Venue	____________________
Seat Number	________
Student Number	|__|__|__|__|__|__|__|__|
Family Name	_____________________
First Name	_____________________

THE UNIVERSITY OF QUEENSLAND AUSTRALIA

This exam paper must not be removed from the venue

School of Mathematics & Physics

EXAMINATION

Semester Two Practice Exam, 2019

STAT7203 Applied Probability and Statistics

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Casio FX82 series or UQ approved (labelled)

One A4 sheet of handwritten notes double sided is permitted

Materials To Be Supplied To Students:

None

Instructions To Students:

Additional exam materials (eg. rough paper) will be provided upon request.

There are 50 marks available on this exam from 5 questions.

Write your answers in the spaces provided on pages 2 – 15 of this examination paper. Show your working and state conclusions where appropriate. Pages 16 – 20 give formulas and statistical tables. Those pages will not be marked.

For Examiner Use Only

Question	Mark

Total ________

1. [10 marks] A pair of random variables (X, Y) has a joint probability density function given by

$$f_{X,Y}(x, y) = \begin{cases} 2 & \text{if } 0 < y < x < 1 \\ 0 & \text{else.} \end{cases}$$

   (a) Determine the marginal probability density function of X. [2 marks]

   (b) Compute $\mathbb{E}[Y \mid X = x]$. [3 marks]

(c) Compute $\text{Cov}(X, Y)$. [3 marks]

(d) Are X and Y independent? Justify your answer. [2 marks]

2. [6 marks] Let $U \sim \mathsf{Uniform}(0, 1)$ and define the random variable

$$X = -\log(\sqrt{1 + 3U} - 1).$$

(a) Find the probability density function of X. [4 marks]

(b) The moment generating function of X is

$$M_X(t) = \frac{2}{3(1-t)} + \frac{2}{3(2-t)}, \qquad t < 1.$$

Using the moment generating function, determine the variance of X. [2 marks]

3. [10 marks] Maximal oxygen consumption is a way to measure the physical fitness of an individual. It is the amount of oxygen in millilitres a person uses per kilogram of body weight per minute (mL/kg/min). A medical researcher is investigating whether athletes have a greater mean maximal oxygen consumption than non-athletes. A total of 24 students was sampled from an American university. Each student was asked whether they were on an athletic scholarship or not, before having their maximal oxygen consumption level measured. The data are summarised in the table below:

	n	$\bar{x}$	s
Athlete	10	57	6.9
Non-athlete	14	49	6.1

(a) Do athletes have higher maximal oxygen consumption than non-athletes? Answer this question by carrying out an appropriate hypothesis test. You should clearly state the null and alternative hypotheses, compute a p-value, and write a conclusion in a manner that can be understood by the medical researcher. [5 marks]

(b) Briefly explain the role of the significance level in hypothesis testing. [1 mark]

(c) Construct an approximate 95% confidence interval for the true mean maximal oxygen consumption for athletes. [4 marks]

4. [10 marks] Energy drinks have become widely popular among adolescents and are also consumed by athletes, particularly those who have just begun their sporting career. A recent paper presented a study on the consumption of energy drinks by teenagers engaged in sports, including quantity consumed and factors that might be associated with consumption. A total of 707 students, selected randomly from sports classes at various schools, completed a questionnaire on energy drink consumption. The following table shows the crosstabulation of regular energy drink consumption by gender:

	Energy Drinks	
Gender	Yes	No
Female	192	90
Male	296	129

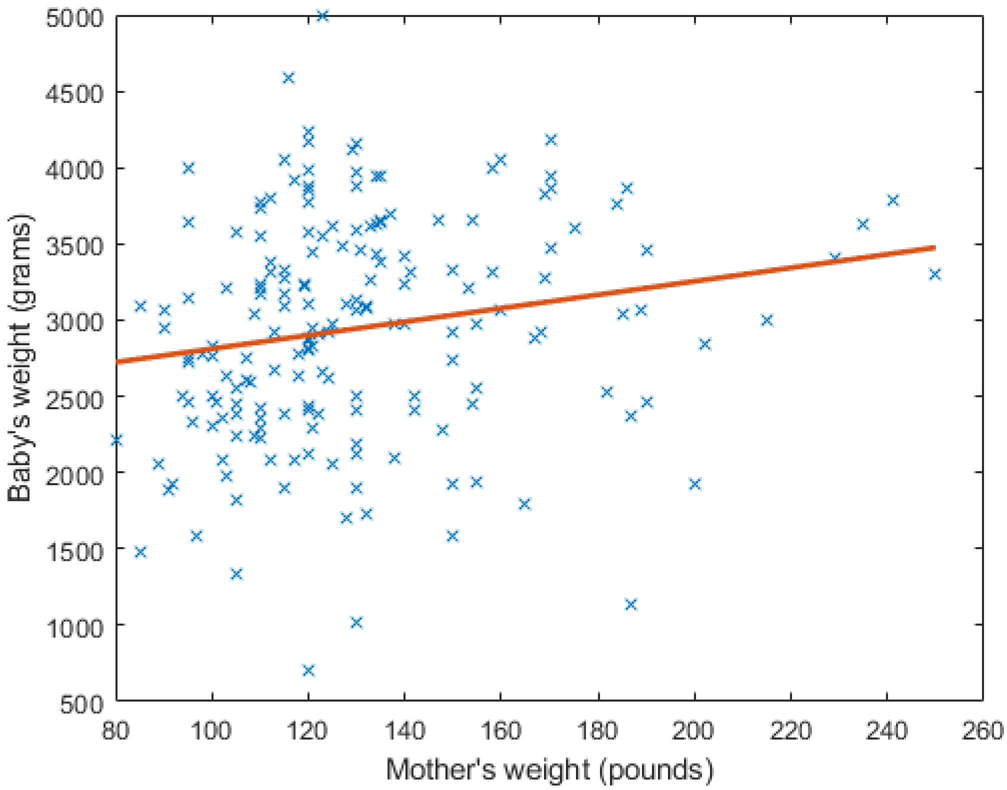
   (a) Assuming this sample is representative of all teenagers engaged in sports, give a 95% confidence interval for the true difference in the proportions of females and males who consume energy drinks. What does the interval say about the difference in energy drink consumption between genders? [4 marks]

(b) Another factor recorded on the questionnaire was the frequency of practising sports. The following table gives the summary of results for the 681 students who indicated they practised at least once a week:

	Energy Drinks	
Practising Sports	Yes	No
Daily	328	146
2-3 times per week	28	13
Once per week	114	52

Based on this table, is there evidence of an association between energy drink consumption and frequency of practising sports? [6 marks]

5. [14 marks] In a study of factors thought to be associated with birth weight, data from 189 births was collected at the Baystate Medical Center, Springfield, Massachusetts during 1986. Two of the variables recorded were the baby's weight (grams) at birth and the mother's weight (pounds) at last menstrual period. The data are plotted below with the least squares regression line



The output on the next page show the results of a linear regression fit in MATLAB for the relationship between the mother's weight (`Mother`) and the baby's weight (`Baby`).

```
Linear regression model:
    Baby ~ 1 + Mother

Estimated Coefficients:
                   Estimate      SE       tStat       pValue
                   ________    ______    ______    __________

    (Intercept)     2369.6     228.49    10.371    3.3968e-20
    Mother          4.4291     1.7135    2.5848      0.010504


Number of observations: 189, Error degrees of freedom: 187
Root Mean Squared Error: 718
R-squared: 0.0345,  Adjusted R-Squared 0.0293
F-statistic vs. constant model: 6.68, p-value = 0.0105
```

