Chapter 8

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Technical Incerto Reading Club - 08-Jun-2021

Contents

One-liners

- Fat Tails make extrapolation dangerous
- Under Fat Tails diversification is harder
- Term structure of likelihood of κ is what we need

Log Glasses

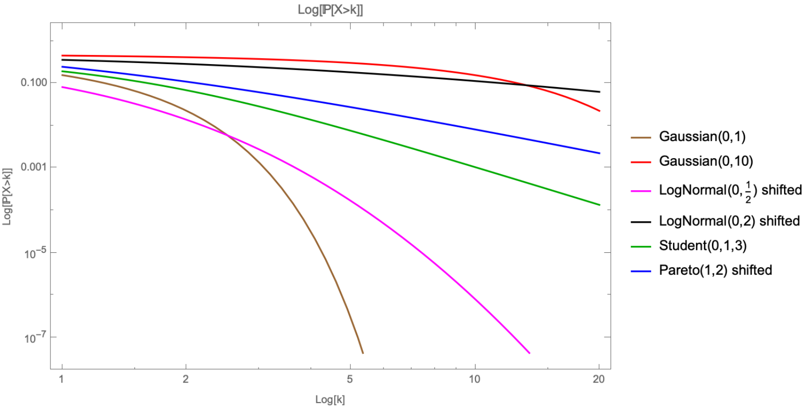

Figure: LogLog plot of $\mathbb{P}[X > k]$

Surprise

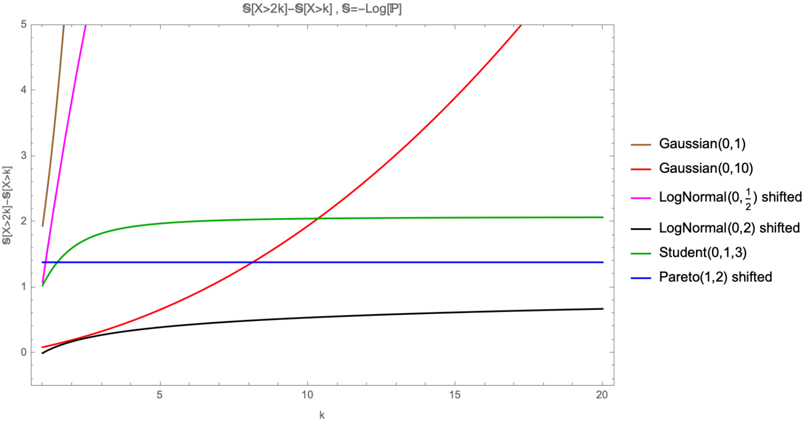

Figure: Surprise

Formulas

- Gaussian: $MAD\,[n,\sigma] = \sqrt{\frac{2}{\pi}}n\sigma$
- StudentT:
  - $MAD\,[n=1,\alpha,\sigma] = \frac{2\sqrt{\alpha}}{\alpha-1}\frac{Gamma\left[\frac{1}{2}+\frac{\alpha}{2}\right]}{\sqrt{\pi}Gamma\left[\frac{\alpha}{2}\right]}\sigma$
  - $MAD\,[n=2,\alpha,\sigma] = \frac{4\sqrt{\alpha}}{2^{\alpha}}\frac{Gamma\left[-\frac{1}{2}+\frac{\alpha}{2}\right]Gamma\left[-\frac{1}{2}+\alpha\right]}{\left(Gamma\left[\frac{\alpha}{2}\right]\right)^3}\sigma$
- LogNormal: $MAD\,[n=1,\mu,\sigma] = 2\exp\left[\mu+\frac{\sigma^2}{2}\right]\,Erf\left[\frac{\sigma}{2\sqrt{2}}\right]$
- Pareto: $MAD\,[n=1,\alpha,L=1] = 2\cdot L\cdot(\alpha-1)^{-2+\alpha}\cdot\alpha^{1-\alpha}$

