

CS1010S Programming Methodology

Lecture 2

Functional Abstraction

24 Jan 2018

Expectations

Tutorial Allocation

Coursemology Survey

- Choose your preferred slots
- As many slots as possible
- Set up classes to suit everyone

Recitation

Appeal on CORS

classes starts

TOMORROW

Late Policy

- < 10 min: OK
- < 30 min: -10%
- < 12 hours: -20%
- < 24 hours: -50%
- > 24 hours: -100%

Ask early for extensions

Submission is Final

But please remember to click

Finalize Submission

Don't Stress

But please do
your work

Try NOT to submit at 23:58

Operators

Assignment

a = 5

Equality testing

a == 5

Not equal

a != 5

Backslash \

Escape character

```
print('That's')  
print('That\'s')
```

#Comments

```
# this is not a hashtag!
```

```
print("Good to go")
```

```
#print("Good to go")
```

```
# whatever is after the # is ignored
```

```
if light == "red": # Check state of light
```

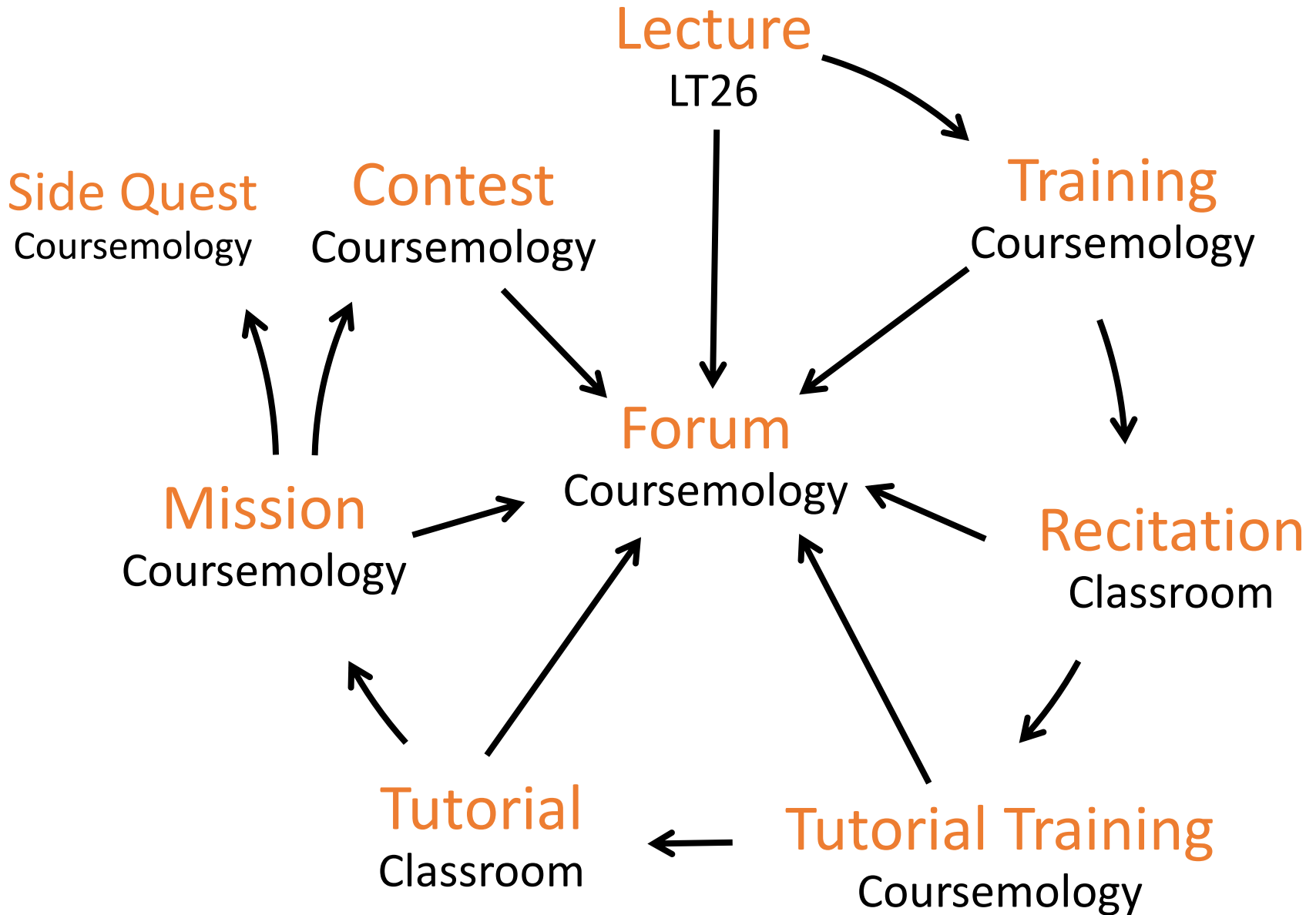
What's this?

Python Imaging Library



```
from PIL import *
```

(Misison 0)



Forums

Post reflections for
EXP

Trainings

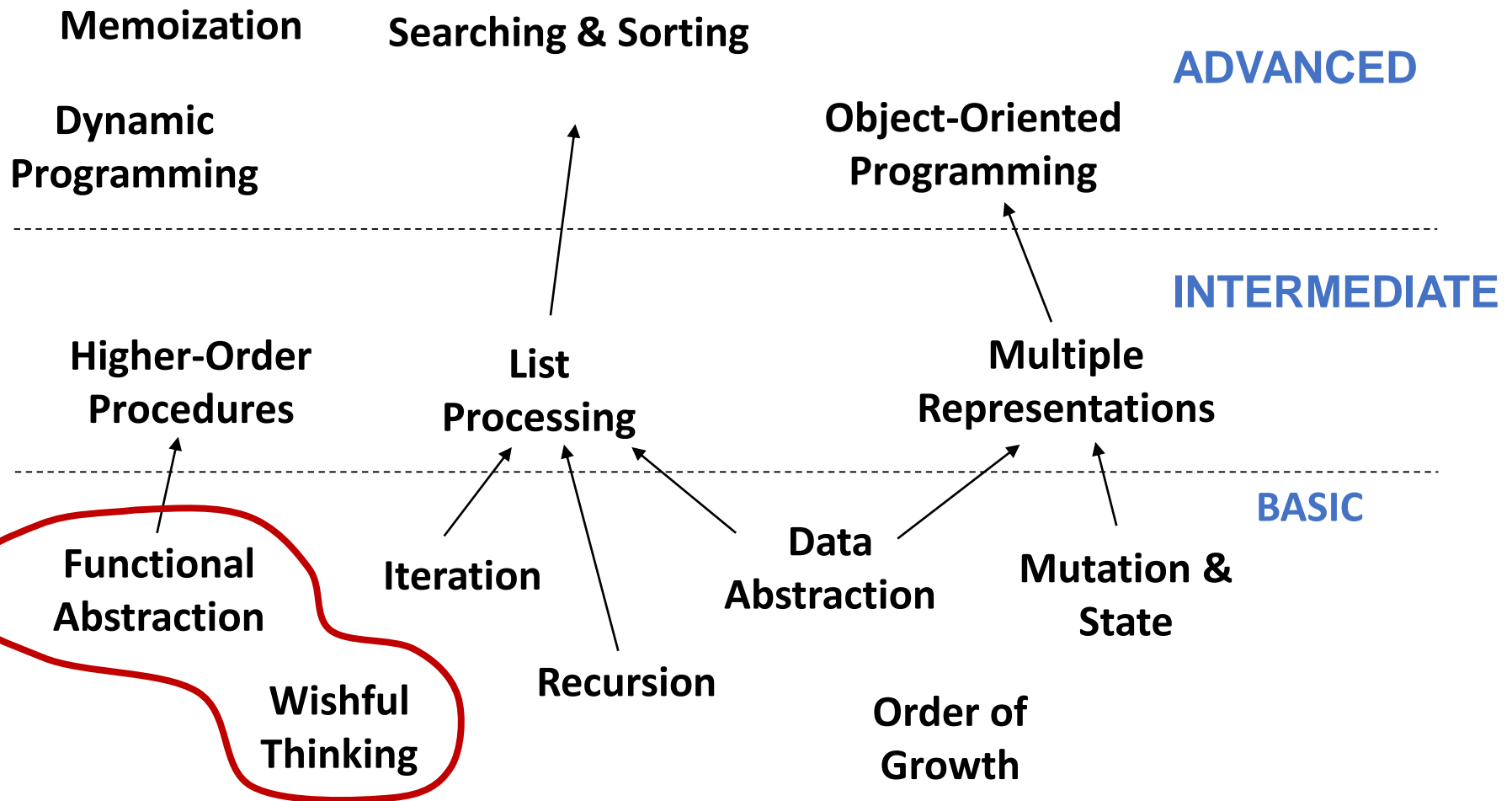
Please don't
anyhow hantam

Computational Thinking

Fasten your seatbelt



CS1010S Road Map



Fundamental concepts of computer programming

Functional Abstraction

WHAT

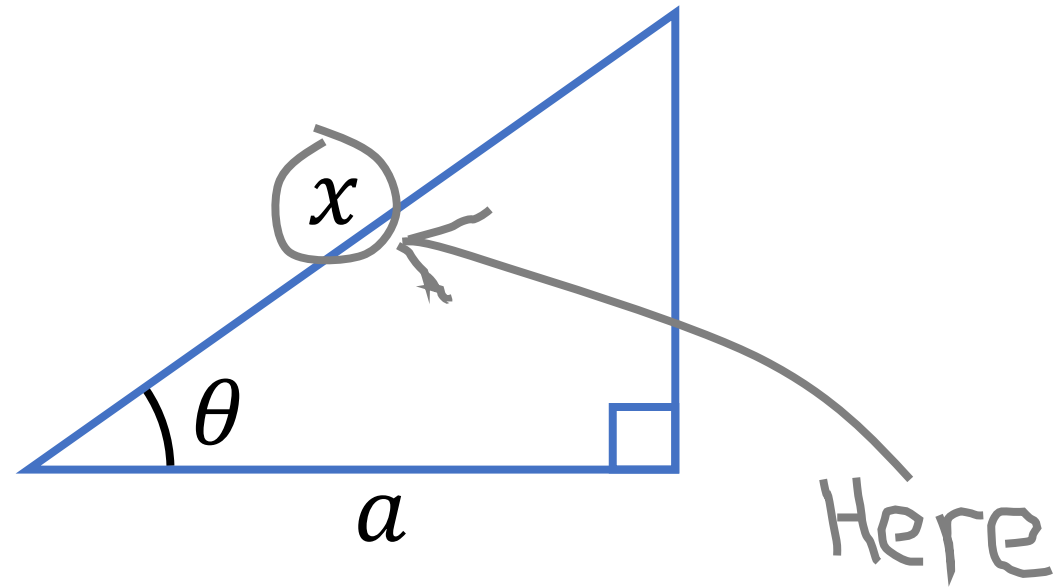
HOW

WHY

What is a function?



Functions are nothing new



Find x ?

$$x = \frac{\cos(\theta)}{a}$$

input

function

Let's start with
something easier

Question

How do we square a
number?

The square function

Define

Name

Input

```
def square(x):  
    return x * x
```

The diagram illustrates the components of the Python function definition `def square(x): return x * x`. It uses color-coding: `def` and `return` are orange, `square` is blue, and `x` is black. Arrows point from labels to the corresponding parts of the code: 'Define' to `def`, 'Name' to `square`, 'Input' to `x`, and 'Return' to `return`. A bracket under the expression `x * x` is labeled 'Output'.

Return

Output

square(21) 441

square(2 + 5) 49

square(square(3)) 81

Another function

```
def sum_of_squares(x, y):  
    return square(x) + square(y)
```

```
sum_of_squares(3, 4)    25
```

And another

```
from math import sqrt
```

```
def hypotenuse(a, b):  
    return sqrt(sum_of_squares(a, b))
```

```
hypotenuse(5, 12)    13
```

General Form

```
def <name> (<formal parameters>):  
    <body>
```

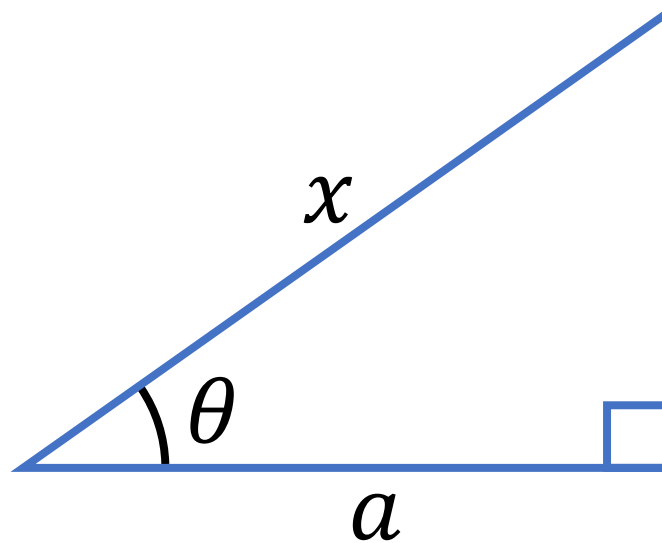
- **name**
 - Symbol associated with the function
- **formal parameters**
 - Names used in the body to refer to the arguments of the function
- **body**
 - The statement(s) to be evaluated
 - Has to be indented (standard is 4 spaces)
 - Can return values as output

Black Box



Don't need to know how it works
Just know what it does

Black Box



$$x = \frac{a}{\cos(\theta)}$$

Do you know how **cos** work?

Black Box



As long as we know what it does,
we can use it.

