Categorical Data Analysis

Chapter 8

Deyuan Li
School of Management
Fudan University

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Outline

8.1 Loglinear Models for Two-way Tables

$I \times J$ contingency table, the numbers in each cell are assumed to be independent and have Poisson distributions with mean $\{\mu_{ij}\}$. The observed cell counts by $\{n_{ij}\}$.

8.1.1 Independence model

The *loglinear independence model* is

(i.e. $\mu_{ij} = \mu\alpha_i\beta_j$), with constraints $\lambda_I^X = \lambda_J^Y = 0$.

The ML estimates are $\{\hat{\mu}_{ij} = n_{i+}n_{+j}/n\}$. Goodness-of-fit can be checked by using X^2 or G^2.

8.1.2 Interpretation of Parameters

We illustrate with independence model for $I \times 2$ tables. In row i, the logit equals

$$\begin{aligned}\mathrm{logit}[P(Y=1|X=i)] &= \log \frac{P(Y=1|X=i)}{P(Y=2|X=i)} \\ &= \log \frac{\mu_{i1}}{\mu_{i2}} = \log \mu_{i1} - \log \mu_{i2} \\ &= (\lambda + \lambda_i^X + \lambda_1^Y) - (\lambda + \lambda_i^X + \lambda_2^Y) = \lambda_1^Y - \lambda_2^Y.\end{aligned}$$

The final term doses not depend on i.

So, independence implies a model of form $\mathrm{logit}[P(Y=1|X=i)] = \alpha$. In each row, the odds of response in column equal $\exp(\lambda_1^Y - \lambda_2^Y)$.

8.1.3 Saturated model

The *saturated model* is $\log \mu_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_{ij}^{XY}$, where $\{\lambda_{ij}^{XY}\}$ are association terms. All $\lambda_{ij}^{XY} = 0 \Rightarrow$ independence.

The constraints are $\lambda_I^X = \lambda_J^Y = 0$ and $\lambda_{Ij}^{XY} = \lambda_{iJ}^{XY} = 0$ for all i, j. So, the number of parameters in saturated model is $1 + (I-1) + (J-1) + (I-1)(J-1) = IJ$, the number of cells.

There exist relationships between log odds and $\{\lambda_{ij}^{XY}\}$. For example, for 2×2 tables,

Thus, $\{\lambda_{ij}^{XY}\}$ determine the association.

8.1.5 Multinomial models for cell probabilities

Conditional on the sum n of the cell counts, Poisson loglinear models for $\{\mu_{ij}\}$ become multinomial models for the cell probabilities $\{\pi_{ij} = \mu_{ij}/\sum\sum\mu_{ab}\}$.

For saturated models, it is

$$\pi_{ij} = \frac{\exp(\lambda + \lambda_i^X + \lambda_j^Y + \lambda_{ij}^{XY})}{\sum_a \sum_b \exp(\lambda + \lambda_a^X + \lambda_b^Y + \lambda_{ab}^{XY})}.$$

Outline

8.2 Logistic Models for Independence and Interaction in Three-Way Tables

In Section 2.3 we introduced three-way contingency tables and related structure such as conditional independence and homogeneous association. Loglinear models for three-way tables describe their independence and association patterns.

8.2.1 Types of independence

$I \times J \times K$-table; X, Y, Z three response variables;

n_{ijk}: the number of observations in cell (i, j, k), independently from Poisson sampling with mean μ_{ijk}.

8.2.1 Types of independence

- *Mutual independence* has loglinear form

  (similar to multinomial sampling, $\pi_{ijk} = \pi_{i++}\pi_{+j+}\pi_{++k}$).
- *Jointly independence* (Y is independent of X and Z) has loglinear form

  (similar to multinominal sampling, $\pi_{ijk} = \pi_{i+k}\pi_{+j+}$).
- *Conditional independence* (X and Y are independent given Z) has loglinear form

  (similar to multinominal sampling, $\pi_{ijk} = \pi_{i+k}\pi_{+jk}/\pi_{++k}$).

8.2.1 Types of independence

Proof of (6).

