

7. Boiled Egg

NUS High School Team 2
Teddy Ong



Question Statement

- ✗ Suggest non-invasive methods to detect the degree to which a hen's egg is cooked by boiling. Investigate the sensitivity of your methods.

Methods
Residual Spin - Manual
Residual Spin - Motor
Microwave
Ultrasound
Degree of Cooked-ness
Predicted Cooking Time



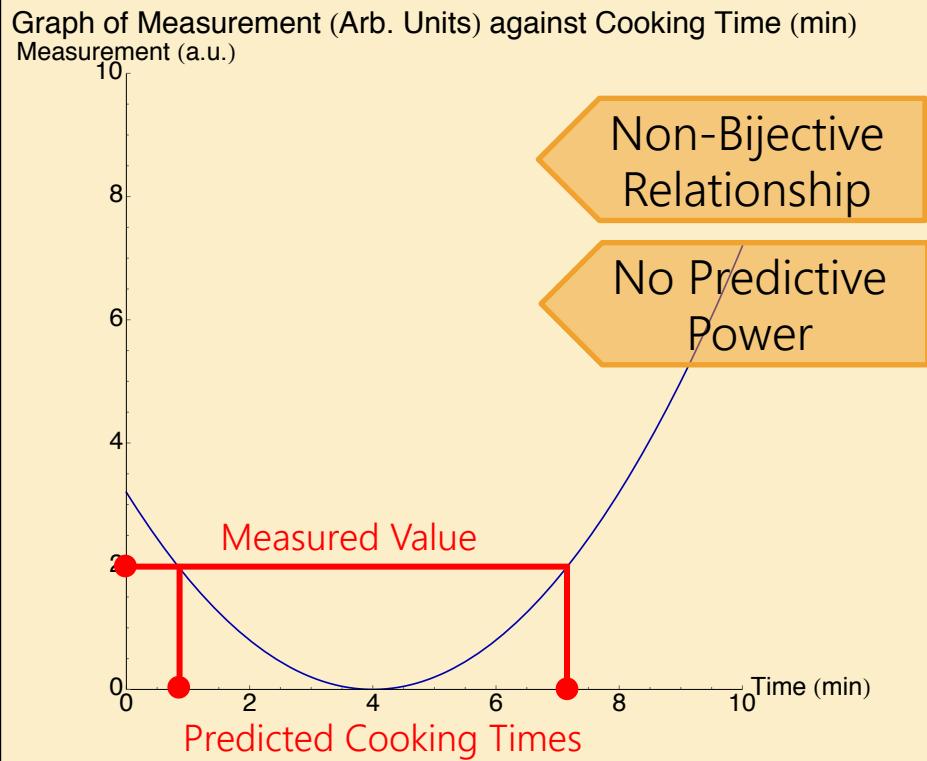
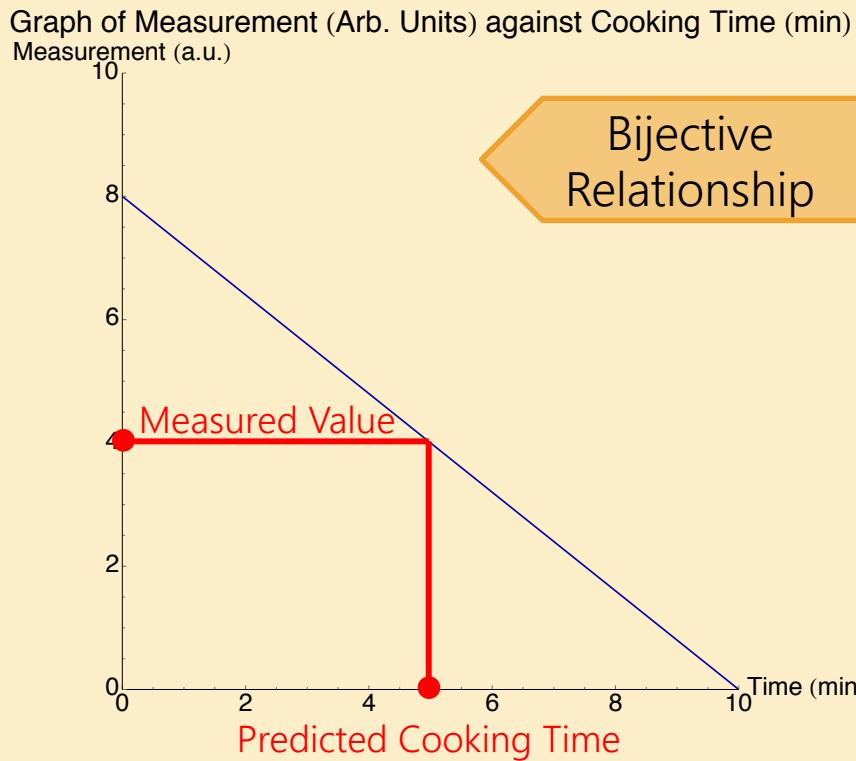
Experimental Methods



Theoretical Methods

Prediction Method

From Theory & Experiment



Agenda

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Theory

Thermodynamic Model

Methods

Residual Spin - Manual

Residual Spin - Motor

Ultrasound & Microwave

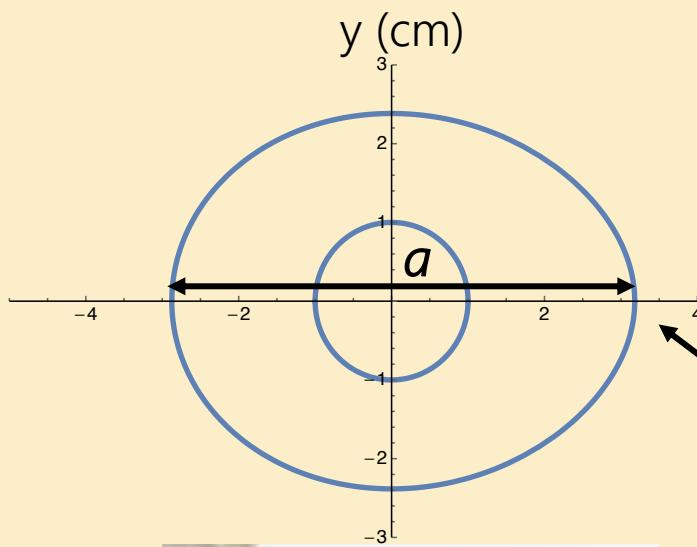
Sensitivity

Error Multiplier

Theoretical Account

Egg Shape

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Cassini Oval Equation

$$y(x) = \sqrt{\frac{x}{2}} \sqrt{(a - b) - 2x + \sqrt{4bx + (a - b)^2}}$$

Measured $a = 5.9 \pm 0.5$ cm
Fitted $b: 3.02$ cm

b controls the acuity of the pointy end



Trace Outline of Egg

Theory

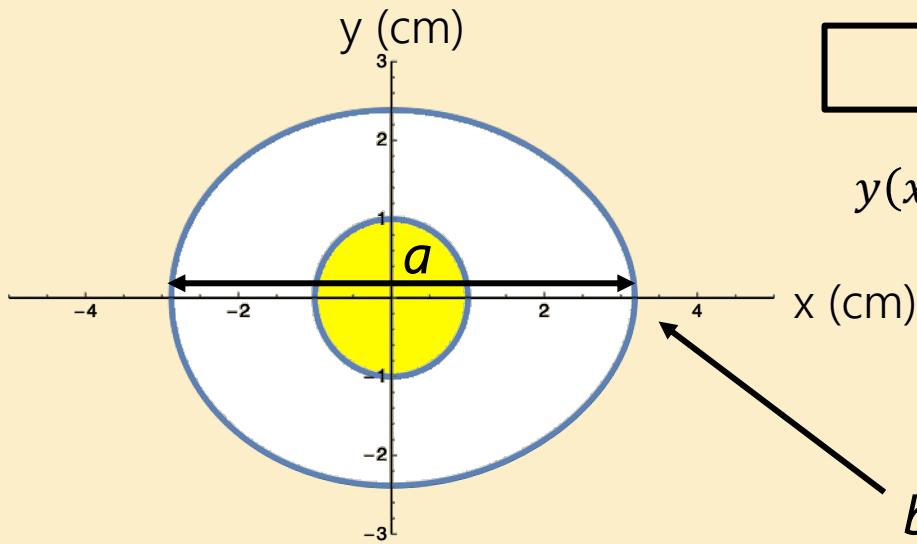
Measurement

Sensitivity

Conclusion

Egg Shape

7



Cassini Oval Equation

$$y(x) = \sqrt{\frac{x}{2}} \sqrt{(a - b) - 2x + \sqrt{4bx + (a - b)^2}}$$

Measured $a = 5.9 \pm 0.5$ cm
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Moment of Inertia

$$\int_{-a}^a \frac{1}{2} \pi \rho_{White} y(x)^4 dx + \frac{2}{5} \frac{4}{3} (\rho_{Yolk} - \rho_{White}) \pi R_{Yolk}^5$$

Egg White

Egg Yolk

Theory

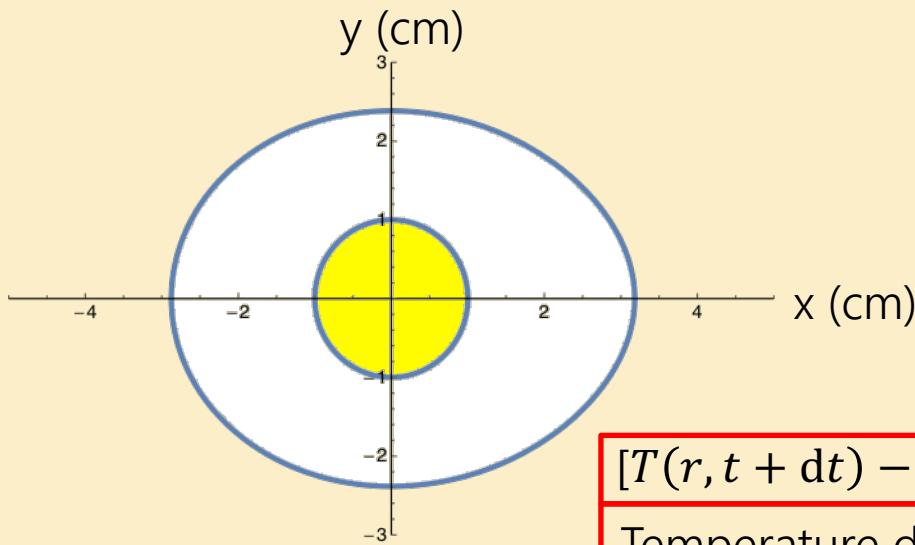
Measurement

Sensitivity

Conclusion

Heat Transfer

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Heat Conduction Equation

$$T(r + dr, t) - T(r, t) = \dot{Q}(r, t) dR(r, t)$$

Temperature difference

Power Transfer

Heat Capacity Equation

$$[T(r, t + dt) - T(r, t)] dC(r, t) = [\dot{Q}(r - dr, t) - \dot{Q}(r, t)] dt$$

Temperature difference

Net Power Transfer

	Heat Capacity	Heat Conductivity	Density
	$c (\text{J g}^{-1}\text{K}^{-1})$	$k (\text{W cm}^{-1}\text{K}^{-1})$	$\rho (\text{g cm}^{-3})$
Egg Yolk	2.7	0.0034	1.032
Egg White	3.7	0.0054	1.038

Boundary Conditions

$$T(r < a, 0) = T_{Egg}$$

$$T(r = a, 0) = T_{Water}$$

$$T(r = \infty, 0) = T_{Water}$$

Theory

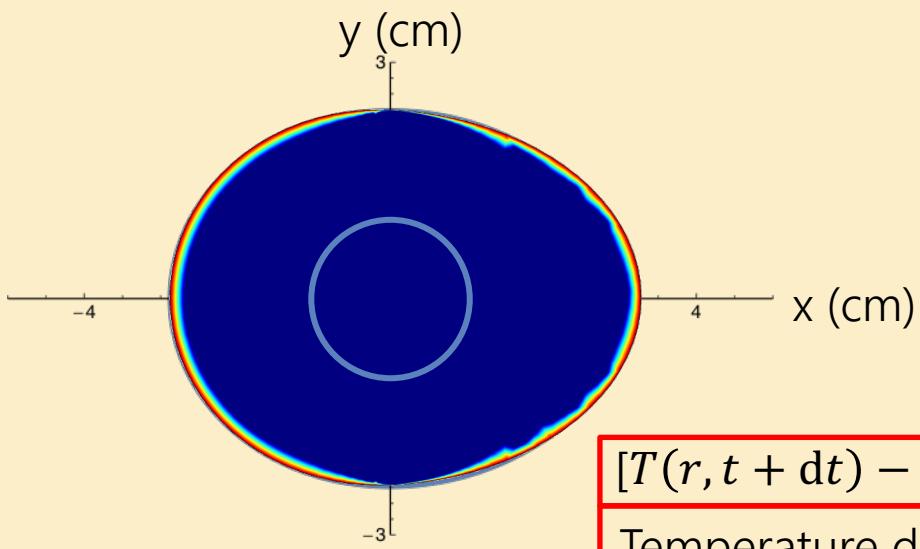
Measurement

Sensitivity

Conclusion

Thermodynamic Model

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Heat Conduction Equation

$$T(r + dr, t) - T(r, t) = \dot{Q}(r, t) dR(r, t)$$

Temperature difference

Power Transfer

Heat Capacity Equation

$$[T(r, t + dt) - T(r, t)]dC(r, t) = [\dot{Q}(r - dr, t) - \dot{Q}(r, t)] dt$$

Temperature difference

Net Power Transfer



Egg Yolk and Egg White are Thermodynamically Similar

Boundary Conditions

$$T(r < a, 0) = T_{Egg}$$

$$T(r = a, 0) = T_{Water}$$

$$T(r = \infty, 0) = T_{Water}$$

Theory

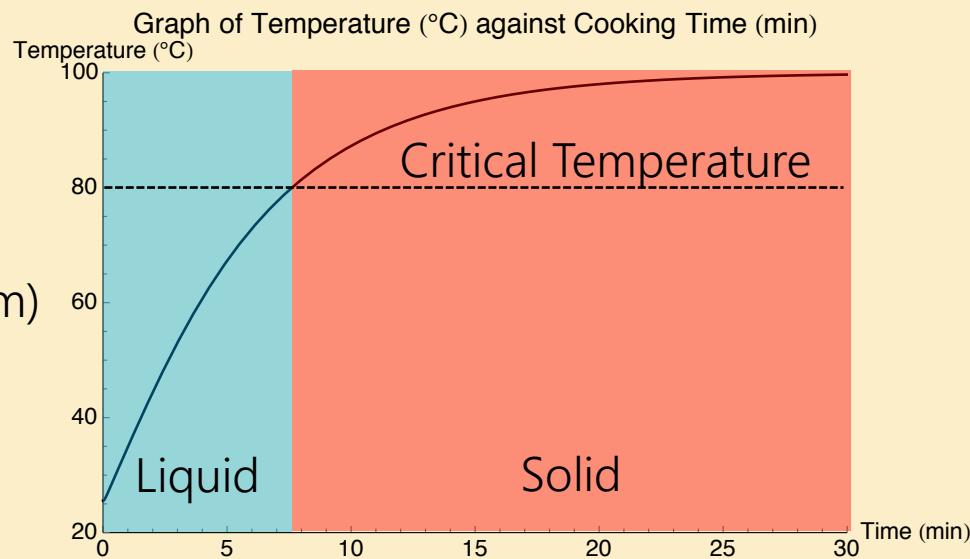
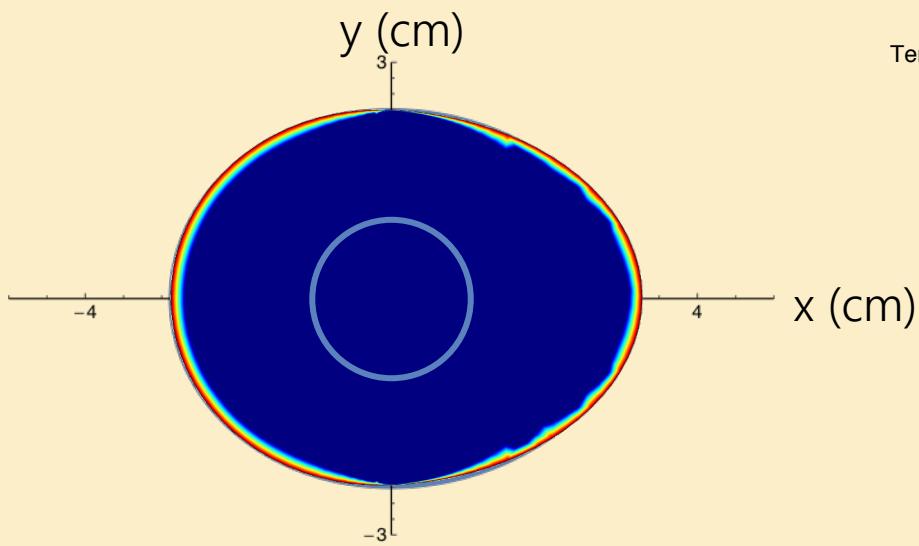
Measurement

Sensitivity

Conclusion

Thermodynamic Model

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Egg Yolk and Egg White are Thermodynamically Similar

Boundary Conditions

$$T(r < a, 0) = T_{Egg}$$

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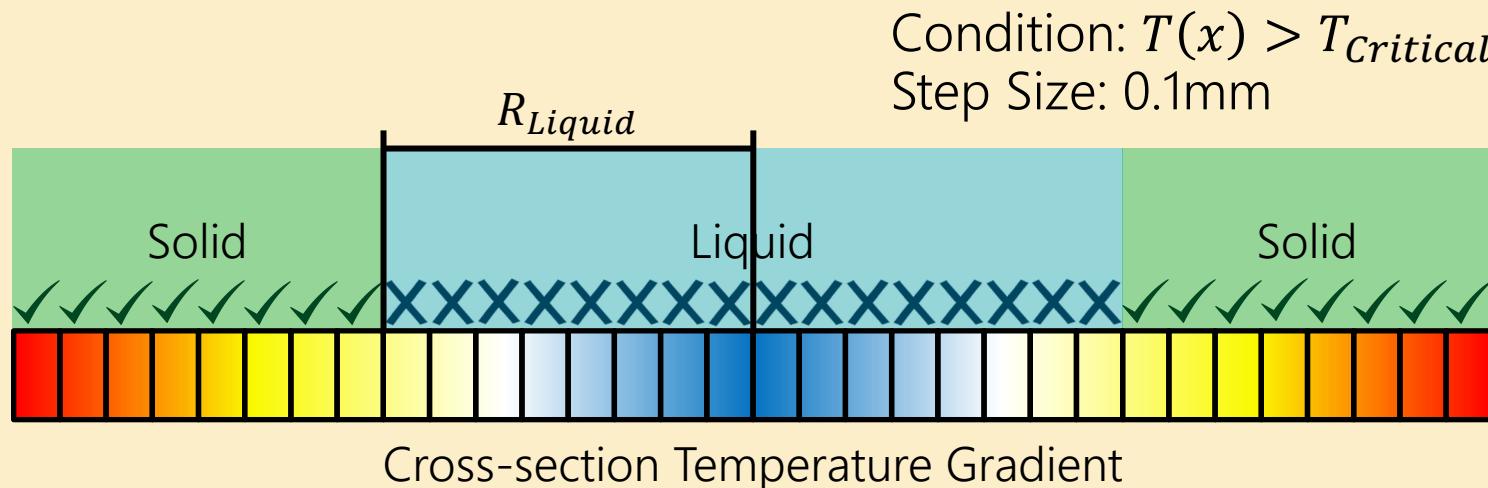
Measurement

Sensitivity

Conclusion

Thermodynamic Model

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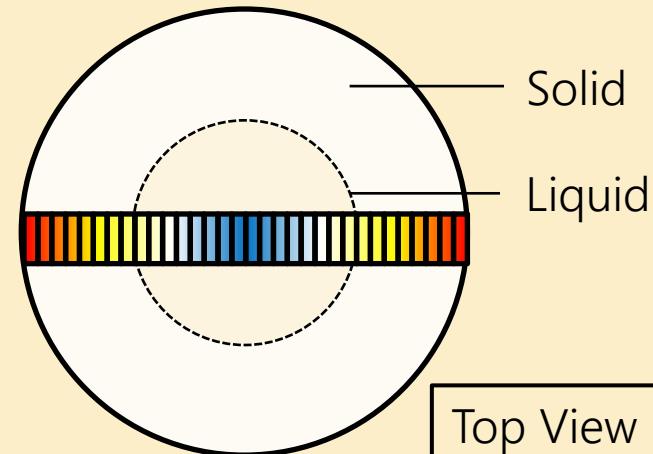
Moment of Inertia of Liquid

$$\int_{-a}^a \frac{1}{2} \pi \rho_{White} R_{Liquid}^4 dx + \frac{2}{5} \frac{4}{3} (\rho_{Yolk} - \rho_{White}) \pi R_{Yolk}^5$$

Egg White

Egg Yolk

When $R_{Liquid} > R_{Yolk}$



Top View

Theory

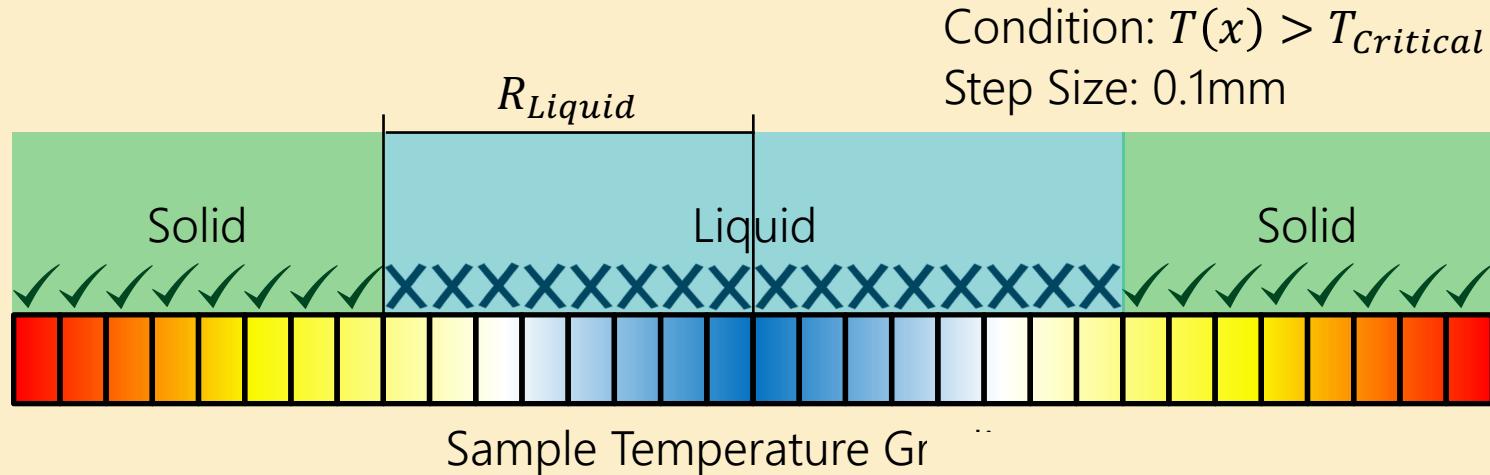
Measurement

Sensitivity

Conclusion

Thermodynamic Model

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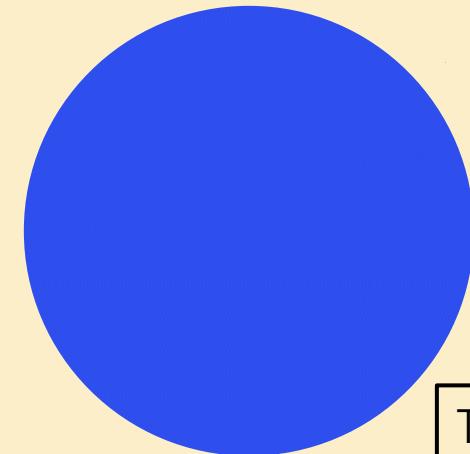
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Egg White

Egg Yolk

When $R_{Liquid} > R_{Yolk}$



Top View

Theory

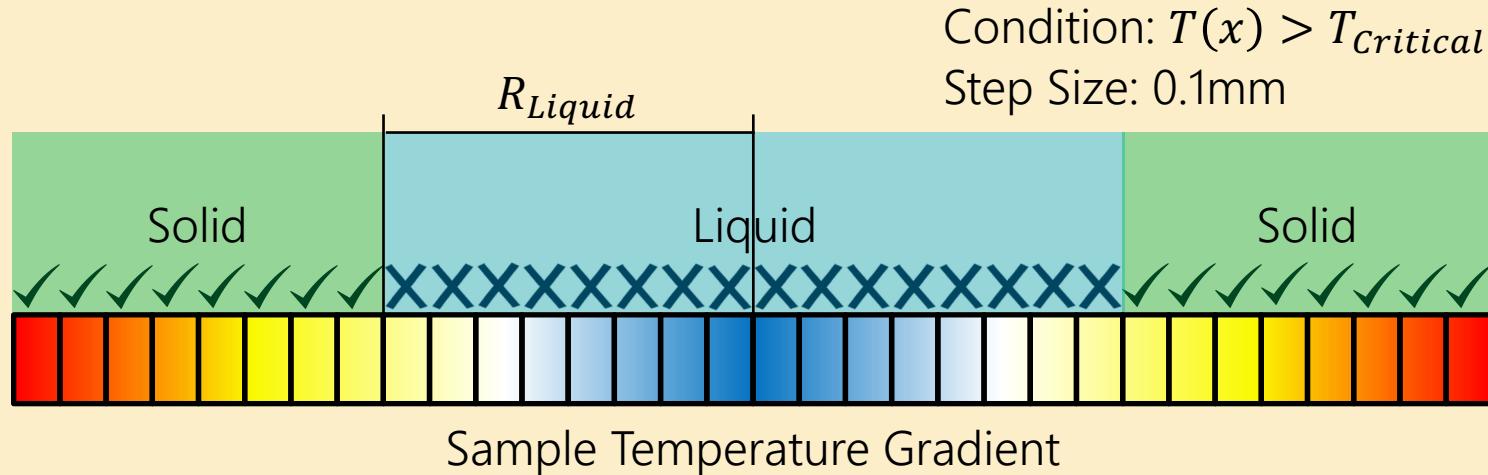
Measurement

Sensitivity

Conclusion

Thermodynamic Model

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Moment of Inertia of Liquid

$$\int_{-a}^a \frac{1}{2} \pi \rho_{White} R_{Liquid}^4 dx +$$

Egg White

$$+ \frac{2}{5} \frac{4}{3} (\rho_{Yolk} - \rho_{White}) \pi R_{Yolk}^5$$

Egg Yolk

Mass of Liquid

$$\int_{-a}^a \pi \rho_{White} R_{Liquid}^2 dx +$$
$$+ \frac{4}{3} (\rho_{Yolk} - \rho_{White}) \pi R_{Yolk}^3$$

Egg White + Yolk

When $R_{Liquid} > R_{Yolk}$

Theory

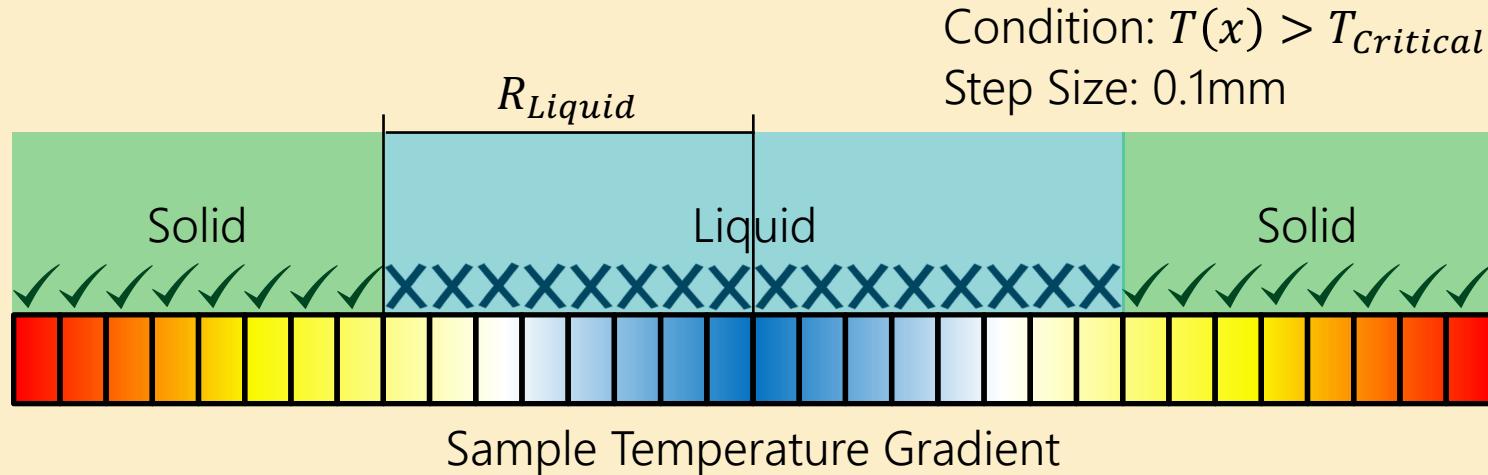
Measurement

Sensitivity

Conclusion

Thermodynamic Model

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Moment of Inertia of Liquid

$$\frac{2}{5} \frac{4}{3} \rho_{Yolk} \pi R_{Liquid}^5$$

Egg Yolk

Mass of Liquid

$$\frac{4}{3} \rho_{Yolk} \pi R_{Liquid}^3$$

Egg Yolk

When $R_{Liquid} < R_{Yolk}$

Theory

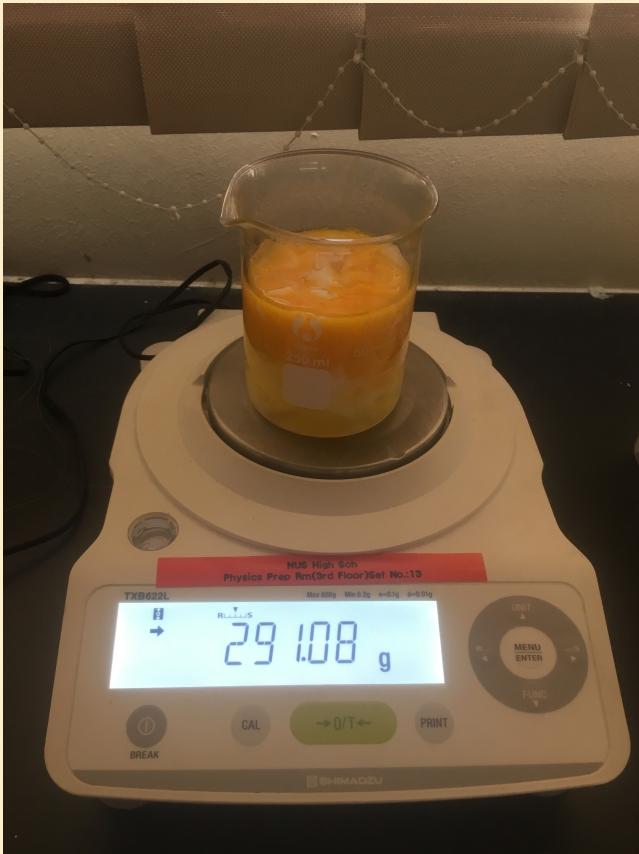
Measurement

Sensitivity

Conclusion

Experimental Set-Up

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Eggs are cracked

Liquid content poured into beaker

Theory

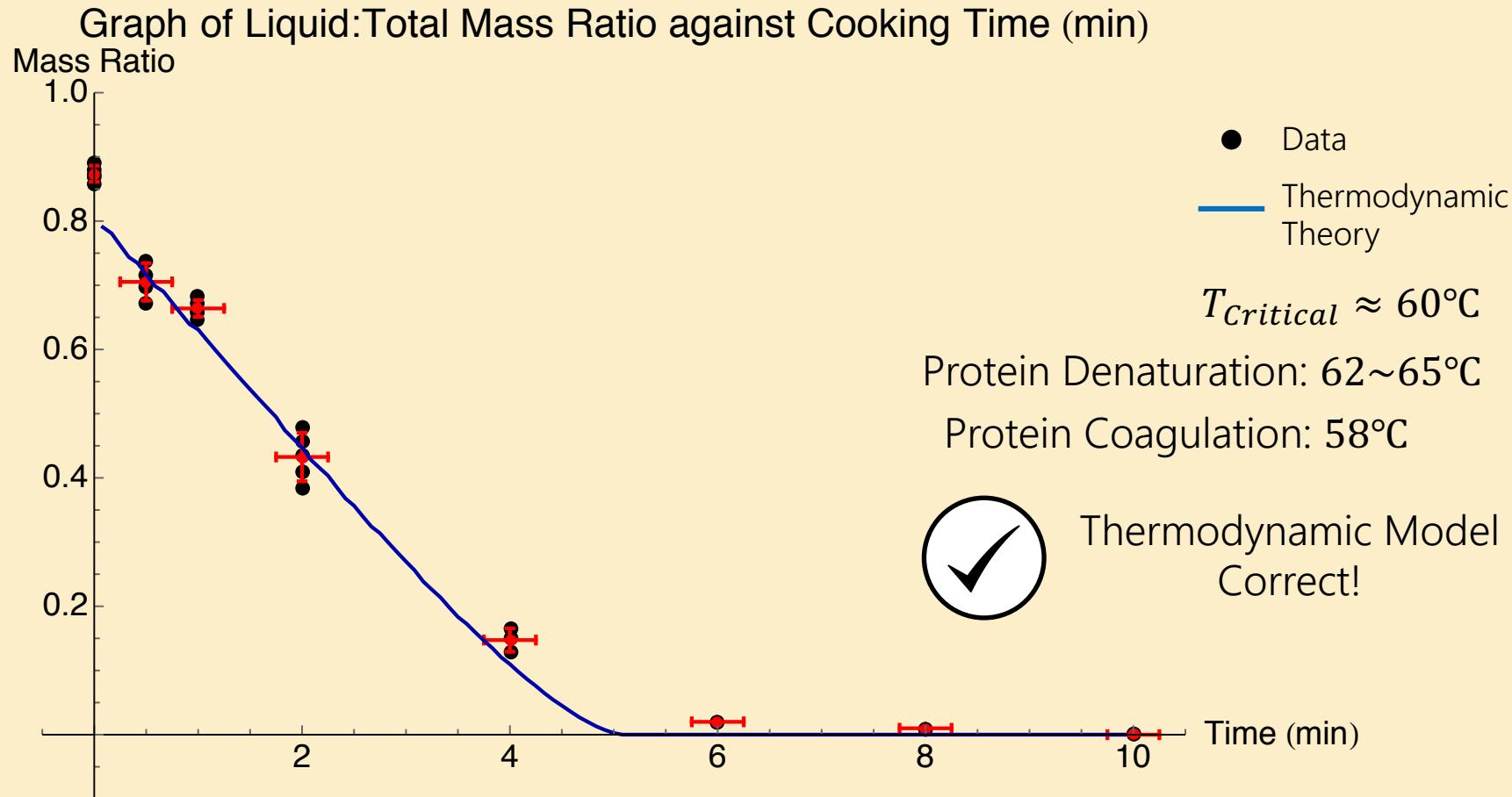
Measurement

Sensitivity

Conclusion

Thermodynamic Model

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Krzysztof J. OPIELINSKI, "ULTRASONIC PARAMETERS OF HEN'S EGG", Molecular and Quantum Acoustics vol. 28 (2007)

