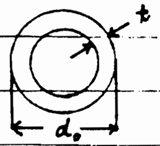
## 問題 1.2

1. 


$t = 2.5^{cm}$, $P = 70\ tf$, $\sigma_w = 800\ Kgf/cm^2$

$$A = \frac{P}{\sigma_w} = \frac{70 \times 10^3}{800} = 87.5^{cm^2}$$

$$A = \frac{\pi}{4}\{d_o^2 - (d_o - 2t)^2\} = \frac{\pi}{4}(4td_o - 4t^2)$$

$$\therefore\ d_o = \left(\frac{4 \times 87.5}{\pi} + 4 \times 2.5^2\right) / 4 \times 2.5 = 13.64^{cm}$$

2. $t = \frac{1}{10} d_o$, $A = \frac{\pi}{4}\left(4 \times \frac{1}{10} d_o^2 - 4 \times \frac{1}{100} d_o^2\right) = \frac{9\pi}{100} d_o^2$

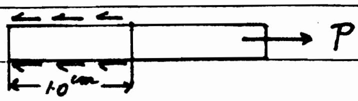
$$\therefore\ d_o = \sqrt{\frac{100}{9\pi} \times 87.5} = 17.592^{cm}$$

3. $\gamma = 7850\ Kg/m^3$, $\sigma_t = 200 \times 10^6\ N/m^2$

$$W = \gamma A l \cdot g$$

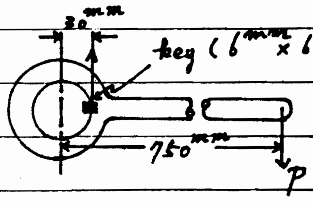
$$\therefore\ \sigma_t = \frac{W}{A} = \frac{\gamma A l \cdot g}{A}$$

$$\therefore\ l = \frac{\sigma_t}{\gamma g} = \frac{200 \times 10^6}{7850 \times 9.801} = 2600\ m$$

4. 


$P = 2400\ Kgf$

$$\tau_{av} = \frac{2400}{2 \times (4 \times 10)} = 30\ Kgf/cm^2$$

5. 


$\tau_{av} = 60\ MN/m^2 = 60 \times 10^6\ N/m^2$

$$\Sigma M = 0$$

$$0.75P = 0.03 \times (6 \times 25) \cdot 10^{-6} \times 60 \times 10^6$$

$$\therefore\ P = 360\ N$$

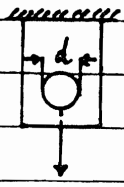
6. $w = 7830\ Kgf/m^3$, $l = 100^m$, $\sigma_w = 400\ Kgf/cm^2$

$$wAl = 7830 \cdot 10^{-6} \times A \times 100 \cdot 10^2 = 78.3A$$

$$S_{max} = (78.3A + 900)\ Kgf$$

$$\sigma_w = S_{max}/A \qquad \therefore\ A = 2.798^{cm^2}$$

7.



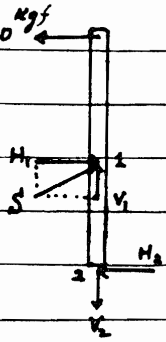

알루미늄의 허용인장응력 $\sigma_w = 20\ N/mm^2$
강철의 허용전단응력 $\tau_w = 40\ N/mm^2$
포오크의 다리의 단면 $50^{mm} \times 6^{mm}$

$$(50-d)\times 6\times 20 = \frac{\pi}{4}d^2\times 40$$

$$\pi d^2 + 12d - 600 = 0$$

$$\therefore d = \frac{-12+\sqrt{12^2+4\pi\times 600}}{2\pi} = 12.04\ mm$$

8.




木材 $10^{cm}\times 10^{cm}$
$\tau_w = 7\ kgf/cm^2$
$\sigma_w = 28\ kgf/cm^2$

Moment equilibrium.

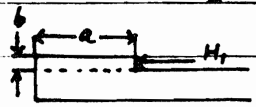
$H_1\times 90 = 120\times 210$ $\qquad$ $H_2\times 90 = 120\times 120$

$\therefore H_1 = 280\ kgf$ $\qquad$ $\therefore H_2 = 160\ kgf$

from Geometry.

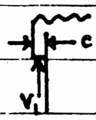
$V_1 = H_1\times\frac{3}{4} = 210\ kgf$ $\qquad$ $S = H_1\times\frac{5}{4} = 350\ kgf$

$V_2 = -V_1 = -210\ kgf.$



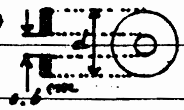

$a\times 10\times 7 = 280$ $\qquad$ $\therefore a = 4^{cm}$

$b\times 10\times 28 = 280$ $\qquad$ $\therefore b = 1^{cm}$




$c\times 10\times 28 = 210$ $\qquad$ $\therefore c = \frac{3}{4}\ cm$

9

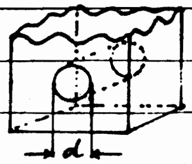



$P = 120^{kgf}$ $\qquad$ bolt의 지름 $= 6^{mm}$
$\sigma_w = 14\ kgf/cm^2$

$$\frac{\pi}{4}(d^2-0.6^2)\times 14 = 120$$

$$d^2 = \frac{2\times 120}{7\pi} + 0.36 = 11.273$$

$$\therefore d = 3.358\ cm$$

10. 

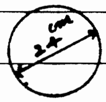
$\sigma_w = 14\ kgf/cm^2$

受压面 $= 10d$

反力 $= \sqrt{H_2^2 + V_2^2} = \sqrt{160^2 + 210^2} \doteqdot 246.008$

$10d \times 14 = 264.008$

$\therefore d = 1.886\ cm$

11 

$\sigma_x$ (압력솥의 응력).

