Exam information	
Course code and name	STAT7203 Applied Probability and Statistics
Semester	Semester 2, 2020
Exam type	Online, non-invigilated
Exam date and time	Please refer to your personalised timetable
Exam duration	Working time: 120 minutes + additional online allowance: 30 minutes. TOTAL exam duration: 2 hrs 30 minutes from the advertised exam commencement time.
Additional time	30 minutes additional time has been incorporated in recognition of the online environment and the different circumstances that students face in their home environments. This includes time for download and upload, and allowances for network or connection issues.
Reading time	Reading time has not been formally allocated for online exams, however students are encouraged to review and plan their approach for the exam before they start. The total exam time should be sufficient to do this.
Exam window	You must commence your exam at the time listed in your personalised timetable. The exam will remain open **only** for the duration of the exam.
Weighting	This exam is weighted at 55% of your total mark for this course.
Permitted materials	This is a Closed Book examination - specified written materials permitted. Materials permitted are: your own notes and any material provided on the STAT7203 Blackboard are permitted. RStudio, MATLAB or a Casio FX82 series or UQ approved calculator may be used for calculation. A bilingual dictionary may be used if needed. You may not make use of any other material. This includes web-sites, books, or any other material or software.
Instructions	There are **55** marks available on this exam from **6** questions. Answer all questions. Show your working and state your conclusions, where appropriate. You can print the exam and write in the exam paper, or write your answers on blank paper (clearly label your solutions so that it is clear which problem it is a solution to), or annotate an electronic file on a suitable device. Scan or photograph your work if necessary and upload your answers to Blackboard as a single PDF file, before the end of the allowed time. You should include your **name and student number** on the first page of the file that you submit.
Who to contact	If you have any concerns or queries about a particular question, or need to make any assumptions to answer the question, state these at the start of your solution to that question. You may also include queries you may have made with respect to a particular question, should you have been able to 'raise your hand' in an examination room. If you experience any **technical difficulties** during the exam, contact the Library AskUs service for advice (open 7am–10pm, 7 days a week, Brisbane time): Chat: https://support.my.uq.edu.au/app/chat/chat_launch_lib/p/45 Phone: +61 7 3506 2615 Email: examsupport@library.uq.edu.au

	You should also ask for an email documenting the advice provided so you can provide this on request. In the event of a **late submission**, you will be required to submit evidence that you completed the exam in the time allowed. We recommend you use a phone camera to take photos (or a video) of every page of your exam. Ensure that the photos are time-stamped. If you submit your exam after the due time then you should send details (including any evidence) to SMP Exams (exams.smp@uq.edu.au) as soon as possible after the end of the exam.
Important exam condition information	The normal academic integrity rules apply. You cannot cut-and-paste material other than your own work as answers. You are not permitted to consult any other person – whether directly, online, or through any other means – about any aspect of this assessment during the period that this assessment is available. If it is found that you have given or sought outside assistance with this assessment then that will be deemed to be cheating and will result in disciplinary action. By undertaking this online assessment you will be deemed to have acknowledged UQ's academic integrity pledge to have made the following declaration: *"I certify that my submitted answers are entirely my own work and that I have neither given nor received any unauthorised assistance on this assessment item"*.

1. [10 marks] Suppose X and Y are two independent random variables such that $X \sim \mathsf{Poisson}(2)$ and $Y \sim \mathsf{Binomial}(5, 0.2)$. Define $W = X + 2Y$.

   (a) Compute $\mathbb{E}[W]$. [2 marks]

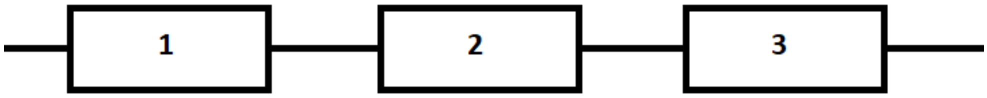
   (b) Compute $\mathrm{Var}(W)$. [2 marks]

   (c) Compute $\mathrm{Cov}(X, W)$. [2 marks]

   (d) Compute $\mathbb{E}[W|X]$. [2 marks]

   (e) Determine the moment generating function of W. [2 marks]

2. [3 marks] Consider the system below comprised of three components. The system is working if there is a path from left to right through working components. The components fail independently of one another and the time to failure (in years) for each component has an $\mathsf{Exponential}(2)$ distribution. Determine the probability that the system is working at time t.