Theory

Measurement

Sensitivity

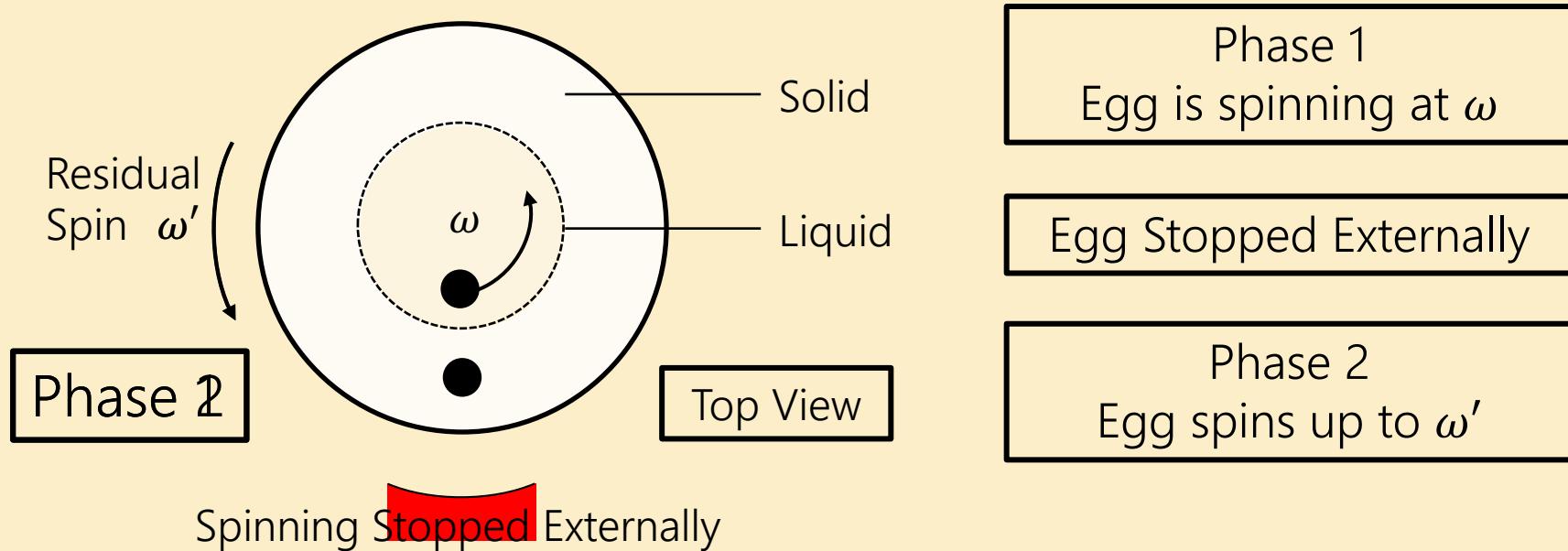
Conclusion

Residual Spin

Manual
Motorized

Residual Spin

18



Assumptions

All the liquid mass are spinning at same ω

No Viscous Losses during Phase 2

No Liquid Boundary Layer

Egg Characterization

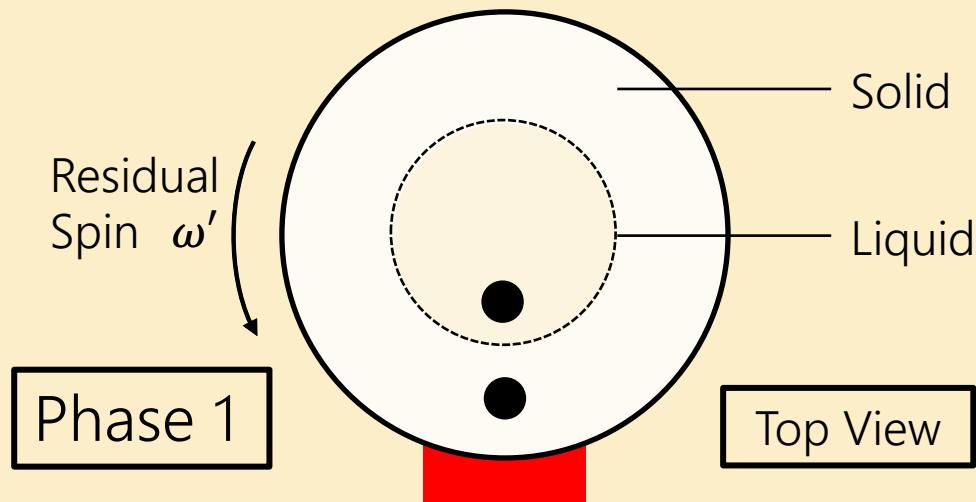
Dynamic Response

Sensitivity

Conclusion

Residual Spin

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Spinning Stopped Externally

Residual Angular Momentum
 $L = I_{Liquid}\omega = I_{Egg}\omega'$

Residual Spin Ratio
 $\frac{\omega'}{\omega} = \frac{I_{Liquid}}{I_{Egg}}$

Moment of Inertia of Liquid

$$I_{Egg} = \int_{-a}^a \frac{1}{2} \pi \rho_{White} y(x)^4 dx + \frac{2}{5} \frac{4}{3} (\rho_{Yolk} - \rho_{White}) \pi R_{Yolk}^5$$

$$I_{Liquid} = \int_{-a}^a \frac{1}{2} \pi \rho_{White} R_{Liquid}^4 dx + \frac{2}{5} \frac{4}{3} (\rho_{Yolk} - \rho_{White}) \pi R_{Yolk}^5$$

Egg Characterization

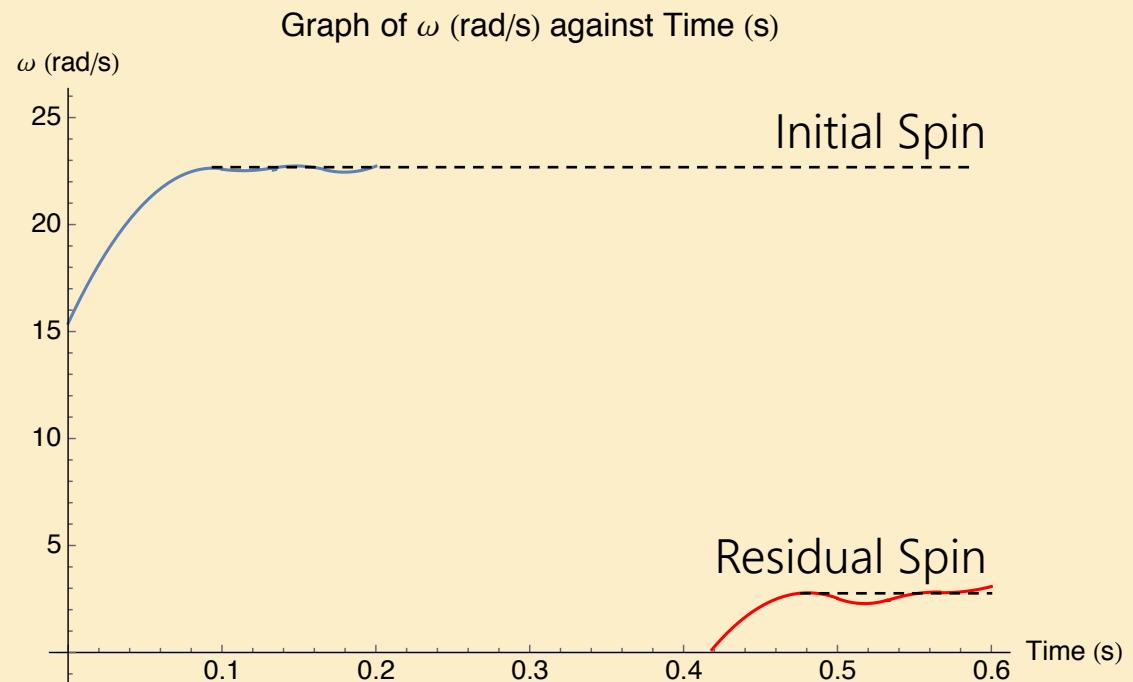
Dynamic Response

Sensitivity

Conclusion

Experimental

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Egg Characterization

Dynamic Response

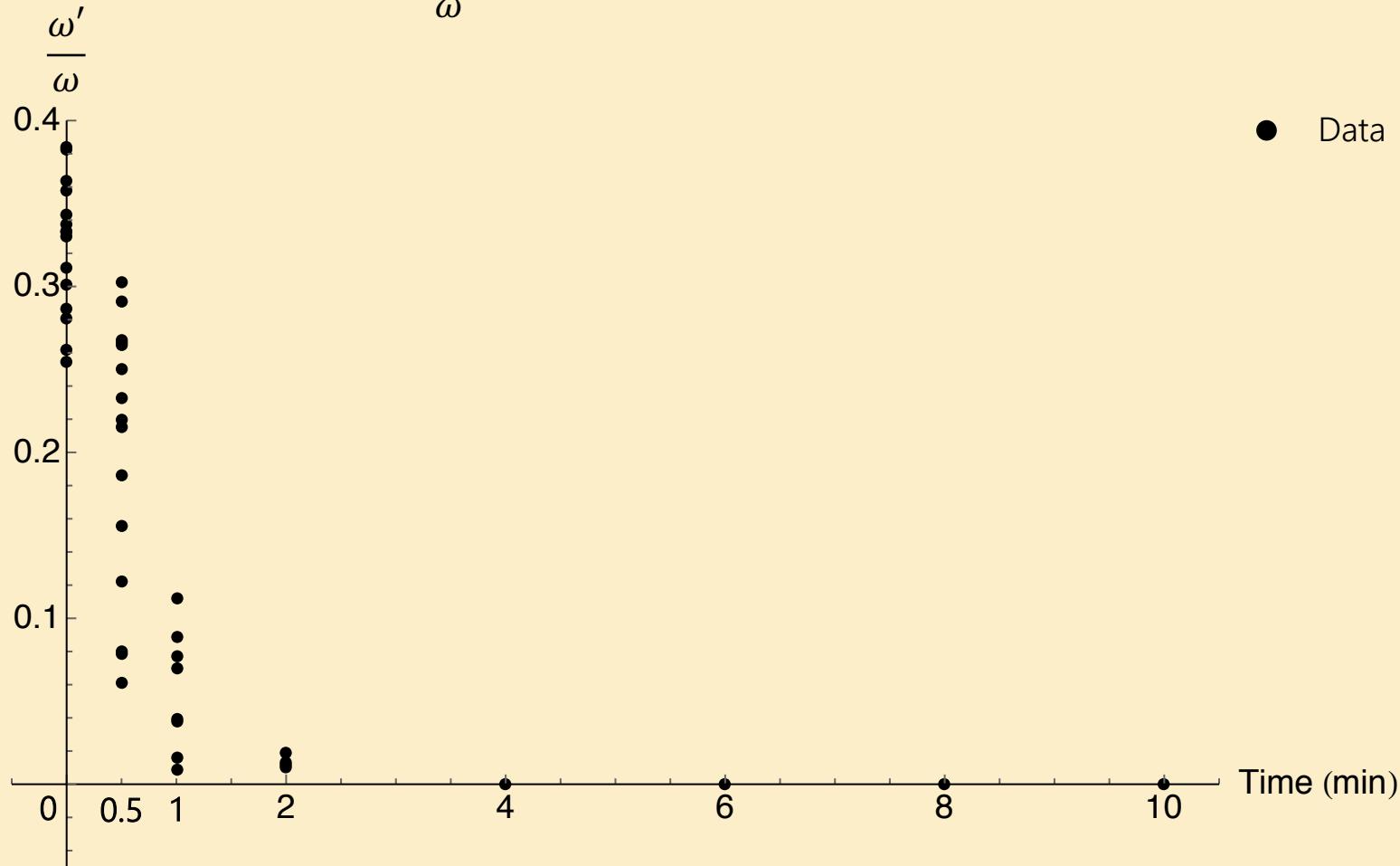
Sensitivity

Conclusion

Residual Spin

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Graph of $\frac{\omega'}{\omega}$ against Cooking Time (min)



Egg Characterization

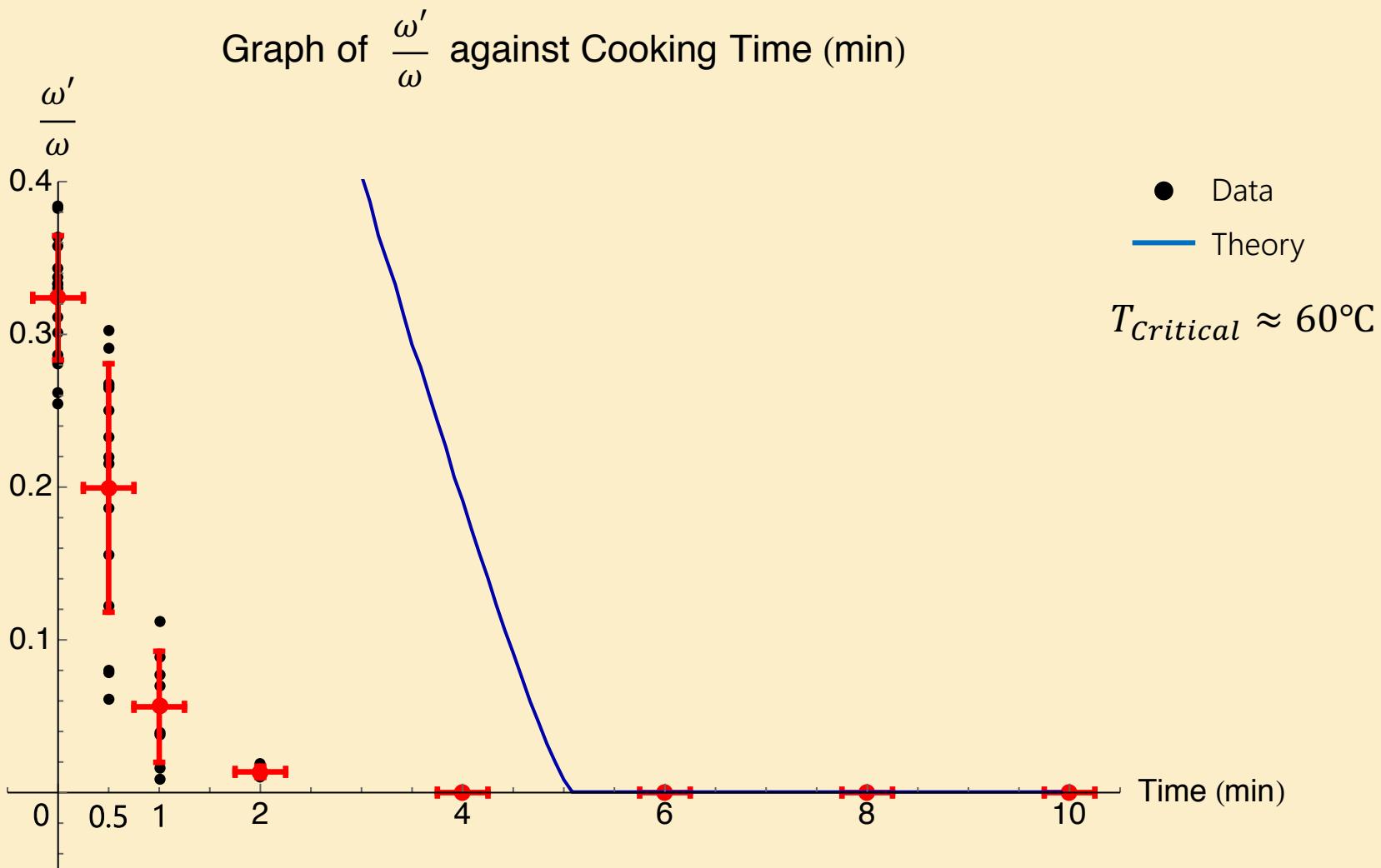
Dynamic Response

Sensitivity

Conclusion

Residual Spin

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Egg Characterization

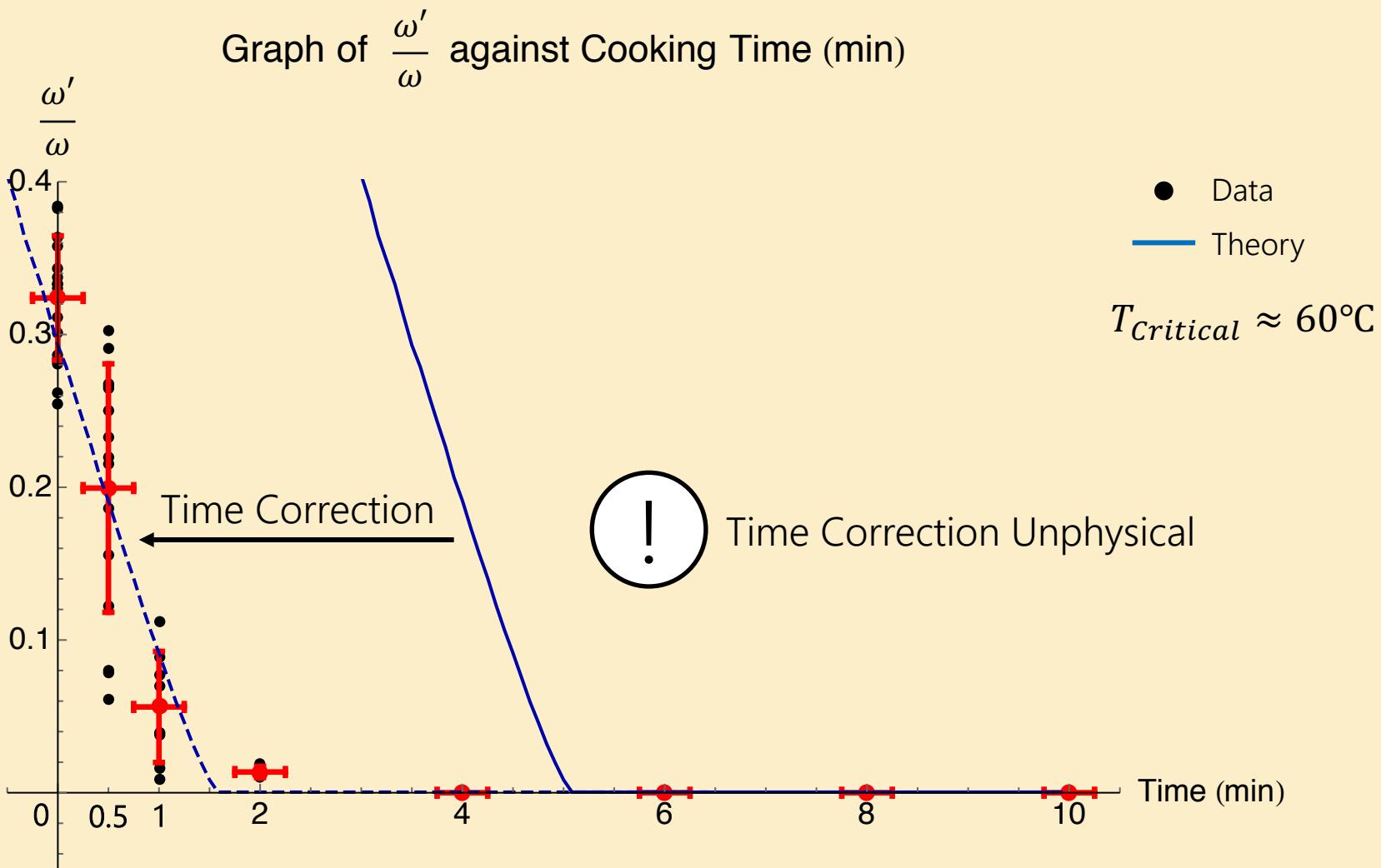
Dynamic Response

Sensitivity

Conclusion

Residual Spin

23



Egg Characterization

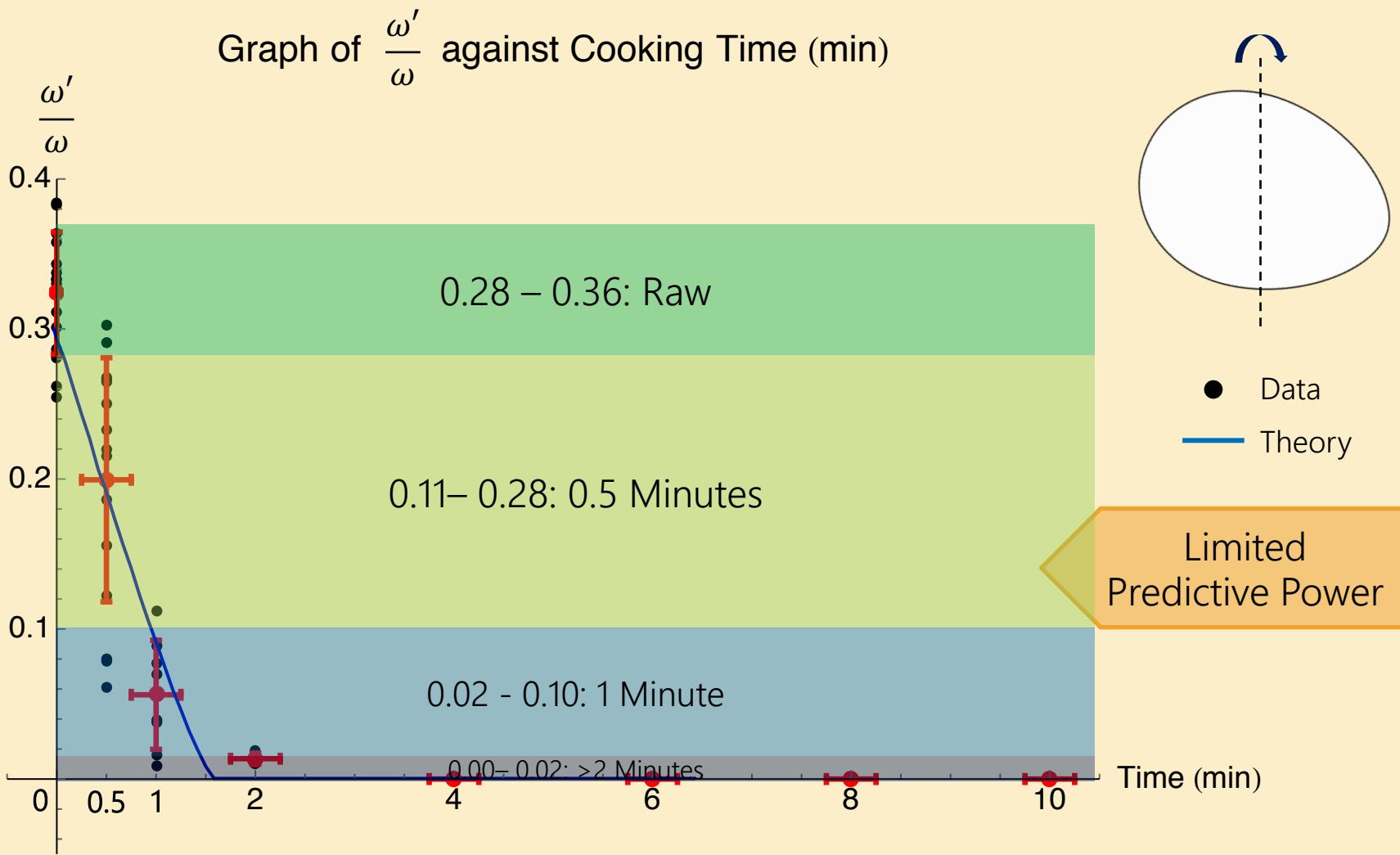
Dynamic Response

Sensitivity

Conclusion

Residual Spin

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Egg Characterization

Dynamic Response

Sensitivity

Conclusion

Rationale for Time Correction

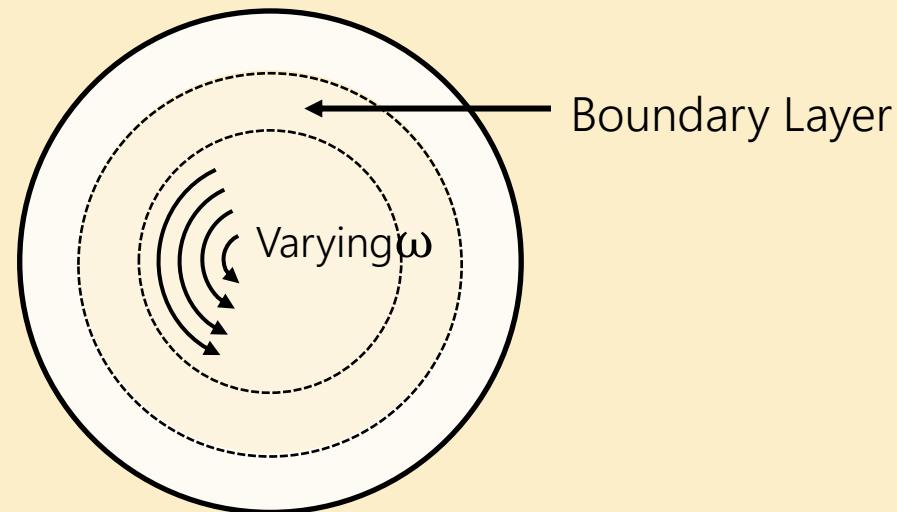
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Assumptions

All the liquid mass are spinning at same ω

No Viscous Losses during Phase 2

No Liquid Boundary Layer



Egg Characterization

Dynamic Response

Sensitivity

Conclusion

Fluid Dynamics

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Continuity Equation

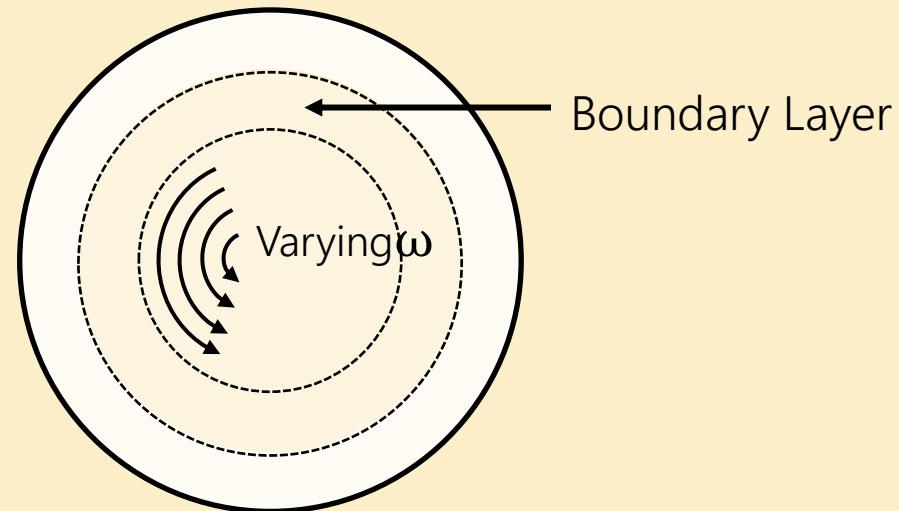
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

Navier-Stokes Equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{\mu}{\rho} \nabla^2 \mathbf{u} = -\nabla w + \mathbf{g}$$

Turbulence

Viscosity



Egg Characterization

Dynamic Response

Sensitivity

Conclusion

Fluid Dynamics

27

Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

1D Navier-Stokes Equation

$$\rho \frac{\partial u_\theta}{\partial t} = \mu \left(\frac{1}{r} \frac{\partial u_\theta}{\partial r} + \frac{\partial^2 u_\theta}{\partial r^2} + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{u_\theta}{r^2} \right)$$

