8, 10$; R_2: $-2, -1$; R_3: $8, 10$ **21.** $\{(1, -1, 2)\}$

22. $\{(1, -1, 1)\}$ **23.** $\{(3, -1, -1)\}$ **24.** $\{(-3, 0, 1)\}$ **25.** $\{(2, -1, 1)\}$ **26.** $\{(4, -3, 2)\}$ **28.** $\{(0, 2, 2)\}$ **30.** $\{(1, 1, 2)\}$

31. $\{(-1, 2, -2)\}$ **32.** $\{(-1, 6, 3)\}$ **33.** $\{(1, 2, -1)\}$ **34.** $\{(2, 0, -1)\}$ **41.** $f(x) = x^3 - 2x^2 + 3$ **42.** $f(x) = -x^3 + 4x^2 - 2$

45. b. 0; The ball hits the ground 3.5 seconds after it is thrown. **46. b.** $s(7) = 118$; The ball's height is 118 feet after 7 seconds. **62.** makes sense

68. For $z = 0$, $(12z + 1, 10z - 1, z)$ is $(1, -1, 0)$; $3(1) - 4(-1) + 4(0) = 7$ is true; $1 - (-1) - 2(0) = 2$ is true; $2(1) - 3(-1) + 6(0) = 5$ is true.

69. For $z = 1$, $(12z + 1, 10z - 1, z)$ is $(13, 9, 1)$; $3(13) - 4(9) + 4(1) = 7$ is true; $13 - 9 - 2(1) = 2$ is true; $2(13) - 3(9) + 6(1) = 5$ is true.

70. a. Answers may vary. **b.** This system has more than one solution.

Exercise Set 8.2

3. $\left\{\left(-2t+2, 2t+\frac{1}{2}, t\right)\right\}$ **4.** $\{(-2+t, -2+t, t)\}$ **5.** $\{(-3, 4, -2)\}$ **6.** $\{(2, -3, 7)\}$ **7.** $\{(5-2t, -2+t, t)\}$ **8.** $\{(-5+7t, 1+3t, t)\}$

15. $\left\{\left(1+\frac{1}{3}t, \frac{1}{3}t, t\right)\right\}$ **16.** $\left\{\left(2, \frac{1}{2}t-\frac{1}{2}, t\right)\right\}$ **17.** $\{(-13t+5, 5t, t)\}$ **18.** $\{(-2t+3, -2t+1, t)\}$ **19.** $\left\{\left(2t-\frac{5}{4}, \frac{13}{4}, t\right)\right\}$ **20.** $\{(98, 2t-43, t)\}$

25. a. $4w-2x+2y-3z=0; 7w-x-y-3z=0; w+x+y-z=0$

26. a. $2w+17x-23y+40z=0; 2w+5x+y+3z=0; x-2y+3z=0$

27. a. $w+2x+5y+5z=-3; w+x+3y+4z=-1; w-x-y+2z=3$

28. a. $w+y+z=0; w-x+2y+3z=0; 3w-2x+5y+7z=0$

33. a. $\begin{cases} w+z=380 \\ w+x=600 \\ x-y=170 \\ y-z=50 \end{cases}$ **b.** $\{(380-t, 220+t, 50+t, t)\}$

34. a. There is no combination of the foods that can satisfy the given requirements.
b. There are many combinations of the foods that satisfy the new requirements.

35. a. The system has no solution, so there is no way to satisfy these dietary requirements with no Food 1 available.
b. 4 oz of Food 1, 0 oz of Food 2, 10 oz of Food 3; 2 oz of Food 1, 5 oz of Food 2, 9 oz of Food 3 (other answers are possible).

36. 7 of product A, 2 of product B, 2 of product C; 7 of product A, 1 of product B, 4 of product C (other answers are possible).

40. a. $\begin{cases} x_1-x_6=100 \\ x_2-x_6+x_7=600 \\ x_3+x_7=900 \\ x_1-x_4=-200 \\ x_2-x_4+x_5=100 \\ x_3+x_5=700 \end{cases}$ **b.** $\{(t+100, t-s+600, -s+900, t+300, s-200, t, s)\}$

Section 8.3

Check Point Exercises

9. a. $\begin{bmatrix} 0 & 3 & 4 \\ 0 & 5 & 2 \end{bmatrix} + \begin{bmatrix} -3 & -3 & -3 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 1 \\ -1 & 4 & 1 \end{bmatrix}$ **b.** $2\begin{bmatrix} 0 & 3 & 4 \\ 0 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 8 \\ 0 & 10 & 4 \end{bmatrix}$

Exercise Set 8.3

1. b. a_{32} does not exist; $a_{23}=-1$ **2. b.** a_{32} does not exist; $a_{23}=\frac{1}{2}$ **3. a.** 3×4 **b.** $a_{32}=\frac{1}{2}; a_{23}=-6$ **4. a.** 3×4 **b.** $a_{32}=0; a_{23}=\pi$

7. $x=4; y=6; z=3$ **8.** $x=12; y=2; z=3$ **9. a.** $\begin{bmatrix} 9 & 10 \\ 3 & 9 \end{bmatrix}$ **b.** $\begin{bmatrix} -1 & -8 \\ 3 & -5 \end{bmatrix}$ **c.** $\begin{bmatrix} -16 & -4 \\ -12 & -8 \end{bmatrix}$ **d.** $\begin{bmatrix} 22 & 21 \\ 9 & 20 \end{bmatrix}$

10. a. $\begin{bmatrix} 6 & 4 \\ 5 & 5 \end{bmatrix}$ **b.** $\begin{bmatrix} -10 & 2 \\ -5 & -3 \end{bmatrix}$ **c.** $\begin{bmatrix} 8 & -12 \\ 0 & -4 \end{bmatrix}$ **d.** $\begin{bmatrix} 10 & 11 \\ 10 & 11 \end{bmatrix}$ **11. a.** $\begin{bmatrix} 3 & 2 \\ 6 & 2 \\ 5 & 7 \end{bmatrix}$ **b.** $\begin{bmatrix} -1 & 4 \\ 0 & 6 \\ 5 & 5 \end{bmatrix}$ **c.** $\begin{bmatrix} -4 & -12 \\ -12 & -16 \\ -20 & -24 \end{bmatrix}$ **d.** $\begin{bmatrix} 7 & 7 \\ 15 & 8 \\ 15 & 20 \end{bmatrix}$

12. a. $\begin{bmatrix} 5 & -2 & 7 \\ -4 & 3 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 1 & 4 & -5 \\ 2 & 1 & 9 \end{bmatrix}$ **c.** $\begin{bmatrix} -12 & -4 & -4 \\ 4 & -8 & -20 \end{bmatrix}$ **d.** $\begin{bmatrix} 13 & -3 & 15 \\ -9 & 8 & 7 \end{bmatrix}$ **13. a.** $\begin{bmatrix} -3 \\ -1 \\ 0 \end{bmatrix}$ **b.** $\begin{bmatrix} 7 \\ -7 \\ 2 \end{bmatrix}$ **c.** $\begin{bmatrix} -8 \\ 16 \\ -4 \end{bmatrix}$

d. $\begin{bmatrix} -4 \\ -6 \\ 1 \end{bmatrix}$ **14. a.** $\begin{bmatrix} 10 & 0 & 0 \end{bmatrix}$ **b.** $\begin{bmatrix} 2 & 4 & -6 \end{bmatrix}$ **c.** $\begin{bmatrix} -24 & -8 & 12 \end{bmatrix}$ **d.** $\begin{bmatrix} 26 & 2 & -3 \end{bmatrix}$ **15. a.** $\begin{bmatrix} 8 & 0 & -4 \\ 14 & 0 & 6 \\ -1 & 0 & 0 \end{bmatrix}$ **b.** $\begin{bmatrix} -4 & -20 & 0 \\ 14 & 24 & 14 \\ 9 & -4 & 4 \end{bmatrix}$

c. $\begin{bmatrix} -8 & 40 & 8 \\ -56 & -48 & -40 \\ -16 & 8 & -8 \end{bmatrix}$ **d.** $\begin{bmatrix} 18 & -10 & -10 \\ 42 & 12 & 22 \\ 2 & -2 & 2 \end{bmatrix}$ **16. a.** $\begin{bmatrix} 3 & 2 & 6 \\ 5 & 2 & -8 \\ -2 & 2 & 3 \end{bmatrix}$ **b.** $\begin{bmatrix} 9 & -8 & 4 \\ 7 & -2 & 4 \\ -6 & 2 & -5 \end{bmatrix}$ **c.** $\begin{bmatrix} -24 & 12 & -20 \\ -24 & 0 & 8 \\ 16 & -8 & 4 \end{bmatrix}$ **d.** $\begin{bmatrix} 12 & 1 & 17 \\ 16 & 4 & -18 \\ -8 & 6 & 5 \end{bmatrix}$

17. $\begin{bmatrix} -8 & -8 \\ 2 & -9 \\ 8 & -4 \end{bmatrix}$ **18.** $\begin{bmatrix} -8 & -8 \\ 2 & -9 \\ 8 & -4 \end{bmatrix}$ **19.** $\begin{bmatrix} -1 & 3 \\ -1 & \frac{9}{2} \\ -1 & -2 \end{bmatrix}$ **20.** $\begin{bmatrix} -\frac{2}{3} & 2 \\ -\frac{2}{3} & 3 \\ -\frac{2}{3} & -\frac{4}{3} \end{bmatrix}$ **21.** $\begin{bmatrix} \frac{1}{3} & \frac{13}{3} \\ -\frac{4}{3} & 6 \\ -\frac{7}{3} & -\frac{4}{3} \end{bmatrix}$ **22.** $\begin{bmatrix} 5 & 17 \\ -5 & \frac{45}{2} \\ -11 & -2 \end{bmatrix}$ **23.** $\begin{bmatrix} 7 & 27 \\ -8 & 36 \\ -17 & -4 \end{bmatrix}$

24. $\begin{bmatrix} 17 & -3 \\ 2 & -9 \\ -7 & 16 \end{bmatrix}$ **25.** $\begin{bmatrix} \frac{27}{2} & \frac{31}{2} \\ -4 & 18 \\ -\frac{29}{2} & 6 \end{bmatrix}$ **26.** $\begin{bmatrix} \frac{29}{2} & \frac{25}{2} \\ -3 & \frac{27}{2} \\ -\frac{27}{2} & 8 \end{bmatrix}$ **27. a.** $\begin{bmatrix} 0 & 16 \\ 12 & 8 \end{bmatrix}$ **b.** $\begin{bmatrix} -7 & 3 \\ 29 & 15 \end{bmatrix}$ **28. a.** $\begin{bmatrix} -10 & 12 \\ 25 & -30 \end{bmatrix}$ **b.** $\begin{bmatrix} 0 & 0 \\ 9 & -40 \end{bmatrix}$

29. a. $[30]$ **b.** $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$ **30. a.** $\begin{bmatrix} -1 & -2 & -3 \\ -2 & -4 & -6 \\ -3 & -6 & -9 \end{bmatrix}$ **b.** $[-14]$ **31. a.** $\begin{bmatrix} 4 & -5 & 8 \\ 6 & -1 & 5 \\ 0 & 4 & -6 \end{bmatrix}$ **b.** $\begin{bmatrix} 5 & -2 & 7 \\ 17 & -3 & 2 \\ 3 & 0 & -5 \end{bmatrix}$

32. a. $\begin{bmatrix} 3 & 4 & -3 \\ -1 & 7 & -4 \\ 7 & 9 & -6 \end{bmatrix}$ **b.** $\begin{bmatrix} 6 & -1 & -1 \\ -4 & -11 & 19 \\ 4 & -7 & 9 \end{bmatrix}$ **33. a.** $\begin{bmatrix} 6 & 8 & 16 \\ 11 & 16 & 24 \\ 1 & -1 & 12 \end{bmatrix}$ **b.** $\begin{bmatrix} 38 & 27 \\ -16 & -4 \end{bmatrix}$ **34. a.** $\begin{bmatrix} 2 & -8 & 20 \\ 8 & 3 & 5 \\ 10 & 2 & 10 \end{bmatrix}$ **b.** $\begin{bmatrix} 12 & 14 \\ 9 & 3 \end{bmatrix}$

35. a. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **b.** $\begin{bmatrix} 4 & -1 & -3 & 1 \\ -1 & 4 & -3 & 2 \\ 14 & -11 & -3 & -1 \\ 25 & -25 & 0 & -5 \end{bmatrix}$ **36. a.** $\begin{bmatrix} 18 & 1 \\ -1 & 15 \end{bmatrix}$ **b.** $\begin{bmatrix} 0 & 1 & -7 & 0 \\ 3 & -1 & 1 & 3 \\ 2 & -3 & 17 & 2 \\ 17 & -6 & 8 & 17 \end{bmatrix}$ **37.** $\begin{bmatrix} 17 & 7 \\ -5 & -11 \end{bmatrix}$ **38.** $\begin{bmatrix} -5 & -7 \\ -1 & 9 \end{bmatrix}$

39. $\begin{bmatrix} 11 & -1 \\ -7 & -3 \end{bmatrix}$ **40.** $\begin{bmatrix} 24 & 0 \\ -33 & -5 \\ -3 & -1 \end{bmatrix}$ **41.** $A - C$ is not defined because A is 3×2 and C is 2×2.

42. $B - A$ is not defined because B is 2×2 and A is 3×2. **43.** $\begin{bmatrix} 16 & -16 \\ -12 & 12 \\ 0 & 0 \end{bmatrix}$ **44.** $\begin{bmatrix} 28 & 12 \\ -56 & -24 \\ -7 & -3 \end{bmatrix}$ **45.** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **46.** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

47. Answers will vary.; Example:

$$A(B + C) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So, $A(B + C) = AB + AC$.

48. Answers will vary.; Example:

$$A + (B + C) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(A + B) + C = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\right) + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, $A + (B + C) = (A + B) + C$.

49. $\begin{bmatrix} x \\ -y \end{bmatrix}$; It changes the sign of the y-coordinate. **50.** $\begin{bmatrix} -x \\ y \end{bmatrix}$; It changes the sign of the x-coordinate.

51. a. $\begin{bmatrix} 1 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 1 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 0 \end{bmatrix}$ **c.** $\begin{bmatrix} 1 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 1 \\ -2 & -2 & -2 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

52. a. $\begin{bmatrix} 1 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 1 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 2 \\ -1 & -1 & -1 \\ 2 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 3 \end{bmatrix}$ **c.** $\begin{bmatrix} 1 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 3 & 3 \\ 0 & 3 & 0 \end{bmatrix}$

53. $\begin{bmatrix} -2 & 1 & 1 & -1 & -1 & -2 \\ -3 & -3 & -2 & -2 & 2 & 2 \end{bmatrix}$

54. $\begin{bmatrix} 2 & 5 & 5 & 3 & 3 & 2 \\ -3 & -3 & -2 & -2 & 2 & 2 \end{bmatrix}$

55. $\begin{bmatrix} 0 & \frac{3}{2} & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 1 & 1 & \frac{3}{2} & \frac{3}{2} & \frac{7}{2} & \frac{7}{2} \end{bmatrix}$

56. $\begin{bmatrix} 0 & \frac{3}{2} & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 2 & 2 & \frac{5}{2} & \frac{5}{2} & \frac{9}{2} & \frac{9}{2} \end{bmatrix}$

57. a. $\begin{bmatrix} 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & -5 & -5 \end{bmatrix}$

b. The effect is a reflection across the x-axis.

58. a. $\begin{bmatrix} 0 & -3 & -3 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix}$

b. The effect is a reflection across the y-axis.

59. a. $\begin{bmatrix} 0 & 0 & -1 & -1 & -5 & -5 \\ 0 & 3 & 3 & 1 & 1 & 0 \end{bmatrix}$

b.

The effect is a 90° counterclockwise rotation about the origin.

60. a. $\begin{bmatrix} 0 & 6 & 6 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix}$

b.

The effect is a horizontal stretch by a factor of 2.

61. b. $B = \begin{bmatrix} 9 & 29 \\ 65 & 77 \end{bmatrix}$

c. $B - A = \begin{bmatrix} 7 & 23 \\ 34 & 31 \end{bmatrix}$

The difference between the percentage of people completing the transition to adulthood in 1960 and 2000 by age and gender

62. a. $M = \begin{bmatrix} 2400 & 2700 & 3000 \\ 2200 & 2500 & 2900 \\ 2000 & 2300 & 2600 \end{bmatrix}$ **b.** $W = \begin{bmatrix} 2000 & 2100 & 2400 \\ 1800 & 2000 & 2200 \\ 1600 & 1800 & 2100 \end{bmatrix}$

c. $M - W = \begin{bmatrix} 400 & 600 & 600 \\ 400 & 500 & 700 \\ 400 & 500 & 500 \end{bmatrix}$

The differences between the basic caloric needs of men and women by age and activity level

63. a. System 1: The midterm and final both count for 50% of the course grade. System 2: The midterm counts for 30% of the course grade and the final counts for 70%.

b. $\begin{bmatrix} 84 & 87.2 \\ 79 & 81 \\ 90 & 88.4 \\ 73 & 68.6 \\ 69 & 73.4 \end{bmatrix}$ System 1 grades are listed first (if different). Student 1: B; Student 2: C or B; Student 3: A or B; Student 4: C or D; Student 5: D or C

64. a. $\begin{bmatrix} 15{,}800 & 18{,}600 \\ 12{,}500 & 14{,}800 \\ 3700 & 4600 \end{bmatrix}$

82. When a matrix of this type is multiplied by itself, each diagonal element of the product is the square of the corresponding element of the original; all other elements are zero. **83.** $AB = -BA$ so they are anticommutative.

87. $\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$

Mid-Chapter 8 Check Point

1. $\{(1, -1, 2)\}$ **6.** $\begin{bmatrix} -4 & -\frac{1}{2} \\ 3 & 3 \end{bmatrix}$ **7.** $\begin{bmatrix} -12 & -2 \\ -21 & -4 \\ 3 & 1 \end{bmatrix}$ **8.** $\begin{bmatrix} 12 & -4 \\ 22 & -7 \\ -4 & 1 \end{bmatrix}$

9. $A + C$ does not exist because A is 3×2 and C is 2×2. **10.** $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -3 & \frac{1}{2} \end{bmatrix}$

Exercise Set 8.4

2. $AB = \begin{bmatrix} -3 & -4 \\ 0 & 1 \end{bmatrix}; BA = \begin{bmatrix} -3 & 0 \\ -4 & 1 \end{bmatrix}; B \neq A^{-1}$ **3.** $AB = \begin{bmatrix} 8 & -16 \\ -2 & 7 \end{bmatrix}; BA = \begin{bmatrix} 12 & 12 \\ 1 & 3 \end{bmatrix}; B \neq A^{-1}$

4. $AB = \begin{bmatrix} -6 & -12 \\ 3 & 6 \end{bmatrix}; BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; B \neq A^{-1}$ **13.** $\begin{bmatrix} \frac{2}{7} & -\frac{3}{7} \\ \frac{1}{7} & \frac{2}{7} \end{bmatrix}$ **14.** $\begin{bmatrix} \frac{1}{6} & \frac{1}{4} \\ \frac{1}{3} & 0 \end{bmatrix}$ **15.** $\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$ **16.** $\begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$

17. A does not have an inverse. **18.** A does not have an inverse. **19.** $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{6} \end{bmatrix}$ **20.** $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{9} \end{bmatrix}$ **21.** $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

22. $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ **23.** $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 3 & 2 & 6 \end{bmatrix}$ **24.** $\begin{bmatrix} \frac{7}{2} & -2 & -2 \\ -\frac{5}{2} & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ **25.** $\begin{bmatrix} -3 & 2 & -4 \\ -1 & 1 & -1 \\ 8 & -5 & 10 \end{bmatrix}$ **26.** $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ **27.** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$

28. $\begin{bmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{4} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ **29.** $\begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 10 \end{bmatrix}$ **30.** $\begin{bmatrix} 7 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 23 \\ 10 \end{bmatrix}$ **31.** $\begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 1 & 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \\ -6 \end{bmatrix}$

32. $\begin{bmatrix} 1 & 4 & -1 \\ 1 & 3 & -2 \\ 2 & 7 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix}$ **37. a.** $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 9 \end{bmatrix}$ **b.** $\{(1, 2, -1)\}$ **38. a.** $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 8 \\ -1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$

b. $\{(-2, -3, 2)\}$ **39. a.** $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 1 \end{bmatrix}$ **b.** $\{(2, -1, 5)\}$ **40. a.** $\begin{bmatrix} 1 & -6 & 3 \\ 2 & -7 & 3 \\ 4 & -12 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 14 \\ 25 \end{bmatrix}$ **b.** $\{(2, -1, 1)\}$

41. a. $\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ -1 & 1 & -1 & 2 \\ 0 & -1 & 1 & -2 \end{bmatrix}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ 2 \\ -4 \end{bmatrix}$ **42. a.** $\begin{bmatrix} 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 \\ -1 & 1 & -2 & 1 \\ 4 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ 4 \\ 6 \end{bmatrix}$ **43.** $\begin{bmatrix} \frac{1}{2}e^{-x} & -\frac{1}{2}e^{-3x} \\ \frac{1}{2}e^{-3x} & \frac{1}{2}e^{-5x} \end{bmatrix}$

44. $\begin{bmatrix} \frac{1}{2}e^{-2x} & \frac{1}{2}e^{-3x} \\ -\frac{1}{2}e^{-x} & \frac{1}{2}e^{-2x} \end{bmatrix}$ **45.** $\begin{bmatrix} \frac{1}{8} & \frac{5}{8} \\ \frac{3}{8} & \frac{7}{8} \end{bmatrix}$ **46.** $\begin{bmatrix} \frac{1}{4} & \frac{5}{8} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$

47. $(AB)^{-1} = \begin{bmatrix} -23 & 16 \\ 13 & -9 \end{bmatrix}$; $A^{-1}B^{-1} = \begin{bmatrix} -3 & 11 \\ 8 & -29 \end{bmatrix}$; $B^{-1}A^{-1} = \begin{bmatrix} -23 & 16 \\ 13 & -9 \end{bmatrix}$; $(AB)^{-1} = B^{-1}A^{-1}$

48. $(AB)^{-1} = \begin{bmatrix} -11 & 26 \\ 19 & -45 \end{bmatrix}$; $A^{-1}B^{-1} = \begin{bmatrix} -79 & 101 \\ 18 & 23 \end{bmatrix}$; $B^{-1}A^{-1} = \begin{bmatrix} -11 & 26 \\ 19 & -45 \end{bmatrix}$; $(AB)^{-1} = B^{-1}A^{-1}$

50. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $\begin{bmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $\begin{cases} aw + by = 1 \\ cw + dy = 0 \end{cases}$ and $\begin{cases} ax + bz = 0 \\ cx + dz = 1 \end{cases}$;
$w = \dfrac{d}{ad - bc}, x = \dfrac{-b}{ad - bc}, y = \dfrac{-c}{ad - bc}, z = \dfrac{a}{ad - bc}$; $A^{-1} = \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

51. The encoded message is 27, −19, 32, −20.; The decoded message is 8, 5, 12, 16 or HELP.
52. The encoded message is 33, −21, 83, −61.; The decoded message is 12, 15, 22, 5 or LOVE.
53. The encoded message is 14, 85, −33, 4, 18, −7, −18, 19, −9.
54. The encoded message is −1, 59, −20, 25, 121, −48, −7, 39, −17.

67. $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$ **68.** $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$ **69.** $\begin{bmatrix} 0 & -1 & 0 & 1 \\ -1 & -5 & 0 & 3 \\ -2 & -4 & 1 & -2 \\ -1 & -4 & 0 & 1 \end{bmatrix}$ **70.** $\begin{bmatrix} \frac{3}{5} & 0 & -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & 0 & \frac{1}{5} & -\frac{1}{10} \\ 0 & 1 & 0 & 0 \\ -\frac{6}{5} & 0 & \frac{4}{5} & \frac{1}{10} \end{bmatrix}$

71. $\{(2, 3, -5)\}$ **72.** $\{(1, 2, -1)\}$ **73.** $\{(1, 2, -1)\}$ **74.** $\{(5, 4, -1)\}$ **79.** does not make sense **86.** false

Exercise Set 8.5

29. $\{(-5, -2, 7)\}$ **30.** $\{(-2, 3, 4)\}$ **31.** $\{(2, -3, 4)\}$ **32.** $\{(2, 3, 5)\}$ **33.** $\{(3, -1, 2)\}$ **34.** $\{(-1, 3, 2)\}$ **35.** $\{(2, 3, 1)\}$ **36.** $\{(0, 4, 2)\}$

53. The equation of the line is $y = -\frac{11}{5}x + \frac{8}{5}$. **54.** The equation of the line is $y = \frac{1}{3}x + \frac{10}{3}$. **64.** −2100 **65.** 13,200

68. does not make sense **70.** does not make sense **71. d.** Each determinant has zeros below the main diagonal and a's everywhere else. **e.** Each determinant equals a raised to the power equal to the order of the determinant. **73.** The sign of the value is changed when 2 columns are interchanged in a 2nd order determinant. **74.** For both systems, $x = \dfrac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$.

75. $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = x(y_1 - y_2) - y(x_1 - x_2) + (x_1y_2 - x_2y_1) = 0$; solving for y, $y = \dfrac{y_1 - y_2}{x_1 - x_2}x + \dfrac{x_1y_2 - x_2y_1}{x_1 - x_2}$, and $m = \dfrac{y_1 - y_2}{x_1 - x_2}$ and $b = \dfrac{x_1y_2 - x_2y_1}{x_1 - x_2}$.

79.

Chapter 8 Review Exercises

3. $\{(1, 3, -4)\}$ **4.** $\{(-2, -1, 0)\}$ **9.** $\{(2t + 4, t + 1, t)\}$ **12. a.** $\begin{cases} x + z = 750 \\ y - z = -250 \\ x + y = 500 \end{cases}$ **b.** $\{(-t + 750, t - 250, t)\}$ **14.** $\begin{bmatrix} 0 & 2 & 3 \\ 8 & 1 & 3 \end{bmatrix}$

15. $\begin{bmatrix} 0 & -4 \\ 6 & 4 \\ 2 & -10 \end{bmatrix}$ **16.** $\begin{bmatrix} -4 & 4 & -1 \\ -2 & -5 & 5 \end{bmatrix}$ **17.** Not possible since B is 3×2 and C is 3×3. **18.** $\begin{bmatrix} 2 & 3 & 8 \\ 21 & 5 & 5 \end{bmatrix}$ **19.** $\begin{bmatrix} -12 & 14 & 0 \\ 2 & -14 & 18 \end{bmatrix}$

20. $\begin{bmatrix} 0 & -10 & -15 \\ -40 & -5 & -15 \end{bmatrix}$ **21.** $\begin{bmatrix} -1 & -16 \\ 8 & 1 \end{bmatrix}$ **22.** $\begin{bmatrix} -10 & -6 & 2 \\ 16 & 3 & 4 \\ -23 & -16 & 7 \end{bmatrix}$ **23.** $\begin{bmatrix} -6 & 4 & -8 \\ 0 & 5 & 11 \\ -17 & 13 & -19 \end{bmatrix}$ **24.** $\begin{bmatrix} 10 & 5 \\ -2 & -30 \end{bmatrix}$

25. Not possible since AB is 2×2 and BA is 3×3. **26.** $\begin{bmatrix} 7 & 6 & 5 \\ 2 & -1 & 11 \end{bmatrix}$ **27.** $\begin{bmatrix} -6 & -22 & -40 \\ 9 & 43 & 58 \\ -14 & -48 & -94 \end{bmatrix}$ **29.** $\begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ **30.** $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix}$

31. $\begin{bmatrix} -2 & 0 & 0 \\ 1 & 1 & -3 \end{bmatrix}$

32. $\begin{bmatrix} 0 & 1 & 1 \\ -2 & -2 & -4 \end{bmatrix}$

33. $\begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

The effect is a reflection over the x-axis

34. $\begin{bmatrix} 0 & -2 & -2 \\ 0 & 0 & -4 \end{bmatrix}$

The effect is a reflection over the y-axis.

35. $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

The effect is a 90° counterclockwise rotation about the origin.

36. $\begin{bmatrix} 0 & 4 & 4 \\ 0 & 0 & -4 \end{bmatrix}$

The effect is a horizontal stretch by a factor of 2.

37. $AB = \begin{bmatrix} 1 & 7 \\ 0 & 5 \end{bmatrix}$; $BA = \begin{bmatrix} 1 & 0 \\ 1 & 5 \end{bmatrix}$; $B \neq A^{-1}$ **41.** $\begin{bmatrix} 3 & 0 & -2 \\ -6 & 1 & 4 \\ 1 & 0 & -1 \end{bmatrix}$ **42.** $\begin{bmatrix} 8 & -8 & 5 \\ -3 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix}$ **43. a.** $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 3 & 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 0 \end{bmatrix}$

b. $\{(-18, 79, -27)\}$ **44. a.** $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -5 \\ 10 \end{bmatrix}$ **b.** $\{(4, -2, 3)\}$ **45.** The encoded message is 96, 135, 46, 63; The decoded message is 18, 21, 12, 5 or RULE. **54.** $\{(23, -12, 3)\}$ **55.** $\{(-3, 2, 1)\}$ **56.** $a = \dfrac{5}{8}$; $b = -50$; $c = 1150$; 30- and 50-year-olds are involved in an average of 212.5 automobile accidents per day.

Chapter 8 Test

3. $\begin{bmatrix} 5 & 4 \\ 1 & 11 \end{bmatrix}$ **4.** $\begin{bmatrix} 5 & -2 \\ 1 & -1 \\ 4 & -1 \end{bmatrix}$ **5.** $\begin{bmatrix} \frac{3}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$ **6.** $\begin{bmatrix} -1 & 2 \\ -5 & 4 \end{bmatrix}$ **8. a.** $\begin{bmatrix} 3 & 5 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -13 \end{bmatrix}$ **b.** $\begin{bmatrix} \frac{3}{19} & \frac{5}{19} \\ \frac{2}{19} & -\frac{3}{19} \end{bmatrix}$

Cumulative Review Exercises (Chapters P–8)

1. $\left\{\dfrac{-1+\sqrt{33}}{4}, \dfrac{-1-\sqrt{33}}{4}\right\}$

10.

$f(x) = \dfrac{x}{x^2 - 16}$

12.

15. $\dfrac{8}{x-3} + \dfrac{-2}{x-2} + \dfrac{-3}{x+2}$

16.

$y = -\dfrac{2}{3}x - 1$

17.

$3x - 5y < 15$

18.

$f(x) = x^2 - 2x - 3$

19.

$(x-1)^2 + (y+1)^2 = 9$

21.

$y = 2\sin 2\pi x,\ 0 \le x \le 2$

23. $\dfrac{\cos 2x}{\cos x - \sin x} = \dfrac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \dfrac{(\cos x + \sin x)(\cos x - \sin x)}{\cos x - \sin x} = \cos x + \sin x$ **24.** $\dfrac{3\pi}{2}$ **25.** $2\mathbf{i} - 13\mathbf{j}$

CHAPTER 9

Section 9.1

Check Point Exercises

1. foci at $(-3\sqrt{3}, 0)$ and $(3\sqrt{3}, 0)$

$\dfrac{x^2}{36} + \dfrac{y^2}{9} = 1$

2. foci at $(0, -\sqrt{7})$ and $(0, \sqrt{7})$

$16x^2 + 9y^2 = 144$

4. foci at $(-1 - \sqrt{5}, 2)$ and $(-1 + \sqrt{5}, 2)$

$\dfrac{(x+1)^2}{9} + \dfrac{(y-2)^2}{4} = 1$

Exercise Set 9.1

1. foci at $(-2\sqrt{3}, 0)$ and $(2\sqrt{3}, 0)$

$\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$

2. foci at $(-3, 0)$ and $(3, 0)$

$\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

3. foci at $(0, -3\sqrt{3})$ and $(0, 3\sqrt{3})$

$\dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$

4. foci at $(0, -\sqrt{33})$ and $(0, \sqrt{33})$

$\dfrac{x^2}{16} + \dfrac{y^2}{49} = 1$

5. foci at $(0, -\sqrt{39})$ and $(0, \sqrt{39})$

$\dfrac{x^2}{25} + \dfrac{y^2}{64} = 1$

6. foci at $(-\sqrt{13}, 0)$ and $(\sqrt{13}, 0)$

$\dfrac{x^2}{49} + \dfrac{y^2}{36} = 1$

7. foci at $(0, -4\sqrt{2})$ and $(0, 4\sqrt{2})$

$\dfrac{x^2}{49} + \dfrac{y^2}{81} = 1$

8. foci at $(0, -6)$ and $(0, 6)$

$\dfrac{x^2}{64} + \dfrac{y^2}{100} = 1$

9. foci at $(0, -2)$ and $(0, 2)$

$\dfrac{x^2}{\frac{9}{4}} + \dfrac{y^2}{\frac{25}{4}} = 1$

10. foci at $\left(-\frac{\sqrt{299}}{4}, 0\right)$ and $\left(\frac{\sqrt{299}}{4}, 0\right)$

$\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{25}{16}} = 1$

11. foci at $\left(-\frac{\sqrt{3}}{2}, 0\right)$ and $\left(\frac{\sqrt{3}}{2}, 0\right)$

$x^2 = 1 - 4y^2$

12. foci at $\left(0, -\frac{\sqrt{3}}{2}\right)$ and $\left(0, \frac{\sqrt{3}}{2}\right)$

$y^2 = 1 - 4x^2$

13. foci at $(0, -\sqrt{21})$ and $(0, \sqrt{21})$

$25x^2 + 4y^2 = 100$

14. foci at $(0, -\sqrt{5})$ and $(0, \sqrt{5})$

$9x^2 + 4y^2 = 36$

15. foci at $(-2\sqrt{3}, 0)$ and $(2\sqrt{3}, 0)$

$4x^2 + 16y^2 = 64$

16. foci at $(-\sqrt{21}, 0)$ and $(\sqrt{21}, 0)$

$4x^2 + 25y^2 = 100$

17. foci at $(0, -\sqrt{2})$ and $(0, \sqrt{2})$

$7x^2 = 35 - 5y^2$

18. foci at $(0, -1)$ and $(0, 1)$

$6x^2 = 30 - 5y^2$

19. $\frac{x^2}{4} + \frac{y^2}{1} = 1$; foci at $(-\sqrt{3}, 0)$ and $(\sqrt{3}, 0)$ **20.** $\frac{x^2}{16} + \frac{y^2}{4} = 1$; foci at $(-2\sqrt{3}, 0)$ and $(2\sqrt{3}, 0)$ **21.** $\frac{x^2}{1} + \frac{y^2}{4} = 1$; foci at $(0, \sqrt{3})$ and $(0, -\sqrt{3})$

22. $\frac{x^2}{4} + \frac{y^2}{16} = 1$; foci at $(0, 2\sqrt{3})$ and $(0, -2\sqrt{3})$ **23.** $\frac{(x+1)^2}{4} + \frac{(y-1)^2}{1} = 1$; foci at $(-1 - \sqrt{3}, 1)$ and $(-1 + \sqrt{3}, 1)$

24. $\frac{(x+1)^2}{1} + \frac{(y+1)^2}{4} = 1$; foci at $(-1, -1 - \sqrt{3})$ and $(-1, -1 + \sqrt{3})$ **25.** $\frac{x^2}{64} + \frac{y^2}{39} = 1$ **26.** $\frac{x^2}{36} + \frac{y^2}{32} = 1$ **27.** $\frac{x^2}{33} + \frac{y^2}{49} = 1$

28. $\frac{x^2}{7} + \frac{y^2}{16} = 1$ **29.** $\frac{x^2}{13} + \frac{y^2}{9} = 1$ **30.** $\frac{x^2}{4} + \frac{y^2}{8} = 1$ **31.** $\frac{x^2}{16} + \frac{y^2}{4} = 1$ **32.** $\frac{x^2}{36} + \frac{y^2}{9} = 1$ **33.** $\frac{(x+2)^2}{4} + \frac{(y-3)^2}{25} = 1$

34. $\frac{(x-2)^2}{25} + \frac{(y+3)^2}{100} = 1$ **35.** $\frac{(x-7)^2}{4} + \frac{(y-6)^2}{9} = 1$ **36.** $\frac{(x-5)^2}{9} + \frac{(y-2)^2}{1} = 1$

37. foci at $(2 - \sqrt{5}, 1)$ and $(2 + \sqrt{5}, 1)$

$\frac{(x-2)^2}{9} + \frac{(y-1)^2}{4} = 1$

38. foci at $(1 - \sqrt{7}, -2)$ and $(1 + \sqrt{7}, -2)$

$\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$

39. foci at $(-3 - 2\sqrt{3}, 2)$ and $(-3 + 2\sqrt{3}, 2)$

$(x+3)^2 + 4(y-2)^2 = 16$

40. foci at $(-1, -2)$ and $(7, -2)$

$(x-3)^2 + 9(y+2)^2 = 18$

41. foci at $(4, 2)$ and $(4, -6)$

$\frac{(x-4)^2}{9} + \frac{(y+2)^2}{25} = 1$

42. foci at $(3, -1 + \sqrt{7})$ and $(3, -1 - \sqrt{7})$

$\frac{(x-3)^2}{9} + \frac{(y+1)^2}{16} = 1$

43. foci at $(0, 2 + \sqrt{11})$, $(0, 2 - \sqrt{11})$

$\frac{x^2}{25} + \frac{(y-2)^2}{36} = 1$

44. foci at $(4, \sqrt{21})$ and $(4, -\sqrt{21})$

$\frac{(x-4)^2}{4} + \frac{y^2}{25} = 1$

45. foci at $(-3 - 2\sqrt{2}, 2)$ and $(-3 + 2\sqrt{2}, 2)$

$\frac{(x+3)^2}{9} + (y-2)^2 = 1$

46. foci at $(-2 - \sqrt{15}, 3)$ and $(-2 + \sqrt{15}, 3)$

$\frac{(x+2)^2}{16} + (y-3)^2 = 1$

47. foci at $(1, -3 + \sqrt{3})$ and $(1, -3 - \sqrt{3})$

$\frac{(x-1)^2}{2} + \frac{(y+3)^2}{5} = 1$

48. foci at $(-1, 3 + \sqrt{3})$ and $(-1, 3 - \sqrt{3})$

$\frac{(x+1)^2}{2} + \frac{(y-3)^2}{5} = 1$

49. foci at $(1, -3 + \sqrt{5})$ and $(1, -3 - \sqrt{5})$

$9(x-1)^2 + 4(y+3)^2 = 36$

50. foci at $(-4, -3 + \sqrt{35})$ and $(-4, -3 - \sqrt{35})$

$36(x+4)^2 + (y+3)^2 = 36$

51. $\frac{(x-2)^2}{25} + \frac{(y+1)^2}{9} = 1$
foci at $(-2, -1)$ and $(6, -1)$

$9x^2 + 25y^2 - 36x + 50y - 164 = 0$

52. $\frac{(x-4)^2}{9} + \frac{(y+2)^2}{4} = 1$
foci at $(4 - \sqrt{5}, -2)$ and $(4 + \sqrt{5}, -2)$

$4x^2 + 9y^2 - 32x + 36y + 64 = 0$

53. $\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$
foci at $(1 - \sqrt{7}, -2)$ and $(1 + \sqrt{7}, -2)$

$9x^2 + 16y^2 - 18x + 64y - 71 = 0$

54. $\frac{(x+5)^2}{16} + \frac{(y-1)^2}{4} = 1$
foci at $(-5 - 2\sqrt{3}, 1)$ and $(-5 + 2\sqrt{3}, 1)$

$x^2 + 4y^2 + 10x - 8y + 13 = 0$

55. $\frac{(x+2)^2}{16} + \frac{(y-3)^2}{64} = 1$
foci at $(-2, 3 + 4\sqrt{3})$ and $(-2, 3 - 4\sqrt{3})$

$4x^2 + y^2 + 16x - 6y - 39 = 0$

56. $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{4} = 1$
foci at $(3 - \sqrt{21}, -2)$ and $(3 + \sqrt{21}, -2)$

$4x^2 + 25y^2 - 24x + 100y + 36 = 0$

57. $\{(0, -1), (0, 1)\}$ **58.** $\{(0, -5), (0, 5)\}$
59. $\{(0, 3)\}$ **60.** $\{(-2, 0)\}$
61. $\{(0, -2), (1, 0)\}$ **62.** $\varnothing$
63.