↖ (the inputs
and output)

Return Type



Output is returned with **return**
Return type can be **None**

Abstract Environment

Picture Language

(runes.py)

Also graphics.py + PyGif.py

Elements of Programming

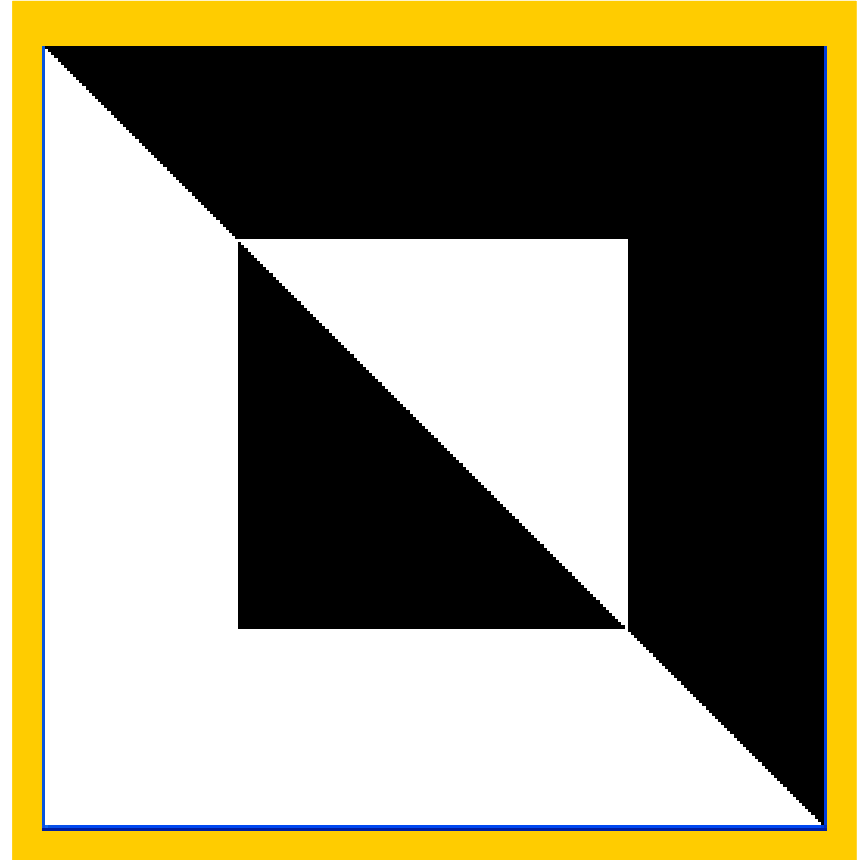
1. Primitives
2. Means of Combination
3. Means of Abstraction
4. Controlling Logic

Primitives building block

`show(rcross_bb)`



Picture object



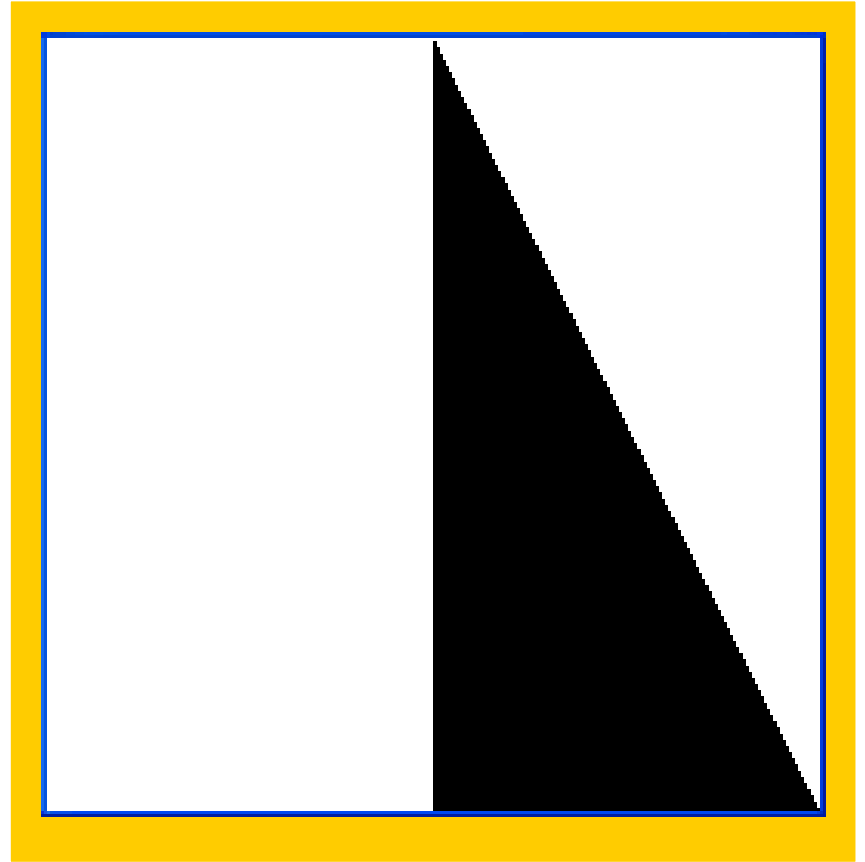
Primitives building block

`show(corner_bb)`



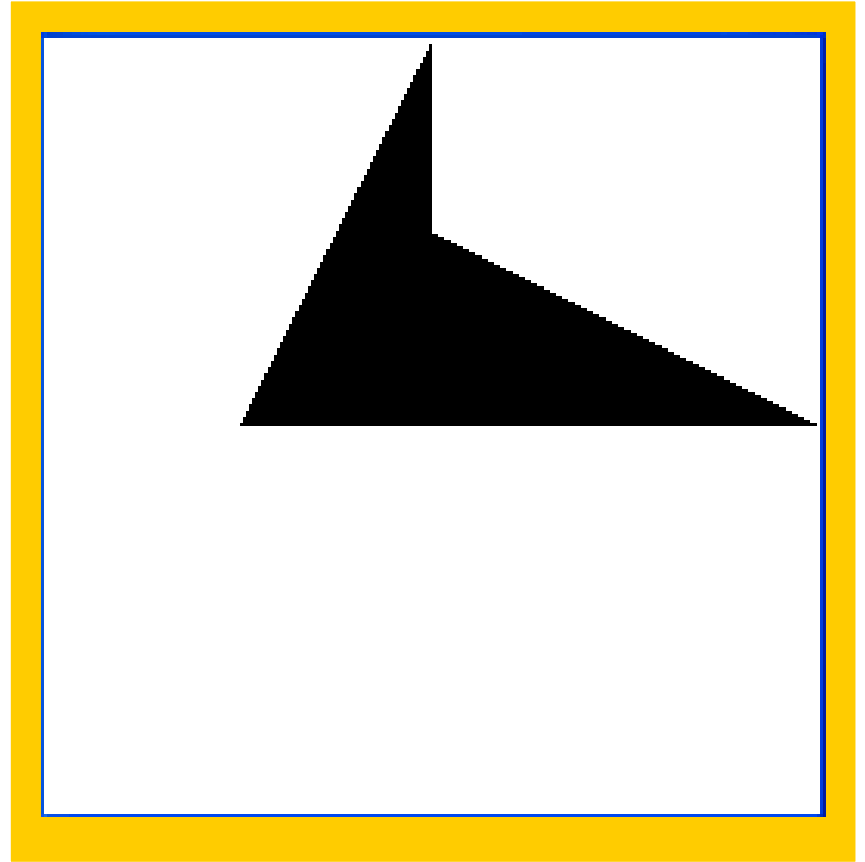
Primitives building block

`show(sail_bb)`



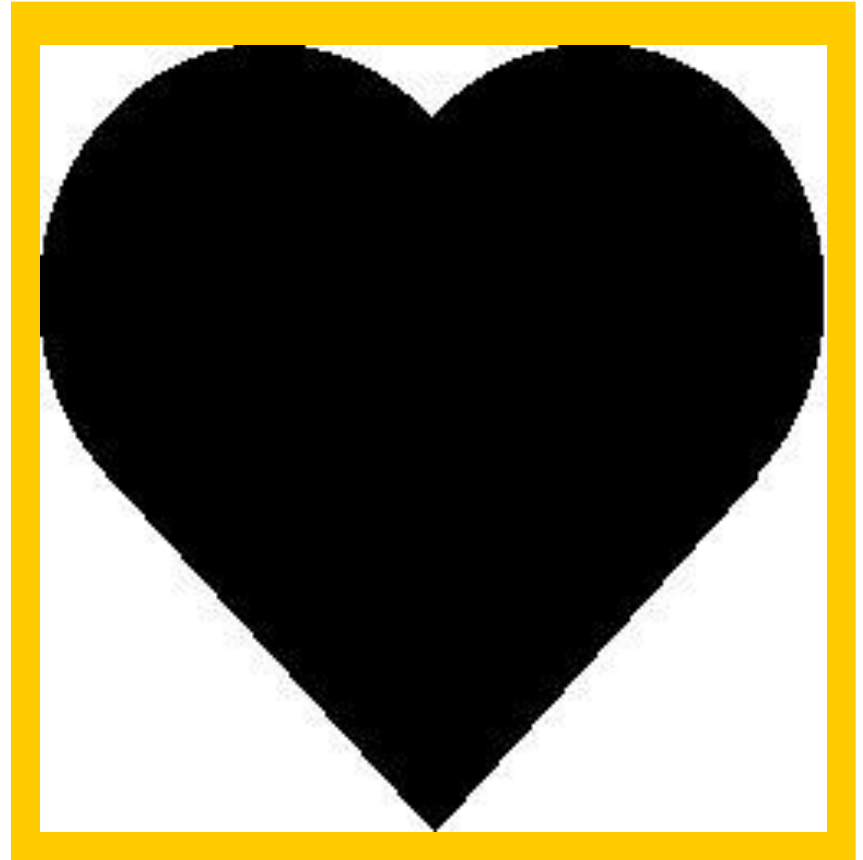
Primitives building block

`show(nova_bb)`



Primitives building block

`show(heart_bb)`



Applying operations

`op(picture)`

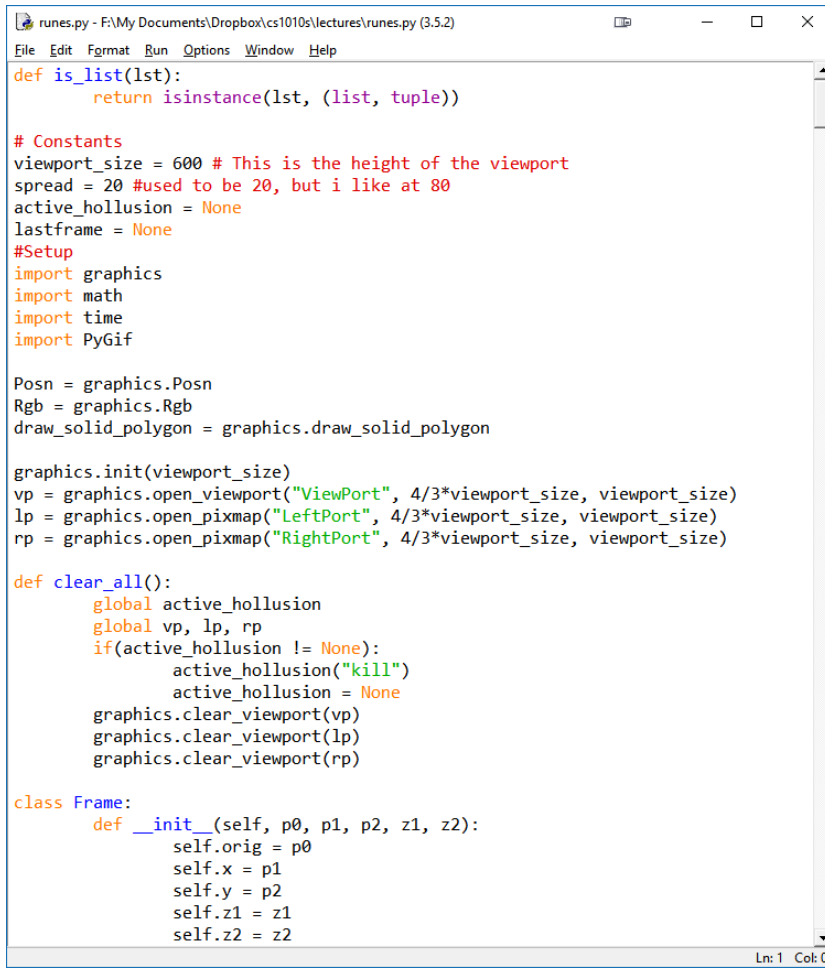
function name

input(s)

Example:

`show(heart_bb)`

Fun with IDLE



```
runes.py - F:\My Documents\Dropbox\cs1010s\lectures\runes.py (3.5.2)
File Edit Format Run Options Window Help
def is_list(lst):
    return isinstance(lst, (list, tuple))

# Constants
viewport_size = 600 # This is the height of the viewport
spread = 20 #used to be 20, but i like at 80
active_hollusion = None
lastframe = None

#Setup
import graphics
import math
import time
import PyGif

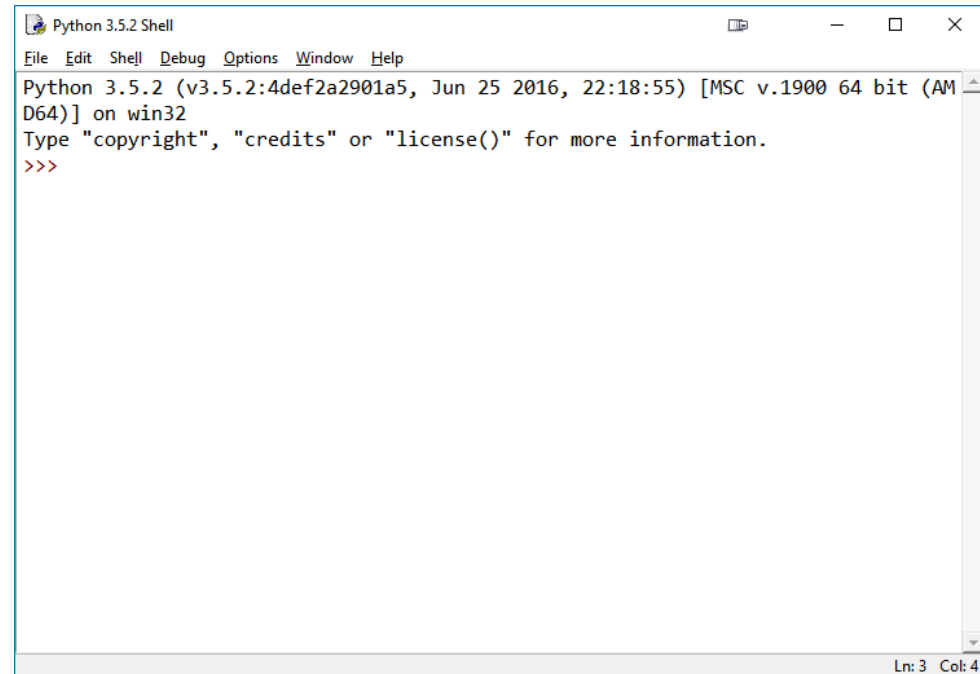
Posn = graphics.Posn
Rgb = graphics.Rgb
draw_solid_polygon = graphics.draw_solid_polygon

graphics.init(viewport_size)
vp = graphics.open_viewport("ViewPort", 4/3*viewport_size, viewport_size)
lp = graphics.open_pixmap("LeftPort", 4/3*viewport_size, viewport_size)
rp = graphics.open_pixmap("RightPort", 4/3*viewport_size, viewport_size)

def clear_all():
    global active_hollusion
    global vp, lp, rp
    if(active_hollusion != None):
        active_hollusion("kill")
        active_hollusion = None
    graphics.clear_viewport(vp)
    graphics.clear_viewport(lp)
    graphics.clear_viewport(rp)

class Frame:
    def __init__(self, p0, p1, p2, z1, z2):
        self.orig = p0
        self.x = p1
        self.y = p2
        self.z1 = z1
        self.z2 = z2
```

Ln: 1 Col: 0



```
Python 3.5.2 Shell
File Edit Shell Debug Options Window Help
Python 3.5.2 (v3.5.2:4def2a2901a5, Jun 25 2016, 22:18:55) [MSC v.1900 64 bit (AMD64)] on win32
Type "copyright", "credits" or "license()" for more information.
>>>
```

Ln: 3 Col: 4

Font matters

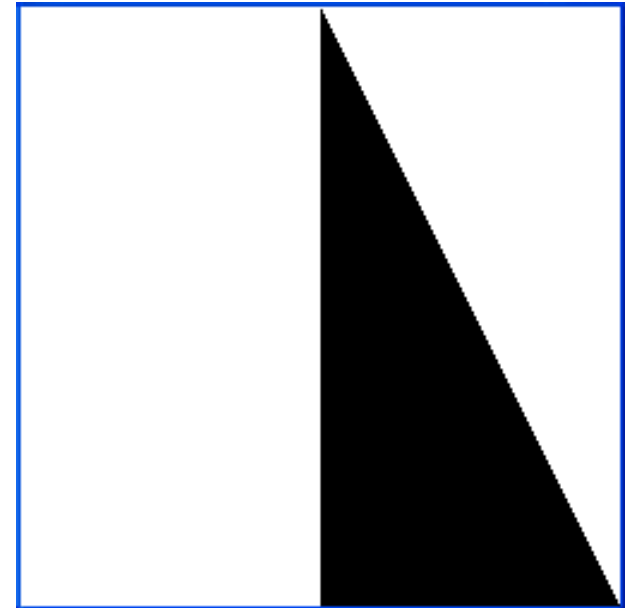
Primitive Operation

Rotating to the Right

`clear_all()` operation picture
`show(quarter_turn_right(sail_bb))`

The diagram shows the code `show(quarter_turn_right(sail_bb))` with orange curly braces. One brace is under `quarter_turn_right` and another is under `sail_bb`. A larger brace is under the entire expression `quarter_turn_right(sail_bb)`. The word `show` is to the left of the first brace.