3. [16 marks] A pair of random variables (X, Y) has a joint probability distribution in which Y has marginal probability density function

$$f_Y(y) = \begin{cases} 6y(1-y), & y \in (0,1) \\ 0, & \text{else} \end{cases}$$

and the conditional probability density function of X given $\{Y = y\}$ is uniform on the interval $(0, y)$.

(a) Write down the joint probability density function of (X, Y), clearly specifying the support of the distribution. [2 marks]

(b) Determine the marginal probability density function of X. [3 marks]

(c) Using the formula $\mathbb{E}[XY] = \mathbb{E}\left[Y\mathbb{E}\left[X \mid Y\right]\right]$ or otherwise, compute $\mathrm{Cov}(X, Y)$.
[5 marks]

(d) Suppose $Z = -\log Y$ (where log denotes the natural logarithm). Determine the probability density function of Z, clearly specifying the support of the distribution of Z. [4 marks]

(e) The moment generating function of Z is

$$M_Z(s) = \frac{6}{6 - 5s + s^2}, \quad s < 2.$$

Using $M_Z(s)$ or otherwise, determine the expected value of Z. [2 marks]

4. [8 marks] A study investigated the effect of playing computer games on heart rate. Twenty four individuals were recruited into the study and randomly assigned to play either an M-rated game or a G-rated game, with twelve participants in each group. Each participant's heart rate was measured before and after playing the video game for 20 minutes. The G-rated video game group had an average change (after − before) in heart rate of 3.2 beats per minute (bpm) and sample standard deviation 4.5 bpm. The M-rated video game group had an average change (after − before) in heart rate of 6.9 beats per minute (bpm) and sample standard deviation 5.7 bpm.

   (a) Do M-rated video games raise the heart rate more than G-rated video games? State the null and alternative hypotheses, and use an appropriate test statistic to determine the P-value. Based on the statistical test, what do you conclude? [5 marks]

   (b) Construct a 95% confidence interval for the mean increase in heart rate after playing a G-rated video game for 20 minutes. [3 marks]

5. [8 marks] A study was conducted in Australia on household use of information technology. As part of this survey, 400 adults aged between 25 and 54 were asked if they have ever experienced loss or damage due to a computer virus.

   (a) Of the 211 males surveyed, 28 had experienced loss or damage due to a computer virus. Of the 188 females surveyed, 17 had experienced loss or damage due to a computer virus. Construct a 99% confidence interval for the difference in the proportion of males and females affected. [3 marks]

(b) The table below relates the respondent's age and whether they experienced loss or damage due to a computer virus.

Age	Yes	No
25 – 34	12	136
35 – 44	12	120
45 – 54	19	101

Based on this table, is there evidence of an association between age and experiencing loss or damage due to a computer virus?

[5 marks]

6. [10 marks] Twenty nine university students were recruited into a study on target acquisition times in 3D gaming environments. Each student was assigned a different target acquisition difficulty level as measured by the Index of Difficulty (IoD) and their average time taken to acquire the target was recorded (TargetTime). The data are displayed in the figure below together with the fitted least squares line.