$y = -\sqrt{16 - 4x^2}$

64.

$y = -\sqrt{4 - 4x^2}$

68. One person should stand at 10 feet along the 100 foot width and the other person should stand at 90 feet.

78. Earth's orbit: $\frac{x^2}{8649} + \frac{y^2}{8630.41} = 1$

Mars's orbit: $\frac{x^2}{20{,}093.0625} + \frac{y^2}{19{,}390.5625} = 1$

81. does not make sense **83.** $\frac{x^2}{\frac{36}{5}} + \frac{y^2}{36} = 1$

86. $\frac{c}{a}$ is close to zero when c is very small. This happens when a and b are nearly equal, or when the shape of the graph is nearly circular.

87. $\frac{x^2}{9} - \frac{y^2}{4} = 1$; Terms are separated by subtraction rather than addition.

88. b. The equation $y^2 = -9$ has no real solutions.

89. b. The equation $x^2 = -16$ has no real solutions.

Section 9.2

Check Point Exercises

3. foci at $(-3\sqrt{5}, 0)$ and $(3\sqrt{5}, 0)$
asymptotes: $y = \pm\frac{1}{2}x$

$\frac{x^2}{36} - \frac{y^2}{9} = 1$

4. foci at $(0, \sqrt{5})$ and $(0, -\sqrt{5})$
asymptotes: $y = \pm 2x$

$y^2 - 4x^2 = 4$

5. foci at $(3 - \sqrt{5}, 1)$ and $(3 + \sqrt{5}, 1)$
asymptotes: $(y - 1) = \pm\frac{1}{2}(x - 3)$

$\frac{(x-3)^2}{4} - \frac{(y-1)^2}{1} = 1$

6. foci at $(3, -5 + \sqrt{13})$ and $(3, -5 - \sqrt{13})$

asymptotes: $(y + 5) = \pm\frac{2}{3}(x - 3)$

$4x^2 - 24x - 9y^2 - 90y - 153 = 0$

Exercise Set 9.2

1. vertices at $(2, 0)$ and $(-2, 0)$; foci at $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$; graph (b)

2. vertices at $(1, 0)$ and $(-1, 0)$; foci at $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$; graph (d)

3. vertices at $(0, 2)$ and $(0, -2)$; foci at $(0, \sqrt{5})$ and $(0, -\sqrt{5})$; graph (a)

4. vertices at $(0, 1)$ and $(0, -1)$; foci at $(0, \sqrt{5})$ and $(0, -\sqrt{5})$; graph (c)

5. $y^2 - \frac{x^2}{8} = 1$ **6.** $\frac{y^2}{4} - \frac{x^2}{32} = 1$ **7.** $\frac{x^2}{9} - \frac{y^2}{7} = 1$ **8.** $\frac{x^2}{25} - \frac{y^2}{24} = 1$ **9.** $\frac{y^2}{36} - \frac{x^2}{9} = 1$ **10.** $\frac{x^2}{16} - \frac{y^2}{64} = 1$

11. $\frac{(x - 4)^2}{4} - \frac{(y + 2)^2}{5} = 1$ **12.** $\frac{(y - 1)^2}{9} - \frac{(x + 2)^2}{16} = 1$

13. foci: $(\pm\sqrt{34}, 0)$

asymptotes: $y = \pm\frac{5}{3}x$

$\frac{x^2}{9} - \frac{y^2}{25} = 1$

14. foci: $(\pm\sqrt{41}, 0)$

asymptotes: $y = \pm\frac{5}{4}x$

$\frac{x^2}{16} - \frac{y^2}{25} = 1$

15. foci: $(\pm 2\sqrt{41}, 0)$

asymptotes: $y = \pm\frac{4}{5}x$

$\frac{x^2}{100} - \frac{y^2}{64} = 1$

16. foci: $(\pm 15, 0)$

asymptotes: $y = \pm\frac{3}{4}x$

$\frac{x^2}{144} - \frac{y^2}{81} = 1$

17. foci: $(0, \pm 2\sqrt{13})$

asymptotes: $y = \pm\frac{2}{3}x$

$\frac{y^2}{16} - \frac{x^2}{36} = 1$

18. foci: $(0, \pm\sqrt{89})$

asymptotes: $y = \pm\frac{5}{8}x$

$\frac{y^2}{25} - \frac{x^2}{64} = 1$

19. foci: $\left(0, \pm\frac{\sqrt{5}}{2}\right)$

asymptotes: $y = \pm\frac{1}{2}x$

$4y^2 - x^2 = 1$

20. foci: $\left(0, \pm\frac{\sqrt{10}}{3}\right)$

asymptotes: $y = \pm\frac{1}{3}x$

$9y^2 - x^2 = 1$

21. foci: $(\pm\sqrt{13}, 0)$

asymptotes: $y = \pm\frac{3}{2}x$

$9x^2 - 4y^2 = 36$

22. foci: $(\pm\sqrt{29}, 0)$

asymptotes: $y = \pm\frac{2}{5}x$

$4x^2 - 25y^2 = 100$

23. foci: $(0, \pm\sqrt{34})$

asymptotes: $y = \pm\frac{5}{3}x$

$9y^2 - 25x^2 = 225$

24. foci: $(0, \pm 5)$

asymptotes: $y = \pm\frac{3}{4}x$

$16y^2 - 9x^2 = 144$

25. foci: $(\pm 2, 0)$

asymptotes: $y = \pm x$

$y = \pm\sqrt{x^2 - 2}$

26. foci: $(\pm\sqrt{6}, 0)$

asymptotes: $y = \pm x$

$y = \pm\sqrt{x^2 - 3}$

27. $\frac{x^2}{9} - \frac{y^2}{25} = 1$

30. $\frac{y^2}{25} - \frac{x^2}{9} = 1$

31. $\frac{(x - 2)^2}{4} - \frac{(y + 3)^2}{9} = 1$

32. $\frac{(x + 1)^2}{4} - \frac{(y + 2)^2}{4} = 1$

33. foci: $(-9, -3), (1, -3)$

asymptotes: $(y + 3) = \pm\frac{4}{3}(x + 4)$

$\frac{(x + 4)^2}{9} - \frac{(y + 3)^2}{16} = 1$

34. foci: $(-2 \pm \sqrt{34}, 1)$

asymptotes: $(y - 1) = \pm\frac{5}{3}(x + 2)$

$\frac{(x + 2)^2}{9} - \frac{(y - 1)^2}{25} = 1$

35. foci: $(-3 \pm \sqrt{41}, 0)$

asymptotes: $y = \pm\frac{4}{5}(x + 3)$

$\frac{(x + 3)^2}{25} - \frac{y^2}{16} = 1$

36. foci: $(-2 \pm \sqrt{34}, 0)$

asymptotes: $y = \pm\frac{5}{3}(x + 2)$

$\frac{(x + 2)^2}{9} - \frac{y^2}{25} = 1$

37. foci: $(1, -2 \pm 2\sqrt{5})$

asymptotes: $(y + 2) = \pm\frac{1}{2}(x - 1)$

$\frac{(y + 2)^2}{4} - \frac{(x - 1)^2}{16} = 1$

38. foci: $(-1, 2 \pm \sqrt{85})$

asymptotes: $(y - 2) = \pm\frac{6}{7}(x + 1)$

$\frac{(y - 2)^2}{36} - \frac{(x + 1)^2}{49} = 1$

39. foci: $(3 \pm \sqrt{5}, -3)$

asymptotes: $(y + 3) = \pm\frac{1}{2}(x - 3)$

$(x - 3)^2 - 4(y + 3)^2 = 4$

40. foci: $(-3 \pm \sqrt{10}, 4)$

asymptotes: $(y - 4) = \pm\frac{1}{3}(x + 3)$

$(x + 3)^2 - 9(y - 4)^2 = 9$

41. foci: $(1 \pm \sqrt{6}, 2)$

asymptotes: $(y - 2) = \pm(x - 1)$

$(x - 1)^2 - (y - 2)^2 = 3$

42. foci: $(-3, 2 \pm \sqrt{10})$

asymptotes: $(y - 2) = \pm(x + 3)$

$(y - 2)^2 - (x + 3)^2 = 5$

43. $(x - 1)^2 - (y + 2)^2 = 1$

foci: $(1 \pm \sqrt{2}, -2)$

asymptotes: $(y + 2) = \pm(x - 1)$

$x^2 - y^2 - 2x - 4y - 4 = 0$

44. $\frac{(x + 4)^2}{4} - \frac{(y - 3)^2}{16} = 1$

foci: $(-4 \pm 2\sqrt{5}, 3)$

asymptotes: $(y - 3) = \pm 2(x + 4)$

$4x^2 - y^2 + 32x + 6y + 39 = 0$

45. $\frac{(y + 1)^2}{4} - \frac{(x + 2)^2}{0.25} = 1$

foci: $(-2, -1 \pm \sqrt{4.25})$

asymptotes: $(y + 1) = \pm 4(x + 2)$

$16x^2 - y^2 + 64x - 2y + 67 = 0$

46. $\frac{(y - 1)^2}{4} - \frac{(x - 3)^2}{9} = 1$

foci: $(3, 1 \pm \sqrt{13})$

asymptotes: $(y - 1) = \pm\frac{2}{3}(x - 3)$

$9y^2 - 4x^2 - 18y + 24x - 63 = 0$

47. $\frac{(x - 2)^2}{9} - \frac{(y - 3)^2}{4} = 1$

foci: $(2 \pm \sqrt{13}, 3)$

asymptotes: $(y - 3) = \pm\frac{2}{3}(x - 2)$

$4x^2 - 9y^2 - 16x + 5y - 101 = 0$

48. $\frac{(x + 1)^2}{\frac{1}{4}} - \frac{(y + 1)^2}{\frac{1}{9}} = 1$

foci: $(-1 \pm \frac{\sqrt{13}}{6}, -1)$

asymptotes: $(y + 1) = \pm\frac{2}{3}(x + 1)$

$4x^2 - 9y^2 + 8x - 18y - 6 = 0$

49. $\frac{y^2}{4} - \frac{(x - 4)^2}{25} = 1$

foci: $(4, \pm\sqrt{29})$

asymptotes: $y = \pm\frac{2}{5}(x - 4)$

$4x^2 - 25y^2 - 32x + 164 = 0$

50. $\frac{(y + 2)^2}{9} - \frac{(x - 2)^2}{16} = 1$

foci: $(2, -7), (2, 3)$

asymptotes: $(y + 2) = \pm\frac{3}{4}(x - 2)$

$9x^2 - 16y^2 - 36x - 64y + 116 = 0$

51. domain: $(-\infty, -3] \cup [3, \infty)$
range: $(-\infty, \infty)$

$\frac{x^2}{9} - \frac{y^2}{16} = 1$

52. domain: $(-\infty, -5] \cup [5, \infty)$
range: $(-\infty, \infty)$

$\frac{x^2}{25} - \frac{y^2}{4} = 1$

53. domain: $[-3, 3]$
range: $[-4, 4]$

$\frac{x^2}{9} + \frac{y^2}{16} = 1$

54. domain: $[-5, 5]$
range: $[-2, 2]$

$\frac{x^2}{25} + \frac{y^2}{4} = 1$

55. domain: $(-\infty, \infty)$
range: $(-\infty, -4] \cup [4, \infty)$

$\frac{y^2}{16} - \frac{x^2}{9} = 1$

56. domain: $(-\infty, \infty)$
range: $(-\infty, -2] \cup [2, \infty)$

$\frac{y^2}{4} - \frac{x^2}{25} = 1$

61. If M_1 is located 2640 feet to the right of the origin on the x-axis, the explosion is located on the right branch of the hyperbola given by the equation $\frac{x^2}{1{,}210{,}000} - \frac{y^2}{5{,}759{,}600} = 1$.

62. a. $\frac{x^2}{5625} - \frac{y^2}{4375} = 1$

65. a. ellipse **b.** $x^2 + 4y^2 = 4$

66. a. hyperbola **b.** $x^2 - y^2 = 1$

77.

; no; two intersecting lines

78.

Answers may vary depending on the choice for a and b. For $a = 2$, $b = 3$, a graph is shown. The two graphs open right/left and up/down, sharing a common set of asymptotes given by $y = \pm\frac{b}{a}x$.

79. $2y^2 + (10 - 6x)y + (4x^2 - 3x - 6) = 0$

$$y = \frac{3x - 5 \pm \sqrt{x^2 - 24x + 37}}{2}$$

The xy-term rotates the hyperbola.

80.

The second equation is a function with domain $(-\infty, \infty)$.

81. does not make sense **82.** makes sense **85.** false **89.** $\frac{c}{a}$ will be large when a is small. When this happens, the asymptotes will be nearly vertical.

90. $\frac{y^2}{36} - \frac{(x-5)^2}{20} = 1$ **91.** Any hyperbola where $a = b$, such as $\frac{x^2}{4} - \frac{y^2}{4} = 1$, has perpendicular asymptotes.

92.

93.

Section 9.3

Check Point Exercises

1. focus: $(2, 0)$
directrix: $x = -2$

2. focus: $(0, -3)$
directrix: $y = 3$

4. vertex: $(2, -1)$; focus: $(2, 0)$
directrix: $y = -2$

5. vertex: $(2, -1)$; focus: $(1, -1)$
directrix: $x = 3$

Exercise Set 9.3

1. focus:$(1, 0)$; directrix: $x = -1$; graph (c)
2. focus: $(0, 1)$; directrix: $y = -1$; graph (a)
3. focus: $(0, -1)$, directrix: $y = 1$; graph (b)
4. focus: $(-1, 0)$; directrix: $x = 1$; graph (d)
5. focus: $(4, 0)$; directrix: $x = -4$

6. focus: $(1, 0)$; directrix: $x = -1$

7. focus: $(-2, 0)$; directrix: $x = 2$

8. focus: $(-3, 0)$; directrix: $x = 3$

9. focus: $(0, 3)$; directrix: $y = -3$

10. focus: $(0, 2)$; directrix: $y = -2$

11. focus: $(0, -4)$; directrix: $y = 4$

12. focus: $(0, -5)$; directrix: $y = 5$

13. focus: $\left(\frac{3}{2}, 0\right)$; directrix: $x = -\frac{3}{2}$

14. focus: $\left(0, \frac{3}{2}\right)$; directrix: $y = -\frac{3}{2}$

15. focus: $\left(0, -\frac{1}{8}\right)$; directrix: $y = \frac{1}{8}$

16. focus: $\left(-\frac{1}{8}, 0\right)$; directrix: $x = \frac{1}{8}$

31. vertex: $(1, 1)$; focus: $(2, 1)$; directrix: $x = 0$; graph (c)
32. vertex: $(-1, -1)$; focus: $(-1, 0)$; directrix: $y = -2$; graph (a)
33. vertex: $(-1, -1)$; focus: $(-1, -2)$; directrix: $y = 0$; graph (d)
34. vertex: $(1, 1)$; focus: $(0, 1)$; directrix: $x = 2$; graph (b)

35. vertex: $(2, 1)$; focus: $(2, 3)$
directrix: $y = -1$

36. vertex: $(-2, -1)$; focus: $(-2, 0)$
directrix: $y = -2$

37. vertex: $(-1, -1)$; focus: $(-1, -3)$
directrix: $y = 1$

38. vertex: $(-2, -2)$; focus: $(-2, -4)$
directrix: $y = 0$

39. vertex: $(-1, -3)$; focus: $(2, -3)$
directrix: $x = -4$

40. vertex: $(-2, -4)$; focus: $(1, -4)$
directrix: $x = -5$

41. vertex: $(0, -1)$; focus: $(-2, -1)$
directrix: $x = 2$

42. vertex: $(0, 1)$; focus: $(-2, 1)$
directrix: $x = 2$

43. $(x - 1)^2 = 4(y - 2)$; vertex: $(1, 2)$
focus: $(1, 3)$; directrix: $y = 1$

44. $(x + 3)^2 = -8(y - 1)$
vertex: $(-3, 1)$; focus: $(-3, -1)$
directrix: $y = 3$

45. $(y - 1)^2 = -12(x - 3)$
vertex: $(3, 1)$; focus: $(0, 1)$
directrix: $x = 6$

46. $(y - 1)^2 = 8x$
vertex: $(0, 1)$; focus: $(2, 1)$
directrix: $x = -2$

47. $(x + 3)^2 = 4(y + 2)$
vertex: $(-3, -2)$; focus: $(-3, -1)$
directrix: $y = -3$

48. $(x + 4)^2 = 4(y + 2)$
vertex: $(-4, -2)$; focus: $(-4, -1)$
directrix: $y = -3$

49. domain: $[-4, \infty)$
range: $(-\infty, \infty)$; not a function

50. domain: $[-6, \infty)$
range: $(-\infty, \infty)$; not a function

51. domain: $(-\infty, \infty)$
range: $(-\infty, 1]$; function

52. domain: $(-\infty, \infty)$
range: $(-\infty, 8]$; function

53. domain: $(-\infty, 3]$
range: $(-\infty, \infty)$; not a function

54. domain: $(-\infty, -2]$
range: $(-\infty, \infty)$; not a function

55. $\{(-4, 2), (0, 0)\}$

56. $\{(2, 3), (3, 2)\}$

57. $\{(-2, 1)\}$

58. $\{(-5, 0), (4, -3), (4, 3)\}$

59. $\varnothing$

60. $\varnothing$

77. $y = -1 \pm \sqrt{6x - 12}$

78. $y = -5 \pm \sqrt{x}$

79. $9y^2 + (-24x - 80)y + 16x^2 - 60x + 100 = 0$

$$y = \frac{12x + 40 \pm 10\sqrt{15x + 7}}{9}$$

80. $3y^2 + (2\sqrt{3}x - 8)y + x^2 + 8\sqrt{3}x + 32 = 0$

$$y = \frac{-\sqrt{3}x + 4 \pm 4\sqrt{-2\sqrt{3}x - 5}}{3}$$

83. makes sense

89. focus: $\left(0, -\dfrac{E}{4A}\right)$; directrix: $y = \dfrac{E}{4A}$

90. $(x + 1)^2 = -8(y - 2)$

92. $(x')^2 + (y')^2 = 2$

93. a.

c. $\sin\theta = \dfrac{4}{5} = 0.8$; $\cos\theta = \dfrac{3}{5} = 0.6$

Mid-Chapter 9 Check Point

1. foci: $(\pm\sqrt{21}, 0)$

$\frac{x^2}{25} + \frac{y^2}{4} = 1$

2. foci: $(0, \pm\sqrt{5})$

$9x^2 + 4y^2 = 36$

3. foci: $(2, 2), (2, -4)$

$\frac{(x-2)^2}{16} + \frac{(y+1)^2}{25} = 1$

4. foci: $(-5, 1), (1, 1)$

$\frac{(x+2)^2}{25} + \frac{(y-1)^2}{16} = 1$

5. foci: $(2 \pm 4\sqrt{2}, -3)$

$x^2 + 9y^2 - 4x + 54y + 49 = 0$

6. foci: $(\pm\sqrt{10}, 0)$
asymptotes: $y = \pm\frac{1}{3}x$

$\frac{x^2}{9} - y^2 = 1$

7. foci: $(0, \pm\sqrt{10})$
asymptotes: $y = \pm 3x$

$\frac{y^2}{9} - x^2 = 1$

8. foci: $(0, \pm 2\sqrt{5})$
asymptotes: $y = \pm 2x$

$y^2 - 4x^2 = 16$

9. foci: $(\pm\sqrt{53}, 0)$
asymptotes: $y = \pm\frac{2}{7}x$

$4x^2 - 49y^2 = 196$

10. foci: $(-3, -2), (7, -2)$
asymptotes: $(y + 2) = \pm\frac{4}{3}(x - 2)$

$\frac{(x-2)^2}{9} - \frac{(y+2)^2}{16} = 1$

11. foci: $(-1, 3 \pm 2\sqrt{5})$;
asymptotes:
$(y - 3) = \pm 2(x + 1)$

$4x^2 - y^2 + 8x + 6y + 11 = 0$

12. focus: $(2, -4)$;
directrix: $y = 2$;

$(x - 2)^2 = -12(y + 1)$

13. focus: $(-\frac{5}{2}, 1)$;
directrix: $x = -\frac{7}{2}$;

14.

$x^2 + y^2 = 4$

15.

$x + y = 4$

16.

$x^2 - y^2 = 4$

17.

$x^2 + 4y^2 = 4$

18.

$(x + 1)^2 + (y - 1)^2 = 4$

19.

$x^2 + 4(y - 1)^2 = 4$

20.

$(x - 1)^2 - (y - 1)^2 = 4$

21.

22. $\frac{x^2}{25} + \frac{y^2}{9} = 1$

23. $\frac{(x-1)^2}{81} + \frac{(y-2)^2}{56} = 1$

24. $\frac{y^2}{4} - \frac{x^2}{5} = 1$

25. $\frac{(x+1)^2}{4} - \frac{(y-5)^2}{5} = 1$

30. a. $\frac{x^2}{1.1025} - \frac{y^2}{7.8975} = 1$

b.

Section 9.4

Check Point Exercises

2.

3.

4.

Exercise Set 9.4

15. $x = \frac{\sqrt{2}}{2}(x' - y'); y = \frac{\sqrt{2}}{2}(x' + y')$

16. $x = \frac{\sqrt{2}}{2}(x' - y'); y = \frac{\sqrt{2}}{2}(x' + y')$

17. $x = \frac{\sqrt{2}}{2}(x' - y'); y = \frac{\sqrt{2}}{2}(x' + y')$

18. $x = \frac{\sqrt{2}}{2}(x' - y'); y = \frac{\sqrt{2}}{2}(x' + y')$

19. $x = \frac{\sqrt{3}x' - y'}{2}; y = \frac{x' + \sqrt{3}y'}{2}$

20. $x = \frac{\sqrt{3}x' - y'}{2}; y = \frac{x' + \sqrt{3}y'}{2}$

21. $x = \frac{3x' - 4y'}{5}; y = \frac{4x' + 3y'}{5}$

22. $x = \frac{3x' - 4y'}{5}; y = \frac{4x' + 3y'}{5}$

23. $x = \sqrt{5}\left(\frac{2x' - y'}{5}\right); y = \sqrt{5}\left(\frac{x' + 2y'}{5}\right)$

24. $x = \sqrt{10}\left(\frac{3x' - y'}{10}\right); y = \sqrt{10}\left(\frac{x' + 3y'}{10}\right)$

25. $x = \frac{4x' - 3y'}{5}; y = \frac{3x' + 4y'}{5}$

26. $x = \sqrt{10}\left(\frac{3x' - y'}{10}\right); y = \sqrt{10}\left(\frac{x' + 3y'}{10}\right)$

27. a. $3x'^2 + y'^2 = 20$

b. $\frac{x'^2}{\frac{20}{3}} + \frac{y'^2}{20} = 1$

c.

28. a. $3x'^2 - y'^2 = 3$

b. $\frac{x'^2}{1} - \frac{y'^2}{3} = 1$

c.

29. a. $-4x'^2 + 16y'^2 = 64$

b. $\frac{y'^2}{4} - \frac{x'^2}{16} = 1$

c.

30. a. $x'^2 + 9y'^2 = 9$

b. $\frac{x'^2}{9} + \frac{y'^2}{1} = 1$

c.

31. a. $64x'^2 - 16y'^2 = 16$

b. $\frac{x'^2}{\frac{1}{4}} - \frac{y'^2}{1} = 1$

c.

32. a. $16x'^2 + 64y'^2 = 64$

b. $\frac{x'^2}{4} + \frac{y'^2}{1} = 1$

c.

33. a. $650x'^2 + 25y'^2 = 225$

b. $\frac{x'^2}{\frac{9}{26}} + \frac{y'^2}{9} = 1$

c.

34. a. $2y'^2 - x' = 0$

b. $y'^2 = \frac{1}{2}x'$

c.

35. a. $50x'^2 - 75y'^2 = 25$

b. $\frac{x'^2}{\frac{1}{2}} - \frac{y'^2}{\frac{1}{3}} = 1$

c.

36. a. $x'^2 - 9y'^2 = -36$

b. $\frac{y'^2}{4} - \frac{x'^2}{36} = 1$

c.

37. a. $625x'^2 + 1250y'^2 = 625$

b. $\frac{x'^2}{1} + \frac{y'^2}{\frac{1}{2}} = 1$

c.

38. a. $500x'^2 + 1500y'^2 = 4500$

b. $\frac{x'^2}{9} + \frac{y'^2}{3} = 1$

c.

54.

55.

56.

57.

58.

59.

60.

66. In a rotated $x'y'$-system, x and y are replaced with x' and y', respectively.

67.
$$\begin{aligned}
A' &= A\cos^2\theta + B\sin\theta\cos\theta + C\sin^2\theta\\
C' &= A\sin^2\theta - B\sin\theta\cos\theta + C\cos^2\theta\\
A' + C' &= A\cos^2\theta + B\sin\theta\cos\theta + C\sin^2\theta + A\sin^2\theta - B\sin\theta\cos\theta + C\cos^2\theta\\
&= A(\cos^2\theta + \sin^2\theta) + B(\sin\theta\cos\theta - \sin\theta\cos\theta) + C(\sin^2\theta + \cos^2\theta)\\
&= A(1) + B(0) + C(1)\\
&= A + C
\end{aligned}$$

68. $B'^2 - 4A'C'$
$$\begin{aligned}
&= [B(\cos^2\theta - \sin^2\theta) + 2(C - A)(\sin\theta\cos\theta)]^2 - 4[A\cos^2\theta + B\sin\theta\cos\theta + C\sin^2\theta][A\sin^2\theta - B\sin\theta\cos\theta + C\cos^2\theta]\\
&= 4A^2[(\sin\theta\cos\theta)^2 - (\sin\theta\cos\theta)^2] + B^2[\cos^4\theta + 2\cos^2\theta\sin^2\theta + \sin^4\theta] + 4C^2[(\sin\theta\cos\theta)^2 - (\sin\theta\cos\theta)^2]\\
&\quad + 4AB[\cos^2\theta(\sin\theta\cos\theta) - \sin^2\theta(\sin\theta\cos\theta) - (\sin\theta\cos\theta)(\cos^2\theta - \sin^2\theta)]\\
&\quad - 4AC[\cos^4\theta + 2(\sin\theta\cos\theta)^2 + \sin^4\theta] + 4BC[(\sin\theta\cos\theta)(\cos^2\theta - \sin^2\theta) - (\sin\theta\cos\theta)(\cos^2\theta - \sin^2\theta)]\\
&= 4A^2[0] + B^2[(\cos^2\theta + \sin^2\theta)^2] + 4C^2[0] + 4AB[\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta - \cos^2\theta + \sin^2\theta)]\\
&\quad - 4AC[(\cos^2\theta + \sin^2\theta)^2] + 4BC[\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta - \cos^2\theta + \sin^2\theta)]\\
&= B^2[(1)^2] + 4AB[0] - 4AC[(1)^2] + 4BC[0] = B^2 - 4AC
\end{aligned}$$

70.

71.

72.

Section 9.5

Check Point Exercises

1.

2.

3.

Exercise Set 9.5

7. $(60\sqrt{3}, 1)$ **8.** $(80\sqrt{2}, -58 + 80\sqrt{2})$

9.

10.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

21. $y = 2x$

22. $y = -2x$

23. $y = (x + 4)^2$

24. $y = (x + 2)^2$

25. $y = x^2 - 1, x \geq 0$

26. $y = x^2 + 1, x \geq 0$

27. $\frac{x^2}{4} + \frac{y^2}{4} = 1$

28. $\frac{x^2}{9} + \frac{y^2}{9} = 1$

29. $\frac{(x - 1)^2}{9} + \frac{(y - 2)^2}{9} = 1$

30. $\frac{(x + 1)^2}{4} + \frac{(y - 1)^2}{4} = 1$

31. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

32. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

33. $\frac{(x - 1)^2}{9} + \frac{(y + 1)^2}{4} = 1, -2 \leq x \leq 4, -1 \leq y \leq 1$

34. $\frac{(x - 2)^2}{16} + \frac{(y + 1)^2}{9} = 1, -2 \leq x \leq 6, -1 \leq y \leq 2$

35. $x^2 - y^2 = 1$

36. $\frac{x^2}{25} - \frac{y^2}{9} = 1$

37. $y = x - 4, x \geq 2, y \geq -2$

38. $y = x - 4, x \geq 2, y \geq -2$

39. $y = \frac{1}{x}, x \geq 1, y \geq 0$

40. $y = \frac{1}{x}, x \geq 1, y \geq 0$

41. $(x - h)^2 + (y - k)^2 = r^2$

42. $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

43. $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$

44. $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

57. a.

b.

c.

d.

58. a.

b.

c.

d.

59.

60.

61.

62.

63.

64.

65.

a. decreasing: $(-\infty, 1)$; increasing: $(1, \infty)$

66.

a. increasing: $(-\infty, 1)$; decreasing: $(1, \infty)$

67.

a. increasing: $(0, 2\pi)$; decreasing: $(2\pi, 4\pi)$
b. maximum of 4 at $x = 2\pi$
minimum of 0 at $x = 0$ and $x = 4\pi$

68.

a. increasing: $(0, 3\pi)$; decreasing: $(3\pi, 6\pi)$
b. maximum of 6 at $x = 3\pi$
minimum of 0 at $x = 0$ and $x = 6\pi$

69. a. $x = (180 \cos 40°)t$; $y = 3 + (180 \sin 40°)t - 16t^2$
b. After 1 second: 137.9 feet in distance, 102.7 feet in height;
After 2 seconds: 275.8 feet in distance, 170.4 feet in height;
After 3 seconds: 413.7 feet in distance, 206.1 feet in height

70. a. $x = (150 \cos 35°)t$; $y = 3 + (150 \sin 35°)t - 16t^2$
b. After 1 second; 122.9 feet in distance, 73.0 feet in height;
After 2 seconds; 245.7 feet in distance, 111.1 feet in height;
After 3 seconds; 368.6 feet in distance, 117.1 feet in height

78.

79.

80.

81.

82.

83.

Window: $[-100, 1500] \times [-100, 500]$;
The maximum height is 419.4 feet at a time of 5.1 seconds.
The range of the projectile is 1174.6 feet horizontally.
It hits the ground at 10.2 seconds.

84.

Window: $[-100, 3000] \times [-100, 500]$;
The maximum height is 462.6 feet at a time of 5.4 seconds.
The range of the projectile is 2642.9 feet horizontally.
It hits the ground at 10.8 seconds.

85. a. $x = (140 \cos 22°)t$; $y = 5 + (140 \sin 22°)t - 16t^2$

b.

Window: $[-100, 500] \times [-10, 60]$

90. $\sqrt[3]{x^2} + \sqrt[3]{y^2} = 1$

92. a. $\sin t = \dfrac{PB}{a} \Rightarrow \sin t = \dfrac{XA}{a} \Rightarrow a \sin t = XA$

OA is the same as the length of arc PA. Since the radius is a, and the central angle is t, the length of the arc is at. So $OA = at$. Therefore,

$$x = OA - XA$$
$$x = at - a \sin t$$
$$x = a(t - \sin t)$$

b. $\cos t = \dfrac{BC}{a} \Rightarrow a \sin t = BC$ and $AC = a$, since a is the radius of the circle. So,

$$y = AC - BC$$
$$y = a - a \cos t$$
$$y = a(1 - \cos t)$$

93. $r = \dfrac{2}{1 + \frac{1}{2}\cos\theta}$

94. $\dfrac{4}{3} \approx 1.33$; 2; $\dfrac{8}{3} \approx 2.67$; 3.09; 3.53; 4;

95. a.

$$r = \frac{1}{3 - 3\cos\theta}$$
$$r(3 - 3\cos\theta) = 1$$
$$3r - 3r\cos\theta = 1$$
$$3r = 1 + 3r\cos\theta$$
$$(3r)^2 = (1 + 3r\cos\theta)^2$$
$$9r^2 = (1 + 3r\cos\theta)^2$$

b. $9y^2 = 1 + 6x$; parabola

Section 9.6

Check Point Exercises

1.

2.

3.

Exercise Set 9.6

1. a. parabola **b.** The directrix is 3 units above the pole, at $y = 3$. **2. a.** parabola **b.** The directrix is 3 units to the right of the pole, at $x = 3$.
3. a. ellipse **b.** The directrix is 3 units to the left of the pole, at $x = -3$. **4. a.** ellipse **b.** The directrix is 3 units to the right of the pole, at $x = 3$.
5. a. parabola **b.** The directrix is 4 units above the pole, at $y = 4$. **6. a.** parabola **b.** The directrix is 4 units below the pole, at $y = -4$.
7. a. hyperbola **b.** The directrix is 3 units to the left of the pole, at $x = -3$. **8. a.** hyperbola **b.** The directrix is 3 units to the right of the pole, at $x = 3$.

9.

10.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

21. $[-3, 15, 1]$ by $[-7, 7, 1]$ **22.** $[-2, 8, 1]$ by $[-4, 4, 1]$ **23.** $[-4, 2, 1]$ by $[-10, 10, 1]$ **24.** $[-2, 4, 1]$ by $[-10, 10, 1]$

25. $[-2, 5, 1]$ by $[-10, 10, 1]$ **26.** $[0, 9, 1]$ by $[-5, 5, 1]$ **27.** $[-4, 4, 1]$ by $[-10, 0.4, 1]$ **28.** $[-4, 4, 1]$ by $[-10, \frac{1}{3}, 1]$

40.

41.

42.

43.

The graph appears to be rotated counterclockwise through an angle of $\frac{\pi}{4}$ radians.

44.

The graph appears to be rotated clockwise through an angle of $\frac{\pi}{3}$ radians.

45. Mercury: $r = \dfrac{(1 - 0.2056^2)(36.0 \times 10^6)}{1 - 0.2056 \cos\theta}$

Earth: $r = \dfrac{(1 - 0.0167^2)(92.96 \times 10^6)}{1 - 0.0167 \cos\theta}$

50.

51. $r = \dfrac{2}{1 - \frac{1}{2}\cos\theta}$ or $r = \dfrac{6}{1 + \frac{1}{2}\cos\theta}$

52. $r = \dfrac{\frac{3}{2}}{1 - \frac{3}{2}\cos\theta}$

53. parabola; using the relationships between rectangular and polar coordinates, $x^2 + y^2 = r^2$ and $x = r\cos\theta$: $y^2 = x + \frac{1}{4}$

57. 2; 5; 10; 17; 26; 37; Sum is 97.