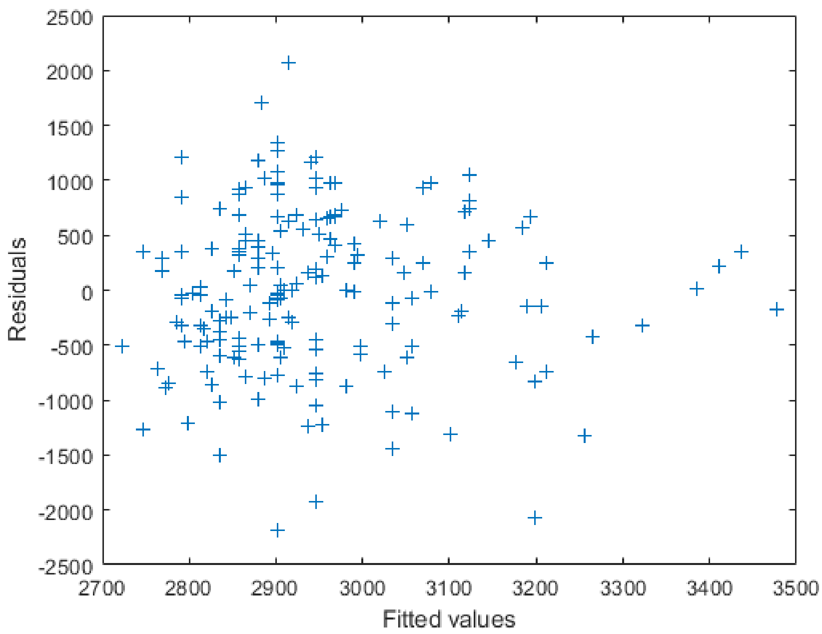
(a) Briefly interpret the value 4.4291 in the regression output. [1 mark]

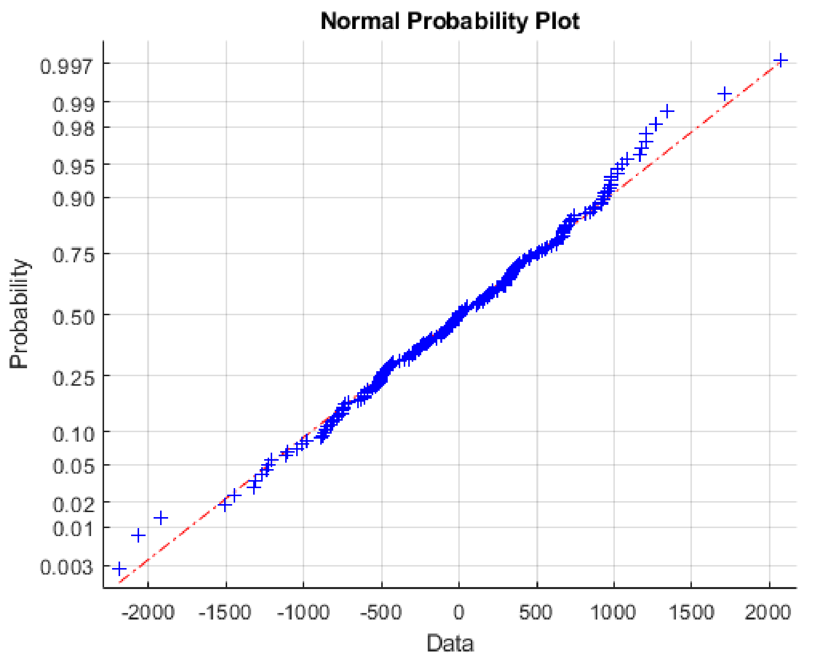
(b) Does the mother's weight explain much of the variation in the baby's weight at birth? Justify your answer. [1 mark]

(c) Give a 90% confidence interval for the intercept term in the linear relationship between the mother's weight and the baby's weight at birth. [2 marks]

(d) Does the data provide evidence of an association between the mother's weight and the baby's weight at birth? State the null and alternative hypotheses and report the appropriate test statistic and P-value from the output. What do you conclude? [3 marks]

