INFS7901
Database Principles

Functional Dependencies and Normal Forms

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Notes

- **ER diagram tool:** You can use MIRO (let's you share ER with your group)
- **Projects**: Come to consultation hours if you have questions about the course content or your projects.
- **Project template and guide**: Make sure that you can run and understand guides 1 to 5 of the project template. We will be focusing on next Guides this week.
- **Midterm:** See sample questions on BB.
- **RiPPLE**: round 1 resources due **is today (3pm)**.
- **Piazza**: Link on course web page.
- **Anonymous Feedback**: click this link to provide feedback. Previous feedback: talk about applications of each module, and also breaks.

Learning Outcomes

Description	Tag
Provide examples of modification, insertion, and deletion anomalies.	Functional-dependencies
Explain the term functional dependency.	
Given a set F of functional dependencies and a functional dependency fd, determine whether fd can be inferred from F using Armstrong's Axioms.	
Given a set of functional dependencies that hold over a table, determine the keys.	
Given a set of functional dependencies that hold over a table, determine the superkeys.	
Given a set F of functional dependencies and X of attributes of a relation, compute the closure of X, which is X+	
Given a relation R, be able to correctly decompose it into two or more relations	Decomposition
Justify why lossless join decompositions are preferred decompositions.	
Explain the benefits of Normalization.	Normalization
Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in 1NF.	
Explain what dependency preservation means.	
Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in 3NF.	
Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in BCNF.	
Describe the process and benefits of Denormalization.	
Reason with the logical foundation of the relational data model and understand the fundamental principles of correct relational database design.	

Design Guidelines

Functional Dependencies

Normalization

Informal Design Guidelines

- Informal measures of relational database schema quality and design guidelines
  - Semantics of the attributes
  - Reducing the redundant values in tuples
  - Reducing the null values in tuples
  - Disallowing fake tuples

Guideline 1

- Design each relation so that it is easy to explain its meaning
- Do not combine attributes from multiple entity types and relationship types into a single relation

Redundant Values in Tuples

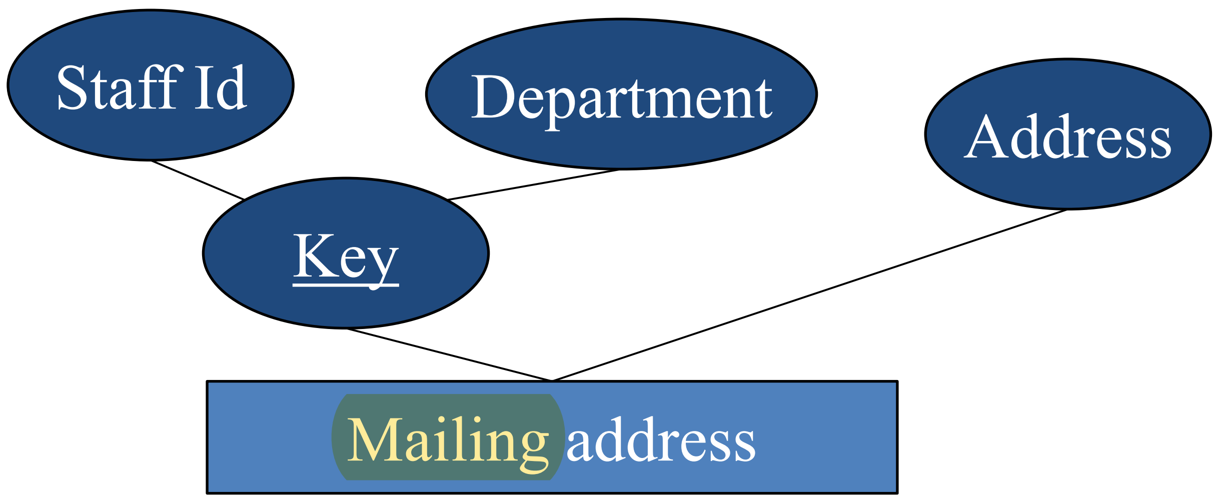
- One design goal is to minimize the storage space that base relations occupy

- The way the grouping of attributes into relation schemas is done, has a significant effect on storage space

- In addition, an incorrect grouping may cause *update anomalies* which may result in inconsistent data or even loss of data

A Motivating Example

- Imagine that we have created the following entity for mailing addresses at a university



Meets all the criteria that we have discussed for an entity so far

What Would an Instance Look Like?

Staff Id	Department	Address
100	Computer Science	78-101 Main Mall
104	Computer Science	78-101 Main Mall
104	Math	69-201 Main Mall
105	Physics	50-205 Main Mall

- **Modification anomaly:** data inconsistency that results from data redundancy.
  - Example: Updating the mailing address of a department.
- **Deletion anomaly:** loss of certain attributes because of the deletion of other attributes.
  - Example: Deleting 105 would lead to deletion of the address of Physics.
- **Insertion anomaly:** Lack of ability to insert some attributes without the presence of other attributes.
  - Example: Storing the mailing address of a department that has no faculty members.

Guideline 2

- Design the base relation schema so that no insertion, deletion, or modification anomalies occur in the relations
- If any do occur, ensure that all applications that access the database update the relations in such a way as to not compromise the integrity of the database

Guideline 3

- As far as possible, avoid placing attributes in a base relation whose values may be null
- If nulls are unavoidable, make sure that they apply in exceptional cases only and that they do not apply to a majority of tuples in the relation

Decomposing a Relation

- A decomposition of R replaces R by two or more relations such that:
  - Each new relation contains a subset of the attributes of R (and no attributes not appearing in R)
  - Every attribute of R appears in at least one new relation.
- Example:

Mailing Address (staff Id, Department, Address)

Academics(staff Id, Department)
Mailing Address(Department, Address)

How should we decompose tables to remove anomalies?

The join

- Definition: $R_1 \bowtie R_2$ is the (natural) join of the two relations
  - each tuple of R_1 is concatenated with every tuple in R_2 having the same values on the common attributes.

R_1

A	B
1	2
4	5
7	2

R_2

B	C
2	3
5	6
2	8

$R_1 \bowtie R_2$

A	B	C
1	2	3
4	5	6
7	2	8
1	2	8
7	2	3

Lossless-Join Decompositions: Definition

Decomposition of R into R_1 and R_2 is a lossless-join w.r.t. a set of FDs F if, for every instance r that satisfies F:

$$R = R_1 \bowtie R_2$$

- Informally: If we break a relation, R, into bits, when we put the bits back together, we should get exactly R back again.

Example Lossy-Join Decomposition

R

A	B	C
1	2	3
4	5	6
7	2	8

Decompose

R_1

A	B
1	2
4	5
7	2

R_2

B	C
2	3
5	6
2	8

Join

R’

A	B	C
1	2	3
4	5	6
7	2	8
1	2	8
7	2	3

- The word loss in lossless refers to loss of information, not to loss of tuples. Maybe a better term here is “addition of fake information”.
- Last two rows are not in the original. Would this have happened if B determined C?

