The Continuity Principle

A Framework for Preserving the Compounding Engine in Portfolio Design

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Positioning Note: Context and Intellectual Lineage

The Continuity Principle builds upon a lineage of ideas in modern portfolio theory, behavioral finance, and adaptive market design, yet extends them into a new framework focused on the preservation of the compounding process itself.

While its logic is grounded in established theory, its application is new: it reframes risk management not as volatility control or capital preservation alone, but as continuity control, the explicit design of portfolios capable of sustaining uninterrupted compounding through stress, liquidity shocks, and regime shifts.

Relation to Paleologo: Behavior as a Metric of Truth

Paleologo's work on fund behavior and the "geometry of performance" emphasized that a portfolio's character is revealed not by its return, but by its behavior under stress. The Continuity Principle shares this behavioral lens, yet shifts it from diagnosis to design. It asks not only how a strategy behaves, but how to engineer a portfolio that behaves well by construction. Where Paleologo measures the endurance of compounding, the Continuity Principle constructs the arithmetic that protects it.

Relation to Andrew Lo: Adaptive Rationality and Survival

Lo's Adaptive Markets Hypothesis reinterprets market efficiency as an evolutionary process: agents survive by adapting to changing environments. The Continuity Principle draws on that same adaptive realism but applies it to portfolio architecture. It argues that portfolios, like organisms, must be built to survive environmental change. Rather than modeling rational adaptation, it prescribes design resilience: portfolios must remain operable even when their environment shifts.

> In Lo's framework, the market adapts.
>
> In the Continuity framework, the portfolio endures adaptation itself.

Relation to Asness: The Asymmetry of Compounding

Cliff Asness and colleagues at AQR have long emphasized the asymmetry between arithmetic and geometric returns: volatility, costs, and behavioral drawdowns destroy compounded wealth even when mean returns look strong. The Continuity Principle translates this observation into a formal design constraint, introducing the continuity budget, recovery-time condition, and sustainable-leverage rule as explicit mathematical guardrails on compounding behavior. It operationalizes what Asness described: the real cost of volatility and liquidity stress on long-term wealth creation.

The Idea: Continuity Before Everything

I used to think portfolio theory was a matter of choosing the best mix of assets, an act of taste disguised as math. You estimate expected returns, estimate risk, run an optimizer, and declare the result "optimal." The story is tidy. It is also incomplete. Compounding is not a tidy story. Compounding is a fragile process living inside an uncertain world.

A portfolio can be "right" on average and still fail as a compounding machine. It can have a strong expected return and still spend long periods unable to recover, stuck below its previous peak, forced to de-risk, forced to liquidate, forced to abandon its own premise. That is the quiet tragedy in investing: not underperformance, but interruption. Not being wrong but being unable to stay in the game long enough for rightness to matter.

The Continuity Principle begins with a simple shift in definition: The objective is not to maximize return. The objective is to maximize long-run growth while preserving the portfolio's ability to remain invested.

Definition: What "Optimal" Really Means

A portfolio is not a forecast, it is a process. Its outcome depends on whether it can stay deployed across regimes where correlations spike, liquidity thins, and conviction is punished by time.

The Continuity Principle replaces the usual one-period objective (maximize expected return for a risk budget) with a multi-period truth: The best portfolio is the one that can keep compounding. Not the one that looks best at a point estimate.

This pushes portfolio theory toward a single, concrete target: geometric growth. For small returns, the long-run compounding rate can be approximated by:

$$g = \mu - \tfrac{1}{2}\sigma^2$$

where μ is arithmetic expected return and σ is volatility. Volatility is not a psychological cost; it is a mathematical tax on compounding.

Return maximization therefore becomes disciplined. You do not maximize μ directly; you maximize g, and you treat discontinuity like events that force liquidation, deleveraging, mandate breach, or long recovery as a first-class constraint.

Continuity Budget: The Two Constraints That Matter

The portfolio breaks compounding in two ways: depth, when drawdowns force action, and time, when recovery is slow enough to destroy optionality.

A) Depth constraint: breach probability of a loss barrier

Model portfolio PnL (or log-wealth proxy) as:

$$dX_t = \mu \, dt + \sigma \, dB_t$$

Let a > 0 be a maximum tolerable cumulative loss barrier. A useful first-pass approximation for the probability of ever hitting −a is:

$$Pr(hit \; -a) = exp(-2\mu a \,/\, \sigma^2)$$

If the portfolio is scaled by leverage L, then $\mu \to L\mu$ and $\sigma \to L\sigma$, yielding:

$$Pr(hit \; -a) = exp(-2\mu a \,/\, (L\sigma^2))$$

Given a tolerated probability p, the maximum sustainable leverage is:

$$\frac{2\mu a}{\sigma^2 \, |ln \, p|}$$

This is not a risk slogan, it is a leverage rule.

B) Time constraint: recovery is a compounding requirement

If the portfolio is d below peak, a simple recovery-time approximation is:

$$\frac{d}{\mu}$$

The point is not precision but design logic: deep drawdowns are costly twice wealth and time. Continuity therefore requires a time budget alongside the depth budget.

Portfolio Construction: Growth Objective + Continuity Constraints

Portfolio expected return and variance:

$$\mu_p(w) = w^{\mathrm{T}} \hat{\mu}$$

$$\sigma_p{}^2(w) = w^{\mathrm{T}} \Sigma \, w$$

A continuity-optimal objective is growth with costs:

$$max\_w \, [\, \mu_p(w) - ½\sigma_p{}^2(w) - TC(w) \,]$$

Constraint 1: Barrier survivability

$$\frac{\mu_p(w)}{\sigma_p{}^2(w)} \geq \frac{|ln \, p|}{2a}$$

Constraint 2: Recovery-time budget

$$\mu_p(w) \geq \frac{d}{\tau *}$$

Implementation: A Concrete Operating Procedure

Step 3: Solve the constrained growth problem

$$\mu_p(w) - ½\sigma_p{}^2(w) - TC(w)$$

subject to the continuity constraints above.

Step 5: Volatility targeting as a continuity governor

$$w_t \leftarrow \kappa_t w_t$$

$$\kappa_t = min(1, \sigma\_target / \sigma_{p, t})$$

Validation: What You Test and What "Working" Looks Like

Metric 1: Realized geometric growth

$$\hat{g} = (1/T)\, \Sigma\, log(1 + r_t)$$

This is the primary objective in realized form.

Continuity portfolios should show higher realized geometric growth, fewer discontinuity events, and faster, more stable recoveries.