# LINEAR ALGEBRA

1) 'P' contains all the polynomials and 'Q' is the subset of P. to be the subspace of Q, then Q should have zero polynomial.

(i) 'P' has zero polynomial as Q.

$$Q = \{ f(x) \mid f(0) = f(1) = 0 \}$$

0 polynomial lies in Q as $f(0) = '0'$.

(ii) Let $g(x), h(x)$ be two polynomials in Q, we have to prove $g(x) + h(x) \in Q$.

$$g(0) = g(1) = 0$$

$$h(0) = h(1) = 0$$

Consider $P(x) = h(x) + g(x)$

Let $P(0) = h(0) + g(0) = 0$

$$P(1) = h(1) + g(1) = 0$$

$$P(1) = 0.$$

$$P(x) \in Q.$$

(iii) Let $\gamma(x) \in Q = \{ f(x) \mid f(0) = f(1) = 0 \}$

Consider $C \in R$.

to prove $C.\gamma(x) \in Q$.

check $x = 0, \quad C.\gamma(0) = 0$

$x = 1, \quad C.\gamma(1) = 0$

$\therefore$ $C.\gamma(x)$ satisfy.

$\therefore$ Q is a subspace as it satisfies all the conditions of subspace.

2) Given $U = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Zero vector in S.

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 0.$$

② $\Rightarrow$ $U + V \in W$.

$S_1 = [a \quad b \quad c]$

$S_2 = [x \quad y \quad z]$

$V = S_1 + S_2 = [a+x \quad b+y \quad c+z]$

$V \cdot U = [a+x \quad b+y \quad c+z] \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$= a + 2b + c + x + 2y + z$

$= S_1 \cdot U + S_2 \cdot U$

hence 2nd condition is satisfied.

3rd case: $S_1 = [a \quad b \quad c]$

$k \rightarrow$ constant.

$P = kS = [ka \quad kb \quad kc]$

to prove $P \in S$.

$P \cdot U = [ka \quad kb \quad kc] \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$= k(a + 2b + c)$

$\therefore P \in S.$

hence 'S' is subspace.

3) Given vectors are $\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-2\\1\end{bmatrix}, \begin{bmatrix}0\\3\\0\end{bmatrix}$

if it spans $\mathbb{R}^3$ then for any $x, y, z \in R$ there exist $c_1, c_2, c_3, c_4$ such that

$$(x, y, z) = c_1(1, 0, 1) + c_2(1, 1, 1) + c_3(1, -2, 1) + c_4(0, 3, 0).$$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 3 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 1 & -2 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ rank of matrix $= 2$.

Since the rank of matrix is 2 it does not span the ~~sp~~ subspace $\mathbb{R}^3$.

4) Given $v = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$, $w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(a) $\{0\}$

Let $v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$a\bar{v}_1 = 0$ [a can be any scalar]

∴ $\{0\}$ is linearly dependent.

[b] $\{v\}$

$v_1 = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$

$a_1(2, 0, 3) = (0, 0, 0)$.

$2a_1 = 0 \mid 3a_1 = 0$

$\therefore a_1 = 0$.

$\{v\}$ in linearly independent.

[c] $\{w, 0\}$

any set which contains a zero vector is linearly dependent.

(d) $\{v, 0\} \rightarrow$ linearly dependent.

(e) $\{v, w\}$.

$av + bw = (0, 0, 0)$.

$a(2, 0, 3) + b(1, 1, 1) = (0, 0, 0)$.

$2a + b = 0 \quad -(1)$

$b = 0 \quad -(2)$

$3a + b = 0 \quad -(3)$

from ①, ②, ③

$a = b = 0$.

$\therefore \{v, w\}$ is linearly independent.

(f) $\{v, w, 0\} \rightarrow$ linearly dependent.

$\therefore$ Linearly dependent vectors :-

a, c, d, f.