$p = 2\ N/mm^2$, bolt의 직경 $= 12\ mm$

$\sigma_w = 80\ N/mm^2$, bolt의 수 $= n$

$P_x$ 上向力 $= \frac{\pi}{4} \times 240^2 \times 2 \ (A \cdot \sigma_x)$

bolt의 저항력 $= \frac{\pi}{4} \times 1.2^2 \times n \times 80$

Equilibrium : $\frac{\pi}{4} \times 240^2 \times 2 = \frac{\pi}{4} \times 12^2 \times n \times 80 = \sigma_w \cdot A \cdot n$.

[illegible] $\therefore n = 10$ 개.

## 問題 1.3

1. $P = 20\ KN$, $E = 69\ GN/m^2 = 69\ KN/mm^2$

$$\delta = \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} = \frac{20}{69}\left(\frac{1000}{30^2} + \frac{2000}{\frac{\pi}{4} \times 30^2}\right)$$

$= 1.142\ mm$

2. $E = 2.1 \times 10^6\ kgf/cm^2$, $l = 100\ m$, $A = 2.27\ cm^2$.

피스톤 저항력 : $P_1 = 900\ kgf$, $P_2 = 90\ kgf$.

크랭크 半径은 피스톤의 上下운동에 따르는 連結棒의 伸縮量을 흡수할 수 있어야 하며,

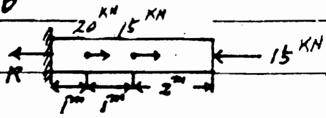
$$\delta = \frac{(P_1 + P_2)l}{AE} = \frac{(900 + 90) \times 100 \times 100}{2.27 \times 2.1 \times 10^6}$$

$= 2.0768\ cm$

이므로, 要求되는 크랭크 半径은

$2r = 20 + 2.0768 = 22.0768\ cm$,

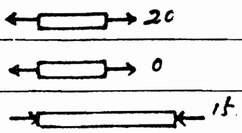
$\therefore r = 11.0384\ cm$.

3. 

$A = 300\ mm^2$, $E = 200\ KN/mm^2$

反力을 R이라 하면

$\Sigma F_x = 0 \qquad -R + 20 + 15 - 15 = 0 \qquad \therefore R = 20\ KN$

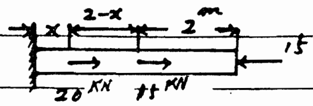


첫구간 $\delta_1 = \dfrac{S_1 L_1}{AE} = \dfrac{20 \times 1000}{300 \times 200} = \dfrac{1}{3}\ mm$

중간구간 $\delta_2 = 0$

셋째구간 $\delta_3 = \dfrac{-S_3 L_3}{AE} = \dfrac{-15 \times 2000}{300 \times 200} = -\dfrac{1}{2}\ mm$

$$\delta = \delta_1 + \delta_2 + \delta_3 = \frac{1}{3} + 0 - \frac{1}{2} = -0.167\ mm$$

4. 

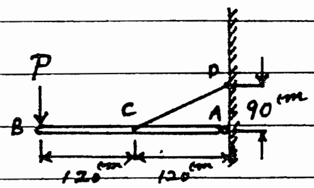
$\delta_1 = \dfrac{20 \times x}{300 \times 200}$

$\delta_2 = 0$

$\delta_3 = \dfrac{-15 \times 2000}{300 \times 200}$

$$\delta = \delta_1 + \delta_2 + \delta_3 = 0$$

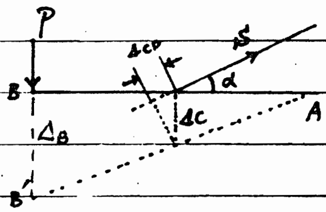
$$\therefore x = \frac{15 \times 1000}{20} = 1500\ mm = 1.5\ m$$

5. 

죄임봉 CD의 断面積 $A = 0.5\ cm^2$

$\sigma_w = 1400\ kgf/cm^2$

$E = 2.1 \times 10^6\ kgf/cm^2$



Equilibrium $\Sigma M_A = 0$

$P \times 240 = S \sin\alpha \times 120$

$\therefore P = \frac{1}{2} \times \frac{3}{5} S = \frac{3}{10} S$

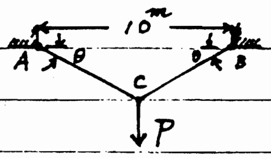
$S = A\sigma_w = 1400 \times 0.5 = 700\ kgf$

$\therefore P = \dfrac{3 \times 700}{10} = 210\ kgf$

$$\Delta C = \frac{5}{3} \Delta_{CD} = \frac{700 \times 150}{0.5 \times 2.1 \times 10^6} \times \frac{5}{3} = \frac{1}{6}\ cm$$

$$\therefore \Delta B = 2\Delta C = \frac{1}{3}\ cm$$

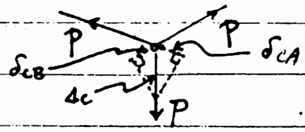
6.




$P = 20\ KN$, $\sigma_w = 80\ MN/m^2$

$\theta = 30°$, $E = 20{,}000\ KN/cm^2$

$A = 2.5\ cm^2$



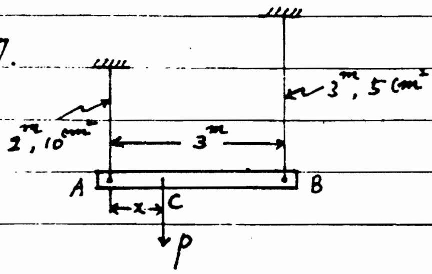

$P_{CA} = P_{CB} = P \quad (\because \theta = 30°)$

$\therefore \delta_{CA} = \delta_{CB} = \dfrac{Pl_{CA}}{AE} = \dfrac{20 \times 1000/\sqrt{3}}{2.5 \times 20000}$

$= 0.231\ cm$

$\therefore \Delta c = \delta_{CA} \times 2 = 0.462\ cm$

7.