Angular Momentum Retained

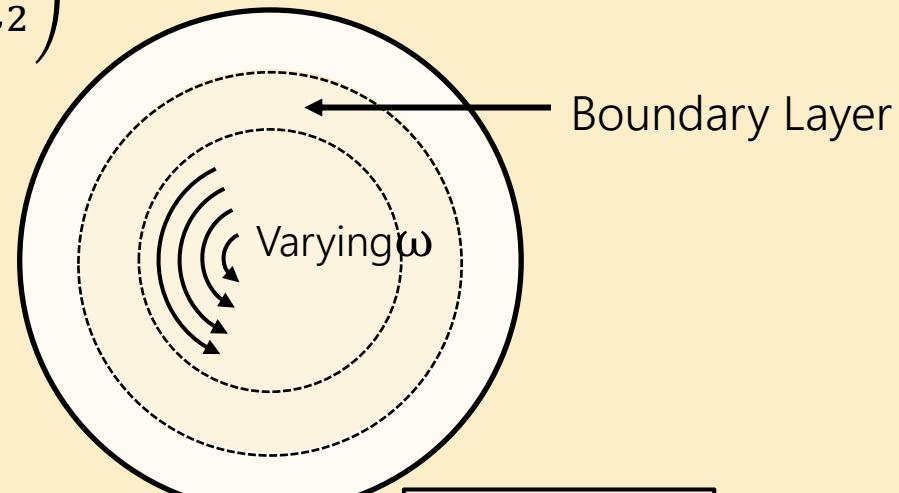
$$L = \int_0^{2\pi} \int_0^{y(x)} \int_0^a \rho r^3 \omega(z, r) dz dr d\theta$$

Navier-Stokes Equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{\mu}{\rho} \nabla^2 \mathbf{u} = -\nabla w + \mathbf{g}$$

Turbulence

Viscosity



Egg Characterization

Dynamic Response

Sensitivity

Conclusion

Fluid Dynamics

28

Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

1D Navier-Stokes Equation

$$\rho \frac{\partial u_\theta}{\partial t} = \mu \left(\frac{1}{r} \frac{\partial u_\theta}{\partial r} + \frac{\partial^2 u_\theta}{\partial r^2} + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{u_\theta}{r^2} \right)$$

Angular Momentum Retained

$$L = \int_0^{2\pi} \int_0^{y(x)} \int_0^a \rho r^3 \omega(z, r) dz dr d\theta$$

Navier-Stokes Equation

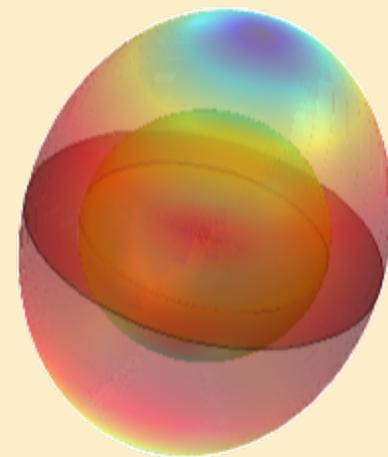
$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{\mu}{\rho} \nabla^2 \mathbf{u} = -\nabla w + \mathbf{g}$$

Turbulence

Viscosity

Residual Spin

$$\frac{\omega'}{\omega} = \frac{I_{Liquid}}{I_{Egg}}$$



Velocity Field

Egg Characterization

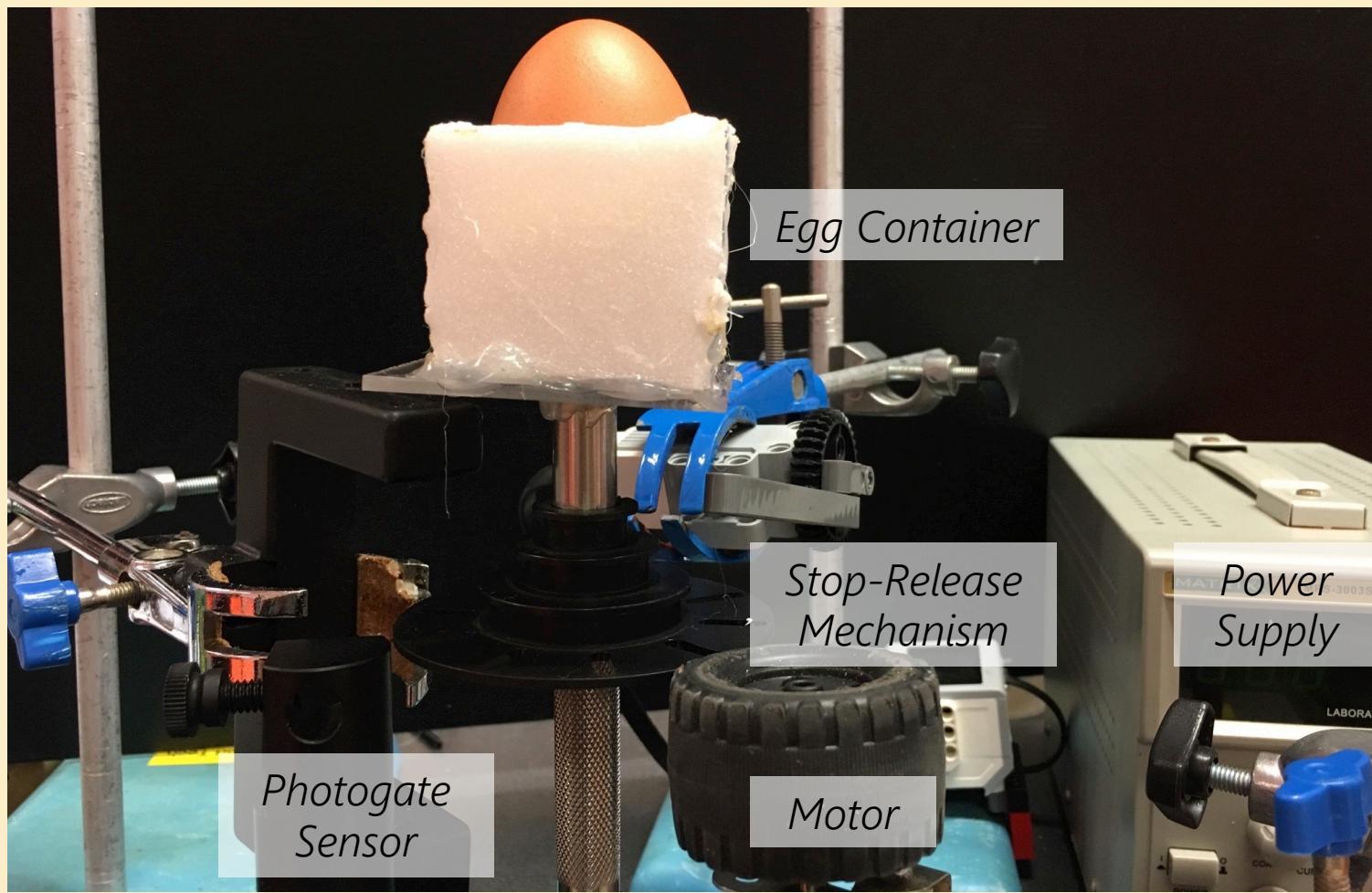
Dynamic Response

Sensitivity

Conclusion

Experimental Set-Up

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Egg Characterization

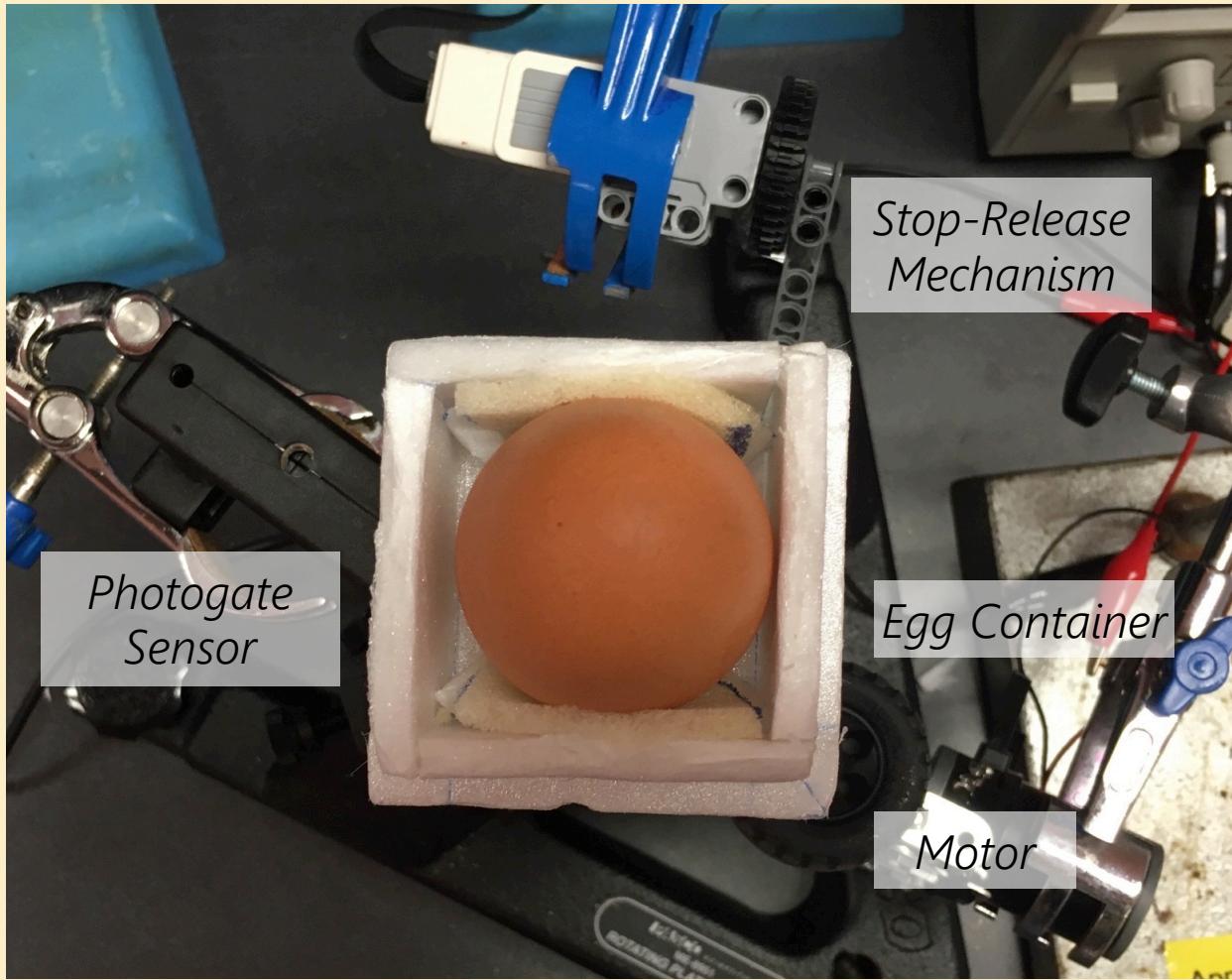
Dynamic Response

Sensitivity

Conclusion

Experimental Set-Up

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Egg Characterization

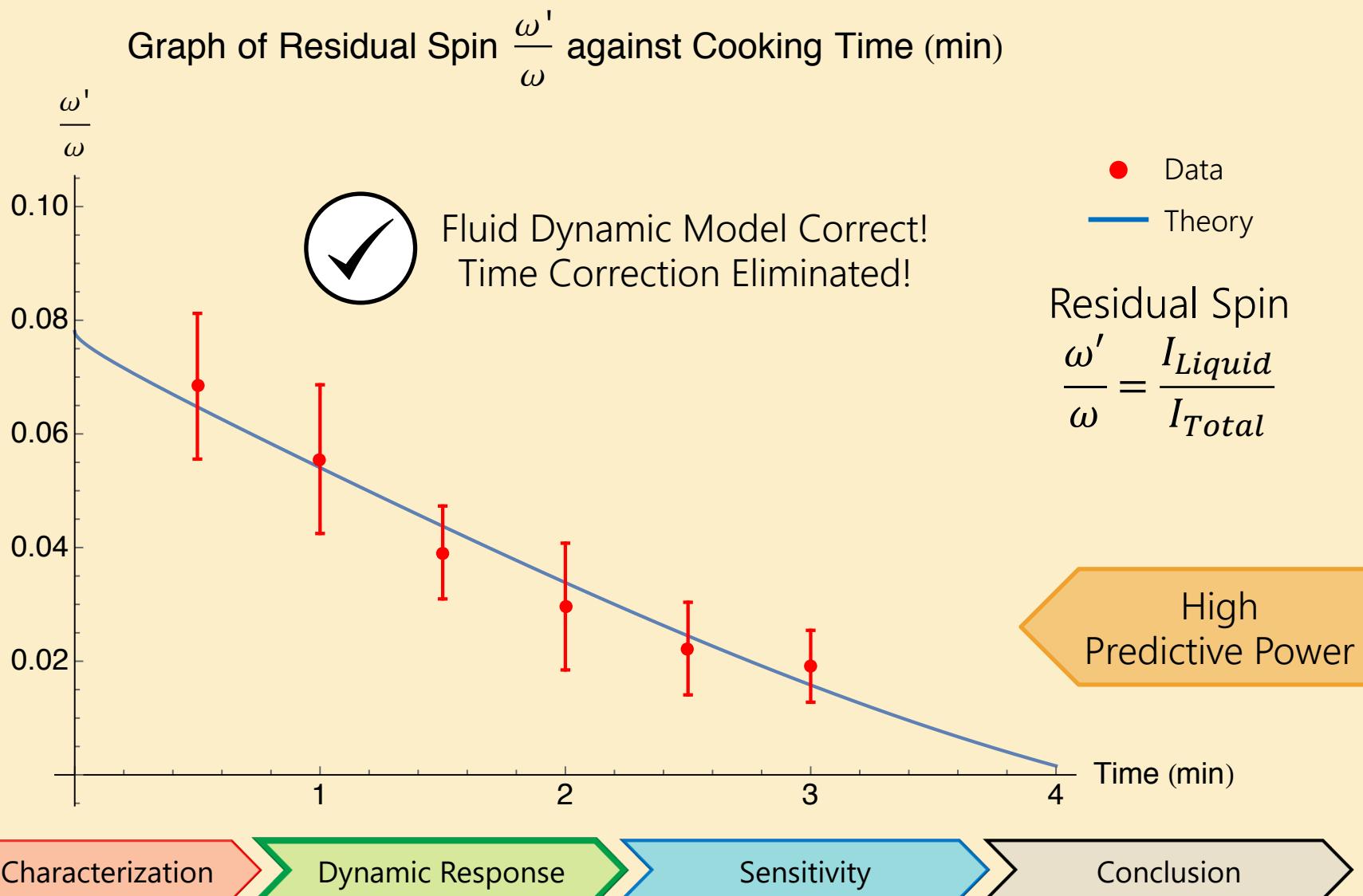
Dynamic Response

Sensitivity

Conclusion

Residual Spin

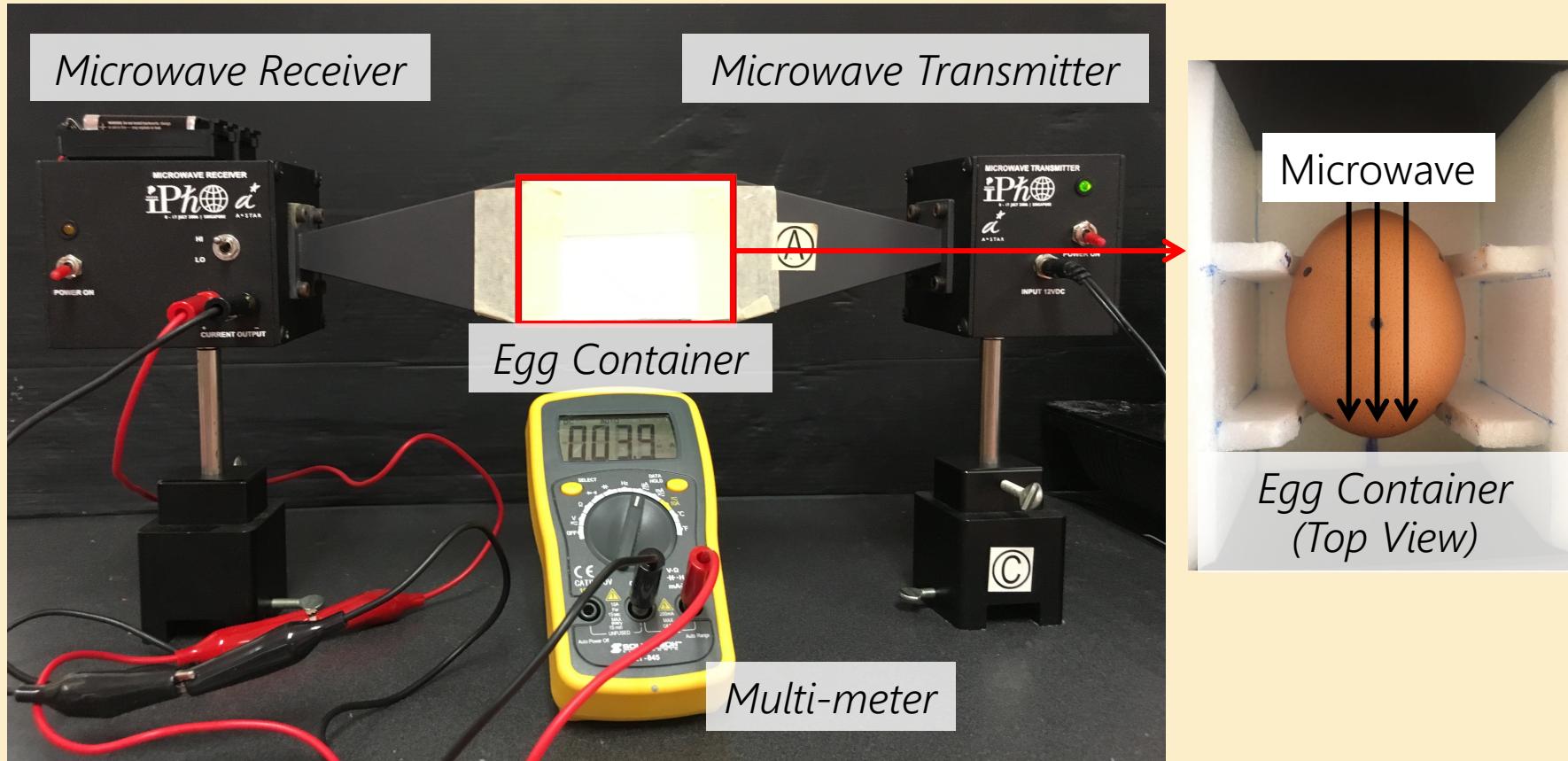
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Microwave & Ultrasound

Experimental Set-Up

33



Egg Characterization

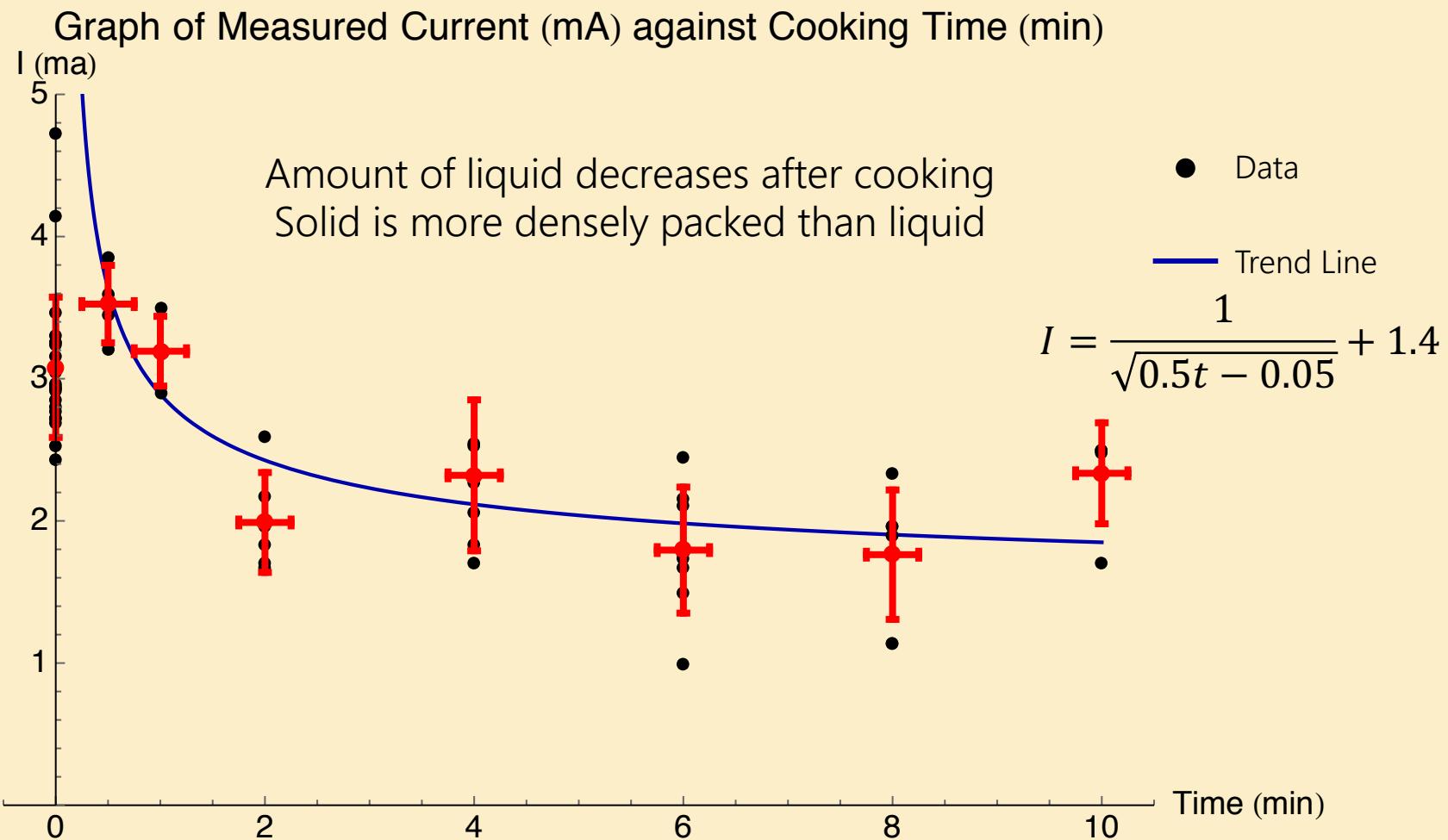
Microwave

Sensitivity

Conclusion

Microwave

34



Egg Characterization

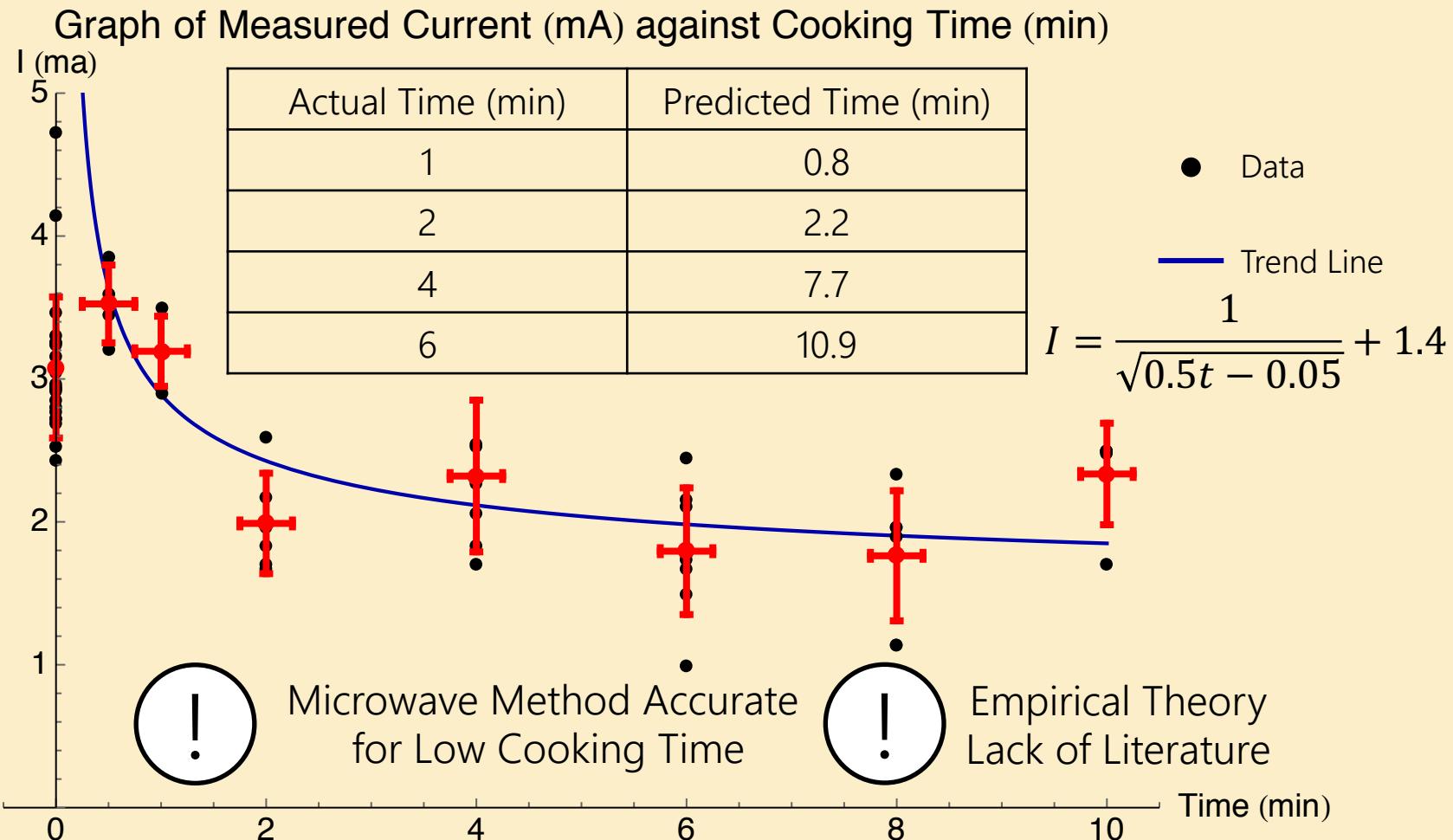
Microwave

Sensitivity

Conclusion

Microwave

35



Egg Characterization

Microwave

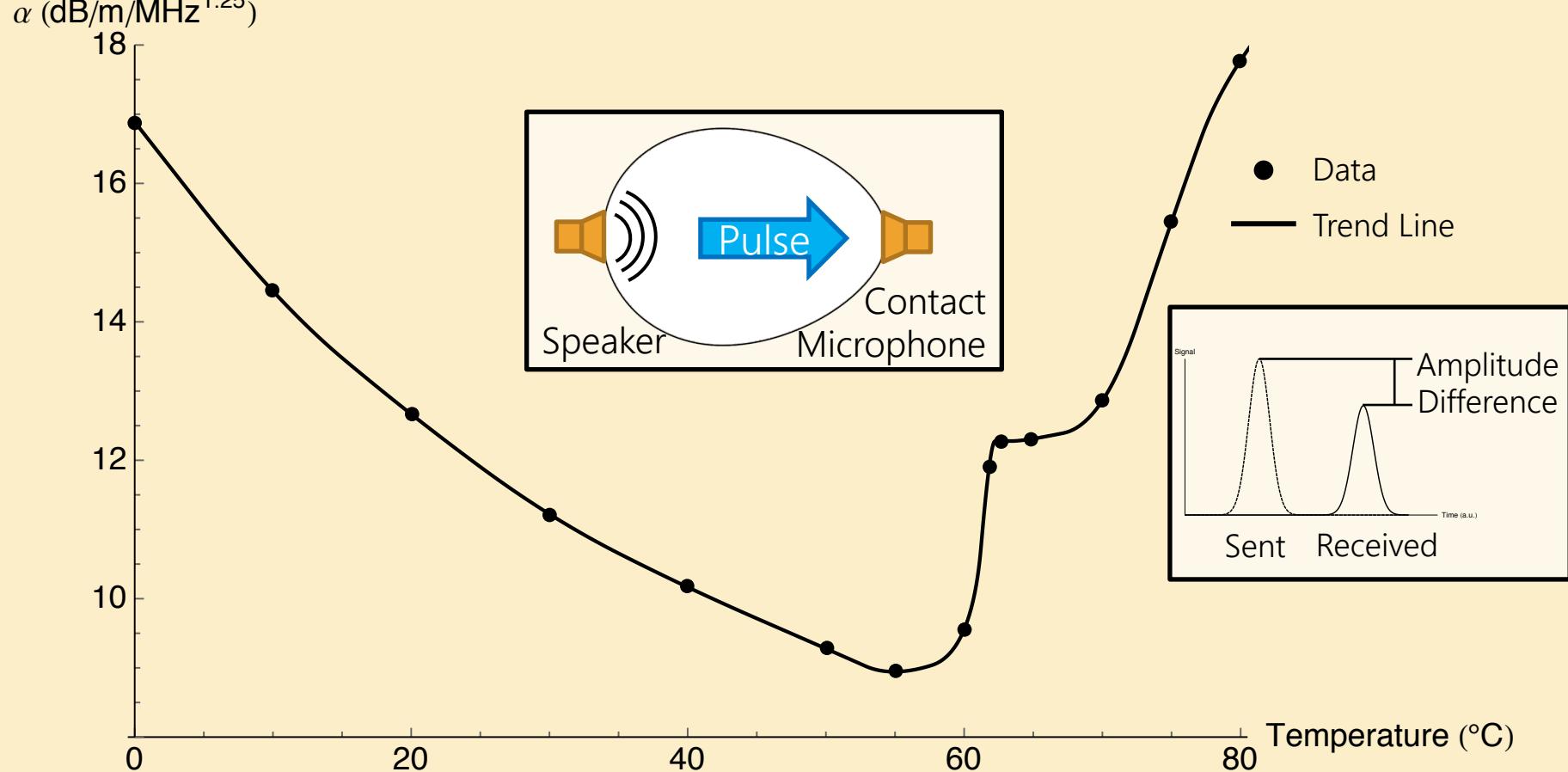
Sensitivity

Conclusion

Ultrasonic Attenuation

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Graph of Attenuation Coefficeint ($\text{dB}/\text{m}/\text{MHz}^{1.25}$) against Temperature ($^{\circ}\text{C}$)



Krzysztof J. OPIELINSKI, "ULTRASONIC PARAMETERS OF HEN'S EGG", Molecular and Quantum Acoustics vol. 28 (2007)

Egg Characterization

Ultrasound

Sensitivity

Conclusion

Protein Denaturation

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Graph of Attenuation Coefficeint ($\text{dB}/\text{m}/\text{MHz}^{1.25}$) against Temperature ($^{\circ}\text{C}$)

α ($\text{dB}/\text{m}/\text{MHz}^{1.25}$)

18

16

14

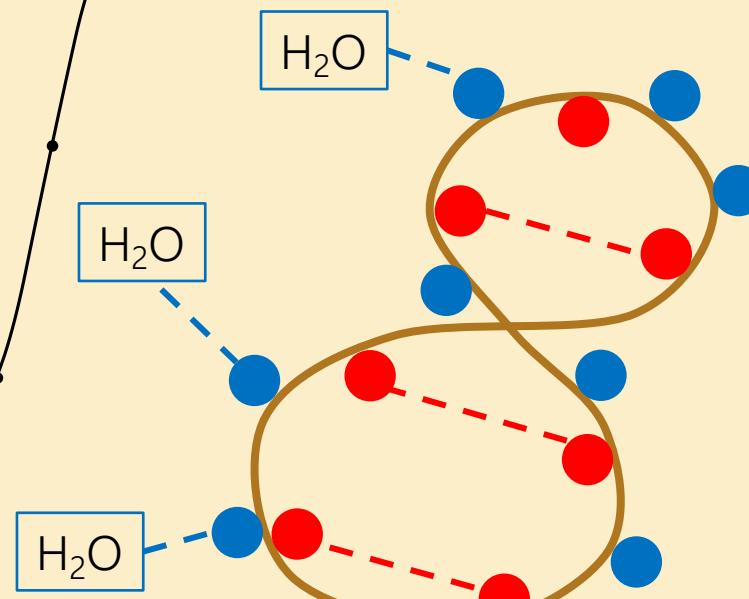
12

10

0

Temperature ($^{\circ}\text{C}$)