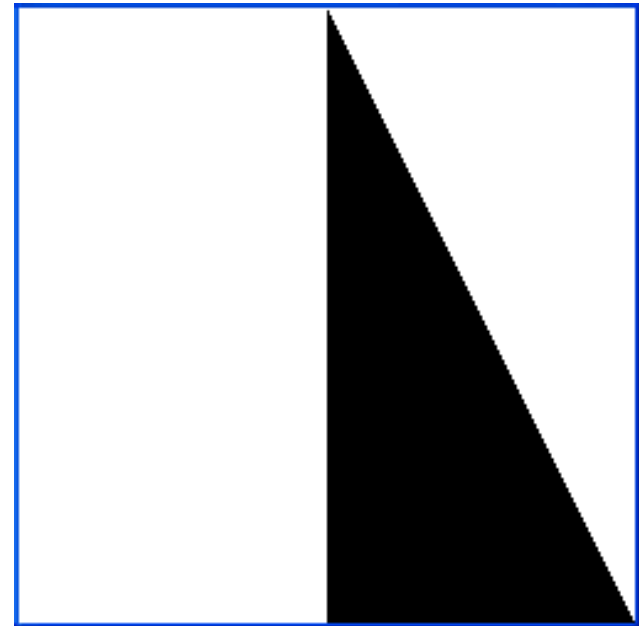
result is
another picture



Derived Operation

Rotating Upside Down

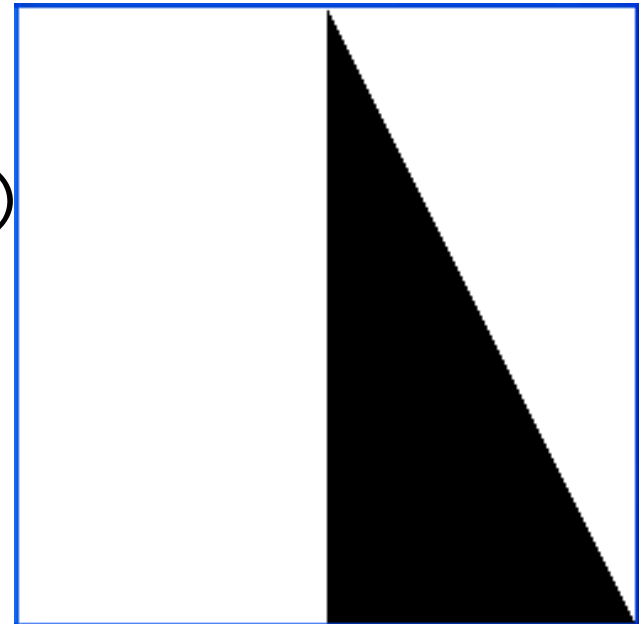
```
def turn_upside_down(pic):  
    return quarter_turn_right(  
        quarter_turn_right(pic))  
  
clear_all()  
show(turn_upside_down(sail_bb))
```



How about Rotating to the Left?

```
def quarter_turn_left(pic):  
    return quarter_turn_right(  
        quarter_turn_upside_down(pic))
```

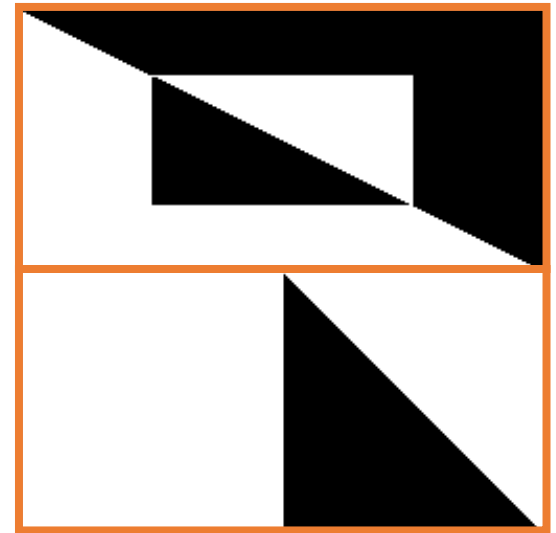
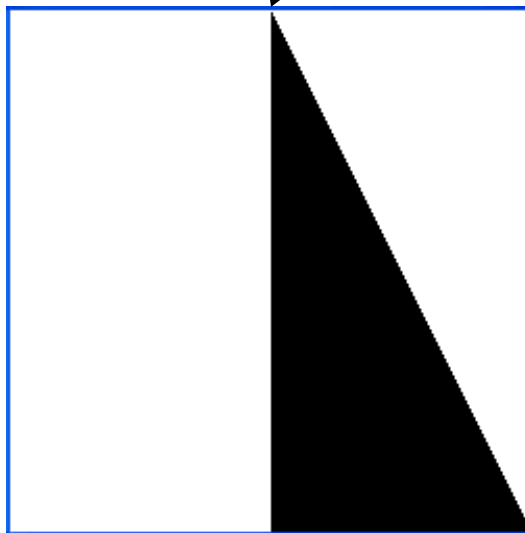
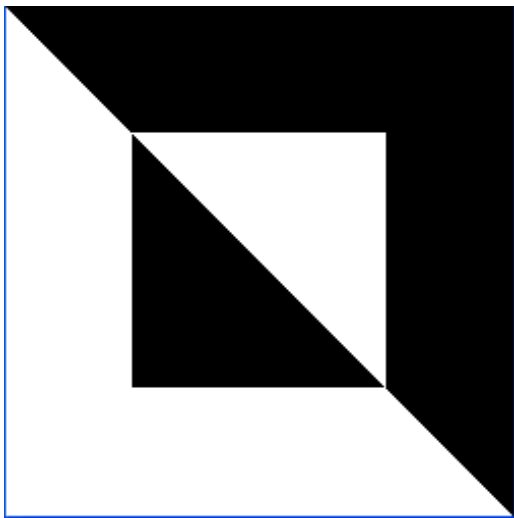
```
clear_all()  
show(quarter_turn_left(sail_bb))
```



Means of Combination

Stacking

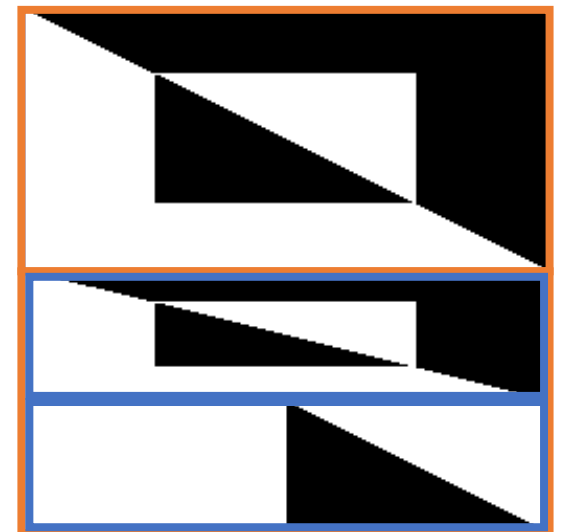
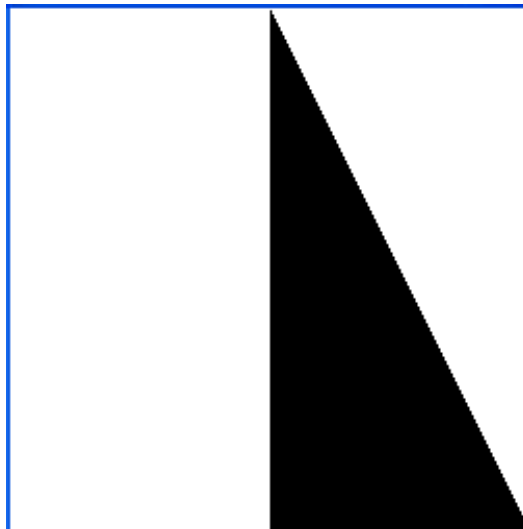
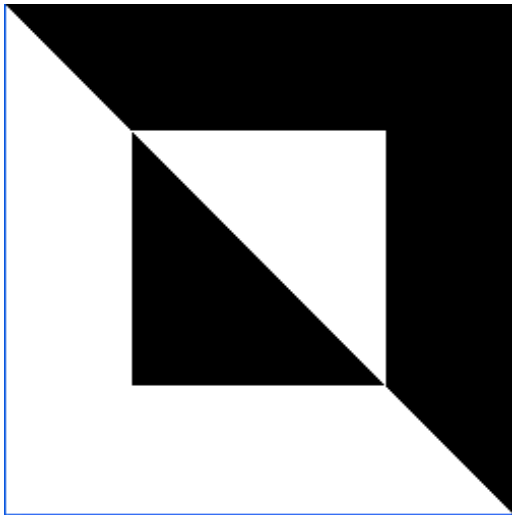
```
clear_all()  
show(stack(rcross_bb, sail_bb))
```



Multiple Stacking

```
clear_all()
```

```
show(stack(rcross_bb,  
          stack(rcross_bb,  
                sail_bb) ) )
```



Means of Combination

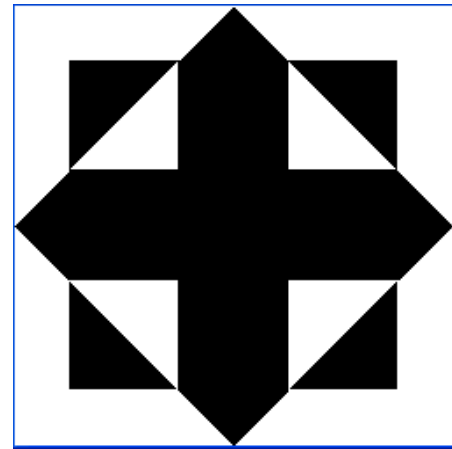
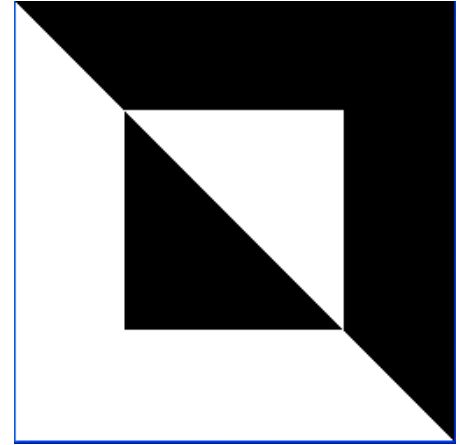
Placing Beside

```
def beside(pic1, pic2):  
    return quarter_turn_right(  
        stack(quarter_turn_left(pic2),  
              quarter_turn_left(pic1)))
```

A complex object

```
clear_all()
show(
  stack(
    beside(
      quarter_turn_right(rcross_bb),
      turn_upside_down(rcross_bb)),
    beside(
      rcross_bb,
      quarter_turn_left(rcross_bb))))
```

Let's give it a name
`make_cross`



```
stack(  
  beside(  
    quarter_turn_right(rcross_bb),  
    turn_upside_down(rcross_bb)),  
  beside(  
    rcross_bb,  
    quarter_turn_left(rcross_bb))))
```