$E_a = 1.05 \times 10^6\ kgf/cm^2$

$E_b = 2.1 \times 10^6\ kgf/cm^2$

Compatibility $\delta_A = \delta_B$

$\delta_A = \dfrac{P_a l_a}{A_a E_a} = \dfrac{200\ P_a}{10 \times 1.05 \times 10^6}$

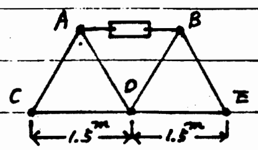
$\delta_B = \dfrac{P_b l_b}{A_b E_b} = \dfrac{300\ P_b}{5 \times 2.1 \times 10^6}$

$\therefore \dfrac{P_a}{P_b} = \dfrac{300}{5 \times 2.1} \times \dfrac{10 \times 10.5}{200} = 1.5$

$\Sigma M_c = 0 \qquad \therefore P_a x = P_b (3 - x)$

$\dfrac{P_a}{P_b} = \dfrac{3-x}{x} = 1.5 \qquad x = 1.2^m$

8

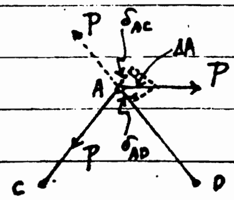



$A = 2.01\ cm^2$ $\quad E = 2.1 \times 10^6\ kgf/cm^2$

pitch $= 0.75^{mm}$ $\quad P = 5000\ kgf$

$n$ 바퀴 돌릴 때 늘어난 길이

$= 2 \times 0.75 \times 10^{-1}\ n\ cm.$




compatibility $\delta_{AC} = -\delta_{AD} = \delta_1$

$\delta_1 = \dfrac{Pl}{AE} = \dfrac{5000 \times 1.5 \times 100}{2.01 \times 2.1 \times 10^6} = 0.178^{cm}$

$\therefore \Delta A = 2\delta_1 = 0.356^{cm}$

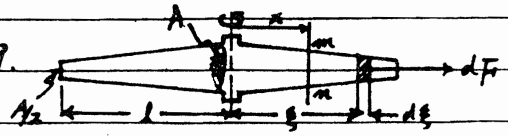
(8-cont.) AB 자체가 P에 의한 축소길이

$$\delta_2 = \frac{Pl}{AE} = 0.178^{cm} = \delta_1$$

$\therefore$ AB 전체가 늘어난 길이 $= 0.178 + 0.356 \times 2 = 0.89^{cm}$

$\therefore\ 2 \times 0.75 \times 10^{-1} n = 0.89$

$\therefore\ n = 5.933$ 바퀴

9. 

$\gamma$ : 단위체적당 중량

$$A_\xi = \frac{A}{2}(2l-\xi)/l$$

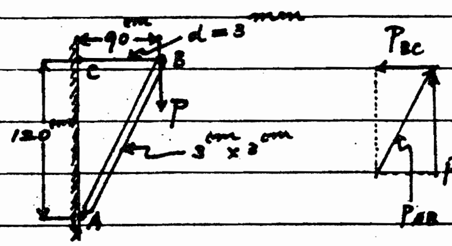
$$dF = \gamma A_\xi\, d\xi \cdot \omega^2 \xi = \frac{\gamma \omega^2 A}{2l}(2l-\xi)\xi\, d\xi$$

$x$ 에서의 원심력 : $S_x$

$$S_x = \int_x^l dF = \frac{\gamma\omega^2 A}{2l}\int_x^l (2l\xi - \xi^2)\, d\xi = \frac{\gamma A \omega^2}{2l}\left[l\xi^2 - \frac{1}{3}\xi^3\right]_x^l$$

$$= \frac{\gamma A \omega^2}{2l}\left\{\frac{2}{3}l^3 - x^2(l-x)\right\}$$

$$S_{max} = \frac{\gamma A \omega^2 l^2}{3} \qquad \therefore\ \sigma_0 = \frac{\gamma \omega^2 l^2}{3}$$

10 

$P = 200\,kgf \quad E_s = 2.1 \times 10^6\, kgf/cm^2$

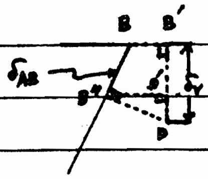
$E_w = 0.1 \times 10^6\, kgf/cm^2$

$P_{AB} = -\frac{5}{4}P = -250\,kgf$

$P_{BC} = \frac{3}{4}P = 150\,kgf$

$$\delta_{AB} = \frac{P_{AB} \cdot 150}{A_w E_w} = \frac{-250 \times 150}{9 \times 0.1 \times 10^6} = -0.042^{cm}$$

$$\delta_{BC} = \frac{P_{BC} \cdot 90}{A_s E_s} = \frac{150 \times 90}{\frac{\pi}{4} \times 9 \times \frac{1}{100} \times 2.1 \times 10^6} = 0.091^{cm}$$

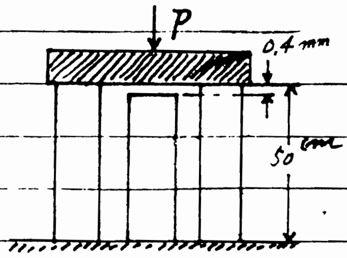


$$\delta_V = \delta_{AB} \times \frac{4}{5} + \left(\delta_{AB} \times \frac{3}{5} + \delta_{BC}\right) \times \frac{3}{4} = 0.1208^{cm}$$

$\therefore\ \delta_H = \delta_{BC} = 0.091^{cm} \qquad \delta_V = 0.121^{cm}$

## 問題 1.4

1.