- Hydrophobic
- Hydrophilic



Egg Characterization

Ultrasound

Sensitivity

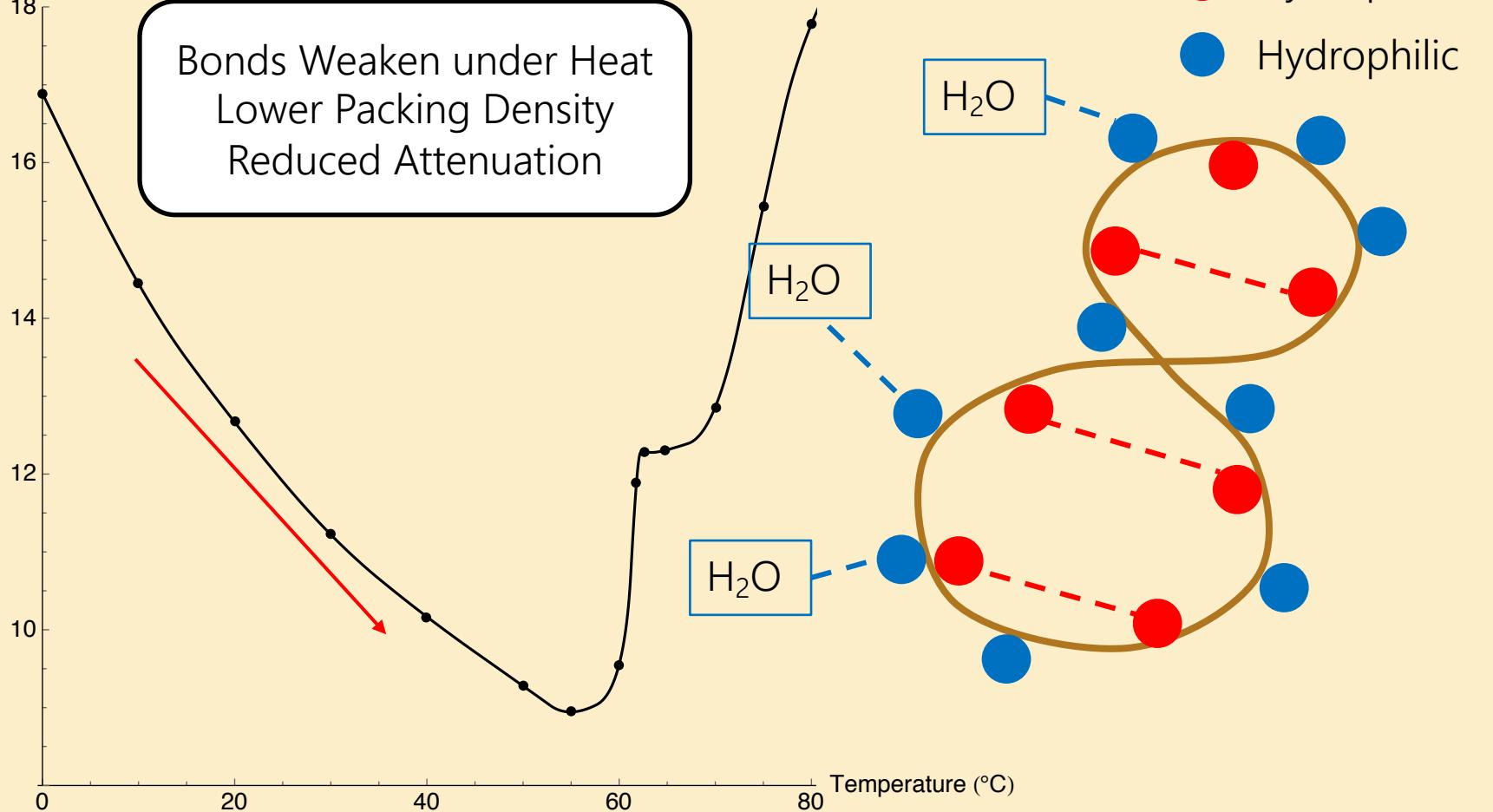
Conclusion

Protein Denaturation

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Graph of Attenuation Coefficeint ($\text{dB}/\text{m}/\text{MHz}^{1.25}$) against Temperature ($^{\circ}\text{C}$)

α ($\text{dB}/\text{m}/\text{MHz}^{1.25}$)



Egg Characterization

Ultrasound

Sensitivity

Conclusion

Protein Denaturation

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Graph of Attenuation Coefficeint ($\text{dB}/\text{m}/\text{MHz}^{1.25}$) against Temperature ($^{\circ}\text{C}$)

α ($\text{dB}/\text{m}/\text{MHz}^{1.25}$)

18

16

14

12

10

0

Bonds Break
Lowest Density Packing
Protein Unwinds

Denaturation

- Hydrophobic
- Hydrophilic

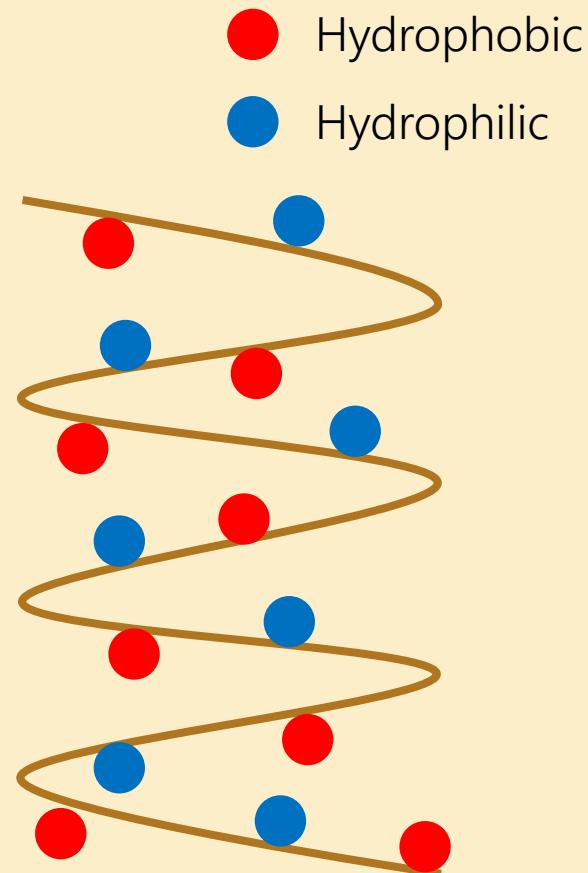
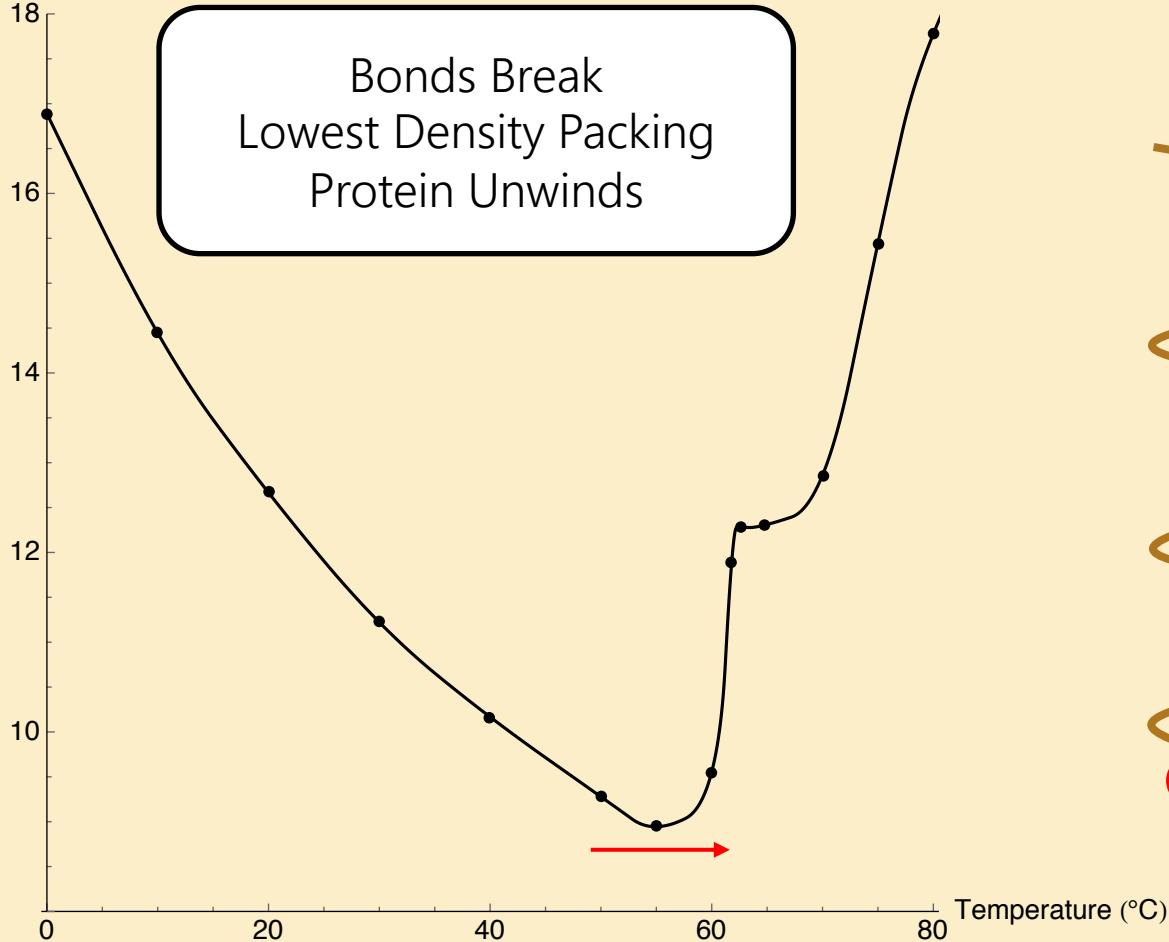
Temperature ($^{\circ}\text{C}$)



Protein Denaturation

40

Graph of Attenuation Coefficeint ($\text{dB}/\text{m}/\text{MHz}^{1.25}$) against Temperature ($^{\circ}\text{C}$)
 α ($\text{dB}/\text{m}/\text{MHz}^{1.25}$)



Egg Characterization

Ultrasound

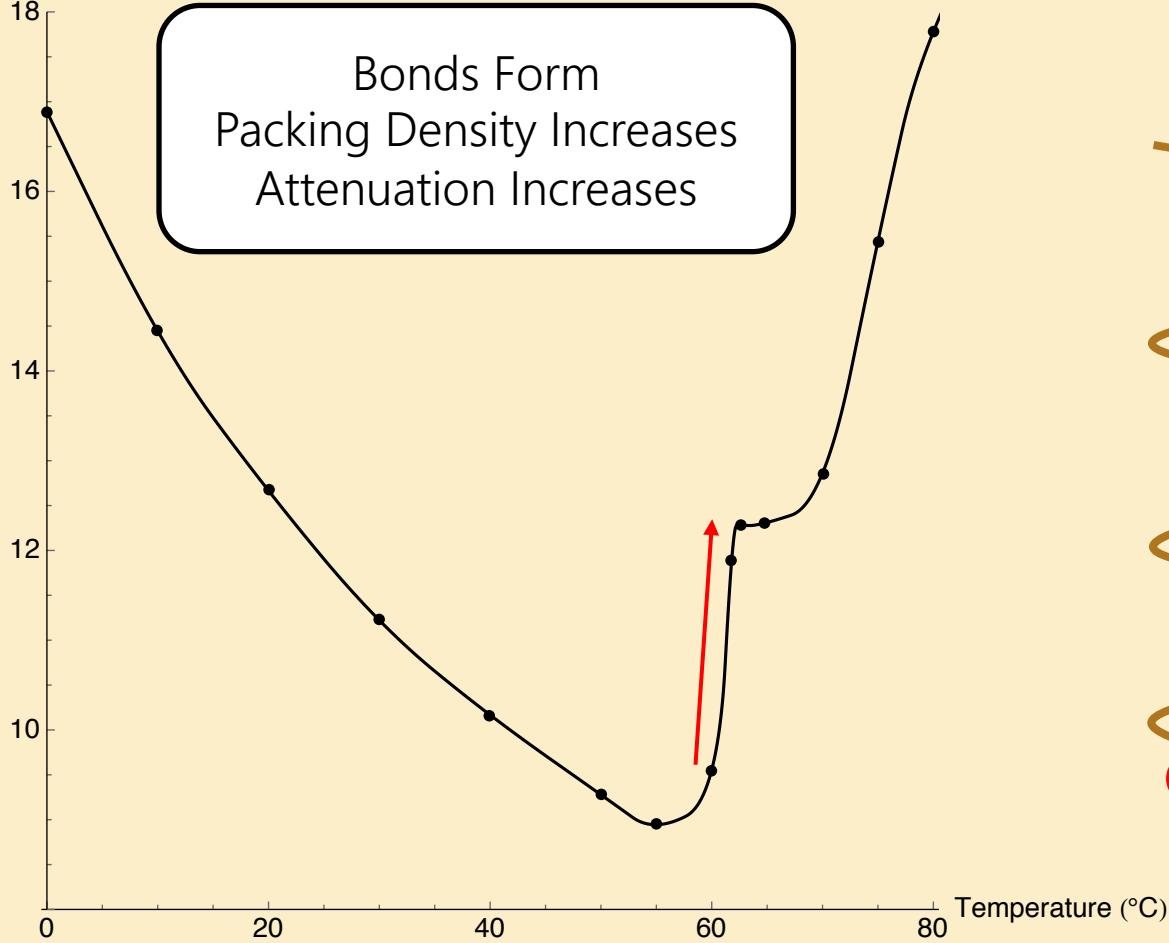
Sensitivity

Conclusion

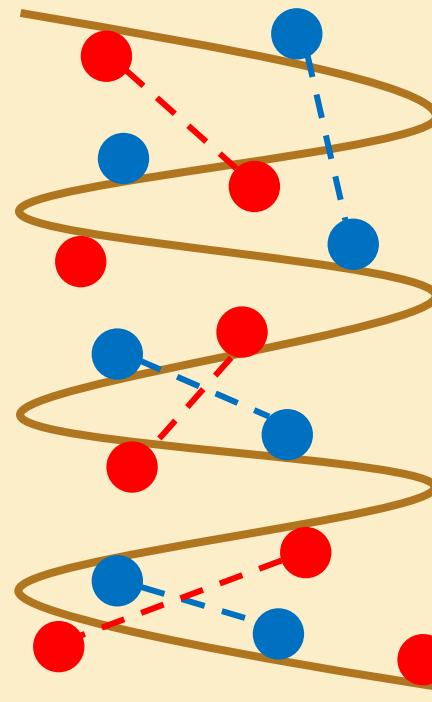
Protein Denaturation

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Graph of Attenuation Coefficeint ($\text{dB}/\text{m}/\text{MHz}^{1.25}$) against Temperature ($^{\circ}\text{C}$)
 α ($\text{dB}/\text{m}/\text{MHz}^{1.25}$)



- Hydrophobic
- Hydrophilic



Egg Characterization

Ultrasound

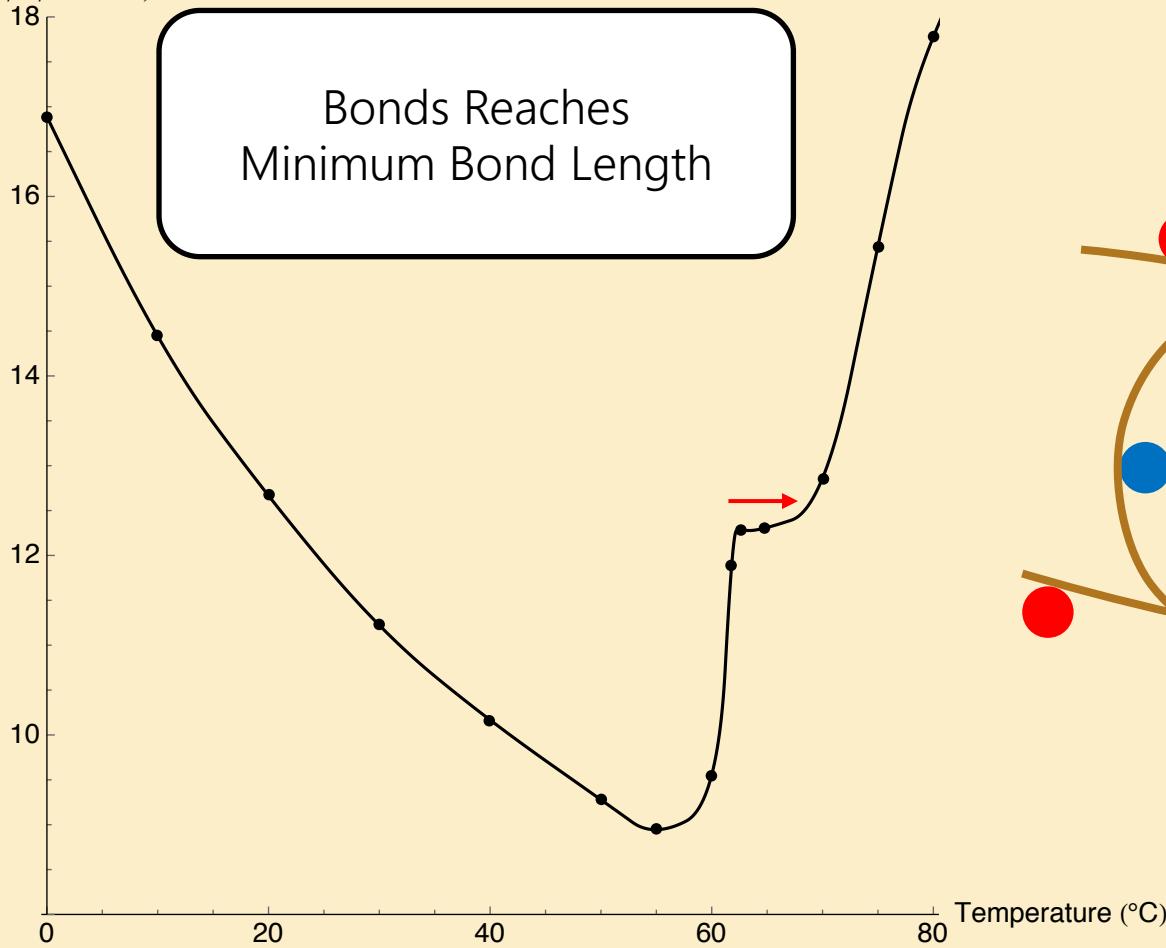
Sensitivity

Conclusion

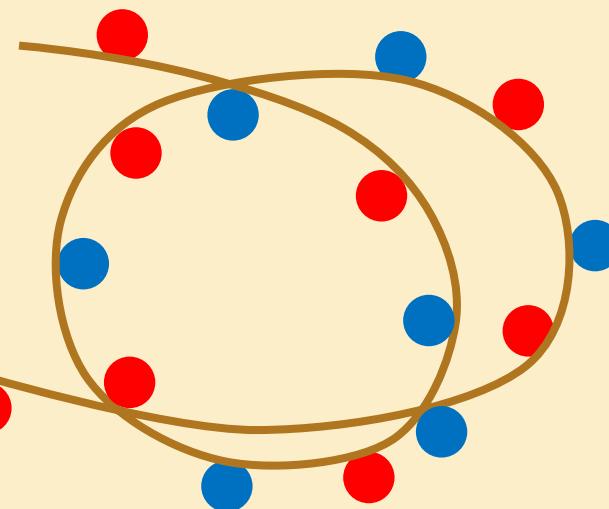
Protein Denaturation

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Graph of Attenuation Coefficeint ($\text{dB}/\text{m}/\text{MHz}^{1.25}$) against Temperature ($^{\circ}\text{C}$)



- Hydrophobic
- Hydrophilic



Egg Characterization

Ultrasound

Sensitivity

Conclusion

Protein Agglutination

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Graph of Attenuation Coefficeint ($\text{dB}/\text{m}/\text{MHz}^{1.25}$) against Temperature ($^{\circ}\text{C}$)

α ($\text{dB}/\text{m}/\text{MHz}^{1.25}$)

18

16

14

12

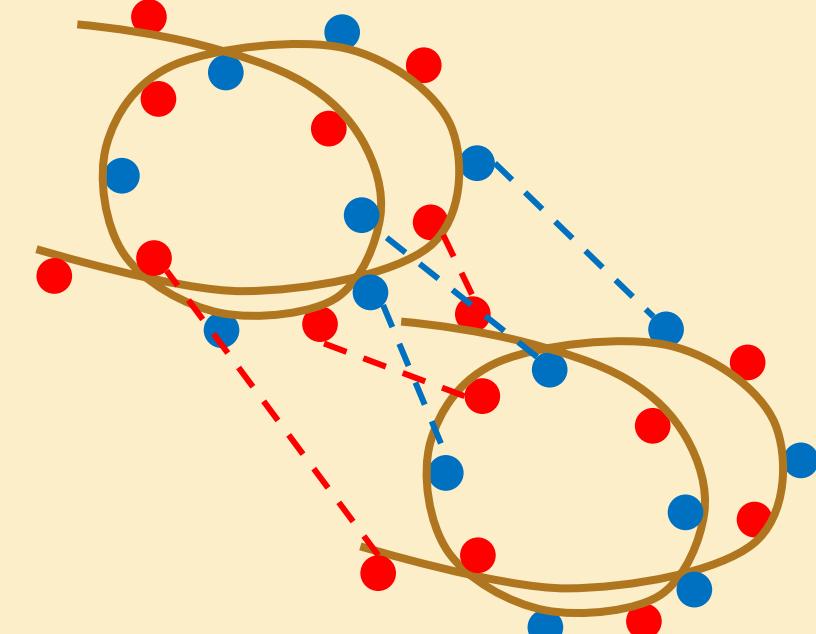
10

0

New Bonds form between Protein Molecules

Agglutination

- Hydrophobic
- Hydrophilic



Temperature ($^{\circ}\text{C}$)

Egg Characterization

Ultrasound

Sensitivity

Conclusion

Protein Agglutination

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Graph of Attenuation Coefficeint ($\text{dB}/\text{m}/\text{MHz}^{1.25}$) against Temperature ($^{\circ}\text{C}$)

α ($\text{dB}/\text{m}/\text{MHz}^{1.25}$)

18

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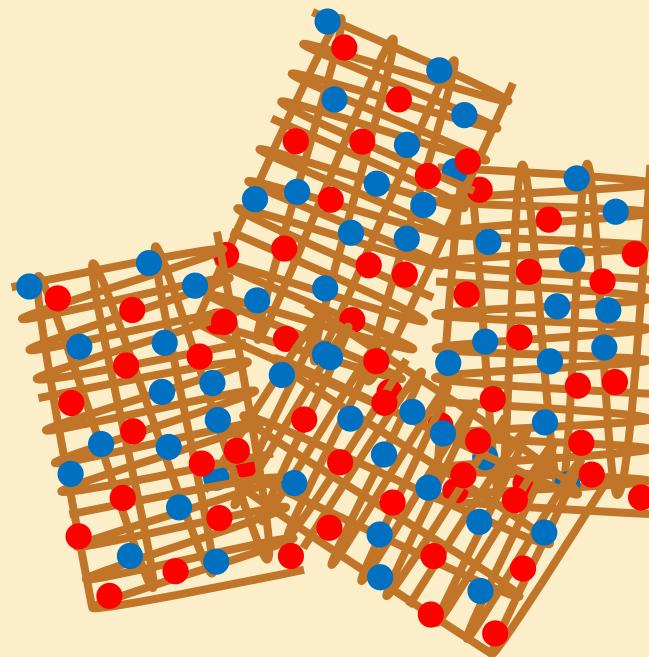
10

0

New Bonds form between Protein Molecules

Temperature ($^{\circ}\text{C}$) Gel-like Product

- Hydrophobic
- Hydrophilic



Egg Characterization

Ultrasound

Sensitivity

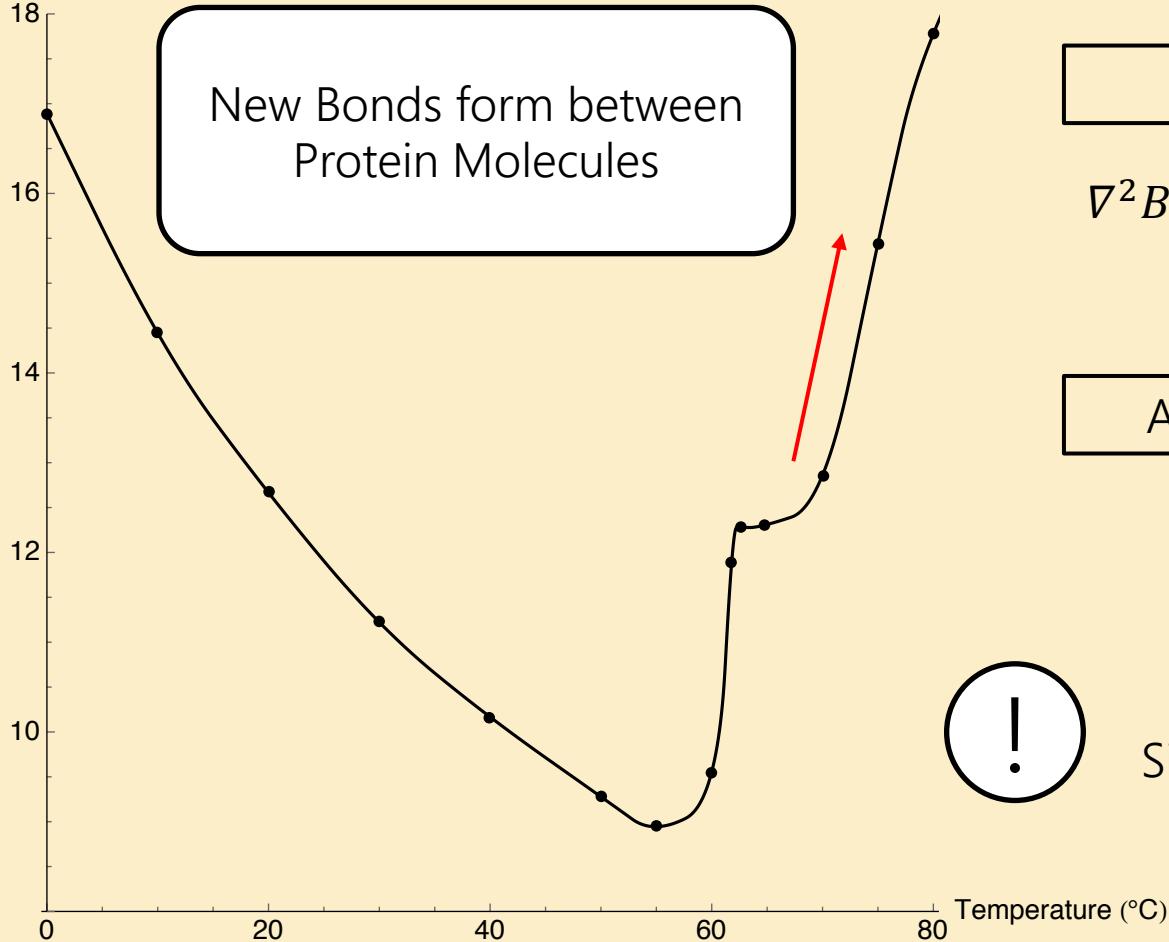
Conclusion

Protein Agglutination

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Graph of Attenuation Coefficeint ($\text{dB}/\text{m}/\text{MHz}^{1.25}$) against Temperature ($^{\circ}\text{C}$)

α ($\text{dB}/\text{m}/\text{MHz}^{1.25}$)



E.M. Wave Equation

$$\nabla^2 B = \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2} \quad \nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$$

Acoustic Wave Equation

$$\nabla^2 P = \frac{1}{c^2} \frac{\partial^2 P}{\partial t^2}$$



Similar Wave Equations
Similar Interaction with Matter

Egg Characterization

Ultrasound

Sensitivity

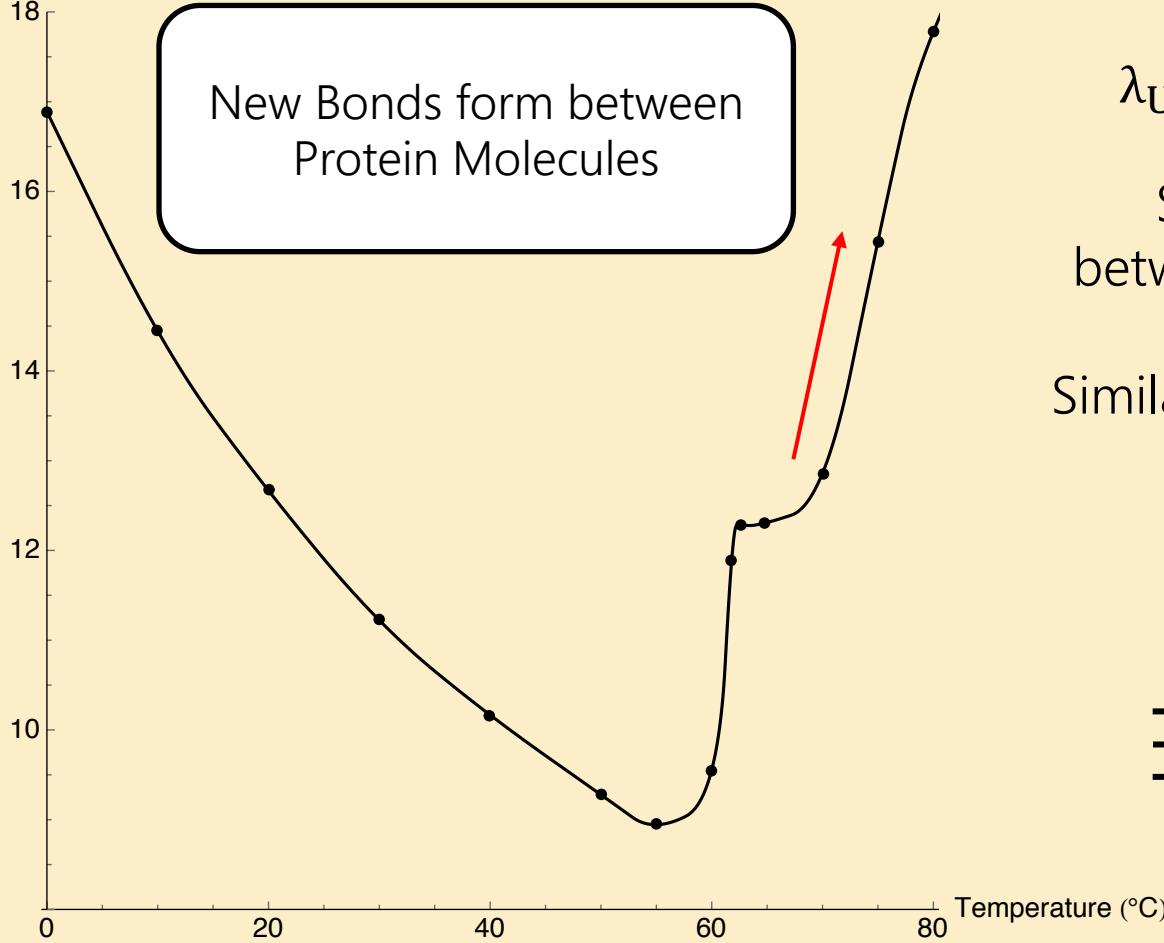
Conclusion

Protein Agglutination

46

Graph of Attenuation Coefficeint ($\text{dB}/\text{m}/\text{MHz}^{1.25}$) against Temperature ($^{\circ}\text{C}$)