LogNormal

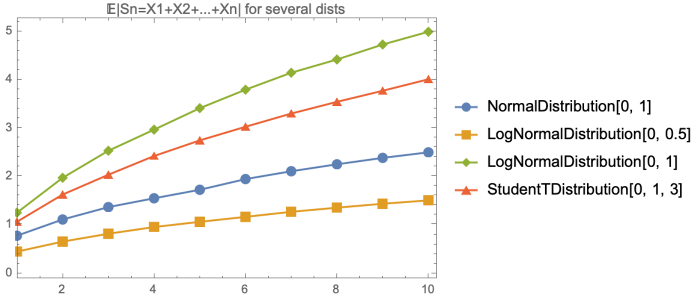

Figure: Mean Deviation for LogNormals

Approximation for the StudentT

- For $\alpha \to \infty$ StudentT is Gaussian
- So fit n^{γ} to the $MAD\left[n\right]$ curve
- $0.5 \leq \gamma \leq 1$

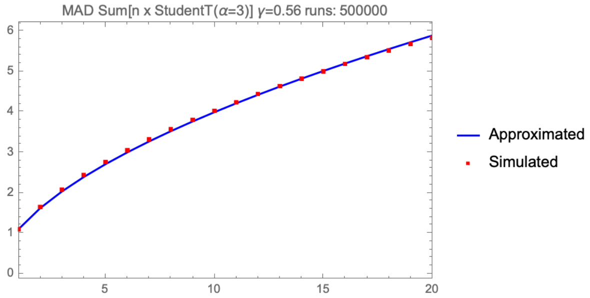

Figure: Fit for $\alpha = 3$

Rough fit

- $\gamma \approx 0.5 + 2^{-1.336 \cdot \alpha - 0.14}$

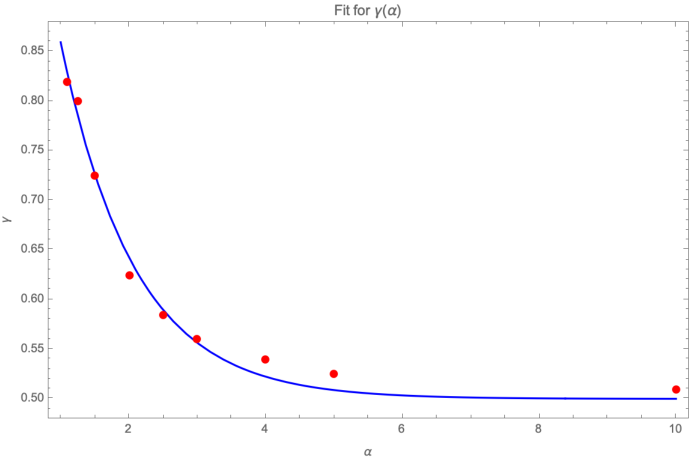

Figure: Fit for γ

Cubic Student T

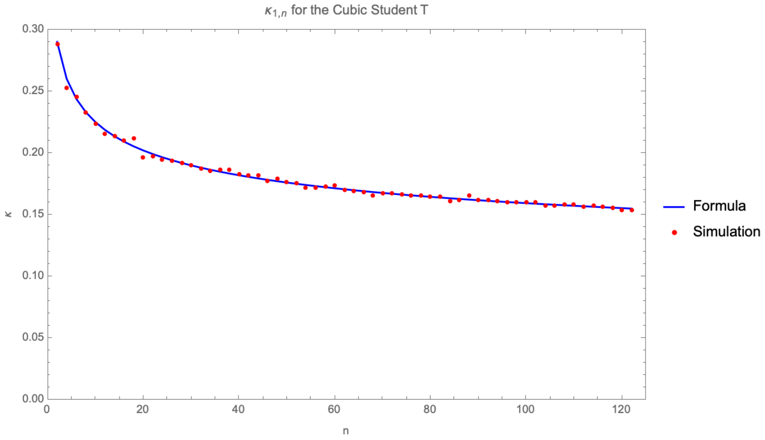

Figure: StudentT $\kappa_{1,n}$

Low σ LogNormal

- $\sigma = 0.5$ (bias at low n)