(e) The following figures were generated in MATLAB to help check the assumptions underlying the linear regression model. State these assumptions and comment on their validity for this data with reference to these figures and the figure on page 9. [3 marks]





(additional space for answer to part (e) — not all this space is needed)

(f) A second linear model was fitted with the additional explanatory variable `Smoke` which is a dummy variable that takes the value 1 when the mother is a smoker and 0 otherwise. The edited output is given below.

```
>> birthlm2 = fitlm(birth,'Baby~Mother+Smoke')

birthlm2 = 


Linear regression model:
    Baby ~ 1 + Mother + Smoke

Estimated Coefficients:
                   Estimate      SE       tStat       pValue
                   ________    ______    ______    __________

    (Intercept)     2501.1     230.84    10.835    1.6091e-21
    Mother          4.2367     1.6899    2.5071      0.013028
    Smoke          -272.08                           0.010749


Number of observations: 189, Error degrees of freedom:
Root Mean Squared Error: 708
R-squared:        ,  Adjusted R-Squared 0.0578
F-statistic vs. constant model: 6.76, p-value = 0.00146
>> birthlm2.CoefficientCovariance

ans =

   1.0e+04 *

    5.3285   -0.0374   -0.5389
   -0.0374    0.0003    0.0008
   -0.5389    0.0008    1.1150

>> var(birth.Baby)

ans =

   5.3175e+05
```

From this output determine (i) the standard error for the estimate of the coefficient of `Smoke`, (ii) the error degrees of freedom and (ii) the R-squared value. [4 marks]

(space for answer to part (f) — not all this space is needed)

END OF EXAMINATION

Formula Sheet

Elementary probability

- **Sum rule:** For disjoint $A_1, A_2, \ldots$: $\mathbb{P}(\bigcup_i A_i) = \sum_i \mathbb{P}(A_i)$.
- $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$.
- $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$.
- **Conditional probability:** $\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$.
- **Law of total probability**: $\mathbb{P}(A) = \sum_{i=1}^n \mathbb{P}(A \mid B_i)\,\mathbb{P}(B_i)$, where $B_1, B_2, \ldots, B_n$ is a partition of Ω.
- **Bayes' Rule:** $\mathbb{P}(B_j|A) = \frac{\mathbb{P}(B_j)\,\mathbb{P}(A|B_j)}{\sum_{i=1}^n \mathbb{P}(B_i)\,\mathbb{P}(A|B_i)}$.
- **Independent events**: $\mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B)$.

Random variables

- **Cdf** of X: $F(x) = \mathbb{P}(X \leqslant x),\ x \in \mathbb{R}$.
- **Pmf** of X: (discrete r.v.) $f(x) = \mathbb{P}(X = x)$.
- **Pdf** of X: (continuous r.v.) $f(x) = F'(x)$.
- For a discrete r.v. X: $\mathbb{P}(X \in B) = \sum_{x \in B} \mathbb{P}(X = x)$.
- For a continuous r.v. X with pdf f: $\mathbb{P}(X \in B) = \int_B f(x)\,dx$.
- In particular (continuous), $F(x) = \int_{-\infty}^x f(u)\,du$.
- **Important discrete distributions**:

Distr.	pmf	suppport
$\mathsf{Ber}(p)$	$p^x(1-p)^{1-x}$	$\{0,1\}$
$\mathsf{Bin}(n,p)$	$\binom{n}{x}\,p^x(1-p)^{n-x}$	$\{0,1,\ldots,n\}$
$\mathsf{Poi}(\lambda)$	$\mathrm{e}^{-\lambda}\,\frac{\lambda^x}{x!}$	$\{0,1,\ldots\}$
$\mathsf{Geom}(p)$	$p(1-p)^{x-1}$	$\{1,2,\ldots\}$

- **Important continuous distributions**:

Distr.	pdf	$x \in$
$\mathsf{U}[a,b]$	$\frac{1}{b-a}$	$[a,b]$
$\mathsf{Exp}(\lambda)$	$\lambda\,\mathrm{e}^{-\lambda x}$	$\mathbb{R}_+$
$\mathsf{N}(\mu,\sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}}\,\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$\mathbb{R}$