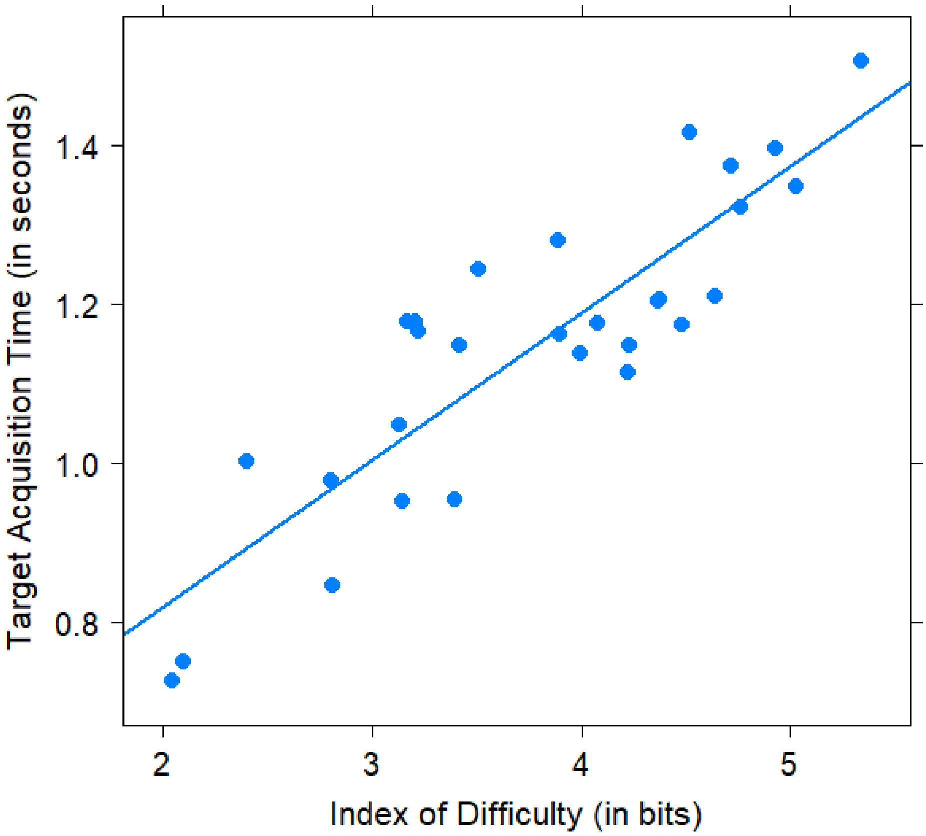


Figure 1: Plot of target acquisition times against index of difficulty.

The results of a linear regression fit for the relationship between TargetTime and IoD are given in the table below.

	Estimate	Std. Error
(Intercept)	0.42225	0.09725
IoD	0.19801	0.02526

(a) Briefly interpret the value 0.19801 in the regression output. [1 mark]

(b) Give a 95% confidence interval for the intercept term of the linear relationship between the time taken to acquire the target and IoD.
[2 marks]

(c) Does the data provide evidence of a relationship between IoD and the time taken to acquire the target? State the null and alternative hypotheses, determine the appropriate test statistic and provide a bound on the P-value. Based on the statistical test, what do you conclude? [4 marks]

(d) The following figures were generated to check the assumptions underlying the linear regression. State the assumptions of the linear regression model and comment on their validity for this data with reference to Figures 1 and 2.

[3 marks]

