STAT2203: Probability Models and Data Analysis for Engineering
Assignment 2

Due by 11:00 am on Tuesday the 24th of September, 2019
via the Electronic Assignment Submission System (62-225)

The marks for each question is indicate by the number in square brackets. There are a total of 12 marks for this assignment.

1. A continuous random variable X has probability density function

$$f_X(x) = c\exp(-2|x| + x) = \begin{cases} ce^{-x}, & x \geqslant 0 \\ ce^{3x}, & x \leqslant 0 \end{cases}$$

   (a) Determine the value of c. [1]

   (b) Determine the moment generating function $M_X(t)$ of X, remembering to state the valid range for t. [1]

   (c) Hence, or otherwise, determine the mean and variance of X. [2]

   (d) Define the random variable $Y := X^4$. Give the probability density function for Y. [2]

2. Let $X_1, X_2, \ldots$ be a sequence of independent random variables, each with a $\mathsf{Geometric}(1/2)$ distribution. Let N be a random variable with a $\mathsf{Geometric}(1/3)$ distribution, independent of $X_1, X_2, \ldots$. Define the random variable

$$Y = \sum_{i=1}^{N} X_i,$$

   where $Y = 0$ if $N = 0$. Determine the probability generating function of Y and identify its distribution. [2]

3. Suppose the random variable X has a $\mathsf{N}(2, 3)$ distribution. Conditional on $\{X = x\}$, the random variable Y has a $\mathsf{N}(1 + x, 2)$ distribution.

   (a) Detemine the probability that $X \geqslant 3$. [1]

   (b) Determine the marginal distribution of Y. [2]

   (c) Determine the correlation between X and Y. [1]

Total [12]