$X \sim \text{pdf } f_X(x)$

$Y = aX + b$

$f_Y(y) = \frac{1}{|a|} f(x)\left(\frac{y-b}{a}\right)$

$Y = g(X)$ g is increasing $\frac{d}{dy} F_Y(y) = y' \times (F_Y'(y)) = y' \cdot f_Y\, y$

$F_Y(y) = \mathbb{P}(g(X) \leq y) = \mathbb{P}(X \leq g^{-1}(y)) = F_X(g^{-1}(y))$

$\cancel{\frac{d}{dy} g^{-1}(y) = \frac{1}{g'(y)}}$

if g is decreasing $\Rightarrow$ $\cancel{\frac{d}{dy} F_Y(y) = \frac{1}{|g^{-1}(y)|}}$ $f_X\left(\frac{g^{-1}(y)}{u}\right)$

$\frac{d}{dy} F_Y(y) = \frac{d}{dy} F_X(g^{-1}(y))$

$= (g^{-1}(y))' F_X'(g^{-1}(y))$

$= (g^{-1}(y))' f_X(g^{-1}(y))$

---

quantile function of

$q_X(x) \rightarrow$ increasing function

format. $F_X(q_X(x)) = x$ $\quad u \sim U[0,1]$, $\mathbb{P}(U \leq x) = x$, $x \in (0,1)$

$F_{XY}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{XY}(x,y)\,dx\,dy$

$F_X(x) = \int_{-\infty}^{x}\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy$

$f(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$

independent X and Y

$F_{XY}(x,y) = F_X(x) \cdot F_Y(y)$

$f_{XY}(x,y) = f_X(x) \cdot f_Y(y)$

$Y = \frac{1}{X}$

$F_Y(y)$

$= \mathbb{P}(Y \leq y)$

$= \mathbb{P}(\frac{1}{X} \leq y)$

$= \mathbb{P}(\frac{1}{y} \leq X)$

$= \mathbb{P}(X \geq \frac{1}{y})$

$= 1 - \mathbb{P}(X < \frac{1}{y})$

$= 1 - \mathbb{P}_X(\frac{1}{y})$

$\mathbb{E}[aX + bY] = a\mathbb{E}[X] + b\mathbb{E}[Y]$

$Var(aX+b) = a^2 Var(X)$

$Var(X+Y) = Var(X) + Var(Y) + 2Cor(X,Y)$

$Cor(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$

X, Y. independent

$\mathbb{E}[XY] = \mathbb{E}[X] \cdot \mathbb{E}[Y]$

$Cor(X,Y) = 0$

$Var(X+Y) = Var(X) + Var(Y)$

$\rho(X,Y) = \frac{Cor(X,Y)}{\sqrt{Var(X)\,Var(Y)}}$

$\begin{bmatrix} U \\ V \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mathbb{E}[U] \\ \mathbb{E}[V] \end{bmatrix}, \begin{bmatrix} Var U & Cor UV \\ Cor VU & Var V \end{bmatrix}\right)$

$Y = 3 + 2X_1 - X_2$

$\mathbb{E}[Y] = 3 + 2\mathbb{E}[X_1] - \mathbb{E}[X_2] = 3 + 2\times(-1) - 1 = 0$

$Var(Y) = 2^2 Var(X_1) + (-1)^2 Var(X_2) + 2\times 2 \times (-1)\, Cor(X_1, X_2)$

$= 4\times 2 + 1\times 3 - 4\times 0 = 11$

$\therefore Y \sim Normal(0, 11)$

X has a multivariate Normal distribution

$Y = a + bX \quad Y \sim Normal(a + b\mu, b\Sigma b^T)$

$X_1 \sim Normal(-1, 2) \quad X_2 \sim Normal(1, 3)$

$Y = 3 + 2X_1 - X_2$

$$Y = \underset{a}{3} + \underset{b}{[2, -1]} \underset{X}{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}$$

$$X \sim Normal\left(\underset{\mu}{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}, \underset{\Sigma_{ij}}{\begin{bmatrix} \overset{\Sigma_{11}}{2} & \overset{\Sigma_{12}}{0} \\ \underset{\Sigma_{21}}{0} & \underset{\Sigma_{22}}{3} \end{bmatrix}}\right)$$

$\mu \sim$ mean

$\Sigma_{ij} = Cor(X_i, X_j)$ 如果 X independent $\Sigma_{ij} = 0$ $\Sigma_{ii} = Var(X_i) = \sigma^2$

$$\underset{b}{[2, -1]} \underset{\Sigma}{\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}} \underset{b^T}{\begin{bmatrix} 2 \\ -1 \end{bmatrix}}$$

$$Y \sim Normal\left(\underset{a}{3} + \underset{b}{[2, -1]} \underset{\mu}{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}, \ldots\right)$$

$(3 + (-2) + (-1), [4, -3]\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$Y \sim Normal(0, 11)$