8.2.1 Types of independence

TABLE 8.1 Summary of Loglinear Independence Models

Model	Probabilistic Form for π_{ijk}	Association Terms in Loglinear Model	Interpretation
(8.6)	$\pi_{i++}\pi_{+j+}\pi_{++k}$	None	Variables mutually independent
(8.8)	$\pi_{i+k}\pi_{+j+}$	λ_{ik}^{XZ}	Y independent of X and Z
(8.10)	$\pi_{i+k}\pi_{+jk}/\pi_{++k}$	$\lambda_{ik}^{XZ} + \lambda_{jk}^{YZ}$	X and Y independent, given Z

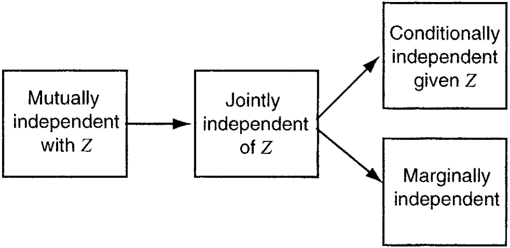

FIGURE 8.1 Relationships among types of XY independence.

8.2.2 Homogeneous association and three-factor interaction

Loglinear models (6), (8) and (10) have three, two and one pair of conditionally independent variables, respectively.

A model that permits all three pairs to be conditionally dependent is

$$\log \mu_{ijk} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ} \tag{11}$$

Model (11) is called the loglinear model of *homogeneous association* or of *no three-factor interaction*.

The general loglinear model for a three-way contingency table is the *saturated model*, i.e.

$$\log \mu_{ijk} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ} + \lambda_{ijk}^{XYZ}. \tag{12}$$

8.2.2 Homogeneous association and three-factor interaction

The number of parameters in saturated model equals $1+(I-1)+(J-1)+(K-1)+(I-1)(J-1)+(I-1)(K-1)+(J-1)(K-1)+(I-1)(J-1)(K-1)=IJK$, the number of cells.

Table 8.2 lists the loglinear models and their symbols.

TABLE 8.2 Loglinear Models for Three-Dimensional Tables

Loglinear Model	Symbol
$\log \mu_{ijk} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z$	(X, Y, Z)
$\log \mu_{ijk} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY}$	(XY, Z)
$\log \mu_{ijk} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{jk}^{YZ}$	(XY, YZ)
$\log \mu_{ijk} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{jk}^{YZ} + \lambda_{ik}^{XZ}$	(XY, YZ, XZ)
$\log \mu_{ijk} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{jk}^{YZ} + \lambda_{ik}^{XZ} + \lambda_{ijk}^{XYZ}$	(XYZ)

8.2.3 Interpreting model parameters

Interpretations of loglinear model parameters use their highest-order terms.

For instance, interpretation for model (11) use the two-factor terms to describe conditional odds ratios. At a fixed level k of Z, the conditional association between X and Y uses $(I-1)(J-1)$ odds ratios, such as the local odds ratios

$$\theta_{ij(k)} = \frac{\pi_{ijk}\pi_{i+1,j+1,k}}{\pi_{i,j+1,k}\pi_{i+1,j,k}}, \quad 1 \le i \le I-1,\ 1 \le j \le J-1.$$

Similarly, $(I-1)(K-1)$ odds ratios $\{\theta_{i(j)k}\}$ describe XZ conditional association, and $(J-1)(K-1)$ odds ratios $\{\theta_{(i)jk}\}$ describe YZ conditional association.

8.2.3 Interpreting model parameters

The two-factor parameters relate directly to the conditional odds ratios.

For model (11) (i.e. model (XY, XZ, YZ)),

$$\log \theta_{ij(k)} = \log \frac{\mu_{ijk}\mu_{i+1,j+1,k}}{\mu_{i+1,jk}\mu_{i,j+1,k}} = \lambda_{ij}^{XY} + \lambda_{i+1,j+1}^{XY} - \lambda_{i,j+1}^{XY} - \lambda_{i+1,j}^{XY}.$$

So, the meaning of parameters are obvious.

8.2.4 Alcohol, cigarette, and marijuana use example

Table 8.3 refers to a 1992 survey by the Wright State University School of Medicine and United Health Service in Dayton Ohio. 2276 students are asked whether using alcohol, cigarettes, or marijuana in their final year of high school. $\Rightarrow 2 \times 2 \times 2$ table.

A: alcohol use; *C*: cigarette use; *M*: marijuana（大麻）use.

TABLE 8.3 Alcohol, Cigarette, and Marijuana Use for High School Seniors

Alcohol Use	Cigarette Use	Marijuana Use Yes	Marijuana Use No
Yes	Yes	911	538
	No	44	456
No	Yes	3	43
	No	2	279

Source: Data courtesy of Harry Khamis, Wright State University.

