Extrapolation of Option Prices

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Abstract

In the FX Options market, the practice of quoting OTC options by delta leads to delta-based interpolation with the well-known effect of the flattening of implied volatilities on the wings. It is also known that pricing of deep out-of-the-money options is a problem if implied volatilities do not rise enough. Following on from [Taleb at al 2019], [Wystup 2012] and [Gatheral 2006], we discuss how the 10-delta points could be treated as the start of the region where pries are calibrated by the power law behavior and show how a logstrike extrapolation (as in the SVI model) can be used to extrapolate from the 10-delta points and reproduce the behavior highlighted in Taleb. Code is available on GitHub at `https://github.com/MarcosCarreira/OptionExtrapolation`.

Keywords: Option pricing; implied volatilities; tail events

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1 Introduction

[Taleb at al 2019] discuss an approach for the extrapolation of option prices assuming a reference price for an OTM option with strike K and a behavior described by a strong Pareto Law (where the log of the survival function is constant after K). Looking at both classical [Gatheral 2006] and recent [Gatheral 2021] works by Gatheral, a logstrike parametrization seems an obvious candidate to produce the desired behavior. And the practice of FX Markets, as described by Wystup in [Wystup 2012] and [Wystup 2017], to quote 25-delta and 10-delta strikes provides us with the ability to start from strikes (10-delta) that are closer to to the tail; most of the efforts in modeling equity options tend to focus on the skew around the ATM strikes.

2 FX Options parametrization

2.1 Deltas

As explained in [Wystup 2012], we will use the forward delta definition as the delta parametrization and ATM delta-neutral for major currencies.

The delta assuming r=q=0 becomes:

$$\frac{1}{2}\cdot\operatorname{erfc}\left(-\frac{\log\left(\frac{f}{K}\right)+\frac{\sigma^2\tau}{2}}{\sqrt{2}\sigma\sqrt{\tau}}\right) \tag{1}$$

And therefore:

$$\frac{\log\left(\frac{f}{K}\right)}{\sigma\sqrt{\tau}}+\frac{\sigma\sqrt{\tau}}{2}=-\sqrt{2}\cdot\operatorname{erfc}^{-1}(2\delta) \tag{2}$$

Isolating the logstrike:

$$\log\left(\frac{K}{f}\right)=\sigma\sqrt{\tau}\cdot\left(\sqrt{2}\cdot\operatorname{erfc}^{-1}(2\delta)+\frac{\sigma\sqrt{\tau}}{2}\right) \tag{3}$$

Plotting the *InverseErfc* function inside the parenthesis and highlighting the {50, 25, 75, 10, 90} delta points on Figure 1:

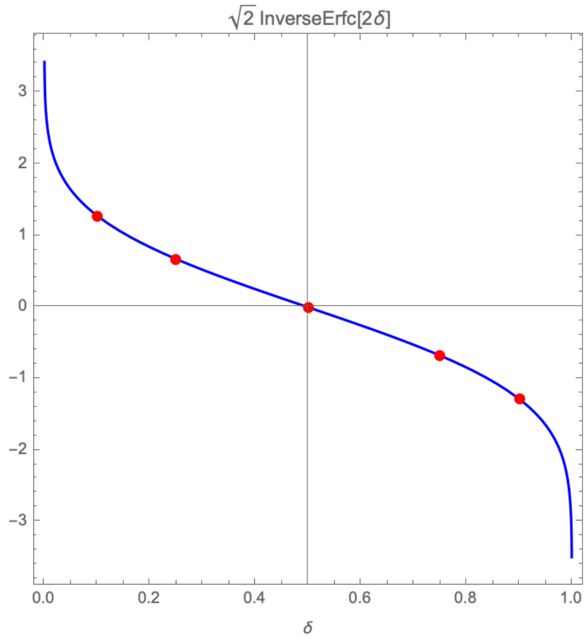

Figure 1: InverseErfc

We can see that within the {10,90} deltas region the logstrike will be very close to linear with respect to δ, as long as σ doesn't change a lot.

