STAT7203: Applied Probability and Statistics
Assignment 3

Due by 11:00 am on Wednesday the 23rd of October, 2019
via the Electronic Assignment Submission System (62-225)

The marks for each question is indicate by the number in square brackets. There are a total of 14 marks for this assignment.

1. A pair of random variables (X, Y) has a joint probability distribution in which $X \sim \mathsf{Uniform}(0, 1)$ and the conditional probability density function of Y given $\{X = x\}$ is

$$f_{Y|X}(y|x) = \begin{cases} 1, & x \le y \le 1 + x \\ 0, & \text{else.} \end{cases}$$

   (a) Determine the marginal probability density function for Y. [2]

   (b) Using the fact that $\mathbb{E}[XY] = \mathbb{E}[X\mathbb{E}[Y|X]]$, compute the covariance between X and Y. [2]

   (c) Suppose that $U \sim \mathsf{Uniform}(0, 1)$ and define the random variable $Z := g(U)$, where

$$g(u) = \begin{cases} \sqrt{2u}, & 0 \le u \le \frac{1}{2}, \\ 2 - \sqrt{2(1-u)}, & \frac{1}{2} < u \le 1. \end{cases}$$

   Show the distribution of Z is the same as the marginal distribution of Y in part (a). [2]

2. Serious gaming technology is increasingly being used as a method of training. A 2010 study compared the efficacy of a serious game 'Triage Trainer' to traditional card-sort exercises in preparing learners for a major incident triage. In this study 91 learners were randomly distributed into one of two training groups: 44 participants practiced triage sieve protocol using a card-sort exercise, whilst the remaining 47 participants used 'Triage Trainer'. After the training sessions, each participant was evaluated by triaging eight casualties in a simulated live exercise. Their performance was assessed in terms of accuracy and speed.

   (a) The performance of the leaners in the evaluation exercise is recorded in the table below giving the number of correctly assigned casualties out of eight. Is there any evidence of an association between training method and accuracy? State the null and alternative hypotheses, and use an appropriate test statistic to determine the P-value. What do you conclude? [3]

	8/8	7/8	6/8 (or less)
Card-sort group	24	11	9
Triage Trainer group	34	9	4

(b) Construct a confidence interval for the difference in the proportion of learners from the two training groups who correctly assign all eight casualties. [1]

(c) The average time taken to triage all eight casualties in the card-sort group was 435s with a sample standard deviation of 74s, whereas the average time taken to triage all eight casualties in the 'Triage Trainer' group was 456s with a sample standard deviation of 62s. Is there any evidence of a difference in the mean time taken to triage casualties between the card-sort group and the 'Triage Trainer' group? State the null and alternative hypotheses, and use an appropriate test statistic to determine the P-value. What do you conclude? [3]

(d) Construct a 95% confidence interval for the mean time taken to triage eight casualties by a person trained using the 'Triage Trainer' game. [1]

Total [14]