8.2.4 Alcohol, cigarette, and marijuana use example

TABLE 8.4 Fitted Values for Loglinear Models Applied to Table 8.3

Alcohol Use	Cigarette Use	Marijuana Use	Loglinear Model[a] (A, C, M)	(AC, M)	(AM, CM)	(AC, AM, CM)	(ACM)
Yes	Yes	Yes	540.0	611.2	909.24	910.4	911
		No	740.2	837.8	438.84	538.6	538
	No	Yes	282.1	210.9	45.76	44.6	44
		No	386.7	289.1	555.16	455.4	456
No	Yes	Yes	90.6	19.4	4.76	3.6	3
		No	124.2	26.6	142.16	42.4	43
	No	Yes	47.3	118.5	0.24	1.4	2
		No	64.9	162.5	179.84	279.6	279

[a] A, alcohol use; C, cigarette use; M, marijuana use.

From Table 8.4, we see that model (AC, AM, CM) is close to the observed data (i.e. saturated model (ACM)). The other models fit poorly.

8.2.4 Alcohol, cigarette, and marijuana use example

Table 8.5 illustrates model association patterns by presenting estimated conditional and marginal odds ratios.

TABLE 8.5 Estimated Odds Ratios for Loglinear Models in Table 8.5

	Conditional Association			Marginal Association		
Model	AC	AM	CM	AC	AM	CM
(A, C, M)	1.0	1.0	1.0	1.0	1.0	1.0
(AC, M)	17.7	1.0	1.0	17.7	1.0	1.0
(AM, CM)	1.0	61.9	25.1	2.7	61.9	25.1
(AC, AM, CM)	7.8	19.8	17.3	17.7	61.9	25.1
(ACM) level 1	13.8	24.3	17.5	17.7	61.9	25.1
(ACM) level 2	7.7	13.5	9.7			

8.2.4 Alcohol, cigarette, and marijuana use example

The *AC* conditional association for the model (AM, CM) is

$$1.0 \simeq \frac{909.24 \times 0.24}{45.76 \times 4.76} \simeq \frac{438.84 \times 179.84}{555.16 \times 142.16}$$

and the *AC* marginal association for the model (AM, CM) is

$$2.7 \simeq \frac{(909.24 + 438.84) \times (0.24 + 179.84)}{(45.76 + 555.16) \times (4.76 + 142.16)}.$$

8.2.4 Alcohol, cigarette, and marijuana use example

Three comments:

1. $1.0 \neq 2.7$: conditional independence does not imply marginal independence;
2. The estimated conditional odds ratios equal to 1.0 for each pairwise term not appearing in a model, such as (AC) association in model (AM, CM).
3. The estimated odds ratios are very dependent on the model, which highlights the importance of good model selection.

Outline

8.3.1 Chi-squared goodness-of-fit tests

As usual, X^2 and G^2 test whether a model holds by comparing cell fitted values and observed counts. The *df* equals to the number of cells minus the number of model parameters.
$df = N - p$.

Table 8.6 shows results of testing fit for several loglinear models for the students survey data (see Table 8.3).

8.3.1 Chi-squared goodness-of-fit tests

TABLE 8.6 Goodness-of-Fit Tests for Loglinear Models in Table 8.4

Model	G^2	X^2	df	P-value[a]
(A, C, M)	1286.0	1411.4	4	< 0.001
(A, CM)	534.2	505.6	3	< 0.001
(C, AM)	939.6	824.2	3	< 0.001
(M, AC)	843.8	704.9	3	< 0.001
(AC, AM)	497.4	443.8	2	< 0.001
(AC, CM)	92.0	80.8	2	< 0.001
(AM, CM)	187.8	177.6	2	< 0.001
(AC, AM, CM)	0.4	0.4	1	0.54
(ACM)	0.0	0.0	0	—

[a] P-value for G^2 statistic.

From Table 8.6, we see that models lack any association term fit poorly ($P - \text{value} < 0.05$). The model (AC, AM, CM) has all pairwise associations and fits well ($P - \text{value} = 0.54$).

8.3.2 Inference about conditional association

Tests about conditional associations are checked by comparing loglinear models. Likelihood-ratio statistics,

$$G^2[M_0|M_1] = G^2(M_0) - G^2(M_1).$$

For model (XY, XZ, YZ), consider the hypothesis of XY conditional independence. This is $H_0 : \lambda_{ij}^{XY} = 0$ for the $(I-1)(J-1)$ association parameters.

The test statistic is

with $df = (I-1)(J-1)$.