Chapter 9 Review Exercises

1. foci: $(\pm\sqrt{11}, 0)$

$\frac{x^2}{36} + \frac{y^2}{25} = 1$

2. foci: $(0, \pm 3)$

$\frac{y^2}{25} + \frac{x^2}{16} = 1$

3. foci: $(0, \pm 2\sqrt{3})$

$4x^2 + y^2 = 16$

4. foci: $(\pm\sqrt{5}, 0)$

$4x^2 + 9y^2 = 36$

5. foci: $(1 \pm \sqrt{7}, -2)$

$\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$

6. foci: $(-1, 2 \pm \sqrt{7})$

$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{16} = 1$

7. foci: $(-3 \pm \sqrt{5}, 2)$

$4x^2 + 9y^2 + 24x - 36y + 36 = 0$

8. foci: $(1, -1 \pm \sqrt{5})$

$9x^2 + 4y^2 - 18x + 8y - 23 = 0$

11. $\dfrac{(x+3)^2}{36} + \dfrac{(y-5)^2}{4} = 1$

14. The hit ball will collide with the other ball.

15. foci: $(\pm\sqrt{17}, 0)$; $y = \pm\frac{1}{4}x$

$\frac{x^2}{16} - y^2 = 1$

16. foci: $(0, \pm\sqrt{17})$; $y = \pm 4x$

$\frac{y^2}{16} - x^2 = 1$

17. foci: $(\pm 5, 0)$; $y = \pm\frac{3}{4}x$

$9x^2 - 16y^2 = 144$

18. foci: $(0, \pm 2\sqrt{5})$; $y = \pm\frac{1}{2}x$

$4y^2 - x^2 = 16$

19. foci: $(2 \pm \sqrt{41}, -3)$

$y + 3 = \pm\frac{4}{5}(x - 2)$

$\frac{(x-2)^2}{25} - \frac{(y+3)^2}{16} = 1$

20. foci: $(3, -2 \pm \sqrt{41})$

$y + 2 = \pm\frac{5}{4}(x - 3)$

$\frac{(y+2)^2}{25} - \frac{(x-3)^2}{16} = 1$

21. foci: $(1, 2 \pm \sqrt{5})$

$y - 2 = \pm 2(x - 1)$

$y^2 - 4y - 4x^2 + 8x - 4 = 0$

22. foci: $(1 \pm \sqrt{2}, -1)$

$y + 1 = \pm(x - 1)$

$x^2 - y^2 - 2x - 2y - 1 = 0$

25. c must be greater than a.

27. vertex: $(0, 0)$; focus: $(2, 0)$
directrix: $x = -2$

28. vertex: $(0, 0)$; focus: $(0, -4)$
directrix: $y = 4$

29. vertex: $(0, 2)$; focus: $(-4, 2)$
directrix: $x = 4$

30. vertex: $(4, -1)$; focus: $(4, 0)$
directrix: $y = -2$

31. vertex: $(0, 1)$; focus: $(0, 0)$

directrix: $y = 2$

32. vertex: $(-1, 5)$; focus: $(0, 5)$

directrix: $x = -2$

33. vertex: $(2, -2)$; focus: $\left(2, -\frac{3}{2}\right)$;
directrix: $y = -\frac{5}{2}$

47. a. $x'^2 - y'^2 = 8$

b. $\dfrac{x'^2}{8} - \dfrac{y'^2}{8} = 1$

c.

48. a. $3x'^2 + y'^2 = 2$

b. $\dfrac{x'^2}{\frac{2}{3}} + \dfrac{y'^2}{2} = 1$

c.

49. a. $18x'^2 - 2y'^2 = 18$

b. $\dfrac{x'^2}{1} - \dfrac{y'^2}{9} = 1$

c.

50. a. $50x'^2 + 150y'^2 = 450$

b. $\dfrac{x'^2}{9} + \dfrac{y'^2}{3} = 1$

c.

51. a. $16x'^2 + 96y' = 0$

b. $x'^2 = -6y'$

c.

52. $y = -\dfrac{1}{2}x + \dfrac{1}{2}$

53. $(y + 1)^2 = x, 0 \le x \le 9,$
$-2 \le y \le 2$

54. $(y - 1)^2 = \dfrac{1}{4}x$

55. $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1, 0 \le x \le 4,$
$-3 < y \le 3$

56. $\dfrac{(x - 3)^2}{4} + \dfrac{(y - 1)^2}{4} = 1$
or $(x - 3)^2 + (y - 1)^2 = 4$

57. $\dfrac{x^2}{9} - \dfrac{y^2}{9} = 1,$
$3 \le x \le 3\sqrt{2}. 0 \le y \le 3$

59. a. $x = (100 \cos 40°)t$; $y = 6 + (100 \sin 40°)t - 16t^2$

b. After 1 second: 76.6 feet in distance, 54.3 feet in height; after 2 seconds: 153.2 feet in distance, 70.6 feet in height; after 3 seconds: 229.8 feet in distance, 54.8 feet in height.

d.

; The ball is at its maximum height at 2.0 seconds. The maximum height is 70.6 feet.

60. a. $r = \dfrac{4}{1 - \sin\theta}$ **b.** $e = 1; p = 4$; parabola

c.

61. a. $r = \dfrac{6}{1 + \cos\theta}$ **b.** $e = 1; p = 6$; parabola

c.

62. a. $r = \dfrac{3}{1 + \frac{1}{2}\sin\theta}$ **b.** $e = \dfrac{1}{2}; p = 6$; ellipse

c.

63. a. $r = \dfrac{\frac{2}{3}}{1 - \frac{2}{3}\cos\theta}$ **b.** $e = \dfrac{2}{3}; p = 1$; ellipse

c.

64. a. $r = \dfrac{2}{1 + 2\sin\theta}$ **b.** $e = 2; p = 1$; hyperbola

c.

65. a. $r = \dfrac{2}{1 + 4\cos\theta}$ **b.** $e = 4; p = \dfrac{1}{2}$; hyperbola

c.

Chapter 9 Test

1. foci: $(\pm\sqrt{13}, 0)$

asymptotes: $y = \pm\dfrac{3}{2}x$

2. vertex: $(0, 0)$; focus: $(0, -2)$

directrix: $y = 2$

3. foci: $(-6, 5), (2, 5)$

4. foci: $(-1, 1 \pm \sqrt{5})$

asymptotes: $y - 1 = \pm 2(x + 1)$

5. vertex: $(-5, 1)$; focus: $(-5, 3)$
directrix: $y = -1$

6. $\dfrac{x^2}{100} + \dfrac{y^2}{51} = 1$

7. $\dfrac{y^2}{49} - \dfrac{x^2}{51} = 1$

14.

15.

16.

17.

Cumulative Review Exercises (Chapters P–9)

11.

12.

13.

14. a. $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{1}{8}, \pm \frac{3}{8}, \pm \frac{1}{16}, \pm \frac{3}{16}, \pm \frac{1}{32}, \pm \frac{3}{32}$

15. a. domain: $(-2, 2)$; range: $[-3, \infty)$ **b.** minimum of -3 at $x = 0$

g.

h.

16. $(g \circ f)(x) = x^2 - 2$

19. The costs will be the same when the number of miles driven is 175 miles. The cost will be \$67.

21. $\dfrac{\csc\theta - \sin\theta}{\sin\theta} = \dfrac{\csc\theta}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta} = \dfrac{\frac{1}{\sin\theta}}{\sin\theta} - 1 = \dfrac{1}{\sin^2\theta} - 1 = \dfrac{1 - \sin^2\theta}{\sin^2\theta} = \dfrac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$

22.

25. $b \approx 14.4$, $C = 44°$, $c \approx 10.5$

CHAPTER 10

Exercise Set 10.1

7. $-4, 5, -6, 7$ **8.** $5, -6, 7, -8$ **12.** $\frac{1}{3}, -\frac{1}{5}, \frac{1}{9}, -\frac{1}{17}$ **20.** $2, \frac{3}{2}, \frac{8}{3}, \frac{15}{2}$ **22.** $-2, -2, -4, -12$ **69. a.** 1515; A total of 1515 thousand, or 1,515,000, autism cases were diagnosed in the United States from 2001 through 2008. **70. a.** 116.5; Average state cigarette tax per pack each year averaged 116.5¢ for the years 2005 through 2010. **82.** 1,307,674,368,000 **83.** 8.109673361 E15
88. $a_{10} = 2.5937$; $a_{100} = 2.7048$; $a_{1000} = 2.7169$; $a_{10,000} = 2.7181$; $a_{100,000} = 2.7183$; As n gets larger, a_n gets closer to $e \approx 2.7183$.

89.

As n gets larger, a_n approaches 1.

90.

As n gets larger, a_n approaches 0.

91.

As n gets larger, a_n approaches 0.

92.

As n gets larger, a_n approaches $\frac{3}{5}$.

96. does not make sense **103.** $-5; -5; -5; -5$; The difference between consecutive terms is always -5.
104. 4; 4; 4; 4; The difference between consecutive terms is always 4.

Exercise Set 10.2

1. 200, 220, 240, 260, 280, 300 **2.** 300, 350, 400, 450, 500, 550 **3.** $-7, -3, 1, 5, 9, 13$ **4.** $-8, -3, 2, 7, 12, 17$ **5.** $300, 210, 120, 30, -60, -150$
6. $200, 140, 80, 20, -40, -100$ **7.** $\frac{5}{2}, 2, \frac{3}{2}, 1, \frac{1}{2}, 0$ **8.** $\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}, -\frac{1}{2}$ **9.** $-9, -3, 3, 9, 15, 21$ **10.** $-7, -3, 1, 5, 9, 13$
11. $30, 20, 10, 0, -10, -20$ **12.** $50, 30, 10, -10, -30, -50$ **23.** $a_n = 4n - 3$; $a_{20} = 77$ **24.** $a_n = 5n - 3$; $a_{20} = 97$
25. $a_n = 11 - 4n$; $a_{20} = -69$ **26.** $a_n = 11 - 5n$; $a_{20} = -89$ **27.** $a_n = 7 + 2n$; $a_{20} = 47$ **28.** $a_n = 3n + 3$; $a_{20} = 63$
29. $a_n = -16 - 4n$; $a_{20} = -96$ **30.** $a_n = -65 - 5n$; $a_{20} = -165$ **31.** $a_n = 1 + 3n$; $a_{20} = 61$ **32.** $a_n = 5n + 1$; $a_{20} = 101$
33. $a_n = 40 - 10n$; $a_{20} = -160$ **34.** $a_n = 36 - 12n$; $a_{20} = -204$ **45.** $8 + 13 + 18 + \cdots + 88$; 816 **46.** $2 + 8 + 14 + \cdots + 116$; 1180
47. $2 - 1 - 4 - \cdots - 85$; -1245 **48.** $4 + 2 + 0 - \cdots - 74$; -1400 **49.** $4 + 8 + 12 + \cdots + 400$; 20,200
50. $-4 - 8 - 12 - \cdots - 200$; -5100 **69.** Company A: \$307,000; Company B: \$324,000; Company B pays the greater total amount.
79. makes sense **85.** $-2; -2; -2; -2$; The ratio of a term to the term that directly precedes it is always -2.
86. 5; 5; 5; 5; The ratio of a term to the term that directly precedes it is always 5.

Exercise Set 10.3

1. 5, 15, 45, 135, 405 **2.** 4, 12, 36, 108, 324 **3.** $20, 10, 5, \frac{5}{2}, \frac{5}{4}$ **4.** $24, 8, \frac{8}{3}, \frac{8}{9}, \frac{8}{27}$ **5.** $10, -40, 160, -640, 2560$ **6.** $10, -30, 90, -270, 810$
7. $-6, 30, -150, 750, -3750$ **8.** $-2, 12, -72, 432, -2592$ **17.** $a_n = 3(4)^{n-1}$; $a_7 = 12{,}288$ **18.** $a_n = 3(5)^{n-1}$; $a_7 = 46{,}875$
19. $a_n = 18\left(\frac{1}{3}\right)^{n-1}$; $a_7 = \frac{2}{81}$ **20.** $a_n = 12\left(\frac{1}{2}\right)^{n-1}$; $a_7 = \frac{3}{16}$ **21.** $a_n = 1.5(-2)^{n-1}$; $a_7 = 96$ **22.** $a_n = 5\left(-\frac{1}{5}\right)^{n-1}$; $a_7 = \frac{1}{3125}$
72. \$1,073,741,823 **77. a.** \$11,617 **78. a.** \$14,163 **86.** \$15 billion

99.

horizontal asymptote: $y = 3$; sum of series: 3

100.

horizontal asymptote: $y = 10$; sum of series: 10

Mid-Chapter 10 Check Point

1. $1, -2, \frac{3}{2}, -\frac{2}{3}, \frac{5}{24}$ **2.** $5, 2, -1, -4, -7$ **3.** $5, -15, 45, -135, 405$ **4.** $3, 1, 3, 1, 3$ **5.** $a_n = 4n - 2$; $a_{20} = 78$ **6.** $a_n = 3(2)^{n-1}$; $a_{10} = 1536$
18. 464 ft; 3600 ft

Section 10.4

Check Point Exercises

1. a. $S_1: 2 = 1(1 + 1)$; $S_k: 2 + 4 + 6 + \cdots + 2k = k(k + 1)$; $S_{k+1}: 2 + 4 + 6 + \cdots + 2(k + 1) = (k + 1)(k + 2)$

   b. $S_1: 1^3 = \frac{1^2(1 + 1)^2}{4}$; $S_k: 1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{k^2(k + 1)^2}{4}$; $S_{k+1}: 1^3 + 2^3 + 3^3 + \cdots + (k + 1)^3 = \frac{(k + 1)^2(k + 2)^2}{4}$

2. $S_1: 2 = 1(1 + 1)$; $S_k: 2 + 4 + 6 + \cdots + 2k = k(k + 1)$; $S_{k+1}: 2 + 4 + 6 + \cdots + 2k + 2(k + 1) = (k + 1)(k + 2)$; S_{k+1} can be obtained by adding $2k + 2$ to both sides of S_k.

3. $S_1: 1^3 = \frac{1^2(1 + 1)^2}{4}$; $S_k: 1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{k^2(k + 1)^2}{4}$; $S_{k+1}: 1^3 + 2^3 + 3^3 + \cdots + k^3 + (k + 1)^3 = \frac{(k + 1)^2(k + 2)^2}{4}$; S_{k+1} can be obtained by adding $k^3 + 3k^2 + 3k + 1$ to both sides of S_k.

4. S_1: 2 is a factor of $1^2 + 1$; S_k: 2 is a factor of $k^2 + k$; S_{k+1}: 2 is a factor of $(k + 1)^2 + (k + 1) = k^2 + 3k + 2$; S_{k+1} can be obtained from S_k by writing $k^2 + 3k + 2$ as $(k^2 + k) + 2(k + 1)$.

Exercise Set 10.4

1. $S_1: 1 = 1^2$; $S_2: 1 + 3 = 2^2$; $S_3: 1 + 3 + 5 = 3^2$
2. $S_1: 3 = \frac{1(6)}{2}$; $S_2: 3 + 4 = \frac{2(7)}{2}$; $S_3: 3 + 4 + 5 = \frac{3(8)}{2}$
3. S_1: 2 is a factor of $1 - 1 = 0$; S_2: 2 is a factor of $2^2 - 2 = 2$; S_3: 2 is a factor of $3^2 - 3 = 6$.
4. S_1: 3 is a factor of $1^3 - 1 = 0$; S_2: 3 is a factor of $2^3 - 2 = 6$; S_3: 3 is a factor of $3^3 - 3 = 24$.
5. $S_k: 4 + 8 + 12 + \cdots + 4k = 2k(k + 1)$; $S_{k+1}: 4 + 8 + 12 + \cdots + (4k + 4) = 2(k + 1)(k + 2)$
6. $S_k: 3 + 4 + 5 + \cdots + (k + 2) = \frac{k(k + 5)}{2}$; $S_{k+1}: 3 + 4 + 5 + \cdots + (k + 3) = \frac{(k + 1)(k + 6)}{2}$
7. $S_k: 3 + 7 + 11 + \cdots + (4k - 1) = k(2k + 1)$; $S_{k+1}: 3 + 7 + 11 + \cdots + (4k + 3) = (k + 1)(2k + 3)$
8. $S_k: 2 + 7 + 12 + \cdots + (5k - 3) = \frac{k(5k - 1)}{2}$; $S_{k+1}: 2 + 7 + 12 + \cdots + (5k + 2) = \frac{(k + 1)(5k + 4)}{2}$
9. S_k: 2 is a factor of $k^2 - k + 2$; S_{k+1}: 2 is a factor of $k^2 + k + 2$.
10. S_k: 2 is a factor of $k^2 - k$; S_{k+1}: 2 is a factor of $k^2 + k$.
11. $S_1: 4 = 2(1)(1 + 1)$; $S_k: 4 + 8 + 12 + \cdots + 4k = 2k(k + 1)$; $S_{k+1}: 4 + 8 + 12 + \cdots + 4(k + 1) = 2(k + 1)(k + 2)$; S_{k+1} can be obtained by adding $4k + 4$ to both sides of S_k.
12. $S_1: 3 = \frac{1(1 + 5)}{2}$; $S_k: 3 + 4 + 5 + \cdots + (k + 2) = \frac{k(k + 5)}{2}$; $S_{k+1}: 3 + 4 + 5 + \cdots + (k + 3) = \frac{(k + 1)(k + 6)}{2}$; S_{k+1} can be obtained by adding $k + 3$ to both sides of S_k.
13. $S_1: 1 = 1^2$; $S_k: 1 + 3 + 5 + \cdots + (2k - 1) = k^2$; $S_{k+1}: 1 + 3 + 5 + \cdots + (2k + 1) = (k + 1)^2$; S_{k+1} can be obtained by adding $2k + 1$ to both sides of S_k.
14. $S_1: 3 = \frac{3(1)(1 + 1)}{2}$; $S_k: 3 + 6 + 9 + \cdots + 3k = \frac{3k(k + 1)}{2}$; $S_{k+1}: 3 + 6 + 9 + \cdots + 3(k + 1) = \frac{3(k + 1)(k + 2)}{2}$; S_{k+1} can be obtained by adding $3k + 3$ to both sides of S_k.
15. $S_1: 3 = 1[2(1) + 1]$; $S_k: 3 + 7 + 11 + \cdots + (4k - 1) = k(2k + 1)$; $S_{k+1}: 3 + 7 + 11 + \cdots + (4k + 3) = (k + 1)(2k + 3)$; S_{k+1} can be obtained by adding $4k + 3$ to both sides of S_k.
16. $S_1: 2 = \frac{1[5(1) - 1]}{2}$; $S_k: 2 + 7 + 12 + \cdots + (5k - 3) = \frac{k(5k - 1)}{2}$; $S_{k+1}: 2 + 7 + 12 + \cdots + (5k + 2) = \frac{(k + 1)(5k + 4)}{2}$; S_{k+1} can be obtained by adding $5k + 2$ to both sides of S_k.
17. $S_1: 1 = 2^1 - 1$; $S_k: 1 + 2 + 2^2 + \cdots + 2^{k-1} = 2^k - 1$; $S_{k+1}: 1 + 2 + 2^2 + \cdots + 2^k = 2^{k+1} - 1$; S_{k+1} can be obtained by adding 2^k to both sides of S_k.
18. $S_1: 1 = \frac{3^1 - 1}{2}$; $S_k: 1 + 3 + 3^2 + \cdots + 3^{k-1} = \frac{3^k - 1}{2}$; $S_{k+1}: 1 + 3 + 3^2 + \cdots + 3^k = \frac{3^{k+1} - 1}{2}$; S_{k+1} can be obtained by adding 3^k to both sides of S_k.
19. $S_1: 2 = 2^{1+1} - 2$; $S_k: 2 + 4 + 8 + \cdots + 2^k = 2^{k+1} - 2$; $S_{k+1}: 2 + 4 + 8 + \cdots + 2^{k+1} = 2^{k+2} - 2$; S_{k+1} can be obtained by adding 2^{k+1} to both sides of S_k.
20. $S_1: \frac{1}{2} = 1 - \frac{1}{2^1}$; $S_k: \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$; $S_{k+1}: \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}$; S_{k+1} can be obtained by adding $\frac{1}{2^{k+1}}$ to both sides of S_k.
21. $S_1: 1 \cdot 2 = \frac{1(1 + 1)(1 + 2)}{3}$; $S_k: 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k(k + 1) = \frac{k(k + 1)(k + 2)}{3}$;

    $S_{k+1}: 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (k + 1)(k + 2) = \frac{(k + 1)(k + 2)(k + 3)}{3}$; S_{k+1} can be obtained by adding $(k + 1)(k + 2)$ to both sides of S_k.
22. $S_1: 1 \cdot 3 = \frac{1(1 + 1)[2(1) + 7]}{6}$; $S_k: 1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + k(k + 2) = \frac{k(k + 1)(2k + 7)}{6}$;

    $S_{k+1}: 1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + (k + 1)(k + 3) = \frac{(k + 1)(k + 2)(2k + 9)}{6}$; S_{k+1} can be obtained by adding $(k + 1)(k + 3)$ to both sides of S_k.

23. S_1: $\frac{1}{1\cdot 2} = \frac{1}{1+1}$; S_k: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}$; S_{k+1}: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$; S_{k+1} can be obtained by adding $\frac{1}{(k+1)(k+2)}$ to both sides of S_k.

24. S_1: $\frac{1}{2\cdot 3} = \frac{1}{2(1)+4}$; S_k: $\frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots + \frac{1}{(k+1)(k+2)} = \frac{k}{2k+4}$; S_{k+1}: $\frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots + \frac{1}{(k+2)(k+3)} = \frac{k+1}{2k+6}$; S_{k+1} can be obtained by adding $\frac{1}{(k+2)(k+3)}$ to both sides of S_k.

25. S_1: 2 is a factor of 0; S_k: 2 is a factor of $k^2 - k$; S_{k+1}: 2 is a factor of $k^2 + k$; S_{k+1} can be obtained from S_k by rewriting $k^2 + k$ as $(k^2 - k) + 2k$.

26. S_1: 2 is a factor of 4; S_k: 2 is a factor of $k^2 + 3k$; S_{k+1}: 2 is a factor of $k^2 + 5k + 4$; S_{k+1} can be obtained from S_k by rewriting $k^2 + 5k + 4$ as $(k^2 + 3k) + 2(k + 2)$.

27. S_1: 6 is a factor of 6; S_k: 6 is a factor of $k(k+1)(k+2)$; S_{k+1}: 6 is a factor of $(k+1)(k+2)(k+3)$; S_{k+1} can be obtained from S_k by rewriting $(k+1)(k+2)(k+3)$ as $k(k+1)(k+2) + 3(k+1)(k+2)$ and noting that either $k+1$ or $k+2$ is even, so 6 is a factor of $3(k+1)(k+2)$.

28. S_1: 3 is a factor of 0; S_k: 3 is a factor of $k(k+1)(k-1)$ or $k^3 - k$; S_{k+1}: 3 is a factor of $(k+1)(k+2)k$ or $k^3 + 3k^2 + 2k$; S_{k+1} can be obtained from S_k by rewriting $k^3 + 3k^2 + 2k$ as $(k^3 - k) + 3(k^2 + k)$.

29. S_1: $5\cdot 6^1 = 6(6^1 - 1)$; S_k: $\sum_{i=1}^{k} 5\cdot 6^i = 6(6^k - 1)$; S_{k+1}: $\sum_{i=1}^{k+1} 5\cdot 6^i = 6(6^{k+1} - 1)$; S_{k+1} can be obtained by adding $5\cdot 6^{k+1}$ to both sides of S_k.

30. S_1: $7\cdot 8^1 = 8(8^1 - 1)$; S_k: $\sum_{i=1}^{k} 7\cdot 8^i = 8(8^k - 1)$; S_{k+1}: $\sum_{i=1}^{k+1} 7\cdot 8^i = 8(8^{k+1} - 1)$; S_{k+1} can be obtained by adding $7\cdot 8^{k+1}$ to both sides of S_k.

31. S_1: $1 + 2 > 1$; S_k: $k + 2 > k$; S_{k+1}: $k + 3 > k + 1$; S_{k+1} can be obtained by adding 1 to both sides of S_k.

32. S_1: If $0 < x < 1$, then $0 < x^1 < 1$; S_k: If $0 < x < 1$, then $0 < x^k < 1$; S_{k+1}: If $0 < x < 1$, then $0 < x^{k+1} < 1$; S_{k+1} can be obtained by multiplying the respective parts of $0 < x < 1$ and $0 < x^k < 1$.

33. S_1: $(ab)^1 = a^1b^1$; S_k: $(ab)^k = a^kb^k$; S_{k+1}: $(ab)^{k+1} = a^{k+1}b^{k+1}$; S_{k+1} can be obtained by multiplying both sides of S_k by (ab).

34. S_1: $\left(\frac{a}{b}\right)^1 = \frac{a^1}{b^1}$; S_k: $\left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}$; S_{k+1}: $\left(\frac{a}{b}\right)^{k+1} = \frac{a^{k+1}}{b^{k+1}}$; S_{k+1} can be obtained by multiplying both sides of S_k by $\left(\frac{a}{b}\right)$.

41. S_3: $3^2 > 2(3) + 1$; S_k: $k^2 > 2k + 1$ for $k \geq 3$; S_{k+1}: $(k+1)^2 > 2(k+1) + 1$ or $k^2 + 2k + 1 > 2k + 3$; S_{k+1} can be obtained from S_k by noting that S_{k+1} is the same as $k^2 > 2$ which is true for $k \geq 3$.

42. S_5: $2^5 > 5^2$; S_k: $2^k > k^2$ for $k \geq 5$; S_{k+1}: $2^{k+1} > (k+1)^2$ or $2(2^k) > k^2 + 2k + 1$; S_{k+1} can be obtained from S_k by multiplying both sides of S_k by 2 and noting that $k^2 > 2k + 1$ for $k \geq 5$.

43. S_1: $\frac{1}{4}$; S_2: $\frac{1}{3}$; S_3: $\frac{3}{8}$; S_4: $\frac{2}{5}$; S_5: $\frac{5}{12}$; S_n: $\frac{n}{2n+2}$; Use S_k to obtain the conjectured formula.

44. S_1: $\frac{1}{2}$; S_2: $\frac{1}{3}$; S_3: $\frac{1}{4}$; S_4: $\frac{1}{5}$; S_5: $\frac{1}{6}$; S_n: $\frac{1}{n+1}$; Use S_k to obtain the conjectured formula.

46. The exponents begin with the exponent on $a + b$ and decrease by 1 in each successive term.

47. The exponents begin with 0, increase by 1 in each successive term, and end with the exponent on $a + b$.

Exercise Set 10.5

9. $x^3 + 6x^2 + 12x + 8$ **10.** $x^3 + 12x^2 + 48x + 64$ **11.** $27x^3 + 27x^2y + 9xy^2 + y^3$ **12.** $x^3 + 9x^2y + 27xy^2 + 27y^3$

13. $125x^3 - 75x^2 + 15x - 1$ **14.** $64x^3 - 48x^2 + 12x - 1$ **15.** $16x^4 + 32x^3 + 24x^2 + 8x + 1$ **16.** $81x^4 + 108x^3 + 54x^2 + 12x + 1$

17. $x^8 + 8x^6y + 24x^4y^2 + 32x^2y^3 + 16y^4$ **18.** $x^8 + 4x^6y + 6x^4y^2 + 4x^2y^3 + y^4$ **19.** $y^4 - 12y^3 + 54y^2 - 108y + 81$

20. $y^4 - 16y^3 + 96y^2 - 256y + 256$ **21.** $16x^{12} - 32x^9 + 24x^6 - 8x^3 + 1$ **22.** $16x^{20} - 32x^{15} + 24x^{10} - 8x^5 + 1$

23. $c^5 + 10c^4 + 40c^3 + 80c^2 + 80c + 32$ **24.** $c^5 + 15c^4 + 90c^3 + 270c^2 + 405c + 243$ **25.** $x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$

26. $x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$ **27.** $243x^5 - 405x^4y + 270x^3y^2 - 90x^2y^3 + 15xy^4 - y^5$

28. $x^5 - 15x^4y + 90x^3y^2 - 270x^2y^3 + 405xy^4 - 243y^5$ **29.** $64a^6 + 192a^5b + 240a^4b^2 + 160a^3b^3 + 60a^2b^4 + 12ab^5 + b^6$

30. $a^6 + 12a^5b + 60a^4b^2 + 160a^3b^3 + 240a^2b^4 + 192ab^5 + 64b^6$ **31.** $x^8 + 16x^7 + 112x^6 + \cdots$ **32.** $x^8 + 24x^7 + 252x^6 + \cdots$

33. $x^{10} - 20x^9y + 180x^8y^2 - \cdots$ **34.** $x^9 - 18x^8y + 144x^7y^2 - \cdots$ **35.** $x^{32} + 16x^{30} + 120x^{28} + \cdots$ **36.** $x^{34} + 17x^{32} + 136x^{30} + \cdots$

37. $y^{60} - 20y^{57} + 190y^{54} - \cdots$ **38.** $y^{63} - 21y^{60} + 210y^{57} - \cdots$ **49.** $x^{12} + 4x^7 + 6x^2 + \frac{4}{x^3} + \frac{1}{x^8}$ **50.** $x^8 + 4x^3 + \frac{6}{x^2} + \frac{4}{x^7} + \frac{1}{x^{12}}$

51. $x - 3x^{1/3} + \frac{3}{x^{1/3}} - \frac{1}{x}$ **52.** $x^2 - 3x + 3 - \frac{1}{x}$ **53.** $4x^3 + 6x^2h + 4xh^2 + h^3$ **54.** $5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4$

68.

f_2, f_3, and f_4 are approaching $f_1 = f_5$.

69.

f_2, f_3, f_4, and f_5 are approaching $f_1 = f_6$.

70. $f_1(x) = x^3 - 3x^2 + 3x - 1$

71. $f_1(x) = x^4 - 8x^3 + 24x^2 - 32x + 16$

83. $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!r!} = \frac{n!}{(n-r)![n-(n-r)]!} = \binom{n}{n-r}$

84. $\binom{n}{r} + \binom{n}{r+1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-(r+1))!} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)r!(n-r-1)!}$

$= \frac{n!(r+1) + n!(n-r)}{(r+1)r!(n-r)!} = \frac{n!(r+1+n-r)}{(r+1)r!(n-r)!} = \frac{n!(n+1)}{(r+1)r!(n-r)!} = \frac{(n+1)!}{(r+1)!(n-r)!} = \frac{(n+1)!}{(r+1)!(n+1-(r+1))!} = \binom{n+1}{r+1}$

85. a. $(a+b)^1 = a+b = \binom{1}{0}a^1 + \binom{1}{1}b^1$ **b.** Assume: $(a+b)^k = \binom{k}{0}a^k + \binom{k}{1}a^{k-1}b + \binom{k}{2}a^{k-2}b^2 + \cdots + \binom{k}{k-1}ab^{k-1}$ $+ \binom{k}{k}b^k$; Prove: $(a+b)^{k+1} = \binom{k+1}{0}a^{k+1} + \binom{k+1}{1}a^{k+1-1}b + \binom{k+1}{2}a^{k+1-2}b^2 + \cdots + \binom{k+1}{k+1-1}ab^{k+1-1} + \binom{k+1}{k+1}b^{k+1}$

c. $(a+b)(a+b)^k = (a+b)\left[\binom{k}{0}a^k + \binom{k}{1}a^{k-1}b + \binom{k}{2}a^{k-2}b^2 + \cdots + \binom{k}{k-1}ab^{k-1} + \binom{k}{k}b^k\right]$ or $(a+b)^{k+1} = \binom{k}{0}a^{k+1}$ $+ \binom{k}{0}a^kb + \binom{k}{1}a^kb + \binom{k}{1}a^{k-1}b^2 + \binom{k}{2}a^{k-1}b^2 + \binom{k}{2}a^{k-2}b^3 + \cdots + \binom{k}{k-1}a^2b^{k-1} + \binom{k}{k-1}ab^k + \binom{k}{k}ab^k + \binom{k}{k}b^{k+1}$

d. $(a+b)^{k+1} = \binom{k}{0}a^{k+1} + \left[\binom{k}{0} + \binom{k}{1}\right]a^kb + \left[\binom{k}{1} + \binom{k}{2}\right]a^{k-1}b^2 + \left[\binom{k}{2} + \binom{k}{3}\right]a^{k-2}b^3 + \cdots + \left[\binom{k}{k-1} + \binom{k}{k}\right]ab^k + \binom{k}{k}b^{k+1}$

e. $(a+b)^{k+1} = \binom{k}{0}a^{k+1} + \binom{k+1}{1}a^kb + \binom{k+1}{2}a^{k-1}b^2 + \binom{k+1}{3}a^{k-2}b^3 + \cdots + \binom{k+1}{k}ab^k + \binom{k}{k}b^{k+1}$

f. $(a+b)^{k+1} = \binom{k+1}{0}a^{k+1} + \binom{k+1}{1}a^kb + \binom{k+1}{2}a^{k-1}b^2 + \binom{k+1}{3}a^{k-2}b^3 + \cdots + \binom{k+1}{k}ab^k + \binom{k+1}{k+1}b^{k+1}$

Exercise Set 10.6

17. combinations

Exercise Set 10.7

27. $\frac{1}{18{,}009{,}460}; \frac{5}{900{,}473}$ **28.** $\frac{1}{593{,}775}; \frac{4}{23{,}751}$ **29. a.** 2,598,960 **b.** 1287 **c.** $\frac{1287}{2{,}598{,}960} \approx 0.0005$ **30.** $\frac{11}{1105} \approx 0.00995$

53. c. $\left(\frac{15}{16}\right)^{10} \approx 0.524$ **d.** $1 - \left(\frac{15}{16}\right)^{10} \approx 0.476$ **66.** does not make sense

73. a. The first person can have any birthday in the year. The second person can have all but one birthday.

76.

; The two pieces of the graph approach the point (4, 2)

$f(x) = \frac{x^2 - 6x + 8}{x - 4}$

77.

$f(x) = \begin{cases} 2x - 4 & \text{if } x \neq 3 \\ -5 & \text{if } x = 3 \end{cases}$

Chapter 10 Review Exercises

10. $\sum_{i=1}^{15} \frac{i}{i+2}$ **11.** $\sum_{i=4}^{13} i^3$ or $\sum_{i=1}^{10} (i+3)^3$ **12.** 7, 11, 15, 19, 23, 27 **13.** −4, −9, −14, −19, −24, −29 **14.** $\frac{3}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1$ **15.** −2, 3, 8, 13, 18, 23

33. 16, −8, 4, −2, 1 **34.** −1, 5, −25, 125, −625 **38.** $a_n = 2^{n-1}; a_8 = 128$ **39.** $a_n = 100\left(\frac{1}{10}\right)^{n-1}; a_8 = \frac{1}{100{,}000}$ **55. a.** \$91,361

57. S_1: $5 = \frac{5(1)(1+1)}{2}$; S_k: $5 + 10 + 15 + \cdots + 5k = \frac{5k(k+1)}{2}$; S_{k+1}: $5 + 10 + 15 + \cdots + 5(k+1) = \frac{5(k+1)(k+2)}{2}$; S_{k+1} can be obtained by adding $5(k+1)$ to both sides of S_k.

58. S_1: $1 = \frac{4^1 - 1}{3}$; S_k: $1 + 4 + 4^2 + \cdots + 4^{k-1} = \frac{4^k - 1}{3}$; S_{k+1}: $1 + 4 + 4^2 + \cdots + 4^k = \frac{4^{k+1} - 1}{3}$; S_{k+1} can be obtained by adding 4^k to both sides of S_k.

59. S_1: $2 = 2(1)^2$; S_k: $2 + 6 + 10 + \cdots + (4k - 2) = 2k^2$; S_{k+1}: $2 + 6 + 10 + \cdots + (4k + 2) = 2k^2 + 4k + 2$; S_{k+1} can be obtained by adding $4k + 2$ to both sides of S_k.

60. S_1: $1 \cdot 3 = \frac{1(1+1)[2(1)+7]}{6}$; S_k: $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + k(k+2) = \frac{k(k+1)(2k+7)}{6}$;

S_{k+1}: $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + (k+1)(k+3) = \frac{(k+1)(k+2)(2k+9)}{6}$; S_{k+1} can be obtained by adding $(k+1)(k+3)$ to both sides of S_k.

61. S_1: 2 is a factor of 6; S_k: 2 is a factor of $k^2 + 5k$; S_{k+1}: 2 is a factor of $k^2 + 7k + 6$; S_{k+1} can be obtained from S_k by rewriting $k^2 + 7k + 6$ as $(k^2 + 5k) + 2(k+3)$.