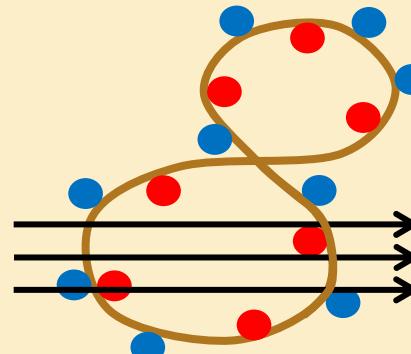
α ($\text{dB}/\text{m}/\text{MHz}^{1.25}$)



$\lambda_{\text{Ultrasound}} \approx \lambda_{\text{Microwave}}$

Similar Interactions
between Wave and Protein

Similar Trend for Attenuation



Egg Characterization

Ultrasound

Sensitivity

Conclusion

Protein Agglutination

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Beer-Lambert Law

$$I = I_0 e^{-\alpha x}$$

Boiled Egg

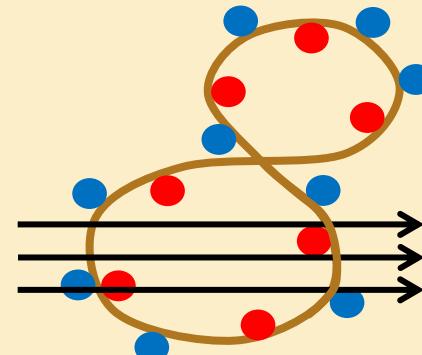
$$I = \prod_{n=1}^N I_0(1 - c)e^{-\alpha \frac{n a}{N}}$$

$$\lambda_{\text{Ultrasound}} \approx \lambda_{\text{Microwave}}$$

Similar Interactions
between Wave and Protein

Similar Trend for Attenuation

I	Output Intensity
I_0	Input Intensity
a	Length of Egg
N	No. of Segments
α	Attenuation Coefficient
c	Reflectance by Egg Shell



Egg Characterization

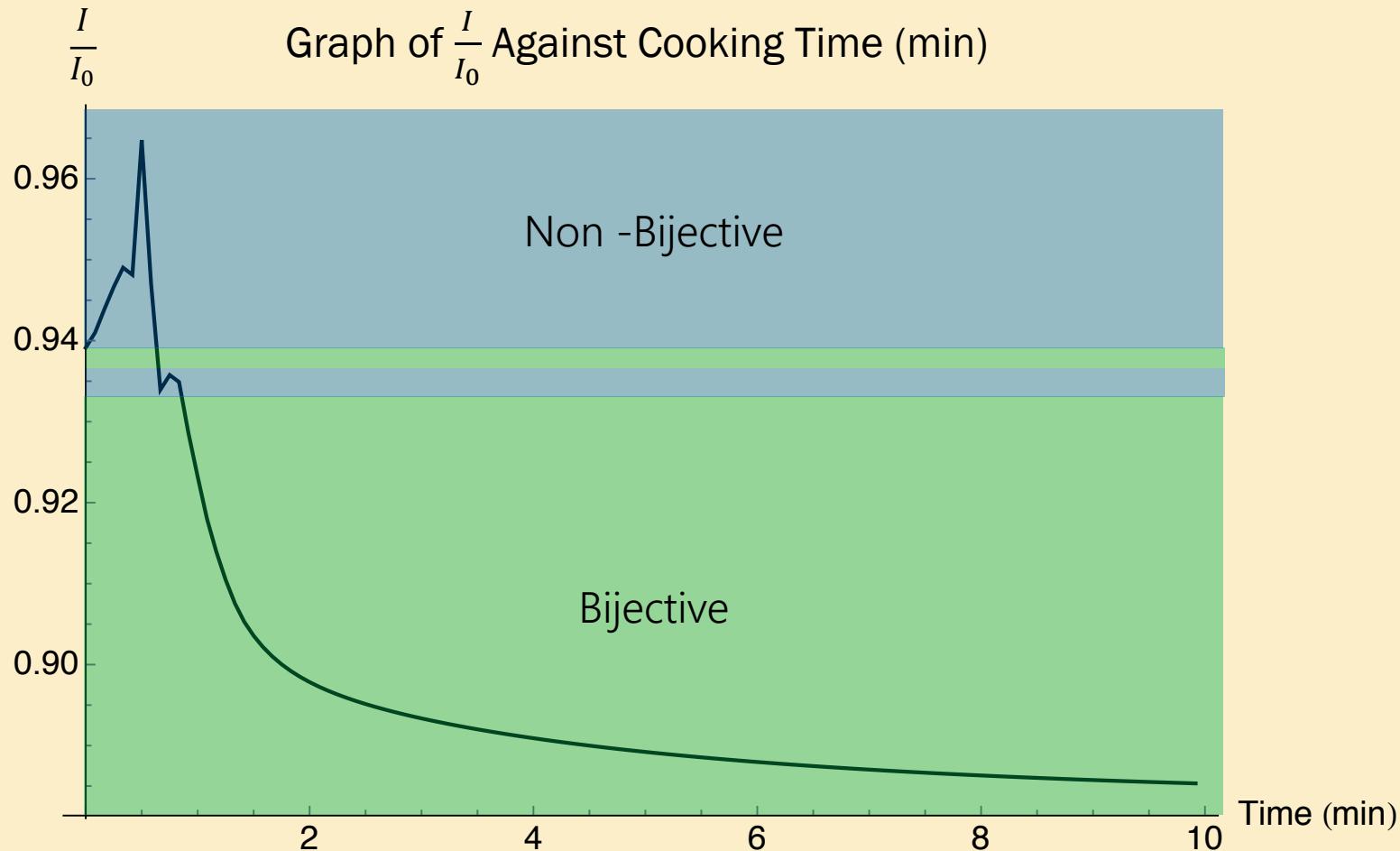
Ultrasound

Sensitivity

Conclusion

Ultrasonic Attenuation

48



Egg Characterization

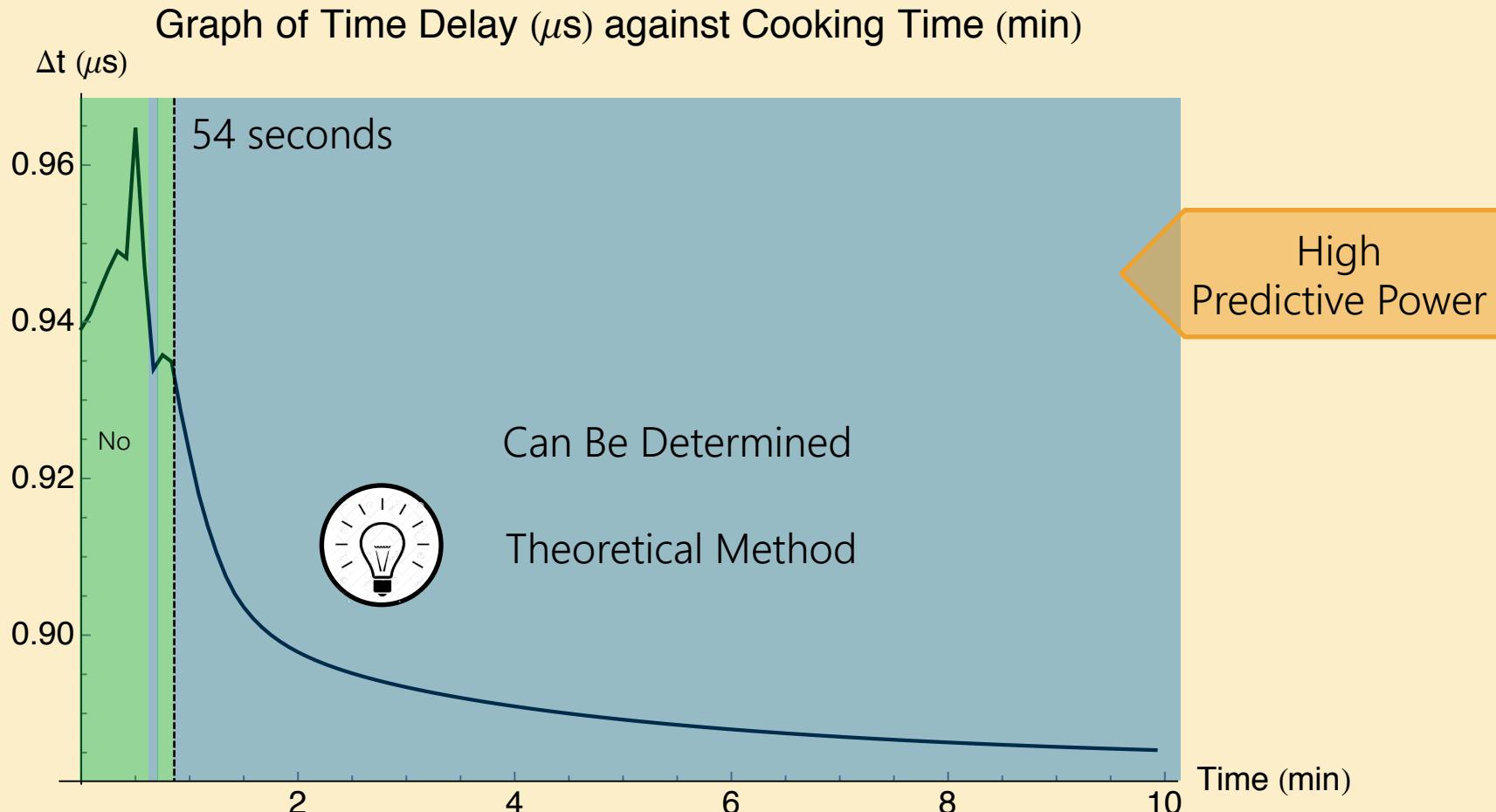
Ultrasound

Sensitivity

Conclusion

Ultrasonic Attenuation

49



Egg Characterization

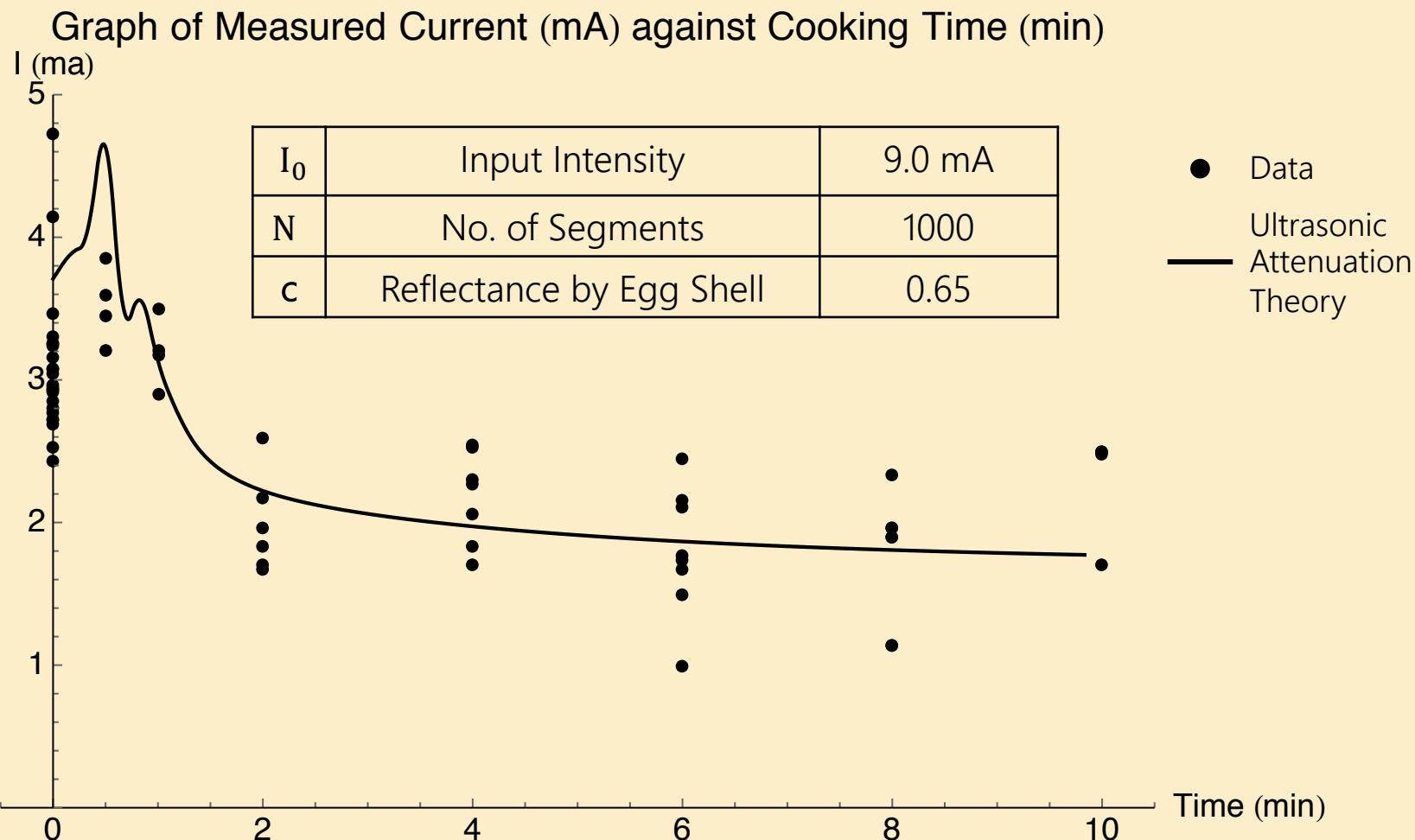
Ultrasound

Sensitivity

Conclusion

Experimental Results

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Egg Characterization

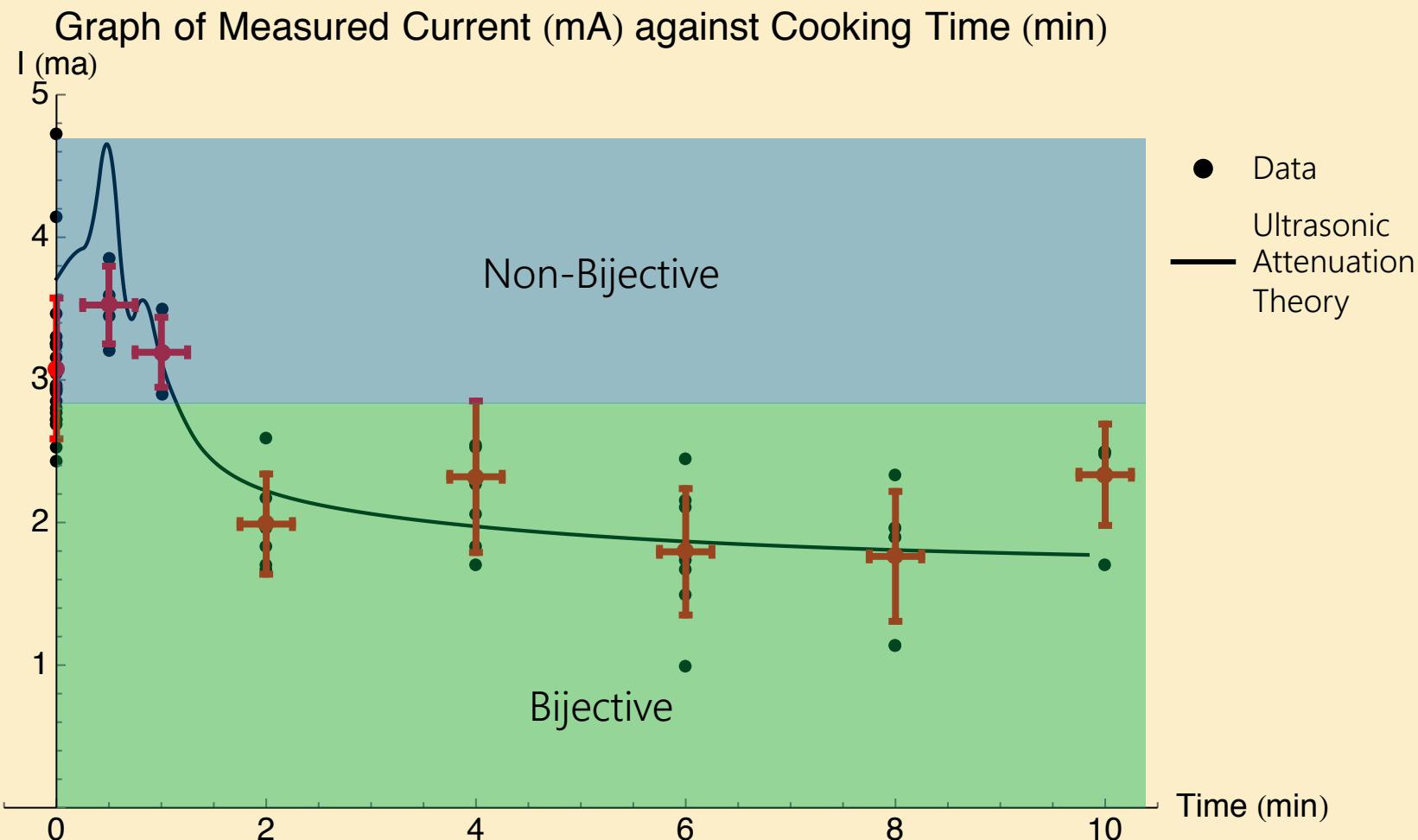
Microwave

Sensitivity

Conclusion

Experimental Results

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Egg Characterization

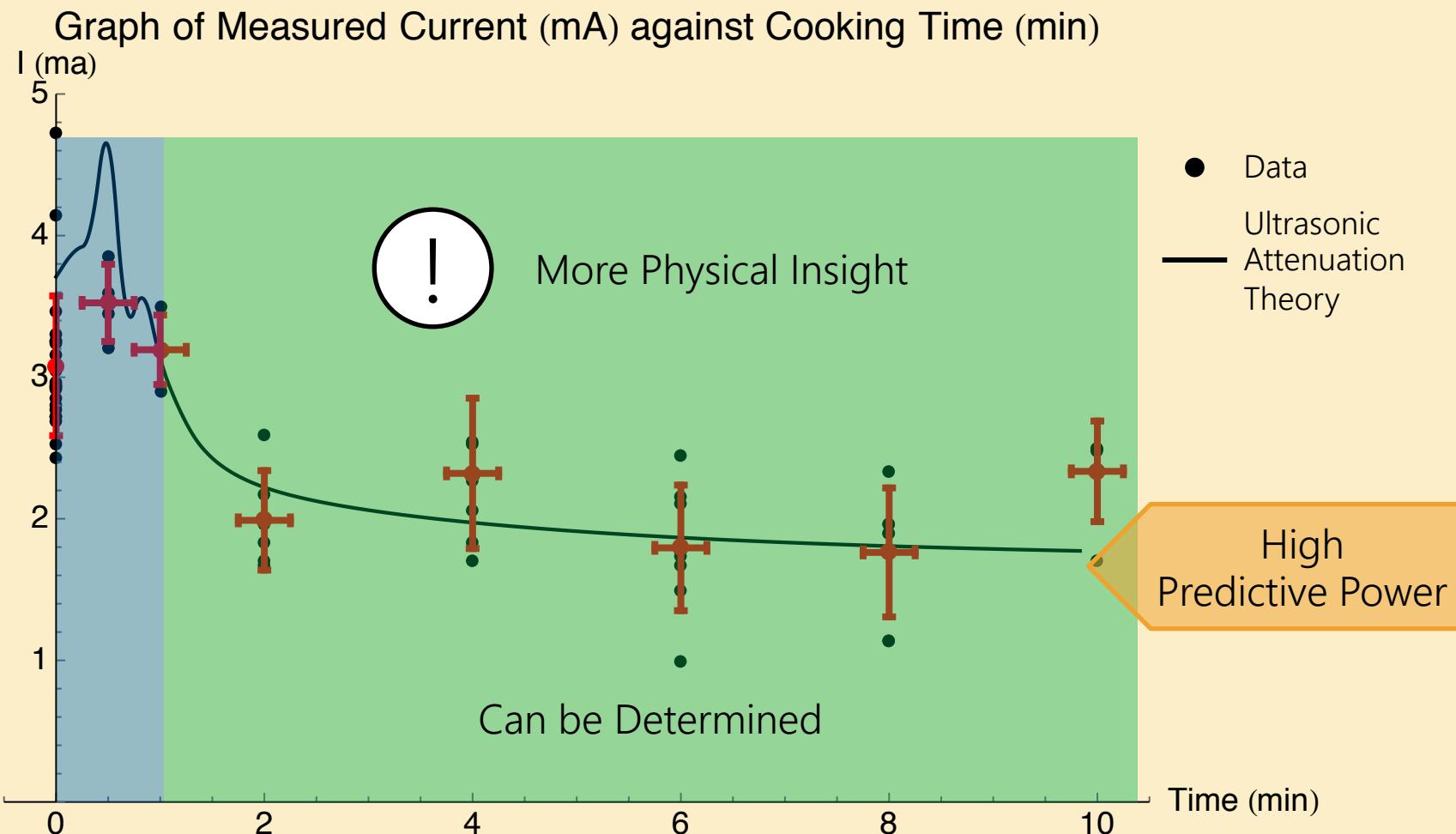
Microwave

Sensitivity

Conclusion

Experimental Results

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Egg Characterization

Microwave

Sensitivity

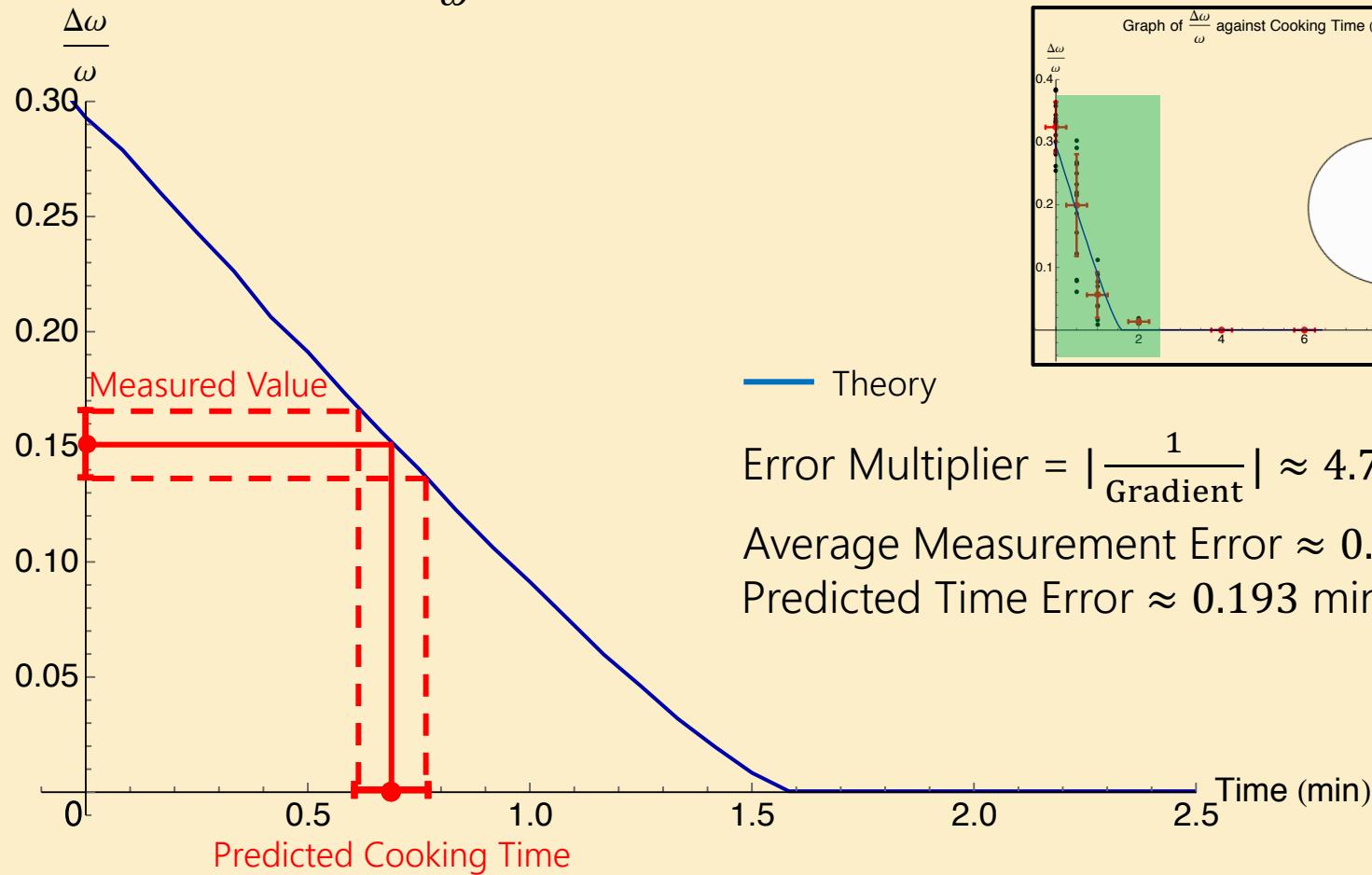
Conclusion

Sensitivity

Method 1: Manual Residual Spin

54

Graph of $\frac{\Delta\omega}{\omega}$ against Cooking Time (min)



Egg Characterization

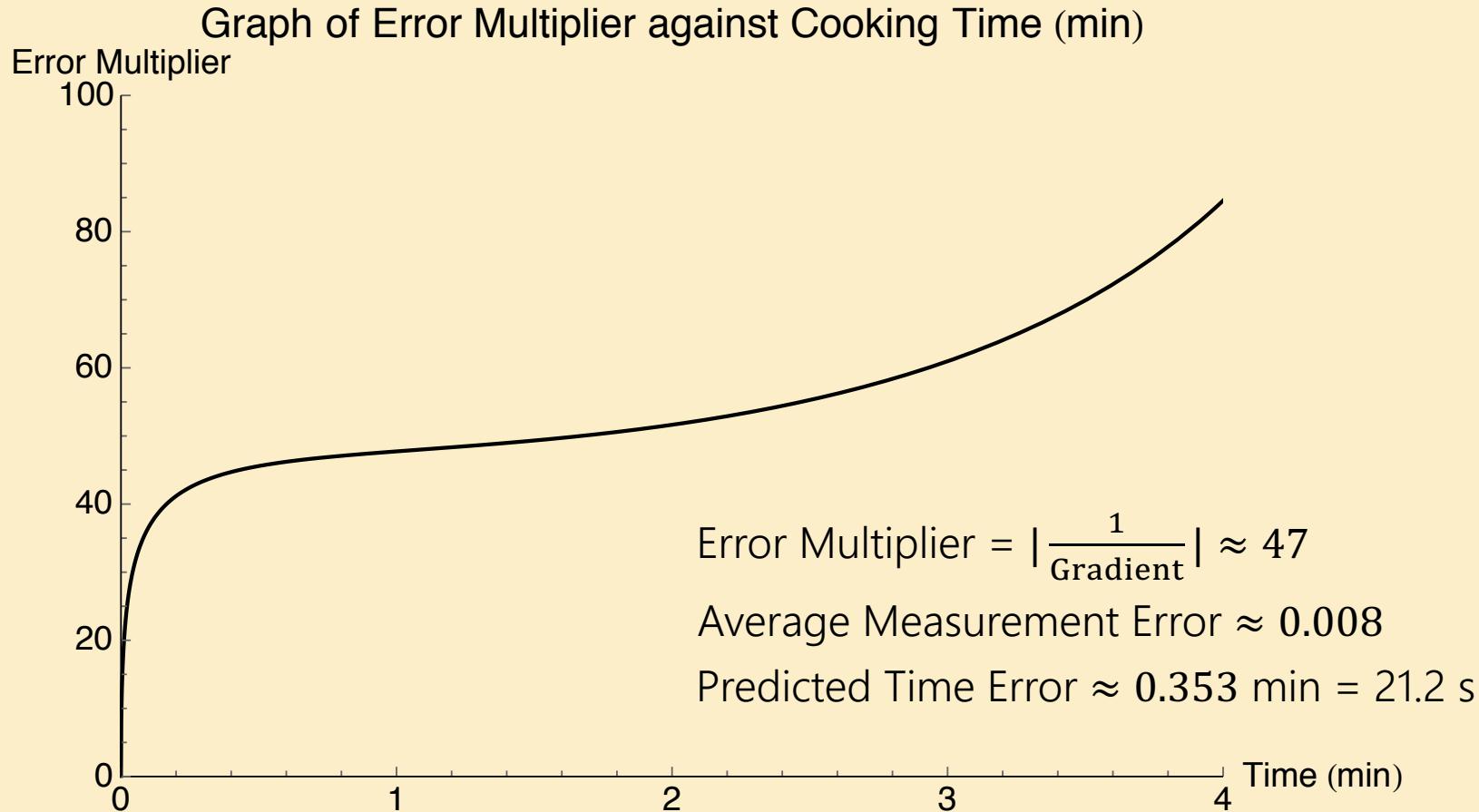
Dynamic Response

Sensitivity

Conclusion

Method 2: Motorized Residual Spin

55



Egg Characterization

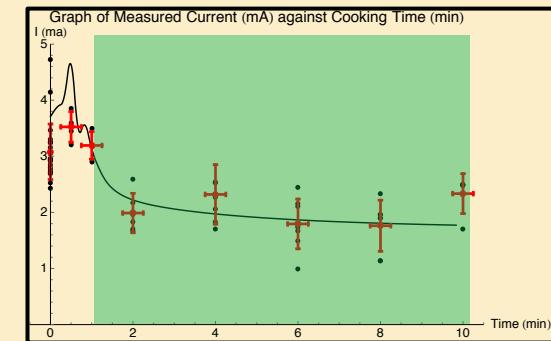
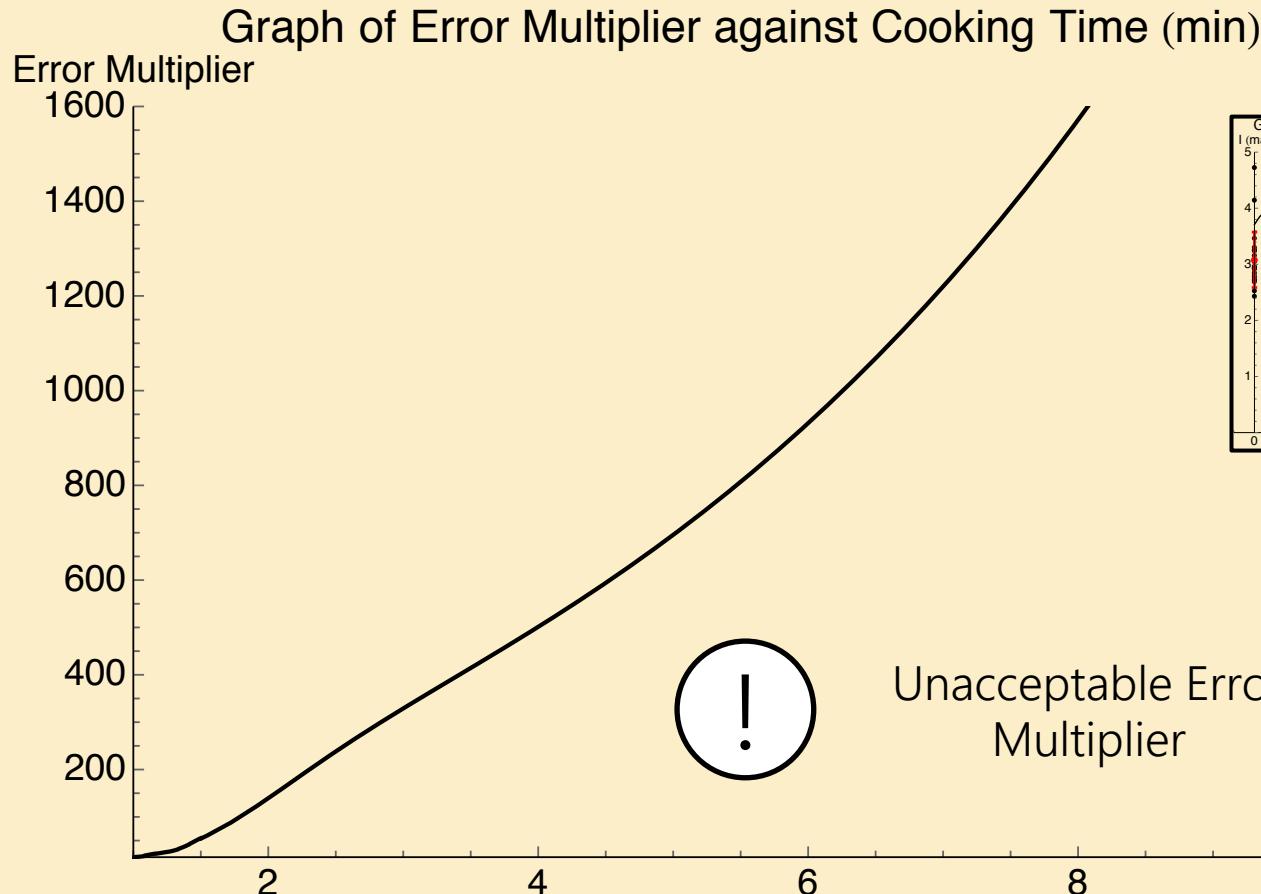
Dynamic Response