8.3.2 Inference about conditional association

For the student survey data, the test of conditional independence between alcohol use and cigarette smoking compares model (AM, CM) with the alternative model (AC, AM, CM). The test statistic is

$$G^2[(AM, CM)|(AC, AM, CM)] = 187.8 - 0.4 = 187.4$$

with $df = 2 - 1 = 1$ ($P < 0.001$).

So, alcohol use and cigarette use are not conditional independence and AC-term is necessary.

8.3.2 Inference about conditional association

Table 8.7 shows output from fitting model (*AC*, *AM*, *CM*) with parameters in the last row and in the last column equal to zero.

TABLE 8.7 Output for Fitting Loglinear Model to Table 8.3

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value / DF
Deviance	1	0.3740	0.3740
Pearson Chi-Square	1	0.4011	0.4011

Parameter			Estimate	Standard Error	Wald Chi-Square	Pr>ChiSq
Intercept			5.6334	0.0597	8903.96	<.0001
a	1		0.4877	0.0758	41.44	<.0001
c	1		−1.8867	0.1627	134.47	<.0001
m	1		−5.3090	0.4752	124.82	<.0001
a*m	1	1	2.9860	0.4647	41.29	<.0001
a*c	1	1	2.0545	0.1741	139.32	<.0001
c*m	1	1	2.8479	0.1638	302.14	<.0001

LR Statistics

Source	DF	Chi-Square	Pr>ChiSq
a*m	1	91.64	<.0001
a*c	1	187.38	<.0001
c*m	1	497.00	<.0001

8.3.2 Inference about conditional association

Consider the conditional *AC* odds ratio, assuming model (AC, AM, CM).

- Table 8.7 reports $\hat{\lambda}_{11}^{AC} = 2.054$ with SE$= 0.174$, which is the estimated conditional log odds ratio (because of the constraints).
- A 95% Wald confidence interval for the true *AC* conditional odds ratio is $\exp[2.054 \pm 1.96(0.174)] = (5.5, 11.0)$.
- Strong positive association exists between cigarette use and alcohol, both for users and nonusers of marijuana.

For the model (AC, AM, CM), the 95% Wald confidence intervals are $(8.0, 49.2)$ for the *AM* conditional odds ratio and $(12.5, 23.8)$ for the *CM* conditional odds ratio. (*for students: How to get the intervals from Table 8.7 ?*)

The intervals are wide, but these associations are strong.

Outline

8.5 Loglinear-Logit Model Connection

Loglinear models treat categorical response variables symmetrically, focusing on association and interactions in their joint distribution.

Logit models, by contrast, describe how a single categorical response depends on explanatory variables.

There exists connection between loglinear and logit models.

- For a loglinear model, forming logits on one response helps to interpret the model.
- Logit models with categorical explanatory variables have equivalent loglinear models.

8.5.1 Using logit models to interpret loglinear models

For example, consider the loglinear model $(XY.XZ, YZ)$. When Y is binary, its logit is

$$\begin{aligned}
\log \frac{P(Y=1|X=i,Z=k)}{P(Y=2|X=i,Z=k)} &= \log \frac{\mu_{i1k}}{\mu_{i2k}} = \log \mu_{i1k} - \log \mu_{i2k} \\
&= (\lambda + \lambda_i^X + \lambda_1^Y + \lambda_k^Z + \lambda_{i1}^{XY} + \lambda_{ik}^{XZ} + \lambda_{1k}^{YZ}) \\
&\quad - (\lambda + \lambda_i^X + \lambda_2^Y + \lambda_k^Z + \lambda_{i2}^{XY} + \lambda_{ik}^{XZ} + \lambda_{2k}^{YZ}) \\
&= (\lambda_1^Y - \lambda_2^Y) + (\lambda_{i1}^{XY} - \lambda_{i2}^{XY}) + (\lambda_{1k}^{YZ} - \lambda_{2k}^{YZ}).
\end{aligned}$$

The first term is constant, not depending on i and k; the second term depends on category i of X; and the third term depends on category k of Z. This logit has the additive form

$$\mathrm{logit}[P(Y=1|X=i,Z=k)] = \alpha + \beta_i^X + \beta_k^Z.$$

Using the notation summarizing logit models by their predictors, we denotes it by $(X+Z)$.

