Dodson & Poston Exercise VII.1.7

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Abstract

In-progress solution. Feel free to add/comment/disparage. See Randy's solution.

(a) Use $f(x) = x^3$ to show that the "if" of Corollary 1.05 cannot be strengthened to "iff".

Solution Corollary 1.05 states that if f is C^1 and $D_x f$ is injective, then there is a neighborhood of x, N, such that $f|_N$ is injective. In the example given, f is C^1 and $f|_N$ is injective for any neighborhood of any x, but $D_0 f$ is not injective, providing a counterexample to the "only if" part of "iff".

(b)

Solution See Figure 1

(c)

Solution

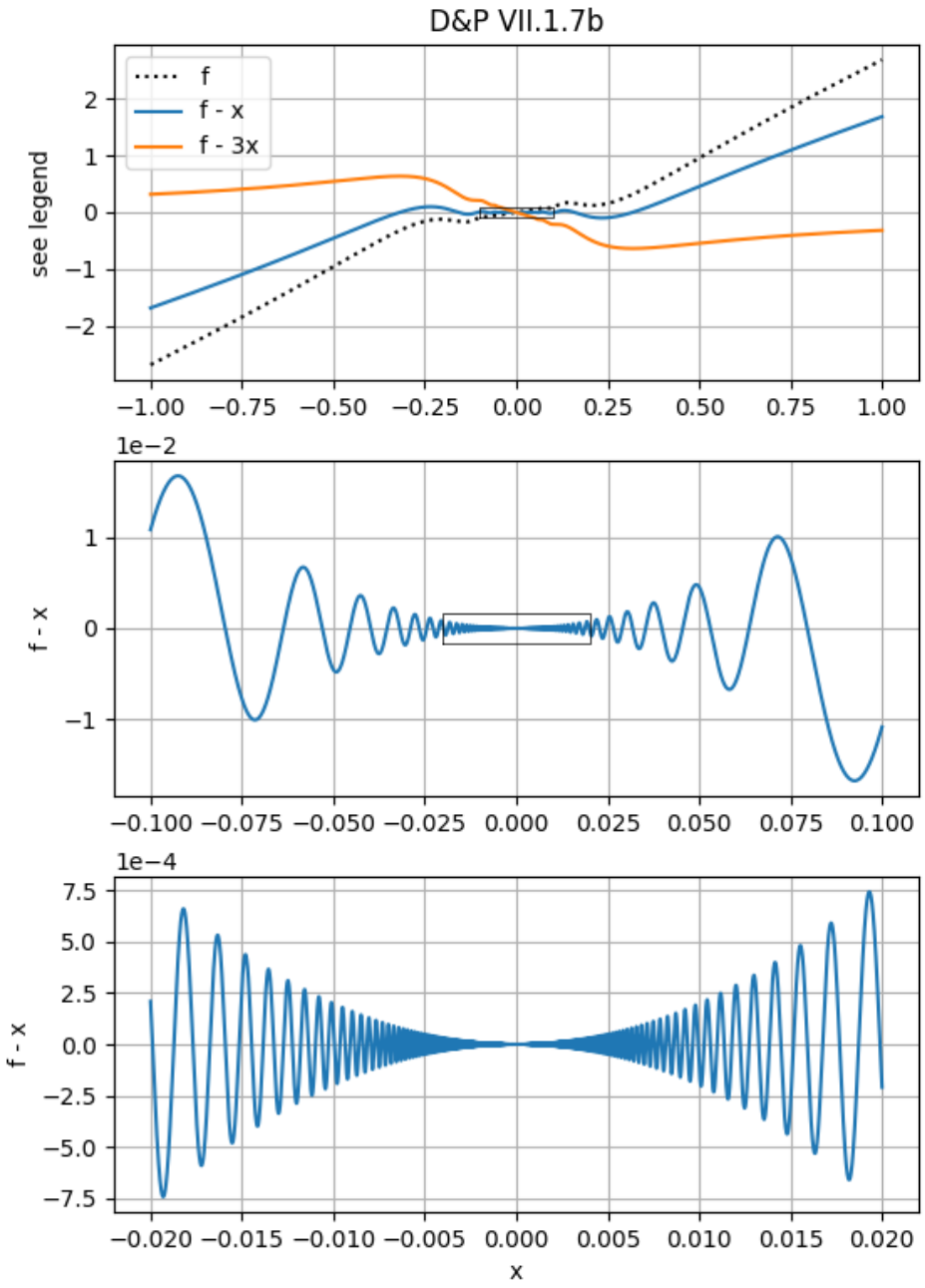

Figure 1: **Top**: f, $f-x$ and $f-3x$ over the range [-1,1]. Near $x=0$, the salient features of f are best shown by f-x. Far from $x=0$, the slope of f asymptotes to 3 (hence the $f-3x$). **Middle**: $f-x$ over the range [-0.1,0.1]. The main features of f start to become evident at this range – diminishing of both amplitude and wavelength as x approaches 0. **Bottom**: At all smaller ranges in x, the picture is the same – a parabolic envelope and ever shortening wavelength as $x \to 0$. Although the slope of f approaches 1 as $x \to 0$, it approaches the asymptotic slope from both above and below the limiting value.