Sensitivity

Conclusion

Method 3: Microwave Attenuation

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Egg Characterization

Dynamic Response

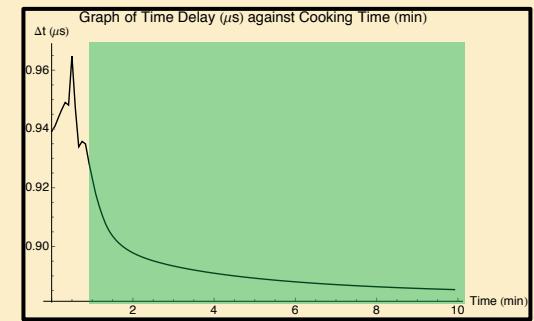
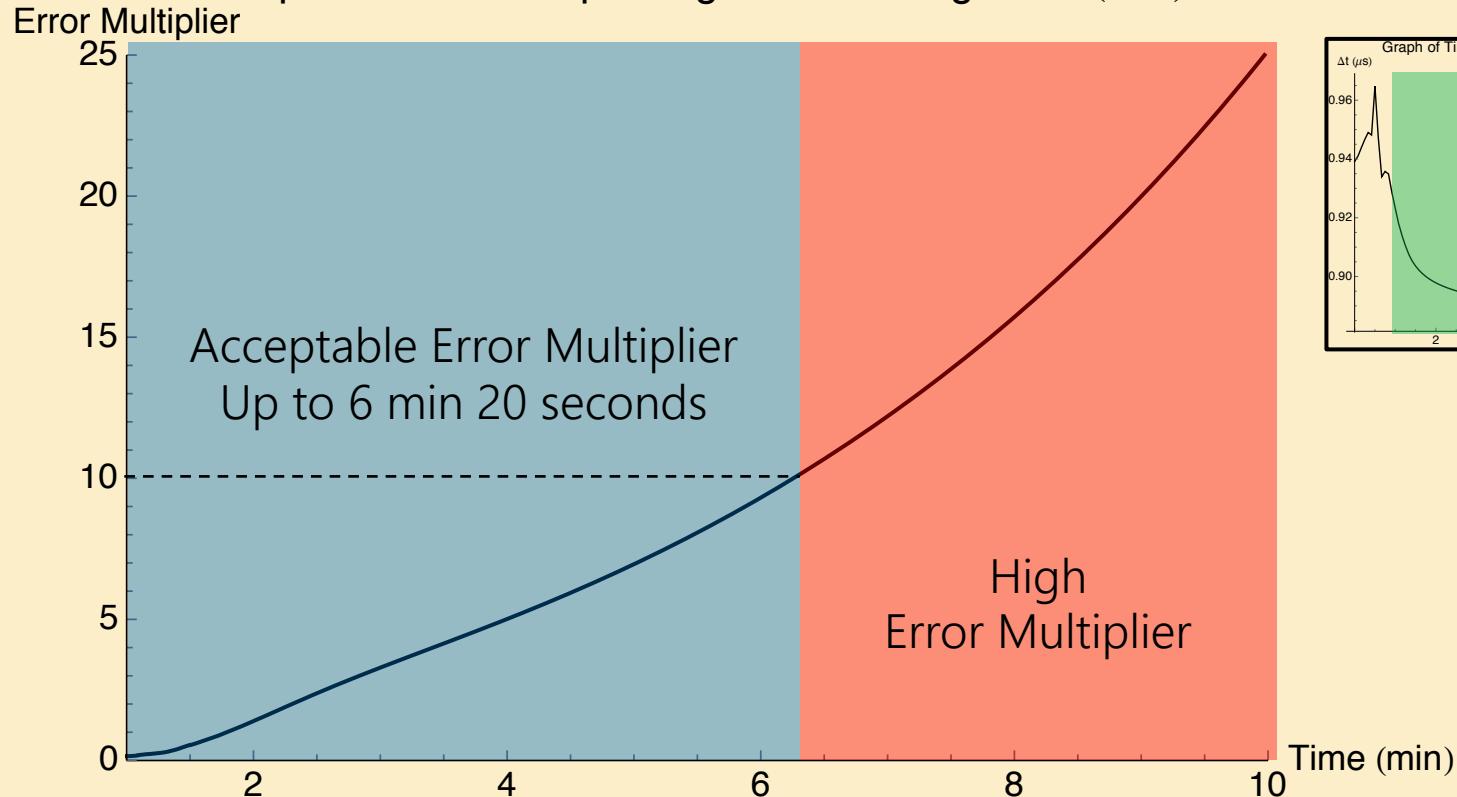
Sensitivity

Conclusion

Method 4: Ultrasonic Attenuation

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Graph of Error Multiplier against Cooking Time (min)



Egg Characterization

Dynamic Response

Sensitivity

Conclusion

Methods

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	Manual Residual Spin	Motorized Residual Spin	Microwave Attenuation	Ultrasound Attenuation
Predicted Time Range (Min)	0 – 1	0 – 4	>1	0 – 2
Error Multiplier & Absolute Error	4.77x 11.5 s	47x 21.2 s	>200x >100 Mins	1.62x
Method Type				

Thank You!

Suggest non-invasive methods to detect the degree to which a hen's egg is cooked by boiling. Investigate the sensitivity of your methods.



Experimental Methods



Theoretical Methods

Measurement

Density Change
Dynamic Response
Ultrasound & Microwave

Experimental Set-Up

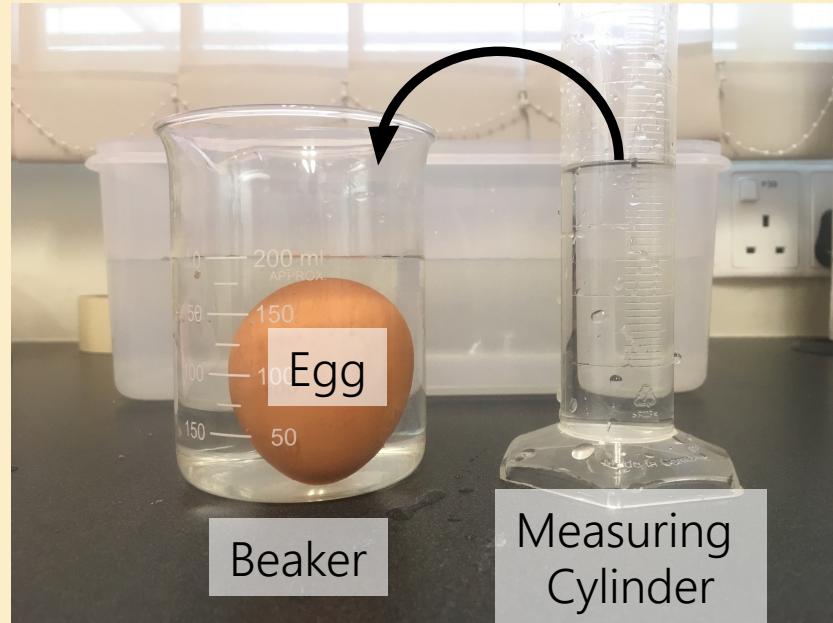
61



Mass

Average Mass:
 $51 \pm 2\text{g}$

Water is poured until the 200ml mark



Volume

Average Volume:
 $57 \pm 7\text{ml}$

Egg Characterization

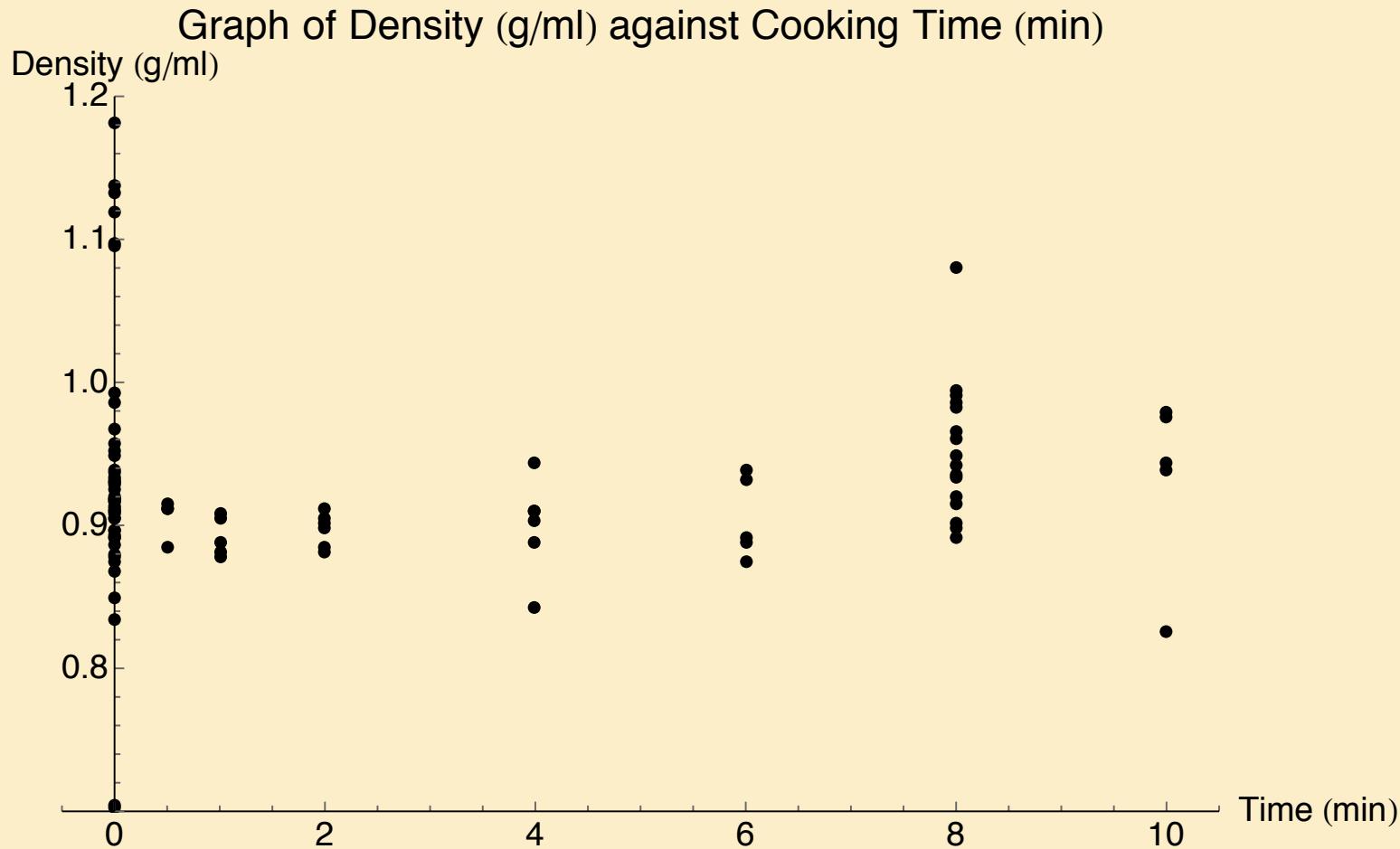
Density Change

Sensitivity

Conclusion

Density Change

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Egg Characterization

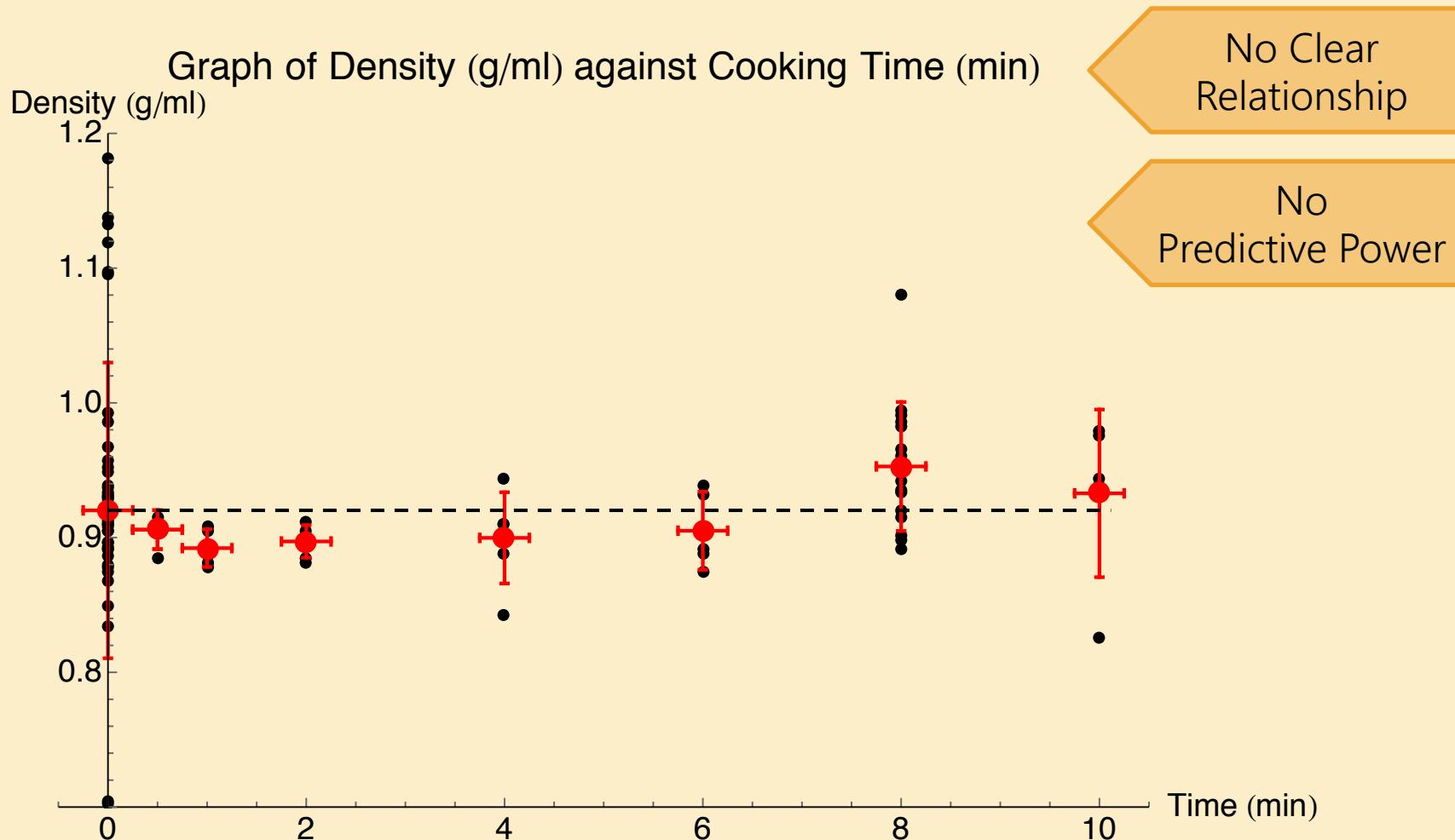
Density Change

Sensitivity

Conclusion

Density Change

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Egg Characterization

Density Change

Sensitivity

Conclusion

Potential Error

64



Diagram of Raw Egg^[1]



Time Lapse Video across 1 hour of
Raw Egg Immersed in Water

Egg Characterization

Density Change

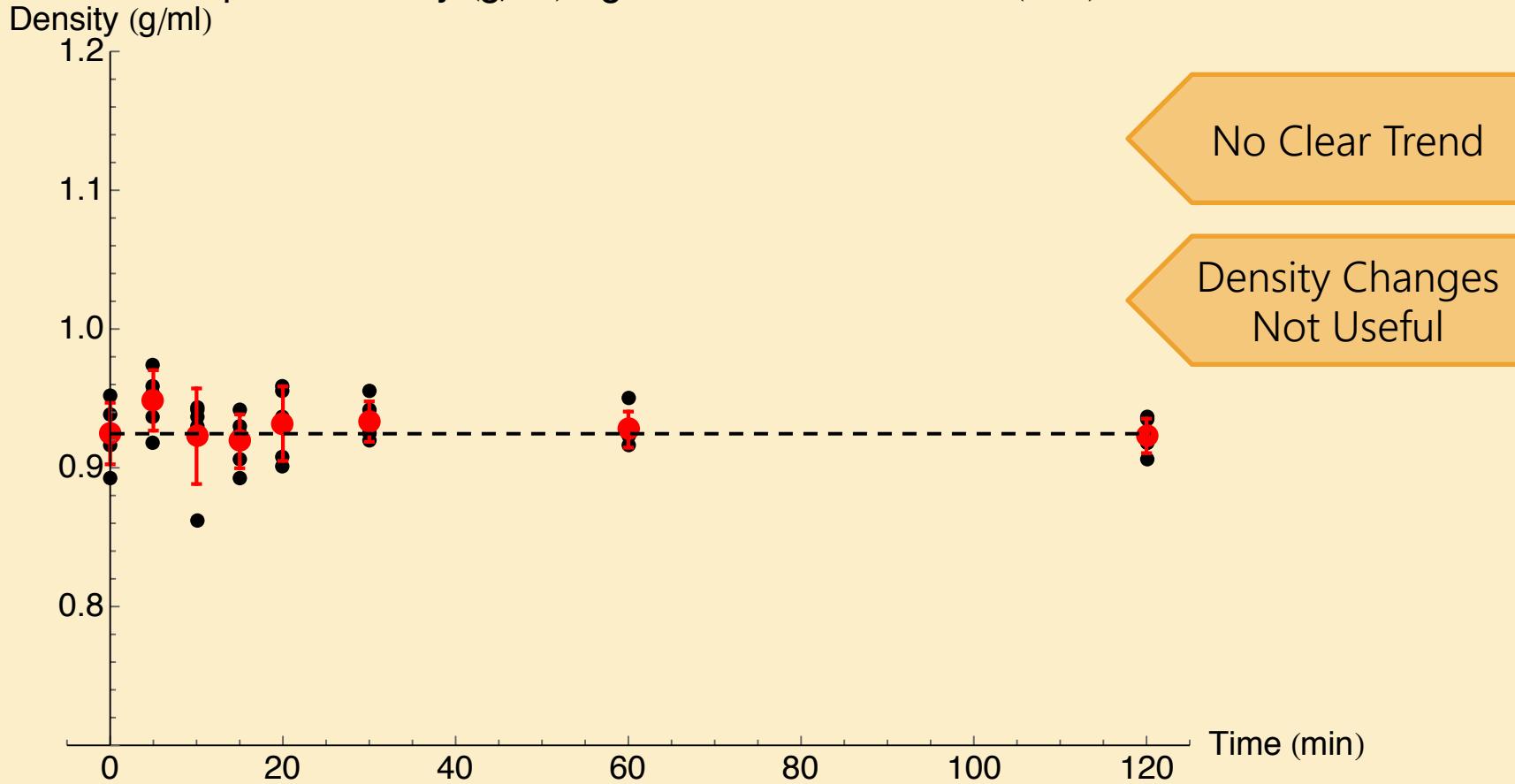
Sensitivity

Conclusion

Absorptivity of Egg Shell

65

Graph of Density (g/ml) against Immersion Time (min)



Egg Characterization

Density Change

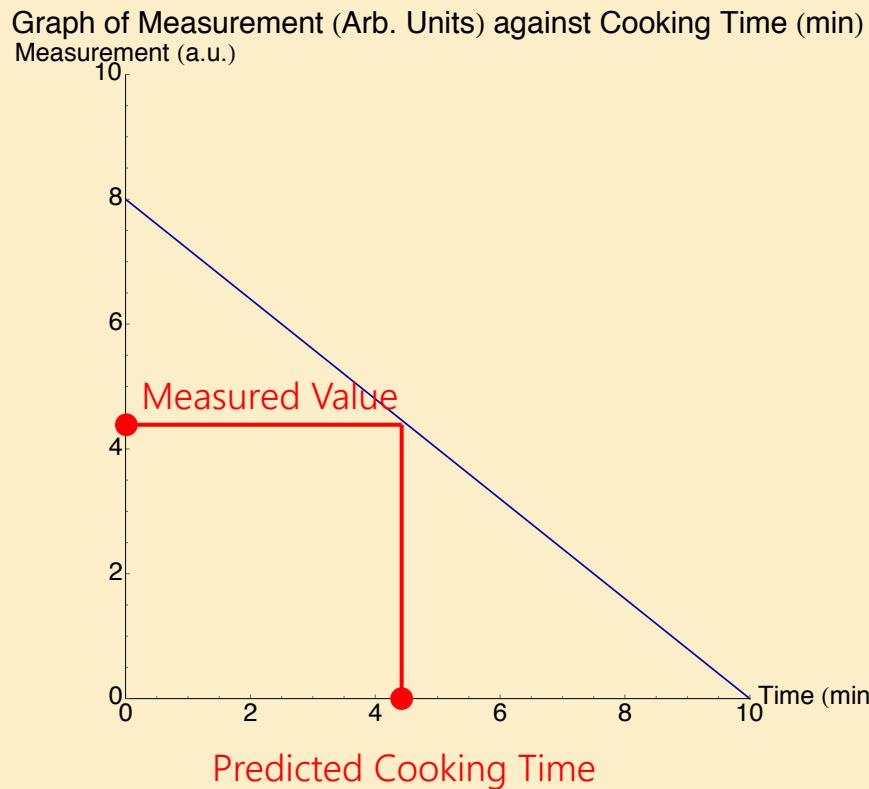
Sensitivity

Conclusion

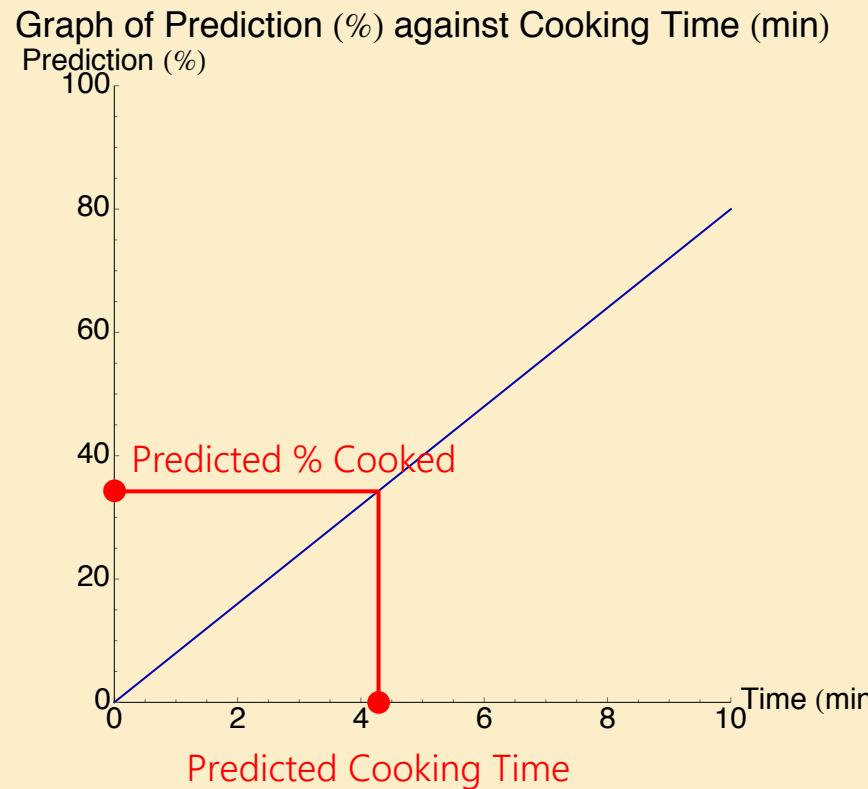
Procedure

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Measurement



Prediction

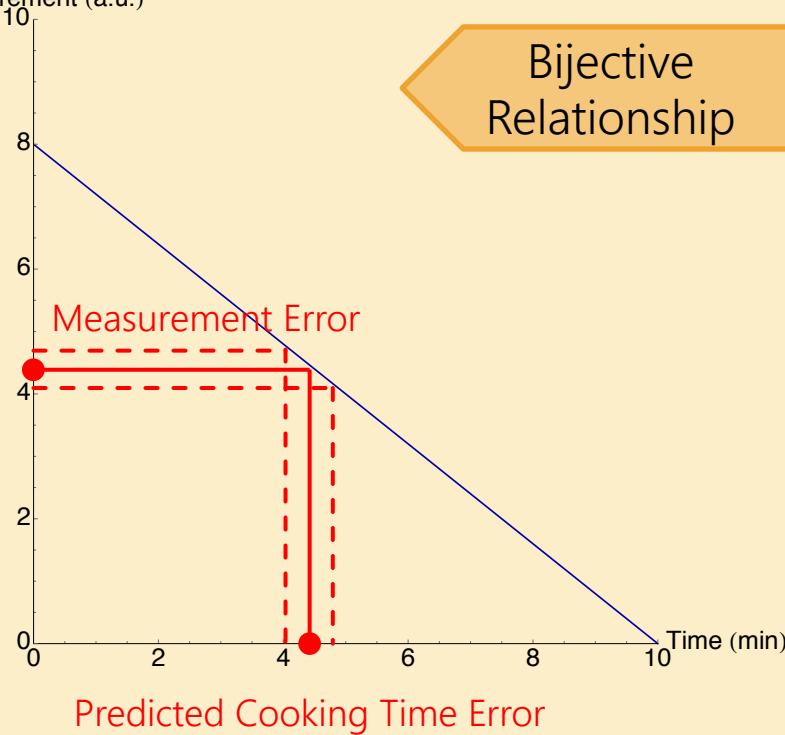


Procedure

67

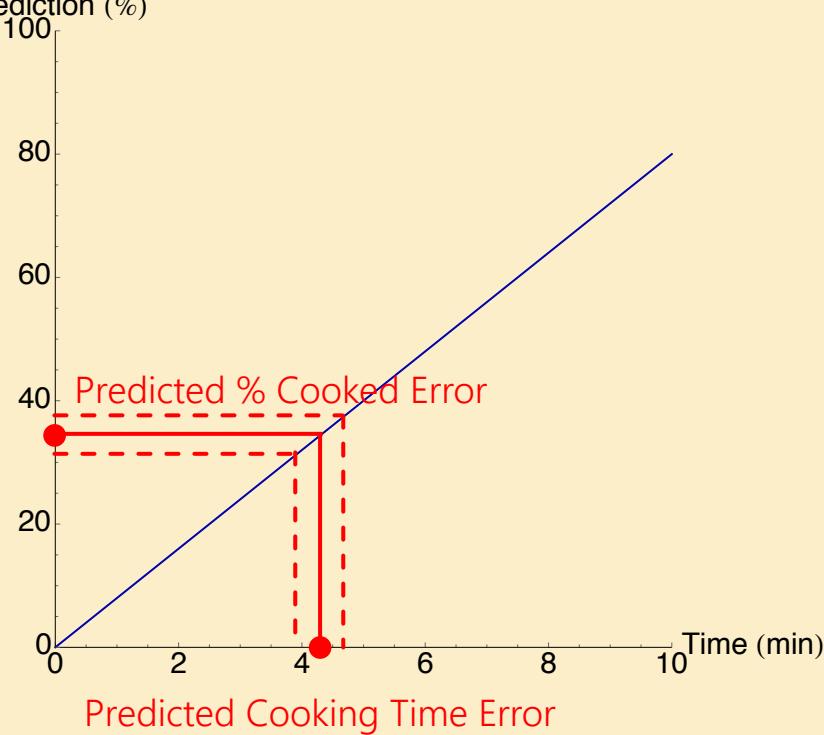
Measurement

Graph of Measurement (Arb. Units) against Cooking Time (min)
Measurement (a.u.)



Prediction

Graph of Prediction (%) against Cooking Time (min)
Prediction (%)

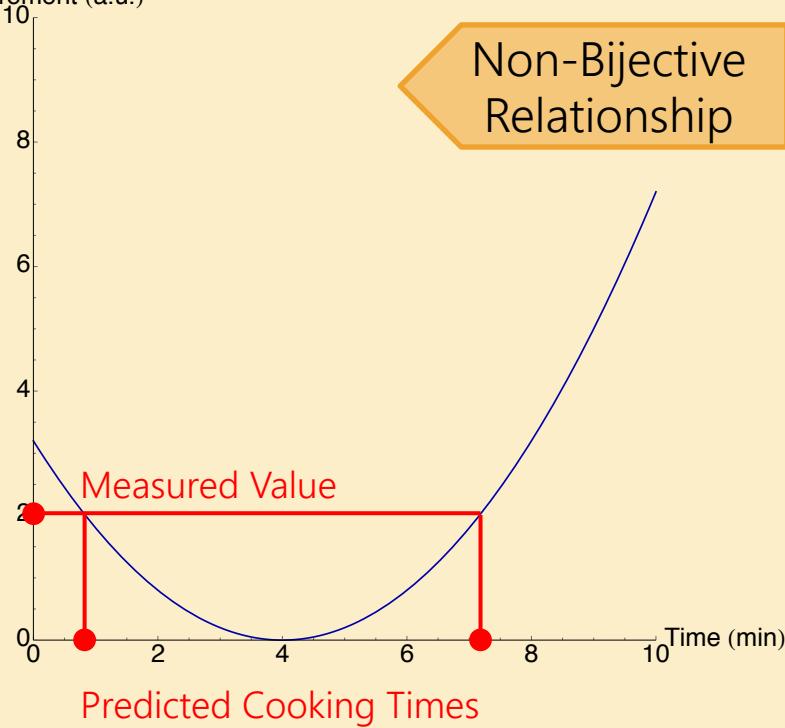


Procedure

68

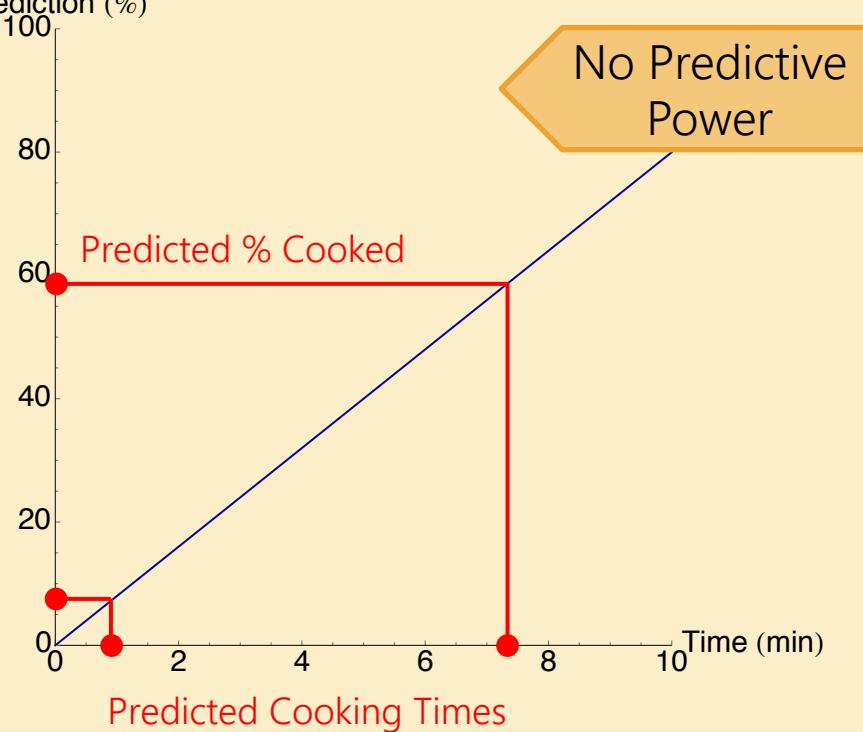
Measurement

Graph of Measurement (Arb. Units) against Cooking Time (min)
Measurement (a.u.)