5) Given $U, W$ are subspaces of $V$.

let $u = (\alpha_1, \alpha_2, \alpha_3)$ [non-zero vector in $U$]

$w = (\beta_1, \beta_2, \beta_3)$ [non zero vector in $V$]

for $\{u, w\}$ to be linearly independent.

$$au + bw = 0 \Leftrightarrow a = 0 \ \& \ b = 0.$$

$$a(\alpha_1, \alpha_2, \alpha_3) + b(\beta_1, \beta_2, \beta_3) = (0, 0, 0).$$

$$(a\alpha_1 + b\beta_1, a\alpha_2 + b\beta_2 + a\alpha_3 + b\beta_3) = (0, 0, 0).$$

$$a\alpha_1 + b\beta_1 = 0$$

$$a\alpha_2 + b\beta_2 = 0.$$

$$a\alpha_3 + b\beta_3 = 0$$

$$\begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ \alpha_3 & \beta_3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 0.$$

rank = 2, as there are no free variables

$a = 0$ & $b = 0$

$\therefore \{u, w\}$ are linearly independent

6) Given $A = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Characteristic equation of matrix A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 4-\lambda & 0 & -2 \\ 2 & 5-\lambda & 4 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)((5-\lambda)^2) = 0$$

$$\lambda = 4, 5, 5$$

Eigen vectors corresponding to $\lambda = 4$.

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} 4-4 & 0 & -2 \\ 2 & 5-4 & 4 \\ 0 & 0 & 5-4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 & -2 \\ 2 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0.$$

$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$X$ is null space of $A - \lambda I$.

$$\therefore 2x_1 + x_2 + 4x_3 = 0$$

$$-2x_3 = 0.$$

Set $x_3 = 0$ & $x_2 = 1$.

$$\therefore 2x_1 + 1 = 0$$

$$x_1 = -\tfrac{1}{2}$$

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eigen vector corresponding to $\lambda = 4$ is

$$x_2 \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix}.$$

eigen vectors corresponding to $\lambda = 5$

$$\begin{bmatrix} 4-5 & 0 & -2 \\ 2 & 5-5 & 4 \\ 0 & 0 & 5-5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0.$$

$$\begin{bmatrix} -1 & 0 & -2 \\ 2 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0.$$

$$\begin{bmatrix} -1 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ p & f & f \end{array}$$

$$x + 2z = 0$$

$y = 1$, $z = 0$ $\Rightarrow x = 0 \Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $z = 1$, $y = 0$ $\Rightarrow \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

eigen vectors corresponding to $\lambda = 5$ are

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ \& } \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

Let $P = \begin{bmatrix} -1 & 0 & -2 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ formed by eigenvectors

and $D$ is the diagonal matrix formed by $A$

$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$P^{-1} = ?$

$$\left[\begin{array}{ccc|ccc} -1 & 0 & -2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\quad P \qquad\qquad I$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 2 & 1 & 4 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\quad I \qquad\qquad P^{-1}$

$$\therefore \quad P^{-1} = \begin{bmatrix} -1 & 0 & -2 \\ 2 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore \quad A = PDP^{-1}$.

$$A = \begin{bmatrix} -1 & 0 & -2 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} -1 & 0 & -2 \\ 2 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$A = \begin{bmatrix} 1 & 0 & 1 & -1 & 1 \\ 2 & 0 & 3 & 1 & 1 \\ 1 & 0 & 0 & -4 & 2 \\ 0 & 0 & 1 & 4 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & -1 & -3 & 1 \\ 0 & 0 & 1 & 4 & 1 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$R_3 \to R_3 + R_2$

$R_1 \to R_1 - R_2$ & $R_3 \leftrightarrow R_4$.

$$\begin{bmatrix} 1 & 0 & 0 & -4 & 2 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 + 4R_3$

$R_2 \to R_2 - 3R_3$.

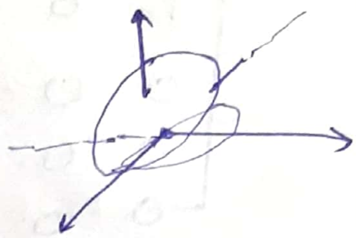
$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Row reduced echelon form]

# Calculus

1) $x^2 + y^2 = 1$

$x + z = 1$



$z$ is varying from $0$ to $1-x$.