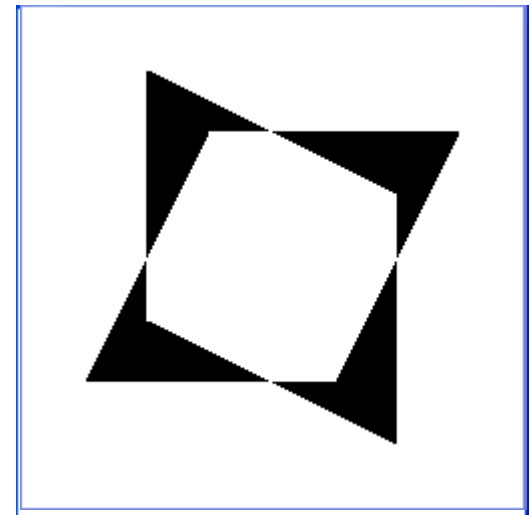
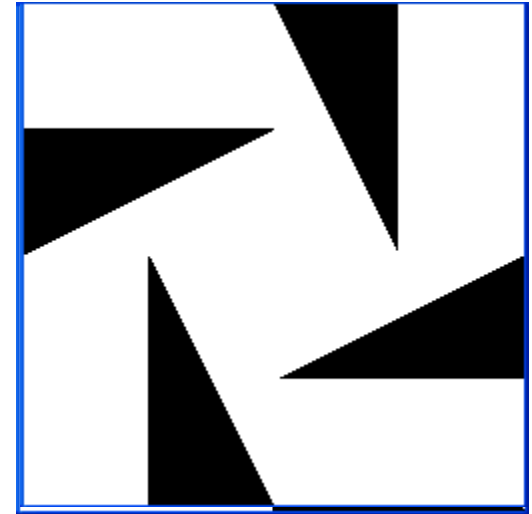
```
stack(  
  beside(  
    quarter_turn_right(pic),  
    turn_upside_down(pic)),  
  beside(  
    pic,  
    quarter_turn_left(pic))))
```

```
def make_cross(pic):  
    return stack(  
        beside(  
            quarter_turn_right(pic),  
            turn_upside_down(pic)),  
        beside(  
            pic,  
            quarter_turn_left(pic))))
```

return vs show

Naming your objects

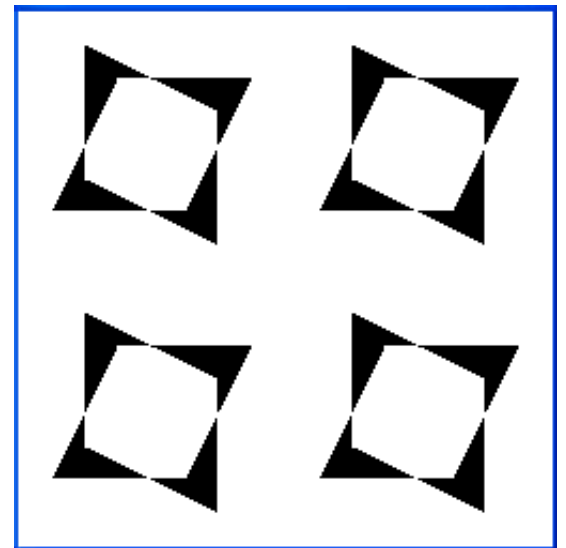
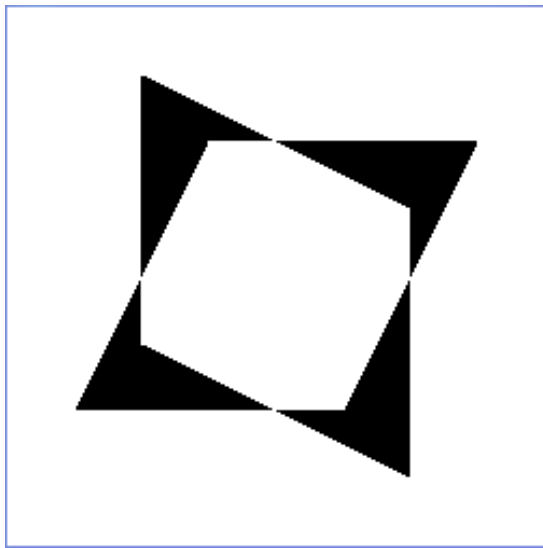
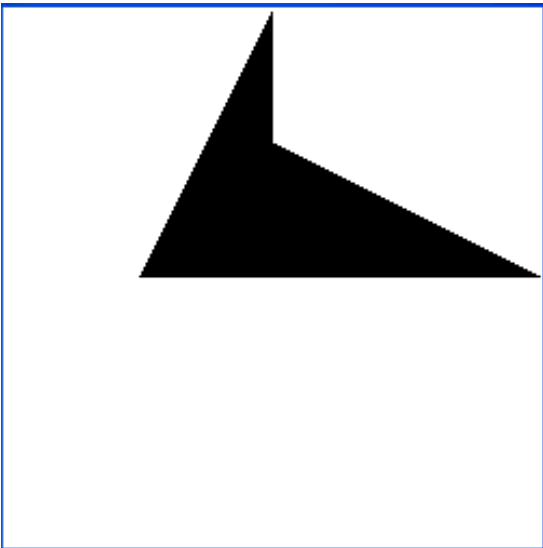
```
clear_all()  
my_pic = make_cross(sail_bb)  
show(my_pic)  
  
my_pic_2 = make_cross(nova_bb)  
show(my_pic_2)
```



Repeating the pattern

```
clear_all()
```

```
show(make_cross(make_cross(nova_bb)))
```



Repeating multiple times

```
clear_all()
```

```
def repeat_pattern(n, pat, pic):
```

```
    if n == 0:
```

```
        return pic
```

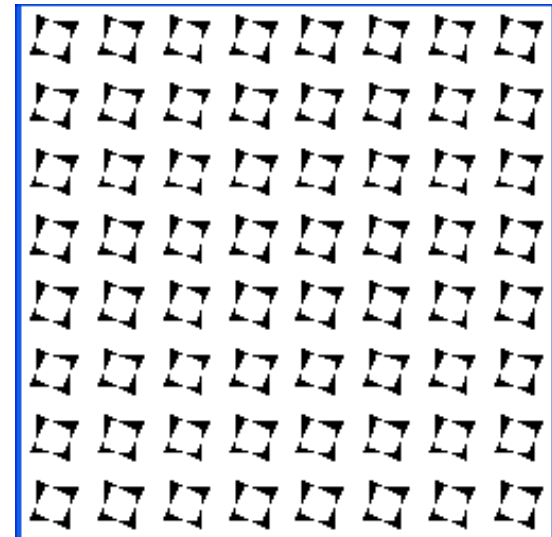
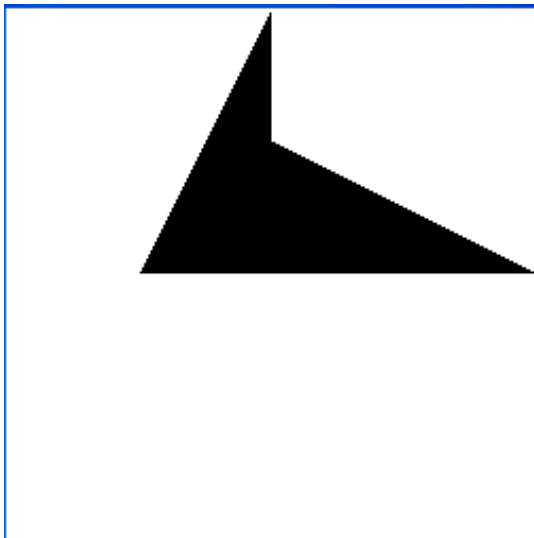
```
    else:
```

```
        return pat(repeat_pattern(n-1, pat, pic))
```

```
show(repeat_pattern(4, make_cross, nova_bb))
```

Qn: What does
`repeat_pattern`
return?

recursion



Anonymous Functions

```
def square(x):  
    return x * x
```

input output

```
foo = lambda x: x * x
```

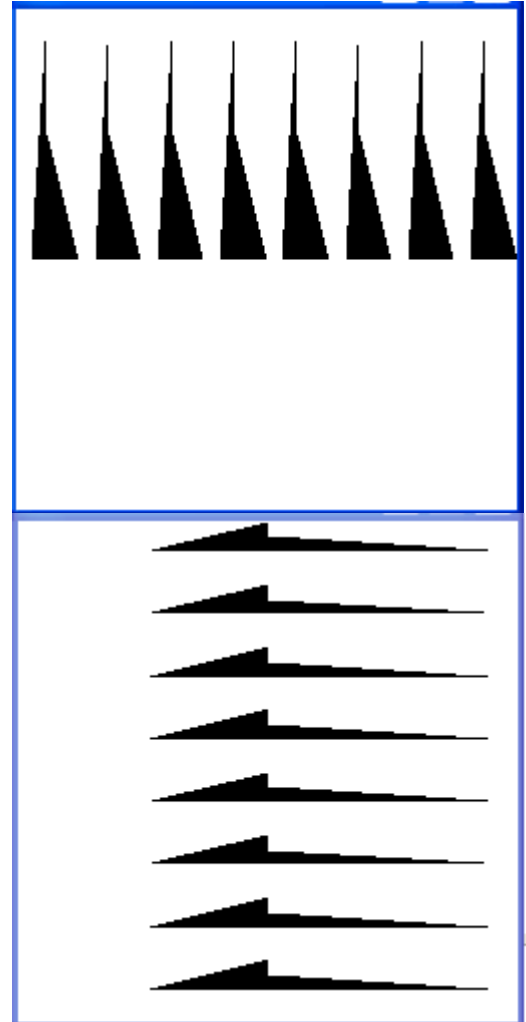
function

```
foo(1)     1  
foo(16)    256
```

New Patterns

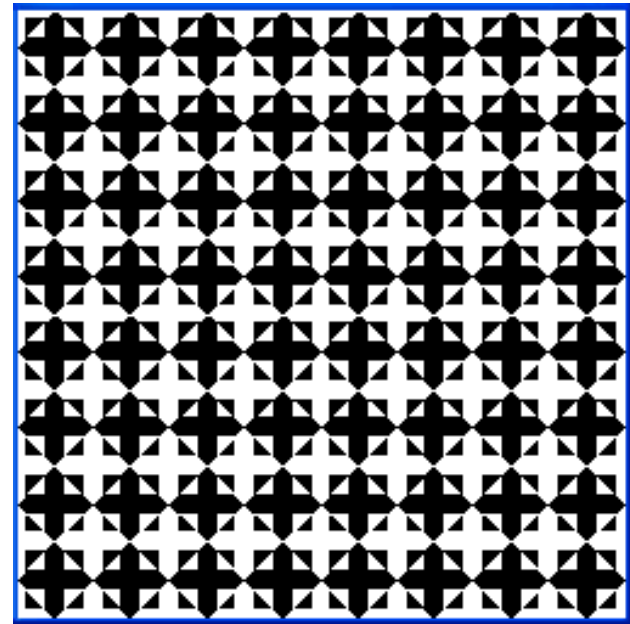
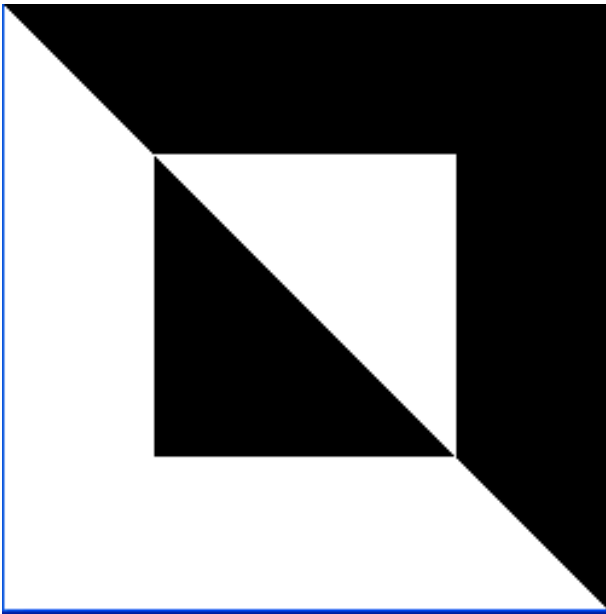
```
clear_all()
show(repeat_pattern(3, anonymous
function
    lambda pic: beside(pic, pic),
    nova_bb))
```

```
clear_all()
show(repeat_pattern(3,
    lambda pic: stack(pic, pic),
    nova_bb))
```



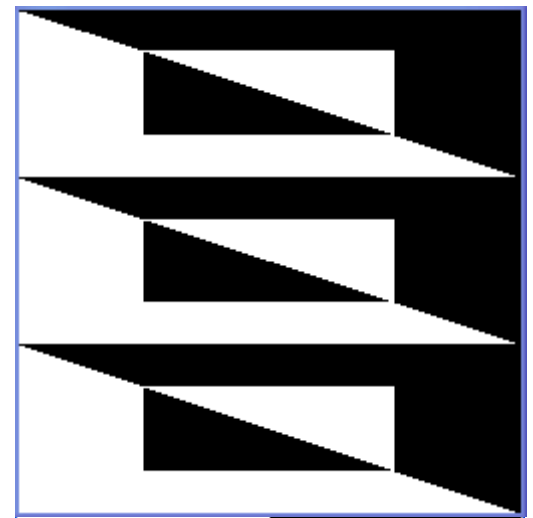
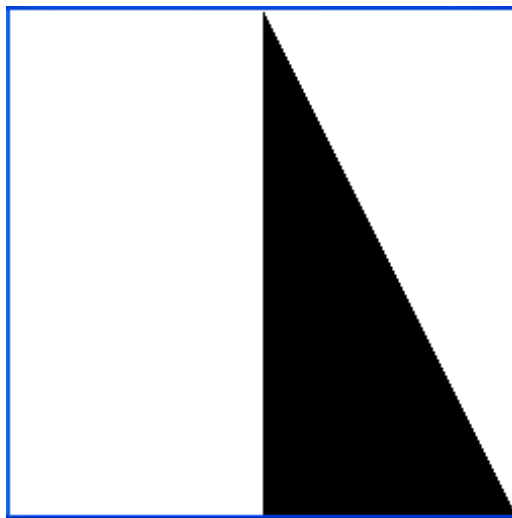
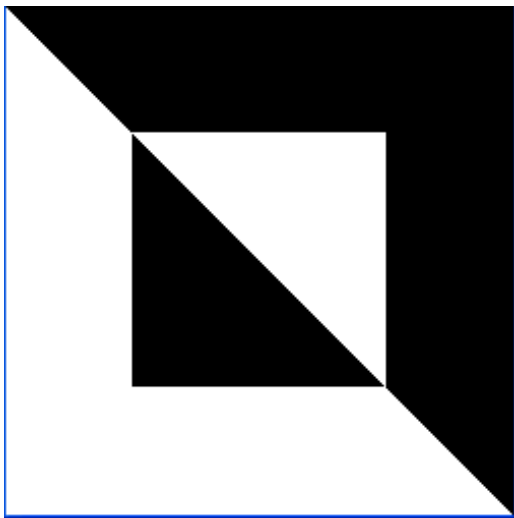
Another nice pattern

```
clear_all()  
show(repeat_pattern(4, make_cross, rcross_bb))
```



What about 3 rows?

```
clear_all()  
show(stack_frac(1/3, rcross_bb, sail_bb))  
clear_all()  
show(stack_frac(1/3, rcross_bb,  
                stack(rcross_bb, rcross_bb)))
```