- **Expectation** (discr.): $\mathbb{E}X = \sum_x x\,\mathbb{P}(X = x)$.
- (of function) $\mathbb{E}\,g(X) = \sum_x g(x)\,\mathbb{P}(X = x)$.
- **Expectation** (cont.): $\mathbb{E}X = \int x\,f(x)\,dx$.
- (of function) $\mathbb{E}\,g(X) = \int g(x) f(x)\,dx$,

- **$\mathbb{E}X$ and Var(X) for discrete distributions**:

	$\mathbb{E}X$	$\mathrm{Var}(X)$
$\mathsf{Ber}(p)$	p	$p(1-p)$
$\mathsf{Bin}(n,p)$	np	$np(1-p)$
$\mathsf{Geom}(p)$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
$\mathsf{Poi}(\lambda)$	λ	λ

- **$\mathbb{E}X$ and Var(X) for continuous distributions**:

	$\mathbb{E}X$	$\mathrm{Var}(X)$
$\mathsf{U}(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$\mathsf{Exp}(\lambda)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$\mathsf{N}(\mu,\sigma^2)$	μ	σ^2

Multiple random variables

- **Joint distribution:** $\mathbb{P}((X,Y) \in B) = \iint_B f_{X,Y}(x,y)\,dx\,dy$.
- **Marginal pdf:** $f_X(x) = \int f_{X,Y}(x,y)\,dy$.
- **Independent r.v.'s:** $f_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = \prod_{k=1}^n f_{X_k}(x_k)$.
- **Expected sum :** $\mathbb{E}(aX + bY) = a\,\mathbb{E}X + b\,\mathbb{E}Y$.
- **Expected product** (if X, Y independent): $\mathbb{E}[X\,Y] = \mathbb{E}X\,\mathbb{E}Y$.
- **Markov inequality:** $\mathbb{P}(X \geqslant x) \leqslant \frac{\mathbb{E}X}{x}$.
- **Covariance:** $\mathrm{cov}(X,Y) = \mathbb{E}(X - \mathbb{E}X)(Y - \mathbb{E}Y)$.
- **Properties of Var and Cov**:

$$\boxed{\begin{array}{l}
\mathrm{Var}(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2.\\
\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X).\\
\mathrm{cov}(X,Y) = \mathbb{E}XY - \mathbb{E}X\mathbb{E}Y.\\
\mathrm{cov}(X,Y) = \mathrm{cov}(Y,X).\\
\mathrm{cov}(aX+bY,Z) = a\,\mathrm{cov}(X,Z) + b\,\mathrm{cov}(Y,Z).\\
\mathrm{cov}(X,X) = \mathrm{Var}(X).\\
\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\,\mathrm{cov}(X,Y).\\
X \text{ and } Y \text{ independent} \Longrightarrow \mathrm{cov}(X,Y) = 0.
\end{array}}$$

- **Conditional pdf:** If $f_X(x) > 0$, $f_{Y\,|\,X}(y\,|\,x) := \frac{f_{X,Y}(x,y)}{f_X(x)}, \quad y \in \mathbb{R}$.
- The corresponding **conditional expectation**: $\mathbb{E}[Y \mid X = x] = \int y\,f_{Y\,|\,X}(y\,|\,x)dy$.
- $\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y \mid X]]$
- **Moment Generating Function (MGF)**: When it exists, for $t \in I \subset \mathbb{R}$, $M(t) = \mathbb{E}\,\mathrm{e}^{tX} = \int_{-\infty}^{\infty} \mathrm{e}^{tx}\,f(x)\,dx$.