$y$ is varying from $-\sqrt{1-x^2}$ to $+\sqrt{1-x^2}$

$x$ is varying from $-1$ to $1$.

$$\therefore \int_{x=-1}^{1} \int_{y=-\sqrt{1-x^2}}^{+\sqrt{1-x^2}} \int_{z=0}^{1-x} dx\, dy\, dz$$

$$= \int_{-1}^{1} \int_{-\sqrt{1-x}}^{\sqrt{1-x^2}} (1-x)\, dx\, dy.$$

$$= \int_{-1}^{1} \left(2\sqrt{1-x^2}\right)(1-x)\, dx.$$

$$= 2\left[\int_{-1}^{1} \sqrt{1-x^2} - \int_{-1}^{1} x\sqrt{1-x^2}\right]$$

$$\left[\int_{-1}^{1} x\sqrt{1-x^2}\, dx = 0 \quad [\because \text{odd function}]\right]$$

$$= 2\int_{-1}^{1} \sqrt{1-x^2}\, dx$$

$$= 4\int_{0}^{1} \sqrt{1-x^2}\, dx$$

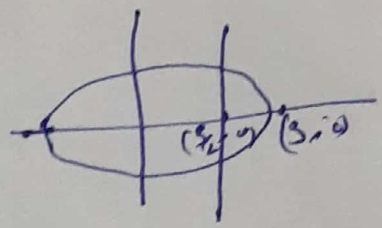
$$= 4\left[\left[\frac{x}{2}\sqrt{1-x^2}\right]_0^1 + \left[\frac{1}{2}\sin^{-1}(x)\right]_0^1\right]$$

$$= 4\left[\frac{1}{2}\sin^{-1}(1)\right] = 4 \times \frac{\pi}{4}$$

$= \pi$ cu.units.

② $x^2+y^2=9$.

$x = 3/2$




$$A = \int_{3/2}^{3} \sqrt{9-x^2}\, dx.$$

$x = \cancel{3\theta}\ 3\cos\theta$ , $\therefore dx = -3\sin\theta\, d\theta$.

$\therefore \frac{3}{2} = 3\cos\theta$ | $3 = 3\cos\theta$

$\theta = \frac{\pi}{3}$ | $\theta = 0$

$$A = -\int_{\pi/3}^{0} \sqrt{9-9\cos^2\theta}\,[3\sin\theta]\, d\theta$$

$$A = \int_{0}^{\pi/3} 9\sin^2\theta\, d\theta.$$

$$A = 9\int_{0}^{\pi/3} \frac{1-\cos 2\theta}{2}\, d\theta$$

$$A = 9\left[\int_{0}^{\pi/3} \frac{1}{2}\, d\theta - \int_{0}^{\pi/3} \frac{\cos 2\theta}{2}\right]$$

$$A = 9\left[\frac{1}{2}\left[\frac{\pi}{3}\right] - \frac{1}{4}\left[\sin 2\theta\right]_0^{\pi/3}\right]$$

$$A = 9\left[\frac{\pi}{6} - 9\times\frac{1}{4}\times\sin\frac{2\pi}{3}\right] = \frac{3\pi}{2} - \frac{9\sqrt{3}}{8}$$

$\because$ it is area above $x$-axis.

$$\therefore A' = 2A = 3\pi - \frac{9\sqrt{3}}{4}.$$

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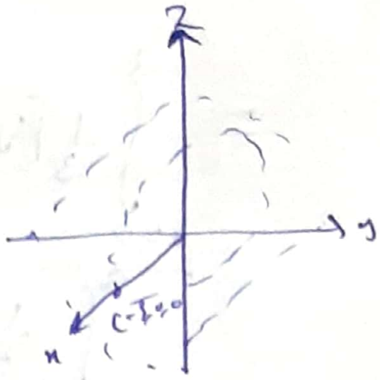
3) $x^2+y^2=9,\ x+z=4$

$z$ is varying from $0$ to $4-x$




$y$ is varying from

$-\sqrt{9-x^2}$ to $+\sqrt{9-x^2}$.