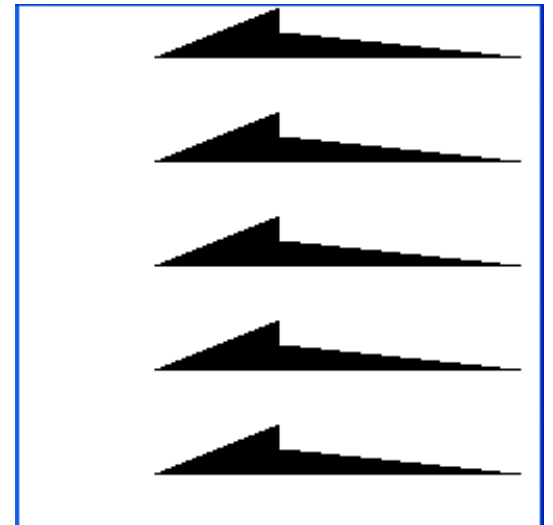
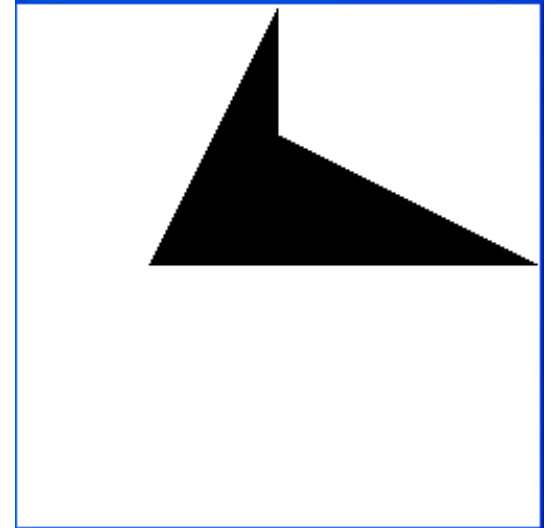


Repeating n times

```
def stackn(n, pic):  
    if n == 1:  
        return pic  
    else:  
        return stack_frac(1/n,  
                           pic,  
                           stackn(n-1, pic))
```

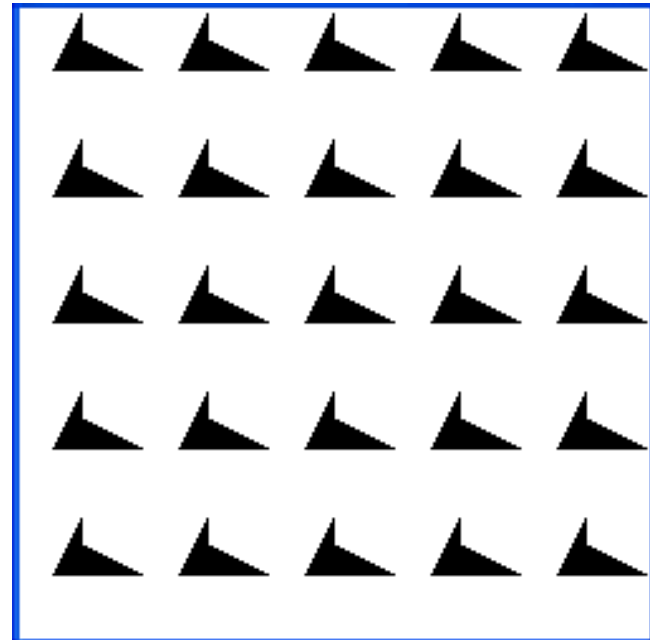
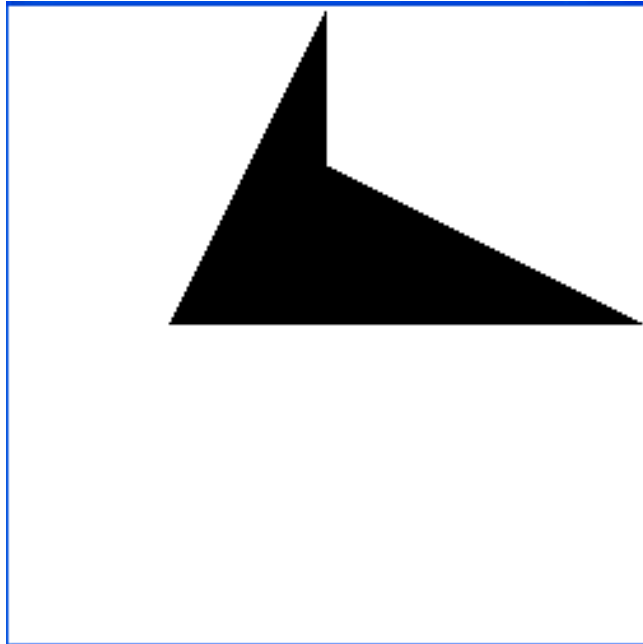
```
clear_all()  
show(stackn(3, nova_bb))
```

```
clear_all()  
show(stackn(5, nova_bb))
```



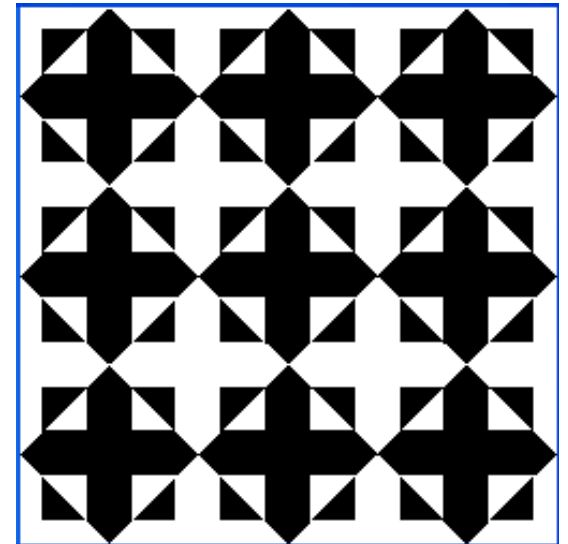
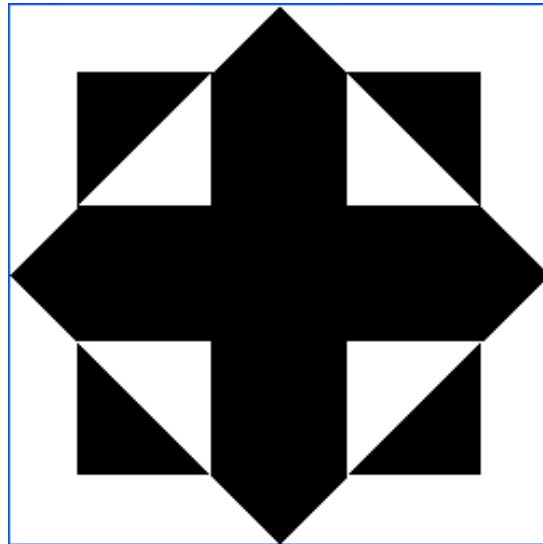
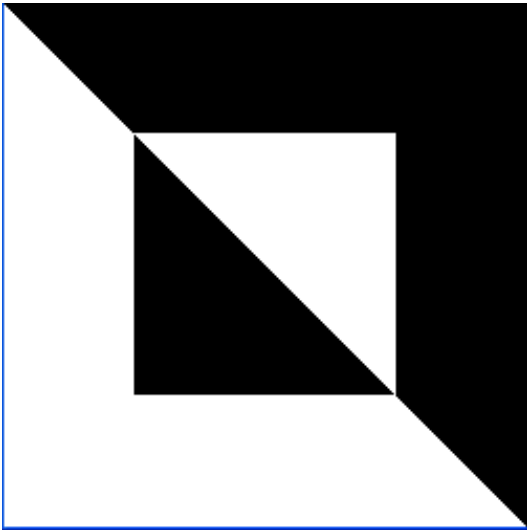
A rectangular quilting pattern

```
clear_all()  
show(stackn(5, quarter_turn_right(  
    stackn(5, quarter_turn_left(nova_bb))))))
```



A rectangular quilting proc

```
def nxn(n, pic):  
    return stackn(n, quarter_turn_right(  
        stackn(n, quarter_turn_left(pic))))  
  
clear_all()  
show(nxn(3, make_cross(rcross_bb)))
```



After all this...

No idea how a
picture is
represented

No idea how the
operations do
their work

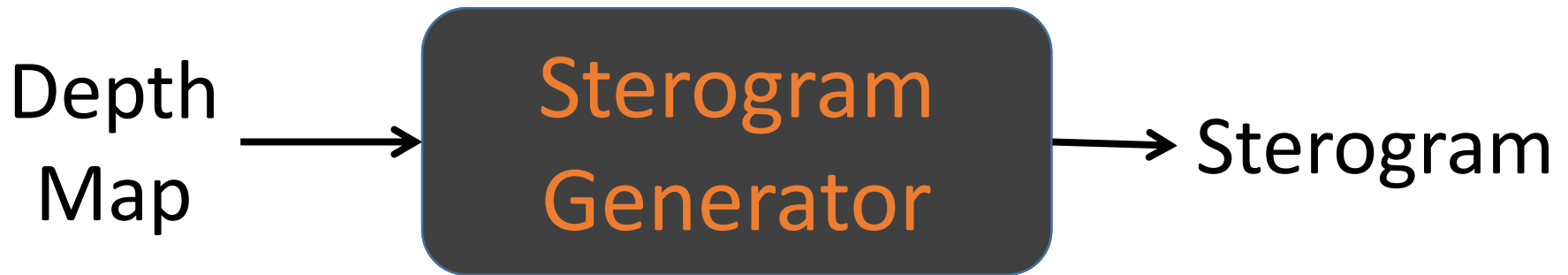
Yet, we can build
complex pictures

This is
Functional
Abstraction

We can make Sterograms!



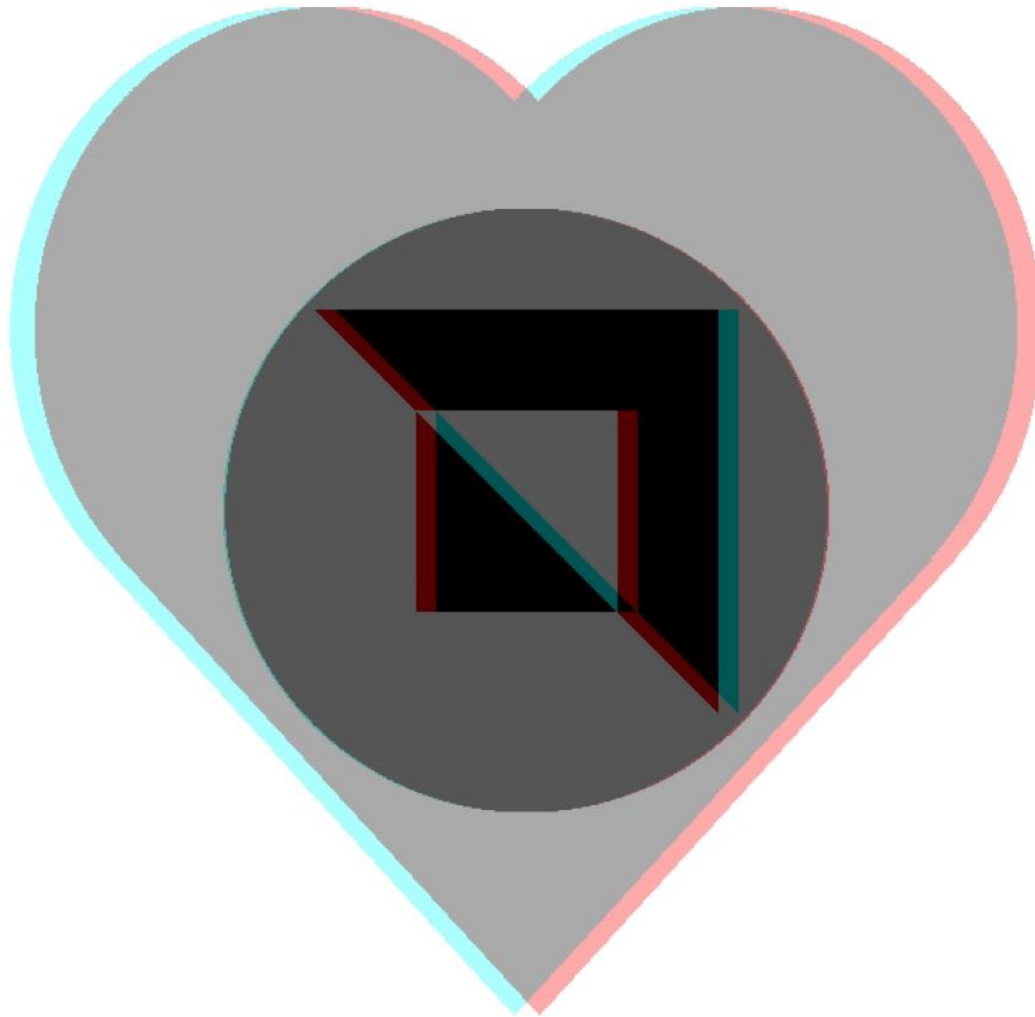
Black Box



Functional Abstraction

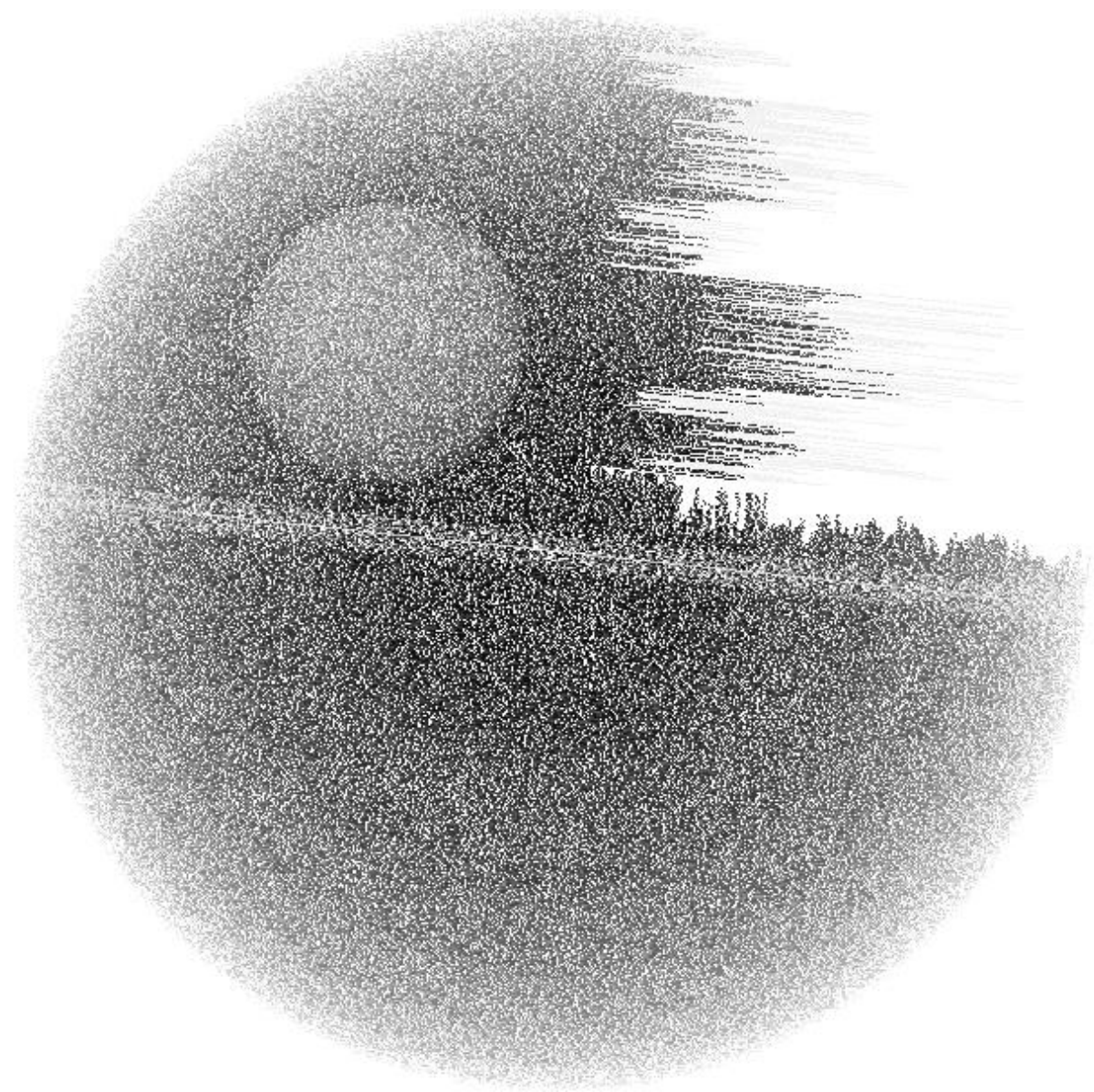
Can't see
stereograms?