- **MGFs for various distributions:**

$\mathsf{U}(a,b)$	$\frac{e^{bt}-e^{at}}{t(b-a)}$
$\mathsf{Exp}(\lambda)$	$\left(\frac{\lambda}{\lambda-t}\right)$
$\mathsf{N}(\mu,\sigma^2)$	$e^{t\mu+\sigma^2 t^2/2}$

- **Moment property**: $\mathbb{E}X^n = M^{(n)}(0)$.
- $M_{X+Y}(t) = M_X(t)\, M_Y(t)$, $\forall t$, if X, Y independent.
- If $X_i \sim \mathsf{N}(\mu_i, \sigma_i^2)$ (independent), then $a + \sum_{i=1}^n b_i X_i \sim \mathsf{N}\left(a + \sum_{i=1}^n b_i\, \mu_i,\ \sum_{i=1}^n b_i^2\, \sigma_i^2\right)$.
- Pdf of the **multivariate Normal** distribution:

$$f_{\boldsymbol{Z}}(\boldsymbol{z}) = \frac{1}{\sqrt{(2\pi)^n\,|\Sigma|}}\, e^{-\frac{1}{2}(\boldsymbol{z}-\boldsymbol{\mu})^T \Sigma^{-1}(\boldsymbol{z}-\boldsymbol{\mu})}\ .$$

  Σ is the covariance matrix, and $\boldsymbol{\mu}$ the mean vector.

- If $\mathbf{X}$ has a multivariate Normal distribution $\mathsf{N}(\boldsymbol{\mu}, \Sigma)$ (dimension n) and $\mathbf{Y} = \boldsymbol{a} + B\mathbf{X}$ (dimension $m \leqslant n$), then $\mathbf{Y} \sim \mathsf{N}(\boldsymbol{a} + B\boldsymbol{\mu}, B^T \Sigma B)$.
- **Central Limit Theorem**:

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{S_n - n\mu}{\sigma\sqrt{n}} \leqslant x\right) = \Phi(x),$$

  where Φ is the cdf of the standard Normal distribution.

- **Normal Approximation to Binomial:** If $X \sim \mathsf{Bin}(n,p)$, then, for large n, $\mathbb{P}(X \leqslant k) \approx \mathbb{P}(Y \leqslant k)$, where $Y \sim \mathsf{N}(np, np(1-p))$.

Statistics

Tests and Confidence Intervals Based on Standard Errors

- Test statistic: $\frac{\text{estimate}-\text{hypothesised}}{\text{se(estimate)}}$.
- Confidence interval:
  estimate $\pm$ (critical value) $\times$ se(estimate).
- $se(\bar{x}) = \frac{s}{\sqrt{n}}$
- $se(\bar{x} - \bar{y}) = s_p\sqrt{\frac{1}{n_x} + \frac{1}{n_y}}$
- (pooled sample variance) $s_p^2 = \frac{(n_x-1)s_x^2 + (n_y-1)s_y^2}{n_x+n_y-2}$.
- $se(\widehat{p}) = \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$
- $se(\widehat{p}_x - \widehat{p}_y) = \sqrt{\frac{\widehat{p}_x(1-\widehat{p}_x)}{n_x} + \frac{\widehat{p}_y(1-\widehat{p}_y)}{n_y}}$
- Use t-distribution for means, correlation and regression. Use normal distribution for proportions.

Chi-squared test

- expected count $= \frac{\text{(row total)}\times\text{(column total)}}{\text{overall total}}$.
- $X^2 = \sum \frac{(\text{observed}-\text{expected})^2}{\text{expected}}$
- degrees of freedom = (#rows − 1) × (#columns − 1).

