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TITLE

CONFERENCE, DATE

Content

- Modal logic and semantics
  - Kripke frames
  - Topological space
  - Neighbourhood frames
- Multimodal logic and product of frames/spaces and logics
  - Notation, Fusion of logics
  - Horizontal and Vertical topology/functions and semantics
  - Product of logics
- Main result and ideas

Modal logic and Kripke frames and models

- Modal logic extends classical propositional logic. Formally:

$$\phi ::= p \mid \bot \mid \neg\phi \mid \phi \vee \phi \mid \Box\phi$$

where $\Box$ is a modal operator and Prop is a set of variable with $p \in$ Prop.

Modal logic and Kripke frames and models

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  where $\Box$ is a modal operator and Prop is a set of variable with $p \in$ Prop.
- A frame $F = (W, R)$ is a pair where
  - W is a non-empty set of worlds
  - $R \subseteq W \times W$ is a binary relation
- A model is a pair $M = (F, R)$ (M is based on F) where
  - F is a frame
  - V is a valuation and is of the form $V : Prop \rightarrow 2^W$

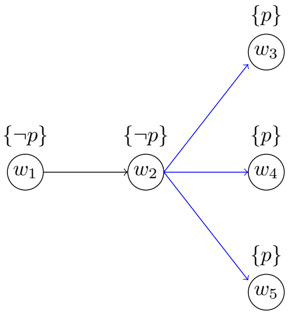
Kripke semantics

- Let $M = (F, V)$ be a model and $w \in W$ a state in M. A formula being true at w is inductively defined as:

$$
\begin{array}{ll}
M, w \Vdash p & \text{iff } w \in V(p) \\
M, w \Vdash \bot & \text{never} \\
M, w \Vdash \neg\phi & \text{iff not } M, w \Vdash \phi \\
M, w \Vdash \phi \vee \psi & \text{iff } M, w \Vdash \phi \vee M, w \Vdash \psi \\
M, w \Vdash \Box\phi & \text{iff } \forall v \in W : wRv \rightarrow M, v \Vdash \phi \\
M, w \Vdash \Diamond\phi & \text{iff } \exists v \in W : wRv \wedge M, v \Vdash \phi
\end{array}
$$

Example

- Let $\phi = \Box p$ and $M = (W, R, V)$ with $W = \{w_1, w_2, w_3, w_4, w_5\}$, $V(p) = \{w_3, w_4, w_5\}$ and $R =$



ϕ is true at w_2

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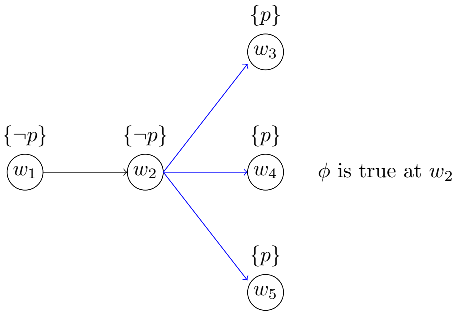

- Kripke semantics has many applications for example epistemic logic, temporal logic,...