Marginal probability

Marginal probability (also called simple probability), P(X), is the probability of an event X occurring, where X is a simple event.

Example: referring to the statistics enrolment example below, What is the probability of selecting a male? What is the probability of selecting a person taking statistics?

	Enrolment status		
Gender	Taking Stats.	Not taking Stats.	Total
Male	40	58	98
Female	32	43	75
Total	72	101	173

$$P(\text{male}) = \frac{\text{No. of males}}{\text{Total sample size}} = \frac{98}{173} = 0.566$$

$$P(\text{taking Stats}) = \frac{\text{No. of taking Stats.}}{\text{Total sample size}} = \frac{72}{173} = 0.416$$

Joint Probability

The probability of occurrence of two or more simple events

Examples:

a. What is the probability of randomly selecting a person who is male **and** taking statistics?
b. What is the probability of randomly selecting a person who is male **and** not taking statistics?

a. $P(\text{male} \cap \text{stats}) = \frac{\text{No. of males and stats}}{\text{Total sample size}} = \frac{40}{173} = 0.231$

b. $P(\text{male} \cap \text{not stats}) = \frac{\text{No. of males and not stats}}{\text{Total sample size}} = \frac{58}{173} = 0.335$

Conditional Probability

Exercise 2: The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent **given that** today is Friday?

$$P(\text{absent} \mid \text{Friday}) = \frac{P(absent \cap Friday)}{P(Friday)} = \frac{0.03}{0.2} = 0.15$$

Binomial Distribution

Example: A biased coin which produces heads only 40% of the time is tossed twice.

a. What is the probability of getting one head?

b. What is the probability of getting at most one head?

Solution: Given: $p = 0.4$; $q = 0.6$; $n = 2$

Formula: $$P(x) = \frac{n!}{x!(n-x)!} p^x . q^{n-x}$$

a. P(one head): $$P(x = 1) = \frac{2!}{1!(2-1)!} \times 0.4^1 \times 0.6^1$$

$$= \frac{2 \times 1}{1 \times 1} \times 0.4 \times 0.6 = 0.48$$

Note: **Excel f_x function:** =binom.dist(1,2,0.4,false)

b. P(at most one head):

$P(x \le 1) = P(x = 0) + P(x = 1)$

$$= \left[\frac{2!}{0!(2-0)!} \times 0.4^0 \times 0.6^2\right] + \left[\frac{2!}{1!(2-1)!} \times 0.4^1 \times 0.6^1\right]$$

$$= \quad 0.36 \quad + \quad 0.48$$

$$= \; 0.84$$

Excel f_x function: =binom.dist(1,2,0.4,true)

Exercise: Forty-five percent of all registered voters in a national election are female. A random sample of 8 voters is selected. What is the probability that the sample contains **2 males**?

Hint:

55% are male and there are two outcomes: male/female, therefore the Binomial distribution equation is applied

n = 8; p = 0.55; q = 1-p = 0.45

Hence $P(x = 2) = \frac{8!}{2!(8-2)!} 0.55^2 (0.45)^{8-2} = 0.0703$

Questions from Lecture 1

Q1. If I sample with replacement, which of the following may be true?

(a) The numerator for the next event's probability changes.

(b) The denominator for the next event's probability changes.

(c) None of the values used in calculating the next event's probability change.

(d) Both the numerator and denominator for the net event's probability change.

Questions from Lecture 1

Q2. Using the given data, answer the following question

	Pass Course (A)	Fail Course (A')
Pass Final (B)	142	34
Fail Final (B')	89	56

What is the probability that a student, taken at random from a class, would have passed the course given that he/she failed the final?

(a) 0.72 (b) 0.55 (c) 0.44 (d) 0.61

Questions from Lecture 1

Q2. Solution

	Pass Course (A)	Fail Course (A')	Total
Pass Final (B)	142	34	176
Fail Final (B')	89	56	145
Total	231	90	321

Question: $P(A \mid B')$

We know that $P(A \mid B') = P(A \text{ and } B')/P(B')$

$= (89/321)/(145/321)$

$= 89/145 = \mathbf{0.61}$