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Visualization

To visualize how each method produces results, we will use some interactive figure such as this oneEvaluation failed.. It represents the problem with two parameters. Each point you see (you can also drag them around) is a measure. It's X and Y coordinates are the value of its two parameters, and its color represent the measured value. Each pixel in the background indicates the result of the interpolation at its position. For this example, we used a very simple interpolation method: the output value is the value of the nearest measure.

Figure 1: Nearest neighbor interpolation

Inverse distance

We can start by trying a very simple method: the inverse distance weighting. If $A_0 ... A_n$ are the measures points and

$a_0 ... a_n$ are their value, and we want to get a result on the point M, we calculate:

$$\frac{\sum\limits_{i=0}^{n} \frac{a_i}{A_i M}}{\sum\limits_{j=0}^{n} \frac{1}{A_j M}}$$

It's a weighted average, where the weight are the inverse distances between the points. This way, the more a point is far from M, the less it will impact the result. What if a distance is zero? Then it means that M is directly on a measured point, so you can take its value instead.

Figure 2: Inverse distance interpolation

Problems with inverse distance

While this method seems at first to be doing well, it has some drawbacks.

First, if there is a lot of points at the same place (if for instance you want more precision around a particular point), their values will impact the result more.

Figure 3: Bad interpolation example

On the right, 4 additional measures have been made around the red point, and it impacts the results for the entire space! (the color of the borders aren't the same) The impact should instead be limited to the zone around the red point.

This method has also another drawback: it takes all the points into account, event when it shouldn't. On the next figure you would expect the center of the screen to be completely blue, but the red point impacted the result and the center is purple-ish.

Figure 4: Another bad interpolation example

Barycentric coordinate in a simplex

As we saw before, the

barycentric coordinate of a vertex A_k is equal to the coeficient x_k of the vector to this vertex $\overrightarrow{A_0A_k}$ in the triangle base. Thus, we just need to find an expretion for that coefficient. If we take all the other vertexs of the triangle, we know that they define a plane P_k, and that A_k is the only point out of it. This means that $\overrightarrow{A_0A_k}$ is the only contributing to the normal direction from the plane P_k (we will represent this direction by the normal vector $\overrightarrow{n}$) We have

$$x_k \times \overrightarrow{A_0A_k}.\overrightarrow{n} = \overrightarrow{A_0M}.\overrightarrow{n}$$

Since the direction is the same, we can take the norm on both sides :

$$x_k \times \left\| \overrightarrow{A_0A_k}.\overrightarrow{n} \right\| = \left\| \overrightarrow{A_0M}.\overrightarrow{n} \right\|$$

$\left\| \overrightarrow{A_0A_k}.\overrightarrow{n} \right\|$ is just the height of the triangle and $\left\| \overrightarrow{A_0M}.\overrightarrow{n} \right\|$ is just the height of M. Thus we have

$$x_k = \frac{\mathrm{Height \quad of \quad M}}{\mathrm{Height of A_k}}$$

Complete proof — Click