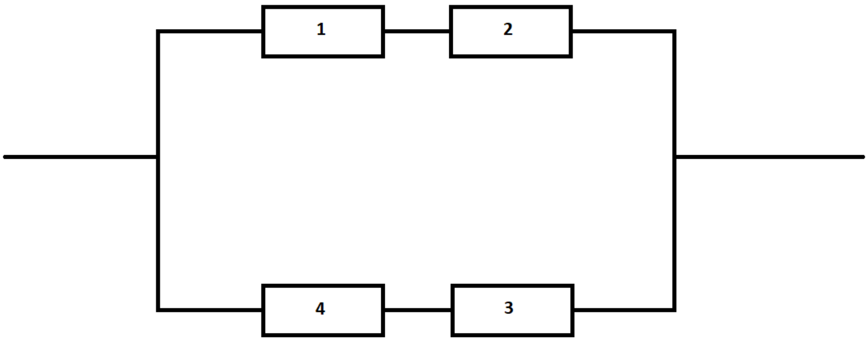
1. Consider the system below comprised of four components. The system is working if there is a path from left to right through working components. The components fail independently of one another. The probability that a given component is working after six months is 0.4.



The probability that the system is working at the end of six months

(a) 0.16

(b) 0.2944

(c) 0.4096

(d) 0.64

(e) 0.84

2. Let A and B be two disjoint events such that $\mathbb{P}(A) > 0$ and $\mathbb{P}(B) > 0$. Which of the following statements is FALSE:

(a) $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

(b) $\mathbb{P}(A^c \cup B^c) = 1$

(c) $\mathbb{P}(A \,|\, B) = 0$

(d) $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$

(e) $\mathbb{P}(A^c \cap B^c) = 1 - \mathbb{P}(A) - \mathbb{P}(B)$

3. A certain genetic disorder occurs in 2% of all newborns in a certain population. A new diagnostic test has been proposed. The new test is positive for the genetic disorder with 0.95 probability when the newborn has the disorder and negative for the disorder with 0.8 probability when the newborn does not have the disorder. The probability that a randomly selected newborn has the genetic disorder given the test is positive is approximately

   (a) 0.019
   (b) 0.024
   (c) 0.088
   (d) 0.093
   (e) 0.215

4. Suppose X is the number of correctly matched numbers in a game of three ball keno. The pmf of X is

x	0	1	2	3
$\mathbb{P}(X = x)$	0.42	0.43	0.14	0.01

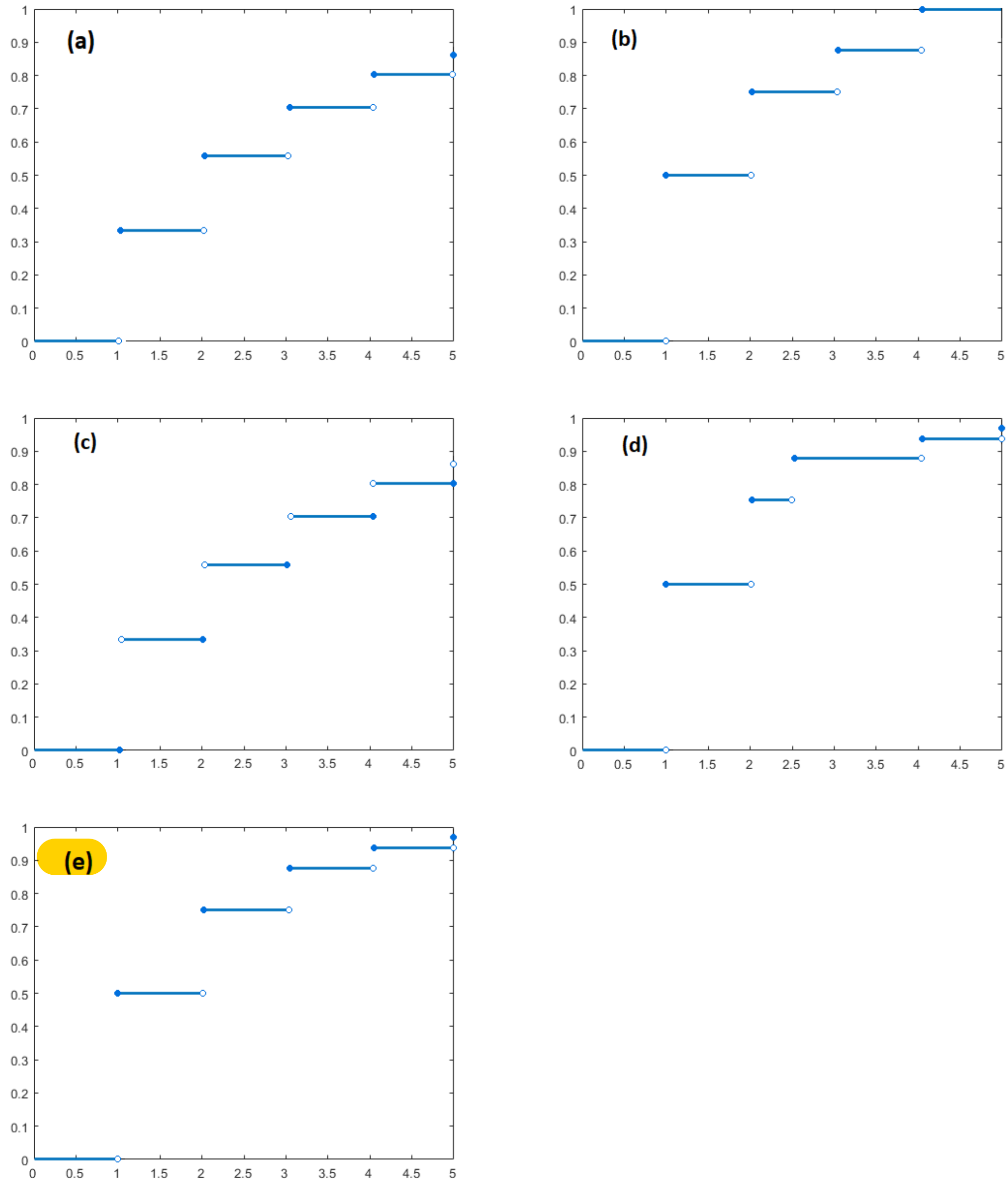
   The variance of the number of correctly matched numbers from one game of keno is