8.5.3 Corresponding between loglinear and logit models

TABLE 8.11 Equivalent Loglinear and Logit Models for a Three-Way Table with Binary Response Variable *Y*

Loglinear Symbol	Logit Model	Logit Symbol
(Y, XZ)	α	(—)
(XY, XZ)	$\alpha + \beta_i^X$	(X)
(YZ, XZ)	$\alpha + \beta_k^Z$	(Z)
(XY, YZ, XZ)	$\alpha + \beta_i^X + \beta_k^Z$	$(X + Z)$
(XYZ)	$\alpha + \beta_i^X + \beta_k^Z + \beta_{ik}^{XZ}$	$(X{*}Z)$

Loglinear models are most natural when at least two variables are response variables.

When only one is a response, it is more sensible to use logit models directly.

8.5.4 Generalized loglinear model

Let $\mathbf{n} = (n_1, n_2, ..., n_N)$ and $\boldsymbol{\mu} = (\mu_1, \mu_2, ..., \mu_N)'$ denote the column vectors of observed and expected accounts for the N cells of a contingency table, with $n = \sum_i n_i$.

For simplicity, we use a single index, but the table may be multidimensional.

Loglinear models for positive Poisson means have the form

$$\log \boldsymbol{\mu} = \mathbf{X}\beta \tag{17}$$

for model $\mathbf{X}$ and column vector β of model parameters.

8.5.4 Generalized loglinear model

For example, for independence model, $\log \mu_{ij} = \lambda + \lambda_i^X + \lambda_j^Y$ of a 2×2 table, with constraints $\lambda_2^X = \lambda_2^Y = 0$, the model is

$$\begin{bmatrix} \log \mu_{11} \\ \log \mu_{12} \\ \log \mu_{21} \\ \log \mu_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ \lambda_1^X \\ \lambda_1^Y \end{bmatrix}.$$

The *generalized loglinear model* is

$$\mathbf{C} \log(\mathbf{A}\boldsymbol{\mu}) = \mathbf{X}\boldsymbol{\beta}$$

for matrices **C** and **A**.

For $\mathbf{C} = \mathbf{A} = \mathbf{I}$, it is model (17); Other special cases include logit models for binary or multicategory responses.

Chapter 10 and 11 will discuss the generalized loglinear models.

Outline

8.6.2 Likelihood equations for loglinear models

For three-way tables, the joint Poisson probability is

$$\prod_i \prod_j \prod_k \frac{e^{-\mu_{ijk}} \mu_{ijk}^{n_{ijk}}}{n_{ijk}!}$$

and the kernel log likelihood is

$$L(\boldsymbol{\mu}) = \sum_i \sum_j \sum_k n_{ijk} \log \mu_{ijk} - \sum_i \sum_j \sum_k \mu_{ijk}. \tag{19}$$

For the general loglinear model (12), this simplifies

$$\begin{aligned} L(\boldsymbol{\mu}) = n\lambda &+ \sum_i n_{i++}\lambda_i^X + \sum_j n_{+j+}\lambda_j^Y + \sum_k n_{++k}\lambda_k^Z \\ &+ \sum_i \sum_j n_{ij+}\lambda_{ij}^{XY} + \sum_i \sum_k n_{i+k}\lambda_{ik}^{XZ} + \sum_j \sum_k n_{+jk}\lambda_{jk}^{YZ} \\ &+ \sum_i \sum_j \sum_k n_{ijk}\lambda_{ijk}^{XYZ} - \sum_i \sum_j \sum_k \exp(\lambda + \cdots + \lambda_{ijk}^{XYZ}). \end{aligned} \tag{20}$$

8.6.2 Likelihood equations for loglinear models

The model is $\log \boldsymbol{\mu} = \mathbf{X}\boldsymbol{\beta}$, for which $\log \mu_i = \sum_j x_{ij}\beta_j$ for all i. Extending (19), for Poisson sampling the log likelihood is

$$\begin{aligned} L(\boldsymbol{\mu}) &= \sum_i n_i \log \mu_i - \sum_i \mu_i \\ &= \sum_i n_i \Big(\sum_j x_{ij}\beta_j \Big) - \sum_i \exp \Big(\sum_j x_{ij}\beta_j \Big). \end{aligned}$$

It follows that

$$\frac{\partial L(\boldsymbol{\mu})}{\partial \beta_j} = \sum_i n_i x_{ij} - \sum_i \mu_i x_{ij}, \quad j = 1, 2, ..., p.$$

8.6.2 Likelihood equations for loglinear models

So, the likelihood equation is

$$\mathbf{X}'\mathbf{n} = \mathbf{X}'\boldsymbol{\mu}.$$

Consider model (XZ, YZ). Its log likelihood is (20) with $\lambda^{XY} = \lambda^{XYZ} = 0$. The log-likelihood derivatives

yields the likelihood equations

$$\hat{\mu}_{i+k} = n_{i+k} \quad \text{for all } i \text{ and } k,$$
$$\hat{\mu}_{+jk} = n_{+jk} \quad \text{for all } j \text{ and } k.$$

8.6.4 Direct versus iterative calculation of fitted values

For some models, we can not get the estimators of μ_{ijk}. For examples, the model (XY, XZ, YZ). For these models we will use iterative methods (Section 8.7) to obtain $\hat{\mu}_{ijk}$.