Anaglyphs



And if you think
this is cool...

You ain't seen
nothing yet!



What have we learnt?

WHAT

Functional Abstraction =
Black-box

HOW

`def` and `lambda`

Functions are
objects
(in Python)

WHY?

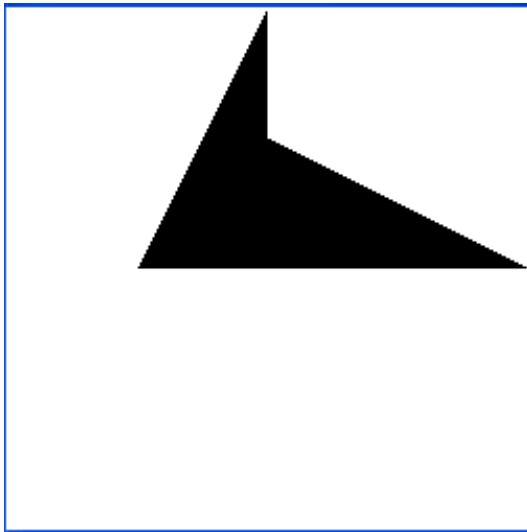
Help us manage
complexity

Allow us to focus
on high-level
problem solving

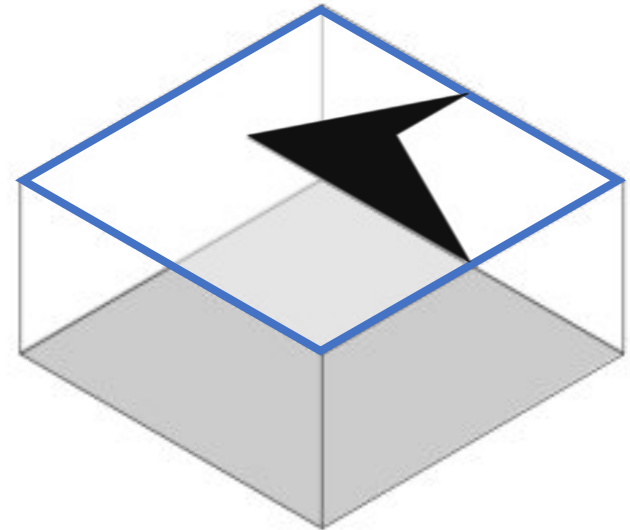
Creating 3D objects

We use greyscale to represent depth

- Black is nearest to you
- White is furthest away



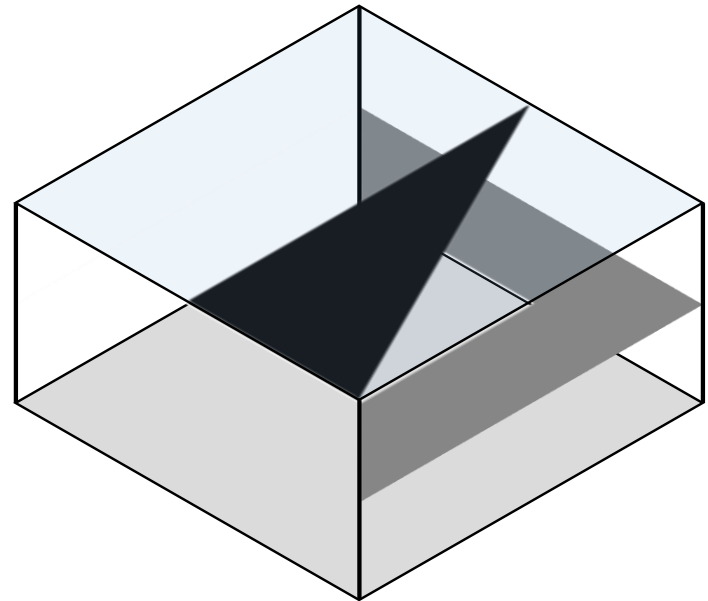
means



Overlay Operation

```
clear_all()
```

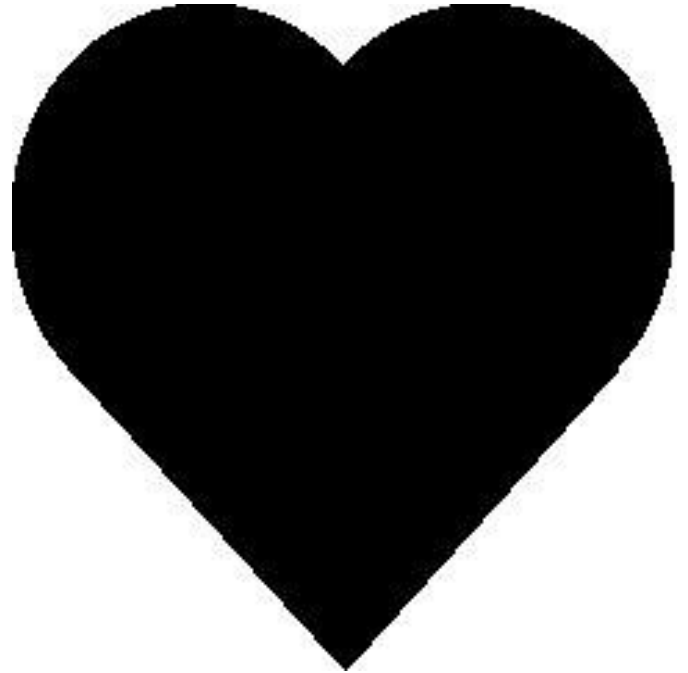
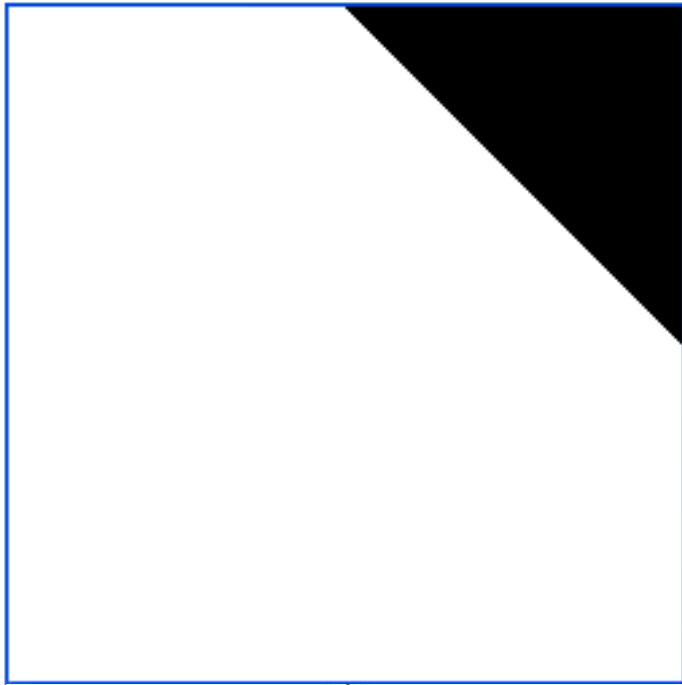
```
show(overlay(sail_bb, rcross_bb))
```



Advanced Overlay Operation

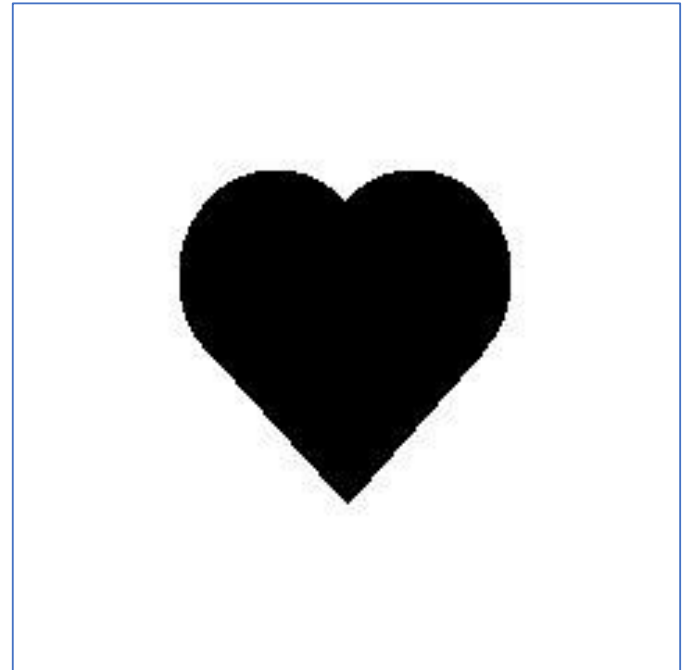
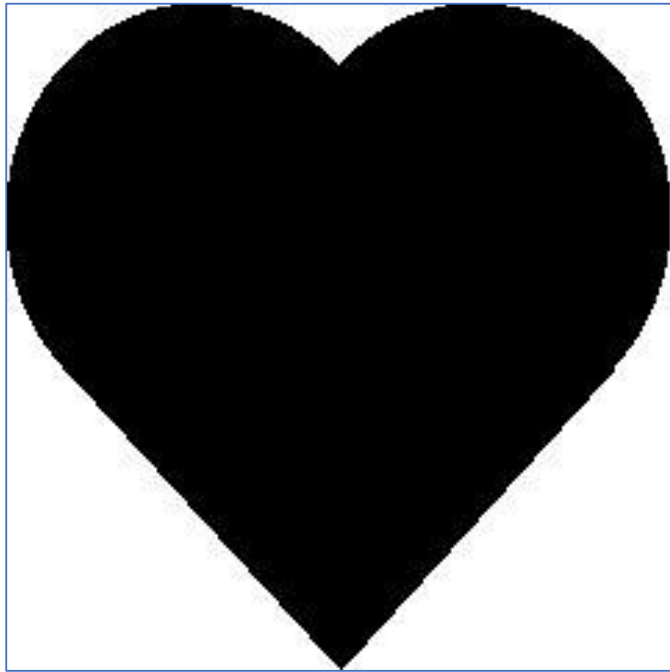
```
clear_all()
```

```
show(overlay_frac(1/4, corner_bb, heart_bb))
```