   (a) 0.5324
   (b) 0.652
   (c) 0.7297
   (d) 0.74
   (e) 1.08

5. Suppose that $X_1, X_2, \ldots, X_{10}$ are independent random variables each having a $\mathsf{Binomial}(5, \frac{2}{3})$ distribution. Let $Y = \frac{1}{10}\sum_{i=1}^{10} X_i$. The variance of Y is approximately

   (a) 0.011
   (b) 0.033
   (c) 0.11
   (d) 1.11
   (e) 3.33

6. Which of the following graphs best depicts the cumulative distribution function of a $\mathsf{Geometric}(\frac{1}{2})$ distribution?



7. Suppose two dice are rolled (one red and one blue). The probability that the face value shown on the blue die is greater than or equal to the face value shown on the red die is

   (a) $\frac{5}{12}$
   (b) $\frac{1}{2}$
   (c) $\frac{7}{12}$
   (d) $\frac{2}{3}$
   (e) $\frac{3}{4}$

8. Consider the pair of random variables (X, Y). Suppose that marginally $X \sim \mathsf{Binomial}(2, \frac{1}{2})$ and $Y \sim \mathsf{Uniform}(\{0, 1, 2\})$. If

$$\mathbb{P}(X = 0,\, Y = 0) = \mathbb{P}(X = 2,\, Y = 2) = \frac{1}{4},$$

   then $\mathbb{P}(X = 1, Y = 2)$ equals

   (a) 0
   (b) $\frac{1}{12}$
   (c) $\frac{1}{8}$
   (d) $\frac{1}{6}$
   (e) $\frac{1}{4}$

9. Let (A_1, A_2, A_3, A_4) be a random permutation of the integers $\{1, 2, 3, 4\}$ with all permutations equally likely. For $i < j$ define the random variables

$$X_{ij} = \begin{cases} 1, & \text{if } A_i > A_j \\ 0, & \text{otherwise.} \end{cases}$$

   Then $\text{Cov}(X_{12}, X_{23})$ equals

   (a) $-\frac{1}{12}$
   (b) 0
   (c) $\frac{1}{6}$
   (d) $\frac{1}{4}$
   (e) $\frac{1}{3}$

10. A manufacturing process produces a batch of 10 components. Each component in the batch may be faulty with probability $\frac{1}{5}$, independent of all other components in the batch. The method used to test whether a component is faulty is not perfect. If the component is working, then the test will identify the component as working with probability one. On the other hand, if the component is faulty, then the test will identify it as faulty with probability $\frac{5}{7}$.
Given that the test identifies no components in the batch as faulty, the expected number of faulty components in the batch is approximately

    (a) 0.571
    (b) 0.667
    (c) 1.4
    (d) 2
    (e) 2.857

11. Suppose $X \sim \mathsf{Bernoulli}(\frac{1}{2})$. Conditional on $\{X = 0\}$, Y has a $\mathsf{Poisson}(2)$ distribution and conditional on $\{X = 1\}$, Y has a $\mathsf{Poisson}(1)$ distribution. The probability generating function of Y is

    (a) $M_Y(s) = \frac{1}{2}(1 + \exp(3(e^s - 1)))$
    (b) $M_Y(s) = \frac{1}{2}(1 + e^s)\exp(3(e^s - 1))$
    (c) $M_Y(s) = \left(\frac{1}{2} + \frac{1}{2}e^s\right)^3$
    (d) $M_Y(s) = \exp(\frac{3}{2}(e^s - 1))$
    (e) $M_Y(s) = \frac{1}{2}(\exp(2(e^s - 1)) + \exp((e^s - 1)))$

12. Suppose the random variable X has probability generating function

$$M_X(s) = \exp(3(e^s + s - 1)).$$

Then $\mathrm{Var}(X)$ is

    (a) $\frac{1}{3}$
    (b) 1
    (c) 3
    (d) 6
    (e) 39

END OF EXAMINATION