$E[Y] = E[E[Y|X]]$

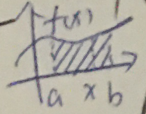
$= \sum_{k=1}^{6} E[Y|X=k] \times P(X=k)$

cdf $0 \le F(x) = P(X \le x) \le 1$

distribution

$P(X=x)=0$ continuous ~~rad~~ random variable.

~~Pdf~~ $F_X(x) = \int_{-\infty}^{x} f_X(u)\,du \rightarrow$ pdf



$P(a \le X \le b) = \int_a^b f_X(x)\,dx$

$f_X(x) = F'_X(x) = \frac{d}{dx}F_X(x)$

$P(x \le X \le x+h) = \int_x^{x+h} f_X(x)\,dx \approx h f_X(x)$

$E[X] = \int_{-\infty}^{\infty} u f_X(u)\,du$ $E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$

$\min\{T_1, T_2\} = P(Y=y) = P(T_1 \le y \cup T_2 \le y)$

$\max\{T_1, T_2\} = P(Y=y) = P(T_1 \le y \cap T_2 \le y)$.

$X \sim Geometric(p)$ 问 the probability that more than n times.

$P(X>n) = (1-p)^n$.

$\because g(X) = g(f_X(x))$ 问 distribution，求 pdf，即 $f_X(x)$.

$\therefore E[g(X)] = \int_{-\infty}^{\infty} g(u) f_X(u)\,du$

$X \sim Uniform[a,b]$

density/pdf $\int_a^b f_X(x)\,dx = \int_a^b \frac{1}{b-a}\,dx$

$E[X] = \frac{a+b}{2}$ $Var(X) = \frac{(a-b)^2}{12}$

$X \sim exp(\lambda)$

pdf $f_X(x) = \lambda e^{-\lambda x}$ $x \ge 0$

cdf $F_X(x) = 1 - e^{-\lambda x}$ $x \ge 0$.

$E[X] = \frac{1}{\lambda}$ $Var(X) = \frac{1}{\lambda^2}$ $P(X>x+y \mid X>y) = P(X>x)$

$\int u(x) v'(x)\,dx = u(x)v(x) - \int v(x) u'(x)\,dx$

eg: $\int_0^\infty x \cdot \lambda e^{-\lambda x}\,dx$, $[u(x) = x, v(x) = -e^{-\lambda x}]$

$= x \cdot (-e^{-\lambda x})\Big|_0^\infty - \int_0^\infty (-e^{-\lambda x}) \cdot 1\,dx = x \cdot (-e^{-\lambda x})\Big|_0^\infty + (-\frac{e^{-\lambda x}}{\lambda})\Big|_0^\infty = \frac{1}{\lambda}$

Standard Normal Distribution

$X \sim Normal(0, 1^2)$ $E[X] = 0$ $Var(X) = 1^2$

pdf $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ $x \in R$

$E[X^2] = Var(X) + E[X]^2 = \sigma^2 + \mu^2$

查表 $P(X \le x) = 1 - P(X>x)$ $X \sim Normal(\mu, \sigma^2)$

$P(X>x) = P(X>x)$

$P(X \le -x) = P(X>x)$ pdf $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{(-\frac{(x-\mu)^2}{2\sigma^2})}$

$P(-x \le X \le x) = 1 - 2P$

μ ~ mean, expectation

σ^2 ~ variance

σ ~ standard deviation

$X \sim Normal(0, 4)$.

求 $P(X \le 3.92)$

令 $Z = \frac{X-\mu}{\sigma}$ Z为 Normal(0,1)

$P(Z \le \frac{3.92-0}{\sqrt{4}}) = P(Z \le 1.96)$

$= 1 - P(Z>1.96) = 0.975$ (查表)

$\frac{X-\mu}{\sqrt{\sigma^2}} \sim Normal(0,1)$.