Geometric Series :- show that the series

$1+r+r^2+r^3+ \dots \infty$. (i) converges if $|r|<1$

ii) diverges if $r \geq 1$, iii) oscillates if $r \leq -1$

Pr Let $S_n = 1+r+r^2+ \dots + r^{n-1}$

case i) when $|r|<1$,

we know that $\lim_{n\to\infty} r^n = 0$

$$S_n = \frac{1-r^n}{1-r} = \frac{1}{1-r} - \frac{r^n}{1-r}$$ So that

$$\lim_{n\to\infty} S_n = \frac{1}{1-r}.$$

So Series is convergent

Case ii :- (i) when $r>1$. $\lim_{n\to\infty} r^n \to \infty$

$$S_n = \frac{r^n-1}{r-1} = \frac{r^n}{r-1} - \frac{1}{r-1}$$

$\lim_{n\to\infty} S_n \to \infty$. as $n \to \infty$

$\therefore$ the Series is divergent

(ii) $r=1$, $S_n = 1+1+ \dots +1 = n$

$\lim_{n\to\infty} S_n \to \infty$

the Series is divergent

case iii :- i) when $r = -1$, then the Series becomes

$1 - 1 + 1 - 1 + \cdots$ which is oscillatory

ii) $r < -1$ let $r = -p$ so that $p > 1$

then $r^n = (-1)^n p^n$

$$S_n = \frac{1 - r^n}{1 - r} = \frac{1 - (-1)^n p^n}{1 + p} =$$

$\because \lim_{n \to \infty} p^n \to \infty$ $(\because p > 1)$

$\therefore \lim_{n \to \infty} S_n \to \infty$ or $-\infty$ as n is odd or even

hence it oscillates

P test :- the series $\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$

i) converges if $p > 1$ (ii) diverges if $p \le 1$

Pf Integral test :- a positive term series which decreases as n increases, converge or diverges according as the integral $\int_1^\infty f(x)$ is finite / infinite

Sol: $p \neq 1$ $\int_1^\infty \frac{dx}{x^p} = \lim_{m \to \infty} \int_1^m \frac{dx}{x^p}$

$$= \lim_{m \to \infty} \int_1^m \frac{dx}{x^p}$$

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$$\lim_{m\to\infty}\left(\int_1^m x^{-p}\,dx\right)$$

$$= \lim_{m\to\infty}\left[\frac{x^{-p+1}}{-p+1}\right]_1^m$$

$$= \lim_{m\to\infty}\frac{m^{1-p}-1}{1-p}$$

$$= \frac{1}{1-p} \quad \text{if } p>1$$

$$= \to \infty \quad \text{if } p<1$$

$$p=1 \Rightarrow \int_1^\infty \frac{dx}{x} = \left[\log x\right]_1^\infty$$

diverges to infinity.

$$\text{if } p> \quad \frac{1}{1-p}\lim_{m\to\infty}\frac{m^{1-p}-1}{\cancel{1-p}}$$

$$\frac{1}{1-p}\left(\lim_{m\to\infty}\frac{1}{m^{(p-1)}} - \lim_{n\to\infty} 1\right)$$

$$= \frac{-1}{1-p}$$

$$= \frac{1}{p-1}$$