Definitions – per CH-01.

Overview – This specification describes a method by which a calculated "tournament-elo" may be calculated for a prototype competition.

Formulation – Tournament elo for each player is represented according to Equation (1):

$$E = E_{ave} + \Delta E \tag{1}$$

Where "E" is the tournament elo, to be processed for seeding/bracket formation activities, "E_{ave}" is the weighted average elo derived from match history, and "ΔE" is the elo correction, calculated from the win rate of the player. The elo average term is expanded upon in Equation (2) below:

$$E_{ave} = \frac{\sum_{i=1}^{n} w_i E_i}{\sum_{i=1}^{n} w_i} \tag{2}$$

Where "n" is the number of points or matches to be considered, "i" is the index of the match, "E_i" is the elo at each index, and "w_i" is the weight or importance of that elo for each index.

<u>Example 1 – A Simple Averaging Method</u>: A competition averages only the past three games for each participant when calculating tournament elo. For a participant whose last three elos were E = [1200, 1300, 1400], calculate average elo for the following situations: 1) No weight, 2) Linear Weight, 3) Quadratic Weight.

For the case of "No Weight", the factors are the index raised to the zeroth power: $w_i = i^0 = 1$. This means that:

$$E_{ave} = \frac{\sum_{i=1}^{n} (1) E_i}{\sum_{i=1}^{n} (1)} = \frac{1}{n} \sum_{i=1}^{n} E_i = \frac{1}{3} [E_1 + E_2 + E_3] = \frac{1200 + 1300 + 1400}{3} = 1300$$

For the case of "Linear Weight", the factors are equal to index: $w_i = i$, with the following effect, taking advantage of summation identities:

$$E_{ave}=\frac{\sum_{i=1}^{n} i\,E_i}{\sum_{i=1}^{n} i}=\frac{2}{n(n+1)}\sum_{i=1}^{n} i\,E_i=\frac{2}{3(3+1)}[(1)E_1+(2)E_2+(3)E_3]=\frac{E_1+2E_2+3E_3}{6}=\frac{1200+2(1300)+3(1400)}{6}$$

$$E_{ave}=\frac{8000}{6}=1333.3$$

For the case of "Quadratic Weight", the factors are equal to index squared: $w_i=i^2$, with the following effect, taking advantage of summation identities:

$$E_{ave}=\frac{\sum_{i=1}^{n} i^2 E_i}{\sum_{i=1}^{n} i^2}=\frac{6}{n(n+1)(2n+1)}\sum_{i=1}^{n} i^2 E_i=\frac{6}{n(n+1)(2n+1)}[(1)^2E_1+(2)^2E_2+(3)^2E_3]=\frac{6[E_1+4E_2+9E_3]}{n(n+1)(2n+1)}$$

$$E_{ave}=\frac{6[1200+4(1300)+9(1400)]}{3(3+1)(2(3)+1)}=\frac{2[1200+5200+12600]}{(4)(6+1)}=\frac{19000}{(2)(7)}=\frac{19000}{14}=1357.1$$

Note that whatever ends up in the denominator must be the sum of all the numbers multiplied by "*E*" values. Also note that as the weight placed on the higher, more recent, elos grows, so too does the weighted average.

The elo correction term "*ΔE*" must account for the following two situations:

1) Immaturity of Match History – the win rate is unnaturally high or low because the starting elo of the player does not reflect the skill level they had when they began competitive play. Continued play will eventually mature to a win-rate that is commensurate with other players of similar elo, once that elo is achieved.
2) Worthy Adversaries Unavailable – the player in question inhabits an extreme of the bell curve (either high or low skill), such that the matchmaker struggles to find worthy opponents within a reasonable waiting period, given the low number of nearby choices. The result is a disproportionate number of matches played above or below the ideal elo, skewing the win rate above or below 50%. Note that since elo is "zero-sum", this does not mean inflation – the occasional upset in these situations results in large elo transfer sufficient to offset aberrant win percentage.

Equation 3, shown below, is based on the science of elo and provides for the correction factor:

$$r=\frac{1}{1+10^{\frac{-\Delta E}{400}}} \qquad (3)$$

Where "*r*" is the win rate and "*ΔE*" is the elo difference. Note that when elo difference is zero, ten raised to the zeroth power is one, and one plus one is two. Thus win rate would be ½ or 50%, as expected.