Scaling

```
clear_all()  
show(scale(1/2, heart_bb))
```



Recall

```
stereogram(scale(1/2, heart_bb))
```



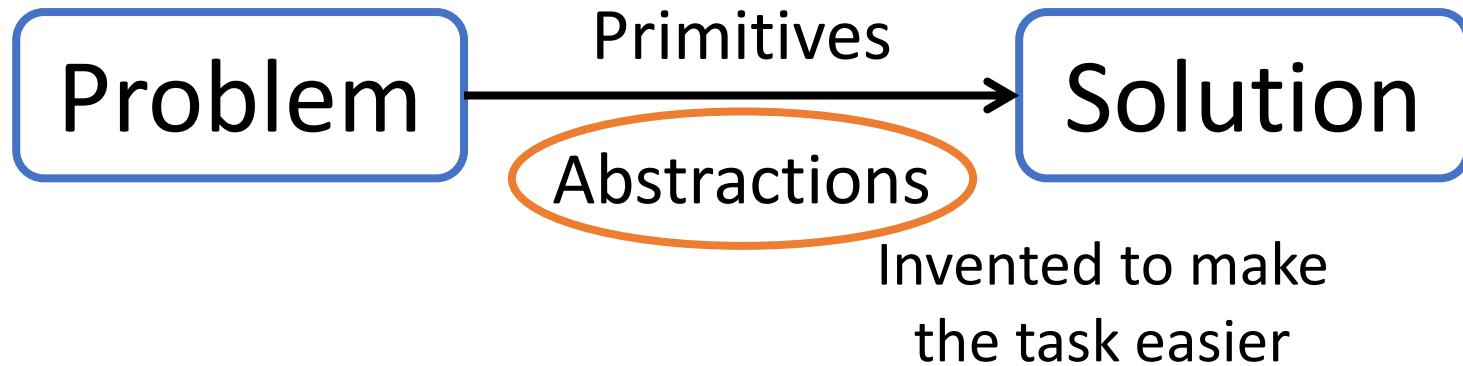


<Break>

Managing Complexity

Computers will
follow orders
precisely

Abstractions

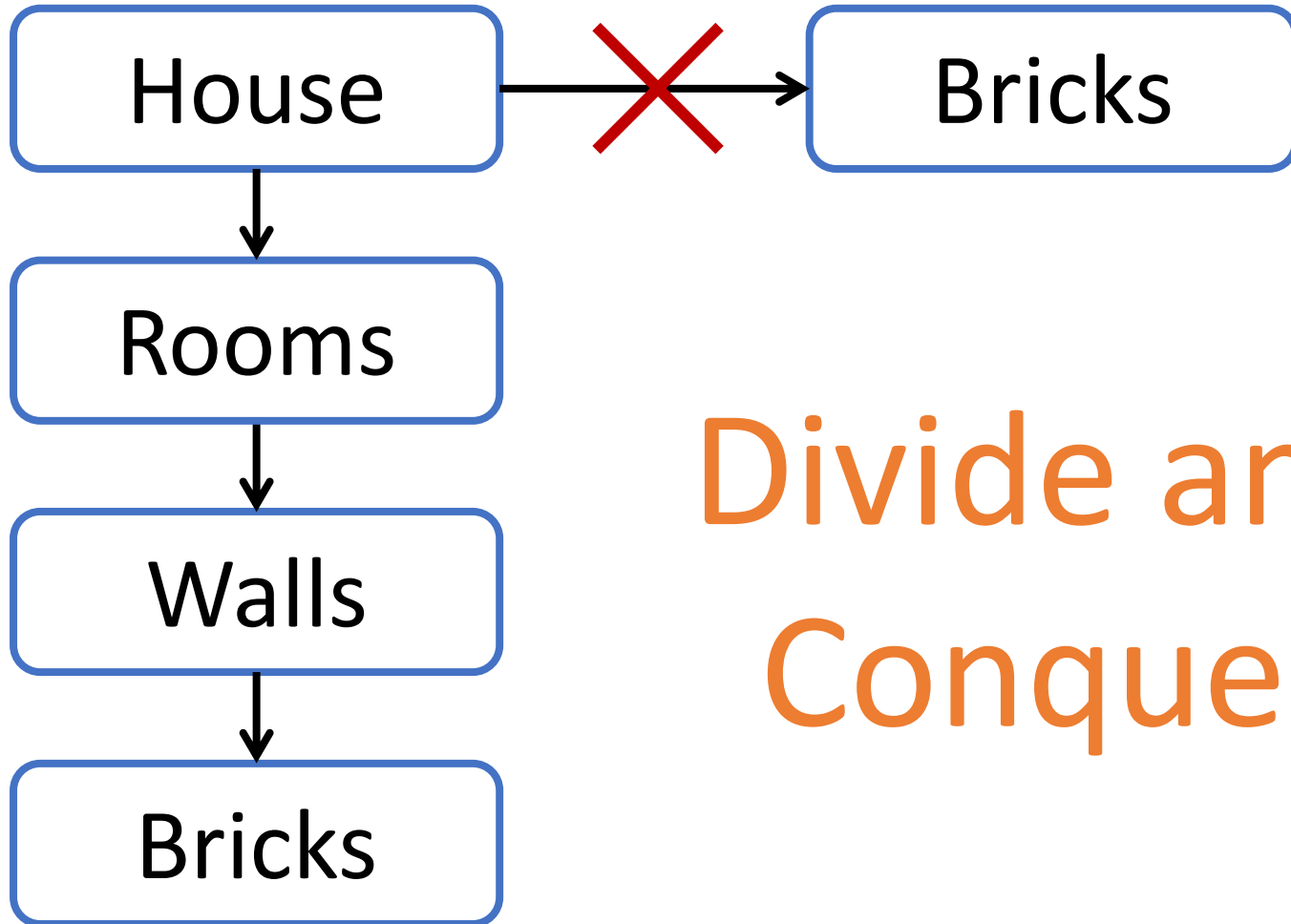


What makes a good abstraction?

Good Abstraction

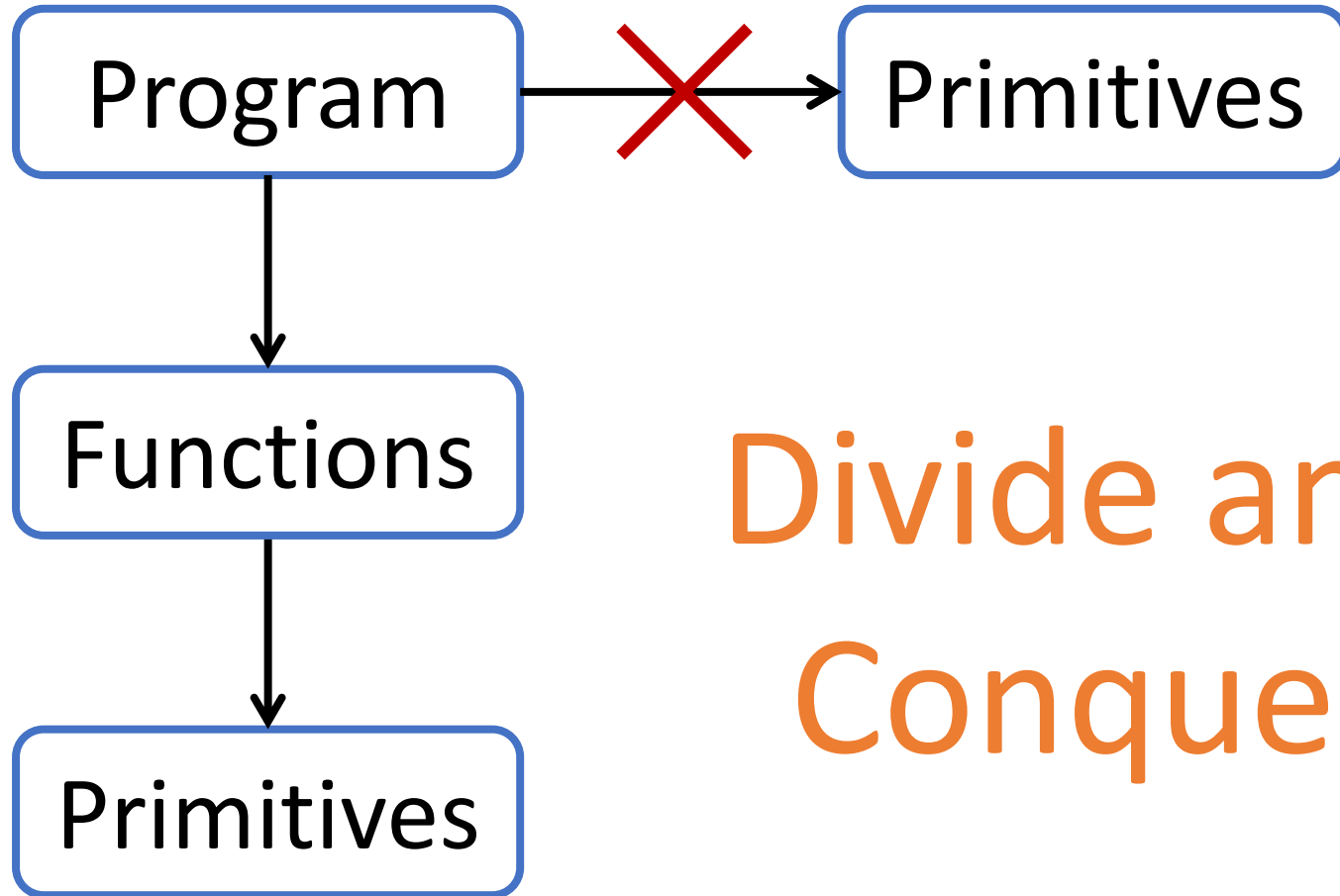
1. Makes it more natural to think about tasks and subtasks

Example



Divide and
Conquer

Programming



Divide and
Conquer

Good Abstraction

2. Makes programs
easier to understand

Compare:

```
def hypotenuse(a, b):  
    return sqrt((a*a) + (b*b))
```

Versus:

```
def hypotenuse(a, b):  
    return sqrt(sum_of_squares(a, b))  
  
def sum_of_squares(x, y):  
    return square(x) + square(y)  
  
def square(x):  
    return x * x
```

Good Abstraction

3. Captures common patterns

```
stack(  
  beside(  
    quarter_turn_right(rcross_bb),  
    turn_upside_down(rcross_bb)),  
  beside(  
    rcross_bb,  
    quarter_turn_left(rcross_bb))))
```

```
stack(  
  beside(  
    quarter_turn_right(pic),  
    turn_upside_down(pic)),  
  beside(  
    pic,  
    quarter_turn_left(pic))))
```

```
def make_cross(pic):  
    return stack(  
        beside(  
            quarter_turn_right(pic),  
            turn_upside_down(pic)),  
        beside(  
            pic,  
            quarter_turn_left(pic))))
```

Allows Code Reuse

Good Abstraction

4. Allows for code reuse

- Function `square` used in `sum_of_squares`.
- `square` can also be used in calculating area of circle.

Another Example

Function to calculate area of circle given the radius

```
pi = 3.14159
```

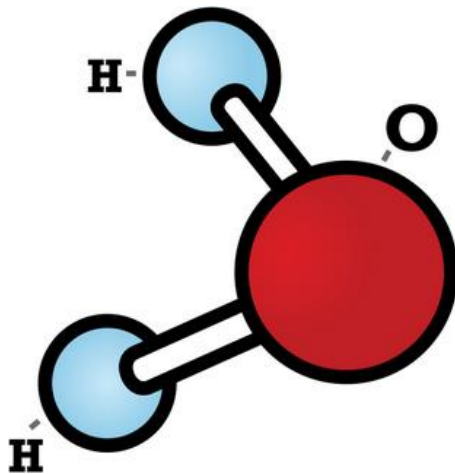
```
def circle_area_from_radius(r):  
    return pi * square(r)
```

given the diameter:

```
def circle_area_from_diameter(d):  
    return circle_area_from_radius(d/2)
```

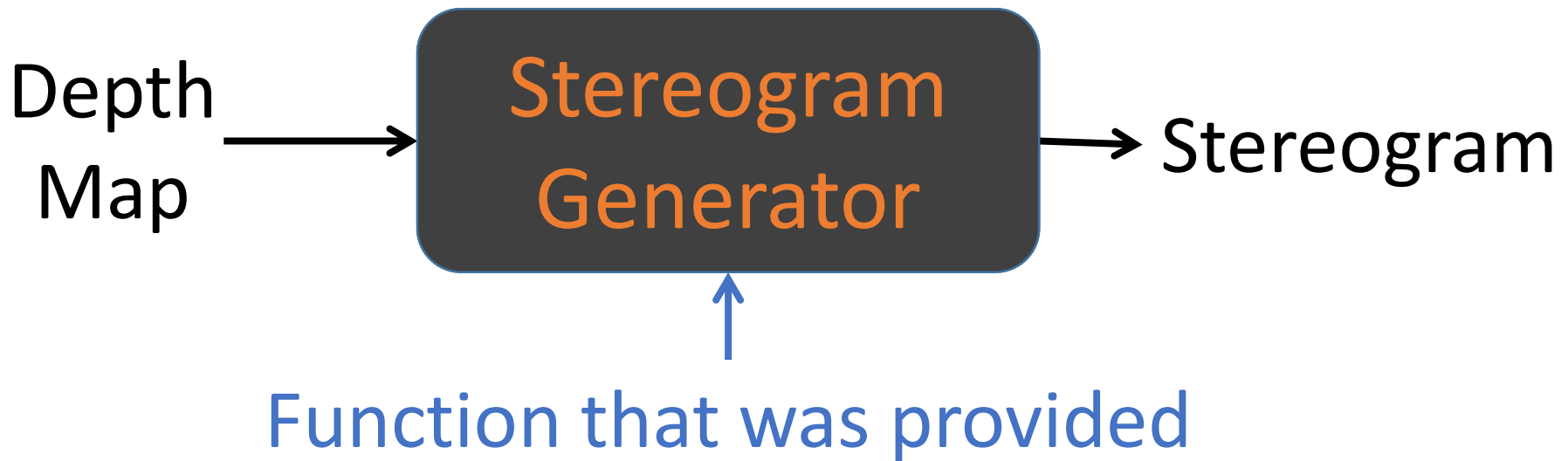

Good Abstraction

5. Hides irrelevant details



Water molecule
represented as 3 balls

Ok for some
chemical analyses,
inadequate for
others.



Good Abstraction

6. Separates
specification from
implementation

Recap

Functional Abstraction

=

Black Box

No need to know how a car
works to drive it!

Functional Abstraction

Separates specification from
implementation

Specification: WHAT

Implementation: HOW

Example

```
def square(x):  
    return x * x
```

```
def square(x):  
    return exp(double(log(x)))
```

```
def double(x): return x + x
```

To think about

Why would we want
to implement a
function in different
ways?

Good Abstraction

7. Makes debugging (fixing errors) easier

Where is the bug?

```
def hypotenuse(a, b):  
    return sqrt(sum_of_squares(a, b))
```

```
def sum_of_squares(x, y):  
    return square(x) + square(y)
```

```
def square(x): return x + x
```

```
def hypotenuse(a, b):  
    return sqrt((a + a) * (b + b))
```


Variable Scope

```
x = 10
```

```
def square(x): return x * x
```

```
def double(x): return x + x
```

```
def addx(y): return y + x
```

```
square(20)
```

```
square(x)
```

```
addx(5)
```

Which x ?

Variable Scope

formal parameter

```
def square(x):  
    return x * x
```

body

A function definition **binds** its formal parameters.

i.e. the formal parameters are visible only **inside the definition (body)**, not outside.

Variable Scope

formal parameter
↓
`def square(x):`
 `return x * x` } body

- Formal parameters are **bound variables**.
- The region where a variable is visible is called the **scope** of the variable.
- Any variable that is not bound is **free**.

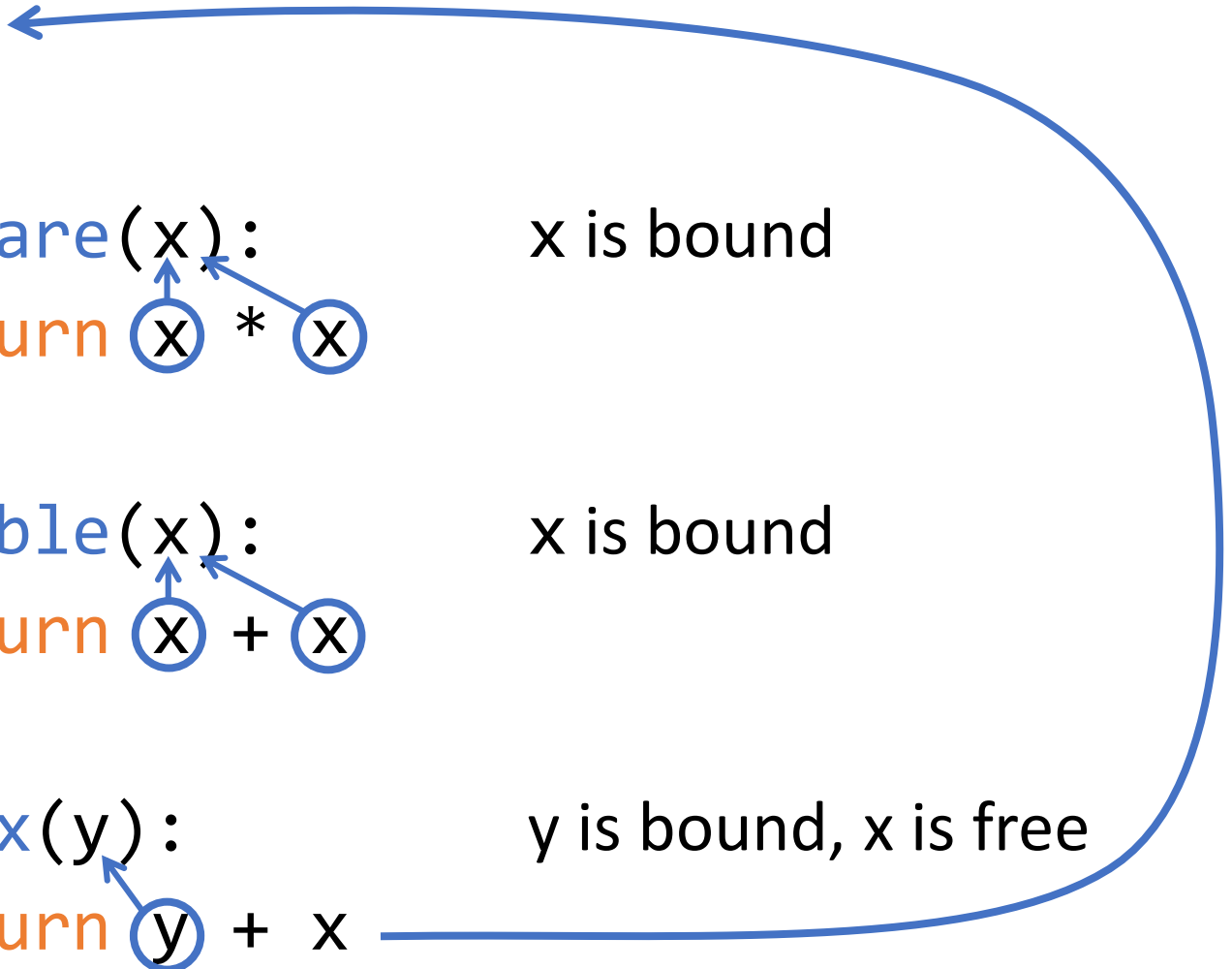
Variable Scope

`x = 10`

`def square(x):` x is bound
`return` $\textcircled{x} * \textcircled{x}$

`def double(x):` x is bound
`return` $\textcircled{x} + \textcircled{x}$

`def addx(y):` y is bound, x is free
`return` $\textcircled{y} + x$



Example

```
pi = 3.14169
```

```
def circle_area_from_radius(r):  
    pi = 22/7    #local pi  
    return pi * square(r)
```

↑
Which pi?