Guideline 4

- Design the relation schemas so that they can be (relationally) *joined* with equality conditions on attributes that are either primary keys or foreign keys in a way that guarantees that no fake tuples are generated

Fixing Anomalies

Staff Id	Department	Address
100	Computer Science	78-101 Main Mall
104	Computer Science	78-101 Main Mall
104	Math	69-201
105	Physics	50-205

Staff Id	Department
100	Computer Science
104	Computer Science
104	Math
105	Physics

Department	Address
Computer Science	78-101 Main Mall
Math	69-201
Physics	50-205

Fixing Anomalies

Staff Id	Department
100	Computer Science
104	Computer Science
104	Math
105	Physics

Department	Address
Computer Science	78-101 Main Mall
Math	69-201
Physics	50-205

Modification anomaly fixed: Updating the mailing address only in one place does not lead to inconsistencies in the database.

Deletion anomaly fixed: Deleting the row with id 105 does not delete the mailing address of Physics.

Insertion anomaly fixed: It is now possible to store the mailing address of a department that has no faculty members.

Fixing Anomalies Beyond our Example

- We were able to fix the anomalies by splitting the Mailing address table.

- In the rest of this module, we will discuss how anomalies can be addressed formally.

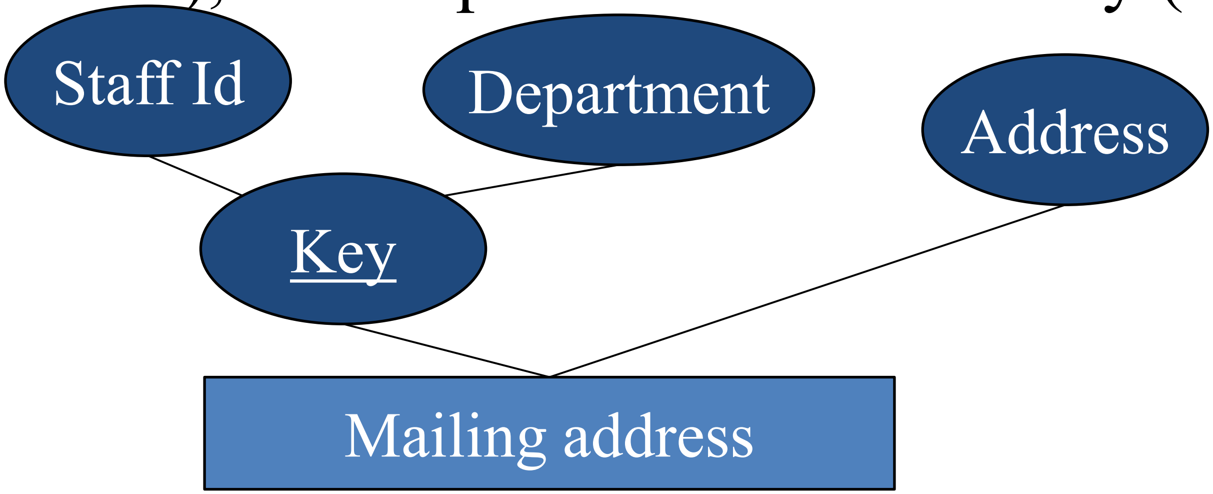
Design Guidelines

Functional Dependencies

Normalization

Functional Dependencies, Informally

- How do I know for sure if departments only have one address?
- Databases allow you to say that one attribute determines another through a functional dependency.
- So if Department determines Address, we say that there is a functional dependency from Department to Address (Dep→ Address), But Department is NOT a key (why?).



Functional Dependencies, Formally

- A **functional dependency** **X→Y** holds if for every legal instance, for all tuples *t1*, *t2*:

$$\text{if} \quad t1.X = t2.X \rightarrow t1.Y = t2.Y$$

- Which means given two tuples in *r*, if the X values agree, then the Y values must also agree.
- Example:

Department → Address
if t1.Department = t2.Department→ t1.Adress= t2.Adress

Identifying Functional Dependencies

- A FD is a statement about *all* allowable instances.
  - Must be identified by application semantics.
  - Given some instance of R, we can check if it violates some FD *f*, but we cannot tell if *f* holds over R!

Staff Id	Department	Address
100	Computer Science	78-101 Main Mall
104	Computer Science	78-101 Main Mall
104	Math	69-201 Main Mall
105	Physics	50-205 Main Mall

- Based on this instance alone, we cannot conclude that Department → Address.

Question

- Given a table with a million rows can you tell if a functional dependency exists from data?

- Given a table can you check if a functional dependency doesn't hold from data?

- Where do functional dependencies come from?

Question

- Consider the relation R with the following instance:

A	B	C	D
1	2	3	4
2	3	4	6
6	7	8	9
1	3	4	5

What FDs cannot be true given the instance above?

A. B→ C

B. B→ D

C. D→B

D. All of the above can be true

E. None of the above can be true

Question

- Consider the relation R with the following instance:

A	B	C	D
1	2	3	4
2	**3**	4	**6**
6	7	8	9
1	**3**	4	**5**

What FDs cannot be true given the instance above?

A. B→ C fine

B. B→ D not possible (2nd and 4th), so correct answer

C. D→B fine

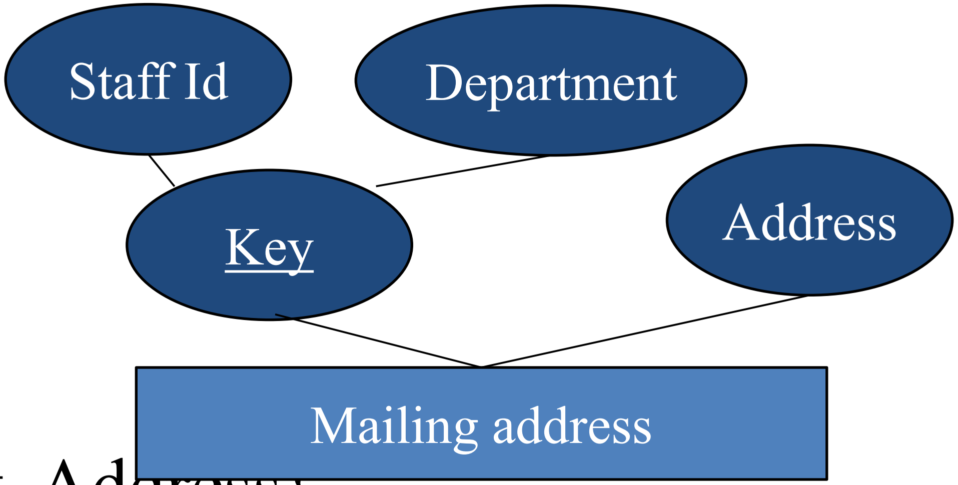
D. All of the above can be true

E. None of the above can be true

Keys

- As a reminder, a key is a minimal set of attributes that uniquely identify a relation
  - i.e., a key is a minimal set of attributes that *functionally determines* all the attributes
- A superkey for a relation uniquely identifies the relation, but does not have to be minimal
  - i.e.,: key $\subseteq$ superkey

  key is a subset of superkey



- Example
  - key {Id, Department}
  - superkey{Id, Department, Address}

Clicker question: Possible Keys

- Assume that the following FDs hold for a relation R(A,B,C,D):
  B→C
  C→B
  D→ABC

Which of the following is a key for the above relation?