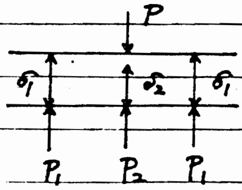

$A = 10 \times 10 = 100\ cm^2,\ l = 50\,cm$

$\sigma_w = 20\ MN/m^2 = 2\ KN/cm^2$

$E_c = 20\ GN/m^2 = 2000\ KN/cm^2$

Equilibrium

$$P = 2P_1 + P_2$$




Hooke's law

$$\delta_1 = \frac{P_1 l}{AE} = \frac{P_1 \times 50}{100 \times 2000}\ cm$$

$$\delta_2 = \frac{P_2 l}{AE} = \frac{P_2 \times 50}{100 \times 2000}\ cm$$

Compatibility ; $\delta_1 = \delta_2 + 0.05$

$$\therefore P_1 - P_2 = \frac{100 \times 2000 \times 0.04}{50} = 160\ KN$$

그런데 $P_1 = A\sigma_w = 10 \times 10 \times 2 = 200\ KN$

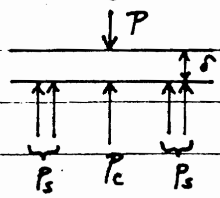
$$\therefore P_2 = P_1 - 160 = 40\ KN$$

$$\therefore P = 2 \times 200 + 40 = 440\ KN$$

2. $\sigma_s = 1400\ Kgf/cm^2 \qquad \sigma_c = 200\ Kgf/cm^2$

$E_s = 2.1 \times 10^6\ Kgf/cm^2 \quad E_c = 0.21 \times 10^6\ Kgf/cm^2$

$A_c = 10 \times 10 - 4\pi = 87.43\ cm^2,\ A_s = \pi\ cm^2,\ l = 20\ cm$




Equilibrium

$$P = 4P_s + P_c \quad \cdots\cdots ①$$

Hooke's law

$$\delta_s = \frac{P_s l}{A_s E_s} = \frac{P_s \times 20}{\pi \times 2.1 \times 10^6}$$

$$\delta_c = \frac{P_c l}{A_c E_c} = \frac{P_c \times 20}{87.43 \quad 0.21 \times 10^6}$$

(2 - cont.) Compatibility : $\delta_s = \delta_c$

$\therefore \; P_c = 2.783 \; P_s$

許容応力値들로 부터 許容荷重을 구해보면

$(P_s)_w = 1400 \times \pi = 4398 \text{ Kgf}$

$(P_c)_w = 200 \times 87.43 = 17486 \text{ Kgf}$

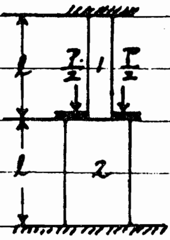
$(P_s)_w$ 기준 ; $P_w = 4P_s + P_c = (P_s)_w(4 + 2.783) = 29830 \text{ Kgf}$

$(P_c)_w$ 기준 ; $P_w = 4P_s + P_c = (P_c)_w\left(\frac{4}{2.783} + 1\right) = 42{,}620 \text{ Kgf}$

위의 두 값중에서 작은 것을 択하여

$\underline{P_w = 29{,}830 \text{ Kgf.}}$

3.




$P = 45 \text{ KN}$ , $A_1 = 6 \text{ cm}^2$ $A_2 = 9 \text{ cm}^2$

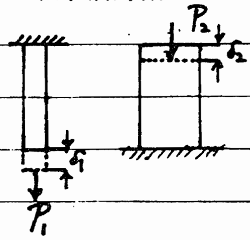
Equilibrium ; $P_1 + P_2 = P$

Compatibility ; $\delta_1 = \delta_2$

Hooke's law

$$\delta_1 = \frac{P_1 l}{A_1 E} = \frac{P_1 l}{6E}$$

$$\delta_2 = \frac{P_2 l}{A_2 E} = \frac{P_2 l}{9E}$$




$\therefore \; P_2 = 1.5 P_1$

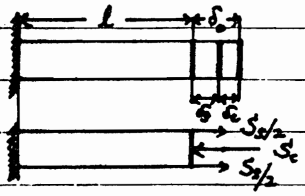
$$P_1 = \frac{P}{2.5} = \frac{45}{2.5} = 18 \text{ KN}$$

$$P_2 = 1.5 \times 18 = 27 \text{ KN}$$

$$\sigma_1 = \frac{P_1}{A_1} = \frac{18}{6} = 3 \text{ KN/cm}^2 = 30 \text{ MN/m}^2 \text{ (tension)}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{27}{9} = 3 \text{ KN/cm}^2 = 30 \text{ MN/m}^2 \text{ (compression)}$$

4.




$E_s/E_c = 12$ $A_s/A_c = 1/15$

(4 - cont.) Equilibrium ; $S_s = S_c$

Compatibility ; $|\delta_s| + |\delta_c| = \delta_o$

Hooke's law ; $\delta_s = \dfrac{S_s l}{A_s E_s}$ , $\delta_c = \dfrac{S_c l}{A_c E_c}$

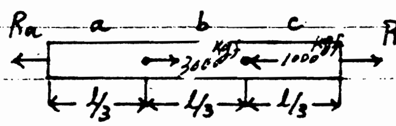
$$\therefore \frac{S_s l}{A_s E_s} + \frac{S_c l}{A_c E_c} = \delta_o = \frac{\sigma_o l}{E_s}$$

$$S_s + S_c \frac{A_s}{A_c} \cdot \frac{E_s}{E_c} = \sigma_o A_s$$

$$\therefore \sigma_s = \frac{S_s}{A_s} = \frac{5}{9}\sigma_o \quad (\text{tension})$$

$$\sigma_c = \sigma_s \cdot \frac{A_s}{A_c} = \frac{1}{27}\sigma_o \quad (\text{compression})$$

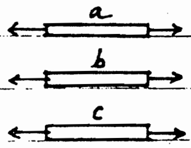
5.




Equilibrium ; 평형방식 ... 시간?

$$-R_a + 3000 - 1000 + R_c = 0$$

$$\therefore R_a - R_c = 2000 \Rightarrow R_c = R_a - 2000$$




$R_a$

$R_a - 3000$ ?

$R_a - 3000 + 1000$

Hooke's law ;

$$\delta_a = \frac{R_a \cdot l/3}{AE}$$

$$\delta_b = \frac{(R_a - 3000)\cdot l/3}{AE}, \quad \delta_c = \frac{(R_a - 2000)\cdot l/3}{AE}$$

Compatibility ; $\delta_a + \delta_b + \delta_c = 0$

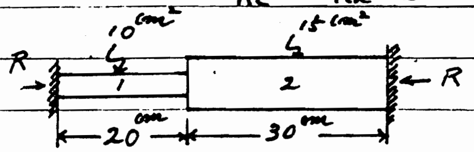
$$\therefore R_a - (3000 - R_a) + (R_a - 2000) = 0$$

$$\therefore R_a = \frac{5000}{3} = 1666.667 \text{ Kgf}$$

$$R_b = R_a - 2000 = -333.333 \text{ Kgf}$$

$$R_c = R_a - 3000 = -1333.333 \text{ Kgf}$$

6.




