STAT7203 & STAT2203
Week 6 Exercises — Practice Mid-semester exam

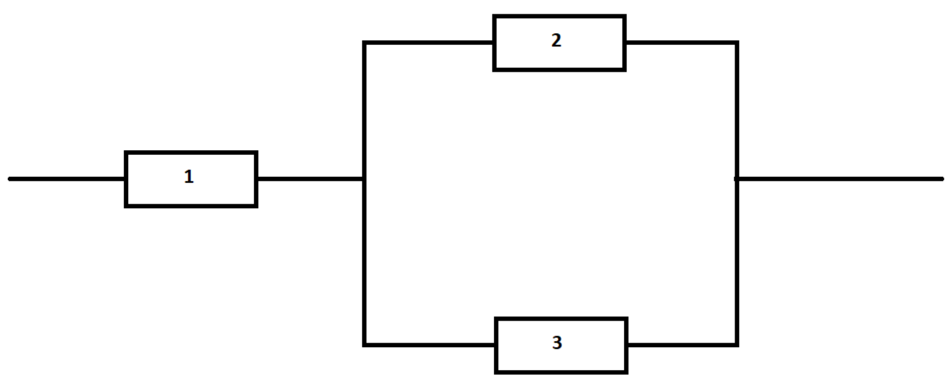
1. Let $(A_1, \ldots, A_4)$ be a random permutation of the integers $\{1, 2, 3, 4\}$ with all permutations equally likely. For $i < j$ define the random variables

$$X_{ij} = \begin{cases} 1, & \text{if } A_i > A_j \\ 0, & \text{otherwise.} \end{cases}$$

   Then $\mathbb{P}(X_{12} = 0, X_{13} = 0)$ equals

   (a) $\frac{1}{24}$
   (b) $\frac{1}{8}$
   (c) $\frac{1}{6}$
   (d) $\frac{1}{4}$
   (e) $\frac{1}{3}$

2. Consider the system below comprised of three components. The system is working if there is a path from left to right through working components. The components fail independently of one another. The probability that a given component has failed in six months is 0.4.



   The probability that the system is working at the end of six months

   (a) 0.256
   (b) 0.36
   (c) 0.504
   (d) 0.64
   (e) 0.84

3. A bank uses machine learning to detect credit card fraud. Their method correctly identifies a fraudulent transaction with 0.9 probability and it correctly identifies non-fraudulent transactions with 0.975 probability. Among the bank's customers fraudulent transactions account for 0.5% of all credit card transactions. The probability that a randomly selected transaction is fraudulent given it has been identified as fraudulent using the machine learning algorithm is approximately