A. B

B. C

C. BD

D. All of the above

E. None of the above

Clicker question: Possible Keys

- Assume that the following FDs hold for a relation R(A,B,C,D):
  B→C
  C→B
  D→ABC

Which of the following is a key for the above relation?

A. B Does not determine all

B. C Does not determine all

C. BD Not minimal

D. All of the above

E. None of the above The right answer

Clicker question: Possible Superkeys

- Assume the same relation R(A,B,C,D) and FDs
  B→C
  C→B
  D→ABC

Which of the following is a superkey for the above relation?

A. D

B. BD

C. BCD

D. All are superkeys

E. None are superkeys

Clicker question: Possible Superkeys

- Assume the same relation R(A,B,C,D) and FDs
  $B \rightarrow C$
  $C \rightarrow B$
  $D \rightarrow ABC$

Which of the following is a superkey for the above relation?

A. D

B. BD

C. BCD

D. All are superkeys

E. None are superkeys

D is a key. Therefore, all of the answers are superkeys

Explicit and Implicit FDs

- Given a set of (explicit) functional dependencies, we can determine implicit ones

studentid → *city*, *city* → *acode* implies *studentid* → *acode*

- A functional dependency fd is implied by a set F of functional dependencies if fd holds whenever all FDs in F hold.

fd= {studentid → acode}

	F
fd1:	studentid → city
fd2:	city → acode

- **Closure of F**: the set of all FDs implied by F.

Armstrong's Axioms

- Armstrong's Axioms (X, Y, Z are sets of attributes):
  - ***Reflexivity***: If $Y \subseteq X$, then $X \rightarrow Y$
    e.g., city,major→city Y={city} X={city, major}
  - ***Augmentation***: If $X \rightarrow Y$, then $X Z \rightarrow Y Z$ for any Z
    e.g., if sid→city, then sid,major → city,major
  - ***Transitivity***: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
    sid→*city*, *city*→*areacode* implies *sid*→*areacode*
- These are *sound (every rule is legit)* and *complete (all legit rules are produced)* inference rules.

1. studentid→acode fd1, fd2, transitivity

F

fd1:	studentid→ city
fd2:	city→ acode

Additional Rules

- Couple of additional rules (that follow from axioms):
  - ***Union***: If X→Y and X→Z, then X→Y Z
    e.g., if sid→acode and sid→city, then sid→acode,city

1.	X→X Y	fd1, augmentation
2.	X Y→Z Y	fd2, augmentation
3.	X→Z Y	1, 2, transitivity

F	
fd1:	X→Y
fd2:	X→Z

  - ***Decomposition***: If X→Y Z, then X→Y and X→Z
    e.g., if sid→acode,city then sid→acode, and sid→city

1	YZ →Y	Reflexivity
2	YZ → Z	Reflexivity
3.	X→Y	fd1, 1, transitivity
5.	X→Z	fd1, 2, transitivity

F	
fd1:	X→YZ

Example: Supplier-Part DB

- Suppliers supply parts to projects.
  - SupplierPart(sname,city,status,p#,pname,qty)
    - supplier attributes: sname, city, status
    - part attributes: p#, pname
    - supplier-part attributes: qty:

- Functional dependencies:
  - fd1: sname → city
  - fd2: city → status
  - fd3: p# → pname
  - fd4: sname, p# → qty

Exercise: Show that (sname, p#) is a superkey

Supplier-Part Key: Part 1:

Exercise: Show that (sname, p#) is a superkey of SupplierPart(sname,city,status,p#,pname,qty)

Proof has two parts:

fd1: sname → city
fd2: city → status
fd3: p# → pname
fd4: sname, p# → qty

a. Show: sname, p# is a (super)key

1. sname, p# → sname, p# — reflex
2. sname → city — fd1
3. sname → status — 2, fd2, trans
4. sname,p# → city, p# — 2, aug
5. sname,p# → status, p# — 3, aug
6. sname,p# → sname, p#, status — 1, 5, union
7. sname,p# → sname, p#, status, city — 4, 6, union
8. sname,p# → sname, p#, status, city, qty — 7, fd4, union
9. sname,p# → sname, p#, status, city, qty, pname — 8, fd3, union

Example: Supplier-Part DB

- Suppliers supply parts to projects.
  - SupplierPart(sname,city,status,p#,pname,qty)
    - supplier attributes: sname, city, status
    - part attributes: p#, pname
    - supplier-part attributes: qty:

- Functional dependencies:
  - fd1: sname → city
  - fd2: city → status
  - fd3: p# → pname
  - fd4: sname, p# → qty

Exercise: Show that (sname, p#) is a key

Supplier-Part Key: Part 2

b. Show: (sname, p#) is a *minimal* key of
SupplierPart(sname,city,status, p#,pname,qty)

1. p# does not appear on the RHS of any FD therefore except for p# itself, nothing else determines p#
3. specifically, sname → p# does not hold
4. therefore, sname is not a key
5. similarly, p# is not a key

fd1: sname → city
fd2: city → status
fd3: p# → pname
fd4: sname, p# → qty

sname,p# →sname, p#, city, status, pname, qty

sname → sname, city, status

p# → p#, pname

Computing the Closure

- Closure for a set of attributes X is denoted by X^+
- X^+ includes all attributes of the relation IFF X is a (super) key

Algorithm for finding Closure of X:
Let Closure = X
    Until Closure doesn't change do
        if $a_1, \ldots, a_n \rightarrow C$ is a FD and $\{a_1, \ldots, a_n\} \in$ Closure
            Then add C to Closure

- Example
  - $Studentid^+ = \{Studentid\}$
  - $Studentid^+ = \{Studentid, city\}$
  - $Studentid^+ = \{Studentid, city, acode\}$

F	
fd1:	studentid $\rightarrow$ city
fd2:	city $\rightarrow$ acode

Supplier-Part Key: Closure

Algorithm for finding Closure of X:
Let Closure = X
Until Closure doesn't change do
if $a_1, \ldots, a_n \rightarrow C$ is a FD and $\{a_1, \ldots, a_n\} \in$ Closure
Then add C to Closure

fd1: sname → city
fd2: city → status
fd3: p# → pname
fd4: sname, p# → qty

Ex: SupplierPart(sname,city,status,p#,pname,qty)

$\{\text{sname,p\#}\}^+ =$ sname,p#, city, status, pname, qty

$\{\text{sname}\}^+ =$ sname, city, status

$\{\text{p\#}\}^+ =$ p#, pname

So seeing if a set of attributes is a superkey means checking to see if it's closure is all the attributes – pretty simple

Question: Finding Key

- Which of the following is a key of the Relation R(ABCDE) with FD's:

	F
fd1:	$D \rightarrow C$
fd2:	$CE \rightarrow A$,
fd3:	$D \rightarrow A$
fd4:	$AE \rightarrow D$.

