Correlation between U and V

$$\mathrm{corr}(U,V)=\frac{\mathrm{Cor}(U,V)}{\sqrt{\mathrm{Var}(U)\cdot\mathrm{Var}(V)}}=\rho(U,V)$$

算 dx 对 y轴作垂线
算 dy 对 X轴作垂线

$$f_{XY}(x,y)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)\cdot\frac{1}{\sqrt{2\pi(1-\rho^2)}}\exp\left(-\frac{(y-\rho x)^2}{2(1-\rho^2)}\right)$$

即 $f_X(x)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)\sim \mathrm{Normal}(0,1).$

$$f_Y(y)=\frac{1}{\sqrt{2\pi(1-\rho^2)}}\exp\left(-\frac{(y-\rho x)^2}{2(1-\rho^2)}\right)\sim \mathrm{Normal}(\rho x,\ 1-\rho^2)$$

$$\mathbb{E}[Y]=\mathbb{E}[\mathbb{E}[Y|X=x]]=\mathbb{E}[Y|X=x]\times\int_{-\infty}^{\infty}f(x)dx=\int y\, f_{Y|X=x}(y|X=x)dy\cdot\int_{-\infty}^{\infty}f(x)dx$$

X, Y independent $Z=X+Y$

$$\mathbb{P}(Z=n)=\mathbb{P}(X+Y=n)=\sum_{k=-\infty}^{\infty}\mathbb{P}(X=n-k)\mathbb{P}(Y=k)$$

$X\sim \mathrm{uniform}(0,1)$
$Y\sim \mathrm{uniform}(X,1)$
$f_{Y|X=x}(y|X=x)=\frac{1}{1-x}$

$$M_X(t)=\mathbb{E}[e^{tX}]=\int_{-\infty}^{\infty}e^{tx}f(x)dx$$

X, Y independent
$M_{X+Y}(t)=M_X(t)\times M_Y(t)$

$X\sim \mathrm{Normal}(\mu,\sigma^2)$ $M_X(t)=e^{\mu t+\frac{1}{2}t^2\sigma^2}$

$$\left.\begin{aligned}M_X'(0)&=\mathbb{E}[X]\\ M_X''(0)&=\mathbb{E}[X^2]\end{aligned}\right\}\text{以 } t \text{ 为变量.}$$

$X\sim \mathrm{Exp}(\lambda)$ $M_X(t)=\frac{\lambda}{\lambda-t}$ $(t<\lambda)$

$Z=X+Y$, X, Y independent.

$$f_Z(z)=\int_{-\infty}^{\infty}f_X(x)f_Y(z-x)dx$$