Practice problems

1. Find order and degree of the following DE's.

(i) $\left(\frac{d^3y}{dx^3}\right)^4 - 6x^2\left(\frac{dy}{dx}\right)^8 = e^x$

(ii) $y = \sqrt{x}\frac{dy}{dx} + \frac{x}{\left(\frac{dy}{dx}\right)^2}$

2. Find the differential equation of following

(a) all the circles of unit radius

(b) $x^3 - 3x^2y = c$

3. Solve the following DE's. Also classify given DE's

(a) $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$

(b) $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

(c) ~~variable separable~~, ~~separable~~ $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

(d) $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

(e) $y^2 = (xy - x^2)\frac{dy}{dx}$

(f) $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$

(g) $\frac{dy}{dx} = \frac{x+y+1}{2x+2y+3}$

Answer Key

1(i) order = 3 degree = 4

(ii) order = 1 degree = 3

2. (a) $\left(1+\left(\frac{dy}{dx}\right)^2\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$ [Hint: general equation circle with unit radius $(x-a)^2+(y-b)^2=1$]

(b) $x\frac{dy}{dx}+2y-x=0$

3. (a) Variable Deparable, $3e^{2y} = 2(e^{3x}+x^3)+c$

b) equations reducible to variable separable.

$1+\tan\left(\frac{x+y}{2}\right) = Ae^x$

(c) $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0 \longrightarrow$ Variable separable Sol: $\sin^{-1}x+\sin^{-1}y=c$

(d) Homogeneous, $y+\sqrt{x^2+y^2} = cx^2$

(e) $y^2 = Ce^{y/x}$, Homogeneous

(f) $(x+1)^2+2(x+1)(y-3)-(y-3)^2=c$, Non homogeneous Case(i)

(g) Non homogeneous Case ii

(h) Linear DE. $y = ce^{-\tan x}+(\tan x-1)$

(i) Linear DE $\frac{x}{y^2}+e^{-y}=c$