$E = 200\ GN/m^2 = 2 \times 10^4\ KN/cm^2$

$\alpha = 12 \times 10^{-6}$ cm/cm/°C

$T_1 = 10°C \quad T_2 = 38°C$

$\therefore \Delta T = 28°C$

(6-cont.) Equilibrium $S_1 = S_2 = S = R$

Compatibility $\delta_1 + \delta_2 = \alpha l \Delta T$

Hooke's law $\delta_1 = \frac{S l_1}{A_1 E}$ $\delta_2 = \frac{S l_2}{A_2 E}$

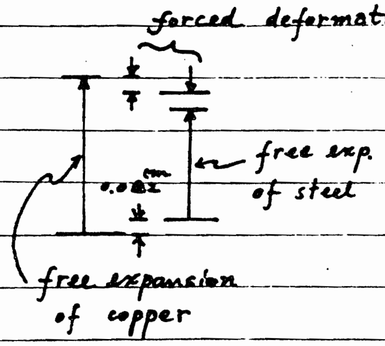
$$\therefore \frac{S}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2}\right) = \alpha l \Delta T$$

$$\therefore S = \frac{\alpha l E \Delta T}{l_1/A_1 + l_2/A_2} = \frac{12\times10^{-6}\times 50 \times 2\times 10^{4} \times 28}{20/10 + 30/15}$$

$$= 84 \text{ KN}$$

$$\therefore \sigma = \frac{S}{A_1} = \frac{84}{10} = 8.4 \text{ KN/cm}^2 = 84 \text{ MN/m}^2$$

7.




$E_c = 1.25\times10^{6}$ kgf/cm²

$E_s = 2.1\times10^{6}$ kgf/cm²

$\alpha_c = 16.5\times10^{-6}$ cm/cm/°c

$\alpha_s = 12\times10^{-6}$ cm/cm/°c

$T_1 = -20$°c , $T_2 = 30$°c

$\therefore \Delta T = 50$°c

Compatibility $\left(\frac{S l_1}{A_{c1}E_c} + \frac{S l_2}{A_{c2}E_c}\right) + \frac{S l}{A_s E_s}$ ?

$$= \alpha_c l \Delta T - 0.002 - \alpha_s l \Delta T$$

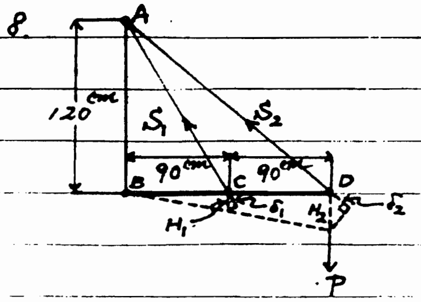
$$S\left(\frac{l_1}{A_{c1}E_c} + \frac{l_2}{A_{c2}E_c} + \frac{l}{A_s E_s}\right) = l\Delta T(\alpha_c - \alpha_s) - 0.002$$

$$S\left(\frac{30}{\pi\times1.25\times10^{6}} + \frac{4\times10}{9\pi\times1.25\times10^{6}} + \frac{90}{12\times2.1\times10^{6}}\right)$$

$$= 90\times50\times(16.5-12)\times10^{-6} - 0.002$$

$$\therefore S = \frac{18250}{18.0014} = 1013.658 \text{ kgf.}$$

8.




部材 BD가 굽는 것을 무시하면

Equilibrium ; $\Sigma M_B = 0$

$$\frac{4}{5} S_1 \times 90 + \frac{2}{\sqrt{13}} S_2 \times 180 = P \times 180 \quad \cdots\cdots\cdots\cdots ①$$

Hooke's law ;

$$\delta_1 = \frac{S_1 l_1}{AE} , \quad \delta_2 = \frac{S_2 l_2}{AE}$$

Compatibility ; $H_1 = \frac{5}{4}\delta_1 \qquad H_2 = \frac{\sqrt{13}}{2}\delta_2$

$$2H_1 = H_2$$

$$\therefore\ 2 \times \frac{5}{4}\delta_1 = \frac{\sqrt{13}}{2}\delta_2$$

$$\therefore\ 5 \times \frac{S_1 l_1}{AE} = \sqrt{13} \times \frac{S_2 l_2}{AE}$$

$l_1 = 150^{cm}$, $l_2 = 60\sqrt{13}^{\ cm}$ 를 代入

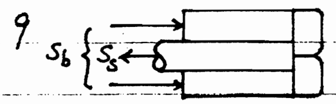
$$\therefore\ S_1 = 1.04\, S_2 \cdots\cdots\cdots\cdots\cdots\cdots ②$$

② → ① $\quad \frac{2}{5} \times 1.04\, S_2 + \frac{2}{\sqrt{13}} S_2 = P$

$$\therefore\ S_2 = 1.0302\, P$$

$$S_1 = 1.0714\, P$$

9




$\sigma_b = 30\ MN/m^2$

$= 3\ KN/cm^2$

$A_s = 4.523^{cm^2}$, $A_b = 4.273^{cm^2}$

$E_s = 200\ GN/m^2$, $E_b = 100\ GN/m^2$

$\alpha_s = 11.5 \times 10^{-6}\ cm/cm/°C$

$\alpha_b = 18.5 \times 10^{-6}\ cm/cm/°C$

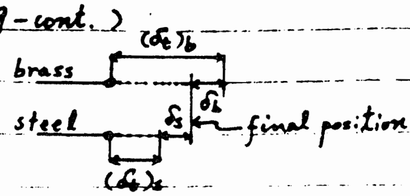
Equilibrium ; $S_b = S_s = S = 3 \times 4.273$

$$= 12.819\ KN$$

Compatibility ; $\delta_b + \delta_c = (\alpha_b - \alpha_s)\,\Delta T \cdot l$

Hooke's law ; $\delta_b = \frac{Sl}{A_b E_b}$ , $\delta_s = \frac{Sl}{A_s E_s}$

(9-cont.)