64. $8x^3 + 12x^2 + 6x + 1$ **65.** $x^8 - 4x^6 + 6x^4 - 4x^2 + 1$ **66.** $x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$

67. $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$ **68.** $x^{16} + 24x^{14} + 252x^{12} + \cdots$ **69.** $x^9 - 27x^8 + 324x^7 - \cdots$

Chapter 10 Test

9. $a_n = 5n - 1; a_{12} = 59$ **10.** $a_n = 16\left(\frac{1}{4}\right)^{n-1}; a_{12} = \frac{1}{262{,}144}$

16. S_1: $1 = \frac{1[3(1)-1]}{2}$; S_k: $1 + 4 + 7 + \cdots + (3k - 2) = \frac{k(3k-1)}{2}$; S_{k+1}: $1 + 4 + 7 + \cdots + (3k + 1) = \frac{(k+1)(3k+2)}{2}$; S_{k+1} can be obtained by adding $3k + 1$ to both sides of S_k.

Cumulative Review Exercises (Chapters P–10)

8.

$g(x) = f(x - 2) + 1$

9.

$h(x) = -f(2x)$

23.

$100x^2 + y^2 = 25$

24.

$4x^2 - 9y^2 - 16x + 54y - 29 = 0$

25.

$f(x) = \dfrac{x^2 - 1}{x - 2}$

26.

$2x - y \geq 4$
$x \leq 2$

27.

$f(x) = x^2 - 4x - 5$

28.

$f(x) = \sqrt[3]{x + 4}$
$f^{-1}(x) = x^3 - 4$

29.

$f(x) = \log_2 x$
$g(x) = -\log_2 (x + 1)$

30. $(f \circ g)(x) = -x^2 + 2$; $(g \circ f)(x) = -x^2 - 2x$

32. $\begin{bmatrix} -2 & 10 \\ -5 & 7 \\ 15 & -15 \end{bmatrix}$

33. $\dfrac{-1}{x - 2} + \dfrac{3x - 2}{x^2 + 2x + 2}$

34. $x^{15} + 10x^{12}y + 40x^9y^2 + 80x^6y^3 + 80x^3y^4 + 32y^5$

41. a. 6 sec

44. a. 10 in. **b.** $\dfrac{3}{8}$ cycles/sec **c.** $\dfrac{8}{3}$ sec

45. $\tan x + \dfrac{1}{\tan x} = \dfrac{\sin x}{\cos x} + \dfrac{1}{\dfrac{\sin x}{\cos x}} = \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \dfrac{1}{\cos x \sin x}$

46. $\dfrac{1 - \tan^2 x}{1 + \tan^2 x} = \dfrac{1 - \dfrac{\sin^2 x}{\cos^2 x}}{1 + \dfrac{\sin^2 x}{\cos^2 x}} \cdot \dfrac{\cos^2 x}{\cos^2 x} = \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} = \dfrac{\cos 2x}{1} = \cos 2x$

47.

$y = -2 \cos (3x - \pi)$

51.

$r = 1 + 2\cos\theta$

52. $B \approx 54°$, $C \approx 92°$, $c \approx 39.3$ or
$B \approx 126°$, $C \approx 20°$, $c \approx 13.5$

53. $y = 2 - x^2$;

CHAPTER 11

Section 11.1

Check Point Exercises

4.

$f(x) = \begin{cases} 3x - 2 & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

Exercise Set 11.1

42. 20 **67. a.** 8; As your nose approaches the fan, the speed of the breeze that your nose feels approaches 8 miles per hour. **b.** Answers may vary. **68.** 124; This means that the recommended safe distance between cars traveling at 60 miles per hour is 124 feet. **71. a.** 30; The cost to rent the car one day and drive it 100 miles is \$30. **72. b.** 450; Yes, the same limit is obtained. The graph shows that the limit is the maximum area. **85.** 0.69315; 0.693147 **86.** 0.25000; 0.250000 **87.** 1.5000; 1.50000 **88.** 0.50000; 0.50000017 **94.** values of x in the interval $\left(\frac{29}{30}, \frac{31}{30}\right)$; values of x in the interval $\left(\frac{299}{300}, \frac{301}{300}\right)$ **95.** 31.544281

96. a.

$f(x) = \begin{cases} x^2 + 5 & \text{if } x < 2 \\ 3x + 1 & \text{if } x \geq 2 \end{cases}$

b. $\lim_{x \to 2^-} f(x) = 9$; $\lim_{x \to 2^+} f(x) = 7$; $\lim_{x \to 2} f(x)$ does not exist.

Exercise Set 11.2

59. b. The length of the starship appears to approach 0. **c.** It is not possible to exceed the speed of light. **60. b.** Relative to a friend on Earth, you will barely grow older. **c.** It is not possible to exceed the speed of light. **80.** 4

86.

$\lim_{x \to 3} f(x) = 5$; $f(3) = 5$

No, it is not necessary to lift your pencil off the paper.

87.

$\lim_{x \to 3} f(x) = 5$; $f(3) = 6$

Yes, it is necessary to lift your pencil off the paper.

88.

$\lim_{x \to 3^-} f(x) = 5$; $\lim_{x \to 3^+} f(x) = 6$; $f(3) = 5$

Yes, it is necessary to lift your pencil off the paper.

Exercise Set 11.3

19. continuous for every number x **20.** continuous for every number x **21.** -1 and 4 **22.** -2 and 5 **25.** continuous for every number x **26.** continuous for every number x **43. h.** As the end of the course approached, the percentage of material learned by the student approached 100%. **44. c.** $\lim_{x \to 3} f(x)$ does not exist; the graph shows a discontinuity at $x = 3$. **d.** The graph jumps at its discontinuities due to per-ounce charges. **53.** 2.7183; 2.71828 **55.** does not make sense **56.** does not make sense **57.** makes sense **59.** No. In Exercise 58, $\lim_{x \to 9} f(x)$ exists. In this exercise, however, $\lim_{x \to 9} \frac{1}{x - 9}$ does not exist. **64.** $-32a - 16h + 48$

Mid-Chapter 11 Check Point

17. does not exist **18. c.** does not exist

Exercise Set 11.4

1. a. 2 **b.** $y = 2x + 3$ **2. a.** 4 **b.** $y = 4x + 2$ **3. a.** -2 **b.** $y = -2x + 3$ **4. a.** -2 **b.** $y = -2x + 6$ **5. a.** -20 **b.** $y = -20x - 20$ **6. a.** -16 **b.** $y = -16x - 16$ **7. a.** 7 **b.** $y = 7x - 8$ **8. a.** 7 **b.** $y = 7x - 3$ **11. a.** $\frac{1}{6}$ **b.** $y = \frac{1}{6}x + \frac{3}{2}$ **12. a.** $\frac{1}{8}$ **b.** $y = \frac{1}{8}x + 2$ **13. a.** -1 **b.** $y = -x + 2$ **14. a.** -2 **b.** $y = -2x + 4$

29. a. & c.

30. a. & c.

31. a. & c.

32. a. & c.

33. a. & c.

34. a. & c.

35. a. & c.

36. a. & c.

43. a. 32 feet per second; −32 feet per second **44. a.** 56 feet per second; −56 feet per second **45. a.** 32 feet per second; −32 feet per second
46. a. 32 feet per second; −32 feet per second **66.** does not make sense **68.** does not make sense

77.
$$\begin{aligned} A'(r) &= \lim_{h\to 0}\frac{A(r+h)-A(r)}{h} \\ &= \lim_{h\to 0}\frac{\pi(r+h)^2-\pi r^2}{h} \\ &= \lim_{h\to 0}\frac{\pi(r^2+2rh+h^2)-\pi r^2}{h} \\ &= \lim_{h\to 0}\frac{\pi r^2+2\pi rh+\pi h^2-\pi r^2}{h} \\ &= \lim_{h\to 0}\frac{h(2\pi r+\pi h)}{h} \\ &= \lim_{h\to 0}(2\pi r+\pi h) = 2\pi r + 0 = 2\pi r \end{aligned}$$

78.
$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{a(x+h)^2+b(x+h)+c-(ax^2+bx+c)}{h} \\ &= \lim_{h\to 0}\frac{ax^2+2axh+ah^2+bx+bh+c-ax^2-bx-c}{h} \\ &= \lim_{h\to 0}\frac{2axh+ah^2+bh}{h} \\ &= \lim_{h\to 0}\frac{h(2ax+ah+b)}{h} \\ &= \lim_{h\to 0}(2ax+ah+b) = 2ax+b \end{aligned}$$

When $2ax + b = 0, x = -\dfrac{b}{2a}$.

Chapter 11 Review Exercises

12. does not exist **16.** does not exist **18.** does not exist **46.** continuous for every number x **47.** 1 and −3 **48.** 0
49. continuous for every number x **56. a.** $-\dfrac{1}{x^2}$ **b.** $-\dfrac{1}{4}; -\dfrac{1}{4}$ **57. a.** $\dfrac{1}{2\sqrt{x}}$ **b.** $\dfrac{1}{12}; \dfrac{1}{18}$ **60. a.** 16 feet per second; −48 feet per second

Cumulative Review Exercises (Chapters P–11)

6.

7.

$f(x) = \dfrac{2x^2-5x+2}{x^2-4}$

8.

$f(x) = \begin{cases} -x+1 & \text{if } -1 \le x < 1 \\ 2 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$

9.

$y = 2\sin\left(2x + \dfrac{\pi}{2}\right)$

10.

$y = \dfrac{1}{2}\sec 2\pi x, 0 \le x \le 2$

11.

12.

$x^2 - 4y^2 - 4x - 24y - 48 = 0$

13.

$f(x) = \sqrt{x}$
$g(x) = \sqrt{x-2} + 1$

14.

15.

19. $x^8 - 12x^6y + 54x^4y^2 - 108x^2y^3 + 81y^4$

23.
$$\begin{aligned}\tan\theta + \cot\theta &= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\\ &= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\\ &= \frac{1}{\cos\theta\sin\theta}\\ &= \sec\theta\csc\theta\end{aligned}$$

24.
$$\begin{aligned}\tan(\theta + \pi) &= \frac{\tan\theta + \tan\pi}{1 - \tan\theta\tan\pi}\\ &= \frac{\tan\theta + 0}{1 - \tan\theta(0)}\\ &= \tan\theta\end{aligned}$$

25. $\begin{bmatrix} 2 & 1 & 3 \\ 8 & 1 & 9 \\ 5 & 1 & 6 \end{bmatrix}$

26.

27. $f(x) = x^9$; $g(x) = x^2 - 3x + 7$

32. 23 cm × 23 cm

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ANSWERS TO SELECTED EXERCISES

CHAPTER P

Section P.1

Check Point Exercises

1. 608 **2.** \$9209 **3.** $\{3, 7\}$ **4.** $\{3, 4, 5, 6, 7, 8, 9\}$ **5. a.** $\sqrt{9}$ **b.** $0, \sqrt{9}$ **c.** $-9, 0, \sqrt{9}$ **d.** $-9, -1.3, 0, 0.\overline{3}, \sqrt{9}$ **e.** $\frac{\pi}{2}, \sqrt{10}$ **f.** $-9, -1.3, 0, 0.\overline{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}$ **6. a.** $\sqrt{2} - 1$ **b.** $\pi - 3$ **c.** 1 **7.** 9 **8.** $38x^2 + 23x$ **9.** $42 - 4x$

Concept and Vocabulary Check

1. expression **2.** b to the nth power; base; exponent **3.** formula; modeling; models **4.** intersection; $A \cap B$ **5.** union; $A \cup B$ **6.** natural **7.** whole **8.** integers **9.** rational **10.** irrational **11.** rational; irrational **12.** absolute value; x; $-x$ **13.** $b + a$; ba **14.** $a + (b + c)$; $(ab)c$ **15.** $ab + ac$ **16.** 0; inverse; 0; identity **17.** inverse; 1; identity **18.** simplified **19.** a

Exercise Set P.1

1. 57 **3.** 10 **5.** 88 **7.** 10 **9.** 44 **11.** 46 **13.** 10 **15.** -8 **17.** 10°C **19.** 60 ft **21.** $\{2, 4\}$ **23.** $\{s, e, t\}$ **25.** $\varnothing$ **27.** $\varnothing$ **29.** $\{1, 2, 3, 4, 5\}$ **31.** $\{1, 2, 3, 4, 5, 6, 7, 8, 10\}$ **33.** $\{a, e, i, o, u\}$ **35. a.** $\sqrt{100}$ **b.** $0, \sqrt{100}$ **c.** $-9, 0, \sqrt{100}$ **d.** $-9, -\frac{4}{5}, 0, 0.25, 9.2, \sqrt{100}$ **e.** $\sqrt{3}$ **f.** $-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}$ **37. a.** $\sqrt{64}$ **b.** $0, \sqrt{64}$ **c.** $-11, 0, \sqrt{64}$ **d.** $-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}$ **e.** $\sqrt{5}, \pi$ **f.** $-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}$ **39.** 0 **41.** Answers may vary; an example is 2. **43.** true **45.** true **47.** true **49.** true **51.** 300 **53.** $12 - \pi$ **55.** $5 - \sqrt{2}$ **57.** -1 **59.** 4 **61.** 3 **63.** 7 **65.** -1 **67.** $|17 - 2|$; 15 **69.** $|5 - (-2)|$; 7 **71.** $|-4 - (-19)|$; 15 **73.** $|-1.4 - (-3.6)|$; 2.2 **75.** commutative property of addition **77.** associative property of addition **79.** commutative property of addition **81.** distributive property of multiplication over addition **83.** inverse property of multiplication **85.** $15x + 16$ **87.** $27x - 10$ **89.** $29y - 29$ **91.** $8y - 12$ **93.** $16y - 25$ **95.** $12x^2 + 11$ **97.** $14x$ **99.** $-2x + 3y + 6$ **101.** x **103.** $>$ **105.** $=$ **107.** $<$ **109.** $=$ **111.** 45 **113.** $\frac{1}{121}$ **115.** 14 **117.** $-\frac{8}{3}$ **119.** $-\frac{1}{2}$ **121.** $x - (x + 4)$; -4 **123.** $6(-5x)$; $-30x$ **125.** $5x - 2x$; $3x$ **127.** $8x - (3x + 6)$; $5x - 6$ **129. a.** 140 beats per minute **b.** 160 beats per minute **131. a.** \$26,317 **b.** overestimates by \$44 **c.** \$30,568 **133. a.** $1200 - 0.07x$ **b.** \$780 **145.** does not make sense **147.** does not make sense **149.** false **151.** false **153.** false **155.** true **157.** $>$ **159. a.** b^7 **b.** b^{10} **c.** Add the exponents. **160. a.** b^4 **b.** b^6 **c.** Subtract the exponents. **161.** It moves the decimal point 3 places to the right.

Section P.2

Check Point Exercises

1. a. $16x^{12}y^{24}$ **b.** $-18x^3y^8$ **c.** $\frac{5y^6}{x^4}$ **d.** $\frac{y^8}{25x^2}$ **2. a.** $-2{,}600{,}000{,}000$ **b.** 0.000003017 **3. a.** 5.21×10^9 **b.** -6.893×10^{-8} **4.** 4.1×10^9 **5. a.** 3.55×10^{-1} **b.** 4×10^8 **6.** \$8300 **7.** $2.5344 \times 10^3 = 2534.4$ centimeters per second

Concept and Vocabulary Check

1. b^{m+n}; add **2.** b^{m-n}; subtract **3.** 1 **4.** $\frac{1}{b^n}$ **5.** false **6.** b^n **7.** true **8.** a number greater than or equal to 1 and less than 10; integer **9.** true **10.** false

Exercise Set P.2

1. 50 **3.** 64 **5.** -64 **7.** 1 **9.** -1 **11.** $\frac{1}{64}$ **13.** 32 **15.** 64 **17.** 16 **19.** $\frac{1}{9}$ **21.** $\frac{1}{16}$ **23.** $\frac{y}{x^2}$ **25.** y^5 **27.** x^{10} **29.** x^5 **31.** x^{21} **33.** $\frac{1}{x^{15}}$ **35.** x^7 **37.** x^{21} **39.** $64x^6$ **41.** $-\frac{64}{x^3}$ **43.** $9x^4y^{10}$ **45.** $6x^{11}$ **47.** $18x^9y^5$ **49.** $4x^{16}$ **51.** $-5a^{11}b$ **53.** $\frac{2}{b^7}$ **55.** $\frac{1}{16x^6}$ **57.** $\frac{3y^{14}}{4x^4}$ **59.** $\frac{y^2}{25x^6}$ **61.** $-\frac{27b^{15}}{a^{18}}$ **63.** 1 **65.** 380 **67.** 0.0006 **69.** $-7{,}160{,}000$ **71.** 0.79 **73.** -0.00415 **75.** $-60{,}000{,}100{,}000$ **77.** 3.2×10^4 **79.** 6.38×10^{17} **81.** -5.716×10^3 **83.** 2.7×10^{-3} **85.** -5.04×10^{-9} **87.** 6.3×10^7 **89.** 6.4×10^4 **91.** 1.22×10^{-11} **93.** 2.67×10^{13} **95.** 2.1×10^3 **97.** 4×10^5 **99.** 2×10^{-8} **101.** 5×10^3 **103.** 4×10^{15} **105.** 9×10^{-3} **107.** 1 **109.** $\frac{y}{16x^8z^6}$ **111.** $\frac{1}{x^{12}y^{16}z^{20}}$ **113.** $\frac{x^{18}y^6}{4}$ **115. a.** 2.17×10^{12} **b.** 3.09×10^8 **c.** \$7023

117. $\$1.0586 \times 10^{10}$ **119.** 1.06×10^{-18} g **121.** 4.064×10^{9} **133.** makes sense **135.** makes sense **137.** true **139.** false **141.** false **143.** true **145.** $A = C + D$ **148. a.** 8 **b.** 8 **c.** $\sqrt{16} \cdot \sqrt{4} = \sqrt{16 \cdot 4}$ **149. a.** 17.32 **b.** 17.32 **c.** $\sqrt{300} = 10\sqrt{3}$ **150. a.** $31x$ **b.** $31\sqrt{2}$

Section P.3

Check Point Exercises

1. a. 9 **b.** -3 **c.** $\frac{1}{5}$ **d.** 10 **e.** 14 **2. a.** $5\sqrt{3}$ **b.** $5x\sqrt{2}$ **3. a.** $\frac{5}{4}$ **b.** $5x\sqrt{3}$ **4. a.** $17\sqrt{13}$ **b.** $-19\sqrt{17x}$ **5. a.** $17\sqrt{3}$ **b.** $10\sqrt{2x}$ **6. a.** $\frac{5\sqrt{3}}{3}$ **b.** $\sqrt{3}$ **7.** $\frac{8(4-\sqrt{5})}{11}$ or $\frac{32-8\sqrt{5}}{11}$ **8. a.** $2\sqrt[3]{5}$ **b.** $2\sqrt[5]{2}$ **c.** $\frac{5}{3}$ **9.** $5\sqrt[3]{3}$ **10. a.** 5 **b.** 2 **c.** -3 **d.** -2 **e.** $\frac{1}{3}$ **11. a.** 81 **b.** 8 **c.** $\frac{1}{4}$ **12. a.** $10x^4$ **b.** $4x^{5/2}$ **13.** $\sqrt{x}$

Concept and Vocabulary Check

1. principal **2.** 8^2 **3.** $|a|$ **4.** $\sqrt{a} \cdot \sqrt{b}$ **5.** $\frac{\sqrt{a}}{\sqrt{b}}$ **6.** $18\sqrt{3}$ **7.** $5; 6\sqrt{3}$ **8.** $7 - \sqrt{3}$ **9.** $\sqrt{10} + \sqrt{2}$ **10.** index; radicand **11.** $(-2)^5$ **12.** $a; |a|$ **13.** $\sqrt[n]{a}$ **14.** 2; 8

Exercise Set P.3

1. 6 **3.** -6 **5.** not a real number **7.** 3 **9.** 1 **11.** 13 **13.** $5\sqrt{2}$ **15.** $3|x|\sqrt{5}$ **17.** $2x\sqrt{3}$ **19.** $x\sqrt{x}$ **21.** $2x\sqrt{3x}$ **23.** $\frac{1}{9}$ **25.** $\frac{7}{4}$ **27.** $4x$ **29.** $5x\sqrt{2x}$ **31.** $2x^2\sqrt{5}$ **33.** $13\sqrt{3}$ **35.** $-2\sqrt{17x}$ **37.** $5\sqrt{2}$ **39.** $3\sqrt{2x}$ **41.** $34\sqrt{2}$ **43.** $20\sqrt{2} - 5\sqrt{3}$ **45.** $\frac{\sqrt{7}}{7}$ **47.** $\frac{\sqrt{10}}{5}$ **49.** $\frac{13(3-\sqrt{11})}{-2}$ **51.** $7(\sqrt{5}+2)$ **53.** $3(\sqrt{5}-\sqrt{3})$ **55.** 5 **57.** -2 **59.** not a real number **61.** 3 **63.** -3 **65.** $-\frac{1}{2}$ **67.** $2\sqrt[3]{4}$ **69.** $x\sqrt[3]{x}$ **71.** $3\sqrt[3]{2}$ **73.** $2x$ **75.** $7\sqrt[5]{2}$ **77.** $13\sqrt[3]{2}$ **79.** $-y\sqrt[3]{2x}$ **81.** $\sqrt{2}+2$ **83.** 6 **85.** 2 **87.** 25 **89.** $\frac{1}{16}$ **91.** $14x^{7/12}$ **93.** $4x^{1/4}$ **95.** x^2 **97.** $5x^2|y|^3$ **99.** $27y^{2/3}$ **101.** $\sqrt{5}$ **103.** x^2 **105.** $\sqrt[3]{x^2}$ **107.** $\sqrt[3]{x^2y}$ **109.** 3 **111.** $\frac{x^2}{7y^{3/2}}$ **113.** $\frac{x^3}{y^2}$ **115.** Paige Fox is bad at math. **117.** $\frac{\sqrt{5}+1}{2}$; 1.62 to 1 **119.** $P = 18\sqrt{5}$ ft; $A = 100$ ft^2 **129.** does not make sense **131.** does not make sense **133.** false **135.** false **137.** Let $\square = 3$. **139.** 4 **141. a.** 8 **b.** $\frac{1}{4}$ **142.** $10x^7y^9$ **143.** $16x^8 + 6x^5$ **144.** $2x^3 + 11x^2 + 22x + 15$

Section P.4

Check Point Exercises

1. a. $-x^3 + x^2 - 8x - 20$ **b.** $20x^3 - 11x^2 - 2x - 8$ **2.** $15x^3 - 31x^2 + 30x - 8$ **3.** $28x^2 - 41x + 15$ **4. a.** $21x^2 - 25xy + 6y^2$ **b.** $4x^2 + 16xy + 16y^2$ **5. a.** $9x^2 + 12x + 4 - 25y^2$ **b.** $4x^2 + 4xy + y^2 + 12x + 6y + 9$

Concept and Vocabulary Check

1. whole **2.** standard **3.** monomial **4.** binomial **5.** trinomial **6.** n **7.** like **8.** distributive; $4x^5 - 8x^2 + 6; 7x^3$ **9.** $5x$; 3; like **10.** $3x^2; 5x; 21x; 35$ **11.** $A^2 - B^2$; minus **12.** $A^2 + 2AB + B^2$; squared; product of the terms; squared **13.** $A^2 - 2AB + B^2$; minus; product of the terms; plus **14.** $n + m$

Exercise Set P.4

1. yes; $3x^2 + 2x - 5$ **3.** no **5.** 2 **7.** 4 **9.** $11x^3 + 7x^2 - 12x - 4; 3$ **11.** $12x^3 + 4x^2 + 12x - 14; 3$ **13.** $6x^2 - 6x + 2; 2$ **15.** $x^3 + 1$ **17.** $2x^3 - 9x^2 + 19x - 15$ **19.** $x^2 + 10x + 21$ **21.** $x^2 - 2x - 15$ **23.** $6x^2 + 13x + 5$ **25.** $10x^2 - 9x - 9$ **27.** $15x^4 - 47x^2 + 28$ **29.** $8x^5 - 40x^3 + 3x^2 - 15$ **31.** $x^2 - 9$ **33.** $9x^2 - 4$ **35.** $25 - 49x^2$ **37.** $16x^4 - 25x^2$ **39.** $1 - y^{10}$ **41.** $x^2 + 4x + 4$ **43.** $4x^2 + 12x + 9$ **45.** $x^2 - 6x + 9$ **47.** $16x^4 - 8x^2 + 1$ **49.** $4x^2 - 28x + 49$ **51.** $x^3 + 3x^2 + 3x + 1$ **53.** $8x^3 + 36x^2 + 54x + 27$ **55.** $x^3 - 9x^2 + 27x - 27$ **57.** $27x^3 - 108x^2 + 144x - 64$ **59.** $7x^2 + 38xy + 15y^2$ **61.** $2x^2 + xy - 21y^2$ **63.** $15x^2y^2 + xy - 2$ **65.** $49x^2 + 70xy + 25y^2$ **67.** $x^4y^4 - 6x^2y^2 + 9$ **69.** $x^3 - y^3$ **71.** $9x^2 - 25y^2$ **73.** $x^2 + 2xy + y^2 - 9$ **75.** $9x^2 + 42x + 49 - 25y^2$ **77.** $25y^2 - 4x^2 - 12x - 9$ **79.** $x^2 + 2xy + y^2 + 2x + 2y + 1$ **81.** $4x^2 + 4xy + y^2 + 4x + 2y + 1$ **83.** $48xy$ **85.** $-9x^2 + 3x + 9$ **87.** $16x^4 - 625$ **89.** $4x^2 - 28x + 49$ **91. a.** \$54,647; overestimates by \$556 **b.** $M - W = 1.8x^3 - 82x^2 + 2644x - 11{,}449$ **c.** \$14,434 **d.** \$15,136; underestimates by \$702 **93.** $4x^3 - 36x^2 + 80x$ **95.** $6x + 22$ **103.** makes sense **105.** makes sense **107.** $x^2 + 2x$ **109.** $2x^3 + 12x^2 + 12x + 10$ **111.** 4 **112.** 2 **113.** 3

Section P.5

Check Point Exercises

1. a. $2x^2(5x - 2)$ **b.** $(x - 7)(2x + 3)$ **2.** $(x + 5)(x^2 - 2)$ **3.** $(x + 8)(x + 5)$ or $(x + 5)(x + 8)$ **4.** $(x - 7)(x + 2)$ or $(x + 2)(x - 7)$ **5.** $(3x - 1)(2x + 7)$ or $(2x + 7)(3x - 1)$ **6.** $(3x - y)(x - 4y)$ or $(x - 4y)(3x - y)$ **7. a.** $(x + 9)(x - 9)$ **b.** $(6x + 5)(6x - 5)$ **8.** $(9x^2 + 4)(3x + 2)(3x - 2)$ **9. a.** $(x + 7)^2$ **b.** $(4x - 7)^2$ **10. a.** $(x + 1)(x^2 - x + 1)$ **b.** $(5x - 2)(25x^2 + 10x + 4)$ **11.** $3x(x - 5)^2$ **12.** $(x + 10 + 6a)(x + 10 - 6a)$ **13.** $\frac{2x - 1}{(x - 1)^{1/2}}$

Concept and Vocabulary Check

1. d **2.** g **3.** b **4.** c **5.** e **6.** a **7.** f **8.** $(x+1)^{1/2}$

Exercise Set P.5

1. $9(2x+3)$ **3.** $3x(x+2)$ **5.** $9x^2(x^2-2x+3)$ **7.** $(x+5)(x+3)$ **9.** $(x-3)(x^2+12)$ **11.** $(x-2)(x^2+5)$ **13.** $(x-1)(x^2+2)$
15. $(3x-2)(x^2-2)$ **17.** $(x+2)(x+3)$ **19.** $(x-5)(x+3)$ **21.** $(x-5)(x-3)$ **23.** $(3x+2)(x-1)$ **25.** $(3x-28)(x+1)$
27. $(2x-1)(3x-4)$ **29.** $(2x+3)(2x+5)$ **31.** $(3x-2)(3x-1)$ **33.** $(5x+8)(4x-1)$ **35.** $(2x+y)(x+y)$ **37.** $(3x+2y)(2x-3y)$
39. $(x+10)(x-10)$ **41.** $(6x+7)(6x-7)$ **43.** $(3x+5y)(3x-5y)$ **45.** $(x^2+4)(x+2)(x-2)$ **47.** $(4x^2+9)(2x+3)(2x-3)$
49. $(x+1)^2$ **51.** $(x-7)^2$ **53.** $(2x+1)^2$ **55.** $(3x-1)^2$ **57.** $(x+3)(x^2-3x+9)$ **59.** $(x-4)(x^2+4x+16)$
61. $(2x-1)(4x^2+2x+1)$ **63.** $(4x+3)(16x^2-12x+9)$ **65.** $3x(x+1)(x-1)$ **67.** $4(x+2)(x-3)$ **69.** $2(x^2+9)(x+3)(x-3)$
71. $(x-3)(x+3)(x+2)$ **73.** $2(x-8)(x+7)$ **75.** $x(x-2)(x+2)$ **77.** prime **79.** $(x-2)(x+2)^2$ **81.** $y(y^2+9)(y+3)(y-3)$
83. $5y^2(2y+3)(2y-3)$ **85.** $(x-6+7y)(x-6-7y)$ **87.** $(x+y)(3b+4)(3b-4)$ **89.** $(y-2)(x+4)(x-4)$
91. $2x(x+6+2a)(x+6-2a)$ **93.** $x^{1/2}(x-1)$ **95.** $\dfrac{4(1+2x)}{x^{2/3}}$ **97.** $-(x+3)^{1/2}(x+2)$ **99.** $\dfrac{x+4}{(x+5)^{3/2}}$ **101.** $-\dfrac{4(4x-1)^{1/2}(x-1)}{3}$
103. $(x+1)(5x-6)(2x+1)$ **105.** $(x^2+6)(6x^2-1)$ **107.** $y(y^2+1)(y^4-y^2+1)$ **109.** $(x+2y)(x-2y)(x+y)(x-y)$
111. $(x-y)^2(x-y+2)(x-y-2)$ **113.** $(2x-y^2)(x-3y^2)$ **115. a.** $(x-0.4x)(1-0.4)=(0.6x)(0.6)=0.36x$ **b.** no; 36%
117. a. $9x^2-16$ **b.** $(3x+4)(3x-4)$ **119. a.** $x(x+y)-y(x+y)$ **b.** $(x+y)(x-y)$ **121.** $4a^3-4ab^2=4a(a+b)(a-b)$
131. makes sense **133.** makes sense **135.** true **137.** false **139.** $-(x+5)(x-1)$ **141.** $-\dfrac{10}{(x-5)^{3/2}(x+5)^{1/2}}$
143. $b=0,3,4,-c(c+4)$, where $c>0$ is an integer **144.** $\dfrac{(x+5)(x+1)}{(x+5)(x-5)}=\dfrac{x+1}{x-5}$ **145.** $\dfrac{2}{3}$ **146.** $\dfrac{7}{6}$

Mid-Chapter P Check Point

1. $12x^2-x-35$ **2.** $-x+12$ **3.** $10\sqrt{6}$ **4.** $3\sqrt{3}$ **5.** $x+45$ **6.** $64x^2-48x+9$ **7.** $\dfrac{x^2}{y^3}$ **8.** $\dfrac{3}{4}$ **9.** $-x^2+5x-6$
10. $2x^3-11x^2+17x-5$ **11.** $-x^6+2x^3$ **12.** $18a^2-11ab-10b^2$ **13.** $\{a,c,d,e,f,h\}$ **14.** $\{c,d\}$ **15.** $5x^2y^3+2xy-y^2$
16. $-\dfrac{12y^{15}}{x^3}$ **17.** $\dfrac{6y^3}{x^7}$ **18.** $\sqrt[3]{x}$ **19.** $16y^2-9x^2-12x-4$ **20.** $x^2-4xy+4y^2-2x+4y+1$ **21.** 1.2×10^{-2} **22.** $2\sqrt[3]{2}$
23. x^6-4 **24.** x^4+4x^2+4 **25.** $10\sqrt{3}$ **26.** $\dfrac{77+11\sqrt{3}}{46}$ **27.** $\dfrac{11\sqrt{3}}{3}$ **28.** $(7x-1)(x-3)$ **29.** prime **30.** $(x^2+3)(x+5)$
31. $(3x-7y)(x+y)$ **32.** $y(4-y)(16+4y+y^2)$ **33.** $2x(5x+1)^2$ **34.** $(x-3-7y)(x-3+7y)$ **35.** $\dfrac{(1-x)^2}{x^{3/2}}$ **36.** $\dfrac{(x-3)(x+3)}{(x^2+1)^{1/2}}$
37. $-11,-\dfrac{3}{7},0,0.45,\sqrt{25}$ **38.** $\sqrt{13}-2$ **39.** $-x^3$ **40.** 1.38×10^9 pounds **41.** 4 times **42. a.** model 2 **b.** overestimates by \$5

Section P.6

Check Point Exercises

1. a. -5 **b.** $6,-6$ **c.** $-2,7$ **2. a.** $x^2, x\neq-3$ **b.** $\dfrac{x-1}{x+1}, x\neq-1$ **3.** $\dfrac{x-3}{(x-2)(x+3)}, x\neq2, x\neq-2, x\neq-3$
4. $\dfrac{3(x-1)}{x(x+2)}, x\neq1, x\neq0, x\neq-2$ **5.** $-2, x\neq-1$ **6.** $\dfrac{2(4x+1)}{(x+1)(x-1)}, x\neq1, x\neq-1$ **7.** $(x-3)(x-3)(x+3)$ or $(x-3)^2(x+3)$
8. $\dfrac{-x^2+11x-20}{2(x-5)^2}, x\neq5$ **9.** $\dfrac{2(2-3x)}{4+3x}, x\neq0, x\neq-\dfrac{4}{3}$ **10.** $-\dfrac{1}{x(x+7)}, x\neq0, x\neq-7$ **11.** $\dfrac{x+1}{x^{3/2}}$ or $\dfrac{x+1}{\sqrt{x^3}}$ **12.** $\dfrac{1}{\sqrt{x+3}+\sqrt{x}}$

Concept and Vocabulary Check

1. polynomials **2.** domain; 0 **3.** factoring; common factors **4.** $\dfrac{x^2}{15}$ **5.** $\dfrac{3}{5}$ **6.** $\dfrac{x^2-x+4}{3}$ **7.** $x+3$ and $x-2$; $x+3$ and $x+1$; $(x+3)(x-2)(x+1)$ **8.** $3x+4$ **9.** complex; complex **10.** $\dfrac{x-(x+3)}{3x(x+3)}=\dfrac{-3}{3x(x+3)}=-\dfrac{1}{x(x+3)}$ **11.** $\sqrt{x}$ **12.** $\sqrt{x+7}+\sqrt{x}$

Exercise Set P.6

1. 3 **3.** $5,-5$ **5.** $-1,-10$ **7.** $\dfrac{3}{x-3}, x\neq3$ **9.** $\dfrac{x-6}{4}, x\neq6$ **11.** $\dfrac{y+9}{y-1}, y\neq1,2$ **13.** $\dfrac{x+6}{x-6}, x\neq6,-6$ **15.** $\dfrac{1}{3}, x\neq2,-3$
17. $\dfrac{(x-3)(x+3)}{x(x+4)}, x\neq0,-4,3$ **19.** $\dfrac{x-1}{x+2}, x\neq-2,-1,2,3$ **21.** $\dfrac{x^2+2x+4}{3x}, x\neq-2,0,2$ **23.** $\dfrac{7}{9}, x\neq-1$ **25.** $\dfrac{(x-2)^2}{x}, x\neq0,-2,2$
27. $\dfrac{2(x+3)}{3}, x\neq3,-3$ **29.** $\dfrac{x-5}{2}, x\neq1,-5$ **31.** $\dfrac{(x+2)(x+4)}{x-5}, x\neq-6,-3,-1,3,5$ **33.** $2, x\neq-\dfrac{5}{6}$ **35.** $\dfrac{2x-1}{x+3}, x\neq0,-3$
37. $3, x\neq2$ **39.** $\dfrac{3}{x-3}, x\neq3,-4$ **41.** $\dfrac{9x+39}{(x+4)(x+5)}, x\neq-4,-5$ **43.** $-\dfrac{3}{x(x+1)}, x\neq-1,0$ **45.** $\dfrac{3x^2+4}{(x+2)(x-2)}, x\neq-2,2$

47. $\frac{2x^2+50}{(x-5)(x+5)}, x \neq -5, 5$ **49.** $\frac{13}{6(x+2)}, x \neq -2$ **51.** $\frac{4x+16}{(x+3)^2}, x \neq -3$ **53.** $\frac{x^2-x}{(x+5)(x-2)(x+3)}, x \neq -5, 2, -3$

55. $\frac{-x^2-2x+1}{(x+1)(x-1)}, x \neq -1, 1$ **57.** $\frac{x-1}{x+2}, x \neq -2, -1$ **59.** $\frac{1}{3}, x \neq 3$ **61.** $\frac{x+1}{3x-1}, x \neq 0, \frac{1}{3}$ **63.** $\frac{1}{xy}, x \neq 0, y \neq 0, x \neq -y$

65. $\frac{x}{x+3}, x \neq -2, -3$ **67.** $-\frac{x-14}{7}, x \neq -2, 2$ **69.** $\frac{x-3}{x+2}, x \neq -2, -1, 3$ **71.** $-\frac{2x+h}{x^2(x+h)^2}, x \neq 0, h \neq 0, x \neq -h$