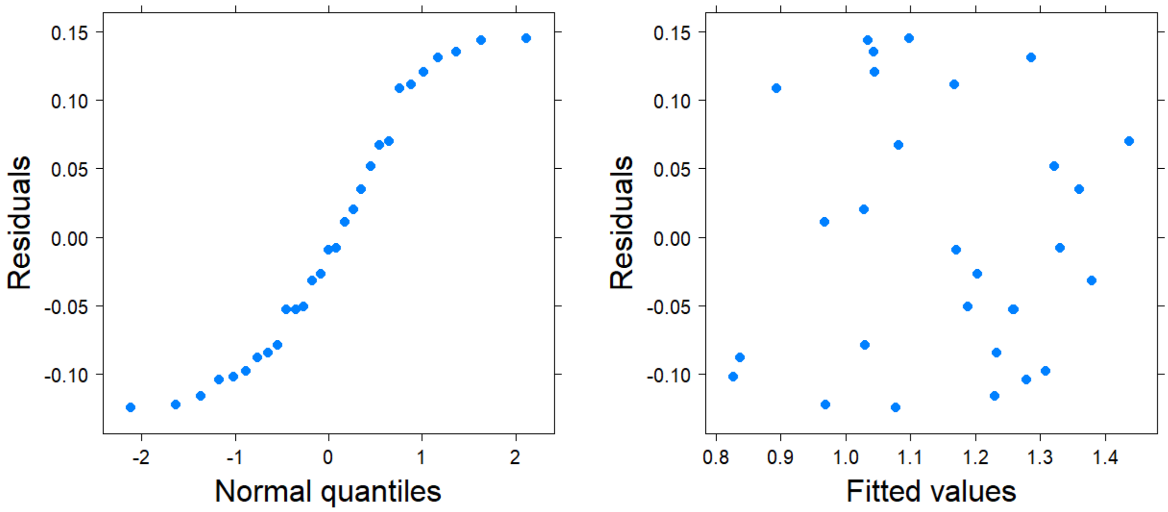


Figure 2: Left: Plot of residuals against quantiles of the standard normal distribution. Right: Plot of residuals against fitted values from the linear regression.

END OF EXAMINATION

Formula Sheet

Elementary probability

- **Sum rule:** For disjoint $A_1, A_2, \ldots$: $\mathbb{P}(\bigcup_i A_i) = \sum_i \mathbb{P}(A_i)$.
- $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$.
- $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$.
- **Conditional probability:** $\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$.
- **Law of total probability**: $\mathbb{P}(A) = \sum_{i=1}^n \mathbb{P}(A \mid B_i)\, \mathbb{P}(B_i)$, where $B_1, B_2, \ldots, B_n$ is a partition of Ω.
- **Bayes' Rule:** $\mathbb{P}(B_j|A) = \frac{\mathbb{P}(B_j)\, \mathbb{P}(A|B_j)}{\sum_{i=1}^n \mathbb{P}(B_i)\, \mathbb{P}(A|B_i)}$.
- **Independent events**: $\mathbb{P}(A \cap B) = \mathbb{P}(A)\, \mathbb{P}(B)$.

Random variables

- **Cdf** of X: $F(x) = \mathbb{P}(X \leqslant x)$, $x \in \mathbb{R}$.
- **Pmf** of X: (discrete r.v.) $f(x) = \mathbb{P}(X = x)$.
- **Pdf** of X: (continuous r.v.) $f(x) = F'(x)$.
- For a discrete r.v. X: $\mathbb{P}(X \in B) = \sum_{x \in B} \mathbb{P}(X = x)$.
- For a continuous r.v. X with pdf f: $\mathbb{P}(X \in B) = \int_B f(x)\, dx$.
- In particular (continuous), $F(x) = \int_{-\infty}^x f(u)\, du$.
- **Important discrete distributions**:

Distr.	pmf	suppport
$\mathsf{Ber}(p)$	$p^x(1-p)^{1-x}$	$\{0,1\}$
$\mathsf{Bin}(n,p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	$\{0,1,\ldots,n\}$
$\mathsf{Poi}(\lambda)$	$\mathrm{e}^{-\lambda} \frac{\lambda^x}{x!}$	$\{0,1,\ldots\}$
$\mathsf{Geom}(p)$	$p(1-p)^{x-1}$	$\{1,2,\ldots\}$

- **Important continuous distributions**:

Distr.	pdf	$x \in$
$\mathsf{U}[a,b]$	$\frac{1}{b-a}$	$[a,b]$
$\mathsf{Exp}(\lambda)$	$\lambda\, \mathrm{e}^{-\lambda x}$	$\mathbb{R}_+$
$\mathsf{N}(\mu,\sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}}\, \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$\mathbb{R}$