Of models in Table 8.12 and 8.13, the only one not having direct estimates is (XY, XZ, YZ). It is not possible to express $\hat{\mu}_{ijk}$ directly in terms of $\{\mu_{ij+}\}, \{\mu_{i+k}\}, \{\mu_{+jk}\}$.

TABLE 8.13 Fitted Values for Loglinear Models in Three-Way Tables

Model[a]	Probabilistic Form	Fitted Value
(X, Y, Z)	$\pi_{ijk} = \pi_{i++}\pi_{+j+}\pi_{++k}$	$\hat{\mu}_{ijk} = \dfrac{n_{i++}\, n_{+j+} n_{++k}}{n^2}$
(XY, Z)	$\pi_{ijk} = \pi_{ij+}\pi_{++k}$	$\hat{\mu}_{ijk} = \dfrac{n_{ij+} n_{++k}}{n}$
(XY, XZ)	$\pi_{ijk} = \dfrac{\pi_{ij+}\pi_{i+k}}{\pi_{i++}}$	$\hat{\mu}_{ijk} = \dfrac{n_{ij+} n_{i+k}}{n_{i++}}$
(XY, XZ, YZ)	$\pi_{ijk} = \psi_{ij}\phi_{jk}\omega_{ik}$	Iterative methods (Section 8.7)
(XYZ)	No restriction	$\hat{\mu}_{ijk} = n_{ijk}$

[a]Formulas for models not listed are obtained by symmetry; for example, for (XZ, Y), $\hat{\mu}_{ijk} =$

8.6.5 Chi-squared goodness-of-fit tests

Model goodness-of-fit statistics compare cell counts to sample counts.

X^2 and G^2 have asymptotic chi-squared distribution. The degree of freedom for different models is listed in Table 8.14.

TABLE 8.14 Residual Degrees of Freedom for Loglinear Models for Three-Way Tables

Model	Degrees of Freedom
(X, Y, Z)	$IJK - I - J - K + 2$
(XY, Z)	$(K-1)(IJ-1)$
(XZ, Y)	$(J-1)(IK-1)$
(YZ, X)	$(I-1)(JK-1)$
(XY, YZ)	$J(I-1)(K-1)$
(XZ, YZ)	$K(I-1)(J-1)$
(XY, XZ)	$I(J-1)(K-1)$
(XY, XZ, YZ)	$(I-1)(J-1)(K-1)$
(XYZ)	0

8.6.6 Covariance matrix of ML parameter estimators

The model is $\log \boldsymbol{\mu} = \mathbf{X}\beta$, i.e. $\log \mu_i = \sum_j x_{ij}\beta_j$ for all i.

For a fixed number of cells, as $n \to \infty$, the ML estimator $\hat{\beta}$ is asymptotically normal with mean β and covariance matrix

$$\mathrm{cov}(\hat{\beta}) = [\mathbf{X}' \mathbf{diag}(\mu)\mathbf{X}]^{-1}.$$

The standard errors are the squared roots of diagonal elements of $\mathrm{cov}(\hat{\beta})$.

TABLE A.13 SAS Code for Fitting Loglinear Models to Drug Survey Data in Table 8.3

```
data drugs;
input a c m count @@;
datalines;
1 1 1 911    1 1 2 538    1 2 1 44    1 2 2 456
2 1 1   3    2 1 2  43    2 2 1  2    2 2 2 279
;
proc genmod;  class a c m;
 model count = a c m a*m a*c c*m / dist = poi link = log lrci type3 obstats;
```

TABLE A.14 SAS Code for Raking Table 8.15

```
data rake;
input school atti count @@;
log_c = log(count); pseudo = 100 / 3;
data lines;
1 1 209    1 2 101    1 3 237
...
;
proc genmod; class school atti;
 model pseudo = school atti / dist = poi link = log offset = log_c obstats;
```