73. $1 - \frac{1}{3x}; x > 0$ **75.** $-\frac{2}{x^2\sqrt{x^2+2}}$ **77.** $\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$ **79.** $\frac{1}{\sqrt{x+5}+\sqrt{x}}$ **81.** $\frac{1}{(x+y)(\sqrt{x}-\sqrt{y})}$ **83.** $\frac{x^2+5x+8}{(x+2)(x+1)}$

85. 2 **87.** $\frac{1}{y(y+5)}$ **89.** $\frac{2d}{a^2+ab+b^2}$ **91. a.** 86.67, 520, 1170; It costs \$86,670,000 to inoculate 40% of the population against this strain of flu, \$520,000,000 to inoculate 80% of the population, and \$1,170,000,000 to inoculate 90% of the population. **b.** $x = 100$ **c.** The cost increases rapidly; it is impossible to inoculate 100% of the population. **93. a.** 2078; underestimates by 22 calories **b.** 2662; underestimates by 38 calories **c.** $\frac{-33x^2+263x+515}{-60x^2+499x+295}$ **95.** $\frac{4x^2+14x}{(x+3)(x+4)}$ **109.** does not make sense **111.** does not make sense **113.** false **115.** true

117. $\frac{1}{x^{2n}-1}$ **119.** $\frac{x-y+1}{(x-y)(x-y)}$ **121.** true **122.** $-x+10$ **123.** -5

Section P.7

Check Point Exercises

1. $\{5\}$ **2.** $\{1\}$ **3.** $\{7\}$ **4.** $\varnothing$ **5.** $q = \frac{pf}{p-f}$ **6.** $\{-2, 3\}$ **7. a.** $\{0, 3\}$ **b.** $\left\{\frac{1}{2}, -1\right\}$ **8. a.** $\{-\sqrt{7}, \sqrt{7}\}$ **b.** $\{-5+\sqrt{11}, -5-\sqrt{11}\}$ **9.** $\{-2+\sqrt{5}, -2-\sqrt{5}\}$ **10.** $\left\{\frac{-1+\sqrt{3}}{2}, \frac{-1-\sqrt{3}}{2}\right\}$ **11.** -56; no real solutions **12.** $\{6\}$

Concept and Vocabulary Check

1. linear **2.** equivalent **3.** apply the distributive property **4.** least common denominator; 12 **5.** 0 **6.** $x \neq 2; x \neq -4$ **7.** $5(x+3)+3(x+4) = 12x+9$ **8.** isolated on one side **9.** factoring **10.** $c; -c$ **11.** $3x-1=7; 3x-1=-7$ **12.** quadratic **13.** $A = 0$ or $B = 0$ **14.** $\pm\sqrt{d}$ **15.** $\pm\sqrt{7}$ **16.** 9 **17.** $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ **18.** $2; 9; -5$ **19.** $1; -4; -1$ **20.** $2 \pm \sqrt{2}$ **21.** b^2-4ac **22.** no **23.** two **24.** the square root property **25.** the quadratic formula **26.** factoring and the zero-product principle **27.** radical **28.** extraneous **29.** $2x+1; x^2-14x+49$

Exercise Set P.7

1. $\{11\}$ **3.** $\{7\}$ **5.** $\{13\}$ **7.** $\{2\}$ **9.** $\{9\}$ **11.** $\left\{\frac{33}{2}\right\}$ **13.** $\{-12\}$ **15.** $\left\{\frac{46}{5}\right\}$ **17. a.** 1 **b.** $\{3\}$ **19. a.** -1 **b.** $\varnothing$ **21. a.** 1 **b.** $\{2\}$ **23. a.** $-1, 1$ **b.** $\{-3\}$ **25. a.** $-2, 4$ **b.** $\varnothing$ **27.** $P = \frac{I}{rt}$ **29.** $p = \frac{T-D}{m}$ **31.** $a = \frac{2A}{h} - b$ **33.** $r = \frac{S-P}{Pt}$ **35.** $S = \frac{F}{B} + V$ **37.** $I = \frac{E}{R+r}$ **39.** $f = \frac{pq}{p+q}$ **41.** $f_1 = -\frac{ff_2}{f-f_2}$ or $f_1 = \frac{ff_2}{f_2-f}$ **43.** $\{-5, 9\}$ **45.** $\{-2, 3\}$ **47.** $\left\{-\frac{5}{3}, 3\right\}$ **49.** $\left\{-\frac{4}{5}, 4\right\}$ **51.** $\varnothing$ **53.** $\left\{\frac{1}{2}\right\}$ **55.** $\{-2, 5\}$ **57.** $\{3, 5\}$ **59.** $\{0, 4\}$ **61.** $\{\pm 3\}$ **63.** $\{\pm\sqrt{10}\}$ **65.** $\{4 \pm \sqrt{5}\}$ **67.** $\{-7, 1\}$ **69.** $\{1+\sqrt{3}, 1-\sqrt{3}\}$ **71.** $\{3+2\sqrt{5}, 3-2\sqrt{5}\}$ **73.** $\{-2+\sqrt{3}, -2-\sqrt{3}\}$ **75.** $\{-5, -3\}$ **77.** $\left\{\frac{-5+\sqrt{13}}{2}, \frac{-5-\sqrt{13}}{2}\right\}$ **79.** $\left\{\frac{3+\sqrt{57}}{6}, \frac{3-\sqrt{57}}{6}\right\}$ **81.** $\left\{\frac{1+\sqrt{29}}{4}, \frac{1-\sqrt{29}}{4}\right\}$ **83.** 36; 2 unequal real solutions **85.** 97; 2 unequal real solutions **87.** 0; 1 real solution **89.** 37; 2 unequal real solutions **91.** $\left\{-\frac{1}{2}, 1\right\}$ **93.** $\left\{\frac{1}{5}, 2\right\}$ **95.** $\{-2\sqrt{5}, 2\sqrt{5}\}$ **97.** $\{1+\sqrt{2}, 1-\sqrt{2}\}$ **99.** $\left\{\frac{-11+\sqrt{33}}{4}, \frac{-11-\sqrt{33}}{4}\right\}$ **101.** $\left\{0, \frac{8}{3}\right\}$ **103.** $\{2\}$ **105.** $\{-2, 2\}$ **107.** $\{2 \pm \sqrt{2}\}$ **109.** $\left\{0, \frac{7}{2}\right\}$ **111.** $\{2+\sqrt{10}, 2-\sqrt{10}\}$ **113.** $\{-5, -1\}$ **115.** $\{6\}$ **117.** $\{6\}$ **119.** $\{-6\}$ **121.** $\{10\}$ **123.** $\{-5\}$ **125.** $\{-2\}$ **127.** $\{-3, 1\}$ **129.** $\{-8, -6, 4, 6\}$ **131.** $\left\{\frac{-1 \pm \sqrt{21}}{2}\right\}$ **133.** $\{8\}$ **135.** $\frac{-2-\sqrt{22}}{2}$ and $\frac{-2+\sqrt{22}}{2}$ **137. a.** underestimates by 2% **b.** 2030 **139.** 125 liters **141.** 33-year-olds and 58-year-olds; The formula models the actual data reasonably well. **143. a.** 48% ± 1% **b.** 47.8% **c.** 2024 **159.** does not make sense **161.** does not make sense **163.** false **165.** true **167.** 2 **169.** $C = \frac{LV-SN}{L-N}$ **171.** $x+150$ **172.** $20+0.05x$ **173.** $4x+400$

Section P.8

Check Point Exercises

1. computer science: \$56 thousand; economics: \$49 thousand; education: \$35 thousand **2.** by 67 years after 1969, or in 2036 **3.** \$1200 **4.** 50 ft by 94 ft **5.** 2 ft **6.** 120 yd **7.** 5 people

Concept and Vocabulary Check

1. $x + 658.6$ **2.** $31 + 2.4x$ **3.** $x - 0.15x$ or $0.85x$ **4.** $x + 5; 2(x + 5) + 2x; x(x + 5)$ **5.** right; hypotenuse; legs
6. right; legs; the square of the length of the hypotenuse **7.** $\frac{10{,}000}{x}; \frac{10{,}000}{x + 2}$

Exercise Set P.8

1. TV: 9 years; sleeping: 28 years **3.** bachelor's: \$55 thousand; master's: \$61 thousand **5.** by 2015 **7. a.** $y = 24{,}000 - 3000x$ **b.** after 5 years
9. 2019; 22,300 students **11.** \$420 **13.** \$150 **15.** \$467.20 **17.** 50 yd by 100 yd **19.** 36 ft by 78 ft **21.** 2 in.
23. length: 9 ft; width: 6 ft **25.** 5 in. **27.** 5 m **29.** 3 ft **31.** 13.2 ft **33.** 13 ft **35.** 21.9 yd **37.** 8 people
39. car: 50 miles per hour; bus: 30 miles per hour **41.** 6 miles per hour **43.** 11 hr **45.** 5 ft 7 in. **47.** 10 **53.** does not make sense
55. does not make sense **57.** 3 miles, 4 miles, 5 miles **59.** Coburn = 60 years old; woman = 20 years old **61.** \$4000 for the mother; \$8000 for the boy; \$2000 for the girl **64.** yes **65.** $\{-3\}$ **66.** $\{14\}$

Section P.9

Check Point Exercises

1. a. $\{x|-2 \le x < 5\}$

b. $\{x|1 \le x \le 3.5\}$

c. $\{x|x < -1\}$

2. a. $(2, 3]$ **b.** $[1, 6)$

3. $[-1, \infty)$ or $\{x|x \ge -1\}$

4. $\{x|x < 4\}$ or $(-\infty, 4)$

5. $\{x|x \ge 13\}$ or $[13, \infty)$

6. $[-1, 4)$ or $\{x|-1 \le x < 4\}$

7. $(-3, 7)$ or $\{x|-3 < x < 7\}$

8. $\left\{x \middle| -\frac{11}{5} \le x \le 3\right\}$ or $\left[-\frac{11}{5}, 3\right]$

9. $\{x|x < -4 \text{ or } x > 8\}$ or $(-\infty, -4) \cup (8, \infty)$

10. more than 720 mi per week

Concept and Vocabulary Check

1. 2; 5; 2; 5 **2.** greater than **3.** less than or equal to **4.** $(-\infty, 9)$; intersection **5.** $(-\infty, 12)$; union **6.** adding 4; dividing; -3; direction; $>$; $<$
7. middle **8.** $-c; c$ **9.** $-c; c$ **10.** $-2 < x - 7 < 2$ **11.** $x - 7 < -2$ or $x - 7 > 2$

Exercise Set P.9

1. $\{x|1 < x \le 6\}$

3. $\{x|-5 \le x < 2\}$

5. $\{x|-3 \le x \le 1\}$

7. $\{x|x > 2\}$

9. $\{x|x \ge -3\}$

11. $\{x|x < 3\}$

13. $\{x|x < 5.5\}$

15. $[-1, 0)$

17. $(-3, 2]$ **19.** $[1, 5)$ **21.** $(-\infty, 8)$ **23.** $(6, \infty)$ **25.** $[3, \infty)$ **27.** $(-\infty, 3)$

29. $\left[\frac{20}{3}, \infty\right)$

31. $(-\infty, -4]$

33. $\left(-\infty, -\frac{2}{5}\right]$

35. $[0, \infty)$

37. $(-\infty, 1)$

39. $[6, \infty)$

41. $[-10, \infty)$

43. $(-\infty, -6)$

45. $[13, \infty)$

47. $(-\infty, -2)$

49. $(3, 5)$ **51.** $[-1, 3)$ **53.** $(-5, -2]$ **55.** $[3, 6)$

57. $(-3, 3)$ **59.** $[-1, 3]$ **61.** $(-1, 7)$ **63.** $[-5, 3]$ **65.** $(-6, 0)$ **67.** $(-\infty, -3) \cup (3, \infty)$ **69.** $(-\infty, -1] \cup [3, \infty)$ **71.** $\left(-\infty, \frac{1}{3}\right) \cup (5, \infty)$
73. $(-\infty, -5] \cup [3, \infty)$ **75.** $(-\infty, -3) \cup (12, \infty)$ **77.** $(-\infty, -1] \cup [3, \infty)$ **79.** $[2, 6]$ **81.** $(-\infty, -3) \cup (5, \infty)$ **83.** $(-\infty, -1] \cup [2, \infty)$
85. $(-1, 9)$ **87.** $\left(-\infty, \frac{1}{3}\right) \cup (1, \infty)$ **89.** $\left(-\infty, -\frac{75}{14}\right) \cup \left(\frac{87}{14}, \infty\right)$ **91.** $(-\infty, -6] \cup [24, \infty)$ **93.** $[6, \infty)$ **95.** $(-\infty, -10] \cup [2, \infty)$
97. $\left(-\infty, -\frac{1}{3}\right] \cup [3, \infty)$ **99.** $(0, 4)$ **101.** intimacy $\ge$ passion or passion $\le$ intimacy **103.** commitment $>$ passion or passion $<$ commitment

105. 9; after 3 years **107. a.** years after 2016 **b.** years after 2020 **c.** years after 2020 **d.** years after 2016 **109.** between 80 and 110 minutes, inclusive **111.** $h \le 41$ or $h \ge 59$ **113.** $15 + 0.08x < 3 + 0.12x$; more than 300 texts **115.** $2 + 0.08x < 8 + 0.05x$; 199 checks or fewer

117. $5.50x > 3000 + 3x$; more than 1200 packages **119.** $245 + 95x \le 3000$; at most 29 bags **121. a.** $\frac{86 + 88 + x}{3} \ge 90$; at least a 96 **b.** $\frac{86 + 88 + x}{3} < 80$; a grade less than 66 **123.** more than 3 and fewer than 15 crossings per three-month period

133. makes sense **135.** makes sense **137.** false **139.** true **141. a.** $|x - 4| < 3$ **b.** $|x - 4| \ge 3$ **143.** 7; 6; 5; 4; 3; 2; 1 **144.** -5; 0; 3; 4; 3; 0; -5 **145.** 3; 2; 1; 0; 1; 2; 3

Chapter P Review Exercises

1. 51 **2.** 16 **3.** 124 ft **4.** $\{a, c\}$ **5.** $\{a, b, c, d, e\}$ **6.** $\{a, b, c, d, f, g\}$ **7.** $\{a\}$ **8. a.** $\sqrt{81}$ **b.** $0, \sqrt{81}$ **c.** $-17, 0, \sqrt{81}$ **d.** $-17, -\frac{9}{13}, 0, 0.75, \sqrt{81}$ **e.** $\sqrt{2}, \pi$ **f.** $-17, -\frac{9}{13}, 0, 0.75, \sqrt{2}, \pi, \sqrt{81}$ **9.** 103 **10.** $\sqrt{2} - 1$ **11.** $\sqrt{17} - 3$ **12.** $|4 - (-17)|$; 21

13. commutative property of addition **14.** associative property of multiplication **15.** distributive property of multiplication over addition **16.** commutative property of multiplication **17.** commutative property of multiplication **18.** commutative property of addition

19. $17x - 15$ **20.** $2x$ **21.** $5y - 17$ **22.** $10x$ **23.** 55; It's the same. **24.** -108 **25.** $\frac{5}{16}$ **26.** $\frac{1}{25}$ **27.** $\frac{1}{27}$ **28.** $-8x^{12}y^9$

29. $\frac{10}{x^8}$ **30.** $\frac{1}{16x^{12}}$ **31.** $\frac{y^8}{4x^{10}}$ **32.** 37,400 **33.** 0.0000745 **34.** 3.59×10^6 **35.** 7.25×10^{-3} **36.** 390,000 **37.** 0.023

38. 1.35×10^{12} **39.** 3.2×10^7 **40.** 42,188 years **41.** $10\sqrt{3}$ **42.** $2|x|\sqrt{3}$ **43.** $2x\sqrt{5}$ **44.** $r\sqrt{r}$ **45.** $\frac{11}{2}$ **46.** $4x\sqrt{3}$

47. $20\sqrt{5}$ **48.** $16\sqrt{2}$ **49.** $24\sqrt{2} - 8\sqrt{3}$ **50.** $6\sqrt{5}$ **51.** $\frac{\sqrt{6}}{3}$ **52.** $\frac{5(6 - \sqrt{3})}{33}$ **53.** $7(\sqrt{7} + \sqrt{5})$ **54.** 5 **55.** -2

56. not a real number **57.** 5 **58.** $3\sqrt[3]{3}$ **59.** $y\sqrt[3]{y^2}$ **60.** $2\sqrt[4]{5}$ **61.** $13\sqrt[3]{2}$ **62.** $x\sqrt[4]{2}$ **63.** 4 **64.** $\frac{1}{5}$ **65.** 5 **66.** $\frac{1}{3}$ **67.** 16

68. $\frac{1}{81}$ **69.** $20x^{11/12}$ **70.** $3x^{1/4}$ **71.** $25x^4$ **72.** $\sqrt{y}$ **73.** $8x^3 + 10x^2 - 20x - 4$; degree 3 **74.** $8x^4 - 5x^3 + 6$; degree 4

75. $12x^3 + x^2 - 21x + 10$ **76.** $6x^2 - 7x - 5$ **77.** $16x^2 - 25$ **78.** $4x^2 + 20x + 25$ **79.** $9x^2 - 24x + 16$ **80.** $8x^3 + 12x^2 + 6x + 1$

81. $125x^3 - 150x^2 + 60x - 8$ **82.** $-x^2 - 17xy - 3y^2$; degree 2 **83.** $24x^3y^2 + x^2y - 12x^2 + 4$; degree 5 **84.** $3x^2 + 16xy - 35y^2$

85. $9x^2 - 30xy + 25y^2$ **86.** $9x^4 + 12x^2y + 4y^2$ **87.** $49x^2 - 16y^2$ **88.** $a^3 - b^3$ **89.** $3x^2(5x + 1)$ **90.** $(x - 4)(x - 7)$

91. $(3x + 1)(5x - 2)$ **92.** $(8 - x)(8 + x)$ **93.** prime **94.** $3x^2(x - 5)(x + 2)$ **95.** $4x^3(5x^4 - 9)$ **96.** $(x + 3)(x - 3)^2$ **97.** $(4x - 5)^2$

98. $(x^2 + 4)(x + 2)(x - 2)$ **99.** $(y - 2)(y^2 + 2y + 4)$ **100.** $(x + 4)(x^2 - 4x + 16)$ **101.** $3x^2(x - 2)(x + 2)$

102. $(3x - 5)(9x^2 + 15x + 25)$ **103.** $x(x - 1)(x + 1)(x^2 + 1)$ **104.** $(x^2 - 2)(x + 5)$ **105.** $(x + 9 + y)(x + 9 - y)$ **106.** $\frac{16(1 + 2x)}{x^{3/4}}$

107. $(x + 2)(x - 2)(x^2 + 3)^{1/2}(-x^4 + x^2 + 13)$ **108.** $\frac{6(2x + 1)}{x^{3/2}}$ **109.** $x^2, x \ne -2$ **110.** $\frac{x - 3}{x - 6}, x \ne -6, 6$ **111.** $\frac{x}{x + 2}, x \ne -2$

112. $\frac{(x + 3)^3}{(x - 2)^2(x + 2)}, x \ne 2, -2$ **113.** $\frac{2}{x(x + 1)}, x \ne 0, 1, -1, -\frac{1}{3}$ **114.** $\frac{x + 3}{x - 4}, x \ne -3, 4, 2, 8$ **115.** $\frac{1}{x - 3}, x \ne 3, -3$

116. $\frac{4x(x - 1)}{(x + 2)(x - 2)}, x \ne 2, -2$ **117.** $\frac{2x^2 - 3}{(x - 3)(x + 3)(x - 2)}, x \ne 3, -3, 2$ **118.** $\frac{11x^2 - x - 11}{(2x - 1)(x + 3)(3x + 2)}, x \ne \frac{1}{2}, -3, -\frac{2}{3}$ **119.** $\frac{3}{x}, x \ne 0, 2$

120. $\frac{3x}{x - 4}, x \ne 0, 4, -4$ **121.** $\frac{3x + 8}{3x + 10}, x \ne -3, -\frac{10}{3}$ **122.** $\frac{25\sqrt{25 - x^2}}{(5 - x)^2(5 + x)^2}$ **123.** $\{-13\}$ **124.** $\{-3\}$ **125.** $\{-1\}$

126. all real numbers except 1 and -1 **127.** $\{7\}$ **128.** $\{-2, 1\}$ **129.** $\left\{\frac{1}{2}, 5\right\}$ **130.** $\left\{-2, \frac{10}{3}\right\}$ **131.** $\left\{\frac{7 + \sqrt{37}}{6}, \frac{7 - \sqrt{37}}{6}\right\}$

132. $\{-3, 3\}$ **133.** $\{3 \pm 2\sqrt{6}\}$ **134.** $\{4\}$ **135.** $\{2\}$ **136.** $\{2\}$ **137.** $g = \frac{s - vt}{t^2}$ **138.** $P = \frac{A}{1 + rT}$ **139.** no real solutions

140. one repeated real solution **141.** *Scream*: 6; *Scream 2*: 8; *Scream 3*: 9 **142.** by 40 years after 1980, or in 2020 **143.** \$60 **144.** \$10,000 in sales **145.** 44 yd by 126 yd **146.** 2022; 32,100 **147.** length: 5 yd; width: 3 yd **148.** approximately 134 m **149.** 2 in. **150.** 10 people

151. $\{x \mid -3 \le x < 5\}$

152. $\{x \mid x > -2\}$

153. $\{x \mid x \le 0\}$

154. $[-1, 1]$ **155.** $(-2, 3)$ **156.** $[1, 3)$ **157.** $(0, 4)$

158. $[-2, \infty)$

159. $\left[\frac{3}{5}, \infty\right)$

160. $\left(-\infty, -\frac{21}{2}\right)$

161. $(-3, \infty)$

162. $(-\infty, -2]$

163. $(2, 3]$

164. $[-9, 6]$

165. $(-\infty, -6)$ or $(0, \infty)$

166. $(-\infty, -3]$ or $[-2, \infty)$

167. $(-\infty, -5] \cup [1, \infty)$

168. no more than 80 miles per day **169.** $[49\%, 99\%)$

Chapter P Test

1. $6x^2 - 27x$ **2.** $-6x + 17$ **3.** $\{5\}$ **4.** $\{1, 2, 5, a\}$ **5.** $\dfrac{5y^8}{x^6}$ **6.** $3r\sqrt{2}$ **7.** $11\sqrt{2}$ **8.** $\dfrac{3(5 - \sqrt{2})}{23}$ **9.** $2x\sqrt[3]{2x}$

10. $\dfrac{x + 3}{x - 2}, x \neq 2, 1$ **11.** 2.5×10^1 **12.** $2x^3 - 13x^2 + 26x - 15$ **13.** $25x^2 + 30xy + 9y^2$ **14.** $\dfrac{2(x + 3)}{x + 1}, x \neq 3, -1, -4, -3$

15. $\dfrac{x^2 + 2x + 15}{(x + 3)(x - 3)}, x \neq 3, -3$ **16.** $\dfrac{11}{(x - 3)(x - 4)}, x \neq 3, 4$ **17.** $\dfrac{2x}{(x + 2)(x + 1)}$ **18.** $\dfrac{10x}{\sqrt{(x^2 + 5)^3}}$ **19.** $(x - 3)(x - 6)$

20. $(x^2 + 3)(x + 2)$ **21.** $(5x - 3)(5x + 3)$ **22.** $(6x - 7)^2$ **23.** $(y - 5)(y^2 + 5y + 25)$ **24.** $(x + 5 + 3y)(x + 5 - 3y)$ **25.** $\dfrac{2x + 3}{(x + 3)^{3/5}}$

26. $-7, -\dfrac{4}{5}, 0, 0.25, \sqrt{4}, \dfrac{22}{7}$ **27.** commutative property of addition **28.** distributive property of multiplication over addition **29.** 7.6×10^{-4}

30. $\dfrac{1}{243}$ **31.** 1.32×10^{10} **32. a.** 43.08%; overestimates by 0.08% **b.** $R = \dfrac{-0.28n + 47}{0.28n + 53}$ **c.** $\dfrac{2}{3}$; Three women will receive bachelor's degrees for every two men.; It describes the projections exactly. **33.** $\{-1\}$ **34.** $\{-6\}$ **35.** $\{5\}$ **36.** $\left\{-\dfrac{1}{2}, 2\right\}$ **37.** $\left\{\dfrac{1 - 5\sqrt{3}}{3}, \dfrac{1 + 5\sqrt{3}}{3}\right\}$

38. $\{1 - \sqrt{5}, 1 + \sqrt{5}\}$ **39.** $\{7\}$ **40.** $\{2\}$ **41.** $\{6, 12\}$ **42.** $\left\{\dfrac{1}{2}, 3\right\}$ **43.** $\{4\}$

44. $(-\infty, 12]$

45. $\left[\dfrac{21}{8}, \infty\right)$

46. $\left[-7, \dfrac{13}{2}\right)$

47. $\left(-\infty, -\dfrac{5}{3}\right] \cup \left[\dfrac{1}{3}, \infty\right)$

48. $h = \dfrac{3V}{lw}$ **49.** $x = \dfrac{y - y_1}{m} + x_1$ **50.** $a = -\dfrac{Rs}{R - s}$ or $a = \dfrac{Rs}{s - R}$ **51.** 2018 **52.** 2018 **53.** quite well

54. average: 59%; good-looking: 27%; strikingly attractive: 2% **55.** 26 yr; \$33,600 **56.** length: 12 ft; width: 4 ft **57.** 10 ft **58.** \$50

59. 20 people **60.** more than 200 text messages

CHAPTER 1

Section 1.1

Check Point Exercises

1.

2.

$y = 4 - x$

3.

$y = |x + 1|$

4. minimum x-value: -100; maximum x-value: 100; distance between tick marks on x-axis: 50; minimum y-value: -100; maximum y-value: 100; distance between tick marks on y-axis: 10

5. a. x-intercept: -3; y-intercept: 5
b. no x-intercept; y-intercept: 4
c. x-intercept: 0; y-intercept: 0

6. a. 65% **b.** 60% **c.** overestimates by 5

Concept and Vocabulary Check

1. x-axis **2.** y-axis **3.** origin **4.** quadrants; four **5.** x-coordinate; y-coordinate **6.** solution; satisfies
7. x-intercept; zero **8.** y-intercept; zero

Exercise Set 1.1

1.

3.

5.

7.

9.

11.

13.

$y = x^2 - 2$

15.

$y = x - 2$

17.

$y = 2x + 1$

19.

$y = -\frac{1}{2}x$

21.

$y = 2|x|$

23.

$y = |x| + 1$

25.

27.

29. c **31.** b **33.** c **35.** no **37.** (2, 0) **39.** (−2, 4) and (1, 1) **41. a.** 2 **b.** −4 **43. a.** 1, − 2 **b.** 2 **45. a.** −1 **b.** none **47.** $y = 2x + 4$

49. $y = 3 - x^2$

51.

53.

55. a. 20% **b.** 19.5%; underestimates by 0.5% **c.** Answers will vary.; approximately 47% **d.** 46.5%; It's less than the estimate, although answers may vary. **e.** 1990; 14%

57. 8; 1 **59.** about 1.9 **67.** makes sense **69.** does not make sense **71.** false **73.** true **75.** I, III **77.** IV **79.** a **81.** b **83.** b **85.** c **87.** set 1

88.

89. a. 3 **b.** −3 and 3 **c.** all real numbers **d.** all real numbers greater than or equal to 1

Section 1.2

Check Point Exercises

1. domain: {0, 10, 20, 30, 40}; range: {9.1, 6.7, 10.7, 13.2, 21.2} **2. a.** not a function **b.** function **3. a.** $y = 6 - 2x$; function **b.** $y = \pm\sqrt{1 - x^2}$; not a function **4. a.** 42 **b.** $x^2 + 6x + 15$ **c.** $x^2 + 2x + 7$

5.

; The graph of g is the graph of f shifted down by 3 units.

6. a. function **b.** not a function **c.** function **d.** not a function **7. a.** 400 **b.** 9 **c.** approximately 425

8. a. domain: $\{x|-2 \le x \le 1\}$ or $[-2, 1]$; range: $\{y|0 \le y \le 3\}$ or $[0, 3]$
b. domain: $\{x|-2 < x \le 1\}$ or $(-2, 1]$; range: $\{y|-1 \le y < 2\}$ or $[-1, 2)$
c. domain: $\{x|-3 \le x < 0\}$ or $[-3, 0)$; range: $\{-3, -2, -1\}$

Concept and Vocabulary Check

1. relation; domain; range **2.** function **3.** f; x **4.** true **5.** false **6.** x; $x + 6$ **7.** ordered pairs **8.** more than once; function **9.** [0, 3); domain **10.** $[1, \infty)$; range **11.** 0; 0; zeros **12.** false

Exercise Set 1.2

1. function; {1, 3, 5}; {2, 4, 5} **3.** not a function; {3, 4}; {4, 5} **5.** function; {3, 4, 5, 7}; {−2, 1, 9} **7.** function; {−3, −2, −1, 0}; {−3, −2, −1, 0} **9.** not a function; {1}; {4, 5, 6} **11.** y is a function of x. **13.** y is a function of x. **15.** y is not a function of x. **17.** y is not a function of x. **19.** y is a function of x. **21.** y is a function of x. **23.** y is a function of x. **25.** y is a function of x. **27. a.** 29 **b.** $4x + 9$ **c.** $-4x + 5$ **29. a.** 2 **b.** $x^2 + 12x + 38$ **c.** $x^2 - 2x + 3$ **31. a.** 13 **b.** 1 **c.** $x^4 - x^2 + 1$ **d.** $81a^4 - 9a^2 + 1$ **33. a.** 3 **b.** 7 **c.** $\sqrt{x} + 3$ **35. a.** $\frac{15}{4}$ **b.** $\frac{15}{4}$ **c.** $\frac{4x^2 - 1}{x^2}$ **37. a.** 1 **b.** −1 **c.** 1

39.

The graph of g is the graph of f shifted up by 3 units.

41.

The graph of g is the graph of f shifted down by 1 unit.

43.

The graph of g is the graph of f shifted up by 1 unit.

45.

The graph of g is the graph of f shifted down by 2 units.

47.

The graph of g is the graph of f shifted up by 2 units.

49.

The graph of g is the graph of f shifted up by 2 units.

51.

The graph of g is the graph of f shifted down by 1 unit.

53.

The graph of g is the graph of f shifted to the right by 1 unit.

55. function **57.** function **59.** not a function **61.** function **63.** function **65.** -4 **67.** 4 **69.** 0 **71.** 2 **73.** 2 **75.** -2 **77. a.** $(-\infty, \infty)$ **b.** $[-4, \infty)$ **c.** -3 and 1 **d.** -3 **e.** $f(-2) = -3$ and $f(2) = 5$ **79. a.** $(-\infty, \infty)$ **b.** $[1, \infty)$ **c.** none **d.** 1 **e.** $f(-1) = 2$ and $f(3) = 4$ **81. a.** $[0, 5)$ **b.** $[-1, 5)$ **c.** 2 **d.** -1 **e.** $f(3) = 1$ **83. a.** $[0, \infty)$ **b.** $[1, \infty)$ **c.** none **d.** 1 **e.** $f(4) = 3$ **85. a.** $[-2, 6]$ **b.** $[-2, 6]$ **c.** 4 **d.** 4 **e.** $f(-1) = 5$ **87. a.** $(-\infty, \infty)$ **b.** $(-\infty, -2]$ **c.** none **d.** -2 **e.** $f(-4) = -5$ and $f(4) = -2$ **89. a.** $(-\infty, \infty)$ **b.** $(0, \infty)$ **c.** none **d.** 1.5 **e.** $f(4) = 6$ **91. a.** $\{-5, -2, 0, 1, 3\}$ **b.** $\{2\}$ **c.** none **d.** 2 **e.** $f(-5) + f(3) = 4$ **93.** -2; 10 **95.** -38 **97.** $-2x^3 - 2x$ **99. a.** {(Philippines, 12), (Spain, 13), (Italy, 14), (Germany, 14), (Russia, 16)} **b.** yes; Each country corresponds to exactly one age. **c.** {(12, Philippines), (13, Spain), (14, Italy), (14, Germany), (16, Russia)} **d.** no; 14 in the domain corresponds to two members of the range, Italy and Germany. **101. a.** 83; The chance that a 60-year-old will survive to age 70 is 83%. **b.** 76; The chance that a 60-year-old will survive to age 70 is 76%. **c.** f **103. a.** 81; In 2010, the wage gap was 81%.; (30, 81) **b.** underestimates by 2% **105.** $C = 100{,}000 + 100x$, where x is the number of bicycles produced; $C(90) = 109{,}000$; It costs \$109,000 to produce 90 bicycles. **107.** $T = \dfrac{40}{x} + \dfrac{40}{x + 30}$, where x is the rate on the outgoing trip; $T(30) = 2$; It takes 2 hours, traveling 30 mph outgoing and 60 mph returning. **119.** does not make sense **121.** does not make sense **123.** false **125.** false **127.** Answers will vary; an example is $\{(1, 1), (2, 1)\}$. **129.** 36; For 100 calling minutes, the monthly cost is \$36.

130. $f(x) = x + 2,\ x \le 1$

131. $4xh + 2h^2 + 3h$

Section 1.3

Check Point Exercises

1. increasing on $(-\infty, -1)$, decreasing on $(-1, 1)$, increasing on $(1, \infty)$ **2. a.** even **b.** odd **c.** neither **3. a.** 20; With 40 calling minutes, the cost is \$20.; (40, 20) **b.** 28; With 80 calling minutes, the cost is \$28.; (80, 28)

4.

$f(x) = \begin{cases} 3 & \text{if } x \le -1 \\ x - 2 & \text{if } x > -1 \end{cases}$

5. a. $-2x^2 - 4xh - 2h^2 + x + h + 5$ **b.** $-4x - 2h + 1, h \ne 0$

Concept and Vocabulary Check

1. $< f(x_2)$; $> f(x_2)$; $= f(x_2)$ **2.** maximum; minimum **3.** $f(x)$; y-axis **4.** $-f(x)$; origin **5.** piecewise **6.** less than or equal to x; 2; -3; 0 **7.** difference quotient; $x + h$; $f(x)$; h; h **8.** false **9.** false

Exercise Set 1.3

1. a. $(-1, \infty)$ **b.** $(-\infty, -1)$ **c.** none **3. a.** $(0, \infty)$ **b.** none **c.** none **5. a.** none **b.** $(-2, 6)$ **c.** none **7. a.** $(-\infty, -1)$ **b.** none **c.** $(-1, \infty)$ **9. a.** $(-\infty, 0)$ or $(1.5, 3)$ **b.** $(0, 1.5)$ or $(3, \infty)$ **c.** none **11. a.** $(-2, 4)$ **b.** none **c.** $(-\infty, -2)$ or $(4, \infty)$ **13. a.** 0; $f(0) = 4$ **b.** $-3, 3$; $f(-3) = f(3) = 0$ **15. a.** -2; $f(-2) = 21$ **b.** 1; $f(1) = -6$ **17.** odd **19.** neither **21.** even **23.** even **25.** even **27.** odd **29.** even **31.** odd **33. a.** $(-\infty, \infty)$ **b.** $[-4, \infty)$ **c.** 1 and 7 **d.** 4 **e.** $(4, \infty)$ **f.** $(0, 4)$ **g.** $(-\infty, 0)$ **h.** 4 **i.** -4 **j.** 4 **k.** 2 and 6 **l.** neither **35. a.** $(-\infty, 3]$ **b.** $(-\infty, 4]$ **c.** -3 and 3 **d.** 3 **e.** $(-\infty, 1)$ **f.** $(1, 3)$ **g.** $(-\infty, -3] \cup \{3\}$ **h.** A relative maximum of 4 occurs at 1. **i.** 1 **j.** positive **37. a.** -1 **b.** 7 **c.** 19 **39. a.** 3 **b.** 3 **c.** 0 **41. a.** 8 **b.** 3 **c.** 6

43. a.

$f(x) = \begin{cases} -x \text{ if } x < 0 \\ x \text{ if } x \ge 0 \end{cases}$

b. $[0, \infty)$

45. a.

$f(x) = \begin{cases} 2x & \text{if } x \le 0 \\ 2 & \text{if } x > 0 \end{cases}$

b. $(-\infty, 0] \cup \{2\}$

47. a.

$f(x) = \begin{cases} x + 3 \text{ if } x < -2 \\ x - 3 \text{ if } x \ge -2 \end{cases}$

b. $(-\infty, \infty)$

49. a.

$f(x) = \begin{cases} 3 \text{ if } x \le -1 \\ -3 \text{ if } x > -1 \end{cases}$

b. $\{-3, 3\}$

51. a.

$f(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x < 1 \\ 2x - 1 & \text{if } x \ge 1 \end{cases}$

b. $[0, \infty)$

53. a.