$x$ is varying from $-3$ to $3$.

$$= \int_{x=-3}^{3}\int_{y=-\sqrt{9-x^2}}^{+\sqrt{9-x^2}}\int_{0}^{4-x} dx\,dy\,dz$$

$$= \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} [4-x]\,dx\,dy.$$

$$= \int_{-3}^{3} 2\sqrt{9-x^2}\,(4-x)\,dx.$$

$$= 2\left[\int_{-3}^{3}(\sqrt{9-x^2})(4) - \int_{-3}^{3} x\sqrt{9-x^2}\right] \quad \left[\int_{-3}^{3} x\sqrt{9-x^2} = 0 \ \because \text{ it is a odd function}\right]$$

$$= 8 \times \int_{-3}^{3}\sqrt{9-x^2}\,dx.$$

$$= 16 \times \int_{0}^{3}\sqrt{9-x^2}\,dx$$

$$= 16\left[\left[\frac{x}{2}\sqrt{9-x^2}\right]_0^3 + \frac{9}{2}\left[\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3\right]$$

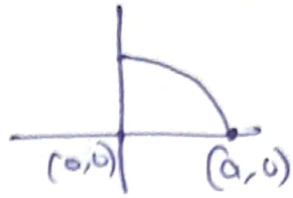
$$= 16\left[0+\frac{9}{2}\sin^{-1}(1)\right]$$

$$= 16\left(\frac{9\pi}{4}\right)$$

$$= 36\pi \text{ cu. units.}$$

4) $x^2+y^2=a^2$

density function $\rho(x,y)=y$




$$\therefore \bar{x} = \frac{\iint_R x\,\rho(x,y)\,dx\,dy}{\iint_R \rho(x,y)\,dx\,dy}.$$

$$\bar{y} = \frac{\iint_R y\,\rho(x,y)\,dx\,dy}{\iint_R \rho(x,y)\,dx\,dy}$$

$$\bar{x} = \frac{\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} x\,y\,dx\,dy}{\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} y\,dx\,dy}.$$

$$\bar{x} = \frac{\frac{1}{2}\int_0^a x\left[y^2\right]_0^{\sqrt{a^2-x^2}}dx}{\frac{1}{2}\int_0^a \left[y^2\right]_0^{\sqrt{a^2-x^2}}dx.}$$

$$= \frac{1}{2}\int_0^a x(a^2-x^2)\,dx \Big/ \frac{1}{2}\int_0^a (a^2-x^2)\,dx$$

$$= \frac{\frac{1}{2}\int_0^a a^2x - x^3\,dx}{\frac{1}{2}\int_0^a a^2-x^2\,dx} = \frac{a^2\left[\frac{x^2}{2}\right]_0^a - \left[\frac{x^4}{4}\right]_0^a}{a^2\left[x\right]_0^a - \left[\frac{x^3}{3}\right]_0^a}$$

$$= \frac{\frac{a^4}{2}-\frac{a^4}{4}}{a^3-\frac{a^3}{3}} = \frac{a^4}{4}\times\frac{3}{2a^3} = \frac{3a}{8}$$

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$$\therefore \boxed{\bar{x} = \frac{3a}{8}}$$

$$\bar{y} = \frac{\iint_R y \times y \, dx\,dy}{\iint_R y \, dx\,dy} = \frac{\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} y^2 \, dx\,dy}{\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} y \, dx\,dy}$$

$$= \frac{\frac{1}{3}\int_{x=0}^{a} [y^3]_0^{\sqrt{a^2-x^2}}}{\frac{1}{2}\left[a^3 - \frac{a^3}{3}\right]}$$