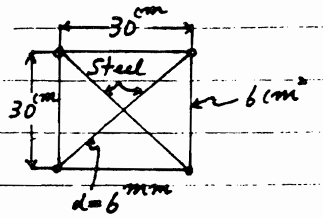

$$\therefore \frac{Sl}{A_bE_b} + \frac{Sl}{A_sE_s} = (\alpha_b - \alpha_s)\, l\, \Delta T$$

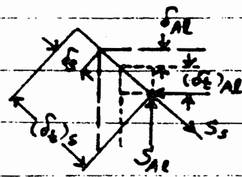
$$\Delta T = S\left(\frac{1}{A_bE_b} + \frac{1}{A_sE_s}\right) \times \frac{1}{\alpha_b - \alpha_s}$$

$$= 3 \times 4.273 \times \left(\frac{1}{4.523 \times 200 \times 10^2} + \frac{1}{4.273 \times 100 \times 10^2}\right) \times \frac{10^6}{18.5 - 11.5}$$

$$= 63.10\ ^\circ C$$

10.




$$E_s = 2.1 \times 10^6\ \mathrm{Kg/cm^2}$$

$$E_{Al} = 0.7 \times 10^6\ \mathrm{Kg/cm^2}$$

$$\alpha_s = 11.5 \times 10^{-6}\ \mathrm{cm/cm/^\circ c}$$

$$\alpha_{Al} = 23 \times 10^{-6}\ \mathrm{cm/cm/^\circ c}$$

$$T_1 = 20^\circ c \qquad T_2 = 75^\circ c$$

Equilibrium

$$\sqrt{2}\, S_{Al} = S_s = S$$

Compatibility ;

$$\sqrt{2}\, \delta_{Al} + \sqrt{2}\, \delta_s = (\delta_t)_s - \sqrt{2}\, (\delta_t)_{Al}$$

Hooke's law

$$\delta_{Al} = \frac{S_{Al}\, l_{Al}}{A_{Al}E_{Al}}, \quad \delta_s = \frac{S_s\, l_s}{A_sE_s}$$

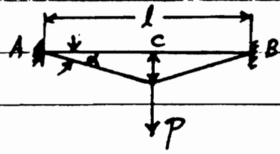
$$A_{Al} = 6\ \mathrm{cm^2}, \quad A_s = \frac{\pi}{4}(0.6)^2\ \mathrm{cm^2}, \quad l_s = \sqrt{2} \times 30\ \mathrm{cm}$$

$$\therefore \sqrt{2} \times \frac{\frac{1}{\sqrt{2}} S \times 30}{6 \times 0.7 \times 10^6} + \frac{4S \times 30\sqrt{2}}{\pi \times 0.6^2 \times 2.1 \times 10^6}$$

$$= 11.5 \times 10^{-6} \times 30\sqrt{2} \times 55 - 23 \times 10^{-6} \times 30 \times 55 \times \sqrt{2}$$

$$\left(\frac{1}{6 \times 0.7} + \frac{4\sqrt{2}}{\pi \times 0.6^2 \times 2.1}\right) S = 11.5 \times 55\sqrt{2} - 23 \times 55\sqrt{2}$$

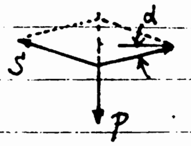
$$\therefore S = -341.4182\ \mathrm{Kg}$$

11. 


Statics ; $S = \frac{P}{2}\cot\alpha$

$\alpha \ll 1$

$\therefore S \doteq \frac{P}{2\alpha} = \frac{Pl}{4\Delta}$




Hooke's law ;

$$\delta = \frac{S\,l/2}{AE} = \frac{Pl^2}{8AE\Delta}$$

그런데 $\delta = \sqrt{\left(\frac{l}{2}\right)^2 + \Delta^2} - \frac{l}{2} = \frac{l}{2}\left\{1 + \left(\frac{2\Delta}{l}\right)^2\right\}^{1/2} - \frac{l}{2}$

$$\approx \frac{l}{2}\left(1 + \frac{1}{2}\cdot\frac{4\Delta^2}{l^2}\right) - \frac{l}{2} = \frac{\Delta^2}{l}$$

$\therefore \frac{\Delta^2}{l} = \frac{Pl^2}{8AE\Delta}$ $\quad\quad \therefore \Delta = \frac{l}{2}\sqrt[3]{\frac{P}{AE}}$

## 問題 1.5

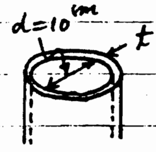
1. $d = 140^{cm}$, $\sigma_u = 170\ MN/m^2$, $\gamma = 7200\ Kg/m^3$

$\sigma = \gamma\,\omega^2 r^2$

$\therefore \omega^2 = \frac{\sigma}{\gamma \cdot r^2} = \frac{170 \times 10^6}{7200 \times 0.7^2} = 48186$

$\therefore \omega = 219.5$ rad/sec

$\therefore n = \frac{\omega}{2\pi} \times 60 = 2096$ rpm.

2. 


$\sigma_w = 2100\ Kgf/cm^2$

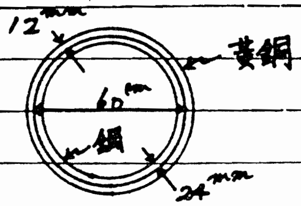
$E = 2.1 \times 10^6\ Kgf/cm^2$

$$\varepsilon = \frac{\Delta d}{d} = \frac{\sigma_w}{E}$$

$$\Delta d = d - d_1 = \frac{\sigma_w}{E}\cdot d = \frac{2100}{2.1\times10^6} \times 10 = 0.01^{cm}$$

$\therefore d_1 = 10 - 0.01 = 9.99^{cm}$

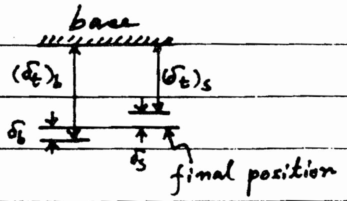
3.




