$^nP_r = \frac{n!}{(n-r)!}$

$^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$

去重就除以它的阶乘

$M_{X+Y}(s) = M_X(s)M_Y(s)$ independent events

$f_{X|Y}(x|y) = f_X(x)$ $\mathbb{P}(A|B) = \mathbb{P}(A)$

$\mathbb{E}[X|Y] = \mathbb{E}[X]$ $\mathbb{P}(A \cap B) = \mathbb{P}(A) \times \mathbb{P}(B)$

$f_{X,Y}(x,y) = f_X(x) f_Y(y)$

$\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$

$Cor(X,Y) = 0$

$Var(X+Y) = Var(X) + Var(Y)$

$\mathbb{E}[g(X)h(Y)] = \mathbb{E}[g(X)]\mathbb{E}[h(Y)]$

等比数例求和公式

$S_n = \frac{a_1(1-q^n)}{1-q}$

$\mathbb{P}(A|B) = \frac{\mathbb{P}(AB)}{\mathbb{P}(B)}$ / $\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$

$\mathbb{P}(B) = \mathbb{P}(B|A)\mathbb{P}(A) + \mathbb{P}(B|A^c)\mathbb{P}(A^c)$

$\mathbb{P}(A \cup B \cup C) = A+B+C-AB-BC-CA+ABC$

$(A \cup B) = A+B-AB$

cdf of X show cdf 即画图

$F(x) = \mathbb{P}(X \le x)$

$0 \le F(x) \le 1$ $F(\Omega) = \sum_{x=0}^{\Omega} \mathbb{P}(X=x) = 1$

$\lim_{x \to +\infty} F(x) = 1$ $\lim_{x \to -\infty} F(x) = 0.$

if $x<y$ then $F(x) \le F(y)$

pmf of X 即画图

$f_X(m) = \mathbb{P}(X=m)$

with replacement

$X \sim DU(A)$ / Uniform($\{1,2,3,4,5,6\}$)

$\mathbb{P}(X) = \frac{1}{|A|}$ 即 $\frac{1}{6}$ Uniform(a,b) $\mathbb{P}(X) = \frac{1}{b-a}$

$exp(\lambda) = e^\lambda$

$X \sim$ Bernoulli(p)

即 $\mathbb{P}(X=1) = p$, $\mathbb{P}(X=0) = 1-p$

$X \sim$ Binomial(n,p) $\rightarrow M_X(s) = (1-p+pe^s)^n$ $\rightarrow \mathbb{E}[X] = np$, $Var(X) = np(1-p)$

$\mathbb{P}(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$ $k = 0,1,2,\cdots$

$X \sim$ Geometric(p) $\longrightarrow \mathbb{P}(X=m+k \mid X>k) = \mathbb{P}(X=m)$

$\mathbb{P}(X=k) = (1-p)^{k-1}p$, $k=1,2,3,4 \cdots$

$\sum_{n=1}^{\infty} (1-p)^{n-1} p = 1$ $\mathbb{E}[X] = \frac{1}{p}$, $Var(X) = \frac{1-p}{p^2}$

large n and small p

$X \sim$ Poisson(λ) $X \sim$ Poisson(np) = Ber(n,p)

$\mathbb{P}(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x \in 0,1,2 \cdots$

$\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = 1$ $\mathbb{E}[X] = \lambda$, $Var(X) = \lambda$

$\mathbb{E}[X(X-1)] = \lambda^2$

$M_X(s) = exp(\lambda(e^s - 1))$

expected. / mean

$\mathbb{E}[X] = \sum_x x\,\mathbb{P}(X=x)$

$\mathbb{E}[X^2] = \sum_x x^2\,\mathbb{P}(X=x)$

if $Y = g(X)$

than $\mathbb{E}[Y] = \sum_x g(x)\mathbb{P}(X=x) = \sum_y y\,\mathbb{P}(Y=y)$

$\cancel{\mathbb{E}[aX] = a\mathbb{E}[X]}$ $\mathbb{E}[Y|X=x] = \sum_y y f_{Y|X=x}(y|x)$

$\mathbb{E}[aX+b] = a\mathbb{E}[X] + b$

Variance

$Var(X) = \mathbb{E}[X - \mathbb{E}[X]]^2$

$= \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

$= \sum_x (x - \mathbb{E}[X])^2 \mathbb{P}(X=x)$

$Var(aX+b) = a^2 Var(X)$

Multiply Random Variables

$B = \{(x,y) : x \in \{1,2,3\}, y \in \{1,2,\cdots 6\}\}$

$f_{XY}(B) = \mathbb{P}((X,Y) \in B) = \sum_{(x,y) \in B} \mathbb{P}(X=x, Y=y)$

$f_X(x) = \mathbb{P}(X=x) = \sum_y \mathbb{P}(X=x, Y=y)$

$= \sum_y f_{XY}(x,y)$

$\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$

$f_X(x) = \sum_y f_{XY}(x,y)$