2.2 Gaussian Kernels estimation

Following Definition 1.5.1 in [Wystup 2012] a gaussian kernel is defined as:

$$K_\lambda(u) = \exp\left(-\frac{u^2}{2\lambda^2}\right) \tag{4}$$

Defining:

$$\Phi_\lambda(x) = \sum_{i=1}^{N} K_\lambda(|x - x_i|) \tag{5}$$

The interpolation function g is given by:

$$g(x) = \frac{1}{\Phi_\lambda(x)} \cdot \sum_{i=1}^{N} \alpha_i \cdot K_\lambda(|x - x_i|) \tag{6}$$

The choice of λ is a function of the distance between the anchor points x_i. If we're using a delta-based interpolation, where Δ is bounded between 0 and 1 and we have 5 points given by {0.10, 0.25, 0.50, 0.75, 0.90} the choice of λ=0.25 made in [Wystup 2012] is well justified.

Things become interesting when we apply the same interpolation method in logstrike-based interpolation. Most of the time the points in logstrike equivalent to the {0.10, 0.25, 0.50, 0.75, 0.90} deltas are (close to) equally spaced, and we can choose:

$$\lambda = \frac{\log(K_{C25\Delta}) - \log(K_{P25\Delta})}{2}$$

But the logstrike space is not bounded, and by limiting ourselves to 5 anchor points we're bound to lose some information in extrapolation.

We use our anchors to determine an estimate of the location of the 25-delta points; although we keep our estimates when solving for the RR, we solve the equation described in [Wystup 2012] for the ST points. We will improve this when dealing with EM currencies.

Using data for EURUSD in 24-May-2021 as in [Gatheral 2021], we have the results of the two different interpolations for the 1M maturity in Figure 2. For these particular values of the smile parameters the interpolation inside the {10,90} deltas is close enough.

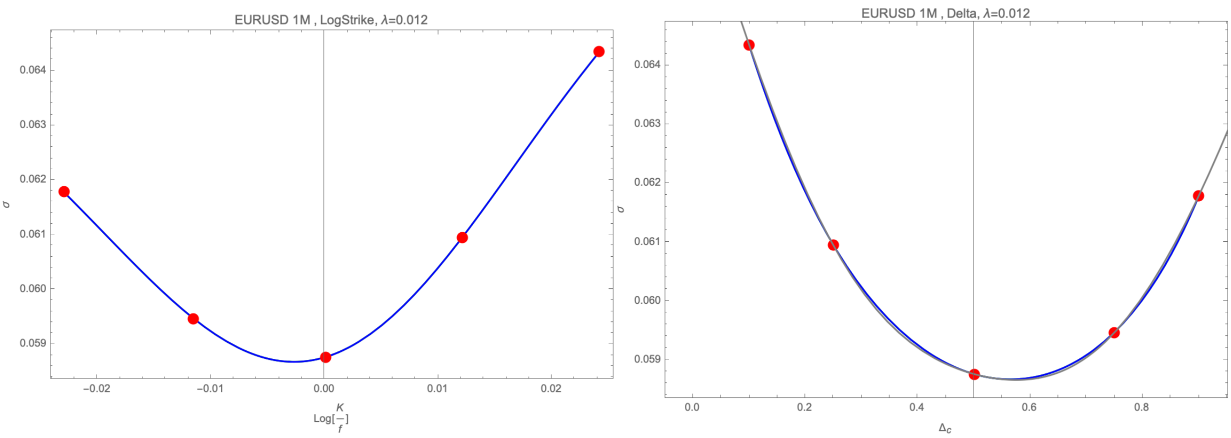

Figure 2: Interpolation EURUSD 1M; logstrike on the left, logstrike (blue) and delta (gray) on the right

2.3 Extrapolation

It is a well-known fact that the extrapolation on the delta space leads to a flattening of the smile in logstrike space.

As an alternative, we can try to fit the behavior highlighted in [Taleb at al 2019]: we want a particular decay for the premium above a certain strike, as seen in Figure 3:

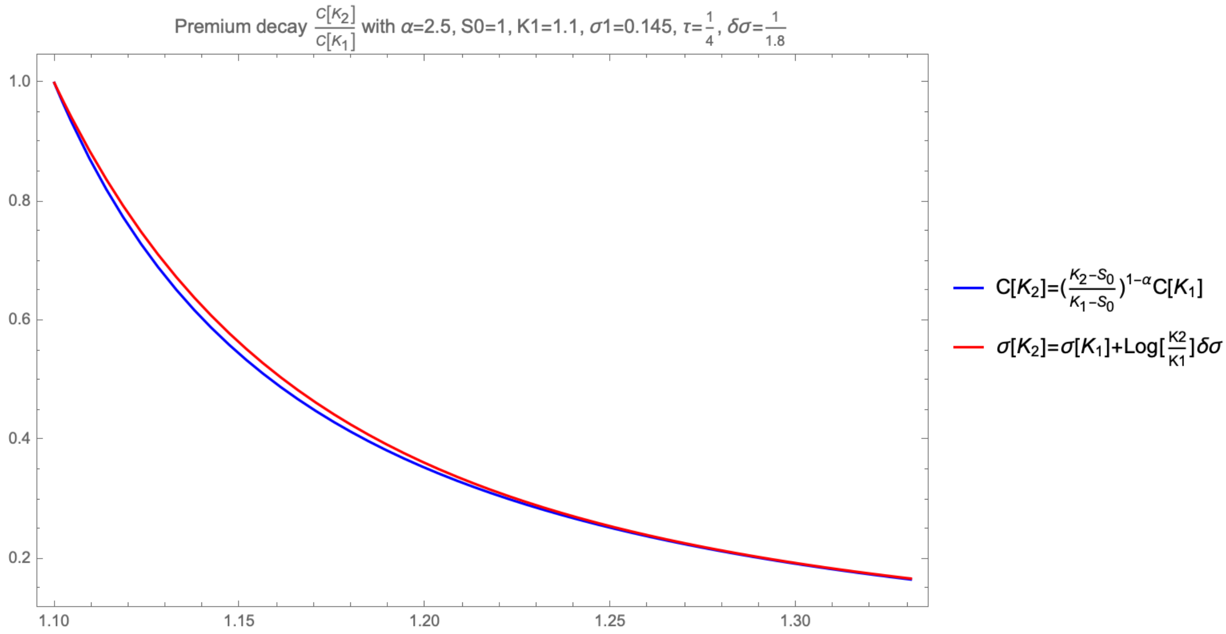

Figure 3: Premium Decay

So the choice of the slope $\delta\sigma$ is given by its fit to α.

An alternative to take into account the continuity issues in section IV ("Arbitrage Boundaries") of [Taleb at al 2019] is to use some measure of the slope of the logstrike interpolation: either the local slope exactly at the 10-deltas (which is going to depend on the ATM vol and is potentially noisier) or the slope between the 10-deltas and the 25-deltas (which is what we use in the following charts). Although this does not ensure a fit to α, it would help to highlight any significant mismatch between the EVT-based estimations of the tail index and the market-implied option prices; although for longer-dated options a significant part of the smile comes from the volatility of volatility, the α-implied slope could be seen as a floor for the slope between the 10-delta strangle premium and the 25-delta strangle premium.

With that, we define 3 different functions for interpolation and extrapolation of implied volatilities; all of them share a g-function slice kernel method for interpolation between 10-delta and 90-delta:

- A delta-based interpolation and extrapolation; where g has {0.10, 0.25, 0.50, 0.75, 0.90} as its anchors and a [0,1] domain. Pictured in gray.
- A logstrike-based interpolation and extrapolation with the same function; where g has the logstrikes of the ATM and RR-implied strikes (as defined in [Wystup 2012]) as its anchors; it typically flattens beyond the 10-deltas on each side. Pictured in blue.
- A logstrike-based interpolation and a constant slopes extrapolation (a different slope on each side) that looks like SVI ([Gatheral 2006]) on the tails. Pictured in green.

3 Results

3.1 EURUSD

3.1.1 1M

Figure 4 shows how different the extrapolation becomes given the goal of matching the tail slope. Data for all maturities is from 24-May-2021, as in [Gatheral 2021].

