Q1 (a) $n = 14$; degrees of freedom $= n-2$

(b) An increase of one ng/g of PCB in maternal milk decreases the IQ of the child by 0.0249

(c) Let β be the true slope for the line relating PCB to mean IQ.

Test $H_0 : \beta = 0$ against $H_1 : \beta \neq 0$

test statistic $t = \dfrac{\text{estimate} - \text{hypothesised}}{\text{s.e. (estimate)}}$

$= \dfrac{-0.024949}{0.0079775} = -3.1274$

p-value $= 2 \times \min\{\mathbb{P}(T_{12} \geq -3.1274), \mathbb{P}(T_{12} \leq -3.1274)\}$

$= 2 \times \mathbb{P}(T_{12} \leq -3.1274)$

$\mathbb{P}(T_{12} \leq -3.055) = 0.005$ and $\mathbb{P}(T_{12} \leq -3.930) = 0.001$

The p-value is between 0.002 and 0.01. (exact value is 0.0087337). This is strong evidence against the null hypothesis, suggesting an association between PCB levels and IQ.

(d) 95% CI estimate $\pm$ $t_{12;0.975}$ $\times$ se. (estimate)

$129.44 \pm 2.179 \times 7.3779$

129.44 ± 16.0764 (IQ points).

e) $129.44 - 0.024949 \times 1400 = 94.5114$ (IQ points)

Q2. Let μ be the mean percentage of platelet aggregation increase after smoking one cigarette.

Test: $H_0: \mu = 0$ against $H_1: \mu > 0$

Test statistic $t = \dfrac{10.5 - 0}{8.27/\sqrt{11}} = 4.211$

degrees of freedom $= 11 - 1 = 10$

p-value $= \mathbb{P}(T_{10} \geq 4.211)$

From tables $\mathbb{P}(T_{10} \geq 4.144) = 0.001$ and $\mathbb{P}(T_{10} \geq 4.587) = 0.0005$

So the p-value is between 0.0005 and 0.001.

There is strong evidence against the null hypothesis, suggesting smoking increases the mean percentage of platelet aggregation.

Q3. First approach –

let p_M and p_F be the proportion of male and female students, respectively, who believe in god.

Test $H_0: p_M = p_F$ against $H_1: p_M \neq p_F$

$\hat{p}_M = \dfrac{90}{90+89} = 0.5028$ $\qquad n_M = 90 + 89 = 179$

$\hat{p}_F = \dfrac{101}{101+76} = 0.5706$ $\qquad n_F = 101 + 76 = 177$

The pooled proportion

$$\hat{p} = \frac{101 + 90}{177 + 179} = 0.5365$$

Test statistic (using pooled proportion)

$$Z = \frac{(\hat{p}_M - \hat{p}_F) - 0}{\sqrt{\hat{p}(1-\hat{p})} \times \sqrt{1/n_M + 1/n_F}} = \frac{0.5028 - 0.5706}{\sqrt{0.5365 \times (1-0.5365)}\sqrt{1/177 + 1/179}}$$

$$= -1.2831$$

Test statistic (without pooled proportion)

$$Z = \frac{(\hat{p}_M - \hat{p}_F) - 0}{\sqrt{\frac{\hat{p}_M(1-\hat{p}_M)}{n_M} + \frac{\hat{p}_F(1-\hat{p}_F)}{n_F}}} = \frac{0.5028 - 0.5706}{\sqrt{\frac{0.5028 \times (1-0.5028)}{179} + \frac{0.5706 \times (1-0.5706)}{177}}}$$

$$= -1.2862$$

(I'll use the test statistic with pooled proportion below.)

p-value $= 2 \times \min\{\mathbb{P}(Z \geq -1.2831), \mathbb{P}(Z \leq -1.2831)\}$

$Z \sim N(0,1)$

$$= 2\,\mathbb{P}(Z \leq -1.2831)$$

$$= 2 \times 0.100 = 0.2$$

(without the pooled proportion, the p-value is 0.198).

There is no evidence against the null hypothesis, suggesting no difference in the proportion of males and females who believe in god.

Second approach –

Test H_0: 'Gender' and 'belief in god' are independent
against H_1: There is some association between 'gender' and 'belief in god'.

Observed Counts

	Yes	No	
Female	101	76	177
Male	90	89	179
	191	165	356

Expected Counts

	Yes	No
Female	94·96	82·04
Male	96·04	82·96

Expected counts

$E_{11} = \dfrac{191 \times 177}{356} = 94{\cdot}96$

$E_{21} = 191 - 94{\cdot}96 = 96{\cdot}04$

$E_{12} = 177 - 94{\cdot}96 = 82{\cdot}04$

$E_{22} = 165 - 82{\cdot}04 = 82{\cdot}96$

$$X^2 = \sum_i \frac{(E_i - O_i)^2}{E_i}$$

$$= \frac{(94{\cdot}96 - 101)^2}{94{\cdot}96} + \frac{(96{\cdot}04 - 90)^2}{96{\cdot}04} + \frac{(82{\cdot}04 - 76)^2}{82{\cdot}04} + \frac{(82{\cdot}96 - 89)^2}{82{\cdot}96}$$

$$= 1{\cdot}6466$$

degrees of freedom = (rows − 1) × (columns − 1) = $(2-1)\times(2-1) = 1$

p-value $= \mathbb{P}(\chi^2_1 \geqslant 1{\cdot}6466)$

From table $\mathbb{P}(\chi^2_1 \geqslant 1{\cdot}323) = 0{\cdot}25$ and $\mathbb{P}(\chi^2_1 \geqslant 2{\cdot}706) = 0{\cdot}10$

The p-value is between 0·10 and 0·25.

There is no evidence against the null hypothesis, suggesting gender and belief in god are independent.