Multiply Random Variable.

$$\mathbb{E}[g(X,Y)] = \sum_x \sum_y g(x,y)\,\mathbb{P}(X=x, Y=y).$$

$$Cov(X,Y) = \mathbb{E}[(X-\mathbb{E}[X])(Y-\mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y] \cancel{\sum_x \sum_y x\cdot y\,\mathbb{P}(X=x,Y=}$$

$$Cov(X,X) = Var(X)$$

$$\mathbb{E}[Y|X_1] \neq \frac{\mathbb{E}[YX_1]}{\mathbb{E}[X_1]}$$

$$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$$

$$\mathbb{E}[X_1|X_1] = X_1$$

$$\mathbb{E}[XY] = \sum_x \sum_y x\cdot y\,\mathbb{P}(X=x, Y=y)$$

$$\mathbb{E}[Y^2|X] = \int_{min(y)}^{max(y)} y^2 f_{Y|X}(y|x)\,dy$$

$$\mathbb{E}[Y|X=x] = \sum_y y\,\mathbb{P}(Y=y|X=x) = \sum_y y f_{Y|X}(y|x)$$

$$\mathbb{E}[Y^2] = \mathbb{E}[\mathbb{E}[Y^2|X]]$$

$$\mathbb{E}[Y] = \sum_x \mathbb{E}[Y|X=x]\,\mathbb{P}(X=x) = \mathbb{E}[\mathbb{E}[Y|X]]$$

MGF $\rightarrow M_X(s) = \mathbb{E}[e^{sX}] = \sum_{n=0}^{n} e^{sn}\,\mathbb{P}(X=n)$, $M_{X_1X_2}(s) = 1-p+p(\exp(\lambda(e^s-1)))$.

$M_{X_1+X_2}(s) = \exp((\lambda+\lambda)(e^s-1))$

$M'(0) = \mathbb{E}[X]$ $\quad X_1 \sim Bernoulli(p)$ $\quad M_{X_1}(s) = 1-p+pe^s$

$M''(0) = \mathbb{E}[X^2]$ $\quad X_2 \sim Poisson(\lambda)$ $\quad M_{X_2}(s) = \exp(\lambda(e^s-1))$.

$$M_X(s) = \mathbb{E}[e^{sX}] = \mathbb{E}[e^{sX}|Y]\times\mathbb{P}(Y) + \mathbb{E}[e^{Xs}|Y^c]\times\mathbb{P}(Y^c)$$

$(u+v)' = u'+v'$	$(x^a)' = ax^{a-1}$
$(u-v)' = u'-v'$	$(e^x)' = e^x$
$(cu)' = cu'$	$(\ln x)' = \frac{1}{x}$
$(uv)' = uv'+u'v$	$f[g(x)] = f'[g(x)]\,g'(x) = f'(u)\,g'(x)$
$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$	eg $(\sin^2 x)' = 2\sin x\cos x$

$X \sim Geometric(p)$ $\quad \rightarrow M_Y(s)$

$Y \sim Binomial(X, p)$

$$M_X(s) = \frac{pe^s}{1-(1-p)e^s} \Rightarrow M_Y(s) = \frac{p(1-p+pe^s)}{1-(1-p)(1-p+pe^s)}$$

$$M_Y(s) = (1-p+pe^s)^X$$