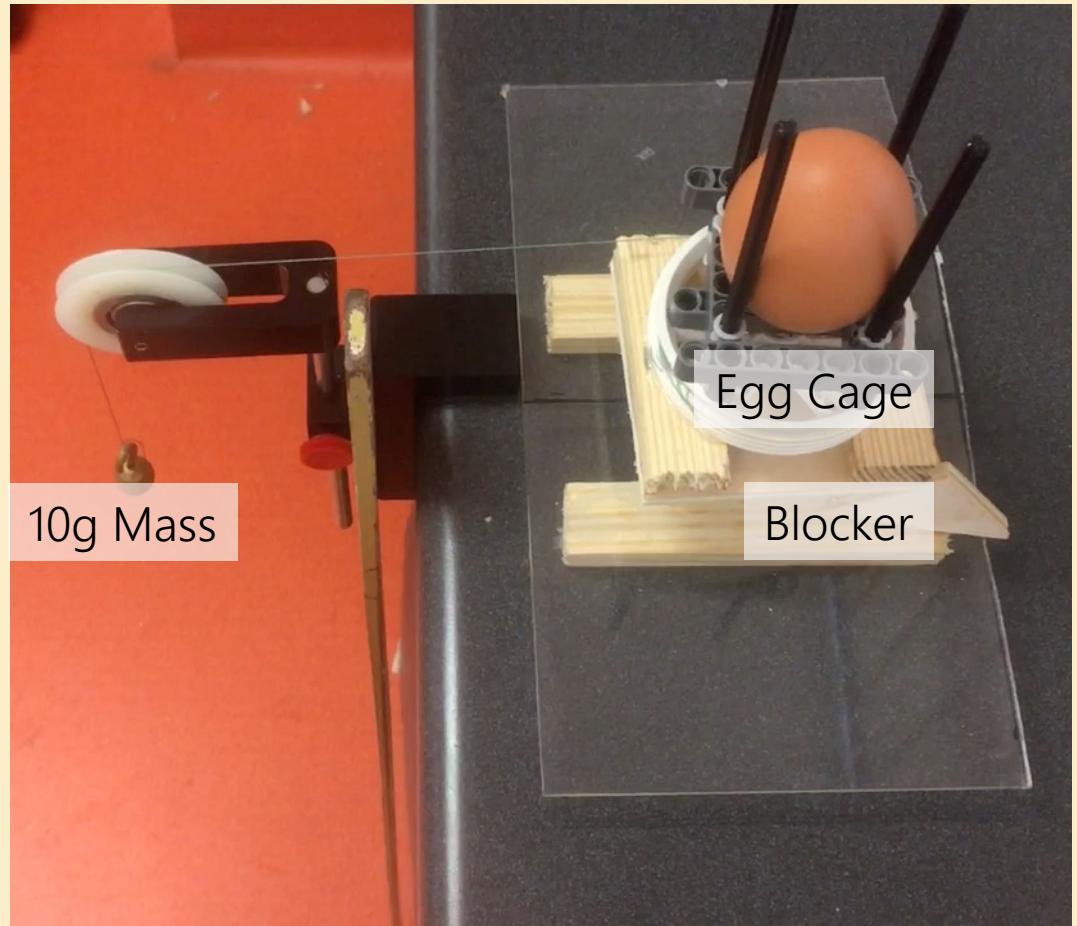
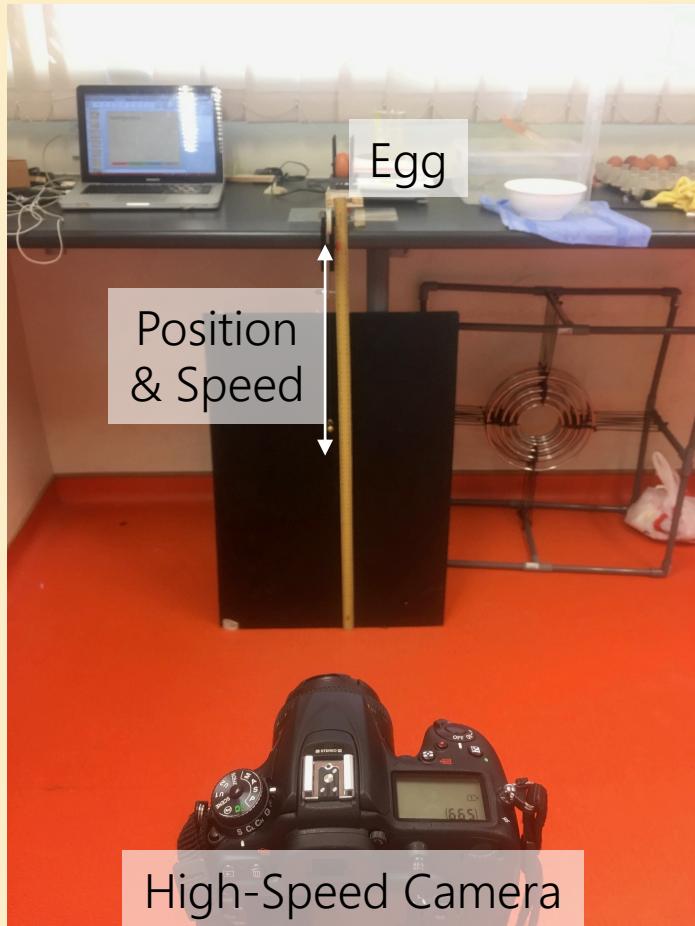
Prediction

Graph of Prediction (%) against Cooking Time (min)
Prediction (%)



Experimental Set-Up

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Egg Characterization

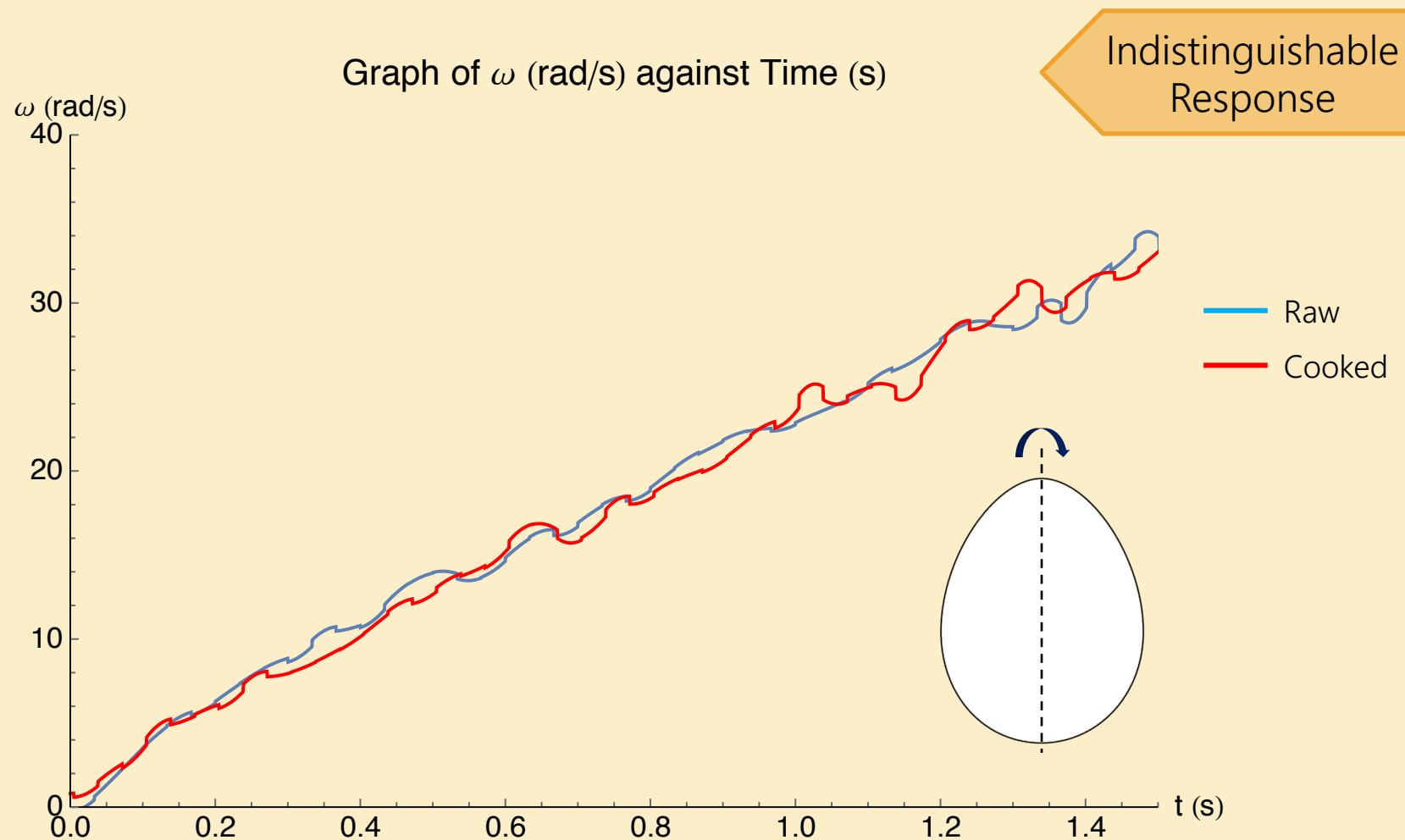
Dynamic Response

Sensitivity

Conclusion

Simple Spin along long axis

70



Egg Characterization

Dynamic Response

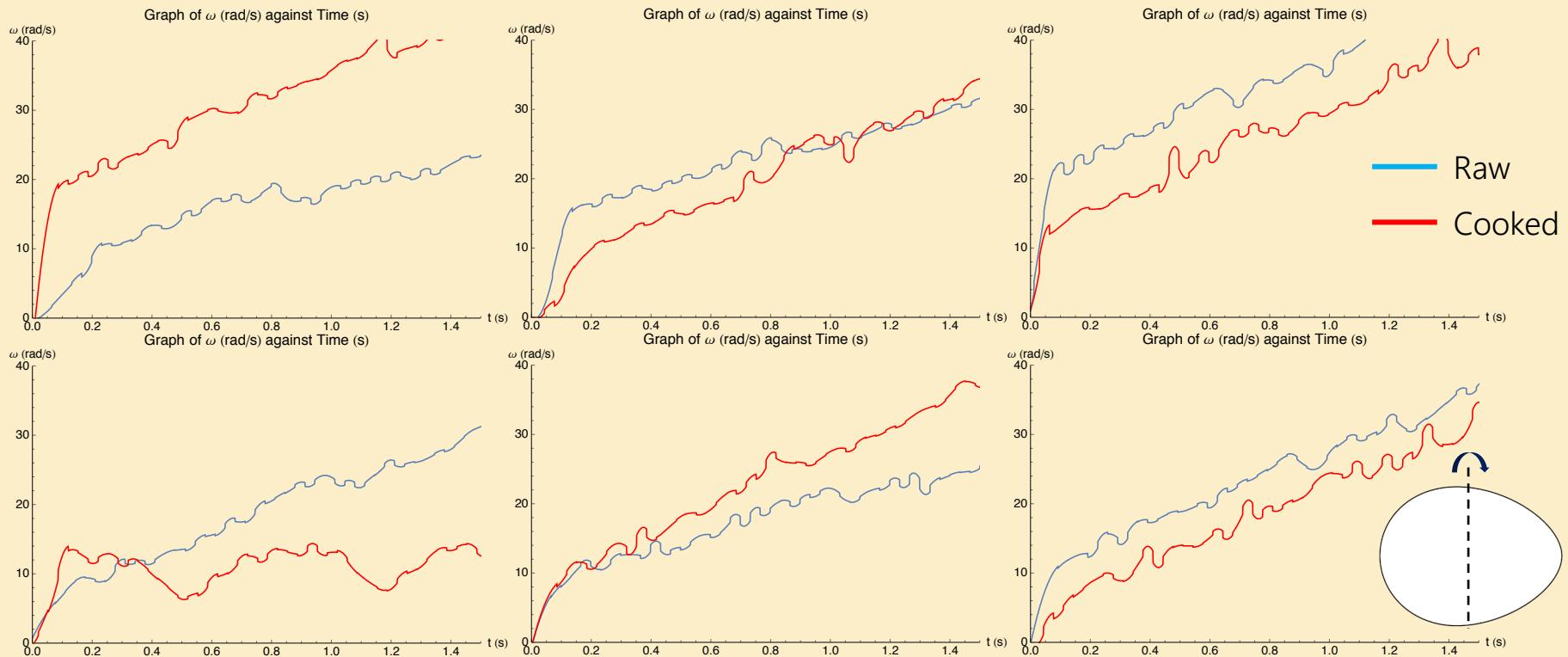
Sensitivity

Conclusion

Simple Spin along short axis

71

Inconsistent
Response



Egg Characterization

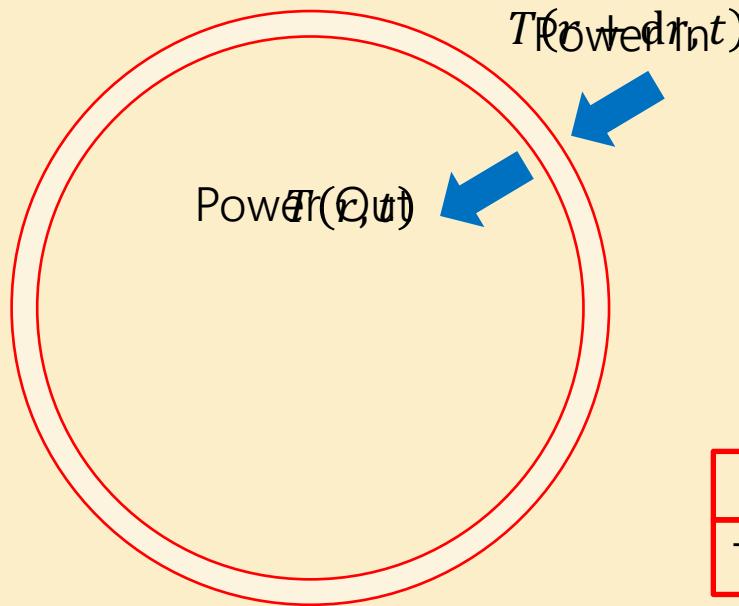
Dynamic Response

Sensitivity

Conclusion

Basic Thermodynamic Model

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Spherical Shell of Egg

Heat Capacity $dC = 4\pi r^2 \rho c dr$

Thermal Resistance $dR = \frac{1}{4\pi r^2 \kappa} dr$

Heat Conduction Equations

$$T(r + dr, t) - T(r, t) = \dot{Q}(r, t) dR(r, t)$$

Temperature difference Power Transfer

$$[T(r, t + dt) - T(r, t)] dC(r, t) = [\dot{Q}(r - dr, t) - \dot{Q}(r, t)] dt$$

Temperature difference

Net Power Input

Theory

Measurement

Sensitivity

Conclusion

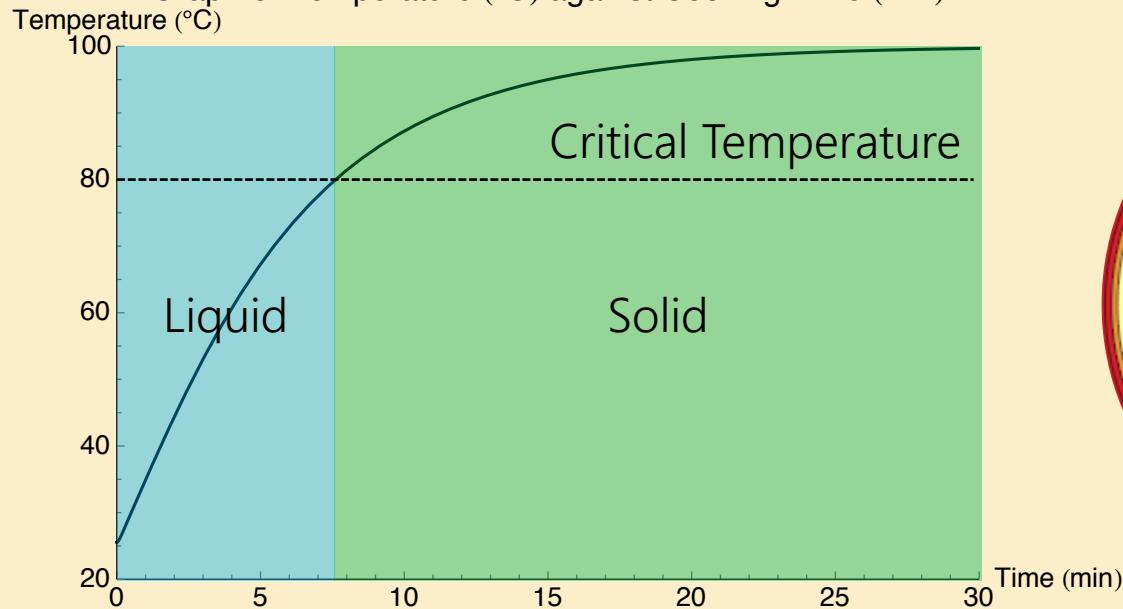
Thermodynamic Model

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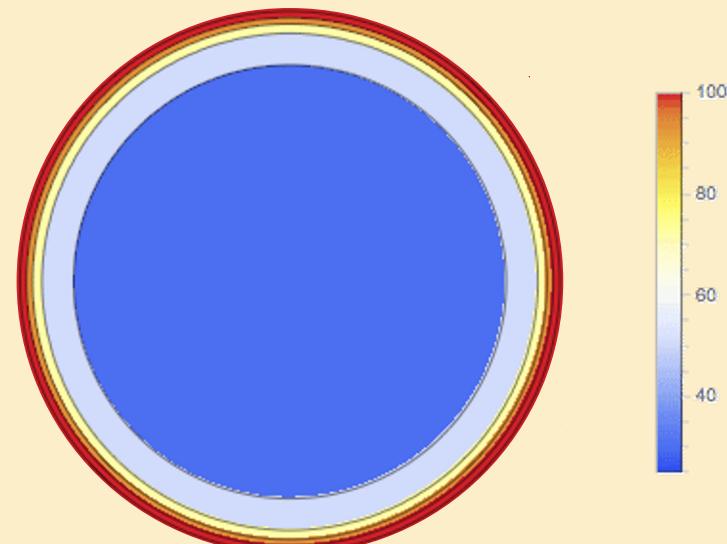
$$T(r, t) = T_{Water} + \left(\frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right) \frac{\zeta \beta a^2 \sum_{N=1}^{\infty} (-1)^{N-1}}{\pi k \sum_{n=1}^{\infty}} \sin\left(\frac{N\pi r}{a}\right) e^{\frac{-N^2 \pi^2 t \kappa}{c \rho a^2}}$$

$T_{Water} = 100^\circ\text{C}$

Graph of Temperature ($^\circ\text{C}$) against Cooking Time (min)



$$T_{Egg} = 25^\circ\text{C}$$



Theory

Measurement

Sensitivity

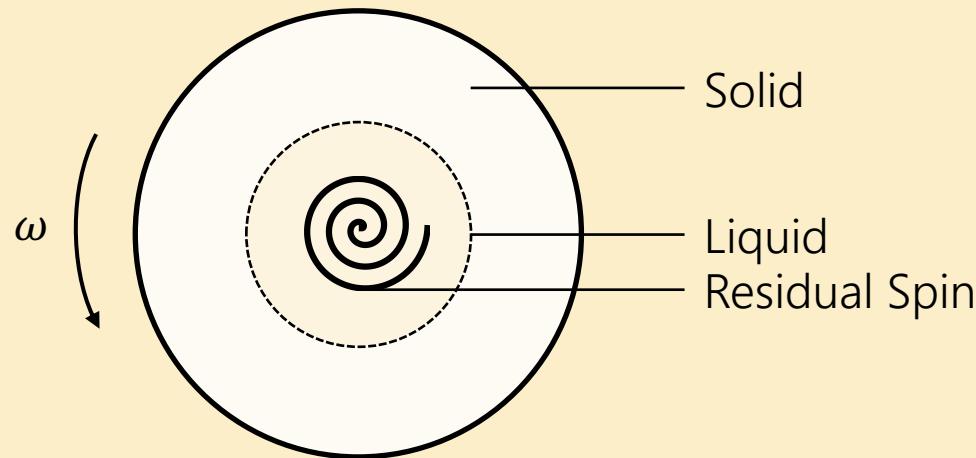
Conclusion

Basic Thermodynamic Model

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$$\text{Rotational Energy Retained by Liquid: } \frac{1}{2} I \omega^2 = \frac{1}{2} \frac{2}{5} m_{Liquid} R_{Liquid}^2 \omega^2 \\ = \frac{1}{2} \frac{2}{5} m_{Egg} R_{Egg}^2 \omega'^2$$

$$\frac{\omega'}{\omega} = \sqrt{\frac{m_{Liquid} R_{Liquid}^2}{m_{Egg} R_{Egg}^2}} = \sqrt{\frac{R_{Liquid}^5}{R_{Egg}^5}} = \frac{R_{Liquid}^{2.5}}{R_{Egg}}$$



Theory

Measurement

Sensitivity

Conclusion

Moment of Inertia

75

$$\text{Rotational Energy Retained by Liquid: } \frac{1}{2} I \omega^2 = \frac{1}{2} \frac{2}{5} m_{Liquid} R_{Liquid}^2 \omega^2 \\ = \frac{1}{2} \frac{2}{5} m_{Egg} R_{Egg}^2 \omega'^2$$

$$\frac{\omega'}{\omega} = \sqrt{\frac{m_{Liquid} R_{Liquid}^2}{m_{Egg} R_{Egg}^2}} = \sqrt{\frac{R_{Liquid}^5}{R_{Egg}^5}} = \frac{R_{Liquid}^{2.5}}{R_{Egg}}$$

$$\frac{m_{Liquid}}{m_{Egg}} = \frac{R_{Liquid}^3}{R_{Egg}}$$

Theory

Measurement

Sensitivity

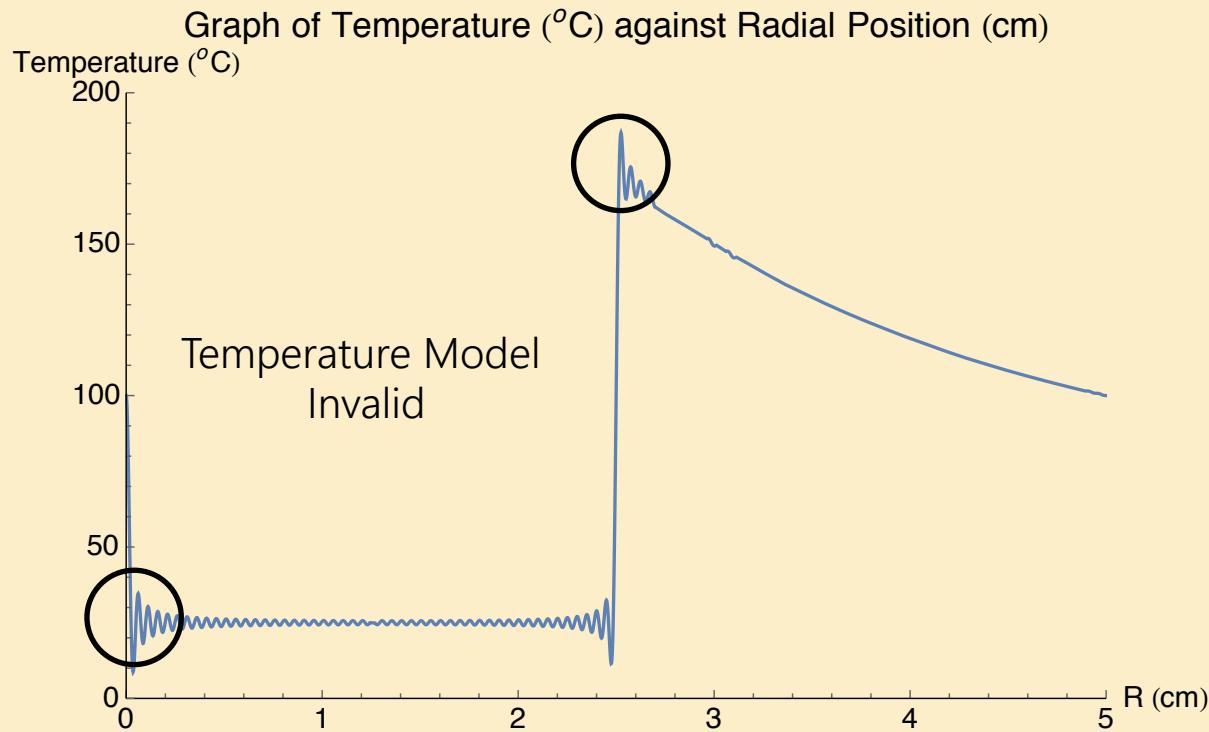
Conclusion

Basic Thermodynamic Model

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Spherical Egg Model Unphysical



Boundary Conditions

$$T(r < a, 0) = T_{Egg}$$

$$T(r = a, 0) = T_{Water}$$

$$T(r = \infty, 0) = T_{Water}$$

Theory

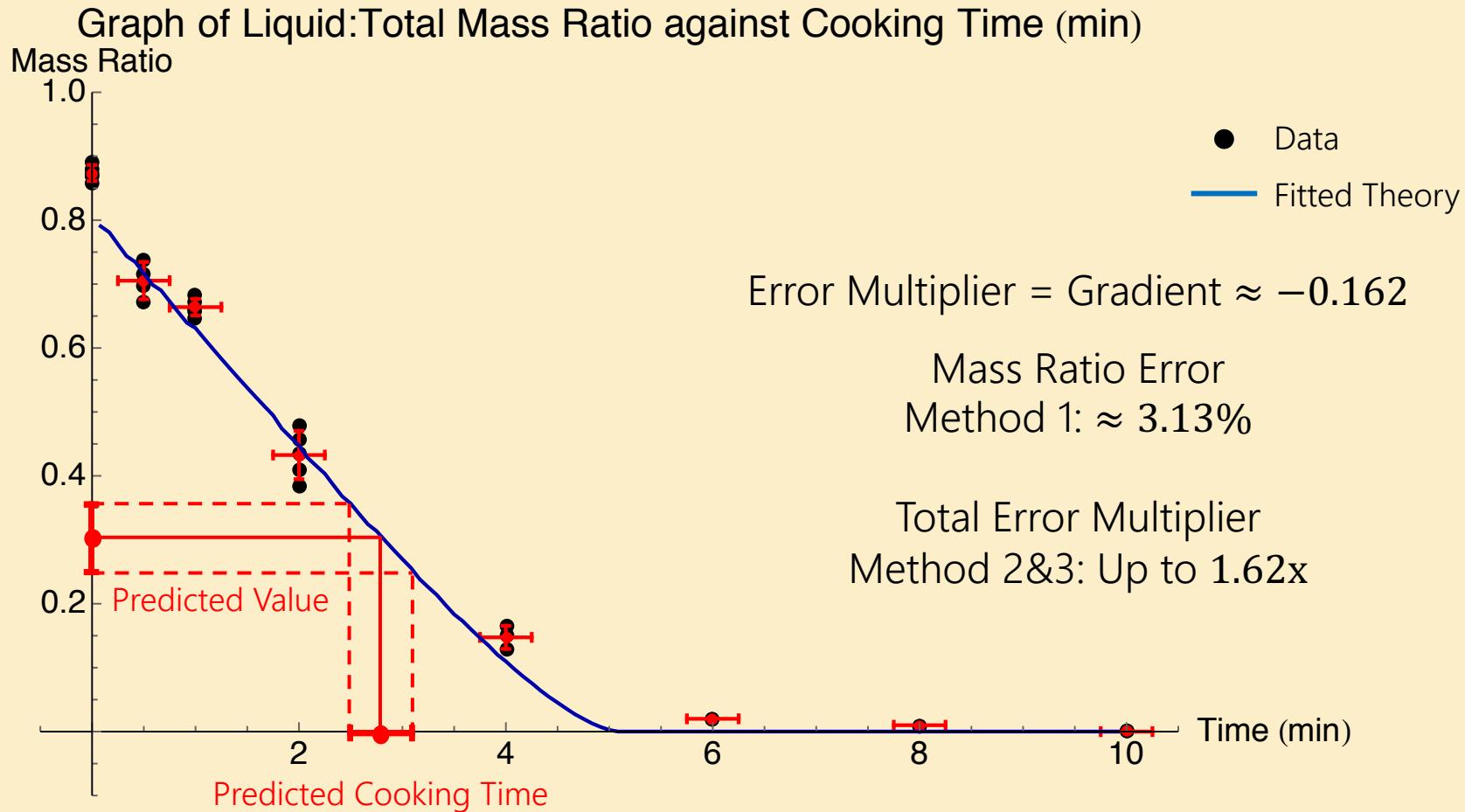
Measurement

Sensitivity

Conclusion

Prediction

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Egg Characterization

Dynamic Response

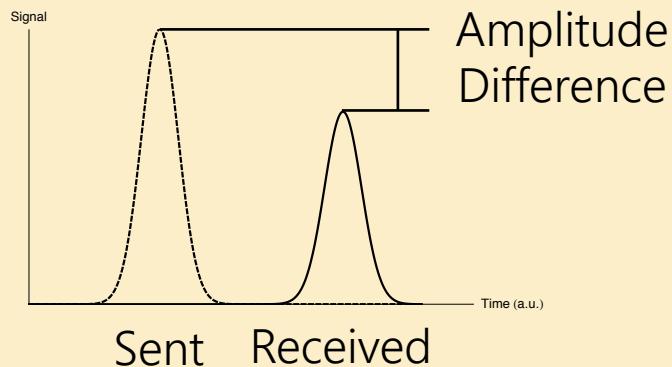
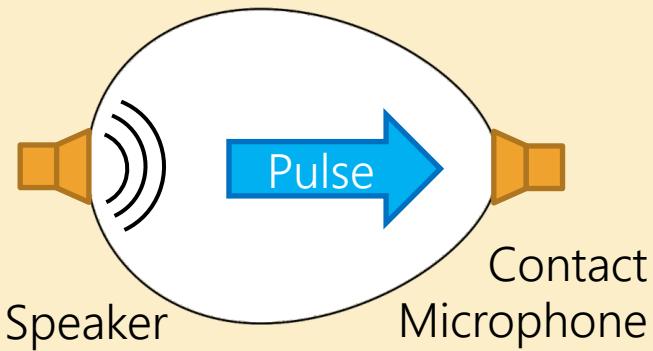
Sensitivity