- **Expectation** (discr.): $\mathbb{E}X = \sum_x x\, \mathbb{P}(X = x)$.
- (of function) $\mathbb{E}\, g(X) = \sum_x g(x)\, \mathbb{P}(X = x)$.
- **Expectation** (cont.): $\mathbb{E}X = \int x\, f(x)\, dx$.
- (of function) $\mathbb{E}\, g(X) = \int g(x) f(x)\, dx$,
- $\mathbb{E}X$ **and** $\mathbf{Var}(X)$ **for discrete distributions**:

	$\mathbb{E}X$	$\mathrm{Var}(X)$
$\mathsf{Ber}(p)$	p	$p(1-p)$
$\mathsf{Bin}(n,p)$	np	$np(1-p)$
$\mathsf{Geom}(p)$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
$\mathsf{Poi}(\lambda)$	λ	λ

- $\mathbb{E}X$ **and** $\mathbf{Var}(X)$ **for continuous distributions**:

	$\mathbb{E}X$	$\mathrm{Var}(X)$
$\mathsf{U}(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$\mathsf{Exp}(\lambda)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$\mathsf{N}(\mu,\sigma^2)$	μ	σ^2

Multiple random variables

- **Joint distribution:** $\mathbb{P}((X,Y) \in B) = \iint_B f_{X,Y}(x,y)\, dx\, dy$.
- **Marginal pdf:** $f_X(x) = \int f_{X,Y}(x,y)\, dy$.
- **Independent r.v.'s:** $f_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = \prod_{k=1}^n f_{X_k}(x_k)$.
- **Expected sum :** $\mathbb{E}(aX + bY) = a\, \mathbb{E}X + b\, \mathbb{E}Y$.
- **Expected product** (if X, Y independent): $\mathbb{E}[X\, Y] = \mathbb{E}X\, \mathbb{E}Y$.
- **Markov inequality:** $\mathbb{P}(X \geqslant x) \leqslant \frac{\mathbb{E}X}{x}$.
- **Covariance:** $\mathrm{cov}(X,Y) = \mathbb{E}(X - \mathbb{E}X)(Y - \mathbb{E}Y)$.
- **Properties of Var and Cov**:

$$
\boxed{\begin{array}{l}
\mathrm{Var}(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2. \\
\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X). \\
\mathrm{cov}(X,Y) = \mathbb{E}XY - \mathbb{E}X\mathbb{E}Y. \\
\mathrm{cov}(X,Y) = \mathrm{cov}(Y,X). \\
\mathrm{cov}(aX+bY,Z) = a\,\mathrm{cov}(X,Z) + b\,\mathrm{cov}(Y,Z). \\
\mathrm{cov}(X,X) = \mathrm{Var}(X). \\
\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\,\mathrm{cov}(X,Y). \\
X \text{ and } Y \text{ independent} \Longrightarrow \mathrm{cov}(X,Y) = 0.
\end{array}}
$$

- **Conditional pdf:** If $f_X(x) > 0$, $f_{Y \mid X}(y \mid x) := \frac{f_{X,Y}(x,y)}{f_X(x)}$, $y \in \mathbb{R}$.
- The corresponding **conditional expectation**: $\mathbb{E}[Y \mid X = x] = \int y\, f_{Y \mid X}(y \mid x) dy$.
- $\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y \mid X]]$
- **Moment Generating Function (MGF)**: When it exists, for $t \in I \subset \mathbb{R}$, $M(t) = \mathbb{E}\, \mathrm{e}^{tX} = \int_{-\infty}^{\infty} \mathrm{e}^{tx}\, f(x)\, dx$.