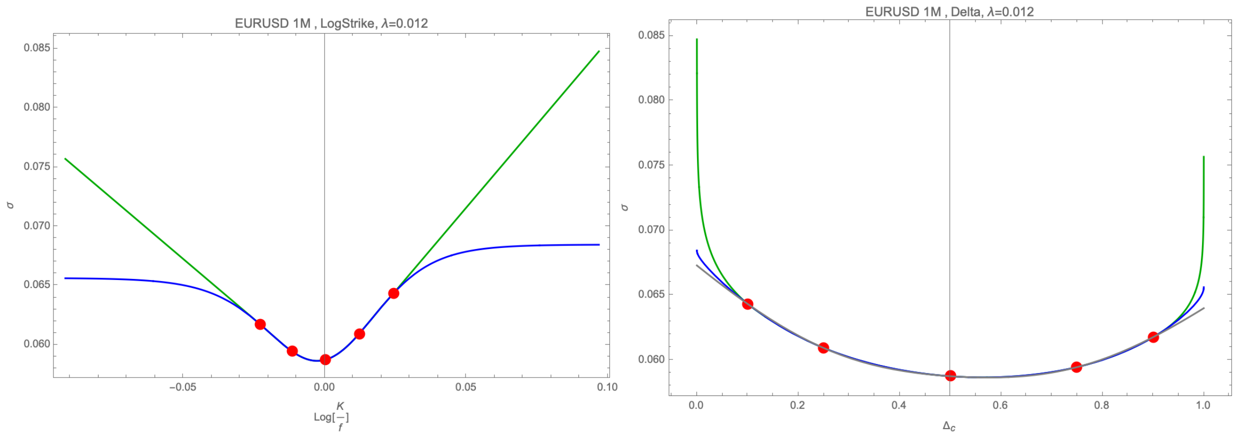

Figure 4: EURUSD 1M implied vols by logstrike - left and by delta - right (delta: gray; logstrike: blue; constant slope: green)

We plot the tail behavior in Figure 5, so we can see that the premium decays are indeed slower (but never violating arbitrage) and that, when seen in delta space, the implied volatilities seem to "avoid" the 0-delta axis - at least much more than the other extrapolations).

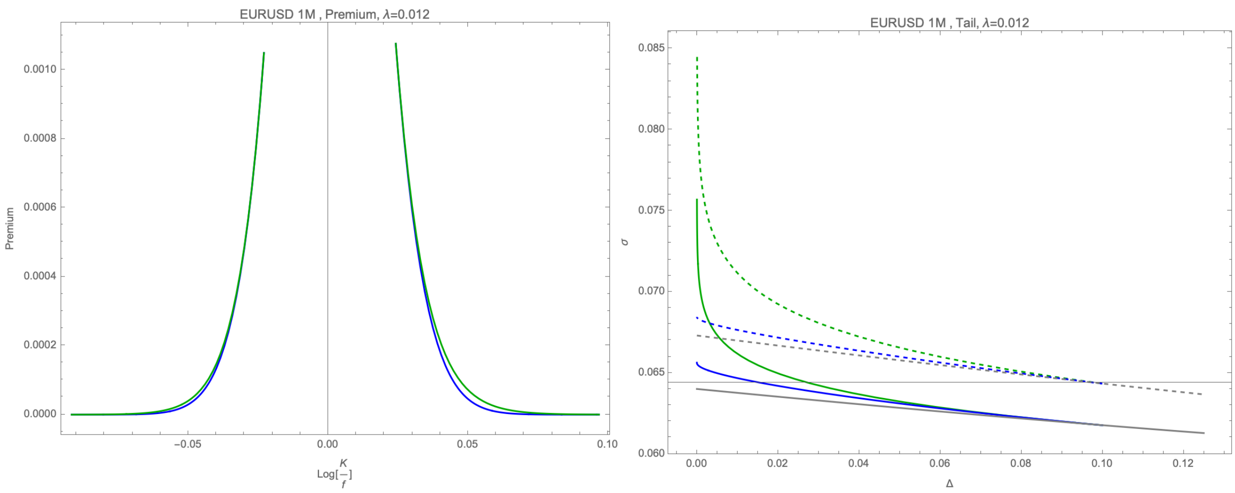

Figure 5: EURUSD 1M tail behavior: premium by logstrike - left and implied vols by delta - right (delta: gray; logstrike: blue; constant slope: green; continuous: puts; dashed: calls)

