Due: 13:00 Monday 18 March 2019 .

Financial Engineering - Assignment 2

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Submission as a group is allowed. Please submit your solutions together with codes on time.

1. (a) Suppose we have process X following geometric Brownian motion dynamics with drift $\mu = 0.1$, $\sigma = 0.1$. By using the random walk approximation as well as the Cholesky decomposition to generate Brownian motion, simulate 1000 paths for the time interval $[0, 1]$. In your computations use $n = 250$ equidistant time discritization points with $t_0 = 0$ and $t_n = T = 1$. Compare the computation times corresponding to the two method. Repeat this exercise for $\mu = 0.4$, $\sigma = 0.4$ (keep the seed fixed in order to see the impact of different parameters). For all cases provide a plot with the simulated paths.

   (b) Simulate 50 paths for a Poisson process with parameter $\lambda = 2$. Now keep the seed fixed and take $\lambda = 0.5$. For both cases provide a plot with the simulated paths.

2. Suppose we want to price a European call option written on a stock with initial value $S_0 = 80$, $\sigma = 0.2$, $\mu = 0.2$. The maturity of the option is in $T = 1$ year and the strike price is $K = 100$. Assume that the risk-free interest rate is $r = 2\%$. Price the option analytically and numerically by using naive Monte Carlo (simulate $n = 10000$ paths). Compute the corresponding Monte Carlo standard error and confidence interval for $\alpha = 0.05$.

3. Suppose we want to price a European call option written on a stock with initial value $S_0 = 80$, $\sigma = 0.2$, $\mu = 0.2$. The maturity of the option is in $T = 1$ year and the strike price is $K = 80$. Assume that the risk-free interest rate is $r = 2\%$. Price the option with antithetic variates (simulate $n = 10000$ paths). Estimate the reduction in the variance relative to naive Monte Carlo. Now take the strike value $K = 40$ (option is deep-in-the-money) and redo your computations (keep the seed fixed). How is the reduction in variance affected?

4. Suppose we want to estimate $\Theta = \mathbb{E}((1 - X^2)^{1/2})$, $X \sim U(0, 1)$.

   (a) Compute naive Monte Carlo estimator of Θ as well as the confidence interval for $\alpha = 0.05$ by taking $n = 10000$.

   (b) Show that it is better to use X^2 instead of X as a control variate. To this, you can estimate the unknown covariances by using computer simulation.

   (c) Use X^2 as a control variate and estimate Θ and obtain the corresponding confidence intervals: take $n = 1000$ in the pilot simulation to come up with optimal c; for the main part of simulation use $n = 10000$ samples.

   (d) Compare your results in (a) and (d)

5. Suppose we have the setting given in Exercises 1. Price the option by using the underlying stock as a control variate. Compare your results to the analytical price. Also, comparing with naive Monte Carlo, calculate the amount of the reduction in the variance. Take $n = 10000$ in your computations. Repeat this exercise for $K = 20$ (keep the seed fixed). What effect can you identify on errors and on the amount of variance reduction?

6. Let the lifetime of a machine is given by Y where Y is exponentially distributed with parameter $\lambda = 1$. We want to estimate $\Theta = \mathbb{P}(X > 20)$ by using simulation.

   (a) Generate 10000samples to estimate Θ with naive Monte Carlo. What is the estimated variance of this estimator?

   (b) Now use importance sampling to estimate Θ where we sample from an exponential density with parameter λ. What would be a good choice of λ? Explain.

   (c) Estimate Θ using 10,000 samples and the importance sampling density you chose in (b). Estimate the variance of your estimator and compare it to your answers in (a).