- **MGFs for various distributions:**

$\mathsf{Bin}(n,p)$	$(1-p+pe^t)^n$
$\mathsf{Poi}(\lambda)$	$\exp(\lambda(e^t-1))$
$\mathsf{U}(a,b)$	$\frac{e^{bt}-e^{at}}{t(b-a)}$
$\mathsf{Exp}(\lambda)$	$\left(\frac{\lambda}{\lambda-t}\right)$
$\mathsf{N}(\mu,\sigma^2)$	$e^{t\mu+\sigma^2t^2/2}$

- **Moment property**: $\mathbb{E}X^n = M^{(n)}(0)$.
- $M_{X+Y}(t) = M_X(t)\,M_Y(t)$, $\forall t$, if X, Y independent.
- If $X_i \sim \mathsf{N}(\mu_i, \sigma_i^2)$ (independent), then $a + \sum_{i=1}^n b_i\,X_i \sim \mathsf{N}\left(a + \sum_{i=1}^n b_i\,\mu_i,\ \sum_{i=1}^n b_i^2\,\sigma_i^2\right)$.
- Pdf of the **multivariate Normal** distribution:

$$f_{\boldsymbol{Z}}(\boldsymbol{z}) = \frac{1}{\sqrt{(2\pi)^n\,|\Sigma|}}\,e^{-\frac{1}{2}(\boldsymbol{z}-\boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{z}-\boldsymbol{\mu})}\ .$$

  Σ is the covariance matrix, and $\boldsymbol{\mu}$ the mean vector.
- If $\mathbf{X}$ has a multivariate Normal distribution $\mathsf{N}(\boldsymbol{\mu},\Sigma)$ (dimension n) and $\mathbf{Y} = \boldsymbol{a} + B\mathbf{X}$ (dimension $m \leqslant n$), then $\mathbf{Y} \sim \mathsf{N}(\boldsymbol{a} + B\boldsymbol{\mu}, B\Sigma B^T)$.
- **Central Limit Theorem**:

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{S_n - n\mu}{\sigma\sqrt{n}} \leqslant x\right) = \Phi(x),$$

  where Φ is the cdf of the standard Normal distribution.
- **Normal Approximation to Binomial:** If $X \sim \mathsf{Bin}(n,p)$, then, for large n, $\mathbb{P}(X \leqslant k) \approx \mathbb{P}(Y \leqslant k)$, where $Y \sim \mathsf{N}(np, np(1-p))$.

Statistics

Tests and Confidence Intervals Based on Standard Errors

- Test statistic: $\frac{\text{estimate}-\text{hypothesised}}{\text{se(estimate)}}$.
- Confidence interval:
  estimate $\pm$ (critical value) $\times$ se(estimate).
- $se(\bar{x}) = \frac{s}{\sqrt{n}}$
- $se(\bar{x}-\bar{y}) = s_p\sqrt{\frac{1}{n_x} + \frac{1}{n_y}}$
- (pooled sample variance) $s_p^2 = \frac{(n_x-1)s_x^2 + (n_y-1)s_y^2}{n_x+n_y-2}$.
- $se(\widehat{p}) = \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$
- $se(\widehat{p}_x - \widehat{p}_y) = \sqrt{\frac{\widehat{p}_x(1-\widehat{p}_x)}{n_x} + \frac{\widehat{p}_y(1-\widehat{p}_y)}{n_y}}$
- Use t-distribution for means, correlation and regression. Use normal distribution for proportions.

Chi-squared test

- expected count $= \frac{\text{(row total)}\times\text{(column total)}}{\text{overall total}}$.
- $X^2 = \sum \frac{(\text{observed}-\text{expected})^2}{\text{expected}}$
- degrees of freedom = (#rows − 1) × (#columns − 1).