A. ABDE

B. BCE

C. CDE

D. All of these are keys

E. None of these are keys

Question: Finding Key

- Which of the following is a key of the Relation R(ABCDE) with FD's:

F

fd1:	$D \rightarrow C$
fd2:	$CE \rightarrow A$,
fd3:	$D \rightarrow A$
fd4:	$AE \rightarrow D$.

A. ABDE Superkey, since D→A

B. BCE Key

C. CDE CDE+=CDEA

D. All of these are keys

E. None of these are keys

Approaching Normality

- Role of FDs in detecting redundancy:
  - Consider a relation R with 3 attributes, A B C.
    - No FDs hold: There is no redundancy here.
    - Given A → B: Several tuples could have the same A value, and if so, they'll all have the same B value!
- **Normalization**: the process of removing redundancy from data

- We were able to fix the anomalies by splitting (decomposing) the Mailing address table, but how should we do this more formally? and how is it related to functional dependencies?

Design Guidelines

Functional Dependencies

Normalization

Normalization

- Normalization is a process that aims at achieving better designed relational database schemas using
  - Functional Dependencies
  - Primary Keys

- The normalization process takes a relational schema through a series of tests to certify whether it satisfies certain conditions
  - The schemas that satisfy certain conditions are said to be in a given *'Normal Form'*.
  - Unsatisfactory schemas are decomposed by breaking up their attributes into smaller relations that possess desirable properties (e.g., no anomalies)

Review of “Key” Concepts

- **Superkey** - A set of attributes such that no two tuples have the same values for these attributes
- **Key** - A minimal Superkey, called a Candidate Key if more than one:
  - **Primary key** - A selected candidate key
  - **Secondary key** - Remaining candidate keys
- **Prime Attribute** - An attribute that is a member of any candidate key
- **Non-prime attribute** - An attribute that is not a member of any candidate key

- Prime attribute

https://es.wikipedia.org/wiki/Llave#/media/Archivo:Standard-lock-key.jpg

- Non-prime attribute

First Normal Form (1NF)

- A relation schema is in 1NF if domains of attributes include only atomic (simple, indivisible) values and the value of an attribute is a single value from the domain of that attribute
- 1NF disallows
  - having a set of values, a tuple of values, or a combination of both as an attribute value for a single tuple
  - “relations within relations” and “relations as attributes of tuples

Customer	Order	Items
Jones	123	Basket, Football
Gupta	876	Hat, Glass, Pencil

A non-1nf relation

Relations in 1NF still have problems

Customer	Order	Item
Jones	123	Basket
Jones	123	Football
Gupta	876	Hat
Gupta	876	Glass
Gupta	876	Pencil

Problem with this design:

Redundancy (Jones, 123)

Customer	Order	Item1	Item2	Item3	Item4
Jones	123	Basket	Football	Null	Null
Gupta	876	Hat	Glass	Pencil	Null

Problem with this design:

Too many Null values

What if an order has >max (4) items

Second Normal Form (2NF)

- A relation is in 2NF, if for every FD $X \rightarrow Y$ where X is a minimal key and Y is a non-prime attribute, then no proper subset of X determines Y.

Not part of any key

- e.g., the address relation is not in 2NF:
  - House#, street, postal_code is a minimal key
  - House#, street, postal_code → Province
  - Postal_code → province

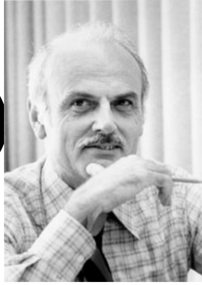
X = House#, street, postal code
Y = province

Redundancy in 2NF

- In 2NF, a member of key shouldn’t determine a non-prime member.

- But still, a non-prime attribute can determine another attribute, which results in redundancy.

Boyce-Codd Normal Form (BCNF)



Raymond Boyce

Ted Codd

A relation R is in BCNF if for all non-trivial dependencies in R:

If X → b then X is a superkey for R

- A dependency is **trivial** if the LHS contains the RHS, e.g., City, Province→ City is a trivial dependency (A, B → A)
- Informally: Whenever a set of attributes of R determine another attribute, it should determine all the attributes of R.

BCNF Example

- Address(<u>House#, Street, </u>City, Province, <u>PostalCode</u>)

House#, Street, PostalCode → City

House#, Street, PostalCode → Province

PostalCode → City

PostalCode → Province

Is Address in BCNF?

$\{PostalCode\}^+ = \{PostalCode, City, Province\}$

No. PostalCode is not a superkey

PostalCode → City is violating BCNF

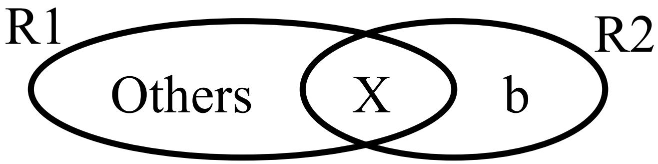
Decomposing into BCNF Losslessly

1. Add implicit FDs to your list of FDs

   e.g. If FDs contain $A \rightarrow B$ and $B \rightarrow C$ add $A \rightarrow C$ to list of FDs

2. Pick any $f \in FD$ that violates BCNF of the form $X \rightarrow b$

3. Decompose R into two relations: $R_1(\text{All-b})$ & $R_2(X \cup b)$



4. Recurse on R_1 and R_2 using FD

Note: answer may vary depending on order you choose. That's okay

How do we decompose into BCNF losslessly?

- The algorithm terminates since all two attribute relations are in BCNF.

R(X,Y)
No FD so no redundancy
$X \rightarrow Y$ so X is key, so in BCNF
$Y \rightarrow X$ so Y is key, so in BCNF
$Y \rightarrow X$ and $X \rightarrow Y$, both X and Y are keys, so in BCNF

BCNF Definition: A relation R is in BCNF if for all non-trivial dependencies in R: If $X \rightarrow b$ then X is a superkey for R

BCNF Decomposition Example

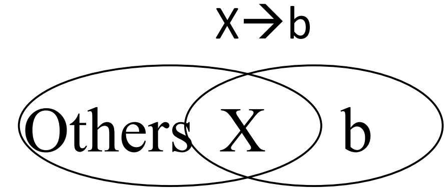
- Relation: R(ABCD)
- Closure and keys

$A^+ = A$ $\qquad$ $B^+ = BC$

$C^+ = C$ $\qquad$ $D^+ = AD$

$BD^+ = BDCA$ is the only key

fd1:		B→C
fd2:		D→A



Considering B→C, is B a superkey in R?

