Solution for CDA2015

- **Solution to 3.6.**

Living arrangement	Stage of breast cancer Local	Advanced	Total
Alone	59	85	144
	(72)	(72)	32.58%
	(-2.638731)	(2.6387308)	
With spouse	109	100	209
	(104.5)	(104.5)	47.29%
	(0.8574376)	(-0.857438)	
With others	53	36	89
	(44.5)	(44.5)	20.14%
	(2.0164045)	(-2.016405)	
Total	221	221	442
	50%	50%	

From SAS output, $X^2 \approx 8.3292$, which is approximated by χ^2_{df} distribution with $df = 2$, and p-value =0.0155; Also $G^2 \approx 8.3752$, approximated by χ^2_{df} distribution with $df = 2$, and p-value=0.0152.

- **Solution to 3.18.**

Drug	Nervousness Yes	No	Total
Loratadine	4	184	188
	(2.4258)	(185.57)	30.57%
	(1.2186962)	(-1.218715)	
Placebo	2	260	262
	(3.3806)	(258.62)	42.26%
	(-0.994638)	(0.9946141)	
Chlorpheniramine	2	168	170
	(2.1935)	(167.81)	27.42%
	(-0.154409)	(0.1543916)	
Total	8	612	620
	1.29%	98.71%	

(a). No. $X^2 = 1.6234 \sim \chi^2(2)$ and $p = 0.4441$; $G^2 = 1.5528 \sim \chi^2(2)$ and $p = 0.4600$.

Alternative method. Since the number of observations in some cells are smaller than 5, we can also use Fisher exact test. By SAS output, the p-value for Fisher exact test is about 0.395, so that we can not reject the independence assumption, i.e. no inferential evidence that nervousness depends on drug.

Note: both methods are correct!

(b). (i) $\hat{\theta} = \frac{n_{11}n_{22}}{n_{12}n_{21}} \approx 2.826$, $\hat{\sigma}(\log \hat{\theta}) = \left(\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}\right)^{1/2} \approx 0.871$. For $\alpha = 0.05$, CI_α for $\log\theta$ is:

$$[\log(\hat{\theta}) - z_{\alpha/2}\hat{\sigma}(\log(\hat{\theta})), \log(\hat{\theta}) + z_{\alpha/2}\hat{\sigma}(\log(\hat{\theta}))] = [-0.668, 2.745]$$

and hence CI_α for θ is:

$$[e^{-0.668}, e^{2.745}] = [0.513, 15.565].$$

(ii) $\hat{\pi}_1 = 4/188$, $\hat{\pi}_2 = 2/262$, $\hat{\pi}_1 - \hat{\pi}_2 \approx 0.01364$, $\hat{\sigma}(\hat{\pi}_1 - \hat{\pi}_2) = \left(\frac{\hat{\pi}_1(1-\hat{\pi}_1)}{n_1} + \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{n_2}\right)^{1/2} \approx 0.0118$. So CI_α for $\pi_1 - \pi_2$ is: $(-0.0094, 0.0368)$.

- **Solution to 3.26.** Recall that, as $n \to \infty$, $\sqrt{n}(\hat{\pi}_i - \pi_i) \xrightarrow{d} N(0, \pi_i(1-\pi_i))$ and $Cov(\hat{\pi}_i, \hat{\pi}_j) = -\pi_i\pi_j/n$ for $i \neq j$. Then by delta method,

$$\begin{aligned}\sqrt{n}(g(\hat{\pi}) - g(\pi)) &= \sum_i \frac{\partial g(\pi)}{\partial \pi_i}\sqrt{n}(\hat{\pi}_i - \pi_i) + o_P(1) \\ &= \sum_i -\frac{\eta_i}{\delta^2}\sqrt{n}(\hat{\pi}_i - \pi_i) + o_P(1) \\ &\xrightarrow{d} N(0,\ \sigma^2),\end{aligned}$$

where

$$\sigma^2 = \sum_i \frac{\eta_i^2}{\delta^4}\pi_i(1-\pi_i) - 2\sum\sum_{i<j}\frac{\eta_i\eta_j}{\delta^4}\pi_i\pi_j = \Big[\sum_i \pi_i\eta_i^2 - \big(\sum_i \pi_i\eta_i\big)^2\Big]/\delta^4.$$

□

- **Solution to 3.30.** Recall the notation

$$\hat{\pi}_1 = y_1/n_1, \quad \hat{\pi}_2 = y_2/n_2, \quad \hat{\pi} = (y_1+y_2)/(n_1+n_2), \quad n = n_1 + n_2.$$

Then

$$z^2 = \frac{(y_1/n_1 - y_2/n_2)^2}{(\frac{y_1+y_2}{n_1+n_2})(1 - \frac{y_1+y_2}{n_1+n_2})(\frac{1}{n_1} + \frac{1}{n_2})} = \frac{(n_2y_1 - n_1y_2)^2(n_1+n_2)}{n_1n_2(y_1+y_2)(n_1+n_2-y_1-y_2)}.$$

For the table

y_1	$n_1 - y_1$
y_2	$n_2 - y_2$

,

$$X^2 = \frac{(y_1 - n_1\hat{\pi})^2}{n_1\hat{\pi}} + \frac{(n_1 - y_1 - n_1(1-\hat{\pi}))^2}{n_1(1-\hat{\pi})} + \frac{(y_2 - n_2\hat{\pi})^2}{n_2\hat{\pi}} + \frac{(n_2 - y_2 - n_2(1-\hat{\pi}))^2}{n_2(1-\hat{\pi})}.$$

Simple calculation implies that $X^2 = z^2$. □