$f(x) = \begin{cases} 0 & \text{if } x < -4 \\ -x & \text{if } -4 \le x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

b. $[0, \infty)$

55. $4, h \neq 0$ **57.** $3, h \neq 0$ **59.** $2x + h, h \neq 0$ **61.** $2x + h - 4, h \neq 0$ **63.** $4x + 2h + 1, h \neq 0$ **65.** $-2x - h + 2, h \neq 0$

67. $-4x - 2h + 5, h \neq 0$ **69.** $-4x - 2h - 1, h \neq 0$ **71.** $0, h \neq 0$ **73.** $-\dfrac{1}{x(x+h)}, h \neq 0$ **75.** $\dfrac{1}{\sqrt{x+h} + \sqrt{x}}, h \neq 0$ **77.** -18

79. $0.30t - 6$

81. $C(t) = \begin{cases} 50 & \text{if } 0 \le t \le 400 \\ 50 + 0.30(t - 400) & \text{if } t > 400 \end{cases}$

83. increasing: (25, 55); decreasing: (55, 75) **85.** 55 years old; 38% **87.** domain: [25, 75]; range: [34, 38] **89.** men
91. 2575; A single taxpayer with taxable income of \$20,000 owes \$2575. **93.** $42{,}449 + 0.33(x - 174{,}400)$
95. 0.85; It costs \$0.85 to mail a 3-ounce first-class letter. **97.** \$0.65

99.

107.

increasing: $(-\infty, 1)$ or $(3, \infty)$
decreasing: $(1, 3)$

109.

increasing: $(2, \infty)$
decreasing: $(-\infty, -2)$
constant: $(-2, 2)$

111.

increasing: $(0, \infty)$
decreasing: $(-\infty, 0)$

113. a.

b.

c. increasing:$(0, \infty)$; decreasing: $(-\infty, 0)$ **d.** $f(x) = x^n$ is increasing for $(-\infty, \infty)$ when n is positive and odd. **e.**

As n increases the steepness increases.

115. makes sense **117.** makes sense **122.** 3 **123.** $\left(\dfrac{3}{2}, 0\right)$ and $(0, -2)$
124. $y = \dfrac{-3x + 4}{2}$ or $y = -\dfrac{3}{2}x + 2$

Section 1.4

Check Point Exercises

1. a. 6 **b.** $-\dfrac{7}{5}$ **2.** $y + 5 = 6(x - 2); y = 6x - 17$ **3.** $y + 1 = -5(x + 2)$ or $y + 6 = -5(x + 1); y = -5x - 11$

4.

$f(x) = \frac{3}{5}x + 1$

5.

6.

7. slope: $-\dfrac{1}{2}$; y-intercept: 2

$3x + 6y - 12 = 0$

8.

$3x - 2y - 6 = 0$

9. $f(x) = 0.016x + 52.0$; 61.6°F

Concept and Vocabulary Check

1. scatter plot; regression **2.** $\dfrac{y_2 - y_1}{x_2 - x_1}$ **3.** positive **4.** negative **5.** zero **6.** undefined **7.** $y - y_1 = m(x - x_1)$
8. $y = mx + b$; slope; y-intercept **9.** $(0, 3)$; 2; 5 **10.** horizontal **11.** vertical **12.** general

Exercise Set 1.4

1. $\frac{3}{4}$; rises **3.** $\frac{1}{4}$; rises **5.** 0; horizontal **7.** -5; falls **9.** undefined; vertical **11.** $y - 5 = 2(x - 3)$; $y = 2x - 1$
13. $y - 5 = 6(x + 2)$; $y = 6x + 17$ **15.** $y + 3 = -3(x + 2)$; $y = -3x - 9$ **17.** $y - 0 = -4(x + 4)$; $y = -4x - 16$
19. $y + 2 = -1\left(x + \frac{1}{2}\right)$; $y = -x - \frac{5}{2}$ **21.** $y - 0 = \frac{1}{2}(x - 0)$; $y = \frac{1}{2}x$ **23.** $y + 2 = -\frac{2}{3}(x - 6)$; $y = -\frac{2}{3}x + 2$
25. using $(1, 2)$, $y - 2 = 2(x - 1)$; $y = 2x$ **27.** using $(-3, 0)$, $y - 0 = 1(x + 3)$; $y = x + 3$ **29.** using $(-3, -1)$, $y + 1 = 1(x + 3)$; $y = x + 2$
31. using $(-3, -2)$, $y + 2 = \frac{4}{3}(x + 3)$; $y = \frac{4}{3}x + 2$ **33.** using $(-3, -1)$, $y + 1 = 0(x + 3)$; $y = -1$ **35.** using $(2, 4)$, $y - 4 = 1(x - 2)$; $y = x + 2$
37. using $(0, 4)$, $y - 4 = 8(x - 0)$; $y = 8x + 4$

39. $m = 2$; $b = 1$

41. $m = -2$; $b = 1$

43. $m = \frac{3}{4}$; $b = -2$

45. $m = -\frac{3}{5}$; $b = 7$

47. $m = -\frac{1}{2}$; $b = 0$

49. $y = -2$

51. $x = -3$

53. $y = 0$

55. $f(x) = 1$

57.

59. a. $y = -3x + 5$
b. $m = -3$; $b = 5$
c.

61. a. $y = -\frac{2}{3}x + 6$
b. $m = -\frac{2}{3}$; $b = 6$
c.

63. a. $y = 2x - 3$
b. $m = 2$; $b = -3$
c.

65. a. $y = 3$
b. $m = 0$; $b = 3$
c.

67.

69.

71.

73. $m = -\frac{a}{b}$; falls
75. undefined slope; vertical
77. $m = -\frac{A}{B}$; $b = \frac{C}{B}$
79. -2

81.

83. 5
85. m_1, m_3, m_2, m_4
87. a. $y - 31.1 = 0.78(x - 10)$ or $y - 38.9 = 0.78(x - 20)$ **b.** $f(x) = 0.78x + 23.3$ **c.** 54.5%

89. a & b. Life Expectancy of United States Males, by Year of Birth; $E(x) = 0.215x + 65.7$

c. 78.6 yr

91. Answers will vary; an example is $y = -2.3x + 255$, where x is the percentage of adult females who are literate and y is under-five mortality per thousand.; Predictions will vary.

101. $m = -3$

103. $m = \frac{3}{4}$

105. does not make sense **107.** does not make sense
109. false **111.** true
113. coefficient of x: -6; coefficient of y: 3
118. $y = 2x + 7$ or $f(x) = 2x + 7$
119. $4x - y - 17 = 0$ **120.** 5

Section 1.5

Check Point Exercises

1. $y - 5 = 3(x + 2)$; $y = 3x + 11$ or $f(x) = 3x + 11$ **2. a.** 3 **b.** $3x - y = 0$ **3.** $m \approx 0.32$; The number of men living alone increased at a rate of 0.32 million per year. The rate of change is 0.32 million men per year. **4. a.** 1 **b.** 7 **c.** 4 **5.** 0.01 mg per 100 ml per hr
6. a. 12 ft/sec **b.** 10 ft/sec **c.** 8.04 ft/sec

Concept and Vocabulary Check

1. the same **2.** -1 **3.** $-\frac{1}{3}$; 3 **4.** -2; $\frac{1}{2}$ **5.** y; x **6.** $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$ **7.** $\frac{s(6) - s(3)}{6 - 3}$

Exercise Set 1.5

1. $y - 2 = 2(x - 4)$; $y = 2x - 6$ or $f(x) = 2x - 6$ **3.** $y - 4 = -\frac{1}{2}(x - 2)$; $y = -\frac{1}{2}x + 5$ or $f(x) = -\frac{1}{2}x + 5$

5. $y + 10 = -4(x + 8)$; $y = -4x - 42$ **7.** $y + 3 = -5(x - 2)$; $y = -5x + 7$ **9.** $y - 2 = \frac{2}{3}(x + 2)$; $2x - 3y + 10 = 0$

11. $y + 7 = -2(x - 4)$; $2x + y - 1 = 0$ **13.** 3 **15.** 10 **17.** $\frac{1}{5}$ **19. a.** 70 ft/sec **b.** 65 ft/sec **c.** 60.1 ft/sec **d.** 60.01 ft/sec

21. $f(x) = 5$ **23.** $f(x) = -\frac{1}{2}x + 1$ **25.** $f(x) = -\frac{2}{3}x - 2$ **27.** $p(x) = -0.25x + 22$

29. 137; There was an average increase of approximately 137 discharges per year. **31. a.** 142 **b.** overestimates by 5 discharges per year
39. a. The product of their slopes is -1.
b.

; no **c.**

; The lines now appear to be perpendicular.

41. makes sense **43.** makes sense **45.** $-\frac{3}{7}$

46. a.

b.

c. The graph in part (b) is the graph in part (a) shifted down 4 units.

47. a.

b.

c. The graph in part (b) is the graph in part (a) shifted to the right 2 units.

48. a.

b.

c. The graph in part (b) is the graph in part (a) reflected across the y-axis.

Mid-Chapter 1 Check Point

1. not a function; domain: $\{1, 2\}$; range: $\{-6, 4, 6\}$ **2.** function; domain: $\{0, 2, 3\}$; range: $\{1, 4\}$ **3.** function; domain: $\{x \mid -2 \le x < 2\}$ or $[-2, 2)$; range: $\{y \mid 0 \le y \le 3\}$ or $[0, 3]$ **4.** not a function; domain: $\{x \mid -3 < x \le 4\}$ or $(-3, 4]$; range: $\{y \mid -1 \le y \le 2\}$ or $[-1, 2]$
5. not a function; domain: $\{-2, -1, 0, 1, 2\}$; range: $\{-2, -1, 1, 3\}$ **6.** function; domain: $\{x \mid x \le 1\}$ or $(-\infty, 1]$; range: $\{y \mid y \ge -1\}$ or $[-1, \infty)$
7. y is a function of x. **8.** y is not a function of x. **9.** No vertical line intersects the graph in more than one point. **10.** $(-\infty, \infty)$
11. $(-\infty, 4]$ **12.** -6 and 2 **13.** 3 **14.** $(-\infty, -2)$ **15.** $(-2, \infty)$ **16.** -2 **17.** 4 **18.** 3 **19.** -7 and 3 **20.** -6 and 2
21. $(-6, 2)$ **22.** negative **23.** neither **24.** -1

25. $y = -2x$

26. $y = -2$

27. $x + y = -2$

28. $y = \frac{1}{3}x - 2$

29. $x = 3.5$

30.

$4x - 2y = 8$

31.

$f(x) = x^2 - 4$

32.

$f(x) = x - 4$

33.

$f(x) = |x| - 4$

34.

$5y = -3x$

35.

$5y = 20$

36.

$f(x) = \begin{cases} -1 \text{ if } x \le 0 \\ 2x + 1 \text{ if } x > 0 \end{cases}$

37. a. $f(-x) = -2x^2 - x - 5$; neither **b.** $-4x - 2h + 1, h \neq 0$ **38. a.** 30 **b.** 50

39. $f(x) = -2x - 5$ **40.** $f(x) = 2x - 3$ **41.** $f(x) = 3x - 13$

42. $f(x) = -\frac{5}{2}x - 13$ **43.** The lines are parallel.

44. a. 0.16 **b.** 0.16; 0.16; minute of brisk walking **45.** 2

Section 1.6

Check Point Exercises

1.

2.

3.

4.

5.

6.

7. a.

$g(x) = f(2x)$

b.

$h(x) = f\left(\frac{1}{2}x\right)$

8.

$y = -\frac{1}{3}f(x + 1) - 2$

9.

$g(x) = 2(x - 1)^2 + 3$

Concept and Vocabulary Check

1. vertical; down **2.** horizontal; to the right **3.** x-axis **4.** y-axis **5.** vertical; y **6.** horizontal; x **7.** false

Exercise Set 1.6

1.

$g(x) = f(x) + 1$

3.

$g(x) = f(x + 1)$

5.

$g(x) = f(x - 1) - 2$

7.

$g(x) = f(-x)$

9.

$g(x) = -f(x) + 3$

11.

$g(x) = \frac{1}{2}f(x)$

13. $g(x) = f\left(\frac{1}{2}x\right)$

15.

$g(x) = -f\left(\frac{1}{2}x\right) + 1$

17.

$g(x) = f(x) - 1$

19.

$g(x) = f(x - 1)$

21.

$g(x) = f(x - 1) + 2$

23.

$g(x) = -f(x)$

25.

$g(x) = f(-x) + 1$

27.

$g(x) = 2f(x)$

29.

$g(x) = f(2x)$

31.

$g(x) = 2f(x + 2) + 1$

33.

$g(x) = f(x) + 2$

35.

$g(x) = f(x + 2)$

37.

$g(x) = -f(x + 2)$

39.

$g(x) = -\frac{1}{2}f(x + 2)$

41.

$g(x) = -\frac{1}{2}f(x + 2) - 2$

43.

$g(x) = \frac{1}{2}f(2x)$

45.

$g(x) = f(x - 1) - 1$

47.

$g(x) = -f(x - 1) + 1$

49.

$g(x) = 2f\left(\frac{1}{2}x\right)$

51.

$g(x) = \frac{1}{2}f(x + 1)$

53.

55.

57.

59.

61.

63.

65.

67.

69.

71.

73.

75.

77.

79.

81.

83.

85.

87.

89.

91.

93.

95.

97.

99.

101.

103.

105.

107.

109.

111.

113.

115.

117.

119.

$g(x) = 2 \text{ int } (x + 1)$

121.

$h(x) = \text{int}\,(-x) + 1$

123. $y = \sqrt{x - 2}$ **125.** $y = (x + 1)^2 - 4$

127. a. First, vertically stretch the graph of $f(x) = \sqrt{x}$ by the factor 2.9; then, shift the result up 20.1 units. **b.** 40.2 in.; very well **c.** 0.9 in. per month **d.** 0.2 in. per month; This is a much smaller rate of change; The graph is not as steep between 50 and 60 as it is between 0 and 10.

135. a.

b.

137. makes sense **139.** does not make sense **141.** false **143.** false **145.** $g(x) = -(x + 4)^2$ **147.** $g(x) = -\sqrt{x - 2} + 2$

149. $(-a, b)$ **151.** $(a + 3, b)$ **153.** $2x^3 + x^2 - 5x + 2$ **154.** $9x^2 - 30x + 30$ **155.** $\dfrac{2x}{3 - x}$

Section 1.7

Check Point Exercises

1. a. $(-\infty, \infty)$ **b.** $(-\infty, -7) \cup (-7, 7) \cup (7, \infty)$ **c.** $[3, \infty)$ **d.** $(-\infty, 8)$ **2. a.** $(f + g)(x) = x^2 + x - 6; (-\infty, \infty)$ **b.** $(f - g)(x) = -x^2 + x - 4; (-\infty, \infty)$ **c.** $(fg)(x) = x^3 - 5x^2 - x + 5; (-\infty, \infty)$ **d.** $\left(\dfrac{f}{g}\right)(x) = \dfrac{x - 5}{x^2 - 1}; (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ **3. a.** $(f + g)(x) = \sqrt{x - 3} + \sqrt{x + 1}$ **b.** $[3, \infty)$ **4. a.** $(B + D)(x) = -3.2x^2 + 56x + 6406$ **b.** 6545.2 thousand **c.** overestimates by 7.2 thousand **5. a.** $(f \circ g)(x) = 10x^2 - 5x + 1$ **b.** $(g \circ f)(x) = 50x^2 + 115x + 65$ **c.** 16 **6. a.** $(f \circ g)(x) = \dfrac{4x}{1 + 2x}$ **b.** $\left(-\infty, -\dfrac{1}{2}\right) \cup \left(-\dfrac{1}{2}, 0\right) \cup (0, \infty)$ **7.** If $f(x) = \sqrt{x}$ and $g(x) = x^2 + 5$, then $h(x) = (f \circ g)(x)$.

Concept and Vocabulary Check

1. zero **2.** negative **3.** $f(x) + g(x)$ **4.** $f(x) - g(x)$ **5.** $f(x) \cdot g(x)$ **6.** $\dfrac{f(x)}{g(x)}; g(x)$ **7.** $(-\infty, \infty)$ **8.** $(2, \infty)$ **9.** $(0, 3); (3, \infty)$ **10.** composition; $f(g(x))$ **11.** $f; g(x)$ **12.** composition; $g(f(x))$ **13.** $g; f(x)$ **14.** false **15.** false **16.** 2

Exercise Set 1.7

1. $(-\infty, \infty)$ **3.** $(-\infty, 4) \cup (4, \infty)$ **5.** $(-\infty, \infty)$ **7.** $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$ **9.** $(-\infty, -7) \cup (-7, 9) \cup (9, \infty)$ **11.** $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ **13.** $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$ **15.** $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$ **17.** $[3, \infty)$ **19.** $(3, \infty)$ **21.** $[-7, \infty)$ **23.** $(-\infty, 12]$ **25.** $[2, \infty)$ **27.** $[2, 5) \cup (5, \infty)$ **29.** $(-\infty, -2) \cup (-2, 2) \cup (2, 5) \cup (5, \infty)$

31. $(f + g)(x) = 3x + 2$; domain: $(-\infty, \infty)$; $(f - g)(x) = x + 4$; domain: $(-\infty, \infty)$; $(fg)(x) = 2x^2 + x - 3$; domain: $(-\infty, \infty)$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x + 3}{x - 1}$; domain: $(-\infty, 1) \cup (1, \infty)$ **33.** $(f + g)(x) = 3x^2 + x - 5$; domain: $(-\infty, \infty)$; $(f - g)(x) = -3x^2 + x - 5$; domain: $(-\infty, \infty)$; $(fg)(x) = 3x^3 - 15x^2$; domain: $(-\infty, \infty)$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{x - 5}{3x^2}$; domain: $(-\infty, 0) \cup (0, \infty)$ **35.** $(f + g)(x) = 2x^2 - 2$; domain: $(-\infty, \infty)$; $(f - g)(x) = 2x^2 - 2x - 4$; domain: $(-\infty, \infty)$; $(fg)(x) = 2x^3 + x^2 - 4x - 3$; domain: $(-\infty, \infty)$; $\left(\dfrac{f}{g}\right)(x) = 2x - 3$; domain: $(-\infty, -1) \cup (-1, \infty)$

37. $(f + g)(x) = 2x - 12$; domain: $(-\infty, \infty)$; $(f - g)(x) = -2x^2 - 2x + 18$; domain: $(-\infty, \infty)$; $(fg)(x) = -x^4 - 2x^3 + 18x^2 + 6x - 45$; domain: $(-\infty, \infty)$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{3 - x^2}{x^2 + 2x - 15}$; domain: $(-\infty, -5) \cup (-5, 3) \cup (3, \infty)$

39. $(f + g)(x) = \sqrt{x} + x - 4$; domain: $[0, \infty)$; $(f - g)(x) = \sqrt{x} - x + 4$; domain: $[0, \infty)$; $(fg)(x) = \sqrt{x}(x - 4)$; domain: $[0, \infty)$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x}}{x - 4}$; domain: $[0, 4) \cup (4, \infty)$ **41.** $(f + g)(x) = \dfrac{2x + 2}{x}$; domain: $(-\infty, 0) \cup (0, \infty)$; $(f - g)(x) = 2$; domain: $(-\infty, 0) \cup (0, \infty)$; $(fg)(x) = \dfrac{2x + 1}{x^2}$; domain: $(-\infty, 0) \cup (0, \infty)$; $\left(\dfrac{f}{g}\right)(x) = 2x + 1$; domain: $(-\infty, 0) \cup (0, \infty)$ **43.** $(f + g)(x) = \dfrac{9x - 1}{x^2 - 9}$; domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$; $(f - g)(x) = \dfrac{x + 3}{x^2 - 9} = \dfrac{1}{x - 3}$; domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$; $(fg)(x) = \dfrac{20x^2 - 6x - 2}{(x^2 - 9)^2}$; domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{5x + 1}{4x - 2}$; domain: $(-\infty, -3) \cup \left(-3, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, 3\right) \cup (3, \infty)$ **45.** $(f + g)(x) = \dfrac{8x^2 + 30x - 12}{(x - 2)(x + 3)}$; domain: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$; $(f - g)(x) = \dfrac{8x^2 + 18x + 12}{(x - 2)(x + 3)}$; domain: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$; $(fg)(x) = \dfrac{48x}{(x - 2)(x + 3)}$; domain: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{4x(x + 3)}{3(x - 2)}$; domain: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$ **47.** $(f + g)(x) = \sqrt{x + 4} + \sqrt{x - 1}$; domain: $[1, \infty)$; $(f - g)(x) = \sqrt{x + 4} - \sqrt{x - 1}$; domain: $[1, \infty)$; $(fg)(x) = \sqrt{x^2 + 3x - 4}$; domain: $[1, \infty)$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x + 4}}{\sqrt{x - 1}}$; domain: $(1, \infty)$

49. $(f+g)(x) = \sqrt{x-2} + \sqrt{2-x}$; domain: $\{2\}$; $(f-g)(x) = \sqrt{x-2} - \sqrt{2-x}$; domain: $\{2\}$; $(fg)(x) = \sqrt{x-2} \cdot \sqrt{2-x}$; domain: $\{2\}$; $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-2}}{\sqrt{2-x}}$; domain: $\varnothing$ **51. a.** $(f \circ g)(x) = 2x + 14$ **b.** $(g \circ f)(x) = 2x + 7$ **c.** $(f \circ g)(2) = 18$ **d.** $(g \circ f)(2) = 11$ **53. a.** $(f \circ g)(x) = 2x + 5$ **b.** $(g \circ f)(x) = 2x + 9$ **c.** $(f \circ g)(2) = 9$ **d.** $(g \circ f)(2) = 13$ **55. a.** $(f \circ g)(x) = 20x^2 - 11$ **b.** $(g \circ f)(x) = 80x^2 - 120x + 43$ **c.** $(f \circ g)(2) = 69$ **d.** $(g \circ f)(2) = 123$ **57. a.** $(f \circ g)(x) = x^4 - 4x^2 + 6$ **b.** $(g \circ f)(x) = x^4 + 4x^2 + 2$ **c.** $(f \circ g)(2) = 6$ **d.** $(g \circ f)(2) = 34$ **59. a.** $(f \circ g)(x) = -2x^2 - x - 1$ **b.** $(g \circ f)(x) = 2x^2 - 17x + 41$ **c.** $(f \circ g)(2) = -11$ **d.** $(g \circ f)(2) = 15$ **61. a.** $(f \circ g)(x) = \sqrt{x-1}$ **b.** $(g \circ f)(x) = \sqrt{x} - 1$ **c.** $(f \circ g)(2) = 1$ **d.** $(g \circ f)(2) = \sqrt{2} - 1$ **63. a.** $(f \circ g)(x) = x$ **b.** $(g \circ f)(x) = x$ **c.** $(f \circ g)(2) = 2$ **d.** $(g \circ f)(2) = 2$ **65. a.** $(f \circ g)(x) = x$ **b.** $(g \circ f)(x) = x$ **c.** 2 **d.** $(g \circ f)(2) = 2$ **67. a.** $(f \circ g)(x) = \frac{2x}{1+3x}$ **b.** $\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, 0\right) \cup (0, \infty)$ **69. a.** $(f \circ g)(x) = \frac{4}{4+x}$ **b.** $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$ **71. a.** $(f \circ g)(x) = \sqrt{x-2}$ **b.** $[2, \infty)$ **73. a.** $(f \circ g)(x) = 5 - x$ **b.** $(-\infty, 1]$ **75.** $f(x) = x^4, g(x) = 3x - 1$ **77.** $f(x) = \sqrt[3]{x}, g(x) = x^2 - 9$ **79.** $f(x) = |x|, g(x) = 2x - 5$ **81.** $f(x) = \frac{1}{x}, g(x) = 2x - 3$ **83.** 5 **85.** -1 **87.** $[-4, 3]$

89.

91. 1 **93.** -6 **95.** 1 and 2 **97. a.** $(M + F)(x) = 2.99x + 235.5$ **b.** 295.3 million **c.** underestimates by 2.7 million

99. $(R - C)(20{,}000) = -200{,}000$; The company loses \$200,000 when 20,000 radios are sold.; $(R - C)(30{,}000) = 0$; The company breaks even when 30,000 radios are sold.; $(R - C)(40{,}000) = 200{,}000$; The company makes a profit of \$200,000 when 40,000 radios are sold. **101. a.** f gives the price of the computer after a \$400 discount. g gives the price of the computer after a 25% discount. **b.** $(f \circ g)(x) = 0.75x - 400$; This models the price of a computer after first a 25% discount and then a \$400 discount. **c.** $(g \circ f)(x) = 0.75(x - 400)$; This models the price of a computer after first a \$400 discount and then a 25% discount. **d.** $f \circ g$ because $0.75x - 400 < 0.75(x - 400)$

109.

Domain: $[0, 4]$

111. makes sense **113.** does not make sense **115.** false **117.** true
119. Answers will vary; One possible answer is $f(x) = x + 1$ and $g(x) = x - 1$.

120. $\{(4, -2), (1, -1), (1, 1), (4, 2)\}$; no **121.** $y = \frac{5}{x-4}$ **122.** $y = \sqrt{x+1}$

Section 1.8

Check Point Exercises

1. $f(g(x)) = 4\left(\frac{x+7}{4}\right) - 7 = x + 7 - 7 = x$; $g(f(x)) = \frac{(4x-7)+7}{4} = \frac{4x}{4} = x$ **2.** $f^{-1}(x) = \frac{x-7}{2}$ **3.** $f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}$

4. $f^{-1}(x) = \frac{3}{x+1}$ **5.** (b) and (c)

6.

7. $f^{-1}(x) = \sqrt{x-1}$

Concept and Vocabulary Check

1. inverse **2.** x; x **3.** horizontal; one-to-one **4.** $y = x$

Exercise Set 1.8

1. $f(g(x)) = x$; $g(f(x)) = x$; f and g are inverses. **3.** $f(g(x)) = x$; $g(f(x)) = x$; f and g are inverses.
5. $f(g(x)) = \frac{5x-56}{9}$; $g(f(x)) = \frac{5x-4}{9}$; f and g are not inverses. **7.** $f(g(x)) = x$; $g(f(x)) = x$; f and g are inverses.
9. $f(g(x)) = x$; $g(f(x)) = x$; f and g are inverses. **11.** $f^{-1}(x) = x - 3$ **13.** $f^{-1}(x) = \frac{x}{2}$ **15.** $f^{-1}(x) = \frac{x-3}{2}$ **17.** $f^{-1}(x) = \sqrt[3]{x-2}$
19. $f^{-1}(x) = \sqrt[3]{x} - 2$ **21.** $f^{-1}(x) = \frac{1}{x}$ **23.** $f^{-1}(x) = x^2, x \geq 0$ **25.** $f^{-1}(x) = \frac{7}{x+3}$ **27.** $f^{-1}(x) = \frac{3x+1}{x-2}; x \neq 2$
29. The function is not one-to-one, so it does not have an inverse function. **31.** The function is not one-to-one, so it does not have an inverse function.
33. The function is one-to-one, so it does have an inverse function.

35.

37.

39. a. $f^{-1}(x) = \dfrac{x+1}{2}$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$; range of f = domain of $f^{-1} = (-\infty, \infty)$

41. a. $f^{-1}(x) = \sqrt{x+4}$

b.

c. domain of f = range of $f^{-1} = [0, \infty)$; range of f = domain of $f^{-1} = [-4, \infty)$

43. a. $f^{-1}(x) = -\sqrt{x} + 1$

b.

d. domain of f = range of $f^{-1} = (-\infty, 1]$; range of f = domain of $f^{-1} = [0, \infty)$

45. a. $f^{-1}(x) = \sqrt[3]{x+1}$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$; range of f = domain of $f^{-1} = (-\infty, \infty)$

47. a. $f^{-1}(x) = \sqrt[3]{x} - 2$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$; range of f = domain of $f^{-1} = (-\infty, \infty)$

49. a. $f^{-1}(x) = x^2 + 1, x \geq 0$

b.

c. domain of f = range of $f^{-1} = [1, \infty)$; range of f = domain of $f^{-1} = [0, \infty)$

51. a. $f^{-1}(x) = (x-1)^3$

b.

c. domain of f = range of $f^{-1} = (-\infty, \infty)$; range of f = domain of $f^{-1} = (-\infty, \infty)$

53. 5 **55.** 1 **57.** 2 **59.** −7 **61.** 3 **63.** 11

65. a. {(Zambia, 4.2), (Colombia, 4.5), (Poland, 3.3), (Italy, 3.3), (United States, 2.5)}
b. {(4.2, Zambia), (4.5, Colombia), (3.3, Poland), (3.3, Italy), (2.5, United States)}; no; The inverse of f is not a function. **67. a.** f is a one-to-one function. **b.** $f^{-1}(0.25)$ is the number of people in a room for a 25% probability of two people sharing a birthday. $f^{-1}(0.5)$ is the number of people in a room for a 50% probability of two people sharing a birthday. $f^{-1}(0.7)$ is the number of people in a room for a 70% probability of two people sharing a birthday.

69. $f(g(x)) = \dfrac{9}{5}\left[\dfrac{5}{9}(x-32)\right] + 32 = x$ and $g(f(x)) = \dfrac{5}{9}\left[\left(\dfrac{9}{5}x + 32\right) - 32\right] = x$

77.

one-to-one

79.

not one-to-one

81.

not one-to-one

83.

not one-to-one

85.

f and g are inverses.

87. makes sense **89.** makes sense **91.** false **93.** false

95. $(f \circ g)^{-1}(x) = \dfrac{x-15}{3}$; $(g^{-1} \circ f^{-1})(x) = \dfrac{x}{3} - 5 = \dfrac{x-15}{3}$

97. No; The spacecraft was at the same height, $s(t)$, for two different values of t-once during the ascent and once again during the descent.

100. $3\sqrt{5}$ **101.**

102. $\{3 \pm \sqrt{13}\}$

Section 1.9

Check Point Exercises

1. $3\sqrt{5} \approx 6.71$ **2.** $\left(4, -\dfrac{1}{2}\right)$ **3.** $x^2 + y^2 = 16$ **4.** $x^2 + (y+6)^2 = 100$

5. a. center: $(-3, 1)$; radius: 2

b.

$(x+3)^2 + (y-1)^2 = 4$

c. domain: $[-5, -1]$
range: $[-1, 3]$

6. $(x+2)^2 + (y-2)^2 = 9$

$x^2 + y^2 + 4x - 4y - 1 = 0$

Concept and Vocabulary Check

1. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **2.** $\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2}$ **3.** circle; center; radius **4.** $(x-h)^2 + (y-k)^2 = r^2$ **5.** general **6.** 4; 16

Exercise Set 1.9

1. 13 **3.** $2\sqrt{29} \approx 10.77$ **5.** 5 **7.** $\sqrt{29} \approx 5.39$ **9.** $4\sqrt{2} \approx 5.66$ **11.** $2\sqrt{5} \approx 4.47$ **13.** $2\sqrt{2} \approx 2.83$ **15.** $\sqrt{93} \approx 9.64$

17. $\sqrt{5} \approx 2.24$ **19.** $(4, 6)$ **21.** $(-4, -5)$ **23.** $\left(\frac{3}{2}, -6\right)$ **25.** $(-3, -2)$ **27.** $(1, 5\sqrt{5})$ **29.** $(2\sqrt{2}, 0)$ **31.** $x^2 + y^2 = 49$

33. $(x - 3)^2 + (y - 2)^2 = 25$ **35.** $(x + 1)^2 + (y - 4)^2 = 4$ **37.** $(x + 3)^2 + (y + 1)^2 = 3$ **39.** $(x + 4)^2 + y^2 = 100$

41. center: $(0, 0)$
radius: 4
domain: $[-4, 4]$
range: $[-4, 4]$

$x^2 + y^2 = 16$

43. center: $(3, 1)$
radius: 6
domain: $[-3, 9]$
range: $[-5, 7]$

$(x - 3)^2 + (y - 1)^2 = 36$

45. center: $(-3, 2)$
radius: 2
domain: $[-5, -1]$
range: $[0, 4]$

$(x + 3)^2 + (y - 2)^2 = 4$

47. center: $(-2, -2)$
radius: 2
domain: $[-4, 0]$
range: $[-4, 0]$

$(x + 2)^2 + (y + 2)^2 = 4$

49. center: $(0, 1)$
radius: 1
domain: $[-1, 1]$
range: $[0, 2]$

$x^2 + (y - 1)^2 = 1$

51. center: $(-1, 0)$
radius: 5
domain: $[-6, 4]$
range: $[-5, 5]$

$(x + 1)^2 + y^2 = 25$

53. $(x + 3)^2 + (y + 1)^2 = 4$
center: $(-3, -1)$
radius: 2

$x^2 + y^2 + 6x + 2y + 6 = 0$

55. $(x - 5)^2 + (y - 3)^2 = 64$
center: $(5, 3)$
radius: 8

$x^2 + y^2 - 10x - 6y - 30 = 0$

57. $(x + 4)^2 + (y - 1)^2 = 25$
center: $(-4, 1)$
radius: 5

$x^2 + y^2 + 8x - 2y - 8 = 0$

59. $(x - 1)^2 + y^2 = 16$

center: $(1, 0)$

radius: 4

$x^2 - 2x + y^2 - 15 = 0$

61. $\left(x - \frac{1}{2}\right)^2 + (y + 1)^2 = \frac{1}{4}$

center: $\left(\frac{1}{2}, -1\right)$

radius: $\frac{1}{2}$

$x^2 + y^2 - x + 2y + 1 = 0$

63. $\left(x + \frac{3}{2}\right)^2 + (y - 1)^2 = \frac{17}{4}$

center: $\left(-\frac{3}{2}, 1\right)$

radius: $\frac{\sqrt{17}}{2}$

$x^2 + y^2 + 3x - 2y - 1 = 0$

65. a. $(5, 10)$ **b.** $\sqrt{5}$
c. $(x - 5)^2 + (y - 10)^2 = 5$

67. $\{(0, -4), (4, 0)\}$

69. $\{(0, -3), (2, -1)\}$

71. 2693 mi **73.** $(x + 2.4)^2 + (y + 2.7)^2 = 900$

83.

85.

87. makes sense
89. makes sense
91. false
93. false

95. a. Distance between (x_1, y_1) and $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$
$$= \sqrt{\left(\frac{x_1 + x_2 - 2x_1}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_1}{2}\right)^2}$$
$$= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$
$$= \sqrt{\frac{x_2^2 - 2x_1x_2 + x_1^2}{4} + \frac{y_2^2 - 2y_1y_2 + y_1^2}{4}}$$
$$= \sqrt{\frac{x_1^2 - 2x_1x_2 + x_2^2}{4} + \frac{y_1^2 - 2y_1y_2 + y_2^2}{4}}$$
$$= \sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2}$$
$$= \sqrt{\left(\frac{x_1 + x_2 - 2x_2}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_2}{2}\right)^2}$$
$$= \sqrt{\left(\frac{x_1 + x_2}{2} - x_2\right)^2 + \left(\frac{y_1 + y_2}{2} - y_2\right)^2}$$

$=$ Distance between (x_2, y_2) and $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

b. $$\sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2} + \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$
$$= 2\sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$
$$= 2\sqrt{\frac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{4}}$$
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$=$ Distance from (x_1, y_1) to (x_2, y_2)

97. $y + 4 = \frac{3}{4}(x - 3)$ **98.** $x - 2000$ **99. a.** perimeter: 140 yd; area: 1200 yd^2 **b.** perimeter: 140 yd; area: 1000 yd^2

100. $h = \frac{22}{\pi r^2}$; $2\pi r^2 + \frac{44}{r}$

Section 1.10

Check Point Exercises

1. a. $f(x) = 0.08x + 15$ **b.** $g(x) = 0.12x + 3$ **c.** 300 text messages **2. a.** $N(x) = -100x + 18{,}000$ **b.** $R(x) = -100x^2 + 18{,}000x$
3. a. $V(x) = x(15 - 2x)(8 - 2x)$ **b.** $\{x|0 < x < 4\}$ or $(0, 4)$ **4.** $A(x) = x(100 - x) = 100x - x^2$ ft^2 **5.** $A(r) = 2\pi r^2 + \frac{2000}{r}$
6. $I(x) = 0.07x + 0.09(25{,}000 - x)$ **7.** $d(x) = \sqrt{x^6 + x^2}$

Concept and Vocabulary Check

1. $4 + 0.15x$ **2.** $x - 300$; $50(x - 300)$; 5000; $50(x - 300)$ **3.** $10 - 2x$; $10 - 2x$; x; $(10 - 2x)$; $(10 - 2x)$; x **4.** xy; $2x + 2y$; $90 - x$; $x(90 - x)$
5. $0.12x + 0.09y$; $30{,}000 - x$; $0.12x + 0.09(30{,}000 - x)$ **6.** $\sqrt{x^2 + y^2}$; $\sqrt{x^2 + x^6}$

Exercise Set 1.10

1. a. $f(x) = 200 + 0.15x$ **b.** 800 mi **3. a.** $M(x) = 239.4 - 0.3x$ **b.** 2152 **5. a.** $f(x) = 1.25x$ **b.** $g(x) = 21 + 0.5x$ **c.** 28 times; \$35 **7. a.** $f(x) = 100 + 0.8x$ **b.** $g(x) = 40 + 0.9x$ **c.** \$600; \$580 **9. a.** $N(x) = -500x + 40{,}000$ **b.** $R(x) = -500x^2 + 40{,}000x$
11. a. $N(x) = -50x + 16{,}500$ **b.** $R(x) = -50x^2 + 16{,}500x$ **13. a.** $Y(x) = -4x + 520$ **b.** $T(x) = -4x^2 + 520x$
15. a. $V(x) = x(24 - 2x)^2$ **b.** $V(2) = 800$; If a 2-in. square is cut from each corner, the volume is 800 cubic in.; $V(3) = 972$; if a 3-in. square is cut from each corner, the volume is 972 cubic in.; $V(4) = 1024$; if a 4-in. square is cut from each corner, the volume is 1024 cubic in.; $V(5) = 980$; if a 5-in. square is cut from each corner, the volume is 980 cubic in.; $V(6) = 864$; if a 6-in. square is cut from each corner, the volume is 864 cubic in. Initially, as x increases, V increases. When $x = 4$, V is a maximum. As x increases beyond 4, V decreases. **c.** $(0, 12)$ **17.** $A(x) = x(20 - 2x)$
19. $P(x) = x(66 - x)$ **21.** $A(x) = x(400 - x)$ **23.** Let x be the length of the side perpendicular to the canal. $A(x) = x(800 - 2x)$
25. $A(x) = \frac{x(1000 - 2x)}{3}$ **27.** $A(r) = r(440 - \pi r)$ **29.** Let x be the length of the interior wall. $C(x) = 475x + \frac{1{,}400{,}000}{x}$
31. $A(x) = \frac{40}{x} + x^2$ **33.** $V(x) = 300x^2 - 4x^3$ **35. a.** $I(x) = 0.15x + 0.07(50{,}000 - x)$ **b.** \$31,250 at 15%, \$18,750 at 7%
37. $I(x) = 0.12x - 0.05(8000 - x)$ **39.** $d(x) = \sqrt{x^4 - 7x^2 + 16}$ **41.** $d(x) = \sqrt{x^2 - x + 1}$ **43. a.** $A(x) = 2x\sqrt{4 - x^2}$
b. $P(x) = 4x + 2\sqrt{4 - x^2}$ **45.** $f(x) = \sqrt{x^2 + 36} + \sqrt{x^2 - 20x + 164}$ **47.** $A(x) = 3x^2 + x - 4$ **49.** $V(x) = 2x^3 + 12x^2 + 12x + 10$
63. does not make sense **65.** does not make sense **67.** $T(x) = \frac{\sqrt{x^2 + 4}}{2} + \frac{6 - x}{5}$ **69.** $A(r) = 12r - \frac{4 + \pi}{2}r^2$ **71.** $15x^2 - 29x - 14$
72. $\sqrt{2}$ **73.** $-\frac{54 + 43\sqrt{2}}{46}$

Chapter 1 Review Exercises

1.