3.1.2 3M

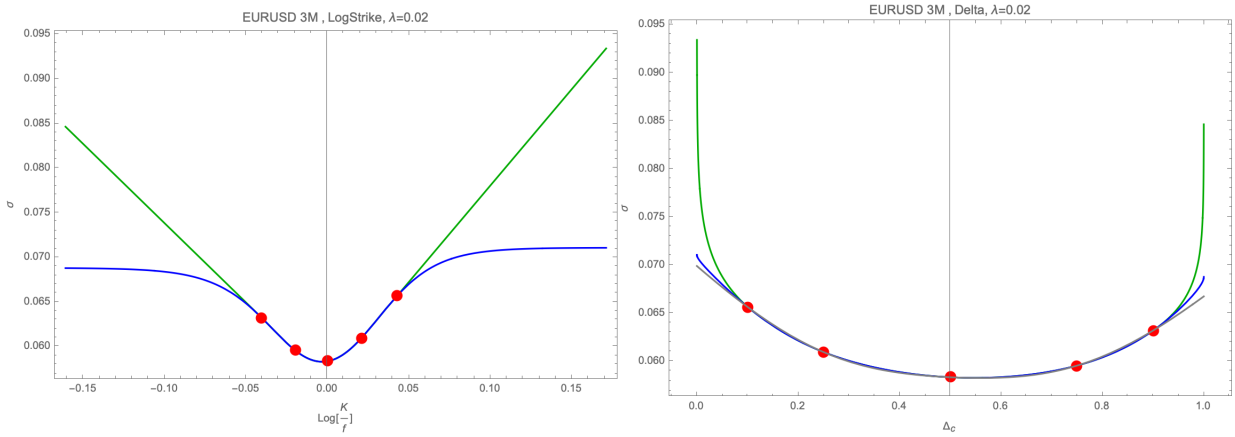

Figure 6: EURUSD 3M implied vols by logstrike - left and by delta - right (delta: gray; logstrike: blue; constant slope: green)

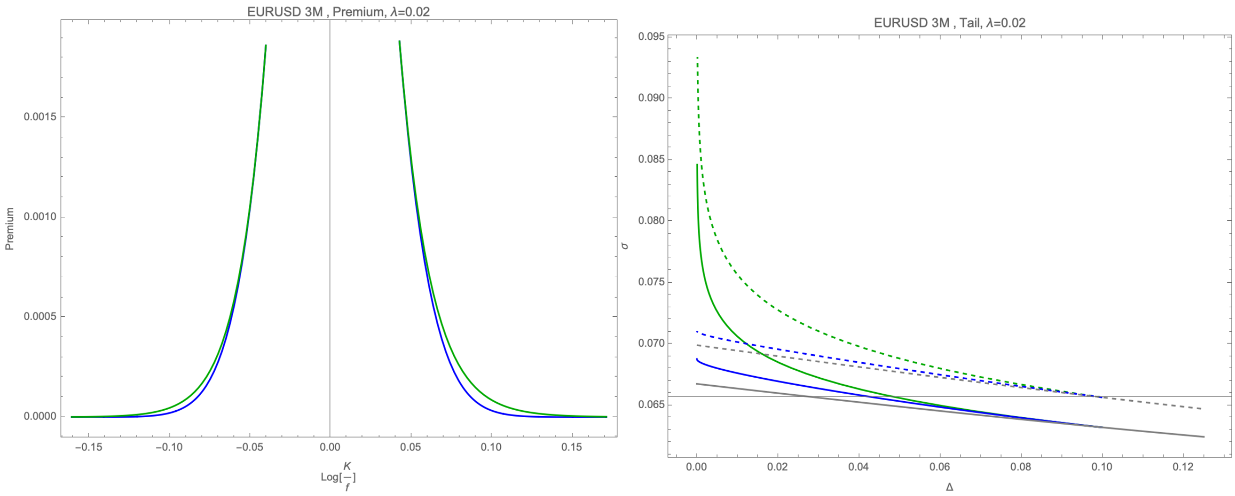

Figure 7: EURUSD 3M tail behavior: premium by logstrike - left and implied vols by delta - right (delta: gray; logstrike: blue; constant slope: green; continuous: puts; dashed: calls)

