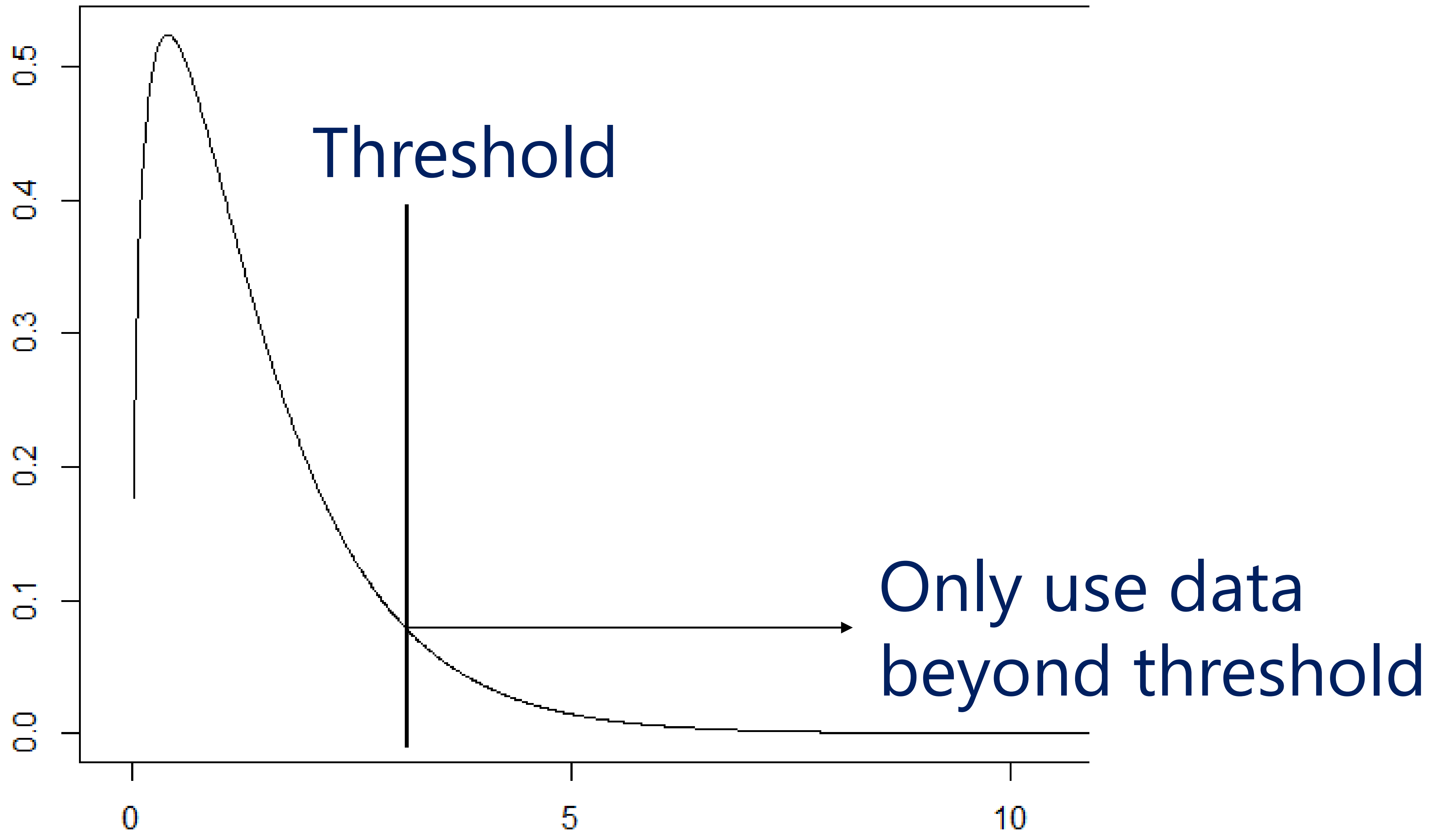
Chapter 9 – 9.1.3 - Peaks over Threshold

- The GPD distribution is used in the POT approach to model data above a certain threshold
- We use the Pickands Balkema de haan theorem to say that data in the tail of a data set follows a GPD distribution.
- Thus GPD is commonly used to model the tail of other distributions. It is defined by three parameters and its cumulative distribution function is:

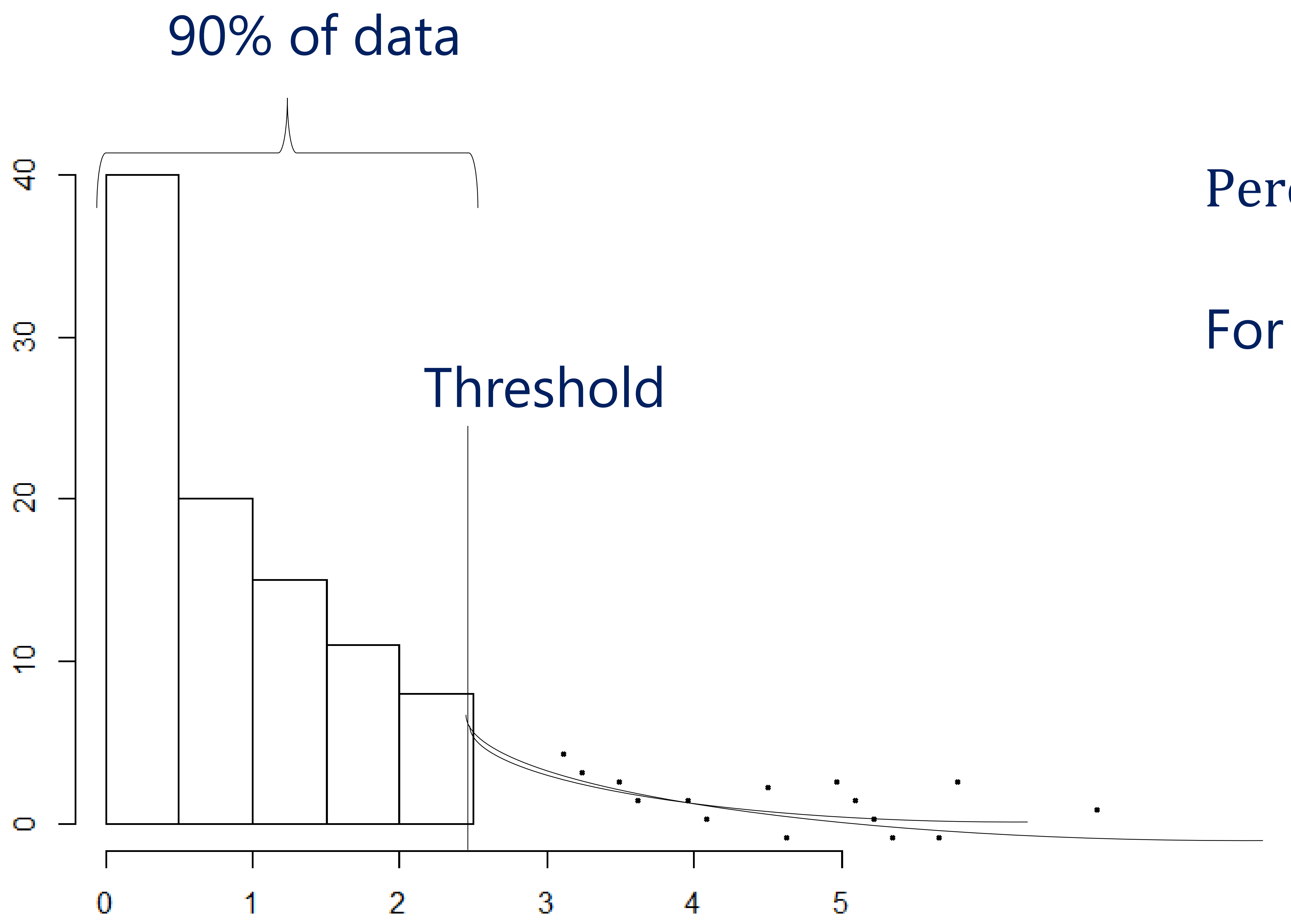
$$F_{(\xi,\mu,\sigma)}(x) = \begin{cases} 1 - \left(1 + \frac{\xi(x-\mu)}{\sigma}\right)^{-1/\xi} & \text{for } \xi \neq 0, \\ 1 - \exp\left(-\frac{x-\mu}{\sigma}\right) & \text{for } \xi = 0. \end{cases}$$

μ is the location parameter;
σ > 0 the scale parameter; and
ξ is the shape parameter.



Chapter 9 – 9.1.3 - Peaks over Threshold

- The GPD tail is fit based on the values above the threshold; ignoring the exact values below the threshold
- We do allow for the proportion of data below the threshold in percentile estimates



$$\text{Percentile}_\alpha = u + \frac{\beta}{\xi}\left(\left(\frac{1-\alpha}{P(X>u)}\right)^{-\xi} - 1\right)$$

For Threshold u, percentile α, GPD parameters ξ, β

Chapter 9 – 9.2 – Invisible tail

- Mathematical specification for “Black Swan”?