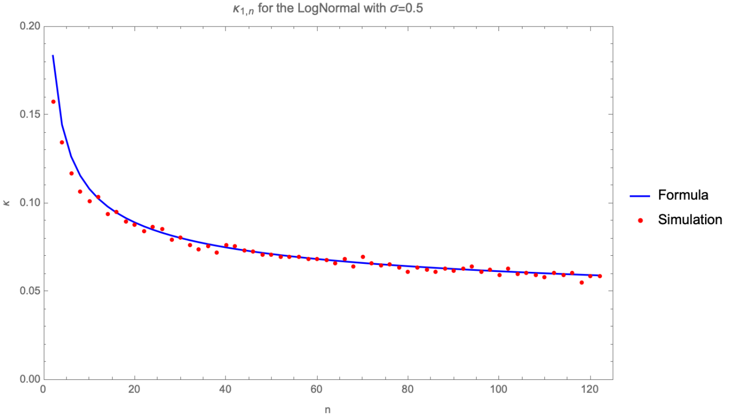

Figure: LogNormal $\sigma = 0.5$ $\kappa_{1,n}$

High σ LogNormal

- $\sigma = 2$ (bias everywhere) - "Did I err?"

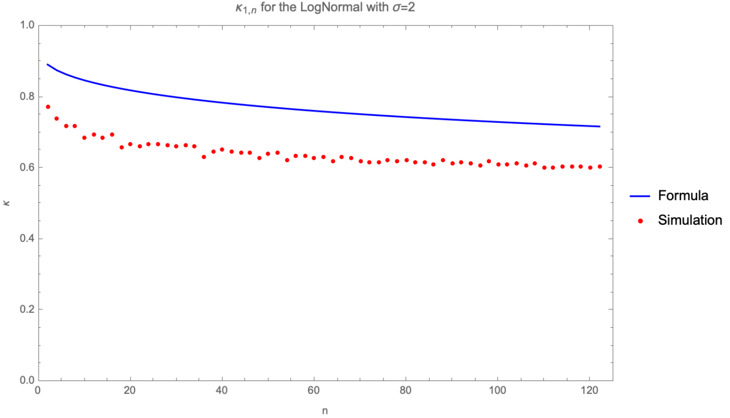

Figure: LogNormal $\sigma = 0.5$ $\kappa_{1,n}$

Kappa dispersion

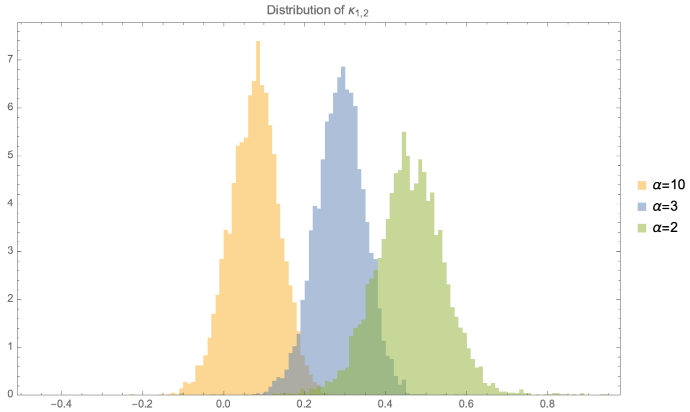

Figure: StudentT $\kappa_{1,2}$

Paired at the start

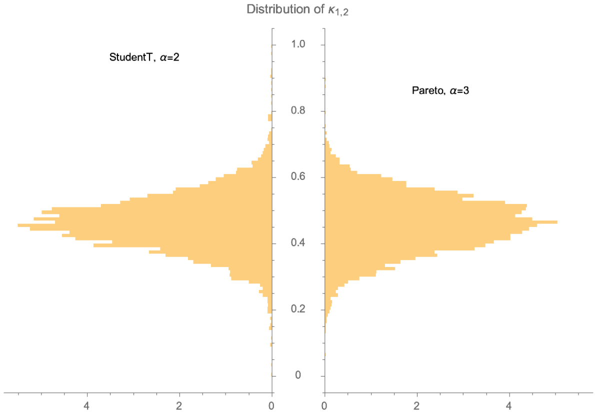

Figure: StudentT(2) x Pareto(3) $\kappa_{1,2}$

StudentT(3) Term Structure

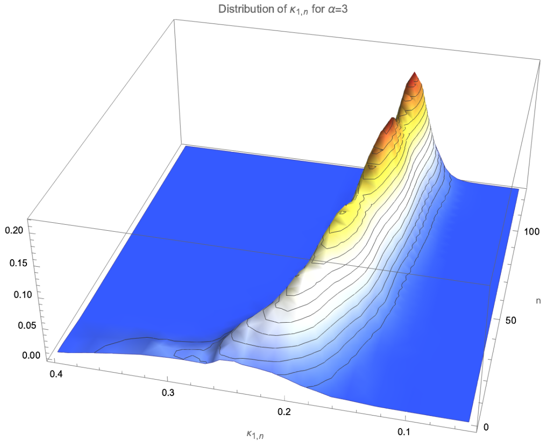

Figure: StudentT $\alpha = 3$ $\kappa_{1,n}$

StudentT(2) Term Structure

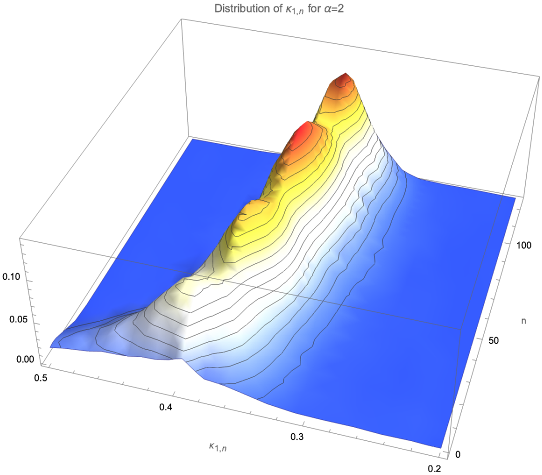

Figure: StudentT $\alpha = 2$ $\kappa_{1,n}$

Pareto Term Structure

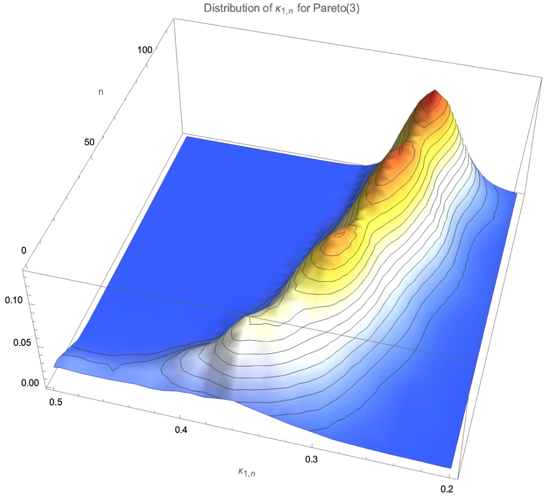

Figure: Pareto $\alpha = 3$ $\kappa_{1,n}$

Decays

- In this case Pareto decays (relatively) faster, the StudentT is at the maximum "time value"

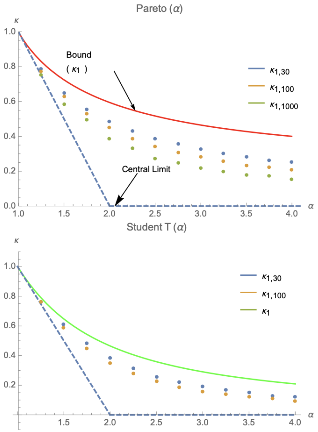

Figure: Decays

Methods and code

- Improve code for performance (large n crashes Mathematica)
- Work on calculations/integrals before asking Mathematica
- Formalize likelihoods

Data and models

- “Truncated” distributions (circuit breakers, etc.)
- Multivariate distributions
- Correlations and autocorrelations

What this talk was about anyway?

Model inference

- Most data doesn't allow for precise estimation of distributions and therefore not even fake precision for estimating parameters
- Understand model properties and develop qualitative measures like kappa