2.

3.

4.

5.

6. x-intercept: -2; y-intercept: 2 **7.** x-intercepts: -2, 2; y-intercept: -4 **8.** x-intercept: 5; no y-intercept
9. (20, 8); 8% of college students anticipated a starting salary of \$20 thousand. **10.** \$30 thousand; 22% **11.** \$70 thousand; 2%
12. \$25 thousand and \$30 thousand **13.** 14% **14.** 19.7%; It's greater than the estimate. **15.** function; domain: {2, 3, 5}; range: {7}
16. function; domain: {1, 2, 13}; range: {10, 500, π} **17.** not a function; domain: {12, 14}; range: {13, 15, 19} **18.** y is a function of x.
19. y is a function of x. **20.** y is not a function of x. **21. a.** $f(4) = -23$ **b.** $f(x + 3) = -7x - 16$ **c.** $f(-x) = 5 + 7x$
22. a. $g(0) = 2$ **b.** $g(-2) = 24$ **c.** $g(x - 1) = 3x^2 - 11x + 10$ **d.** $g(-x) = 3x^2 + 5x + 2$ **23. a.** $g(13) = 3$
b. $g(0) = 4$ **c.** $g(-3) = 7$ **24. a.** -1 **b.** 12 **c.** 3 **25.** not a function **26.** function **27.** function **28.** not a function
29. not a function **30.** function **31. a.** $[-3, 5)$ **b.** $[-5, 0]$ **c.** -3 **d.** -2 **e.** increasing: $(-2, 0)$ or $(3, 5)$; decreasing: $(-3, -2)$ or $(0, 3)$ **f.** $f(-2) = -3$ and $f(3) = -5$ **32. a.** $(-\infty, \infty)$ **b.** $(-\infty, 3]$ **c.** -2 and 3 **d.** 3 **e.** increasing: $(-\infty, 0)$; decreasing: $(0, \infty)$ **f.** $f(-2) = 0$ and $f(6) = -3$ **33. a.** $(-\infty, \infty)$ **b.** $[-2, 2]$ **c.** 0 **d.** 0 **e.** increasing: $(-2, 2)$; constant: $(-\infty, -2)$ or $(2, \infty)$ **f.** $f(-9) = -2$ and $f(14) = 2$ **34. a.** 0; $f(0) = -2$ **b.** $-2, 3$; $f(-2) = -3$, $f(3) = -5$
35. a. 0; $f(0) = 3$ **b.** none **36.** odd; symmetric with respect to the origin **37.** even; symmetric with respect to the y-axis
38. odd; symmetric with respect to the origin

39.

$f(x) = \begin{cases} 5 \text{ if } x \le -1 \\ -3 \text{ if } x > -1 \end{cases}$; $\{-3, 5\}$

40.

$f(x) = \begin{cases} 2x \text{ if } x < 0 \\ -x \text{ if } x \ge 0 \end{cases}$; $(-\infty, 0]$

41. 8 **42.** $-4x - 2h + 1$ **43. a.** yes; The graph passes the vertical line test. **b.** (3, 12); The eagle descended.
c. (0, 3) and (12, 17); The eagle's height held steady during the first 3 seconds and the eagle was on the ground for 5 seconds.
d. (17, 30); The eagle was ascending.

44.

45. $-\frac{1}{2}$; falls **46.** 1; rises **47.** 0; horizontal **48.** undefined; vertical
49. $y - 2 = -6(x + 3)$; $y = -6x - 16$ **50.** using (1, 6), $y - 6 = 2(x - 1)$; $y = 2x + 4$
51. $y + 7 = -3(x - 4)$; $y = -3x + 5$ **52.** $y - 6 = -3(x + 3)$; $y = -3x - 3$ **53.** $x + 6y + 18 = 0$

54. Slope: $\frac{2}{5}$; y-intercept: -1 **55.** Slope: -4; y-intercept: 5 **56.** Slope: $-\frac{2}{3}$; y-intercept: -2 **57.** Slope: 0; y-intercept: 4

$y = \frac{2}{5}x - 1$

$f(x) = -4x + 5$

$2x + 3y + 6 = 0$

$2y - 8 = 0$

58.

$2x - 5y - 10 = 0$

59. $2x - 10 = 0$

60. a. $y - 2.3 = 0.116(x - 15)$ or $y - 11 = 0.116(x - 90)$ **b.** $f(x) = 0.116x + 0.56$ **c.** approximately 5 deaths per 100,000 persons
d. 4.3 deaths per 100,000 persons; underestimates by 0.7 death per 100,000 persons; The line passes below the point for France.
61. a. -0.48 **b.** 0.48; -0.48%; year **62.** 10 **63. a.** 32 ft/sec **b.** -32 ft/sec **c.** The positive sign means the ball is moving up on (0, 2). The negative sign means the ball is moving down on (2, 4).

64.

65.

66.

$y = g(x)$

67.

68.

$y = g(x)$

69.

70.

71.

72.

73.

74.

75.

76.

77.

78.

79.

80.

81.

82.

83.

84.

85. $(-\infty, \infty)$ **86.** $(-\infty, 7) \cup (7, \infty)$ **87.** $(-\infty, 4]$ **88.** $(-\infty, -7) \cup (-7, 3) \cup (3, \infty)$
89. $[2, 5) \cup (5, \infty)$ **90.** $[1, \infty)$ **91.** $(f + g)(x) = 4x - 6$; domain: $(-\infty, \infty)$; $(f - g)(x) = 2x + 4$; domain: $(-\infty, \infty)$; $(fg)(x) = 3x^2 - 16x + 5$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{3x - 1}{x - 5}$; domain: $(-\infty, 5) \cup (5, \infty)$
92. $(f + g)(x) = 2x^2 + x$; domain: $(-\infty, \infty)$; $(f - g)(x) = x + 2$; domain: $(-\infty, \infty)$; $(fg)(x) = x^4 + x^3 - x - 1$; domain: $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{x^2 + x + 1}{x^2 - 1}$; domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

93. $(f+g)(x) = \sqrt{x+7} + \sqrt{x-2}$; domain: $[2, \infty)$; $(f-g)(x) = \sqrt{x+7} - \sqrt{x-2}$; domain: $[2, \infty)$; $(fg)(x) = \sqrt{x^2+5x-14}$; domain: $[2, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+7}}{\sqrt{x-2}}$; domain: $(2, \infty)$ **94. a.** $(f \circ g)(x) = 16x^2 - 8x + 4$ **b.** $(g \circ f)(x) = 4x^2 + 11$ **c.** $(f \circ g)(3) = 124$

95. a. $(f \circ g)(x) = \sqrt{x+1}$ **b.** $(g \circ f)(x) = \sqrt{x} + 1$ **c.** $(f \circ g)(3) = 2$ **96. a.** $(f \circ g)(x) = \frac{1+x}{1-2x}$ **b.** $(-\infty, 0) \cup \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

97. a. $(f \circ g)(x) = \sqrt{x+2}$ **b.** $[-2, \infty)$ **98.** $f(x) = x^4, g(x) = x^2 + 2x - 1$ **99.** $f(x) = \sqrt[3]{x}, g(x) = 7x + 4$ **100.** $f(g(x)) = x - \frac{7}{10}$; $g(f(x)) = x - \frac{7}{6}$; f and g are not inverses of each other. **101.** $f(g(x)) = x$; $g(f(x)) = x$; f and g are inverses of each other. **102.** $f^{-1}(x) = \frac{x+3}{4}$

103. $f^{-1}(x) = \sqrt[3]{\frac{x-1}{8}}$ or $\frac{\sqrt[3]{x-1}}{2}$ **104.** $f^{-1}(x) = \frac{2}{x-5}$ **105.** Inverse function exists. **106.** Inverse function does not exist.

107. Inverse function exists. **108.** Inverse function does not exist.

109.

110. $f^{-1}(x) = \sqrt{1-x}$

111. $f^{-1}(x) = (x-1)^2, x \geq 1$

112. 13 **113.** $2\sqrt{2} \approx 2.83$ **114.** $(-5, 5)$ **115.** $\left(-\frac{11}{2}, -2\right)$ **116.** $x^2 + y^2 = 9$ **117.** $(x+2)^2 + (y-4)^2 = 36$

118. center: $(0, 0)$; radius: 1
domain: $[-1, 1]$; range: $[-1, 1]$

$x^2 + y^2 = 1$

119. center: $(-2, 3)$; radius: 3
domain: $[-5, 1]$; range: $[0, 6]$

$(x+2)^2 + (y-3)^2 = 9$

120. center: $(2, -1)$; radius: 3
domain: $[-1, 5]$; range: $[-4, 2]$

$x^2 + y^2 - 4x + 2y - 4 = 0$

121. a. $W(x) = 15x + 567$ **b.** 2009 **122. a.** $f(x) = 0.05x + 15$ **b.** $g(x) = 0.07x + 5$ **c.** 500 texts
123. a. $N(x) = 640 - 2x$ **b.** $R(x) = x(640 - 2x)$ **124. a.** $V(x) = x(16 - 2x)(24 - 2x)$ **b.** $(0, 8)$
125. $A(x) = x\left(\frac{400 - 3x}{2}\right)$ **126.** $A(x) = 2x^2 + \frac{32}{x}$ **127.** $I(x) = 0.08x + 0.12(10{,}000 - x)$

Chapter 1 Test

1. b, c, d **2. a.** $f(4) - f(-3) = 5$ **b.** $(-5, 6]$ **c.** $[-4, 5]$ **d.** $(-1, 2)$ **e.** $(-5, -1)$ or $(2, 6)$ **f.** 2; $f(2) = 5$ **g.** -1; $f(-1) = -4$
h. -4, 1, and 5 **i.** -3 **3. a.** -2 and 2 **b.** -1 and 1 **c.** 0 **d.** even **e.** no **f.** relative minimum

g.

h.

i.

j. $-\frac{1}{3}$

4. $x + y = 4$

domain: $(-\infty, \infty)$;
range: $(-\infty, \infty)$

5. $x^2 + y^2 = 4$

domain: $[-2, 2]$;
range: $[-2, 2]$

6. $f(x) = 4$

domain: $(-\infty, \infty)$;
range: $\{4\}$

7.

$f(x) = -\frac{1}{3}x + 2$

domain: $(-\infty, \infty)$;
range: $(-\infty, \infty)$

8.

$(x+2)^2 + (y-1)^2 = 9$

domain: $[-5, 1]$;
range: $[-2, 4]$

9.

$f(x) = \begin{cases} 2 \text{ if } x \leq 0 \\ -1 \text{ if } x > 0 \end{cases}$

domain: $(-\infty, \infty)$;
range: $\{-1, 2\}$

10.

$x^2 + y^2 + 4x - 6y - 3 = 0$

domain: $[-6, 2]$;
range: $[-1, 7]$

11.

domain of f = domain of $g = (-\infty, \infty)$;
range of $f = [0, \infty)$; range of $g = [-2, \infty)$

12.

domain of f = domain of $g = (-\infty, \infty)$;
range of $f = [0, \infty)$; range of $g = (-\infty, 4]$

13.

domain of f = range of $f^{-1} = (-\infty, \infty)$;
range of f = domain of $f^{-1} = (-\infty, \infty)$

14.

domain of f = range of $f^{-1} = (-\infty, \infty)$;
range of f = domain of $f^{-1} = (-\infty, \infty)$

15.

16. $f(x-1) = x^2 - 3x - 2$ **17.** $2x + h - 1$ **18.** $(g - f)(x) = -x^2 + 3x - 2$
19. $\left(\frac{f}{g}\right)(x) = \frac{x^2 - x - 4}{2x - 6}$; $(-\infty, 3) \cup (3, \infty)$ **20.** $(f \circ g)(x) = 4x^2 - 26x + 38$ **21.** $(g \circ f)(x) = 2x^2 - 2x - 14$
22. -10 **23.** $f(-x) = x^2 + x - 4$; neither **24.** using $(2, 1)$, $y - 1 = 3(x - 2)$; $y = 3x - 5$
25. $y - 6 = 4(x + 4)$; $y = 4x + 22$ **26.** $2x + y + 24 = 0$ **27. a.** using $(1, 4571)$, $y - 4571 = 433(x - 1)$
b. $f(x) = 433x + 4138$ **c.** 8468 deaths **28.** 48 **29.** $g(-1) = 4$; $g(7) = 2$ **30.** $(-\infty, -5) \cup (-5, 1) \cup (1, \infty)$

domain of f = range of $f^{-1} = [0, \infty)$;
range of f = domain of $f^{-1} = [-1, \infty)$

31. $[1, \infty)$ **32.** $\frac{7x}{2 - 4x}$; domain: $(-\infty, 0) \cup \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$ **33.** $f(x) = x^7$, $g(x) = 2x + 3$ **34.** 5; $\left(\frac{7}{2}, 0\right)$ or $(3.5, 0)$
35. a. $T(x) = 41.78 - 0.19x$ **b.** 2012 **36. a.** $Y(x) = 50 - 1.5(x - 30)$ **b.** $T(x) = x(50 - 1.5(x - 30))$ **37.** $A(x) = x(300 - x)$
38. $A(x) = 2x^2 + \frac{32{,}000}{x}$

CHAPTER 2

Section 2.1

Check Point Exercises

1. a. $8 + i$ **b.** $-10 + 7i$ **2. a.** $63 + 14i$ **b.** $58 - 11i$ **3.** $\frac{16}{17} + \frac{21}{17}i$ **4. a.** $7i\sqrt{3}$ **b.** $1 - 4i\sqrt{3}$ **c.** $-7 + i\sqrt{3}$ **5.** $\{1 \pm i\}$

Concept and Vocabulary Check

1. $\sqrt{-1}$; -1 **2.** complex; imaginary; real **3.** $-6i$ **4.** $14i$ **5.** 18; $-15i$; $12i$; $-10i^2$; 10 **6.** $2 + 9i$ **7.** $2 + 5i$ **8.** i; i; $2i\sqrt{5}$ **9.** $-1 \pm i\frac{\sqrt{6}}{2}$

Exercise Set 2.1

1. $8 - 2i$ **3.** $-2 + 9i$ **5.** $24 - 3i$ **7.** $-14 + 17i$ **9.** $21 + 15i$ **11.** $-19 + 7i$ **13.** $-29 - 11i$ **15.** 34 **17.** 26 **19.** $-5 + 12i$
21. $\frac{3}{5} + \frac{1}{5}i$ **23.** $1 + i$ **25.** $-\frac{24}{25} + \frac{32}{25}i$ **27.** $\frac{7}{5} + \frac{4}{5}i$ **29.** $3i$ **31.** $47i$ **33.** $-8i$ **35.** $2 + 6i\sqrt{7}$ **37.** $-\frac{1}{3} + i\frac{\sqrt{2}}{6}$ **39.** $-\frac{1}{8} - i\frac{\sqrt{3}}{24}$
41. $-2\sqrt{6} - 2i\sqrt{10}$ **43.** $24\sqrt{15}$ **45.** $\{3 \pm i\}$ **47.** $\left\{-1 \pm \frac{3}{2}i\right\}$ **49.** $\left\{\frac{4}{3} \pm i\frac{\sqrt{5}}{3}\right\}$ **51.** $-11 - 5i$ **53.** $-5 + 10i$ **55.** $0 + 47i$ or $47i$
57. 0 **59.** $\frac{20}{13} + \frac{30}{13}i$ **61.** $(47 + 13i)$ volts **63.** $(5 + i\sqrt{15}) + (5 - i\sqrt{15}) = 10$; $(5 + i\sqrt{15})(5 - i\sqrt{15}) = 25 - 15i^2 = 25 + 15 = 40$
73. makes sense **75.** does not make sense **77.** false **79.** false **81.** $\frac{14}{25} - \frac{2}{25}i$ **83.** $\frac{8}{5} + \frac{16}{5}i$ **84.** $\{1, 5\}$ **85.** $\{-1 \pm \sqrt{2}\}$
86.

$f(x) = x^2$
$g(x) = (x + 3)^2 + 1$

Section 2.2

Check Point Exercises

1.

$f(x) = -(x - 1)^2 + 4$

2.

$f(x) = (x - 2)^2 + 1$

3.

$f(x) = -x^2 + 4x + 1$

domain: $(-\infty, \infty)$; range: $(-\infty, 5]$

4. a. minimum **b.** Minimum is 984 at $x = 2$.
c. domain: $(-\infty, \infty)$; range: $[984, \infty)$
5. a. 205 ft; 200 ft **b.** 402 ft
c.

6. 4, -4; -16 **7.** 30 ft by 30 ft; 900 sq ft

Concept and Vocabulary Check

1. standard; parabola; (h, k); > 0; < 0 **2.** $-\frac{b}{2a}$; $f\left(-\frac{b}{2a}\right)$; $-\frac{b}{2a}$; $f\left(-\frac{b}{2a}\right)$ **3.** true **4.** false **5.** true **6.** $x - 8$; $x^2 - 8x$
7. $40 - x$; $-x^2 + 40x$

Exercise Set 2.2

1. $h(x) = (x - 1)^2 + 1$ **3.** $j(x) = (x - 1)^2 - 1$ **5.** $h(x) = x^2 - 1$ **7.** $g(x) = x^2 - 2x + 1$ **9.** $(3, 1)$ **11.** $(-1, 5)$
13. $(2, -5)$ **15.** $(-1, 9)$

17. domain: $(-\infty, \infty)$
range: $[-1, \infty)$
axis of symmetry: $x = 4$

$f(x) = (x - 4)^2 - 1$

19. domain: $(-\infty, \infty)$
range: $[2, \infty)$
axis of symmetry: $x = 1$

$f(x) = (x - 1)^2 + 2$

21. domain: $(-\infty, \infty)$
range: $[1, \infty)$
axis of symmetry: $x = 3$

$y - 1 = (x - 3)^2$

23. domain: $(-\infty, \infty)$
range: $[-1, \infty)$
axis of symmetry: $x = -2$

$f(x) = 2(x + 2)^2 - 1$

25. domain: $(-\infty, \infty)$
range: $(-\infty, 4]$
axis of symmetry: $x = 1$

$f(x) = 4 - (x - 1)^2$

27. domain: $(-\infty, \infty)$
range: $[-4, \infty)$
axis of symmetry: $x = 1$

$f(x) = x^2 - 2x - 3$

29. domain: $(-\infty, \infty)$
range: $\left[-\frac{49}{4}, \infty\right)$
axis of symmetry: $x = -\frac{3}{2}$

$f(x) = x^2 + 3x - 10$

31. domain: $(-\infty, \infty)$
range: $(-\infty, 4]$
axis of symmetry: $x = 1$

$f(x) = 2x - x^2 + 3$

33. domain: $(-\infty, \infty)$
range: $[-6, \infty)$
axis of symmetry: $x = -3$

$f(x) = x^2 + 6x + 3$

35. domain: $(-\infty, \infty)$
range: $[-5, \infty)$
axis of symmetry: $x = -1$

$f(x) = 2x^2 + 4x - 3$

37. domain: $(-\infty, \infty)$
range: $(-\infty, -1]$
axis of symmetry: $x = 1$

$f(x) = 2x - x^2 - 2$

39. a. minimum **b.** Minimum is -13 at $x = 2$. **c.** domain: $(-\infty, \infty)$; range: $[-13, \infty)$ **41. a.** maximum **b.** Maximum is 1 at $x = 1$.
c. domain: $(-\infty, \infty)$; range: $(-\infty, 1]$ **43. a.** minimum **b.** Minimum is $-\frac{5}{4}$ at $x = \frac{1}{2}$. **c.** domain: $(-\infty, \infty)$; range: $\left[-\frac{5}{4}, \infty\right)$
45. domain: $(-\infty, \infty)$; range: $[-2, \infty)$ **47.** domain: $(-\infty, \infty)$; range: $(-\infty, -6]$ **49.** $f(x) = 2(x - 5)^2 + 3$ **51.** $f(x) = 2(x + 10)^2 - 5$
53. $f(x) = -3(x + 2)^2 + 4$ **55.** $f(x) = 3(x - 11)^2$ **57. a.** 18.35 ft; 35 ft **b.** 77.8 ft **c.** 6.1 ft

59. a. 7.8 ft; 1.5 ft **b.** 4.6 ft **c.**

61. 8 and 8; 64 **63.** 8, -8; -64

65. length: 300 ft; width: 150 ft; maximum area: 45,000 sq ft **67.** 12.5 yd by 12.5 yd; 156.25 sq yd **69.** 150 ft by 100 ft; 15,000 sq ft
71. 5 in.; 50 sq in. **73.** \$65; \$422,500 **75.** 25; 1250 lb

85. (80, 1600)

87. (−4, 520)

89. a & e.

b. $f(x) = -0.68x^2 + 7.94x + 6.78$
c. 6; 29.9 million
d. The greatest number of viewers actually occurred in Season 5, not Season 6, and the model underestimates the greatest number by 0.5 million.

91. makes sense **93.** does not make sense **95.** false **97.** false **99.** $x = 3$; (0, 11) **101.** $f(x) = -2(x + 3)^2 - 1$
103. \$95; \$21,675 **106.** $(x + 3)(x + 1)(x - 1)$ **107.** $f(2) = -1$; $f(3) = 16$; The graph passes through $(2, -1)$, which is below the x-axis, and $(3, 16)$, which is above the x-axis. Since the graph of f is continuous, it must cross the x-axis somewhere between 2 and 3 to get from one of these points to the other. **108.** even; symmetric with respect to the y-axis

Section 2.3

Check Point Exercises

1. The graph rises to the left and to the right. **2.** The graph falls to the left and rises to the right. **3.** Since n is odd and the leading coefficient is negative, the graph falls to the right. Since the ratio cannot be negative, the model won't be appropriate. **4.** No; the graph should fall to the left but doesn't appear to do so. **5.** −2 and 2 **6.** −2, 0, and 2 **7.** $-\frac{1}{2}$ with multiplicity 2 and 5 with multiplicity 3; touches and turns at $-\frac{1}{2}$ and crosses at 5
8. $f(-3) = -42$; $f(-2) = 5$ **9.**

$f(x) = x^3 - 3x^2$

10.

$f(x) = 2(x + 2)^2(x - 3)$

Concept and Vocabulary Check

1. 5; −2 **2.** false **3.** end; leading **4.** falls; rises **5.** rises; falls **6.** rises; rises **7.** falls; falls **8.** true **9.** true
10. x-intercept **11.** turns around; crosses **12.** 0; Intermediate Value **13.** $n - 1$

Exercise Set 2.3

1. polynomial function; degree: 3 **3.** polynomial function; degree: 5 **5.** not a polynomial function **7.** not a polynomial function
9. not a polynomial function **11.** could be polynomial function **13.** not a polynomial function **15.** b **17.** a
19. falls to the left and rises to the right **21.** rises to the left and to the right **23.** falls to the left and to the right
25. $x = 5$ has multiplicity 1; The graph crosses the x-axis; $x = -4$ has multiplicity 2; The graph touches the x-axis and turns around.
27. $x = 3$ has multiplicity 1; The graph crosses the x-axis; $x = -6$ has multiplicity 3; The graph crosses the x-axis.
29. $x = 0$ has multiplicity 1; The graph crosses the x-axis; $x = 1$ has multiplicity 2; The graph touches the x-axis and turns around.
31. $x = 2, x = -2$ and $x = -7$ have multiplicity 1; The graph crosses the x-axis. **33.** $f(1) = -1$; $f(2) = 5$ **35.** $f(-1) = -1$; $f(0) = 1$
37. $f(-3) = -11$; $f(-2) = 1$ **39.** $f(-3) = -42$; $f(-2) = 5$

41. a. $f(x)$ rises to the right and falls to the left.
b. $x = -2, x = 1, x = -1$;
$f(x)$ crosses the x-axis at each.
c. The y-intercept is −2.
d. neither
e.

$f(x) = x^3 + 2x^2 - x - 2$

43. a. $f(x)$ rises to the left and the right.
b. $x = 0, x = 3, x = -3$;
$f(x)$ crosses the x-axis at −3 and 3;
$f(x)$ touches the x-axis at 0.
c. The y-intercept is 0.
d. y-axis symmetry
e.

$f(x) = x^4 - 9x^2$

45. a. $f(x)$ falls to the left and the right.
b. $x = 0, x = 4, x = -4$;
$f(x)$ crosses the x-axis at −4 and 4;
$f(x)$ touches the x-axis at 0.
c. The y-intercept is 0.
d. y-axis symmetry
e.

$f(x) = -x^4 + 16x^2$

47. a. $f(x)$ rises to the left and the right.
b. $x = 0, x = 1$;
$f(x)$ touches the x-axis at 0 and 1.
c. The y-intercept is 0.
d. neither
e.

$f(x) = x^4 - 2x^3 + x^2$

49. a. $f(x)$ falls to the left and the right.
b. $x = 0, x = 2$;
$f(x)$ crosses the x-axis at 0 and 2.
c. The y-intercept is 0.
d. neither
e.

$f(x) = -2x^4 + 4x^3$

51. a. $f(x)$ rises to the left and falls to the right.
b. $x = 0, x = \pm\sqrt{3}$;
$f(x)$ crosses the x-axis at 0;
$f(x)$ touches the x-axis at $\sqrt{3}$ and $-\sqrt{3}$.
c. The y-intercept is 0.
d. origin symmetry
e.

$f(x) = 6x^3 - 9x - x^5$

53. a. $f(x)$ rises to the left and falls to the right.
b. $x = 0, x = 3$;
$f(x)$ crosses the x-axis at 3;
$f(x)$ touches the x-axis at 0.
c. The y-intercept is 0.
d. neither
e.

$f(x) = 3x^2 - x^3$

55. a. $f(x)$ falls to the left and the right.
b. $x = 1, x = -2, x = 2$;
$f(x)$ crosses the x-axis at -2 and 2;
$f(x)$ touches the x-axis at 1.
c. The y-intercept is 12.
d. neither
e.

$f(x) = -3(x - 1)^2 (x^2 - 4)$

57. a. $f(x)$ rises to the left and the right.
b. $x = -2, x = 0, x = 1$;
$f(x)$ crosses the x-axis at -2 and 1;
$f(x)$ touches the x-axis at 0.
c. The y-intercept is 0.
d. neither
e.

$f(x) = x^2 (x - 1)^3 (x + 2)$

59. a. $f(x)$ falls to the left and the right.
b. $x = -3, x = 0, x = 1$;
$f(x)$ crosses the x-axis at -3 and 1;
$f(x)$ touches the x-axis at 0.
c. The y-intercept is 0.
d. neither
e.

$f(x) = -x^2 (x - 1)(x + 3)$

61. a. $f(x)$ falls to the left and the right.
b. $x = -5, x = 0, x = 1$;
$f(x)$ crosses the x-axis at -5 and 0;
$f(x)$ touches the x-axis at 1.
c. The y-intercept is 0.
d. neither
e.

$f(x) = -2x^3 (x - 1)^2 (x + 5)$

63. a. $f(x)$ rises to the left and the right.
b. $x = -4, x = 1, x = 2$;
$f(x)$ crosses the x-axis at -4 and 1;
$f(x)$ touches the x-axis at 2.
c. The y-intercept is -16.
d. neither
e.

$f(x) = (x - 2)^2 (x + 4)(x - 1)$

65. a. -2, odd; 1, odd; 4, odd **b.** $f(x) = (x + 2)(x - 1)(x - 4)$ **c.** 8 **67. a.** -1, odd; 3, even **b.** $f(x) = (x + 1)(x - 3)^2$ **c.** 9 **69. a.** -3, even; 2, even **b.** $f(x) = -(x + 3)^2(x - 2)^2$ **c.** -36 **71. a.** -2, even; -1, odd; 1, odd **b.** $f(x) = (x + 2)^2(x + 1)(x - 1)^3$ **c.** -4 **73. a.** 3167; The world tiger population in 2010 was approximately 3167.; (40, 3167) **b.** underestimates by 33 **c.** rises to the right; no; The model indicates an increasing world tiger population that will actually decrease without conservation efforts. **75. a.** from 1 through 4 min and from 8 through 10 min **b.** from 4 through 8 min and from 10 through 12 min **c.** 3 **d.** 4 **e.** negative; The graph falls to the left and falls to the right. **f.** 116 $\pm$ 1 beats per min; 10 min **g.** 64 $\pm$ 1 beats per min; 8 min

95.

97.

99.

101. does not make sense **103.** makes sense **105.** false **107.** false **109.** $f(x) = x^3 - 2x^2$

110. $35\frac{2}{21}$ **111.** $6x^3 - x^2 - 5x + 4$ **112.** $(x - 3)(2x - 1)(x + 2)$

Section 2.4

Check Point Exercises

1. $x + 5$ **2.** $2x^2 + 3x - 2 + \frac{1}{x - 3}$ **3.** $2x^2 + 7x + 14 + \frac{21x - 10}{x^2 - 2x}$ **4.** $x^2 - 2x - 3$ **5.** -105 **6.** $\left\{-1, -\frac{1}{3}, \frac{2}{5}\right\}$

Concept and Vocabulary Check

1. $2x^3 + 0x^2 + 6x - 4$ **2.** $6x^3; 3x; 2x^2; 7x^2$ **3.** $2x^2; 5x - 2; 10x^3 - 4x^2; 10x^3 + 6x^2$ **4.** $6x^2 - 10x; 6x^2 + 8x; 18x; -4; 18x - 4$

5. $9; 3x - 5; 9; 3x - 5 + \frac{9}{2x + 1}$ **6.** divisor; quotient; remainder; dividend **7.** 4; 1; 5; −7; 1 **8.** −5; 4; 0; −8; −2 **9.** true

10. $f(c)$ **11.** $x - c$

Exercise Set 2.4

1. $x + 3$ **3.** $x^2 + 3x + 1$ **5.** $2x^2 + 3x + 5$ **7.** $4x + 3 + \frac{2}{3x - 2}$ **9.** $2x^2 + x + 6 - \frac{38}{x + 3}$ **11.** $4x^3 + 16x^2 + 60x + 246 + \frac{984}{x - 4}$

13. $2x + 5$ **15.** $6x^2 + 3x - 1 - \frac{3x - 1}{3x^2 + 1}$ **17.** $2x + 5$ **19.** $3x - 8 + \frac{20}{x + 5}$ **21.** $4x^2 + x + 4 + \frac{3}{x - 1}$

23. $6x^4 + 12x^3 + 22x^2 + 48x + 93 + \frac{187}{x - 2}$ **25.** $x^3 - 10x^2 + 51x - 260 + \frac{1300}{x + 5}$ **27.** $x^4 + x^3 + 2x^2 + 2x + 2$ **29.** $x^3 + 4x^2 + 16x + 64$

31. $2x^4 - 7x^3 + 15x^2 - 31x + 64 - \frac{129}{x + 2}$ **33.** −25 **35.** −133 **37.** 240 **39.** 1 **41.** $x^2 - 5x + 6; x = -1, x = 2, x = 3$

43. $\left\{-\frac{1}{2}, 1, 2\right\}$ **45.** $\left\{-\frac{3}{2}, -\frac{1}{3}, \frac{1}{2}\right\}$ **47.** 2; The remainder is zero; $\{-3, -1, 2\}$ **49.** 1; The remainder is zero; $\left\{\frac{1}{3}, \frac{1}{2}, 1\right\}$

51. a. The remainder is 0. **b.** 3 mm **53.** $0.5x^2 - 0.4x + 0.3$ **55. a.** 70; When the tax rate is 30%, tax revenue is \$700 billion.; (30, 70)

b. $80 + \frac{800}{x - 110}$; $f(30) = 70$; yes **c.** No, f is a rational function because it is a quotient of two polynomials. **67.** makes sense

69. does not make sense **71.** true **73.** false **75.** $x - 2$ **79.** $\{-2 \pm \sqrt{5}\}$ **80.** $\{-2 \pm i\sqrt{2}\}$ **81.** −3

Section 2.5

Check Point Exercises

1. $\pm 1, \pm 2, \pm 3, \pm 6$ **2.** $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{3}{2}, \pm\frac{3}{4}$ **3.** −5, −4, and 1 **4.** $2, \frac{-3 - \sqrt{5}}{2}$, and $\frac{-3 + \sqrt{5}}{2}$ **5.** $\{1, 2 - 3i, 2 + 3i\}$

6. $f(x) = x^3 + 3x^2 + x + 3$ **7.** 4, 2, or 0 positive zeros, no possible negative zeros

Concept and Vocabulary Check

1. $a_0; a_n$ **2.** true **3.** false **4.** n **5.** $a - bi$ **6.** $-6; (x + 6)(2x^2 - x - 1) = 0$ **7.** $n; 1$ **8.** false **9.** true **10.** true

Exercise Set 2.5

1. $\pm 1, \pm 2, \pm 4$ **3.** $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{3}, \pm\frac{2}{3}$ **5.** $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{3}{2}, \pm\frac{3}{4}$ **7.** $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ **9. a.** $\pm 1, \pm 2, \pm 4$

b. −2, −1, or 2 **c.** −2, −1, and 2 **11. a.** $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$ **b.** $-2, \frac{1}{2}$, or 3 **c.** $-2, \frac{1}{2}$, and 3 **13. a.** $\pm 1, \pm 2, \pm 3, \pm 6$

b. −1 **c.** $-1, \frac{-3 - \sqrt{33}}{2}$, and $\frac{-3 + \sqrt{33}}{2}$ **15. a.** $\pm 1, \pm\frac{1}{2}, \pm 2$ **b.** −2 **c.** $-2, \frac{-1 + i}{2}$, and $\frac{-1 - i}{2}$ **17. a.** $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b. −3, 1, or 4 **c.** $\{-3, 1, 4\}$ **19. a.** $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ **b.** −2 **c.** $\{-2, 1 + \sqrt{7}, 1 - \sqrt{7}\}$ **21. a.** $\pm 1, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

b. $-5, \frac{1}{3}$, or $\frac{1}{2}$ **c.** $\left\{-5, \frac{1}{3}, \frac{1}{2}\right\}$ **23. a.** $\pm 1, \pm 2, \pm 4$ **b.** −2 or 2 **c.** $\{-2, 2, 1 + \sqrt{2}, 1 - \sqrt{2}\}$ **25.** $f(x) = 2x^3 - 2x^2 + 50x - 50$

27. $f(x) = x^3 - 3x^2 - 15x + 125$ **29.** $f(x) = x^4 + 10x^2 + 9$ **31.** $f(x) = x^4 - 9x^3 + 21x^2 + 21x - 130$ **33.** no positive real roots; 3 or 1 negative real roots **35.** 3 or 1 positive real roots; no negative real roots **37.** 2 or 0 positive real roots; 2 or 0 negative real roots

39. −2, 5, and 1 **41.** $\left\{-\frac{1}{2}, \frac{1 + \sqrt{17}}{2}, \frac{1 - \sqrt{17}}{2}\right\}$ **43.** $-1, 2 + 2i$, and $2 - 2i$ **45.** $\{-1, -2, 3 + \sqrt{13}, 3 - \sqrt{13}\}$

47. $-1, 2, -\frac{1}{3}$, and 3 **49.** $\left\{1, -\frac{3}{4}, i\sqrt{2}, -i\sqrt{2}\right\}$ **51.** $\left\{-2, \frac{1}{2}, \sqrt{2}, -\sqrt{2}\right\}$

53. a. −4, 1, and 4

b.