Conclusion

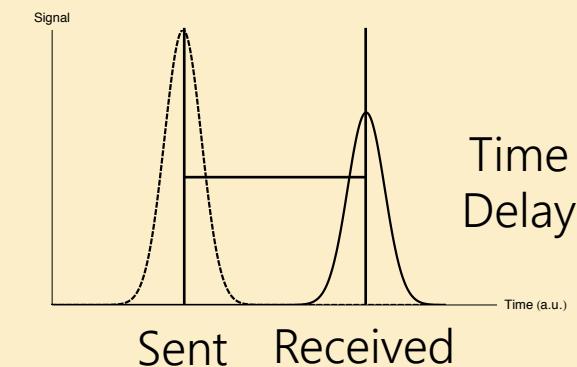
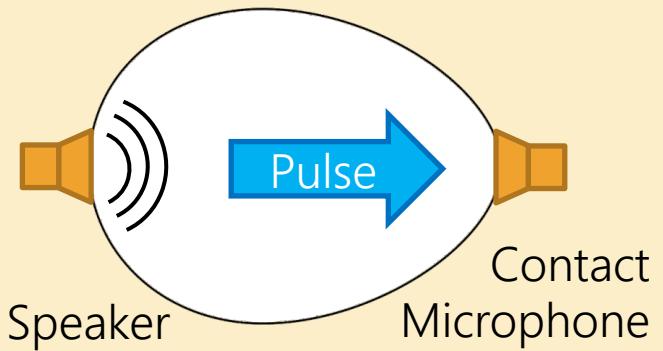
Proposed Experimental Set-Up

78

Ultrasonic Attenuation



Ultrasonic Pulse



Egg Characterization

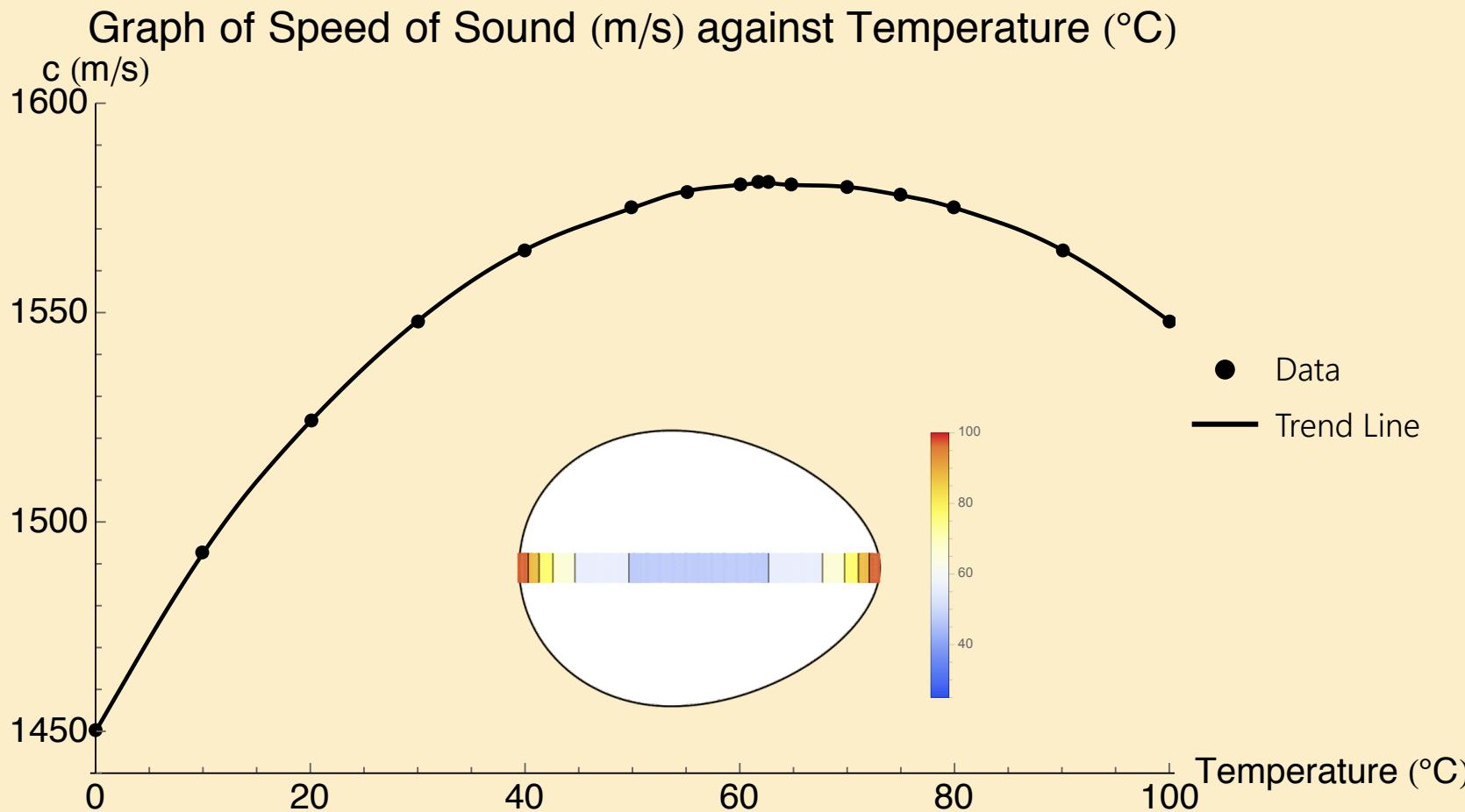
Ultrasound

Sensitivity

Conclusion

Ultrasonic Pulse

79



Krzysztof J. OPIELINSKI, "ULTRASONIC PARAMETERS OF HEN'S EGG", Molecular and Quantum Acoustics vol. 28 (2007)

Egg Characterization

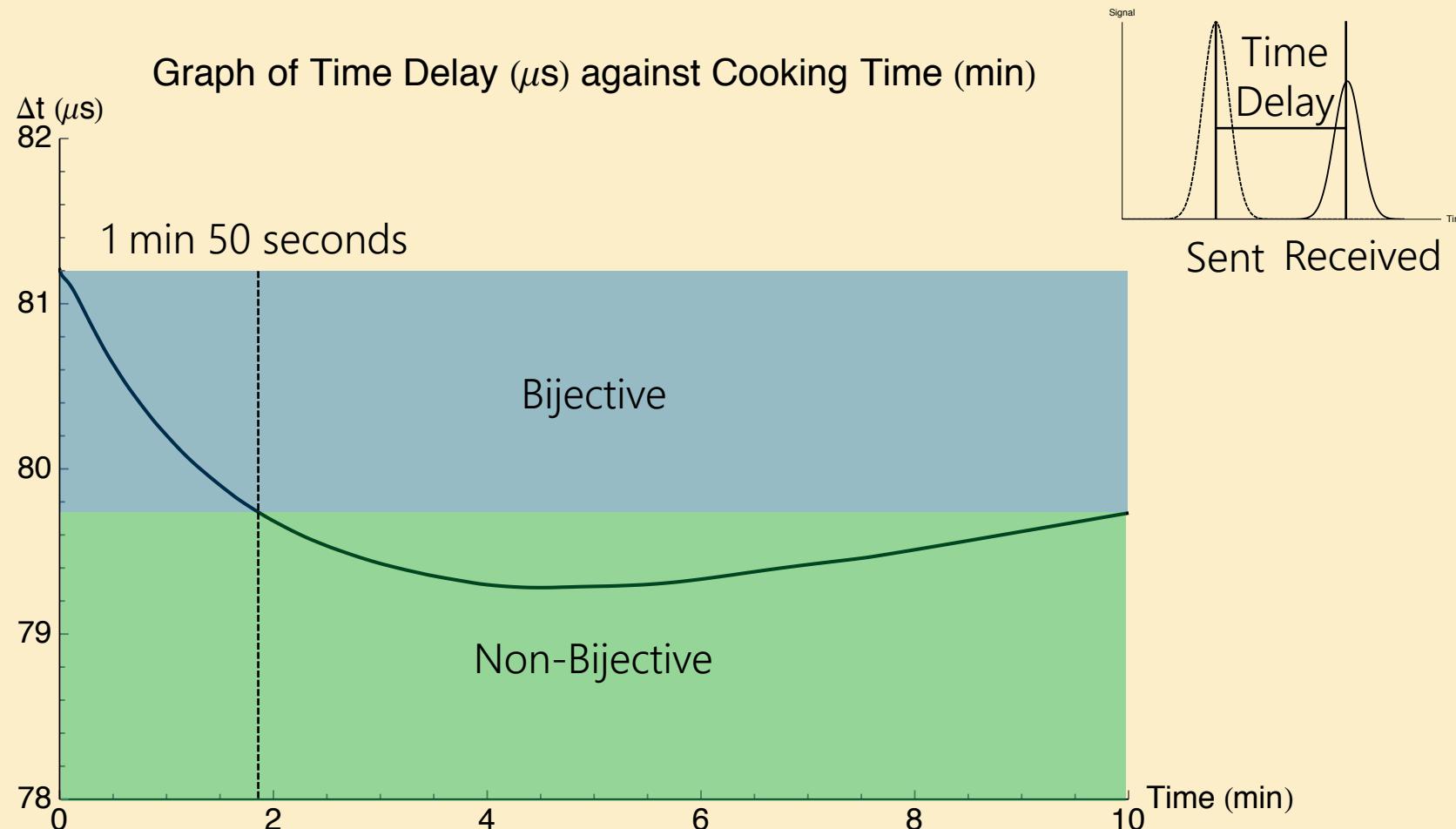
Ultrasound

Sensitivity

Conclusion

Time Delay

80



Egg Characterization

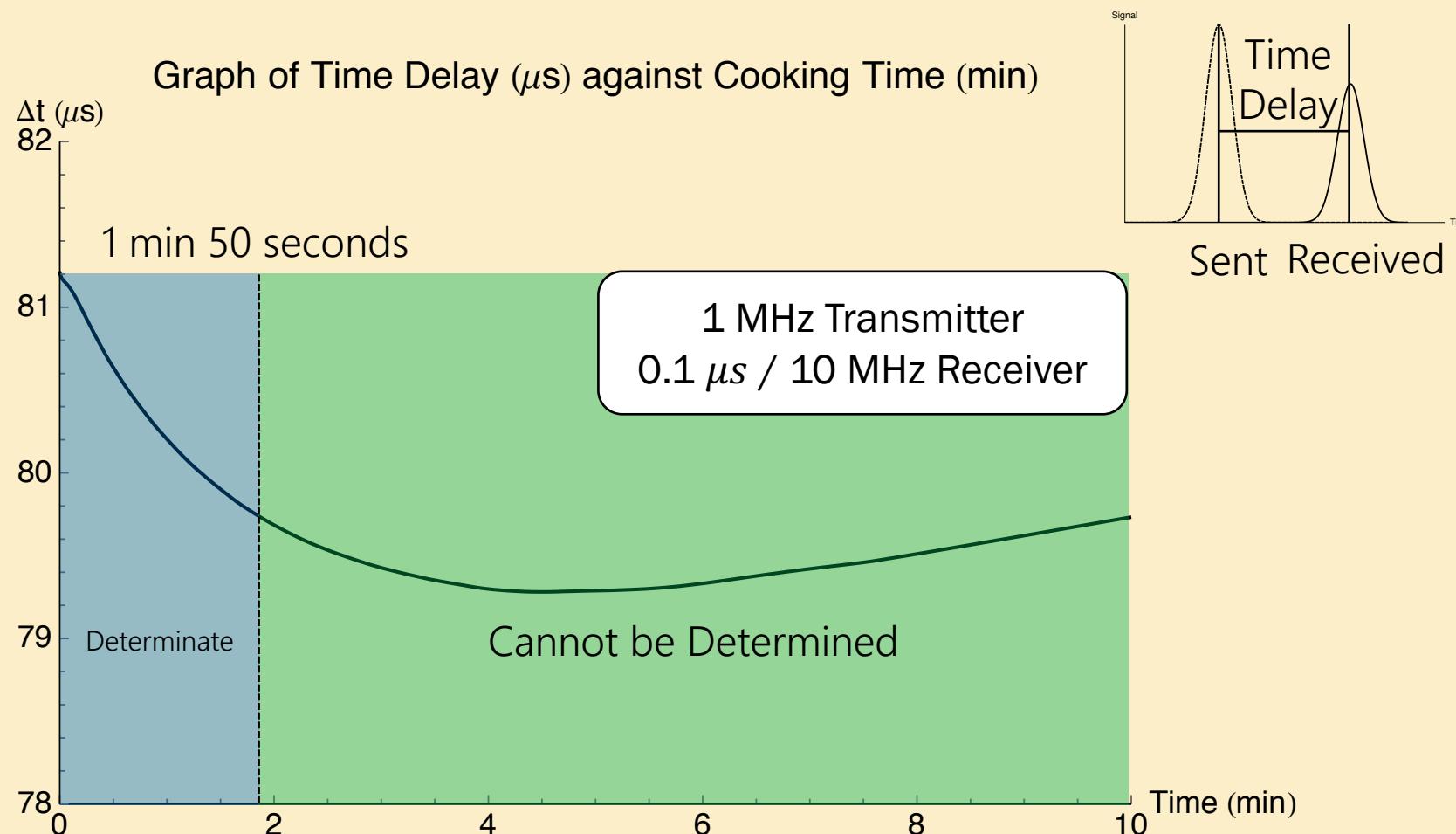
Ultrasound

Sensitivity

Conclusion

Predictive Power

81



Egg Characterization

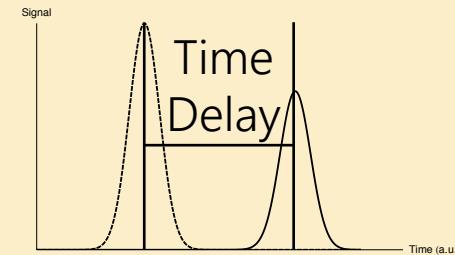
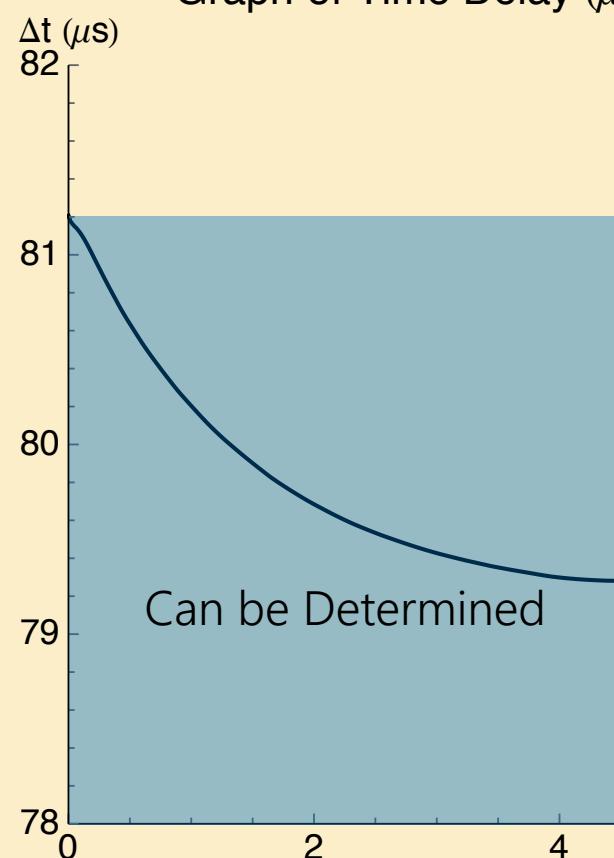
Ultrasound

Sensitivity

Conclusion

Predictive Power

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Cooking Time
 $< 4.5 \text{ min}$

Egg Characterization

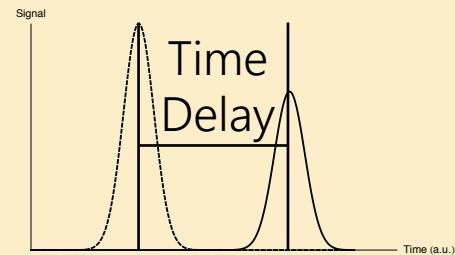
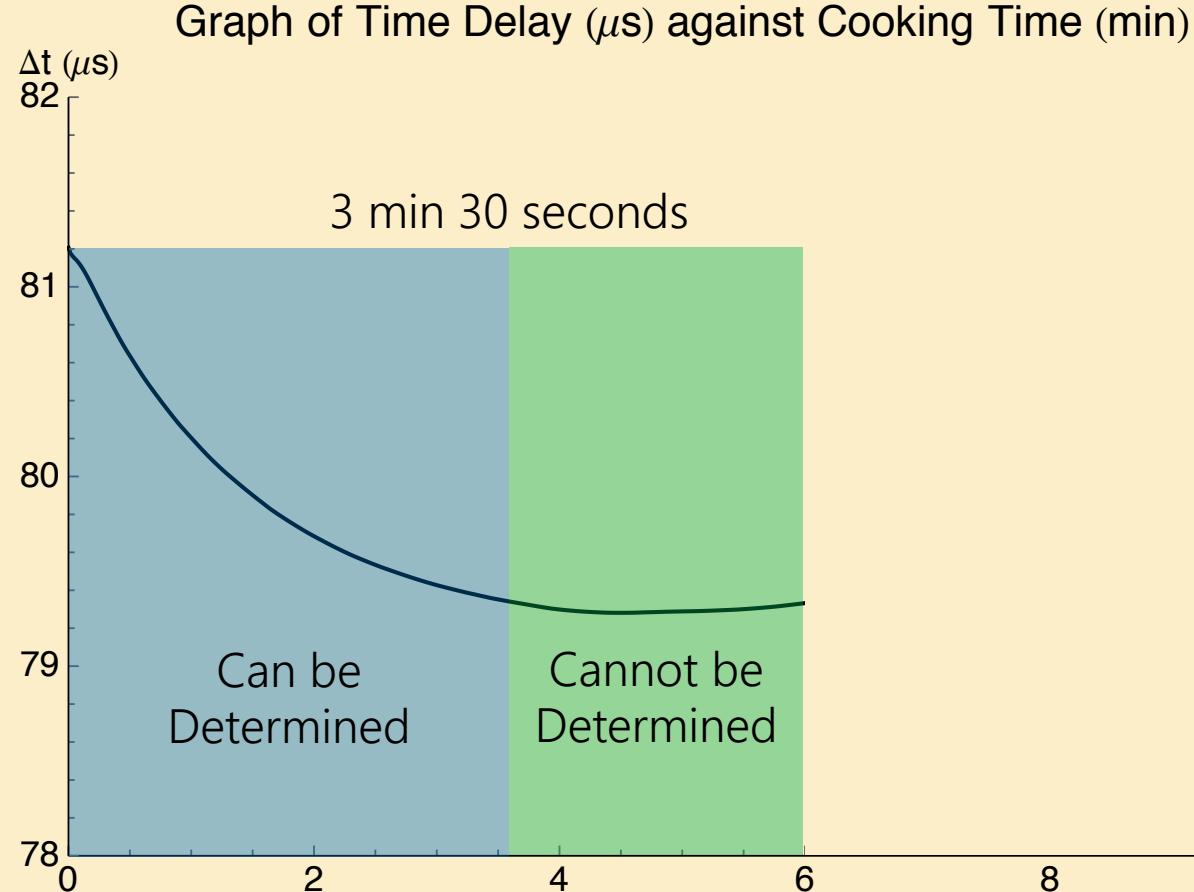
Ultrasound

Sensitivity

Conclusion

Predictive Power

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Sent Received

Cooking Time
 $< 6 \text{ min}$

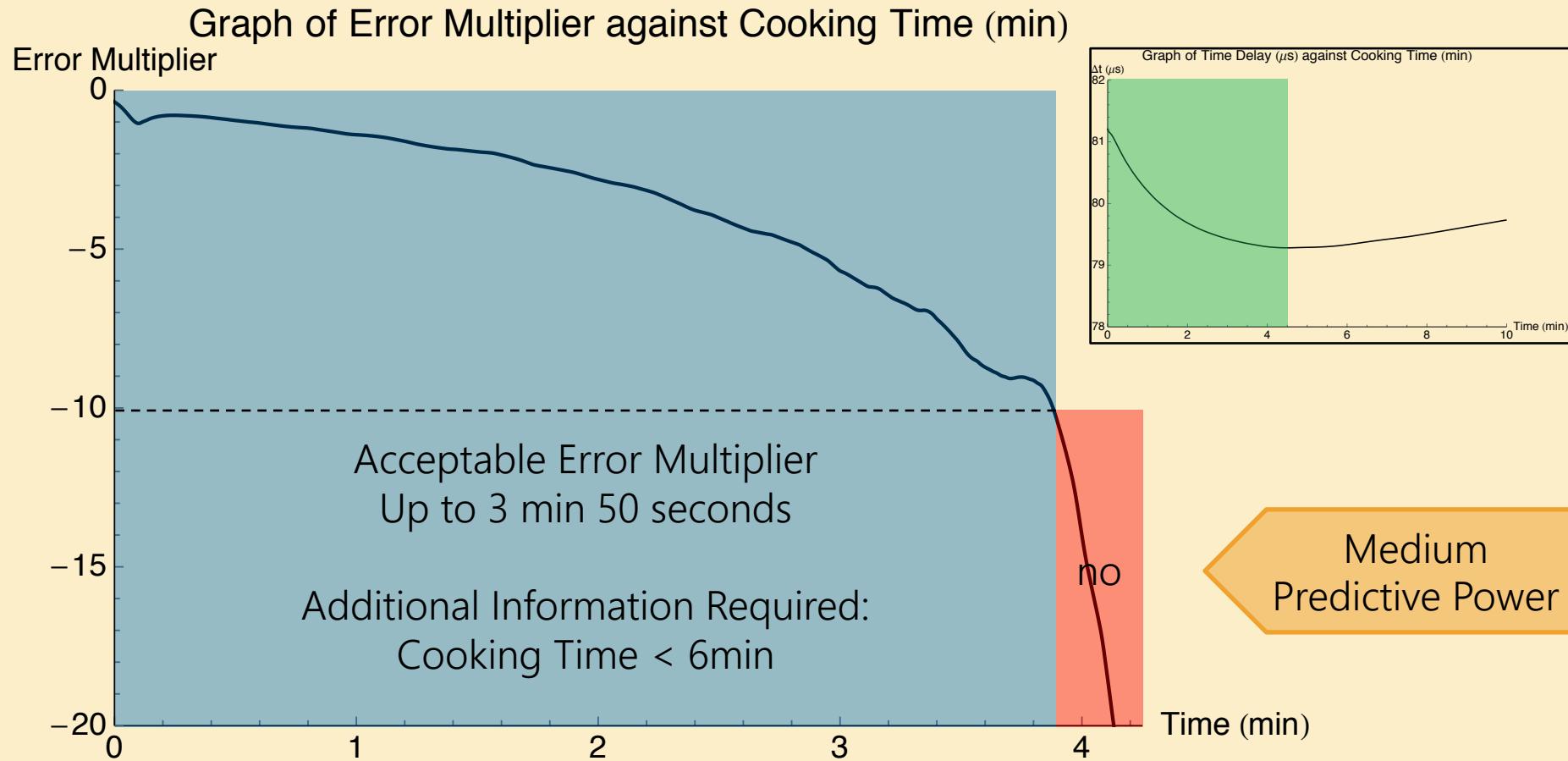
Extra Information
Needed

Limited
Predictive Power



Method 2: Ultrasonic Propagation

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Egg Characterization

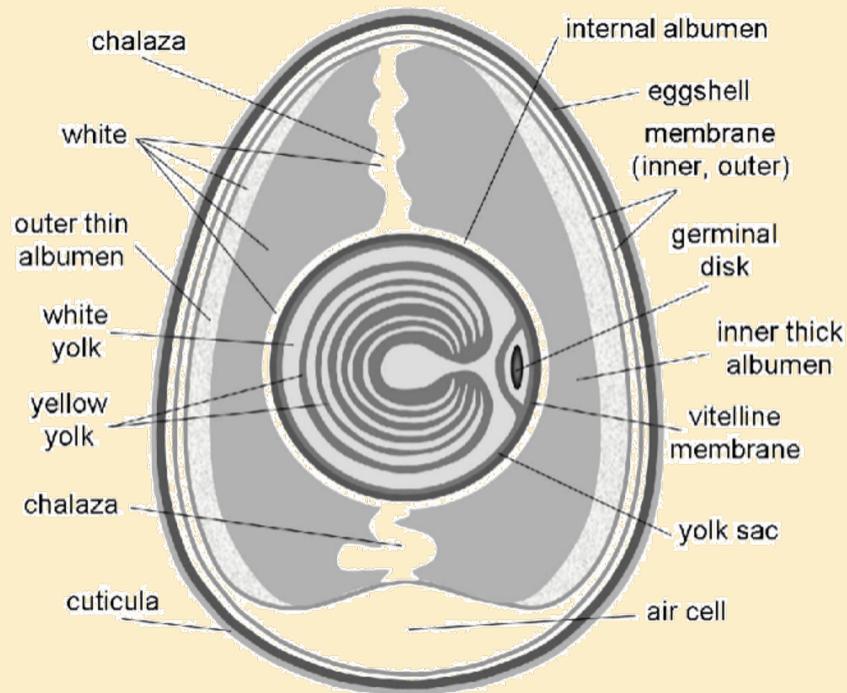
Dynamic Response

Sensitivity

Conclusion

Egg Structure

85



Egg White

85% Water
10% Proteins
5% Others

Egg Yolk

50% Water
15% Proteins
30% Lipid
5% Others

Diagram of Raw Egg^[1]

Proteins include albumins, mucoproteins, and globulins

Egg Characterization

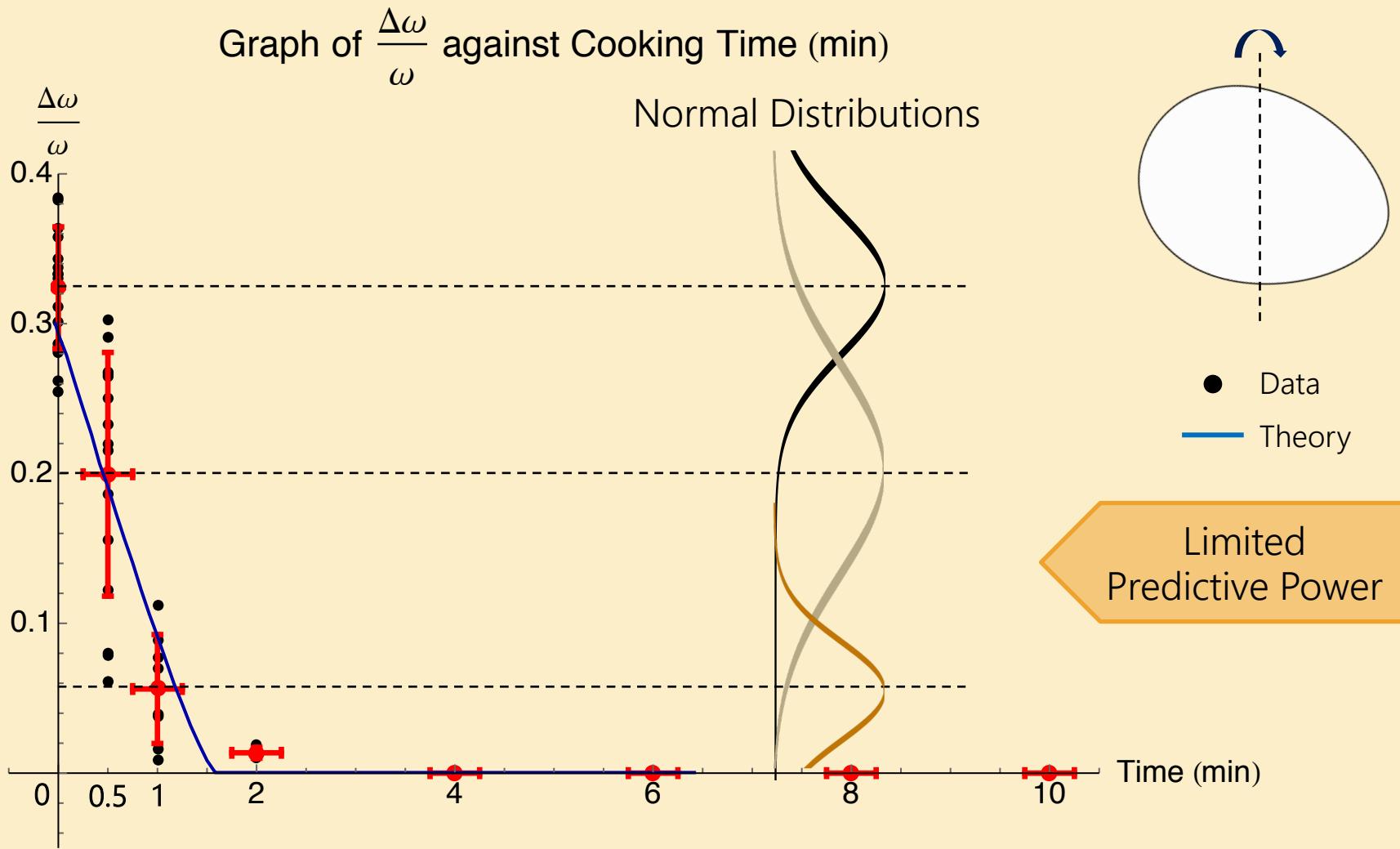
Measurement

Sensitivity

Conclusion

Probabilistic Bin

86



Egg Characterization

Dynamic Response

Sensitivity

Conclusion

Fluid Dynamics

87

Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

Navier-Stokes Equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{\mu}{\rho} \nabla^2 \mathbf{u} = -\nabla w + \mathbf{g}$$

Turbulence Viscosity

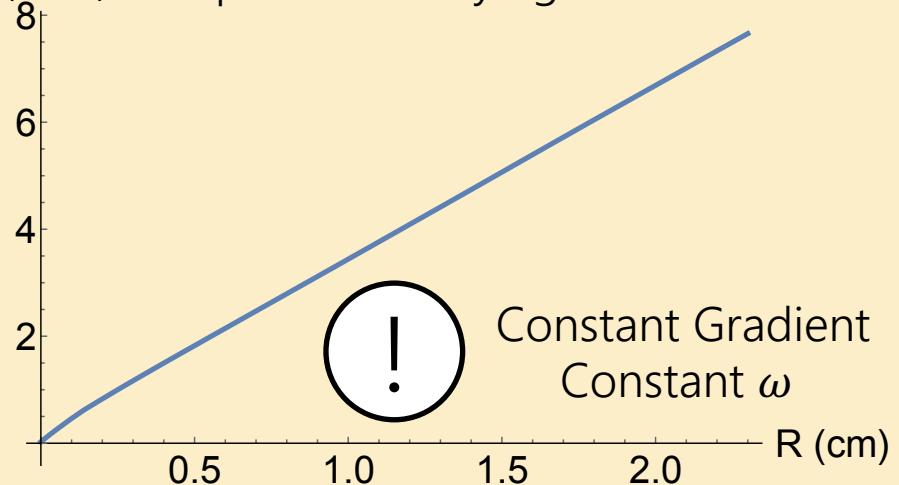
Angular Momentum Retained

$$L = \int_0^{2\pi} \int_0^y(x) \int_0^a \rho r^3 \omega(z, r) dz dr d\theta$$

Residual Spin

$$\frac{\omega'}{\omega} = \frac{I_{Liquid}}{I_{Egg}}$$

v (cm/s) Graph of Velocity against Radius



Egg Characterization

Dynamic Response

Sensitivity

Conclusion