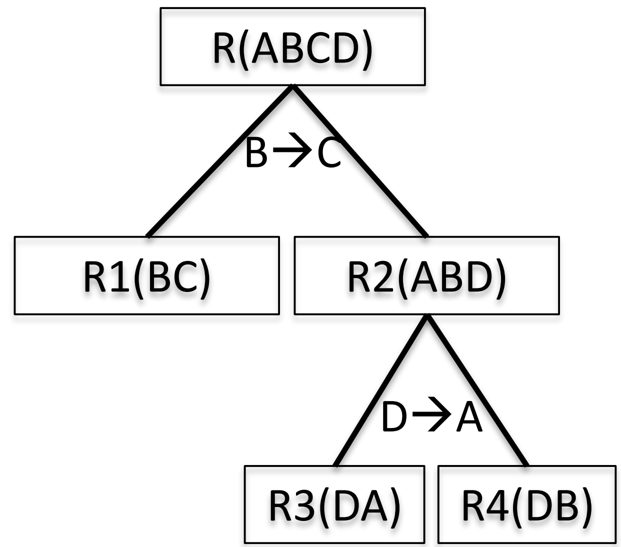
No. Decompose

R1(B,C), R2(A,B,D)

Considering D→ A, is D a superkey for R2?

No. Decompose

R3(D,A), R4(D,B)



Final answer: R1(B,C), R3(D,A), R4(D,B).
What does this mean?

Test if a given FD applies

For an FD X→b, if the decomposed relation S contains {X U b}, and b ∈ X^+ then the FD holds for S.

- For example. Consider relation R(A,B,C,D,E) with functional dependencies AB → C, BC → D, CD → E, DE → A, and AE → B.
- Project these FD's onto the relation S(A,B,C,D).
- Does AB→D hold?
  - First check if A, B and D are all in S? They are
  - Find AB^+= ABCDE
  - Then yes AB→ D does hold in S.

- Does CD→E hold?
  - No (Why? Because S doesn't contain E)

Question

- Consider relation R(A,B,C,D,E) with functional dependencies $AB \rightarrow C$, $BC \rightarrow D$, $CD \rightarrow E$, $DE \rightarrow A$, and $AE \rightarrow B$. Project these FD's onto the relation S(A,B,C,D).
- Which of the following hold in S?

A. $A \rightarrow B$

B. $AB \rightarrow E$

C. $AE \rightarrow B$

D. $BCD \rightarrow A$

E. None of the above

Question

- Consider relation R(A,B,C,D,E) with functional dependencies $AB \rightarrow C$, $BC \rightarrow D$, $CD \rightarrow E$, $DE \rightarrow A$, and $AE \rightarrow B$. Project these FD's onto the relation S(A,B,C,D).
- Which of the following hold in S?

A. $A \rightarrow B$

B. $AB \rightarrow E$

C. $AE \rightarrow B$

D. $BCD \rightarrow A$

E. None of the above

$A^+=A$, so $A \rightarrow B$ does not hold

E is not in S, so $AB \rightarrow E$ does not hold

E is not in S, so $AE \rightarrow B$ does not hold

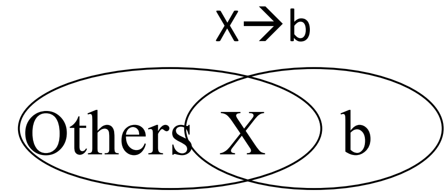
Yes. BCD+=ABCDE; all in S

Don't use the given FD to find closure

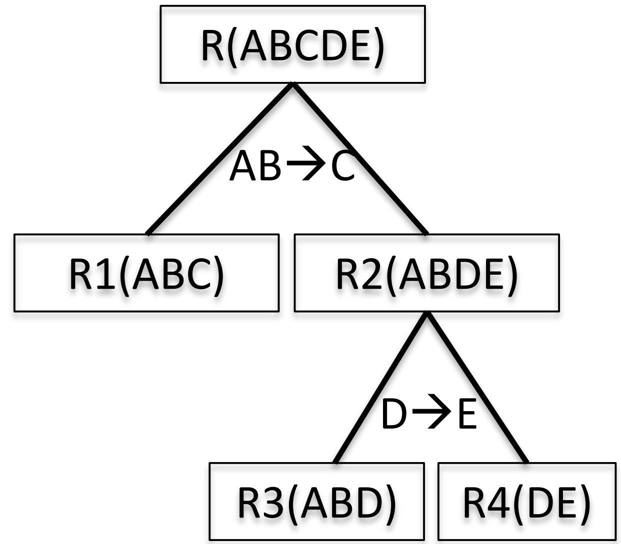
Note that we use all FDs for finding closures, so for D we use $DC \rightarrow E$ Even though E is not present in S.

Another BCNF Example

fd1:	AB→C
fd2:	D→E



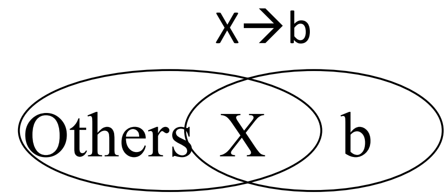
- R(ABCDE)
- Find closure of the following
  - $AB^+ = ABC$
  - $D^+ = DE$
- AB→C violates BCNF in R
  - R1(ABC), R2(ABDE)
- D→E violates BCNF in R2
  - R3(ABD), R4(DE)



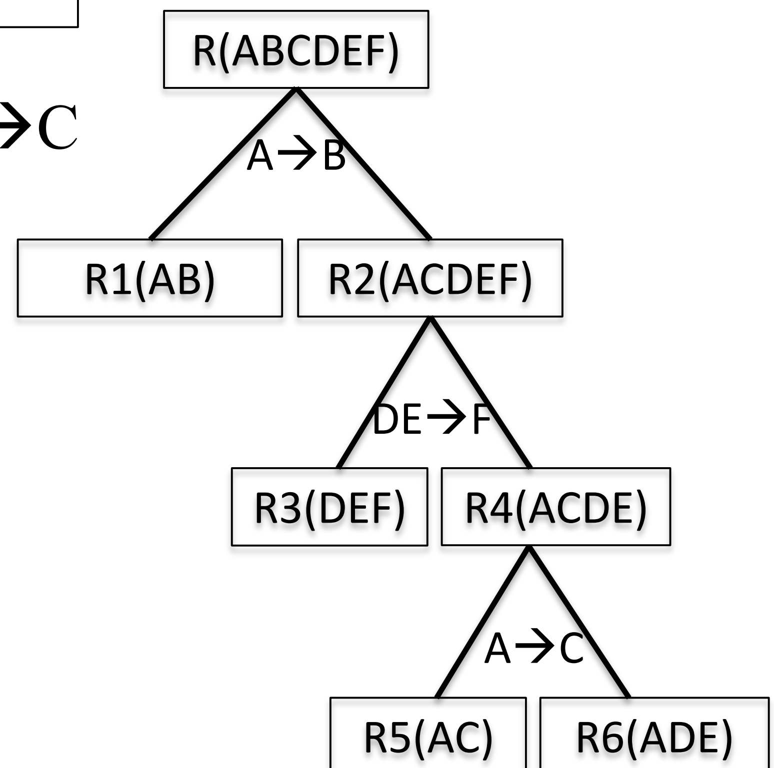
Final answer: R1(ABC), R3(ABD), R4(DE)