   (a) 0.0045
   (b) 0.0046
   (c) 0.153
   (d) 0.48
   (e) 0.9

4. Suppose that X has the below probability mass function

x	0	1	2	3
$\mathbb{P}(X = x)$	0.3	0.5	0.1	0.1

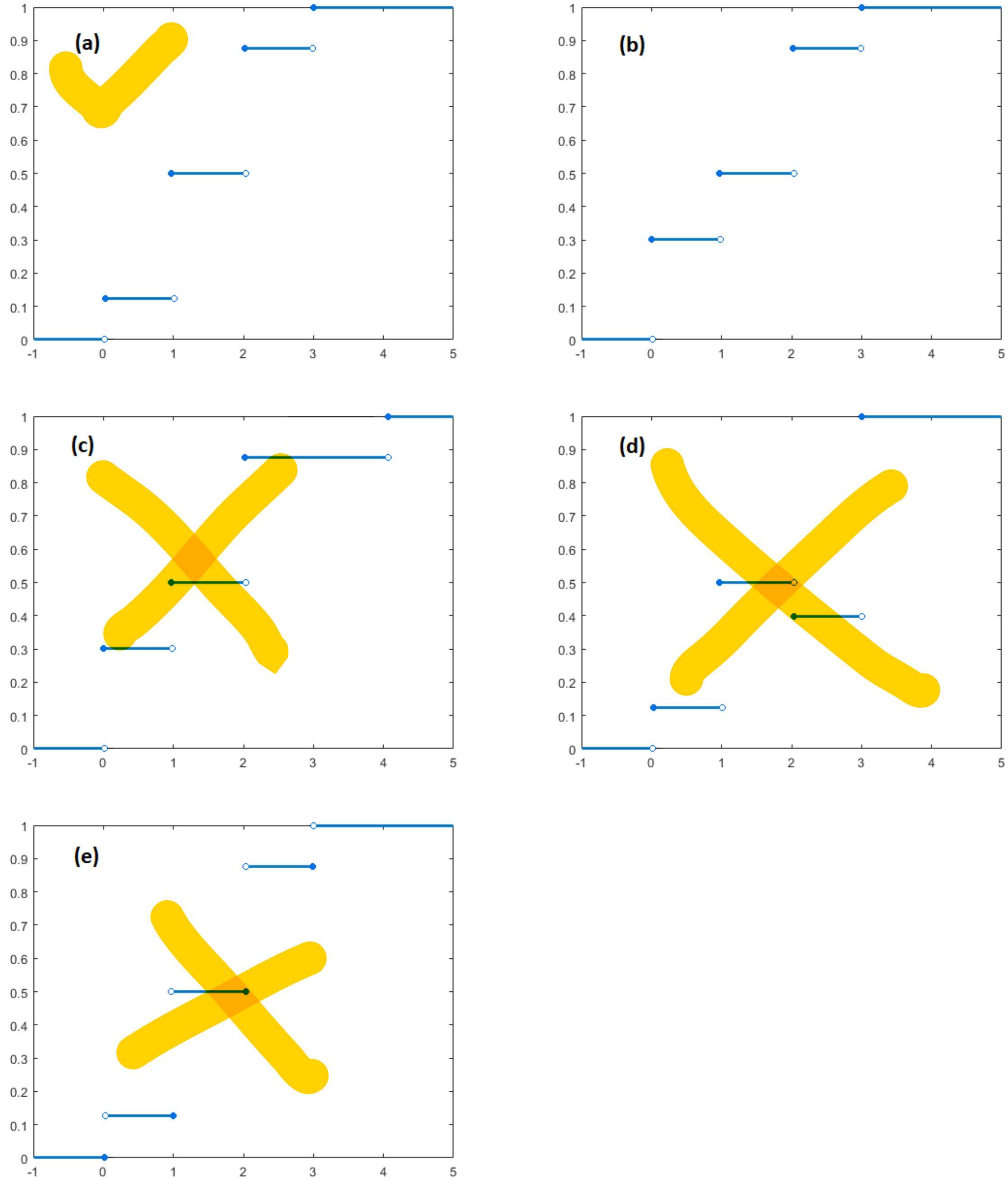
   The variance of X is

   (a) 0.8
   (b) 0.89
   (c) 1
   (d) 1.34
   (e) 1.8

5. Suppose that $X_1, X_2, \ldots, X_{20}$ are independent random variables each having a $\mathsf{Geometric}(\frac{1}{4})$ distribution. Let $Y = \frac{1}{20}\sum_{i=1}^{20} X_i$. The standard deviation of Y is approximately

   (a) 0.173
   (b) 0.6
   (c) 0.775
   (d) 2.683
   (e) 12

6. Which of the following graphs depicts the cumulative distribution function of a Binomial$(3, \frac{1}{2})$ distribution?



7. Suppose X_1 and X_2 are two independent random variables each having a uniform distribution on the integers $1, 2, \ldots, 6$. The probability that $X_1 - X_2 = 0$ given that $X_1 + X_2 = 6$ is

   (a) $\frac{1}{36}$
   (b) $\frac{1}{12}$
   (c) $\frac{1}{6}$
   (d) $\frac{1}{5}$
   (e) $\frac{1}{4}$

8. Consider the pair of random variables (X, Y). Suppose that marginally $X \sim \mathsf{Binomial}(2, \frac{1}{2})$ and $Y \sim \mathsf{Binomial}(2, \frac{3}{4})$. If $\mathbb{P}(X > Y) = 0$ and $\mathbb{P}(X = 0, Y = 2) = \frac{1}{16}$, then $\mathbb{P}(X = 1, Y = 1)$ equals

   (a) $\frac{1}{16}$
   (b) $\frac{1}{8}$
   (c) $\frac{3}{16}$
   (d) $\frac{1}{4}$
   (e) $\frac{5}{16}$

9. Suppose X_1, X_2 and X_3 independent random variables, each having a $\mathsf{Uniform}(\{1, 2, 3\})$ distribution. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 + X_3$. The covariance between Y_1 and Y_2 is

   (a) 0
   (b) 0.667
   (c) 2
   (d) 4
   (e) 4.667

10. Suppose $X_1 \sim \mathsf{Poisson}(\frac{3}{2})$ and $X_2 \sim \mathsf{Poisson}(\frac{5}{2})$ are two independent random variables. Let $Y = X_1 + X_2$. The conditional expectation $\mathbb{E}[X_1 \mid Y = 3]$ is

   (a) 1
   (b) 1.125
   (c) 1.25
   (d) 1.375
   (e) 1.5

11. Suppose X and Y are two independent random variables such that $X \sim \mathsf{Bernoulli}(p)$ and $Y \sim \mathsf{Poisson}(\lambda)$. Define $Z = XY$. Then the moment generating function of Z is:

    (a) $M_Z(s) = \exp(p\lambda(e^s - 1))$
    (b) $M_Z(s) = 1 - p + p\exp(\lambda(e^s - 1))$
    (c) $M_Z(s) = (1 - p + ps)\exp(\lambda(e^s - 1))$
    (d) $M_Z(s) = (1 - p + pe^s)^\lambda$
    (e) $M_Z(s) = \left(\frac{pe^s}{1-p+pe^s)}\right)^\lambda$

12. Suppose the random variable X has probability generating function

$$M_X(s) = \frac{1 + e^s}{4 - 2e^s}.$$

    Then the expected value of X is:

    (a) -1
    (b) 0.375
    (c) 1
    (d) 1.5
    (e) 3