Integration of Vectors

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1 Evaluation of Line Integrals

In a line integral, we integrate a given function $f(x, y, z)$ along a curve C in space from point a at location $r(\vec{a})$ to point b at location $r(\vec{b})$.
In order to achieve this, we describe the curve C by its parametric representation in Cartesian coordinates: $r(\vec{t}) = (x(t), y(t), z(t))$. The curve C is called the path of integration. $P = r(\vec{a})$ is its start point $Q = r(\vec{b})$ is its end point. The curve C is oriented positively in the direction from P to Q and is denoted by an arrow. If the points P and Q coincide the path is **closed**.

1.1 Line integral of vector field

The line integral of a vector field $\vec{F}$ over a curve C with parametric representation $r(\vec{t})$ is:

$$\int_C \vec{F}(\vec{r}) \cdot d\vec{r} = \int_a^b \vec{F}(r(\vec{t})) \cdot \frac{dr(\vec{t})}{dt} dt \tag{1}$$

where $d\vec{r}$ is the curve's displacement vector or line element