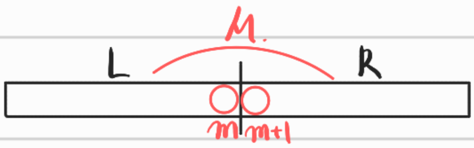
분할정복 과제 설명

최대구간합.



(i) L에서의 최대구간
(ii) R에서의 최대구간
(iii) M에서의 최대구간

} max (L, R, M).

rotating List → 교재 확인.

정렬된 2차원 배열에서의 K 탐색. (p. 67)

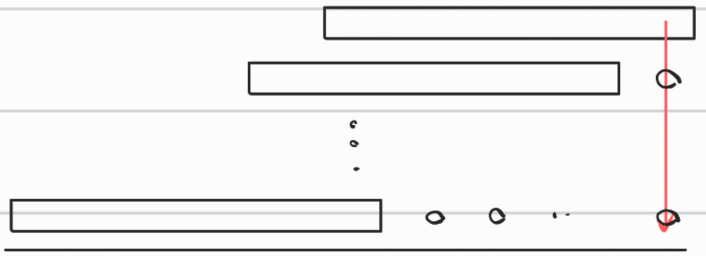
3	5	9	12
6	8 x	10	15
11	15	18 y	21
13	17	20	25

(n×n)
n=4.

큰 수 곱셈

- School method $O(n^2)$

$$\begin{array}{r} a_1 \quad a_2 \quad \cdots \quad a_n \\ \times \quad b_1 \quad b_2 \quad \cdots \quad b_n \\ \hline \end{array}$$



⊗ n회 ⊕ n회

- 분할정복 : n 자리 × n 자리

$u = x\cdot 10^{n/2} + y$ $\quad$ $v = w \times 10^{n/2} + z$

$$u \times v = \underbrace{\boxed{x\cdot w}}_{n/2 \;\; n/2} \cdot 10^{n} + (\boxed{x\cdot z} + \boxed{w\cdot y})\cdot 10^{n/2} + \boxed{y\cdot z}$$

최대 n번의 덧셈 연산

$T(n)$ = n 자리 × n 자리에 필요한 기본 연산 횟수.

$$= 4\cdot T(n/2) + cn$$

$$= 4\left(4\cdot T(n/2^2) + c\cdot \frac{n}{2}\right) + cn$$

$$= 4^2\cdot T(n/2^2) + (2cn + cn)$$

$$\cdots$$

$$= 4^k\cdot T(n/2^k) + cn\,(1 + 2^1 + 2^2 + \cdots + 2^{k-1})$$

$$= c\cdot 4^k + cn\left(\frac{2^k - 1}{2-1}\right) \le c\cdot 4^k + cn\cdot 2^k \leftarrow n = 2^k$$

$$\rightarrow O(4^k) = O(n^{\log_2 4}) = O(n^2)$$

- Karatsuba (개선)

$$u \times v = \boxed{\underset{n/2\ \ n/2}{x \cdot w}} \cdot 10^{n} + \boxed{(\boxed{x \cdot z} + \boxed{w \cdot y})} \cdot 10^{n/2} + \boxed{y \cdot z}$$

$$\uparrow$$

$$\underbrace{(\underset{n/2+1}{x+y})(\underset{n/2+1}{w+z})}_{T(n/2+1)} - xw - yz$$

$$T(n) = 2T(n/2) + T(n/2 \underbrace{+1}_{\text{k}\times}) + cn$$

(∵) Big-O에 영향 같기 때문.

$$= 3 \cdot T(n/2) + cn.$$

$$= 3\left(3 \cdot T(n/2^2) + c \cdot \frac{n}{2}\right) + cn$$

$$= 3^2\, T(n/2^2) + \frac{3}{2} cn + cn$$

$$\cdots$$

$$= 3^k \cdot T(n/2^k) + cn\left(1 + \frac{3}{2} + \left(\frac{3}{2}\right)^2 + \cdots + \left(\frac{3}{2}\right)^{k-1}\right).$$

$$= c \cdot 3^k + cn\left(\frac{(3/2)^k - 1}{3/2 - 1}\right) \le c \cdot 3^k + \cancel{cn}\left(\frac{3^k}{\cancel{2^k}}\right) \to O(3^k) = O(n^{\log_2 3})$$

$n = 2^k$

$$= O(n^{1.5\cdots})$$

⊛ Let $a \in \mathbb{N}$

$$T(n) = a\,T(n/2) + cn. \to O(n^{\log_2 a})$$

→ Master Theorem