Consider

$$I = \int_{x=0}^{a} \frac{1}{3}[y^3]_0^{\sqrt{a^2-x^2}} \, dx$$

$$= \frac{1}{3}\int_0^a (a^2-x^2)^{3/2} \, dx.$$

Let $x = a\cos\theta$ | $\theta \to \frac{\pi}{2}$ to $0$

$dx = -a\sin\theta$

$$= \frac{1}{3}\int_0^{\frac{\pi}{2}} (a^2 - a^2\cos^2\theta)^{3/2} \, a\sin\theta$$

$$= \frac{1}{3}\int_0^{\frac{\pi}{2}} a^4 \sin^4\theta.$$

$$\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \frac{n-1}{n} \times \frac{n-3}{n-2} \times \dots \frac{1}{2} \times \frac{\pi}{2}.$$

$$\therefore I = \frac{a^4}{3} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi a^4}{16}$$

$$\bar{y} = \frac{\frac{\pi a^4}{16}}{\frac{1}{2}\left[\frac{2a^3}{3}\right]}$$

$$\bar{y} = \frac{3a\pi}{16}.$$

$\therefore$ Center of mass $= \left(\frac{3a}{8}, \frac{3a\pi}{16}\right).$

5) $I = \int_0^5 \frac{1}{(2-x)^{1/3}} \cdot dx$

it is an improper integral of type 2.

$$I = \int_0^2 \frac{1}{(2-x)^{1/3}} dx + \int_2^5 \frac{1}{(2-x)^{1/3}} dx$$

$$\downarrow \qquad\qquad \downarrow$$

$$I_1 \qquad\qquad I_2$$

$$I_1 = \int_0^2 \frac{1}{(2-x)^{1/3}} dx$$

$$= \lim_{c \to 2^-} \int_0^c \frac{1}{(2-x)^{1/3}} dx$$

$2-x = t$
$-dx = dt$

and at $x = 0$
$t = 2$
at $x = c$
$t = 2-c$.

$$\lim_{c \to 2^-} \left[ - \int_2^{2-c} \frac{1}{t^{1/3}} dt \right]$$

$$\lim_{c \to 2^-} \int_{2-c}^{2} t^{-1/3} dt = \lim_{c \to 2^-} \frac{\left[ t^{1-1/3} \right]_{2-c}^{2}}{1-1/3}$$

$$\frac{3}{2} \lim_{c \to 2^-} \left[ 2^{2/3} - (2-c)^{2/3} \right] = \frac{3}{2} \times 2^{2/3}.$$

$$I_2 = \int_2^5 \frac{1}{(2-x)^{1/3}} dx$$

$$I_2 = \lim_{c \to 2^+} \int_c^5 \frac{1}{(2-x)^{1/3}} dx.$$

$$2 - x = t$$

$$I_2 = \lim_{c \to 2^+} \left[ - \int_{2-c}^{-3} t^{-1/3} \, dt \right]$$

$$I_2 = \lim_{c \to 2^+} \left[ \int_{-3}^{2-c} t^{-1/3} \, dt \right]$$

$$I_2 = \lim_{c \to 2^+} \left[ t^{2/3} \right]_{-3}^{2-c} \times \frac{3}{2}$$

$$I_2 = \frac{3}{2} \left[ \lim_{c \to 2^+} \left[ (2-c)^{2/3} - (-3)^{2/3} \right] \right]$$

$$I_2 = -\frac{3}{2} \times 9^{1/3}$$

$$I_1 = \frac{3}{2} \times 2^{2/3}$$

$$\therefore I = I_1 + I_2$$

$$= \frac{3}{2} \left[ 2^{2/3} - 9^{1/3} \right]$$

6) given $f(x) = x^3$ and $g(x) = \tan^{-1}(x)$.

in intaral $[0, 1]$

Cauchy's mem value theorem.

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}.$$

$$\frac{1 - 0}{\frac{\pi}{4}} = \frac{3c^2}{\frac{1}{1+c^2}}$$

$$\frac{4}{\pi} = 3c^2(1 + c^2).$$

$$3c^2 + 3c^4 - \frac{4}{\pi} = 0.$$

Put $c^2 = t$

$$3t^2 + 3t - \frac{4}{\pi} = 0.$$

$$t = \frac{-3 \pm \sqrt{9 + \frac{4 \times 3 \times 4}{\pi}}}{6}$$

t should be positive.

$$\therefore t = \frac{-3 + \sqrt{9 + \frac{48}{\pi}}}{6}$$

$c = \sqrt{t}$

$$\boxed{c = \sqrt{\frac{-3 + \sqrt{9 + \frac{48}{\pi}}}{6}}.}$$

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