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TITLE

CONFERENCE, DATE

Content

- Modal logic and semantics
  - Kripke frames
  - Topological space
  - Neighbourhood frames
- Multimodal logic and product of frames/spaces and logics
  - Notation, Fusion of logics
  - Horizontal and Vertical topology/functions and semantics
  - Product of logics
- Main result and ideas

Modal logic and Kripke frames and models

- Modal logic extends classical propositional logic. Formally:

$$\phi ::= p \mid \bot \mid \neg\phi \mid \phi \vee \phi \mid \Box\phi$$

  where $\Box$ is a modal operator and Prop is a set of variable with $p \in$ Prop.

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  where $\Box$ is a modal operator and Prop is a set of variable with $p \in$ Prop.
- A frame $F = (W, R)$ is a pair where
  - W is a non-empty set of worlds
  - $R \subseteq W \times W$ is a binary relation
- A model is a pair $M = (F, R)$ (M is based on F) where
  - F is a frame
  - V is a valuation and is of the form $V : Prop \rightarrow 2^W$

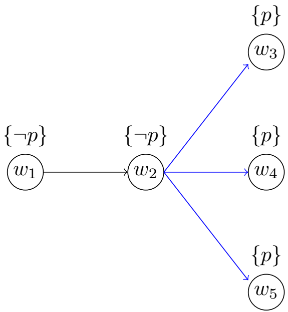
Kripke semantics

- Let $M = (F, V)$ be a model and $w \in W$ a state in M. A formula being true at w is inductively defined as:

$$
\begin{array}{ll}
M, w \Vdash p & \text{iff } w \in V(p) \\
M, w \Vdash \bot & \text{never} \\
M, w \Vdash \neg\phi & \text{iff not } M, w \Vdash \phi \\
M, w \Vdash \phi \vee \psi & \text{iff } M, w \Vdash \phi \vee M, w \Vdash \psi \\
M, w \Vdash \Box\phi & \text{iff } \forall v \in W : wRv \rightarrow M, v \Vdash \phi \\
M, w \Vdash \Diamond\phi & \text{iff } \exists v \in W : wRv \wedge M, v \Vdash \phi
\end{array}
$$

Example

- Let $\phi = \Box p$ and $M = (W, R, V)$ with $W = \{w_1, w_2, w_3, w_4, w_5\}$, $V(p) = \{w_3, w_4, w_5\}$ and $R =$



ϕ is true at w_2

Example

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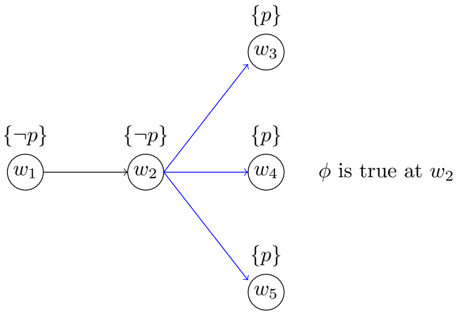

- Kripke semantics has many applications for example epistemic logic, temporal logic,...

Topological space

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Topological space

- We want to reason not just about "what is true" but "where is true"
- Topological space deals with open sets, it can describes which points are "nearby"
- applications: spatial reasoning for example reasoning about neighboring regions or reachability (AI, robotics)
- A topological space is a pair (X, τ), where τ (called topology) is a collection of subsets of X (open sets) such that:
  - $\emptyset$ and X are open
  - the union of arbitrary collection of open sets is open
  - the intersection of finite collection of open sets is open
- A topological model is a structure $M = (X, \tau, \upsilon)$ where (X, τ) is a topological space and υ a valuation of the form $\upsilon : Prop \rightarrow 2^X$

Topological semantics

Let $M = (X, \tau, \upsilon)$ be a topological model and $x \in X$ a point in M. A formula being true at x is inductively defined as:

$M, x \vDash p$	iff $x \in v(p)$
$M, x \vDash \bot$	never
$M, x \vDash \neg\phi$	iff $M, x \nvDash \phi$
$M, x \vDash \phi \vee \psi$	iff $M, x \vDash \phi$ or $M, x \vDash \psi$
$M, x \vDash \Box\phi$	iff $\exists U \in \tau$ such that $x \in U$ and $\forall u \in U,\ M, u \vDash \phi$
$M, x \vDash \Diamond\phi$	iff $\forall U \in \tau$: if $x \in U \rightarrow \exists u \in U,\ M, u \vDash \phi$

Example

- Let (X, τ) be a topological space with $X = \{1, 2, 3\}$, $\tau = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, W\}$ and $V(p) = \{1, 2\}$. Furthermore, let $\phi = \Box p$.

Example

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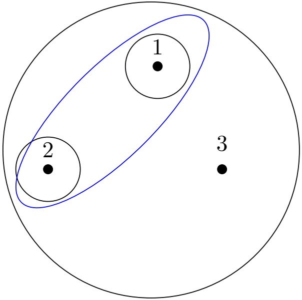

$$M, 1 \Vdash \phi$$
$$M, 3 \nVdash \phi$$

Neighbourhood frames

- generalize Kripke semantics
- captures non-normal modal logics
- application in Epistemic logic

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- generalize Kripke semantics
- captures non-normal modal logics
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- A neighbourhood frame is a pair (X, τ) where τ is a function $\tau : X \to 2^{2^X}$.
- A neighbourhood model is a structure $M = (X, \tau, \upsilon)$, where υ is a valuation of the form $\upsilon : \mathit{Prop} \to 2^X$