$= 30^{mm}$

$T_1 = 220°C \qquad T_2 = 20°C$

$\therefore \Delta T = -200°C$




$E_s = 200\ GN/m^2 = 200\ KN/mm^2$

$E_b = 97\ GN/m^2 = 97\ KN/mm^2$

$\alpha_s = 11.5 \times 10^{-6}$

$\alpha_b = 18.5 \times 10^{-6}$

Compatibility ; $\delta_b + \delta_s = (\delta_t)_b - (\delta_t)_s$

Equilibrium ; $q_b = q_s = q$

Hooke's law ; $\delta_b = \dfrac{qr}{A_b E_b}$ , $\delta_s = \dfrac{qr}{A_s E_s}$

$(\delta_t)_b = \alpha_b \cdot \Delta T \cdot r \qquad (\delta_t)_s = \alpha_s \cdot \Delta T \cdot r$

$$\therefore \frac{qr}{A_b E_b} + \frac{qr}{A_s E_s} = (\alpha_b - \alpha_s)\Delta T$$

$$\therefore q = (\alpha_b - \alpha_s)\Delta T \cdot \frac{1}{\dfrac{1}{A_b E_b} + \dfrac{1}{A_s E_s}} \cdot \frac{1}{r}$$

$$= (18.5 - 11.5) \times 10^{-6} \times 200 \times \frac{1}{\dfrac{1}{12 \times 30 \times 97} + \dfrac{1}{24 \times 30 \times 200}} \times \frac{1}{300}$$

$$= 0.1312\ KN/mm = 131.2\ N/mm$$

$$\sigma_b = \frac{qr}{A_b} = \frac{131.2 \times 300}{12 \times 30} = 109.3\ N/mm^2 = 109.3\ MN/m^2$$

4. $\gamma_s = 7.85 \times 10^{-6}\ Kg/mm^3 \qquad \gamma_b = 8.40 \times 10^{-6}\ Kg/mm^3$

$E_s = 200 \times 10^6\ \dfrac{Kg\text{-}mm}{sec^2}, \quad E_b = 97 \times 10^6\ \dfrac{Kg\text{-}mm}{sec^2}$

(組立円環이 回転하여 各円環의 遠心力으로 因한 strain의 差)

=(溫度降下로 因한 strain의 差)

$\therefore\ \frac{1}{E_b}\gamma_b^2 w^2 r^2 - \frac{1}{E_s}\gamma_s^2 w^2 r^2 = \alpha_b \Delta T - \alpha_s \Delta T$

$w^2 = (\alpha_b - \alpha_s)\Delta T \cdot \frac{1}{r^2} \cdot \frac{1}{\frac{\gamma_b}{E_b} - \frac{\gamma_s}{E_s}}$ ⑦

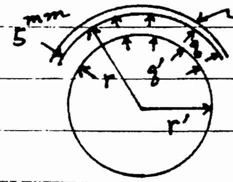
$= (18.5 - 11.5)\times 10^{-6} \times 200 \times \frac{1}{300^2} \times \frac{10^{12}}{\frac{8.4}{97} - \frac{7.85}{200}}$

$= 328537.$

$\therefore\ w = 573.2$ rad/sec

$\sigma_b = \gamma_b w^2 r^2 = 8.4\times 10^{-6} \times 328537 \times 300 \times 300 = 248374\ \frac{Kg\cdot mm/sec^2}{mm^2}$

$= 248.374\ N/mm^2 = 248.374\ MN/m^2$

5. 


band (두께 6mm)  $\sigma = \frac{qr}{A}$  $f = 0.4$

$A = 4\times 1 = 4\ cm^2$

mean $r = 12 + 0.6 + 0.5 = 13.1\ cm$

$\sigma = 1200\ Kgf/cm^2$

$r' = \frac{24}{2} = 12\ cm$

$q = \frac{\sigma A}{r} = \frac{1200\times 4}{13.1} = 366.4\ Kgf/cm$ (band)

$q' = q\cdot\frac{12}{13.1} = 335.64\ Kgf/cm$ (drum)

Breaking Torque/unit length of Periphery

$T_1 = f q' r' = 0.4 \times 335.64 \times 12 = 1611.1\ Kgf\cdot cm/cm$

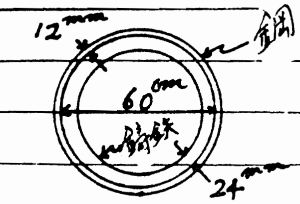
Total torque $= T_1 \times 2\pi r'$

$= 1611.1 \times 2\pi \times 12$

$= 12473.18\ Kgf\cdot cm$

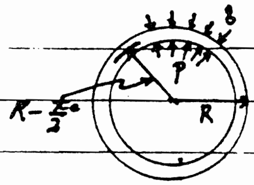
$= 124.7\ Kgf\cdot m$

6.



