# Previous Questions

2025 年 6 月 28 日

# 1　环境配置

见 1.py

# 2　向量与矩阵

## 2.1　旋转矩阵

代码实现部分见 2-1.py

相当于将向量 $v$ 顺时针旋转 $\theta$

## 2.2　矩阵指数

代码实现部分见 2-2.py

考虑泰勒展开 $e^{i\theta\hat{P}}$，易计算验证泡利矩阵的平方都为 $I$，即 $\hat{P}^2 = I$。故

$$
\begin{aligned}
e^{i\theta\hat{P}} &= \sum_{k=0}^{+\infty} \frac{i^k \theta^k \hat{P}^k}{k!} = \sum_{k=0}^{+\infty} (-1)^k \frac{\theta^{2k}}{(2k)!} I + \sum_{k=0}^{+\infty} (-1)^k i \frac{\theta^{2k+1}}{(2k+1)!} \hat{P} \\
&= \cos\theta I + i\sin\theta \hat{P}
\end{aligned}
\tag{1}
$$

只需 $\hat{P}^2 = I$ 即可。

## 2.3 矩阵关于向量的期望

代码实现部分见 2-3.py

事实上泡利矩阵还满足厄米性质，即 $\hat{P}^\dagger = \hat{P}$，故

$$
\begin{aligned}
v^\dagger \hat{Q} v &= (1,0)(\cos\theta/2 I + i\sin\theta/2 \hat{P})^\dagger \hat{Q}(\cos\theta/2 I + i\sin\theta/2 \hat{P})(1,0)^T \\
&= (1,0)(\cos\theta/2 \hat{Q} - i\sin\theta/2 \hat{P}\hat{Q})(\cos\theta/2 I + i\sin\theta/2 \hat{P})(1,0)^T \\
&= (1,0)(\cos^2\theta/2 \hat{Q} + \sin^2\theta/2 \hat{P}\hat{Q}\hat{P} + i\sin\theta/2\cos\theta/2(\hat{Q}\hat{P} - \hat{P}\hat{Q}))(1,0)^T
\end{aligned}
\tag{2}
$$

有两种情况：

1. $\hat{P} = \hat{Q}$，则 $\hat{P}\hat{Q}\hat{P} = \hat{Q}, \hat{P}\hat{Q} = \hat{Q}\hat{P}$。

2. $\hat{P}\hat{Q} = \pm i\hat{R}$，则 $\hat{P}\hat{Q}\hat{P} = \pm i\hat{R}\hat{P} = -\hat{Q}, \hat{P}\hat{Q} - \hat{Q}\hat{P} = \pm 2i\hat{R}$（$\hat{R}$ 也为泡利矩阵）

故

$$
v^\dagger \hat{Q} v = \begin{cases} \hat{Q}_{1,1} & (\hat{P} = \hat{Q}) \\ \cos\theta \hat{Q}_{1,1} \pm \sin\theta \hat{R}_{1,1} & (\hat{P} \neq \hat{Q}) \end{cases}
\tag{3}
$$

总结一下可得

$$
v^\dagger \hat{Q} v = \begin{cases} 1 & (\hat{P} = \hat{Q} = \sigma_z) \\ \cos\theta & (\hat{P} = \sigma_{x,y}, \hat{Q} = \sigma_z) \\ \sin\theta & (\hat{P} = \sigma_x, \hat{Q} = \sigma_y) \\ -\sin\theta & (\hat{P} = \sigma_y, \hat{Q} = \sigma_x) \\ 0 & (otherwise.) \end{cases}
\tag{4}
$$

## 2.4 张量积

代码实现部分见 2-4.py

## 2.5 狄拉克符号

### 2.5.1 旋转矩阵

$|v\rangle = |0\rangle, |v'\rangle = R(\theta)|v\rangle$

### 2.5.2   矩阵指数

### 2.5.3   矩阵关于向量的期望

$$|v_0\rangle = |0\rangle, |v\rangle = e^{i\frac{\theta}{2}\hat{P}}|v_0\rangle$$
$$E = \langle v|\hat{Q}|v\rangle$$

### 2.5.4   张量积

$$E = \langle 0^n|H|0^n\rangle$$

## 2.6   使用Tensorsircuit后端

代码实现部分见 2-*-tc.py

# 3   导数与梯度下降

## 3.1   数值微分

代码实现部分见 3-1.py （假设 $f(x) = \sum x_i^3, x = (11, 45, 14)$）

## 3.2   三角函数数值微分

$f(x) = A\sin(x + B) + C$，则 $f'(x) = A\cos(x + B), f(x + \delta) - f(x - \delta) = A(\sin(x + B + \delta) - \sin(x + B - \delta)) = 2A\cos(x + B)\sin\delta$。取 $\tau = 2\sin\delta$ 即有 $f'(x) = \frac{f(x+\delta)-f(x-\delta)}{\tau}$

## 3.3   单比特参数平移

$f(\theta) = \langle 0|(\cos\theta/2I - i\sin\theta/2\hat{P}_1)\hat{P}_2(\cos\theta/2I + i\sin\theta/2\hat{P}_1)|0\rangle$ ，与 2.4 表达式基本相同，故可类似化简为：

$$f(\theta) = \begin{cases} 1 & (\hat{P}_1 = \hat{P}_2 = \sigma_z) & (A,B,C) = (0,0,1) \\ \cos\theta & (\hat{P}_1 = \sigma_{x,y}, \hat{P}_2 = \sigma_z) & (A,B,C) = (1,\pi/2,0) \\ \sin\theta & (\hat{P}_1 = \sigma_x, \hat{P}_2 = \sigma_y) & (A,B,C) = (1,0,0) \\ -\sin\theta & (\hat{P}_1 = \sigma_y, \hat{P}_2 = \sigma_x) & (A,B,C) = (-1,0,0) \\ 0 & (otherwise.) & (A,B,C) = (0,0,0) \end{cases} \tag{5}$$

有显著的周期性，且符合 3.2 中的形式（后面为对应的 $A,B,C$），故参数平移法给出的导数是正确的。

## 3.4 梯度下降

代码实现部分见 3-4.py

# 4 测量