Example: Implicit FDs matter

fd1	A→B
fd2	DE→ F
fd3	B→C



- R(A,B,C,D,E,F)
  - $A^+ = ABC$
  - $B^+ = BC$
  - $DE^+ = DEF$
  - A^+ contains C so an implicit FD A→C holds. We add fd4 as A→C
- A→B is violating BCNF in R
  - R1(AB), R2(ACDEF)
  - R1 is BCNF, but R2 is not in BCNF
- DE→ F is violating BCNF in R2
  - R3(DEF), R4(ACDE)
  - R3 is in BCNF, is R4 in BCNF?
- A→C is violating BCNF in R4
  - R5(AC), R6(ADE)



Final answer R1 (AB), R3(DEF), R5(AC), R6(ADE)

Clicker Exercise: More BCNF

- Let R(ABCD) be a relation with functional dependencies $A \rightarrow B$, $C \rightarrow D$, $AD \rightarrow C$, $BC \rightarrow A$
- Decompose into BCNF. Which of the following is a lossless-join decomposition of R into Boyce-Codd Normal Form (BCNF)?

A. {AB, AC, BD}

B. {AB, AC, CD}

C. {AB, AC, BCD}

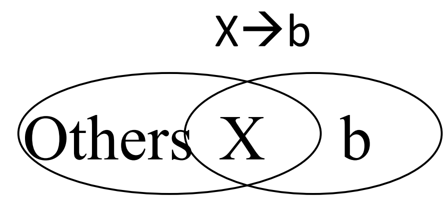
D. All of the above

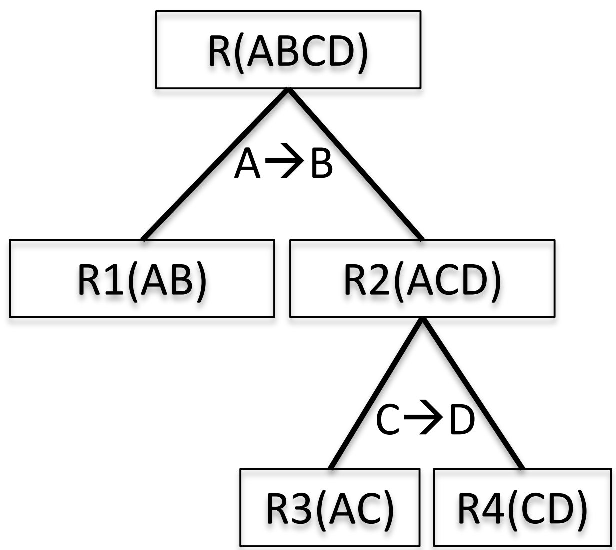
E. None of the above

Clicker Exercise: More BCNF

- Let R(ABCD)
- Decompose into BCNF.
  - $A^+ = AB$
  - $C^+ = CD$
  - $AD^+ = ADBC$
  - $BC^+ = BCAD$
- $A \rightarrow B$ is violating BCNF
  - R1 (AB), R2 (ACD)
- $C \rightarrow D$ is violating BCNF
  - R3(AC), R4(CD)

fd1	$A \rightarrow B$
fd2	$C \rightarrow D$
fd3	$AD \rightarrow C$
fd4	$BC \rightarrow A$





Final answer R1 (AB), R3(AC), R4(CD)

Clicker Exercise: More BCNF

- Let R(ABCD) be a relation with functional dependencies $A \rightarrow B$, $C \rightarrow D$, $AD \rightarrow C$, $BC \rightarrow A$
- Decompose into BCNF. Which of the following is a lossless-join decomposition of R into Boyce-Codd Normal Form (BCNF)?

A. {AB, AC, BD}

Let's see what happens if you randomly decompose

B. {AB, AC, CD}

R1 (<u>A</u>B), R3(<u>AC</u>), R4(<u>C</u>D)

C. {AB, AC, BCD}

C^+=CD, so C is not key for (BCD)

D. All of the above

E. None of the above

Clicker Exercise: Option 'A' exposed

- R(ABCD) and $A \rightarrow B$, $C \rightarrow D$, $AD \rightarrow C$, $BC \rightarrow A$

A. Is {AB, AC, BD} a lossless join?

Imagine tuples:

A	B	C	D
1	2	5	6
1	2	3	7
8	2	9	4

decompose

A	B
1	**2**
8	2

A	C
1	5
1	**3**
8	9

B	D
2	6
2	7
2	**4**

join

A	B	C	D
1	2	5	6
1	2	3	7
8	2	9	4
1	**2**	**3**	**4**

BCNF is great, but…

- Guaranteed that there will be no redundancy of data
- Easy to understand (just look for superkeys)
- Easy to do.
- So what is the main problem with BCNF?
  - For one thing, BCNF may not preserve all dependencies

Dependency Preservation

A functional dependency $X \rightarrow Y$ is preserved in a relation R, if R contains all of the attributes of X and Y.

Example:

- R(A,B,C) fd1 $A \rightarrow B$, fd2 $A \rightarrow D$

fd1 is preserved in R and fd2 is not preserved in R

An Illustrative BCNF example

Unit	Company	Product

Unit → Company
Company, Product → Unit

Is unit a key?

Unit	Company

Unit	Product

Unit → Company

The second functional dependency is no longer preserved.

So What's the Problem?

<u>Unit</u>	Company
SKYWill	ABC
Team Meat	ABC

Unit	Product
SKYWill	Databases
Team Meat	Databases

Unit $\rightarrow$ Company

No problem so far. All *local* FD's are satisfied.
Let's join the relations back into a single table again:

Unit	Company	Product
SKYWill	ABC	Databases
Team Meat	ABC	Databases

Violates the FD: Company, Product $\rightarrow$ Unit

Third Normal Form (3NF)

A relation R is in 3NF if for all non-trivial dependencies in R:
If X → b then
X is a superkey for R OR
b is a prime attribute of R

- A dependency is trivial if the LHS contains the RHS, e.g., City, Province→ City is a trivial dependency

Example: R(Unit,Company, Product)

- Unit → Company
- Company, Product → Unit
  Keys: {Company, Product}, {Unit,Product}

How to find this key?

