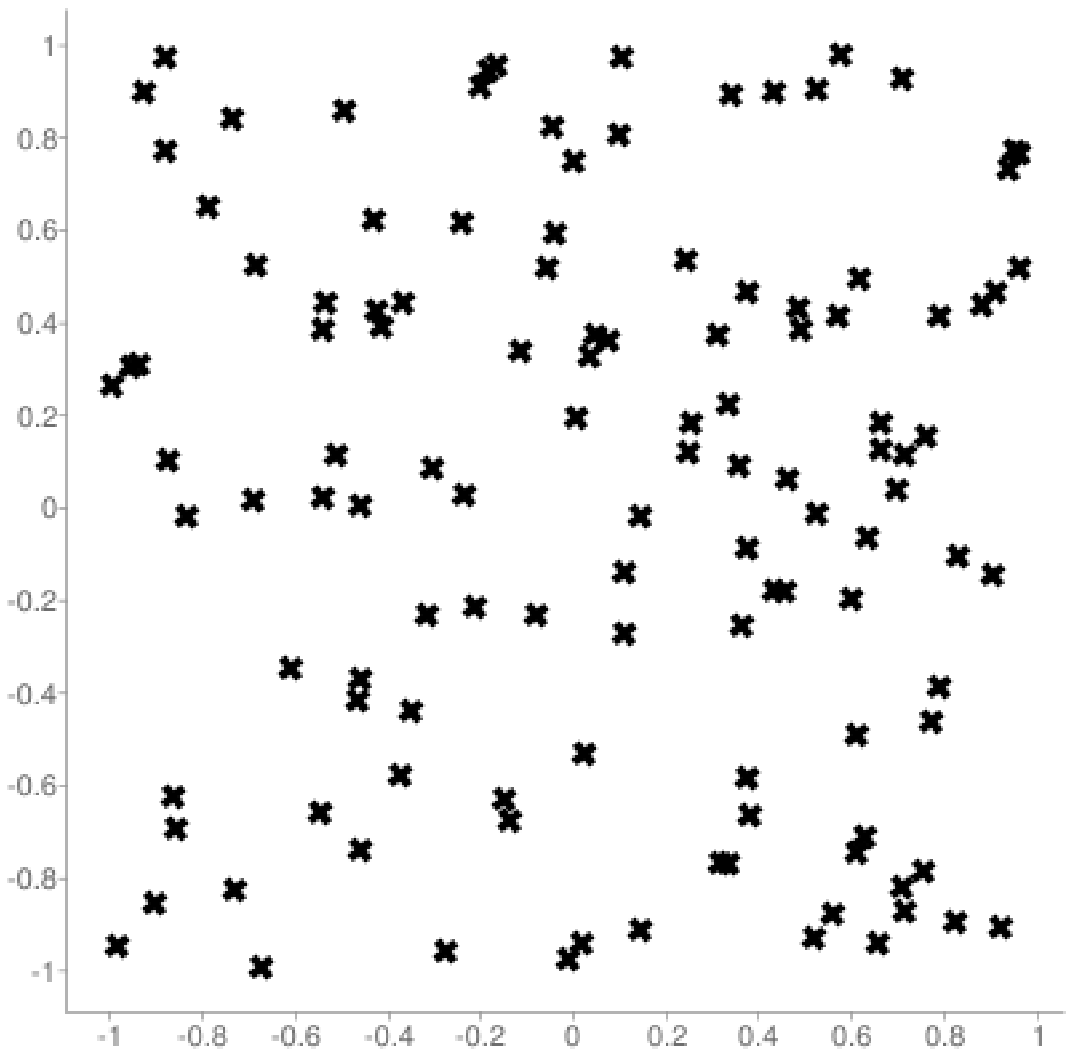
Stochastic Processes 160B, Week 3

Ted Tinker, 3223468

Feburary 2, 2017

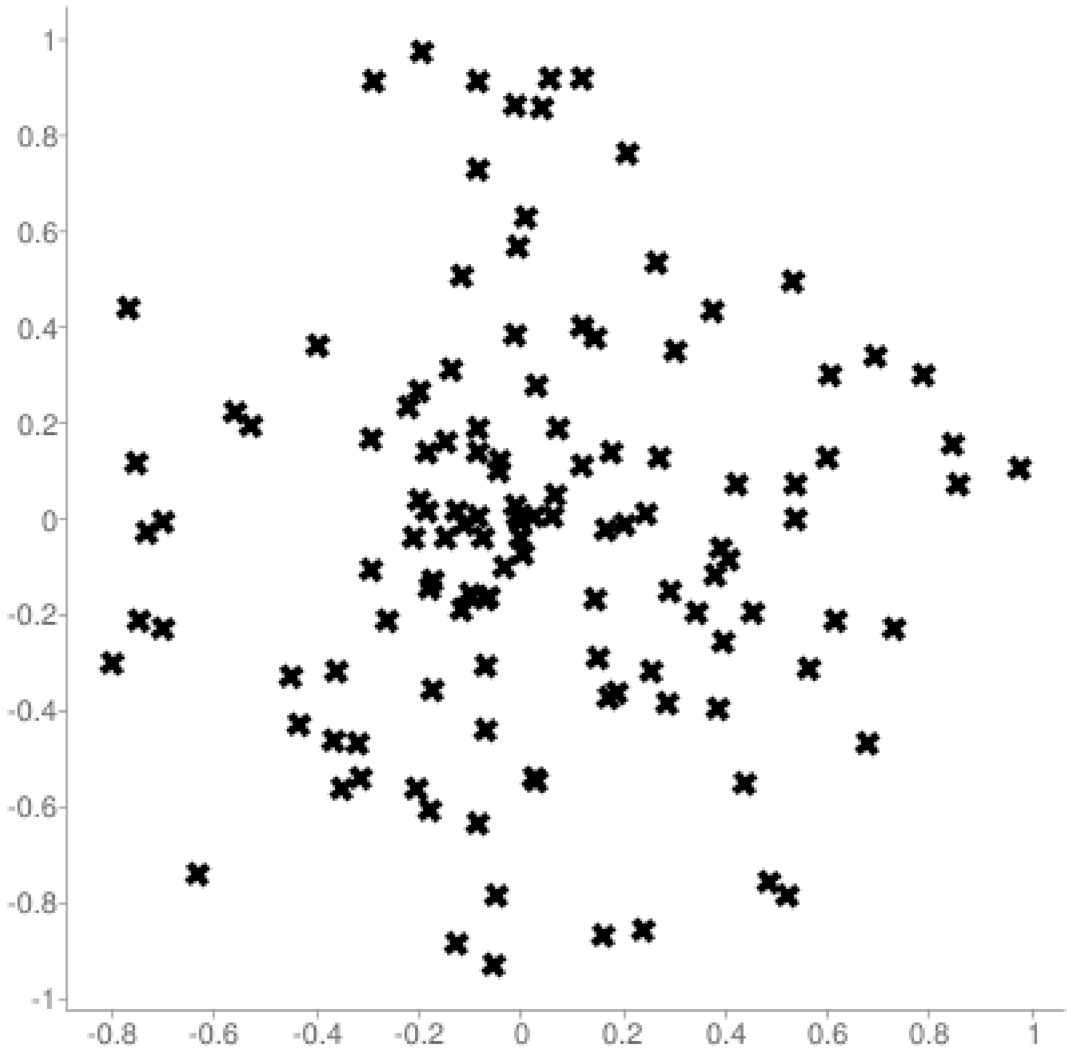
Part A

For this section, my generates a Poisson random variable of parameter 120 to determine how many Poisson Points to generate. This means $\lambda = 30$, as the region $[-1, 1] \times [-1, 1]$ has area of four units. Then it generates that many points, with their x and y positions uniformly random over the intervals. This is a scatterplot of the points my program generated:



Part B

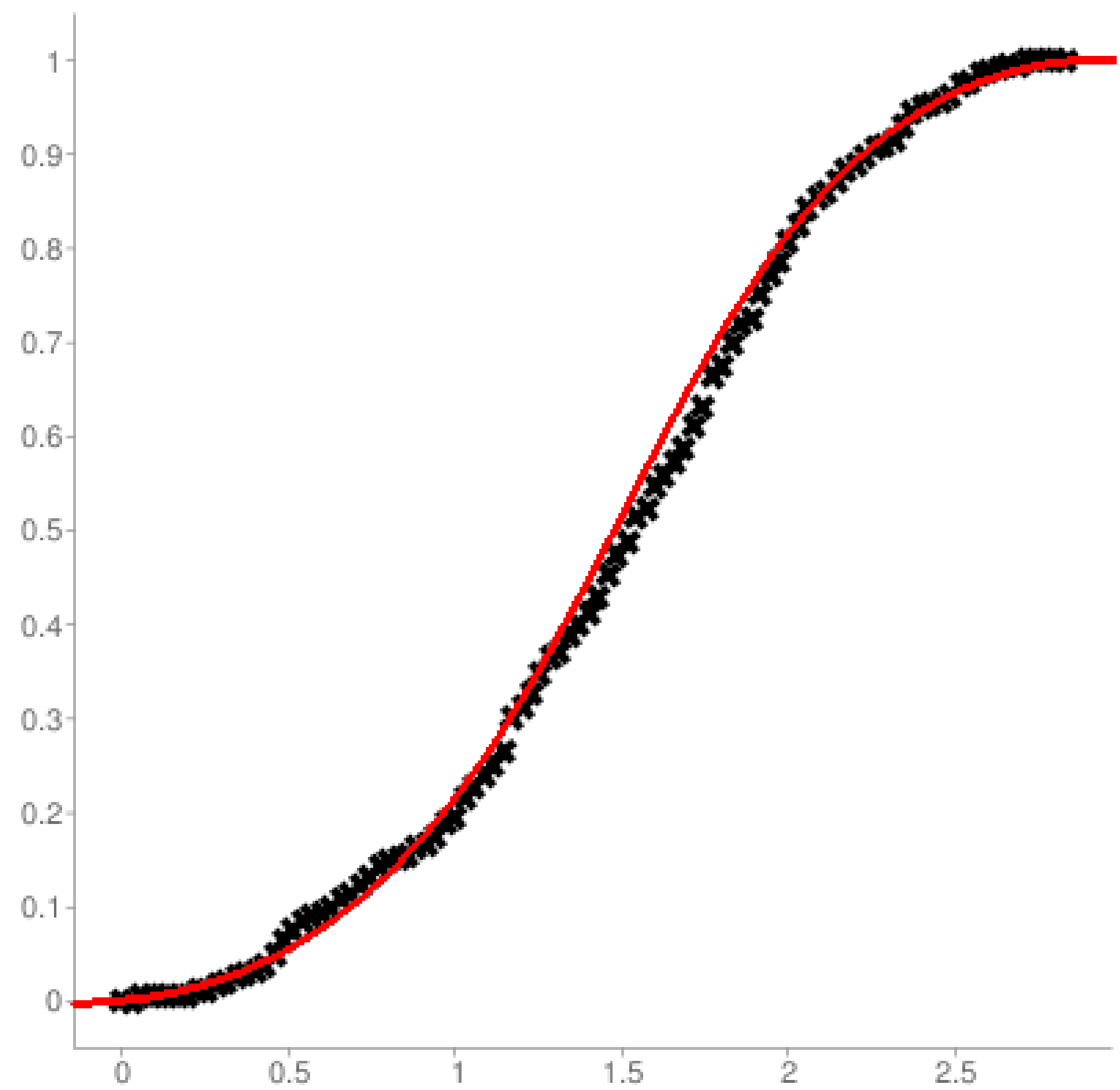
Part B works similarly to Part A, except that it generates values uniformly from $[0, 1]$ for the unit radius and $[0, 2\pi]$ for the angle. With the radius and angle, we convert from polar to cartesean coordinates and graph them:



In comparison to the other scatterplot, we can see a general circular shape.

Part C

In order to simulate a process "on the whole plane," we use the setup in part A, but over the region $[-2, 2] \times [-2, 2]$. This results in a number of events equal to a Poisson random variable with parameter 480, generating a Point Poisson process with parameter $480/16 = 30$. Also, instead of holding the actual coordinates of the points, we store the distance from each point to the origin. The graph below shows the proportion of values below x for x being 100 values between 0 and $2\sqrt{2}$:



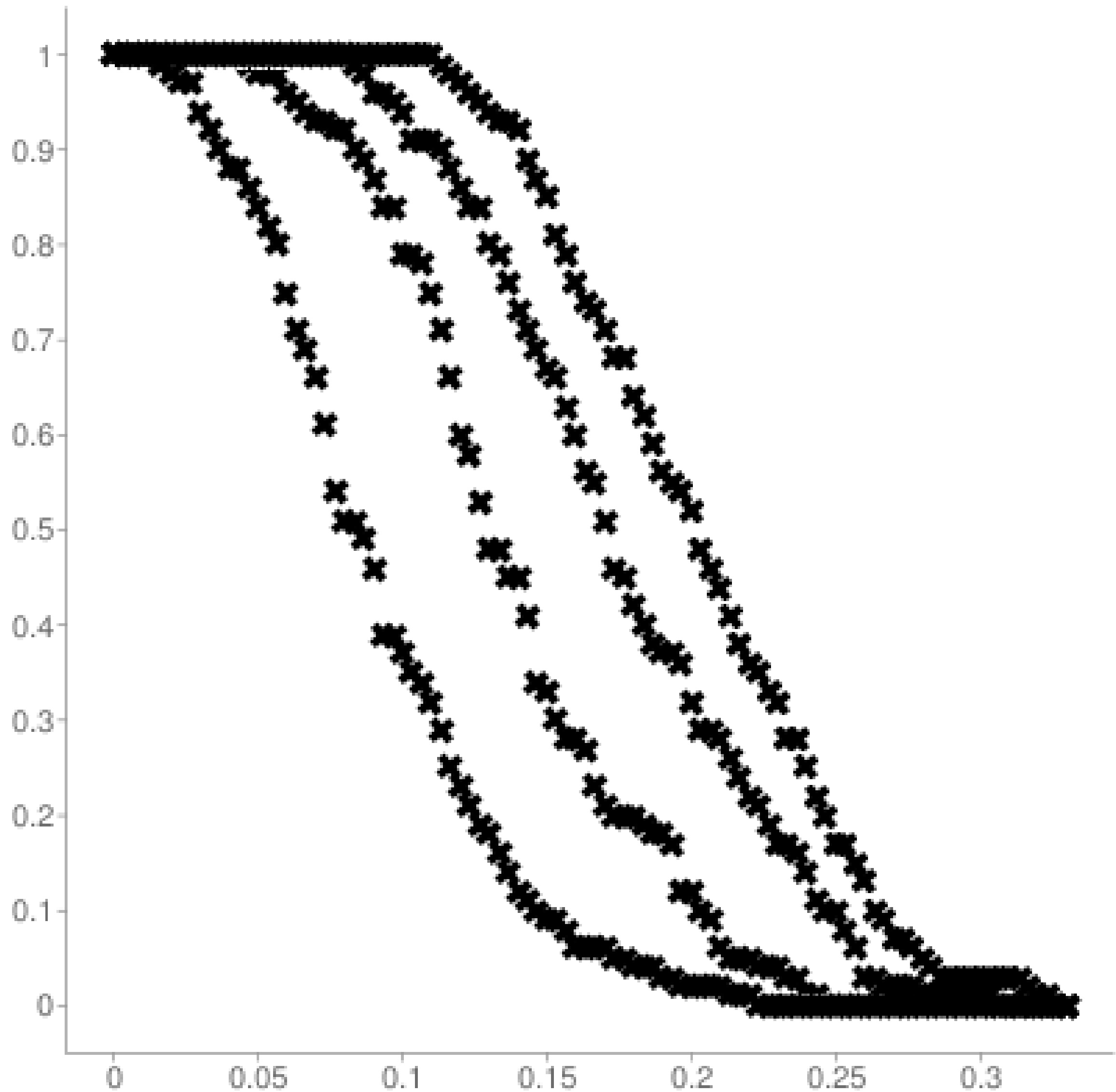
In red, atop the black data points, is a hand-drawn line of best fit. It looks roughly like a sigmoid with the equation (roughly)

$$F(x) = \frac{1}{1 + e^{k(x-1.25)}}$$

so that it has minimum 0, maximum 1, midpoint 1.25, and steepness k.

Part D

This this section, we begin by simulating 100 Poisson point processes and recording the four closest points to the origin in each. Then, for 100 x values between 0 and .33, we check the proportion of these lowest values higher than x.



Above, find G_1, G_2, G_3, and G_4, from left to right. For obvious reasons, it was more common for the lowest values in our Poisson Point processes to be higher than only the smallest values of x. On the other hand, the fourth-lowest values were above almost all values of x up to .1.