Block Structure

```
def hypotenuse(a, b):
```

```
    def sum_of_squares():  
        return square(a) + square(b)
```

```
    return math.sqrt(sum_of_squares())
```

The variables `a` and `b` in `sum_of_squares` refer to the formal parameters of `hypotenuse`.

Hides irrelevant details (`sum_of_squares`) from the user of `hypotenuse`.

Wishful Thinking

WHAT

Top-down design
approach:

Pretend you have
whatever you need

WHY

Easier to think with in
the goal in mind

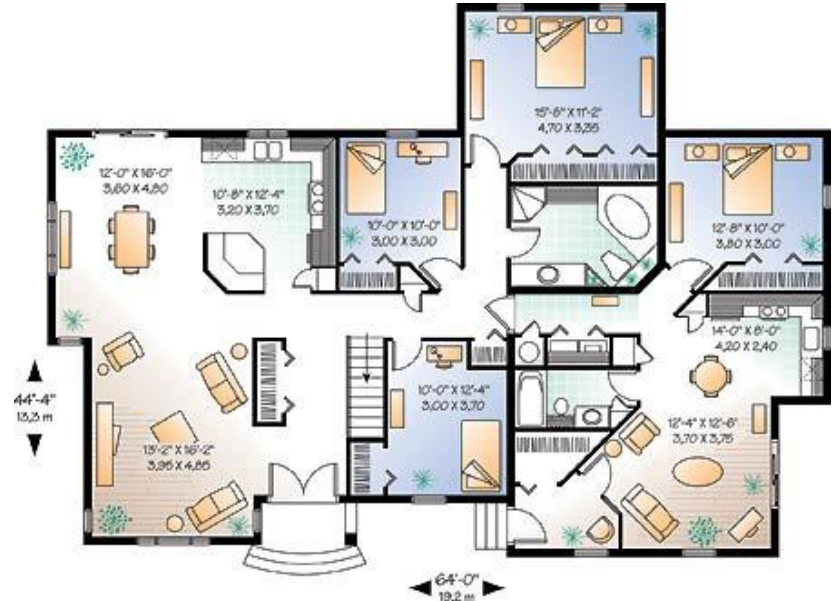
Analogy

Suppose you are to build a house.
Where do you start?

Individual
bricks



Building plan



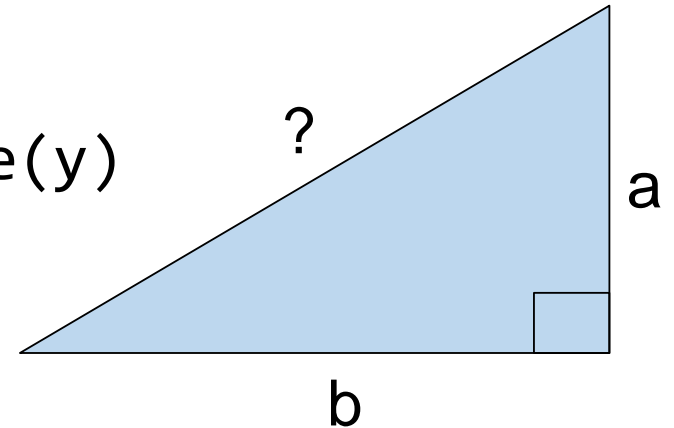
Example

Suppose you want to compute hypotenuse

```
def hypotenuse(a, b):  
    return sqrt(sum_of_squares(a, b))
```

```
def sum_of_squares(x, y):  
    return square(x) + square(y)
```

```
def square(x):  
    return x * x
```



Another Example

Comfort Delgro, the largest taxi operator in Singapore, determines the taxi fare based on distance traveled as follows:

- For the first kilometre or less: \$2.40
- Every 200 metres thereafter or less up to 10 km: \$0.10
- Every 150 metres thereafter or less after 10 km: \$0.10

Problem:

Write a Python function that computes the taxi fare from distance travelled.

How do we
start?

Formulate the problem



Function

Needs a name

Pick an appropriate name
(not foo)

Formulate the problem



- What data do you need?
(be thorough)
- Where would you get it?
(argument/
computed?)

Results should be
unambiguous

- What other abstractions
may be useful?
- Ask the same questions for
each abstraction.

How can the result be computed from data?

1. Try simple examples
2. Strategize step by step
3. Write it down and refine

Solution

- What to call the function? `taxi_fare`
- What data are required? `distance`
- Where to get the data? `function argument`
- What is the result? `fare`

How can the result be computed from data?

- e.g. #1: distance = 800 m, fare = \$2.40
- e.g. #2: distance = 3,300 m
fare = \$2.40 + $\lceil 2300/200 \rceil \times \0.10
= \$3.60
- e.g. #3: distance = 14,500 m
fare = \$2.40 + $\lceil 9000/200 \rceil \times \0.10 +
 $\lceil 4500/150 \rceil \times \0.10 = \$9.90

Pseudocode

Case 1: distance \leq 1000

fare = \$2.40

Case 2: $1000 < \text{distance} \leq 10,000$

fare = \$2.40 + \$0.10 * $\lceil (\text{distance} - 1000) / 200 \rceil$

Case 3: distance $> 10,000$

fare = \$6.90 + \$0.10 * $\lceil (\text{distance} - 10,000) / 150 \rceil$

What's this?

Note: the Python function `ceil` rounds up its argument. `math.ceil(1.5) = 2`

Solution

```
def taxi_fare(distance): # distance in metres
    if distance <= 1000:
        return 2.4
    elif distance <= 10000:
        return 2.4 + (0.10 * ceil((distance - 1000) / 200))
    else:
        return 6.9 + (0.10 * ceil((distance - 10000) / 150))

# check: taxi_fare(3300) = 3.6
```

Can we improve this solution?

Coping with Change

What if...

1. the starting fare increases?
2. stage distance changes?
3. increment amount changes?

CAB CONFUSION

Singapore has many different types of taxis plying the roads, all with different flag-down rates. LIM YONG and BRYANDT LYN help sort through the choices available.

Flag-down rates for first kilometre or less. Figures in brackets denote subsequent fare rates for:

■ Every 400m thereafter or less up to 10km ■ Every 350m thereafter or less after 10km ■ Every 45 seconds of waiting or less

NOTE: Fare comparisons do not take into account surcharges, which vary by company, time and location. Fares correct as at Nov 20.

\$3		\$3.20		\$3.40		\$3.50	
Comfort and CityCab Toyota Crown (22¢) 		Comfort and CityCab Hyundai Sonata (22¢) 		Prime Toyota Axio (22¢) 		Comfort and CityCab Toyota Camry Hybrid (22¢) 	
Trans-Cab Toyota Crown (22¢) 		Premier Kia Magentis (22¢) 		Trans-Cab Toyota Wish (22¢) 		Prime Toyota Prius 1.5 (22¢) 	
SMRT Toyota Crown (22¢) 		SMRT Chevrolet Epica (22¢) 		Premier Toyota Prius (22¢) 		Premier Toyota Prius (22¢) 	
Premier Toyota Crown (22¢) 		Hyundai i30 Wagon (22¢) 		Toyota Premio (22¢) 		Skoda Superb (22¢) 	
Nissan Cedric (22¢) 		Honda Fit (22¢) 		Hyundai Avante (22¢) 		SMRT Chevrolet Epica (22¢) 	
		Honda Airwave (22¢) 		Toyota Fielder (22¢) 		SMRT Chevrolet Epica (22¢) 	
		Honda Partner (22¢) 		Honda Stream (22¢) 			
\$3.60		\$3.70		\$3.80		\$3.90	
Trans-Cab Chevrolet Epica II (22¢) 		Comfort and CityCab Hyundai i40 (22¢) 		SMRT Toyota Prius (22¢) 		Comfort and CityCab Limousine (30¢) 	
		Premier Kia Optima (22¢) 		Hyundai Azera (CNG) (22¢) 		Premier Kia Carnival (30¢) 	
		Prime Toyota Camry/Camry Hybrid (22¢) 		SMRT Mercedes-Benz (22¢) 		Trans-Cab Mercedes-Benz (30¢) 	
		Toyota Estima (22¢) 		London cab (22¢) 		Renault Latitude (22¢) 	
		Honda Stepwagon (22¢) 		Ssangyong Space (22¢) 		Hyundai Starex (22¢) 	
		Toyota Prius 1.8 (22¢) 					
\$4.50		\$5		\$5		\$5	
Prime Toyota Vellfire (33¢) 		Premier Mercedes-Benz E-220 (30¢) 		SMRT Chrysler 300C (33¢) 			

Sources: COMFORT TRANSPORTATION AND CITYCAB, PREMIER TAXI, PRIME CAR RENTAL & TAXI SERVICES, SMRT, TRANS-CAB TAXI

PHOTOS: ST FILE, COMFORT, PREMIER TAXI, PRIME TAXI, SMRT, TRANS-CAB TAXI

Avoid Magic Numbers


It is a terrible idea to hardcode numbers (magic numbers):

- Hard to make changes in future

Define abstractions to hide them!

Solution v2

```
def taxi_fare(distance): # distance in metres
    if distance <= stage1:
        return start_fare
    elif distance <= stage2:
        return start_fare +
            (increment * ceil((distance - stage1) / block1))
    else:
        return taxi_fare(stage2) +
            (increment * ceil((distance - stage2) / block2))
```

A blue oval highlights the recursive call `taxi_fare(stage2)` in the `else` branch. A blue line points from the text "recursive call" to this oval.

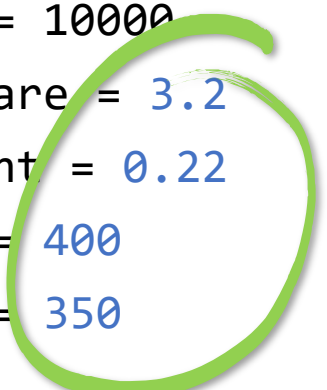
recursive call

```
stage1 = 1000
stage2 = 10000
start_fare = 2.4
increment = 0.1
block1 = 200
block2 = 150
```

In 2017

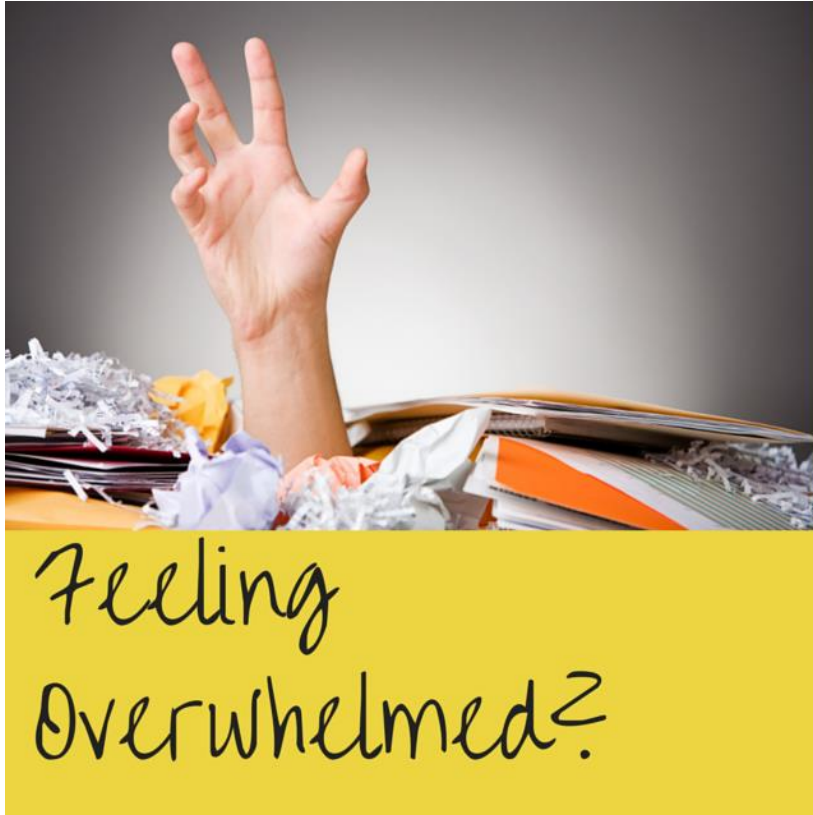
```
def taxi_fare(distance): # distance in metres
    if distance <= stage1:
        return start_fare
    elif distance <= stage2:
        return start_fare +
            (increment * ceil((distance - stage1) / block1))
    else:
        return taxi_fare(stage2) +
            (increment * ceil((distance - stage2) / block2))

stage1 = 1000
stage2 = 10000
start_fare = 3.2
increment = 0.22
block1 = 400
block2 = 350
```



Summary

- Functional Abstraction
- Good Abstractions
- Variable Scoping
- Wishful Thinking



Recitation
tomorrow/Friday