3.1.3 6M

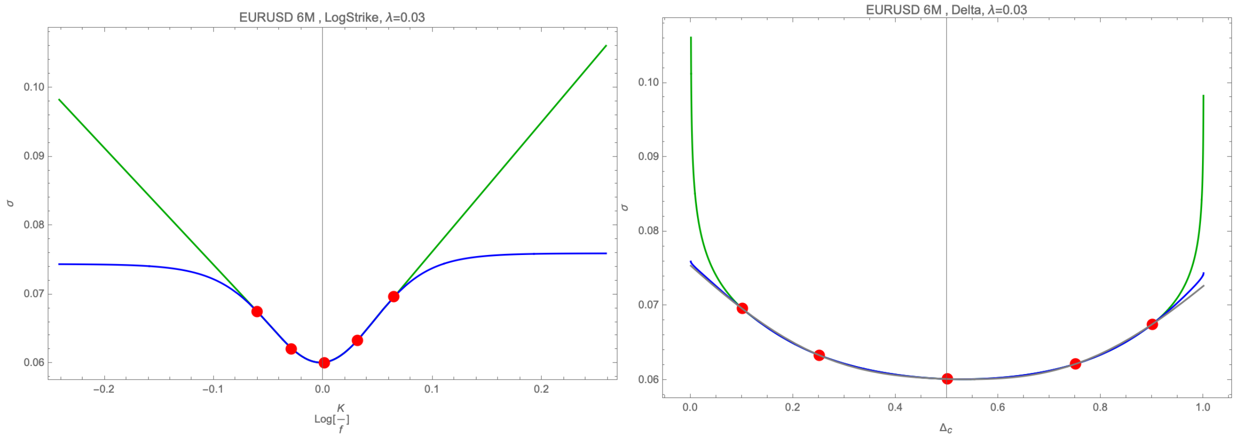

Figure 8: EURUSD 6M implied vols by logstrike - left and by delta - right (delta: gray; logstrike: blue; constant slope: green)

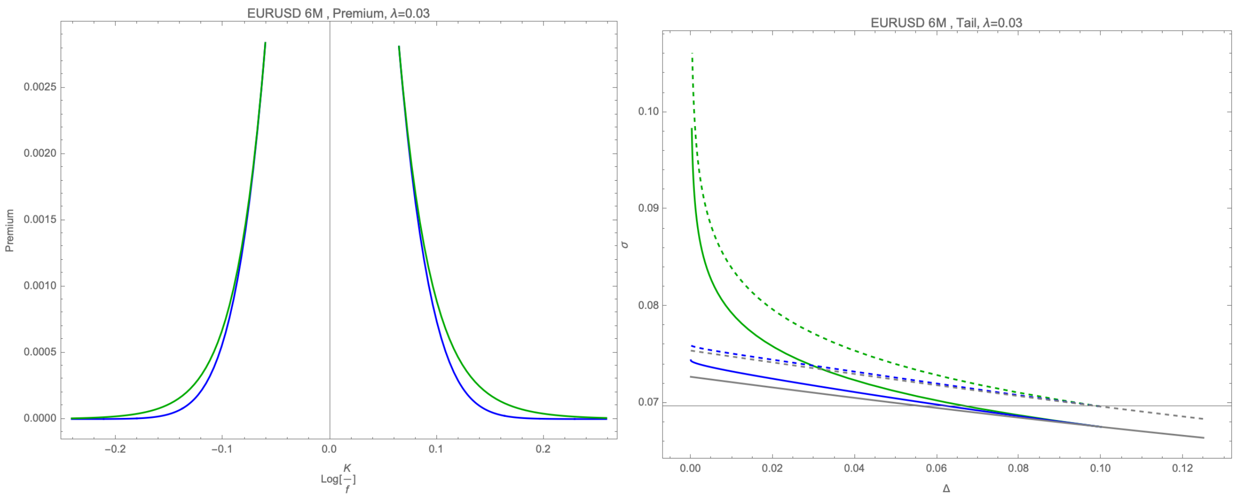

Figure 9: EURUSD 6M tail behavior: premium by logstrike - left and implied vols by delta - right (delta: gray; logstrike: blue; constant slope: green; continuous: puts; dashed: calls)

3.1.4 1Y

There is an interesting reversion of the risk reversal from positive at 9M to negative at 1Y.