$f(x) = -x^3 + x^2 + 16x - 16$

55. a. −1 and $\frac{3}{2}$

b.

$f(x) = 4x^3 - 8x^2 - 3x + 9$

57. a. $\frac{1}{2}, 3, -1 \pm i$

b.

$f(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$

59. a. $-2, -1, -\frac{2}{3}$, 1, and 2

b.

$f(x) = 3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$

61. 7.8 in., 10 in. **63. a.** (7.8, 2000), (10, 2000) **b.** (0, 15) **73.** $\left\{\frac{1}{2}, \frac{2}{3}, 2\right\}$ **75.** $\left\{\pm\frac{1}{2}\right\}$ **77.** 5, 3, or 1 positive real zeros; no negative real zeros

79.

1 real zero, 2 nonreal complex zeros

81.

2 real zeros, 2 nonreal complex zeros

83. makes sense **85.** makes sense **87.** false
89. true **91.** 3 in. **93.** 3 **95.** 5
98. $x = 1$ and $x = 2$ **99.** $x = 1$ **100.** $y = 0$

Mid-Chapter 2 Check Point

1. $-1 - i$ **2.** $-3 + 6i$ **3.** $7 + i$ **4.** i **5.** $3i\sqrt{3}$ **6.** $1 - 4i\sqrt{3}$ **7.** $\frac{3}{4} \pm i\frac{\sqrt{23}}{4}$

8.

domain: $(-\infty, \infty)$
range: $[-4, \infty)$

9.

domain: $(-\infty, \infty)$
range: $(-\infty, 5]$

10.

domain: $(-\infty, \infty)$
range: $(-\infty, 9]$

11.

domain: $(-\infty, \infty)$
range: $[-2, \infty)$

12. -1 and 2

13. -1 and 2

14. -2, 1, and 2

15. $-2, -1$, 1, and 2

16. -1

17. $-\frac{1}{3}, \frac{1}{2}$, and 1

18. -1, 0, and 1

19. $0, 1 \pm 5i$

20. $3, 1 \pm \sqrt{2}$

21. $\{-2, 1\}$ **22.** $\left\{\frac{1}{3}, \frac{1}{2}, 1\right\}$ **23.** $\left\{-\frac{1}{2}, \frac{2}{3}, \frac{7}{2}\right\}$ **24.** $\left\{-10, -\frac{5}{2}, 10\right\}$ **25.** $\{-3, 4, \pm i\}$

26. $\left\{-3, \frac{1}{2}, 1 \pm \sqrt{3}\right\}$ **27.** 75 cabinets per day; \$1200 **28.** $-9, -9$; 81 **29.** 10 in.; 100 sq in.

30. $2x^2 - x - 3 + \frac{x + 1}{3x^2 - 1}$ **31.** $2x^3 - 5x^2 - 3x + 6$ **32.** $f(x) = -2x^3 + 2x^2 - 2x + 2$

33. $f(x) = x^4 - 4x^3 + 13x^2 - 36x + 36$ **34.** yes

Section 2.6

Check Point Exercises

1. a. $\{x \mid x \neq 5\}$ or $(-\infty, 5) \cup (5, \infty)$ **b.** $\{x \mid x \neq -5, x \neq 5\}$ or $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$ **c.** all real numbers or $(-\infty, \infty)$
2. a. $x = 1, x = -1$ **b.** $x = -1$ **c.** none **3. a.** $y = 3$ **b.** $y = 0$ **c.** none

4.

5.

6.

7.

8. $y = 2x - 1$ **9. a.** $C(x) = 500{,}000 + 400x$ **b.** $\overline{C}(x) = \dfrac{500{,}000 + 400x}{x}$ **c.** $\overline{C}(1000) = 900$: The average cost per wheelchair of producing 1000 wheelchairs per month is \$900.; $\overline{C}(10{,}000) = 450$: The average cost per wheelchair of producing 10,000 wheelchairs per month is \$450.; $\overline{C}(100{,}000) = 405$: The average cost per wheelchair of producing 100,000 wheelchairs per month is \$405. **d.** $y = 400$; The cost per wheelchair approaches \$400 as more wheelchairs are produced. **10.** $T(x) = \dfrac{20}{x} + \dfrac{20}{x - 10}$

Concept and Vocabulary Check

1. polynomial **2.** false **3.** true **4.** vertical asymptote; $x = -5$ **5.** horizontal asymptote; $y = 0$; $y = \dfrac{1}{3}$ **6.** true **7.** left; down **8.** one more than **9.** $y = 3x + 5$ **10.** $x - 20$; $\dfrac{30}{x - 20}$

Exercise Set 2.6

1. $\{x \mid x \neq 4\}$ **3.** $\{x \mid x \neq 5, x \neq -4\}$ **5.** $\{x \mid x \neq 7, x \neq -7\}$ **7.** all real numbers **9.** $-\infty$ **11.** $-\infty$ **13.** 0 **15.** $+\infty$ **17.** $-\infty$ **19.** 1 **21.** vertical asymptote: $x = -4$; no holes **23.** vertical asymptotes: $x = -4, x = 0$; no holes **25.** vertical asymptote: $x = -4$; hole at $x = 0$ **27.** no vertical asymptotes; no holes **29.** no vertical asymptotes; hole at $x = 3$ **31.** vertical asymptote: $x = -3$; hole at $x = 3$ **33.** vertical asymptote: $x = 3$; hole at $x = -7$ **35.** no vertical asymptotes; hole at $x = -7$ **37.** $y = 0$ **39.** $y = 4$ **41.** no horizontal asymptote **43.** $y = -\dfrac{2}{3}$

45.

$g(x) = \dfrac{1}{x - 1}$

47.

$h(x) = \dfrac{1}{x} + 2$

49.

$g(x) = \dfrac{1}{x + 1} - 2$

51.

$g(x) = \dfrac{1}{(x + 2)^2}$

53.

$h(x) = \dfrac{1}{x^2} - 4$

55.

$h(x) = \dfrac{1}{(x - 3)^2} + 1$

57.

$f(x) = \dfrac{4x}{x - 2}$

59.

$f(x) = \dfrac{2x}{x^2 - 4}$

61.

$f(x) = \dfrac{2x^2}{x^2 - 1}$

63.

$f(x) = \dfrac{-x}{x + 1}$

65.

$f(x) = -\dfrac{1}{x^2 - 4}$

67.

$f(x) = \dfrac{2}{x^2 + x - 2}$

69.

$f(x) = \dfrac{2x^2}{x^2 + 4}$

71.

$f(x) = \dfrac{x + 2}{x^2 + x - 6}$

73.

$f(x) = \dfrac{x - 2}{x^2 - 4}$

75.

$f(x) = \dfrac{x^4}{x^2 + 2}$

77.

$f(x) = \dfrac{x^2 + x - 12}{x^2 - 4}$

79.

$f(x) = \dfrac{3x^2 + x - 4}{2x^2 - 5x}$

81. a. Slant asymptote: $y = x$

b.

$f(x) = \dfrac{x^2 - 1}{x}$

83. a. Slant asymptote: $y = x$

b.

$f(x) = \dfrac{x^2 + 1}{x}$

85. a. Slant asymptote: $y = x + 4$

b.

$f(x) = \dfrac{x^2 + x - 6}{x - 3}$

87. a. Slant asymptote: $y = x - 2$

b.

$f(x) = \dfrac{x^3 + 1}{x^2 + 2x}$

89.

$f(x) = \dfrac{x + 2}{2x(x - 2)}$

91.

$f(x) = \dfrac{x - 6}{2(x - 3)}$

93.

$f(x) = \dfrac{x - 2}{x + 2}$

95. $g(x) = \dfrac{1}{x + 3} + 2$

$g(x) = \dfrac{1}{x + 3} + 2$

97. $g(x) = \dfrac{-1}{x - 2} + 3$

$g(x) = \dfrac{-1}{x - 2} + 3$

99. a. $C(x) = 100x + 100{,}000$ **b.** $\overline{C}(x) = \dfrac{100x + 100{,}000}{x}$ **c.** $\overline{C}(500) = 300$, when 500 bicycles are produced, it costs \$300 to produce each bicycle; $\overline{C}(1000) = 200$, when 1000 bicycles are produced, it costs \$200 to produce each bicycle; $\overline{C}(2000) = 150$, when 2000 bicycles are produced, it costs \$150 to produce each bicycle; $\overline{C}(4000) = 125$, when 4000 bicycles are produced, it costs \$125 to produce each bicycle. **d.** $y = 100$; The cost per bicycle approaches \$100 as more bicycles are produced. **101. a.** 6.0 **b.** after 6 minutes; about 4.8 **c.** 6.5 **d.** $y = 6.5$; Over time, the pH level rises back to normal. **e.** It quickly drops below normal and then slowly begins to approach the normal level. **103.** 90; An incidence ratio of 10 means 90% of the deaths are smoking related. **105.** $y = 100$; The percentage of deaths cannot exceed 100% as the incidence ratios increase. **107. a.** $f(x) = \dfrac{1.75x^2 - 15.9x + 160}{2.1x^2 - 3.5x + 296}$ **b.** 67% **c.** 66%; underestimates by 1% **d.** $y = \dfrac{1.75}{2.1}$; 83% **109.** $T(x) = \dfrac{10}{x} + \dfrac{5}{x}$ **111.** $A(x) = 2x + \dfrac{50}{x} + 52$

123.

The graph approaches the horizontal asymptote faster and the vertical asymptote slower as n increases.

125. a.

b. The graph increases and reaches a maximum of about 356 arrests per 100,000 drivers at age 25.

c. at age 25, about 356 arrests

127. does not make sense **129.** does not make sense **131.** true **133.** true **138.** $\left\{-3, \dfrac{5}{2}\right\}$ **139.** $\{-2, -1, 2\}$ **140.** $\dfrac{-x - 5}{x + 3}$ or $-\dfrac{x + 5}{x + 3}$

Section 2.7

Check Point Exercises

1. $\{x \mid x < -4 \text{ or } x > 5\}$ or $(-\infty, -4) \cup (5, \infty)$

2. $\left\{x \,\middle|\, \dfrac{-3 - \sqrt{7}}{2} \le x \le \dfrac{-3 + \sqrt{7}}{2}\right\}$ or $\left[\dfrac{-3 - \sqrt{7}}{2}, \dfrac{-3 + \sqrt{7}}{2}\right]$

3. $\{x \mid x \le -3 \text{ or } -1 \le x \le 1\}$ or $(-\infty, -3] \cup [-1, 1]$

4. $\{x \mid x < -1 \text{ or } x \ge 1\}$ or $(-\infty, -1) \cup [1, \infty)$

5. between 1 and 4 seconds, excluding $t = 1$ and $t = 4$

Concept and Vocabulary Check

1. $x^2 + 8x + 15 = 0$; boundary **2.** $(-\infty, -5)$; $(-5, -3)$; $(-3, \infty)$ **3.** true **4.** true **5.** $(-\infty, -2) \cup [1, \infty)$

Exercise Set 2.7

1. $(-\infty, -2) \cup (4, \infty)$

3. $[-3, 7]$

5. $(-\infty, 1) \cup (4, \infty)$

7. $(-\infty, -4) \cup (-1, \infty)$

9. $\varnothing$

11. $\left[-4, \frac{2}{3}\right]$

13. $\left(-3, \frac{5}{2}\right)$

15. $\left(-1, -\frac{3}{4}\right)$

17. $\left[-2, \frac{1}{3}\right]$

19. $(-\infty, 0] \cup [4, \infty)$

21. $\left(-\infty, -\frac{3}{2}\right) \cup (0, \infty)$

23. $[0, 1]$

25. $[2 - \sqrt{2}, 2 + \sqrt{2}]$

27. $\varnothing$

29. $[1, 2] \cup [3, \infty)$

31. $[0, 3] \cup [5, \infty)$

33. $(-\infty, 2) \cup \left(2, \frac{7}{2}\right)$

35. $[-2, -1] \cup [1, \infty)$

37. $(-\infty, -3)$

39. $(-1, \infty)$

41. $\{0\} \cup [9, \infty)$

43. $(-\infty, -3) \cup (4, \infty)$

45. $(-4, -3)$

47. $[2, 4)$

49. $\left(-\infty, -\frac{4}{3}\right) \cup [2, \infty)$

51. $(-\infty, 0) \cup (3, \infty)$

53. $(-\infty, -4] \cup (-2, 1]$

55. $(-\infty, -5) \cup (-3, \infty)$

57. $\left(-\infty, \frac{1}{2}\right) \cup \left[\frac{7}{5}, \infty\right)$

59. $(-\infty, -6] \cup (-2, \infty)$

61. $\left(-\infty, \frac{1}{2}\right] \cup [2, \infty)$

63. $(-\infty, -1) \cup [1, \infty)$

65. $(-\infty, -8) \cup (-6, 4) \cup (6, \infty)$

67. $(-3, 2)$

69. $(-\infty, -1) \cup (1, 2) \cup (3, \infty)$

71. $\left[-6, -\frac{1}{2}\right] \cup [1, \infty)$ **73.** $(-\infty, -2) \cup [-1, 2)$ **75.** between 0 and $\frac{1}{2}$ second **77. a.** dry: 160 ft; wet: 185 ft **b.** dry pavement: graph (b); wet pavement: graph (a) **c.** extremely well; Function values and data are identical. **d.** speeds exceeding 76 miles per hour; points on (b) to the right of (76, 540) **79.** The sides (in feet) are in (0, 6] or [19, 25). **87.** $\left[-3, \frac{1}{2}\right]$ **89.** $(1, 4]$ **91.** $(-4, -1) \cup [2, \infty)$

93. a. $f(x) = 0.1375x^2 + 0.7x + 37.8$ **b.** speeds exceeding 52 miles per hour **95.** does not make sense **97.** does not make sense

99. false **101.** true **103.** Answers may vary. One possible solution is $\frac{x-3}{x+4} \geq 0$. **105.** $\{2\}$ **107.** $(-\infty, 2) \cup (2, \infty)$

109.
$$\begin{aligned} 27 - 3x^2 &\geq 0 \\ 3x^2 &\leq 27 \\ x^2 &\leq 9 \\ -3 \leq x &\leq 3 \end{aligned}$$

110. a. 16 **b.** $y = 16x^2$ **c.** 400 **111. a.** 96 **b.** $y = \frac{96}{x}$ **c.** 32 **112.** 8

Section 2.8

Check Point Exercises

1. 66 gal **2.** 9375 lb **3.** 512 cycles per second **4.** 24 min **5.** 96π cubic feet

Concept and Vocabulary Check

1. $y = kx$; constant of variation **2.** $y = kx^n$ **3.** $y = \frac{k}{x}$ **4.** $y = \frac{kx}{z}$ **5.** $y = kxz$ **6.** directly; inversely **7.** jointly; inversely

Exercise Set 2.8

1. 156 **3.** 30 **5.** $\frac{5}{6}$ **7.** 240 **9.** 50 **11.** $x = kyz$; $y = \frac{x}{kz}$ **13.** $x = \frac{kz^3}{y}$; $y = \frac{kz^3}{x}$ **15.** $x = \frac{kyz}{\sqrt{w}}$; $y = \frac{x\sqrt{w}}{kz}$

17. $x = kz(y + w)$; $y = \frac{x - kzw}{kz}$ **19.** $x = \frac{kz}{y - w}$; $y = \frac{xw + kz}{x}$ **21.** 5.4 ft **23.** 80 in. **25.** about 607 lb **27.** 32°

29. 90 milliroentgens per hour **31.** This person has a BMI of 24.4 and is not overweight. **33.** 1800 Btu per hour

35. $\frac{1}{4}$ of what it was originally **37. a.** $C = \frac{kP_1P_2}{d^2}$ **b.** $k \approx 0.02$; $C = \frac{0.02\,P_1P_2}{d^2}$ **c.** 39,813 daily phone calls

39. a.

b. Current varies inversely as resistance. **c.** $R = \dfrac{6}{I}$

49. does not make sense **51.** makes sense **53.** The destructive power is four times as much. **55.** Reduce the resistance by a factor of $\dfrac{1}{3}$.

58.

59.

60.

Chapter 2 Review Exercises

1. $-9 + 4i$ **2.** $-12 - 8i$ **3.** $17 + 19i$ **4.** $-7 - 24i$ **5.** 113 **6.** $\dfrac{15}{13} - \dfrac{3}{13}i$ **7.** $\dfrac{1}{5} + \dfrac{11}{10}i$ **8.** $i\sqrt{2}$ **9.** $-96 - 40i$
10. $2 + i\sqrt{2}$ **11.** $\{1 + i\sqrt{3}, 1 - i\sqrt{3}\}$ **12.** $\left\{\dfrac{3}{2} + \dfrac{1}{2}i, \dfrac{3}{2} - \dfrac{1}{2}i\right\}$

13.

axis of symmetry; $x = -1$
domain: $(-\infty, \infty)$; range: $(-\infty, 4]$

14.

axis of symmetry: $x = -4$
domain: $(-\infty, \infty)$; range: $[-2, \infty)$

15.

axis of symmetry: $x = 1$
domain: $(-\infty, \infty)$; range: $(-\infty, 4]$

16.

axis of symmetry: $x = 1$
domain: $(-\infty, \infty)$; range: $[-8, \infty)$

17. a. maximum is -57 at $x = 7$ **b.** domain: $(-\infty, \infty)$; range: $(-\infty, -57]$ **18. a.** minimum is 685 at $x = -3$ **b.** domain: $(-\infty, \infty)$; range: $[685, \infty)$
19. a. 16 ft; 20 yd **b.** 6 ft **c.** 45.3 yd **d.**

20. 250 yd by 500 yd; 125,000 sq yard **21.** -7 and 7; -49

22. $x = 166\dfrac{2}{3}$ ft by $y = 125$ ft **23.** 36; 5256 lb **24.** c **25.** b **26.** a **27.** d **28. a.** rises to the right **b.** no; The model indicates increasing deforestation despite a declining rate in which the forest is being cut down. **c.** falls to the right **d.** no; The model indicates the amount of forest cleared, in square kilometers, will eventually be negative, which is not possible. **29.** No; the graph falls to the right, so eventually there would be a negative number of thefts, which is not possible.
30. $x = 1$, multiplicity 1, crosses; $x = -2$, multiplicity 2, touches; $x = -5$, multiplicity 3, crosses
31. $x = -5$, multiplicity 1, crosses; $x = 5$, multiplicity 2, touches
32. $f(1)$ is negative and $f(2)$ is positive, so by the Intermediate Value Theorem, f has a real zero between 1 and 2.

33. a. The graph falls to the left and rises to the right.
b. no symmetry
c.

34. a. The graph rises to the left and falls to the right.
b. origin symmetry
c.

35. a. The graph falls to the left and rises to the right.
b. no symmetry
c.

36. a. The graph falls to the left and to the right.
b. y-axis symmetry
c.

$f(x) = -x^4 + 25x^2$

37. a. The graph falls to the left and to the right.
b. no symmetry
c.

$f(x) = -x^4 + 6x^3 - 9x^2$

38. a. The graph rises to the left and to the right.
b. no symmetry
c.

$f(x) = 3x^4 - 15x^3$

39.

$f(x) = 2x^2(x-1)^3(x+2)$

40.

$f(x) = -x^3(x+4)^2(x-1)$

41. $4x^2 - 7x + 5 - \dfrac{4}{x+1}$

42. $2x^2 - 4x + 1 - \dfrac{10}{5x-3}$

43. $2x^2 + 3x - 1$ **44.** $3x^3 - 4x^2 + 7$

45. $3x^3 + 6x^2 + 10x + 10 + \dfrac{20}{x-2}$ **46.** -5697

47. $2, \frac{1}{2}, -3$ **48.** $\{4, -2 \pm \sqrt{5}\}$ **49.** $\pm 1, \pm 5$ **50.** $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{8}{3}, \pm\frac{4}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}$ **51.** 2 or 0 positive real zeros; no negative real zeros

52. 3 or 1 positive real zeros; 2 or 0 negative real zeros **53.** No sign variations exist for either $f(x)$ or $f(-x)$, so no real roots exist.

54. a. $\pm 1, \pm 2, \pm 4$ **b.** 1 positive real zero; 2 or no negative real zeros **c.** -2 or 1 **d.** $-2, 1$

55. a. $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}$ **b.** 2 or 0 positive real zeros; 1 negative real zero **c.** $-1, \frac{1}{3}$, or $\frac{1}{2}$ **d.** $-1, \frac{1}{3}, \frac{1}{2}$

56. a. $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{8}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{3}{8}, \pm\frac{5}{2}, \pm\frac{5}{4}, \pm\frac{5}{8}, \pm\frac{15}{2}, \pm\frac{15}{4}, \pm\frac{15}{8}$ **b.** 3 or 1 positive real solutions; no negative real solutions **c.** $\frac{1}{2}, \frac{3}{2}$, or $\frac{5}{2}$ **d.** $\left\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right\}$

57. a. $\pm 1, \pm\frac{1}{2}$ **b.** 2 or 0 positive real solutions; 1 negative solution **c.** $\frac{1}{2}$ **d.** $\left\{\frac{1}{2}, \frac{-5-\sqrt{29}}{2}, \frac{-5+\sqrt{29}}{2}\right\}$

58. a. $\pm 1, \pm 2, \pm 3, \pm 6$ **b.** 2 or zero positive real solutions; 2 or zero negative real solutions **c.** $-2, -1, 1$, or 3 **d.** $\{-2, -1, 1, 3\}$

59. a. $\pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{4}$ **b.** 1 positive real root; 1 negative real root **c.** $-\frac{1}{2}$ or $\frac{1}{2}$ **d.** $\left\{-\frac{1}{2}, \frac{1}{2}, i\sqrt{2}, -i\sqrt{2}\right\}$

60. a. $\pm 1, \pm 2, \pm 4, \pm\frac{1}{2}$ **b.** 2 or no positive zeros; 2 or no negative zeros **c.** $-2, -1, \frac{1}{2}$, or 2 **d.** $-2, -1, \frac{1}{2}, 2$

61. $f(x) = x^3 - 6x^2 + 21x - 26$ **62.** $f(x) = 2x^4 + 12x^3 + 20x^2 + 12x + 18$ **63.** $-2, \frac{1}{2}, \pm i$; $f(x) = (x-i)(x+i)(x+2)(2x-1)$

64. $-1, 4$; $g(x) = (x+1)^2(x-4)^2$ **65.** 4 real zeros, one with multiplicity two **66.** 3 real zeros; 2 nonreal complex zeros

67. 2 real zeros, one with multiplicity two; 2 nonreal complex zeros **68.** 1 real zero; 4 nonreal complex zeros

69.

$g(x) = \dfrac{1}{(x+2)^2} - 1$

70.

$h(x) = \dfrac{1}{x-1} + 3$

71. Vertical asymptotes: $x = 3$ and $x = -3$
horizontal asymptote: $y = 0$

$f(x) = \dfrac{2x}{x^2 - 9}$

72. Vertical asymptote: $x = -3$
horizontal asymptote: $y = 2$

$f(x) = \dfrac{2x-4}{x+3}$

73. Vertical asymptotes: $x = 3$ and $x = -2$
horizontal asymptote: $y = 1$

$h(x) = \dfrac{x^2 - 3x - 4}{x^2 - x - 6}$

74. Vertical asymptote: $x = -2$
horizontal asymptote: $y = 1$

$r(x) = \dfrac{x^2 + 4x + 3}{(x+2)^2}$

75. Vertical asymptote: $x = -1$
no horizontal asymptote
slant asymptote: $y = x - 1$

$y = \dfrac{x^2}{x+1}$

76. Vertical asymptote: $x = 3$
no horizontal asymptote
slant asymptote: $y = x + 5$

$y = \dfrac{x^2 + 2x - 3}{x - 3}$

77. No vertical asymptote
no horizontal asymptote
slant asymptote: $y = -2x$

78. Vertical asymptote: $x = \frac{3}{2}$
no horizontal asymptote
slant asymptote: $y = 2x - 5$

79. a. $C(x) = 25x + 50{,}000$
b. $\overline{C}(x) = \dfrac{25x + 50{,}000}{x}$
c. $\overline{C}(50) = 1025$, when 50 calculators are manufactured, it costs \$1025 to manufacture each; $\overline{C}(100) = 525$, when 100 calculators are manufactured, it costs \$525 to manufacture each; $\overline{C}(1000) = 75$, when 1000 calculators are manufactured, it costs \$75 to manufacture each; $\overline{C}(100{,}000) = 25.5$, when 100,000 calculators are manufactured, it costs \$25.50 to manufacture each.
d. $y = 25$; costs will approach \$25.

80. $y = 3000$; The number of fish in the pond approaches 3000.
81. $y = 0$; As the number of years of education increases the percentage rate of unemployment approaches zero.
82. a. $P(x) = 2.99x + 235.5$ **b.** $R(x) = \dfrac{1.53x + 114.8}{2.99x + 235.5}$ **c.** $y = 0.51$; Over time the percentage of men in the U.S. population will approach 51%.
83. $T(x) = \dfrac{4}{x + 3} + \dfrac{2}{x}$ **84.** $P(x) = 2x + \dfrac{2000}{x}$

85. $\left(-3, \frac{1}{2}\right)$

86. $(-\infty, -4] \cup \left[-\frac{1}{2}, \infty\right)$

87. $(-3, 0) \cup (1, \infty)$

88. $(-\infty, -2) \cup (6, \infty)$

89. $[-1, 1) \cup [2, \infty)$

90. $(-\infty, 4) \cup \left[\frac{23}{4}, \infty\right)$

91. a. 261 ft; overestimates by 1 ft **b.** speeds exceeding 40 miles per hour **92.** from 1 to 2 sec **93.** 134.4 cm^3 **94.** 1600 ft
95. 440 vibrations per second **96.** 112 decibels **97.** 16 hr **98.** 800 ft^3 **99. a.** $L = \dfrac{1890}{R}$ **b.** an approximate model **c.** 70 yr

Chapter 2 Test

1. $47 + 16i$ **2.** $2 + i$ **3.** $38i$ **4.** $\{2 \pm i\}$

5.

axis of symmetry: $x = -1$
domain: $(-\infty, \infty)$; range: $[4, \infty)$

6.

axis of symmetry: $x = 1$
domain: $(-\infty, \infty)$; range: $[-4, \infty)$

7. a. maximum of 2 at $x = 3$;
b. domain: $(-\infty, \infty)$; range: $(-\infty, 2]$
8. 23 computers;
maximum daily profit = \$16,900
9. 7 and 7; 49

10. a. 5, 2, −2
b.

11. Since the degree of the polynomial is odd and the leading coefficient is positive, the graph of f should fall to the left and rise to the right. The x-intercepts should be −1, 0, and 1.
12. a. 2 **b.** $\frac{1}{2}, \frac{2}{3}$ **13.** $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$
14. 3 or 1 positive real zeros; no negative real zeros

15. $\{-3, -3 - \sqrt{11}, -3 + \sqrt{11}\}$ **16. a.** $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$ **b.** $-\sqrt{5}, -1, \frac{3}{2}$, and $\sqrt{5}$ **17.** $(x - 1)(x + 2)^2$
18. $f(x) = 2x^4 - 2$ **19.** -1 and $\frac{2}{3}$ **20.** domain: $\{x | x \neq -3\}$ **21.** domain: $\{x | x \neq 1\}$

22. domain: $\{x \mid x \neq 4, x \neq -4\}$

$f(x) = \dfrac{x}{x^2 - 16}$

23. domain: $\{x \mid x \neq 2\}$

$f(x) = \dfrac{x^2 - 9}{x - 2}$

24. domain: $\{x \mid x \neq -3, x \neq 1\}$

$f(x) = \dfrac{x + 1}{x^2 + 2x - 3}$

25. domain: all real numbers

$f(x) = \dfrac{4x^2}{x^2 + 3}$

26. a. $\overline{C}(x) = \dfrac{300{,}000 + 10x}{x}$ **b.** $y = 10$; As the number of satellite radio players increases, the average cost approaches \$10.

27. $(-3, 4)$

28. $(-\infty, 3) \cup [10, \infty)$

29. 45 foot-candles

Cumulative Review Exercises (Chapters P–2)

1. domain: $(-2, 2)$; range: $[0, \infty)$ **2.** -1 and 1, both of multiplicity 2 **3.** 0 **4.** 3 **5.** $x \to -2^+; x \to 2^-$

6.

$g(x) = f(x + 2) + 1$

7. $\{2, -1\}$ **8.** $\left\{\dfrac{5 + \sqrt{13}}{6}, \dfrac{5 - \sqrt{13}}{6}\right\}$ **9.** $\left\{\dfrac{1}{3}, -\dfrac{2}{3}\right\}$ **10.** $\{-3, -1, 2\}$ **11.** $(-\infty, 1) \cup (4, \infty)$

12. $(-\infty, -1) \cup \left(\dfrac{5}{3}, \infty\right)$

13.

$f(x) = x^3 - 4x^2 - x + 4$

14.

$f(x) = x^2 + 2x - 8$

15.

$f(x) = x^2(x - 3)$

16.

$f(x) = \dfrac{x - 1}{x - 2}$

17.

18.

$x^2 + y^2 - 2x + 4y - 4 = 0$

19. $(f \circ g)(x) = 32x^2 - 20x + 2$ **20.** $4x + 2h - 1$

CHAPTER 3

Section 3.1

Check Point Exercises

1. approximately \$160; overestimates by \$11

2.

$f(x) = 3^x$

3.

$f(x) = \left(\dfrac{1}{3}\right)^x$

4.

$f(x) = 3^x$
$g(x) = 3^{x-1}$

5.

$f(x) = 2^x$
$g(x) = 2^x + 1$

6. approximately 4446 **7. a.** \$14,859.47 **b.** \$14,918.25

Concept and Vocabulary Check

1. b^x; $(-\infty, \infty)$; $(0, \infty)$ **2.** x; $y = 0$; horizontal **3.** e; natural; 2.72 **4.** A; P; r; n **5.** semiannually; quarterly; continuous

Exercise Set 3.1

1. 10.556 **3.** 11.665 **5.** 0.125 **7.** 9.974 **9.** 0.387

11.

$f(x) = 4^x$

13.

$g(x) = \left(\dfrac{3}{2}\right)^x$

15.

$h(x) = \left(\dfrac{1}{2}\right)^x$

17.

$f(x) = (0.6)^x$

19. $H(x) = -3^{-x}$
21. $F(x) = -3^x$
23. $h(x) = 3^x - 1$

25.

$f(x) = 2^x$
$g(x) = 2^{x+1}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

27.

$f(x) = 2^x$
$g(x) = 2^x - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$
range: $(-1, \infty)$

29.

$f(x) = 2^x$
$h(x) = 2^{x+1} - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$
range: $(-1, \infty)$

31.

$f(x) = 2^x$
$g(x) = -2^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(-\infty, 0)$

33.

$f(x) = 2^x$
$g(x) = 2 \cdot 2^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

35.

$g(x) = e^{x-1}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

37.

$g(x) = e^x + 2$

asymptote: $y = 2$
domain: $(-\infty, \infty)$
range: $(2, \infty)$

39.

$h(x) = e^{x-1} + 2$

asymptote: $y = 2$
domain: $(-\infty, \infty)$
range: $(2, \infty)$

41.

$h(x) = e^{-x}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

43.

$g(x) = 2e^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$
range: $(0, \infty)$

45.

$h(x) = e^{2x} + 1$

asymptote: $y = 1$
domain: $(-\infty, \infty)$
range: $(1, \infty)$

47.

$f(x) = 3^x$
$g(x) = 3^{-x}$

asymptote of f: $y = 0$
asymptote of g: $y = 0$

49.

$f(x) = 3^x$
$g(x) = \frac{1}{3} \cdot 3^x$

asymptote of f: $y = 0$
asymptote of g: $y = 0$

51.

$f(x) = \left(\frac{1}{2}\right)^x$
$g(x) = \left(\frac{1}{2}\right)^{x-1} + 1$

asymptote of f: $y = 0$
asymptote of g: $y = 1$

53. **a.** \$13,116.51
b. \$13,140.67
c. \$13,157.04
d. \$13,165.31

55. 7% compounded monthly

57. $(0, 1)$

59.

61. $y = 4^x$ **63.** $y = -e^x$ **65.** **a.** 574 million **b.** 1148 million **c.** 2295 million **d.** 4590 million **e.** It appears to double. **67.** \$832,744
69. 3.249009585; 3.317278183; 3.321880096; 3.321995226; 3.321997068; $2^{\sqrt{3}} \approx 3.321997085$; The closer the exponent is to $\sqrt{3}$, the closer the value is to $2^{\sqrt{3}}$.
71. **a.** \$15,166 **b.** \$16,037 **c.** linear model **73.** **a.** 100% **b.** $\approx 68.5\%$ **c.** $\approx 30.8\%$ **d.** $\approx 20\%$ **75.** the function g

81. **a.** $A = 10{,}000\left(1 + \frac{0.05}{4}\right)^{4t}$; $A = 10{,}000\left(1 + \frac{0.045}{12}\right)^{12t}$

b.

5% interest compounded quarterly

83. does not make sense
85. does not make sense
87. false
89. false
91. $y = 3^x$ is (d); $y = 5^x$ is (c); $y = \left(\frac{1}{3}\right)^x$ is (a); $y = \left(\frac{1}{5}\right)^x$ is (b).

93. **a.** $\cosh(-x) = \frac{e^{-x} + e^{-(-x)}}{2} = \frac{e^{-x} + e^x}{2} = \frac{e^x + e^{-x}}{2} = \cosh x$

b. $\sinh(-x) = \frac{e^{-x} - e^{-(-x)}}{2} = \frac{e^{-x} - e^x}{2} = -\frac{e^x - e^{-x}}{2} = -\sinh x$

c. $\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 \stackrel{?}{=} 1$

$\frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} \stackrel{?}{=} 1$

$\frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4} \stackrel{?}{=} 1$

$\frac{4}{4} \stackrel{?}{=} 1$

$1 = 1$

94. We don't know how to solve $x = 2^y$ for y. **95.** $\frac{1}{2}$ **96.** $(-\infty, 3) \cup (3, \infty)$

Section 3.2

Check Point Exercises

1. a. $7^3 = x$ **b.** $b^2 = 25$ **c.** $4^y = 26$ **2. a.** $5 = \log_2 x$ **b.** $3 = \log_b 27$ **c.** $y = \log_e 33$ **3. a.** 2 **b.** -3 **c.** $\frac{1}{2}$ **d.** $\frac{1}{7}$

4. a. 1 **b.** 0 **5. a.** 8 **b.** 17 **6.**

7. $(5, \infty)$ **8.** 80% **9.** 4.0 **10. a.** $(-\infty, 4)$ **b.** $(-\infty, 0) \cup (0, \infty)$ **11.** 34°; extremely well

Concept and Vocabulary Check

1. $b^y = x$ **2.** logarithmic; b **3.** 1 **4.** 0 **5.** x **6.** x **7.** $(0, \infty)$; $(-\infty, \infty)$ **8.** y; $x = 0$; vertical **9.** up 5 units **10.** to the left 5 units **11.** x-axis **12.** y-axis **13.** $5 - x > 0$ **14.** common; $\log x$ **15.** natural; $\ln x$

Exercise Set 3.2

1. $2^4 = 16$ **3.** $3^2 = x$ **5.** $b^5 = 32$ **7.** $6^y = 216$ **9.** $\log_2 8 = 3$ **11.** $\log_2 \frac{1}{16} = -4$ **13.** $\log_8 2 = \frac{1}{3}$ **15.** $\log_{13} x = 2$

17. $\log_b 1000 = 3$ **19.** $\log_7 200 = y$ **21.** 2 **23.** 6 **25.** -1 **27.** -3 **29.** $\frac{1}{2}$ **31.** $-\frac{1}{2}$ **33.** $\frac{1}{2}$ **35.** 1 **37.** 0 **39.** 7 **41.** 19

43.

45.

47. $H(x) = 1 - \log_3 x$ **49.** $h(x) = \log_3 x - 1$ **51.** $g(x) = \log_3(x - 1)$

53.

vertical asymptote: $x = -1$
domain: $(-1, \infty)$
range: $(-\infty, \infty)$

55.

vertical asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

57.

vertical asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

59.

asymptote: $x = 1$
domain: $(1, \infty)$
range: $(-\infty, \infty)$

61.

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

63.

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

65.

asymptote: $x = -2$
domain: $(-2, \infty)$
range: $(-\infty, \infty)$

67.

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

69.

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

71.

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

73.

asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

75. $(-4, \infty)$ **77.** $(-\infty, 2)$ **79.** $(-\infty, 2) \cup (2, \infty)$ **81.** 2 **83.** 7 **85.** 33 **87.** 0 **89.** 6 **91.** -6 **93.** 125 **95.** $9x$

97. $5x^2$ **99.** $\sqrt{x}$ **101.** $3^2 = x - 1$; $\{10\}$ **103.** $4^{-3} = x$; $\left\{\frac{1}{64}\right\}$ **105.** 0 **107.** 2 **109.** $(-\infty, -1) \cup (2, \infty)$ **111.** $(-\infty, -1) \cup (5, \infty)$

113. 95.4% **115. a.** 25.5%; underestimates by 0.7% **b.** 24.2% **117.** $\approx$188 db; yes