STAT7203: Applied Probability and Statistics
Week 2 Questions

1. Warren Buffet challenges Bill Gates to a game of dice. Gates picks one of the following four sided dice, then Buffet chooses one of the remaining dice. Whoever rolls the highest number wins. The three dice have the following numbers on their faces:

   (die A): 12, 10, 3, 1

   (die B): 9, 8, 7, 2

   (die C): 11, 6, 5, 4

   P(MN)=(P(A)+P(B)+P(C))/3=(1+5/8+7/8)/3=5/6
   P(N)=1/4
   P(MIN)=?

   What is the probability of Buffett winning the game, assuming he chooses his die to maximise his probability of winning?[1]

2. We draw at random 5 numbers from 1,. . . 100, *with replacement* (for example, drawing number 9 twice is possible). What is the probability that exactly 3 numbers are even?

   (50*50*50*50*50)/(100*100*100*100*100)=1/32=0.03125

3. We draw at random 5 numbers from 1,. . . 100, *without replacement*. What is the probability that exactly 3 numbers are even?

   5P5*(50C3+{*}50C2)/100P5=0.00104134

4. Let Y be a number drawn at random from $\{1, 2, \ldots, 60\}$ (all numbers are equally likely to be drawn).

   (a) What is the probability that Y is divisible by 2? 30/60=1/2

   P(A)=//2=1/2,P(B)=//3=1/3
   P(AorB)=P(A)+P(B)-P(AB)=
   1/2+1/3-(1/2)*(1/3)=2/3

   (b) What is the probability that Y is divisible by 2 or 3?

   P(A)=1/2,P(B)=1/3,P(C)=1/5
   P(AorBorC)=P(A)+P(B)+P(C)
   -P(AB)-P(BC)-P(CA)+P(ABC)
   =11/15=0.733

   (c) What is the probability that Y is divisible by 2 or 3 or 5?

5. Let $(a_1, \ldots, a_n)$ be a random permutation of the integers $\{1, 2, \ldots, n\}$ with all permuations equally likely.

   (a) What is the probability that $a_1 = 1$? 1/nPn

   P(A)=1/nPn,P(B)=1/nPn
   P(AorB)=P(A)+P(B)-P(AB)=
   (2*nPn-1)/(nPn*nPn)

   (b) What is the probability that $a_1 = 1$ or $a_2 = 2$?

   P(A)=1/nPn,P(B)=1/nPn,P(C)=1/nPn
   P(AorBorC)=P(A)+P(B)+P(C)
   -P(AB)-P(BC)-P(CA)+P(ABC)
   =(3*nPn**2-3*nPn+1)/nPn**3

   (c) What is the probability that $a_1 = 1$ or $a_2 = 2$ or $a_3 = 3$?

6. Testing whether a multivariate polynomial is equal to zero is a standard problem in computational complexity. We will consider the following simpler setting. Let G be a polynomial of degree at most d and assume G is not identically zero. Let X be a number drawn at random from $\{1, 2, \ldots, 100d\}$, that is all numbers are equally likely to be drawn. Show that $\mathbb{P}(G(X) \neq 0) \geq 0.99$. (Hint: What does the fundamental theorem of algebra say?)

[1] Adapted from Janet Lowe (1998) *Bill Gates Speaks*, John Wiley & Sons