- R is not in BCNF but is in 3NF.

To decompose into 3NF we rely on the *minimal cover*
Note: if a table is in 3NF form will be assessed.

Minimal Cover for a Set of FDs

Further reading: will not be assessed

- Goal: Transform FDs to be as small as possible.

Minimal cover G for a set of FDs F:
1. Closure of F = closure of G (i.e., imply the same FDs)
2. Right hand side of each FD in G is a single attribute
3. If we delete an FD in G or delete attributes from an FD in G, the closure changes

- Informally: Every FD in G is needed, and is "*as small as possible*" in order to get the same closure as F

Example:
- A→B, ABCD→E, EF→GH, ACDF→EG has the following minimal cover:
  - A→B, ACD→E, EF→G and EF→H

Finding Minimal Covers of FDs

Further reading: will not be assessed

1. Put FDs in standard form (have only one attribute on RHS)
2. Minimize LHS of each FD
3. Delete Redundant FDs

Example:

$A \rightarrow B$, $ABCD \rightarrow E$, $EF \rightarrow G$, $EF \rightarrow H$, $ACDF \rightarrow EG$

- Replace last rule with
  - $ACDF \rightarrow E$
  - $ACDF \rightarrow G$

Finding Minimal Covers of FDs

Further reading: will not be assessed

1. Put FDs in standard form (have only one attribute on RHS)
2. Minimize LHS of each FD
3. Delete Redundant FDs

Example:

A→B, ABCD→E, EF→G, EF→H, ACDF → E, ACDF → G

- Can we take anything away from the LHS? (try attributes one by one BCD+, ACD+, BCD+ see if closure is the same, clue: A→B so we don't need B in ABCD+)
  - $ABCD^+ = ABCDE$
  - $ACD^+ = ABCDE$, so remove B from the FD

Finding Minimal Covers of FDs

Further reading: will not be assessed

1. Put FDs in standard form (have only one attribute on RHS)
2. Minimize LHS of each FD
3. Delete Redundant FDs

Example:

A→B, ACD→E, EF→G, EF→H, ACDF → E, ACDF → G

- Let's find $ACDF^+$ without considering the highlighted FDs
  - $ACDF^+$= ACDFEBGH, so I can remove the highlighted rules
- Final answer: A→B, ACD→E, EF→G, EF→H

Minimal Example Cover

Further reading: will not be assessed

- Consider the relation R(CSJDPQV) with FDs C→SJDPQV, JP→C, SD→P, J→S

Find a minimal cover

Another Minimal Cover Example

Further reading: will not be assessed

- Consider the relation R(CSJDPQV) with FDs
  C→SJDPQV, JP→C, SD→P, J→S

Put FDs in standard form (have only one attribute on RHS)
Minimize LHS of each FD
Delete Redundant FDs

Find a minimal cover

- C→S, C→J, C→D, C→P, C→Q, C→V, JP→C, SD→P, J→S

Can we shorten any FDs?

Let's consider shortening JP→C

$JP^+ = CSJDPQV$

$J^+ = JS$

$P^+ = P$

Let's consider shortening SD→P

$SD^+ = SDP$

$S^+ = S$ and $D^+ = D$

Another Minimal Cover Example

Further reading: will not be assessed

- Consider the relation R(CSJDPQV) with FDs C→SJDPQV, JP→C, SD→P, J→S

Find a minimal cover

- C→S, C→J, C→D, C→P, C→Q, C→V, JP→C, SD→P, J→S

Can we shorten any FDs? No

Can we remove any FDs?

- Let's consider C→S and find C^+ without considering this rule
  - C^+ = SJDPQV, so we can delete this FD
- Let's consider C→P and find C^+ without considering this rule
  - C^+ = SJDQVP, so we can delete this FD

Minimize LHS before deleting redundant FDs

Further reading: will not be assessed

- If step 3 is done prior to step 2, the final set of FDs could still contain redundant FDs

- ABCD→E, E→D, A→B, AC→D
  - Let's delete redundant FDs.
  - None of the FDs are redundant.

~~1. One attribute on RHS~~
~~2. Delete Redundant FDs~~
~~3. Minimise LHS of each FD~~

Does not work, must first minimise then delete,
Order of procedures
is important.

- ABCD→E, E→D, A→B, AC→D
  - Now let's shorten FDs
  - ABCD→E can be replaced by AC→E
- However the current set of FDs are not minimal
  - AC→E, E→D, A→B, AC→D
  - The highlighted FD can be deleted

Decomposition into 3NF Using Minimal Cover

Further reading: will not be assessed

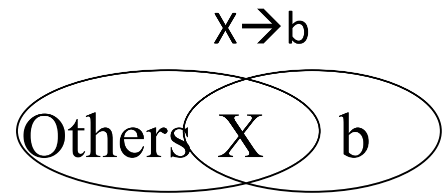
- Decomposition into 3NF:
  1. Given the FDs F, compute F': the *minimal cover for F*
  2. Decompose using F' if violating 3NF similar to how it was done for BCNF
  3. After each decomposition identify the set of dependencies N in F' that are not preserved by the decomposition.
  4. For each X→a in N create a relation $R_n(X \cup a)$ and add it to the decomposition

Put FDs in standard form
Minimize LHS of each FD
Delete Redundant FDs

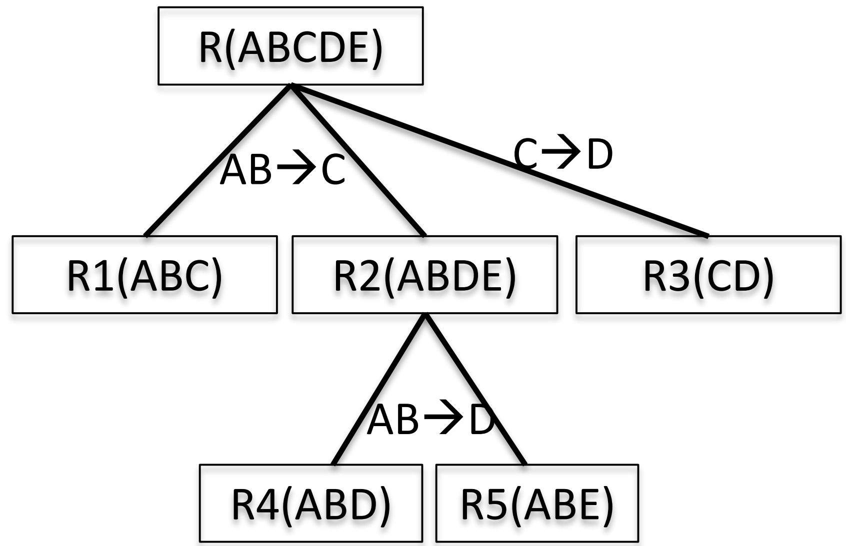
3NF Example

Further reading: will not be assessed

fd1 AB→C
fd2 C→ D



- Example: R(ABCDE)
  - Cover already minimal
    - » $ABE^+ = ABCDE$ only key
  - AB→C is violating 3NF
    - $R_1(ABC)$, $R_2(ABDE)$
  - Are all FDs preserved?
    - We lost C→D so add $R_3(CD)$
  - AB→D (transitivity) is violating 3NF
    - R4(ABD), R5(ABE)



