Stochastic Processes 160B, Week 3

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Part A

Without the Lambda function, we use the first method discussed in class to create a time non-homogeneous Poisson process. After generating the process, my program prints the value of $N(t)$ at $t = 0, \ldots, 10$ to give the user an understanding of the function's shape:

```
N(t) represents the number of events before time t in a non-homogeneous Poisson process
with mean function m(t) = 4 log(1 + t).


N(0) =  0
N(1) =  4
N(2) =  7
N(3) =  9
N(4) =  9
N(5) =  10
N(6) =  10
N(7) =  12
N(7) =  12
N(8) =  12
N(9) =  14
N(10) =  15
```

Part B

For each of 10,000 trials, the program re-randomizes the number of events and the times of their occurrences. Using numpy, we can store the first-event-times in a list and find their mean and variance:

```
In 10,000 trials, the average time of the first event was  0.38442 .

The variance seems to be  0.2119732636 .
```

Part C

We approach Part C with the same methodology as Part A, but we must simulate more of the uniform distribution used to generate $N(t)$. This is because while $4 \times \log(1+t)$ on $t = (0, 10)$ ranges from 0 to about 4.16, $t^2 + 2t$ ranges from 0 to 120; we need to simulate 120 time units of homogeneous distribution to simulate 10 time units non-homogeneous distribution. This gives me a better understanding of how this method works; we're dilating or contracting the time variable to impose a structure on the uniform data.

```
N(t) represents the number of events before time t in a non-homogeneous Poisson process
with mean function m(t) = t^2 + 2t.


N(0) =  0
N(1) =  5
N(2) =  12
N(3) =  19
N(4) =  24
N(5) =  35
N(6) =  49
N(7) =  66
N(7) =  66
N(8) =  77
N(9) =  92
N(10) =  112
```

Part D

To see how many trials would be sufficient, I ran 10,000 trials. As repeated compilations of the program give somewhat consistent results, I decided that was enough:

```
In 10,000 trials,  216  processes had exactly 5 events between time 4 and time 5.
That's  0.0216  of the trials.
```