Re: D&P p.46, Lem. 1.05 — PHM 2020-05-24

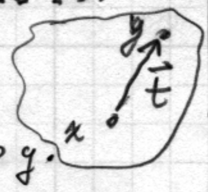
Answer to, "what is $d(d_{x'}^{\leftarrow}(\vec{t}), d_{x'}^{\leftarrow}(\vec{t} + d(x';y')))$?"

First, recall that $\mathbf{d}$ is a mapping from $X \times X \to T$, where objects (like x) in X are <u>points</u> and $\vec{t} \in T$ is a vector, in the familiar sense.

So $d^{\leftarrow}$ is a mapping from T back into $X \times X$.

Formally, $d: X \times X \to T$, $(x,y) \mapsto \vec{t}$

let $x, y \in X$, and $\vec{t} \in T$. our picture is:



where $\vec{t} = d(x,y)$, the vector from x to y.

Next, d_x is a restricted mapping from $\{x\} \times X \to T$ with mapping rule $(x,y) \mapsto \vec{t}$. Since the x in the first position is proscribed, we may write the rule as $y \mapsto \vec{t}$.

$$\boxed{\begin{aligned} d_x: \{x\} \times X &\to T \\ T_x X &\to T \\ (x,y) &\mapsto \vec{t} \\ y &\mapsto \vec{t} \end{aligned}}$$

$d_x^{\leftarrow}(\vec{t})$, then, is a mapping from a vector $\vec{t} \in T$ back to either the pair of points, (x,y), where $y = x + \vec{t}$, ~~or~~ just $y = x + \vec{t}$, since the x is "understood." So $\boxed{d_x^{\leftarrow}(\vec{t}) = x + \vec{t} = y \in X}$

Now going back to part 2 of the proof of Lemma 1.05 on p46. the second line says $d(x'', y'') = d(d_{x'}^{\leftarrow}(\vec{t}), d_{x'}^{\leftarrow}(\vec{t} + d(x', y')))$.

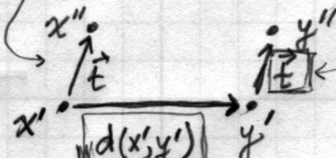
Note that X', X'' are affine subspaces of X, and $x', y' \in X'$, $x'', y'' \in X''$.

Furthermore, $X'' = X' + \vec{t}$, $\vec{t} \in T$, so $\boxed{x'' = x' + \vec{t}}$ and $\boxed{y'' = y' + \vec{t}}$.

It should be clear that $x'' = d_{x'}^{\leftarrow}(\vec{t}) = x' + \vec{t}$

For the second term, a picture helps.

(this is the same as D&P's fig 1.5)



$$d_{x'}^{\leftarrow}(\vec{t} + d(x',y')) = \underbrace{x' + \vec{t} + d(x',y')}_{\text{this is } y'' \in X'' \text{ from the diagram.}}$$

From fig 1.5, it is obvious that $d(x'', y'') = d(x', y')$.

Algebraically we subst. in the "long" forms of x'', y'':

$$\underline{d(x'', y'')} = d(\underset{\uparrow}{x' + \vec{t}}, \underset{\uparrow}{x' + \vec{t}} + d(x', y')) = \underline{\underline{d(x', y')}} \quad \square$$

$\boxed{\text{same starting point, } x'' = x' + \vec{t}}$