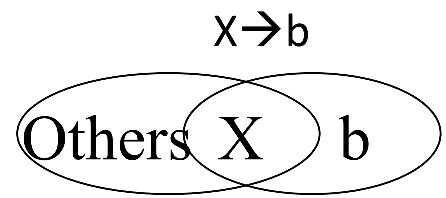
Final answer: R1(ABC), R3(CD), R4(ABD), R5(ABE)

Another 3NF Example

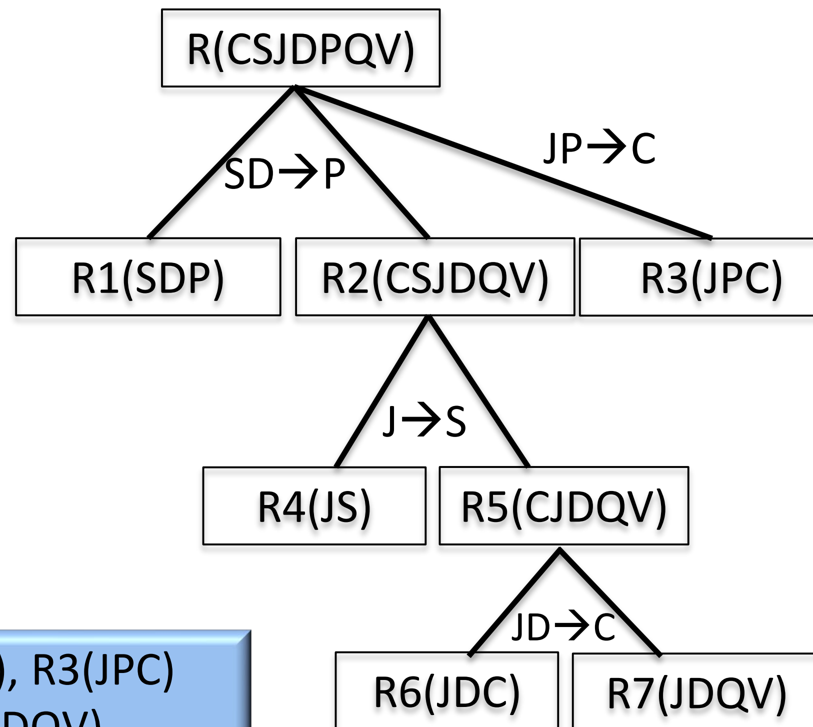
Further reading: will not be assessed

- Let R(CSJDPQV) be a relation with the following FDs. Decompose R into 3NF
  - Already in minimal cover
  - $SD^+ = SDP$
  - $JP^+ = JPSC$
  - $J^+ = JS$
  - $JD^+ = JDSPC$
  - $JDQV^+ = CSJDPQV$ Key

fd1	SD→P
fd2	JP→C
fd3	J→S



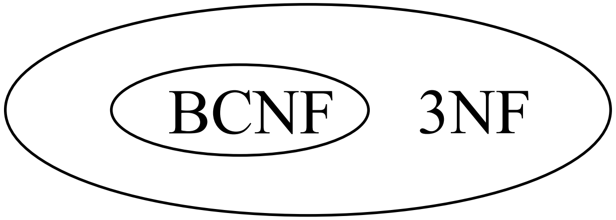
- SD→P violates 3NF in R
  - R1(SDP) , R2(CSJDQV)
- Are all FDs preserved?
  - We lost JP→C so add R3 (JPC)
- J→S violates 3NF in R2
  - R4 (JS), R5 (CJDQV)
- JD→C (implicit)
  - R6 (JDC), R7 (JDQV)



Final answerR1(SDP), R3(JPC)
R4(JS), R6(JDC), R7(JDQV)

BCNF & 3NF

- BCNF guarantees removal of all anomalies
- 3NF has some anomalies, but preserves all dependencies
- If a relation R is in BCNF it is in 3NF.
- A 3NF relation R may not be in BCNF.



- Most organizations go to BCNF or 3NF.

Clicker Question: BCNF and 3NF

- Consider the following relation and functional dependencies:
  R(ABCD) FD's: ACD → B ; AC → D ; D → C ; AC → B
  Which of the following is true:

A. R is in neither BCNF nor 3NF

B. R is in BCNF but not 3NF

C. R is in 3NF but not in BCNF

D. R is in both BCNF and 3NF

Clicker Question: BCNF and 3NF

- Consider the following relation and functional dependencies:
  R(ABCD) FD's: ACD $\rightarrow$ B ; AC $\rightarrow$ D ; D $\rightarrow$ C ; AC $\rightarrow$ B
  Which of the following is true:

A. R is in neither BCNF nor 3NF
B. R is in BCNF but not 3NF
C. R is in 3NF but not in BCNF
D. R is in both BCNF and 3NF

ACD^+=ABCD
AC^+=ACDB
D^+=DC
AD^+=ADCB
Keys: AC, AD

D→C (D not key so NOT BCNF)

C is part of a minimal key so R is in 3NF

Denormalization

- Process of intentionally violating a normal form to gain performance improvements
- Performance improvements:
  1. Fewer joins
  2. Reduces number of foreign keys

- Useful in data analysis or if certain queries often require (joined) results, and the queries are frequent enough.

Learning Outcomes

Description	Tag
Provide examples of modification, insertion, and deletion anomalies.	Functional-dependencies
Explain the term functional dependency.	
Given a set F of functional dependencies and a functional dependency fd, determine whether fd can be inferred from F using Armstrong's Axioms.	
Given a set of functional dependencies that hold over a table, determine the keys.	
Given a set of functional dependencies that hold over a table, determine the superkeys.	
Given a set F of functional dependencies and X of attributes of a relation, compute the closure of X, which is X+	
Given a relation R, be able to correctly decompose it into two or more relations	Decomposition
Justify why lossless join decompositions are preferred decompositions.	
Explain the benefits of Normalization.	Normalization
Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in 1/2NF.	
Explain what dependency preservation means.	
Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in 3NF.	
Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in BCNF.	
Describe the process and benefits of Denormalization.	
Reason with the logical foundation of the relational data model and understand the fundamental principles of correct relational database design.	