Linear regression

- $\mathbf{Y} \sim \mathsf{N}(\mathbf{X}\beta, \sigma^2 I)$
- estimator $\widehat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$
- $\mathrm{Cov}(\widehat{\beta}) = \sigma^2(\mathbf{X}^T\mathbf{X})^{-1}$
- $s^2 = \frac{(\mathbf{Y}-\mathbf{X}\widehat{\beta})^T(\mathbf{Y}-\mathbf{X}\widehat{\beta})}{n-p}$

Other Mathematical Formulas

- Factorial. $n! = n\,(n-1)\,(n-2)\cdots 1$. Gives the number of *permutations* (orderings) of $\{1,\dots,n\}$.
- Binomial coefficient. $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$. Gives the number *combinations* (no order) of k different numbers from $\{1,\dots,n\}$.
- Newton's binomial theorem: $(a+b)^n = \sum_{k=0}^n a^k\,b^{n-k}$.
- Geometric sum: $1 + a + a^2 + \cdots + a^n = \frac{1-a^{n+1}}{1-a}$ $(a \neq 1)$.
  If $|a| < 1$ then $1 + a + a^2 + \cdots = \frac{1}{1-a}$.
- Logarithms:
  1. $\log(x\,y) = \log x + \log y$.
  2. $e^{\log x} = x$.
- Exponential:
  1. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$.
  2. $e^x = \lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n$.
  3. $e^{x+y} = e^x\,e^y$.
- Differentiation:
  1. $(f+g)' = f' + g'$
  2. $(fg)' = f'g + fg'$
  3. $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
  4. $\frac{d}{dx}x^n = n\,x^{n-1}$
  5. $\frac{d}{dx}e^x = e^x$
  6. $\frac{d}{dx}\log(x) = \frac{1}{x}$
- Chain rule: $(f(g(x)))' = f'(g(x))\,g'(x)$.
- Integration: $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$, where $F' = f$.
- Integration by parts: $\int_a^b f(x)\,G(x)\,dx = [F(x)\,G(x)]_a^b - \int_a^b F(x)\,g(x)\,dx$. (Here $F' = f$ and $G' = g$.)

Standard Normal distribution

	Second decimal place of z									
z	0	1	2	3	4	5	6	7	8	9
0.0	0.500	0.496	0.492	0.488	0.484	0.480	0.476	0.472	0.468	0.464
0.1	0.460	0.456	0.452	0.448	0.444	0.440	0.436	0.433	0.429	0.425
0.2	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.390	0.386
0.3	0.382	0.378	0.374	0.371	0.367	0.363	0.359	0.356	0.352	0.348
0.4	0.345	0.341	0.337	0.334	0.330	0.326	0.323	0.319	0.316	0.312
0.5	0.309	0.305	0.302	0.298	0.295	0.291	0.288	0.284	0.281	0.278
0.6	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
0.7	0.242	0.239	0.236	0.233	0.230	0.227	0.224	0.221	0.218	0.215
0.8	0.212	0.209	0.206	0.203	0.200	0.198	0.195	0.192	0.189	0.187
0.9	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
1.0	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.140	0.138
1.1	0.136	0.133	0.131	0.129	0.127	0.125	0.123	0.121	0.119	0.117
1.2	0.115	0.113	0.111	0.109	0.107	0.106	0.104	0.102	0.100	0.099
1.3	0.097	0.095	0.093	0.092	0.090	0.089	0.087	0.085	0.084	0.082
1.4	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
1.5	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
1.6	0.055	0.054	0.053	0.052	0.051	0.049	0.048	0.047	0.046	0.046
1.7	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037
1.8	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.030	0.029
1.9	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
2.0	0.023	0.022	0.022	0.021	0.021	0.020	0.020	0.019	0.019	0.018
2.1	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
2.2	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
2.3	0.011	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.008
2.4	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
2.5	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.6	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.7	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.8	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2.9	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
3.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.2	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.3										

This table gives $P(Z \geq z)$ for $Z \sim \text{Normal}(0,1)$. Critical values of the Normal distribution, the z^* values such that $P(Z \geq z^*) = p$ for a particular p, can be found from the ∞ row of Table 14.2.

