

Exercise 3.8

Consider the set $A = \{x \in \mathbb{R} : 1 < x < 2\}$.

- (a) Show that A is bounded from above. Find the supremum. Is this supremum a maximum of A ?
- (b) Show that A is bounded from below. Find the infimum. Is this infimum a minimum of A ?

Exercise 3.9

Consider the set $A = \{x > 0 : x^2 > 4\} = \{x > 0 : x > 2\}$.

- (a) What is a lower bound of A ?
- (b) Let L be a lower bound of A such that $L > 2$. Let $y = \frac{L+2}{2}$. Show that $2 < y < L$.
- (c) Show that $y \in A$ and $L \leq y$. Show that this leads to a contradiction. Hence, we must have $L \leq 2$ which means that 2 is the infimum of A .

Exercise 3.14

For each of the following sets S find $\sup\{S\}$ and $\inf\{S\}$ if they exist. You do not need to justify your answer.

- (a) $S = \{x \in \mathbb{R} : x^2 < 5\}$.
- (b) $S = \{x \in \mathbb{R} : x^2 > 7\}$.
- (c) $S = \{-\frac{1}{n} : n \in \mathbb{N}\}$.

Exercise 3.11

Let a and b be any two real numbers such that $a < b$.

- (a) Let w be a fixed positive irrational number. Show that there is a rational number r such that $a < wr < b$.
- (b) Show that wr is irrational. Hence, between any two distinct real numbers there is an irrational number.

Example 3 of page no. 18 from the prescribed textbook

Exercise on pg no 18 from problems from 1 to 4 from the text book

Find the GLB and LUB of the above listed sets

	GLB	LUB
$\{x \mid 0 < x \leq 3\}$		
$\{x \mid x^2 - 3 < 0\}$		
$\{y \mid y = \frac{x}{x+1}, x \geq 0\}$		
$\{y \mid y = \frac{3}{x^2}, x \neq 0\}$		
$\{y \mid y = \frac{3}{x^2}, x \in \mathbb{Z}, x \neq 0\}$		

Find the Supremum and infimum of the following sets

1. $\{x^2 \mid x \in \mathbb{Z}^+ \text{ and } -3 \leq \frac{3x}{2} < 27\}$
2. $\{\frac{-2}{x} \mid x \in \mathbb{Z} \text{ and } x \neq 0 \text{ and } |x| < 5\}$
3. $\{v \mid v \in \mathbb{Q} \text{ and } v^2 < 5\}$
4. $\{n \mid n = \frac{6}{x^2+2}, x \in \mathbb{R}\}$
5. $\{\sqrt{9-x^2} \mid x \in \mathbb{R}\}$