<u>Example 2 – Underdog Odds</u>: A tournament promises its participants that no opponent in their division will be 200 elo higher than they are. Calculate the minimum and maximum mathematical odds of a participant winning their first game of the tournament.

Application of Equation (3) is straightforward, plugging in -200 for "*ΔE*":

$$r = \frac{1}{1+10^{\frac{-\Delta E}{400}}} = \frac{1}{1+10^{\frac{200}{400}}} = \frac{1}{1+10^{\frac{1}{2}}} = \frac{1}{1+\sqrt{10}} = 0.24 = 24\%$$

For the maximum, "*ΔE*" is 200:

$$r = \frac{1}{1+10^{\frac{-\Delta E}{400}}} = \frac{1}{1+10^{\frac{-200}{400}}} = \frac{1}{1+10^{\frac{-1}{2}}} = \frac{1}{1+\frac{1}{\sqrt{10}}} = 0.76 = 76\%$$

The maximum probability could also have been achieved by subtracting the minimum from 100%.

Equation 3 may be re-written to solve for the elo gap, given a known win-rate:

$$\Delta E = -400\log_{10}\left(\frac{1}{r}-1\right) \quad (4)$$

<u>Example 3 – Young Account</u>: A player at 1267 elo has a 64% win rate. Calculate the elo of opponents at which such a player would have a 50% win rate.

Converted to a decimal, 64% can be substituted into Equation (4) and elo correction may be calculated:

$$\Delta E = -400\log_{10}\left(\frac{1}{r}-1\right) = -400\log_{10}\left(\frac{1}{0.64}-1\right) = -400\log_{10}(0.5625) = 100$$

$$E_{act} = E_{cur} + \Delta E = 1267 + 100 = 1367$$

It is estimated that the player in question could have a 50% win rate against opponents of 1367 elo.

The calculation in Example 3 is excellent if both of the following assumptions are true:

1) Win rate has a high degree of confidence, and not 100% or derived from only a few games. Tournaments will specify a required number of games from each participants, so that elo corrections calculated per Equation (4) would be regarded as accurate.
2) Equation (4) is regarded as correct when the ideal win rate is 50%, but some players have "steady-state" elos that do not converge at this win rate. To account for such a discrepancy, a modified formula is presented as Equation (5).

This modification is made by letting $r = \frac{1}{2}+\delta = \frac{1+2\delta}{2}$, where "$\delta$" represents deviation "above" or "below" normal, – a relative value. This allows the equation to be "tricked" into accounting for extreme elo on the bell curve, both in high and low skill.

$$\Delta E = -400\log_{10}\left(\frac{1}{r}-1\right) = -400\log_{10}\left(\frac{2}{1+2\delta}-1\right) = -400\log_{10}\left(\frac{2}{1+2\delta}-\frac{1+2\delta}{1+2\delta}\right) = -400\log 10\left(\frac{1-2\delta}{1+2\delta}\right) \quad (5)$$

If Equation 5 is to be used, then an understanding of "normal" win rate for a given elo must be developed. Figure 1 below presents an example for Age of Empires II: Definitive Edition, generated from data compiled by the relic API on September 30th, 2023:

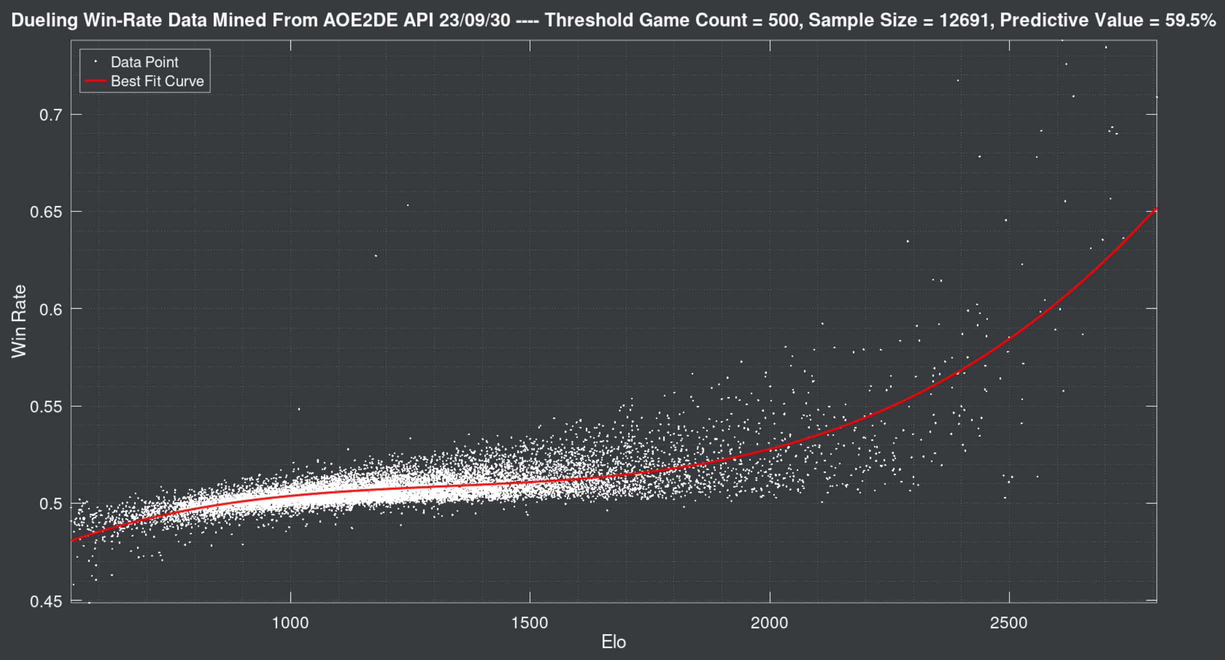

Figure 1: Effect of Elo on Expected Win Rate

A third-order polynomial of best fit (red) was derived for this curve is shown as Equation (6) below:

$$r_{nor} = 0.8756\left(\frac{E}{E_{max}}\right)^3 - 1.246\left(\frac{E}{E_{max}}\right)^2 + 0.6215\left(\frac{E}{E_{max}}\right) + 0.4009 \quad (6)$$

For this particular dataset, the maximum elo, represented by "E_{max}" is equal to 2808, and the equation for the red curve was calculated after dividing elos by this maximum. Once typical win-rate is known from Equation 6, then it may be used to determine "δ" per Equation (7):

$$\delta = r - r_{nor} \quad (7)$$

Then, with a value for win-rate deviation ("δ"), an elo modification may be determined.

Example 4 – Elo Modification for a Top Player: A player with 2132 average elo has a 70% win rate. Calculate the tournament elo in if the maximum elo on the ranked ladder is 2808.

The first step is to determine the win rate that would be normal for a 2132 elo player. To do this, Equation (6) is used with 2132 substituted for "*E*":

$$r_{nor} = 0.8756\left(\frac{E}{E_{max}}\right)^3 - 1.246\left(\frac{E}{E_{max}}\right)^2 + 0.6215\left(\frac{E}{E_{max}}\right) + 0.4009$$

$$r_{nor} = 0.8756\left(\frac{2132}{2808}\right)^3 - 1.246\left(\frac{2132}{2808}\right)^2 + 0.6215\left(\frac{2132}{2808}\right) + 0.4009 = 0.5377$$

If the player in question has a 70% win rate, but should have a 53.8% win rate, then deviation from normal, δ = 16.2%. Substituting this into Equation (5):

$$\Delta E = -400\log 10\left(\frac{1-2\delta}{1+2\delta}\right) = -400\log 10\left(\frac{1-2(0.162)}{1+2(0.162)}\right) = -400\log 10\left(\frac{0.6760}{1.324}\right)$$

$$\Delta E = -400\log 10(0.5106) = 116.8$$

Therefore, to calculate a final tournament elo:

$$E = E_{ave} + \Delta E = 2132 + 117 = 2249$$

Conclusion – This specification outlines a method for calculating "tournament-elo". This tournament elo accounts for the following two factors:

1) "Big picture" averaging that can incorporate elo history and weigh matches according to how recently they were played.
2) Unsteady win rates for players who have not yet found their true elo.

It is not claimed that these methods will resolve the serious issue of "smurfing" in eSports, but they make cheating more difficult and competition seeding more fair. For this reason, this specification is a recommended component in prototype tournaments.

Revision	Description	Change Document	Date
Original Issue	---	N/A	2023/10/26