Linear regression

- $\mathbf{Y} \sim \mathsf{N}(\mathbf{X}\beta, \sigma^2 I)$
- estimator $\widehat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$
- $\text{Cov}(\widehat{\beta}) = \sigma^2(\mathbf{X}^T\mathbf{X})^{-1}$
- $s^2 = \frac{(\mathbf{Y}-\mathbf{X}\widehat{\beta})^T(\mathbf{Y}-\mathbf{X}\widehat{\beta})}{n-p}$

Other Mathematical Formulas

- Factorial. $n! = n\,(n-1)\,(n-2)\cdots 1$. Gives the number of *permutations* (orderings) of $\{1,\ldots,n\}$.
- Binomial coefficient. $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$. Gives the number *combinations* (no order) of k different numbers from $\{1,\ldots,n\}$.
- Newton's binomial theorem: $(a+b)^n = \sum_{k=0}^n a^k\, b^{n-k}$.
- Geometric sum: $1 + a + a^2 + \cdots + a^n = \frac{1-a^{n+1}}{1-a}$ $(a \neq 1)$.
  If $|a| < 1$ then $1 + a + a^2 + \cdots = \frac{1}{1-a}$.
- Logarithms:
  1. $\log(x\,y) = \log x + \log y$.
  2. $e^{\log x} = x$.
- Exponential:
  1. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$.
  2. $e^x = \lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n$.
  3. $e^{x+y} = e^x\, e^y$.
- Differentiation:
  1. $(f+g)' = f' + g'$
  2. $(fg)' = f'g + fg'$
  3. $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
  4. $\frac{d}{dx}x^n = n\,x^{n-1}$
  5. $\frac{d}{dx}e^x = e^x$
  6. $\frac{d}{dx}\log(x) = \frac{1}{x}$
- Chain rule: $(f(g(x)))' = f'(g(x))\, g'(x)$.
- Integration: $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$, where $F' = f$.
- Integration by parts: $\int_a^b f(x)\,G(x)\,dx = [F(x)\,G(x)]_a^b - \int_a^b F(x)\,g(x)\,dx$. (Here $F' = f$ and $G' = f$.)

Table 12.1: Standard Normal distribution

	Second decimal place of z									
z	0	1	2	3	4	5	6	7	8	9
0.0	0.500	0.496	0.492	0.488	0.484	0.480	0.476	0.472	0.468	0.464
0.1	0.460	0.456	0.452	0.448	0.444	0.440	0.436	0.433	0.429	0.425
0.2	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.390	0.386
0.3	0.382	0.378	0.374	0.371	0.367	0.363	0.359	0.356	0.352	0.348
0.4	0.345	0.341	0.337	0.334	0.330	0.326	0.323	0.319	0.316	0.312
0.5	0.309	0.305	0.302	0.298	0.295	0.291	0.288	0.284	0.281	0.278
0.6	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
0.7	0.242	0.239	0.236	0.233	0.230	0.227	0.224	0.221	0.218	0.215
0.8	0.212	0.209	0.206	0.203	0.200	0.198	0.195	0.192	0.189	0.187
0.9	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
1.0	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.140	0.138
1.1	0.136	0.133	0.131	0.129	0.127	0.125	0.123	0.121	0.119	0.117
1.2	0.115	0.113	0.111	0.109	0.107	0.106	0.104	0.102	0.100	0.099
1.3	0.097	0.095	0.093	0.092	0.090	0.089	0.087	0.085	0.084	0.082
1.4	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
1.5	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
1.6	0.055	0.054	0.053	0.052	0.051	0.049	0.048	0.047	0.046	0.046
1.7	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037
1.8	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.030	0.029
1.9	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
2.0	0.023	0.022	0.022	0.021	0.021	0.020	0.020	0.019	0.019	0.018
2.1	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
2.2	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
2.3	0.011	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.008
2.4	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
2.5	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.6	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.7	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.8	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2.9	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
3.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.2	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.3										

This table gives $P(Z \geq z)$ for $Z \sim \text{Normal}(0, 1)$. Critical values of the Normal distribution, the z^* values such that $P(Z \geq z^*) = p$ for a particular p, can be found from the ∞ row of Table 14.2.

