hello

IP = PSPACE

CONFERENCE, DATE

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- $\exists x \exists y \neg(x \wedge y)$ is not in NNF but $\exists x \exists y(\neg x \vee \neg y)$
- QBF $\leq_m^P$ QBF-Truth$_{NNF}$ can be easily done by the verifier
- it suffices to show QBF' $\in$ IP because we can reduce any problem in PSPACE to QBF in polynomial time

How do we find an algorithm for QBF' s.t it satisfies the IP conditions?

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x	y	$x \wedge y = x * y$	$x \wedge y = x + y$	$\neg(x \wedge y) = 1 - (x * y)$
0	0	0	0	1
0	1	0	1	1
1	0	0	1	1
1	1	1	2	0

$$\phi(x_1, x_2, ..., x_n) = 1 \Leftrightarrow \phi_{arith}(x_1, x_2, ..., x_n) > 0$$

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- $\phi_{arith} = 2$

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- $\phi_{arith} = 2$
- If ϕ is true then $\phi_{arith} > 0$
- If ϕ is false then $\phi_{arith} = 0$
- This can be shown by structural induction

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- $\phi = x$. If ϕ is true then $\phi_{arith} = 1$ and if ϕ is true then $\phi_{arith} = 0$
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- Suppose ϕ_1 and ϕ_2 are true, that means $\phi_{1arith} > 0$ and $\phi_{2arith} > 0$ (it's similar for the other case)
- $\phi_1 \wedge \phi_2$, $\phi_1 \vee \phi_2$ also holds
- For $\forall\,\phi_1$, we have by induction that $\phi_1[x := 0], \phi_1[x := 1]$ is true, so the multiplication of two positive value is positive (same for $\exists\phi_1$)

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$\phi_{arith} = \prod_{x_1 \in \{0,1\}} \cdots \prod_{x_m \in \{0,1\}} \phi'_{arith} = 4^{2^m}$

- It holds that for formula ϕ with string length n : $\phi_{arith} \leq 2^{2^n}$
- We solve this problem by using modulo with a suitable value

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- k must be presentable in linear many bits
- the calculation mod k must preserve ">0" for valid and "=0" for invalid formulas
- Number Theory Theorem : for any $a \leq 2^{2^n}, a > 0$, there exist a prime number $k \in [2^n, 2^{3n}]$ s.t $a \not\equiv 0$ (mod k)

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- If $\phi = \phi_1 \wedge \phi_2$, then ask prover to send a_1 and a_2 and check $c = a_1 * a_2$. If it's true then ask the prover to prove that the of ϕ_1 is a_1 and ϕ_2 is a_2
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- For $\phi = \phi_1 \vee \phi_2$, we ask for a_1 and a_2 s.t c = $a_1 + a_2$
- In case $\phi = \forall x \phi_1$ we asked for a polynomial p(x) that represents the arithmetic presentation of ϕ_1 where x is free and we check c = p(0) * p(1)
- If it is true, the verifier sends randomly a number d between $\{0, ..., k-1\} = GF(K)$ and caluclate p(d). Now the verifier expects the prover to prove the value of $\phi_1[x := d]$ is p(d)
- The same process happens when we have $\phi = \exists \phi_1$, but we check $c = p(0) + p(1)$

Protocol continue

- When every variable got a number in GF(K), say $y_1, ..., y_n$ the verifier calculates $\phi_{arith}(y_1, ..., y_n)$ and accept if its equal to $q(y_1, ..., y_n)$ (last polynomial sent by prover) else reject

Example

$$\phi = \forall x \exists y (\neg x \lor y) \land \exists z \exists w \, (z \lor w)$$

$$\phi_{arith} = \underbrace{(\prod_x \prod_y (1 - x) + y)}_{\phi_{\mathbf{1} arith}} * \underbrace{(\sum_z \sum_w z + w)}_{\phi_{\mathbf{2} arith}} \quad x, y, z, w \in \{0, 1\}$$

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Prover		Verifier
	c=8,k=11,b $\longrightarrow$	check c>0, 8, b
	asks for $a_1 = \phi_1$ and $a_2 = \phi_2$ $\longleftarrow$
	sends $a_1 = 2$, $a_2 = 4$ $\longrightarrow$	check $c = a_1 * a_2$
	prove $a_1 = \phi_1, a_2 = \phi_2$ $\longleftarrow$

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	sends $a_1 = 2$, $a_2 = 4$ $\longrightarrow$	check $c = a_1 * a_2$
	$\longleftarrow$ prove $a_1 = \phi_1, a_2 = \phi_2$ send me $p_1(x)$ for ϕ_1 and $p_2(x)$ for ϕ_2	

Example continue

Prover		Verifier
calculate $\phi_{1arith}(x)$, $\phi_{2arith}(z)$	$\xrightarrow[p_1(x) = \prod_y (1-x)+y = x^2-3x+2]{\text{sends } p_1(x) = \phi_{1arith}(x),\ \phi_{2arith}(z)}$	check $a_1 = p_1(0) * p_1(1)$ check $a_2 = p_2(0) + p_2(1)$

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	sends $p_1(d) = 0, p_2(g) = 7$ $\longleftarrow$ ask for $p_1'(d, y), p_2'(g, w)$	Choose randomly $d, g \in GF(k)$,say d=2,g=3

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$$\xrightarrow[\text{sends } p_2'(g,w)=\phi_{2arith}(g,w)]{\text{sends } p_1'(d,y)=\phi_{1arith}(d,y)}$$

$p_1'(d,0) * p_1'(d,1) = p_1(d)$

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	sends $p_1(d) = 0, p_2(g) = 7$ $\longleftarrow$ ask for $p'_1(d, y), p'_2(g, w)$	Choose randomly $d, g \in GF(k)$,say d=2,g=3
	sends $p'_1(d, y) = \phi_{1arith}(d, y)$ $\longrightarrow$ sends $p'_2(g, w) = \phi_{2arith}(g, w)$	$p'_1(d, 0) * p'_1(d, 1) = p_1(d)$ $p'_2(g, 0) * p'_2(g, 1) = p_2(g)$ Choose c,k $\in$ GF(k)

The verifier check $\phi_{arith}(d, c, g, k) = p'_1(d, c) * p'_2(g, k)$. It accepts.

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Prover $\xrightarrow{\text{c= 3, p = 131, b}}$ Verifier

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$\xrightarrow[\text{prover has to lie}]{\text{sends } p_1(x) = 2x + x}$ check $p_1(0) * p_1(1) = c$

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If $\phi_{arith}(d) = p_1(d = 3) = 7$ then we lost. Chances should be low

The verifier check $\phi_{arith}(d, g) = p_1'(d, g)$. $12 \neq 28$. Verfier rejects.

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Prover → Verifier: sends $p_1'(d, y) = 7 * y$

Verifier: check $p_1'(d, 0) + p_1'(d, 1) = p_1(d)$. Chooses g = 4

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Simple QBF

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$$Pr[p(a_1, ..., a_m) = 0] \leq \frac{d}{|S|}$$

- Prover sends wrong polynomial $p \neq h = \phi_{arith}$. Verifier chooses randomly $c \in GF(p)$ where $p \geq 2^n$. Furthermore we have $deg(g - h) \leq 2n$.

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- Prover sends wrong polynomial $p \neq h = \phi_{arith}$. Verifier chooses randomly $c \in GF(p)$ where $p \geq 2^n$. Furthermore we have $deg(g - h) \leq 2n$. Then $Pr[p(c) = h(c)] = Pr[p(c) - h(c) = 0] \leq \frac{2n}{2^n}$.
-