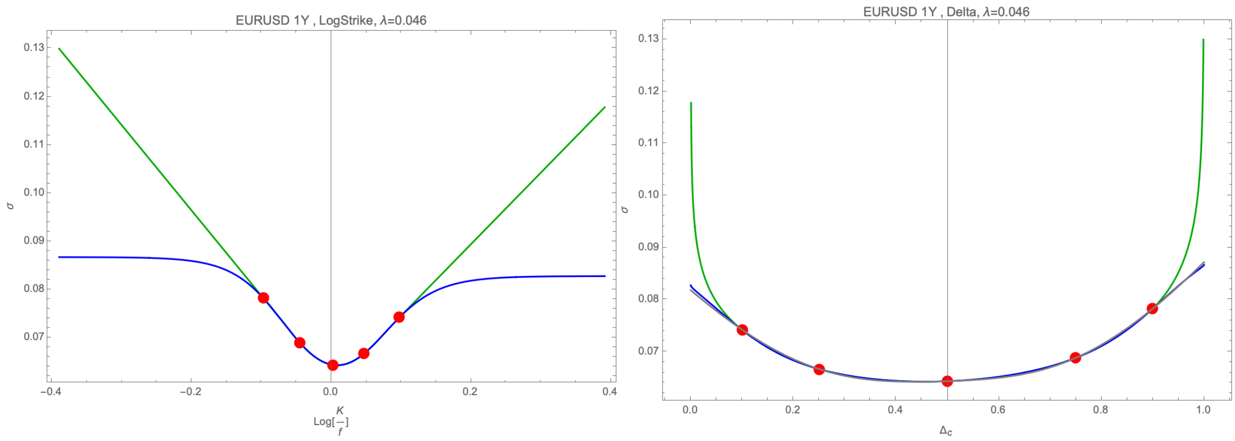

Figure 10: EURUSD 1Y implied vols by logstrike - left and by delta - right (delta: gray; logstrike: blue; constant slope: green)

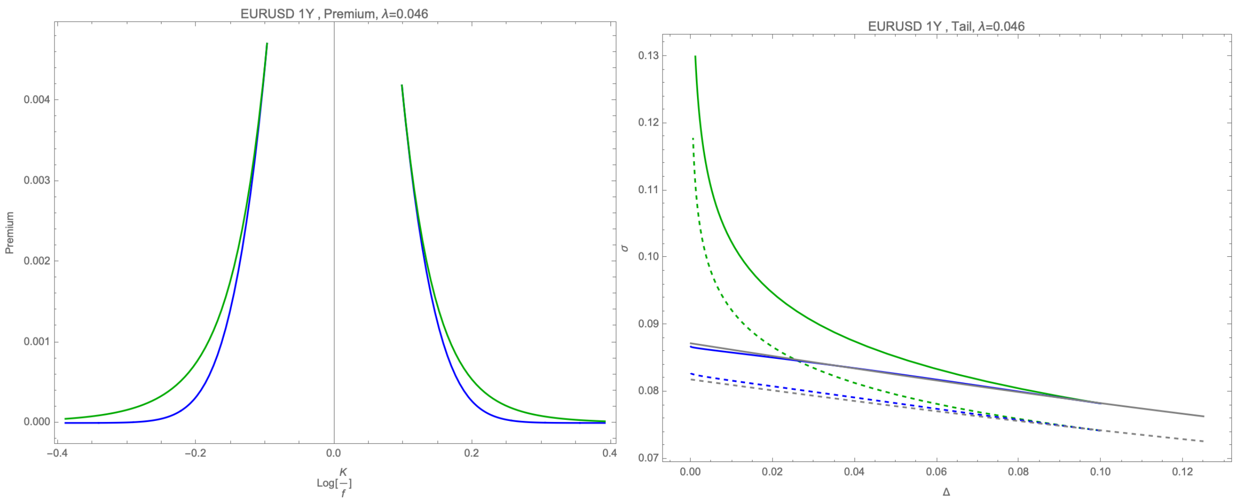

Figure 11: EURUSD 1Y tail behavior: premium by logstrike - left and implied vols by delta - right (delta: gray; logstrike: blue; constant slope: green; continuous: puts; dashed: calls)

3.2 Other currencies

Work to be done:

- Adjust functions to deal with inverted currencies
- Look carefully at EM currencies (high RR, higher ST, strike ATMF all distort smile)

4 Conclusions

We try to implement a simple extrapolation formula in FX options that keeps the good features of the delta-based interpolation between 10-delta and 90-delta while trying to improve the extrapolation on the tails according to [Taleb at al 2019] and avoiding the typical flattening of the smile in FX by using a constant slope in logstrike like the SVI model ([Gatheral 2006]).

References

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