Table 14.2: Critical values of Student's T distribution

df	Probability p								
	0.25	0.10	0.05	0.025	0.01	0.005	0.001	0.0005	0.0001
1	1.000	3.078	6.314	12.71	31.82	63.66	318.3	636.6	3183.1
2	0.816	1.886	2.920	4.303	6.965	9.925	22.33	31.60	70.70
3	0.765	1.638	2.353	3.182	4.541	5.841	10.21	12.92	22.20
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610	13.03
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869	9.678
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959	8.025
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7.063
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041	6.442
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781	6.010
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.694
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.453
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318	5.263
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221	5.111
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.985
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.880
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.791
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965	4.714
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.648
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883	4.590
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850	4.539
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.493
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792	4.452
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768	4.415
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745	4.382
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.352
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707	4.324
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690	4.299
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674	4.275
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659	4.254
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.234
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.094
50	0.679	1.299	1.676	2.009	2.403	2.678	3.261	3.496	4.014
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460	3.962
70	0.678	1.294	1.667	1.994	2.381	2.648	3.211	3.435	3.926
80	0.678	1.292	1.664	1.990	2.374	2.639	3.195	3.416	3.899
90	0.677	1.291	1.662	1.987	2.368	2.632	3.183	3.402	3.878
100	0.677	1.290	1.660	1.984	2.364	2.626	3.174	3.390	3.862
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.719

This table gives t^* such that $P(T \geq t^*) = p$, where $T \sim \text{Student(df)}$.

Table 22.4: χ^2 distribution

	Probability p								
df	0.975	0.95	0.25	0.10	0.05	0.025	0.01	0.005	0.001
1	0.001	0.004	1.323	2.706	3.841	5.024	6.635	7.879	10.83
2	0.051	0.103	2.773	4.605	5.991	7.378	9.210	10.60	13.82
3	0.216	0.352	4.108	6.251	7.815	9.348	11.34	12.84	16.27
4	0.484	0.711	5.385	7.779	9.488	11.14	13.28	14.86	18.47
5	0.831	1.145	6.626	9.236	11.07	12.83	15.09	16.75	20.52
6	1.237	1.635	7.841	10.64	12.59	14.45	16.81	18.55	22.46
7	1.690	2.167	9.037	12.02	14.07	16.01	18.48	20.28	24.32
8	2.180	2.733	10.22	13.36	15.51	17.53	20.09	21.95	26.12
9	2.700	3.325	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	3.247	3.940	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	3.816	4.575	13.70	17.28	19.68	21.92	24.72	26.76	31.26
12	4.404	5.226	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	5.009	5.892	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	5.629	6.571	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	6.262	7.261	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	6.908	7.962	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	7.564	8.672	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	8.231	9.390	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	8.907	10.12	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	9.591	10.85	23.83	28.41	31.41	34.17	37.57	40.00	45.31
21	10.28	11.59	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	10.98	12.34	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	11.69	13.09	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	12.40	13.85	28.24	33.20	36.42	39.36	42.98	45.56	51.18
25	13.12	14.61	29.34	34.38	37.65	40.65	44.31	46.93	52.62
26	13.84	15.38	30.43	35.56	38.89	41.92	45.64	48.29	54.05
27	14.57	16.15	31.53	36.74	40.11	43.19	46.96	49.64	55.48
28	15.31	16.93	32.62	37.92	41.34	44.46	48.28	50.99	56.89
29	16.05	17.71	33.71	39.09	42.56	45.72	49.59	52.34	58.30
30	16.79	18.49	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	24.43	26.51	45.62	51.81	55.76	59.34	63.69	66.77	73.40
50	32.36	34.76	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	40.48	43.19	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	48.76	51.74	77.58	85.53	90.53	95.02	100.4	104.2	112.3
80	57.15	60.39	88.13	96.58	101.9	106.6	112.3	116.3	124.8
90	65.65	69.13	98.65	107.6	113.1	118.1	124.1	128.3	137.2
100	74.22	77.93	109.1	118.5	124.3	129.6	135.8	140.2	149.4

This table gives x^* such that $P(X^2 \geq x^*) = p$, where $X^2 \sim \chi^2(\text{df})$.