- Mini course - https://www.youtube.com/watch?v=64WDwef5jKI (4 minutes)

- Mean (i.e. p=1) above K: $\frac{\alpha\left(L-L^{\alpha}K^{1-\alpha}\right)}{\alpha-1}$
- Pareto Mean = $\frac{\alpha L}{\alpha-1}$
- $\frac{E\left(\mu_{k,1}\right)}{E(\mu)} = 1 - L^{\alpha-1}K^{1-\alpha}$

Chapter 9 – 9.3 – Tallest Mountain

- An historic example of the BMM approach is to consider the October 17th 1987 Black Monday S&P 500 one day fall of 20.4%

- Many models based on fitting the Normal distribution and extrapolating to the tails said this event was 1 in a “Trillion+” year event

- To apply the BMM approach we take a data set of daily returns from 1960 to October 16th 1987 and divide it into blocks of 1 year. This includes 28 annual maxima. The largest fall in the data set is just 6.7%.

- We fit a Generalised Extreme Value distribution to these 28 data points. This gives a 1 in 50 year event of a 24% fall, which is in excess of the 20.4% seen on October 17th 1987

- This shows the BMM gives far more plausible model. It avoid the problem of only preparing for the worst event seen in history (i.e. thinking the tallest mountain is the tallest one personally seen)

[Example from McNeil 1998 On extremes and crashes, Risk Magazine Jan 1999]

Chapter 9 – D – Pareto Calibration

Full article: Estimation of the Pareto and related distributions – A reference-intrinsic approach (tandfonline.com)

Reference	Area of application	Calibration types used
Moharram, Gosain, and Kapoor (1993)	Hydrology	MOM, PWM, MLE, LS
Castillo and Hadi (1997)	Hydrology	MOM, PWM, EPM
McNeil (1997)	Insurance	MLE
Rootzén and Tajvidi (1997)	Insurance	PWM, MLE
Dupuis and Tsao (1998)	Hydrology	OBRE
Holmes and Moriarty (1999)	Weather	LS to the empirical mean excess function
Shi et al. (1999)	Engineering	MLE
Peng and Welsh (2001)	Hydrology	MM, MLE, OBRE
Pandey, Van Gelder, and Vrijling (2001)	Weather	L-moments, MoM
Frigessi, Haug, and Rue (2002)	Insurance	MLE
Diebolt, Guillou, and Worms (2003)	Hydrology	Bayesian and MLE
de Zea Bermudez and Turkman (2003)	Insurance, Hydrology	Bayesian, PWM and EPM
Pisarenko and Sornette (2003)	Seismology	MLE
Coles, Pericchi, and Sisson (2003)	Weather	Bayesian and MLE
Castillo et al. (2004)	Engineering	MOM, PWM, EPM
La Cour (2004)	Engineering	MLE
Engeland, Hisdal, and Frigessi (2004)	Hydrology	PWM, MLE
Behrens, Lopes, and Gamerman (2004)	Economics	Bayesian, ML
Pandey, Van Gelder, and Vrijling (2004)	Hydrology	L-Moments, MoM
Juárez and Schucany (2004)	Weather	MM, OBRE, MLE, MDPD
Goldstein, Morris, and Yen (2004)	Network theory	MLE, LS
Keylock (2005)	Environment	MLE
Diebolt et al. (2005)	Insurance	Bayesian, MLE
Öztekin (2005)	Hydrology	MOM, PWM. MLE, LS, ME
Tancredi, Anderson, and O'Hagan (2006)	Hydrology	Bayesian
Lana et al. (2006)	Weather	L-moments
Jagger and Elsner (2006)	Weather	Bayesian and MLE
Fawcett and Walshaw (2006)	Weather	Bayesian and MLE
Zagorski and Wnek (2007)	Engineering	MLE, LS to empirical mean excess function
Castellanos and Cabras (2007)	Hydrology	Bayesian and MLE
Moisello (2007)	Hydrology, Weather	PWM
Vilar-Zanón and Lozano-Colomer (2007)	Insurance	Bayesian
White, Enquist, and Green (2008)	Ecology	MLE, binning
Krehbiel and Adkins (2008)	Economics	MLE
de Zea Bermudez et al. (2009)	Wildfires	PWM, MLE
Mendes et al. (2010)	Wildfires	Bayesian
Akhundjanov and Chamberlain (2019)	Agricultural land size	MLE, Hill and KS