Due: 18:00 Monday 8 April 2019 .

Financial Engineering - Assignment 3

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Submission as a group is allowed. Please submit your solutions together with codes on time.

1. Suppose we have short rate r_t following a Vasicek process with $r_0 = 0.02$, $\bar{r} = 0.05$, $\gamma = 0.2$, $\sigma = 0.015$. Take $T = 1$ and number of steps equal to 250. Generate 40 paths and plot your results (you can use exact simulation or Euler discretization). Now keep everything fixed and change the parameter γ to 5. Visualize the generated paths in a separate plot.

2. Suppose we have a short rate r following CIR process with $r_0 = 0.02$, $\bar{r} = 0.05$, $\gamma = 1.2$, $\alpha = 0.04$. Take $T = 1$ and number of steps equal to 250. Use Euler discretization where you use truncation or reflection to avoid negative values. Generate 40 paths and plot your results.

3. Price a Lookback call option for which we know that the maturity $T = 2$ years, the current stock price is $S_0 = 10$, the stock's volatility $\sigma = 0.2$. Use monthly monitoring, i.e. $\Delta = 1/12$ and use Monte Carlo simulation with 10000 scenarios.

4. a) Suppose we have the setting given in page 116 of the slide set. Solve this exercise by using Euler discretization and come up with a Monte Carlo estimate and the corresponding errors.

   b) Now repeat this exercise where you use the Milstein scheme to price the same call option. Compute the errors corresponding to different n values and compare them with the one you have obtained above.

5. a) Suppose we have the setting given in page 129 of the slide set. Price this Asian option by using control variates method where you choose the standard European call option as the control variate ($Y_1 = e^{-rT}(S_T - K)^+$). Provide the corresponding 95% confidence interval.

   b) Now take the geometric-average Asian option as control variate ($Y_2 = \max(A_{dg} - K, 0)^+$) and compute the price of the arithmetic-average Asian option again. Provide the corresponding 95% confidence interval and compare this with the result you have obtained above. Which control variate results in more variance reduction? Justify your answer.

6. a) We have a stock price following a GBM with $S_0 = 60$, $r = 0.01$, $\sigma = 0.4$. Suppose we have a down-and-out put option with maturity in 3 months (i.e., $T = 3/12$), strike $K = 50$ and barrier level $b = 40$. Assume daily monitoring and compute the Monte Carlo price of this option.

   b) Explain in which situations and how one can use importance sampling to improve the Monte Carlo price estimates for a barrier option.