Critical values of Student's T distribution

	Probability p								
df	0.25	0.10	0.05	0.025	0.01	0.005	0.001	0.0005	0.0001
1	1.000	3.078	6.314	12.71	31.82	63.66	318.3	636.6	3183.1
2	0.816	1.886	2.920	4.303	6.965	9.925	22.33	31.60	70.70
3	0.765	1.638	2.353	3.182	4.541	5.841	10.21	12.92	22.20
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610	13.03
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869	9.678
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959	8.025
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7.063
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041	6.442
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781	6.010
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.694
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.453
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318	5.263
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221	5.111
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.985
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.880
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.791
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965	4.714
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.648
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883	4.590
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850	4.539
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.493
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792	4.452
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768	4.415
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745	4.382
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.352
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707	4.324
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690	4.299
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674	4.275
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659	4.254
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.234
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.094
50	0.679	1.299	1.676	2.009	2.403	2.678	3.261	3.496	4.014
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460	3.962
70	0.678	1.294	1.667	1.994	2.381	2.648	3.211	3.435	3.926
80	0.678	1.292	1.664	1.990	2.374	2.639	3.195	3.416	3.899
90	0.677	1.291	1.662	1.987	2.368	2.632	3.183	3.402	3.878
100	0.677	1.290	1.660	1.984	2.364	2.626	3.174	3.390	3.862
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.719

This table gives t^* such that $P(T \geq t^*) = p$, where $T \sim$ Student(df).

χ^2 distribution

	Probability p								
df	0.975	0.95	0.25	0.10	0.05	0.025	0.01	0.005	0.001
1	0.001	0.004	1.323	2.706	3.841	5.024	6.635	7.879	10.83
2	0.051	0.103	2.773	4.605	5.991	7.378	9.210	10.60	13.82
3	0.216	0.352	4.108	6.251	7.815	9.348	11.34	12.84	16.27
4	0.484	0.711	5.385	7.779	9.488	11.14	13.28	14.86	18.47
5	0.831	1.145	6.626	9.236	11.07	12.83	15.09	16.75	20.52
6	1.237	1.635	7.841	10.64	12.59	14.45	16.81	18.55	22.46
7	1.690	2.167	9.037	12.02	14.07	16.01	18.48	20.28	24.32
8	2.180	2.733	10.22	13.36	15.51	17.53	20.09	21.95	26.12
9	2.700	3.325	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	3.247	3.940	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	3.816	4.575	13.70	17.28	19.68	21.92	24.72	26.76	31.26
12	4.404	5.226	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	5.009	5.892	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	5.629	6.571	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	6.262	7.261	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	6.908	7.962	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	7.564	8.672	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	8.231	9.390	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	8.907	10.12	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	9.591	10.85	23.83	28.41	31.41	34.17	37.57	40.00	45.31
21	10.28	11.59	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	10.98	12.34	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	11.69	13.09	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	12.40	13.85	28.24	33.20	36.42	39.36	42.98	45.56	51.18
25	13.12	14.61	29.34	34.38	37.65	40.65	44.31	46.93	52.62
26	13.84	15.38	30.43	35.56	38.89	41.92	45.64	48.29	54.05
27	14.57	16.15	31.53	36.74	40.11	43.19	46.96	49.64	55.48
28	15.31	16.93	32.62	37.92	41.34	44.46	48.28	50.99	56.89
29	16.05	17.71	33.71	39.09	42.56	45.72	49.59	52.34	58.30
30	16.79	18.49	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	24.43	26.51	45.62	51.81	55.76	59.34	63.69	66.77	73.40
50	32.36	34.76	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	40.48	43.19	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	48.76	51.74	77.58	85.53	90.53	95.02	100.4	104.2	112.3
80	57.15	60.39	88.13	96.58	101.9	106.6	112.3	116.3	124.8
90	65.65	69.13	98.65	107.6	113.1	118.1	124.1	128.3	137.2
100	74.22	77.93	109.1	118.5	124.3	129.6	135.8	140.2	149.4

This table gives x^* such that $P(X^2 \geq x^*) = p$, where $X^2 \sim \chi^2(\text{df})$.