CAUCHY'S Root Test :-

In a positive Series $\sum u_n$, if $\underset{n\to\infty}{Lt} (u_n)^{1/n} = k$

then the Series converges for $k<1$, and

diverges for $k>1$ and fails when $k=1$.

ex $\sum (\log n)^{-2n}$.

let $u_n = \sum (\log n)^{-2n}$.

$$(u_n^{1/n}) = (\log n^{-2n})^{1/n}$$

$$= (\log n)^{-2}$$

now $\underset{n\to\infty}{Lt}\ u_n^{1/n} = \underset{n\to\infty}{Lt} (\log n)^{-2}$

$$\underset{n\to\infty}{Lt} \frac{1}{(\log n)^2}$$

$$= 0 \ (<1)$$

So given Series is convergent.

Series of positive terms and negative terms.

Def 1) If the Series of ~~ab~~ arbitrary terms be such that

$u_1+u_2+u_3+ \dots u_n + \dots$

the Series $|u_1|+|u_2|+ \dots + |u_n| + \dots$ is convergent.

Then the Series $\sum u_n$ is absolutely convergent

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2) If $\Sigma |u_n|$ is divergent but Σu_n is convergent then Σu_n is said to be conditionally convergent

ex the Series $\Sigma u_n = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \cdots$

is absolutely Convergent, Since the

Series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \Sigma \frac{1}{n^2}$ is convergent

ex now Consider $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \cdots$ is

convergent but $1 + \frac{1}{2} + \frac{1}{3} + \cdots = \Sigma u_n = \Sigma \frac{1}{n}$ is

divergent, so the original series is conditionally convergent.

Leibintz's Test for alternating Series:-

· An alternating Series $u_1 - u_2 + u_3 - u_4 + \cdots$ converges if (i) each term is numerically less than preceding term i.e., $u_{n+1} < u_n, \forall n$

and (ii) $\lim_{n \to \infty} u_n = 0$.

ex $\Sigma u_n = \Sigma \frac{(-1)^n}{n}$ is convergent Since $\lim_{n \to \infty} \frac{1}{n} = 0$

& $\frac{1}{n+1} < \frac{1}{n}, \forall n$

Note An absolutely convergent Series is (necessarily) convergent but any convergent Series need not be absolutely convergent.

eg $\sum \frac{(-1)^n}{n}$ is ~~absolutely~~ convergent but not absolutely convergent as $\sum \frac{1}{n}$ is divergent.