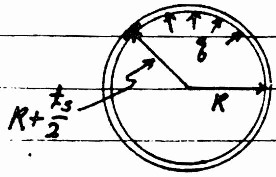

$$\sigma_c = \frac{(p-q)(R-\frac{t_c}{2})}{t_c}$$

$$\varepsilon_c = \frac{(p-q)(R-\frac{t_c}{2})}{E_c t_c}$$




$$\sigma_s = \frac{q(R+\frac{t_s}{2})}{t_s}$$

$$\varepsilon_s = \frac{q(R+\frac{t_s}{2})}{E_s t_s}$$




Compatibility

$$\frac{\varepsilon_c}{\varepsilon_s} = \frac{(R+\frac{t_s}{2})}{(R-\frac{t_c}{2})}$$

$$\frac{\dfrac{(p-q)(R-\frac{t_c}{2})}{E_c t_c}}{\dfrac{q(R+\frac{t_s}{2})}{E_s t_s}} = \frac{R+\frac{t_s}{2}}{R-\frac{t_c}{2}}$$

since $E_c t_c = E_s t_s$

$$\frac{p-q}{q} = \frac{(R+\frac{t_s}{2})^2}{(R-\frac{t_c}{2})^2} = \left(\frac{30.6}{28.8}\right)^2 = 1.1289$$

$(\because R = 30^{cm} \quad t_c = 2.4^{cm} \quad t_s = 1.2^{cm})$

$\therefore p = 2.1289q \;,\; q = 0.4697p$

For cast iron ( $\sigma_c = 30\ MN/m^2 = 30\times10^{-4}\ MN/cm^2$ )

$$\sigma_c = \frac{0.5303p\times28.8}{2.4} \Rightarrow p = \frac{2.4\times30\times10^{-4}}{0.5303\times28.8} = 4.714\times10^{-4}\ MN/cm^2$$

$= 4.714\ MN/m^2 = 4.714\ MPa$

For steel ( $\sigma_s = 140\ MN/m^2 = 140\times10^{-4}\ MN/cm^2$ )

$$\sigma_s = \frac{0.4697p\times30.6}{1.2} \Rightarrow p = \frac{1.2\times140\times10^{-4}}{0.4697\times30.6} = 11.69\ MPa$$

$\therefore p_w = 4.71\ MPa.$

## 問題 2.1

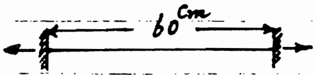
1. $d = 14^{mm},\ l = 50^{mm},\ \delta = 0.05^{mm},$

$E = 200\ GN/m^2 = 200 \times 10^3\ MN/m^2, \quad \varepsilon = \frac{0.05}{50} = 1 \times 10^{-3}\ mm/mm$

$\sigma = E\varepsilon = (200 \times 10^3)(1 \times 10^{-3}) = 200\ MN/m^2$

$\therefore \tau_{max} = \frac{1}{2}\sigma = 100\ MN/m^2$

2.




$d = 1.6^{mm},\ l = 60^{cm},\ P = 15\ kgf,\ \Delta T = -30^\circ C$

$\alpha_b = 2.0 \times 10^{-5}\ mm/mm/^\circ c\ , \quad E_b = 0.98 \times 10^4\ kg/mm^2$

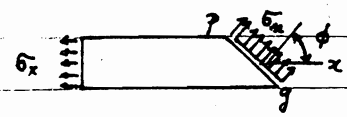
$(\sigma)_p = \frac{P}{A} = \frac{15}{\frac{1}{4}\pi(1.6)^2} = 7.459\ kg/mm^2$

$\sigma_t = E\alpha\Delta T = 0.98 \times 10^4 \times 2.0 \times 10^{-5} \times 30 = 5.88\ kg/mm^2$

$\therefore \sigma = \sigma_p + \sigma_t = 13.339\ kg/mm^2$

$\therefore \tau_{max} = 6.6695\ kg/mm^2$

3.




$\sigma_n = 84\ MN/m^2, \quad \tau = 28\ MN/m^2.$

$pg$ 의 面積을 A라고 하면

Equilibrium : $\sigma_x A \cos\phi = \sigma_n A \cos\phi + \tau A \sin\phi$

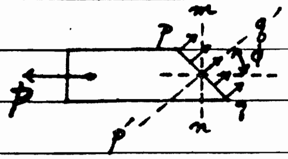
$\sigma_n A \sin\phi = \tau A \cos\phi$

$\therefore \tan\phi = \frac{\tau}{\sigma_n} = \frac{28}{84} = 0.333 \quad \therefore \phi = 18^\circ 26'$

$\sigma_x = \sigma_n + \tau \cdot \tan\phi = 84 + 28 \times 0.333$

$= 93.33\ MN/m^2.$

4.



$A = 1\,cm^2$ , $P = 1{,}000\,kgf$ , $\phi = 30^\circ$

Equilibrium: $P \cdot 1 = \sigma_n \cdot \frac{1}{\cos\phi} \cdot \cos\phi + \tau \frac{1}{\cos\phi} \cdot \sin\phi$

$$\sigma_n \cdot \frac{1}{\cos\phi} \cdot \sin\phi = \tau \frac{1}{\cos\phi} \cdot \cos\phi$$

$$\therefore \ \sigma_n + \tau \tan\phi = 1000 , \quad \tau = \sigma_n \tan\phi$$

$$\therefore \ \sigma_n = \frac{1000}{1 + \tan^2\phi} = 750 \ kgf/cm^2$$

$$\tau = 750 \times 0.5774 = 433 \ kgf/cm^2$$

$$\sigma_n' = \frac{1000}{1 + \tan^2(\phi + 90^\circ)} = 250 \ kgf/cm^2$$

$$\tau' = \sigma_n' \tan(\phi + 90^\circ) = -433 \ kgf/cm^2$$

5. $P = 150\,KN$ , $\tau_w = 60\,MN/m^2 = 6\,KN/cm^2$

직경을 $d$라 하면 $\qquad A = \frac{\pi}{4} d^2$

$$(\tau)_{max} = \frac{1}{2}\sigma = \frac{1}{2} \cdot \frac{P}{A} = \frac{1}{2} \times \frac{150}{\frac{\pi}{4} d^2} = 6$$

$$\therefore \ d^2 = \frac{4 \times 150}{2 \times \pi \times 6} = 15.915\,cm^2$$

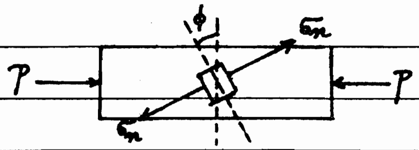
$$\therefore \ d = 3.989\,cm$$

6. 問 2-1-4 를 참조하면

$$\frac{\tau}{\sigma_n} = \tan\phi = 1 \qquad \therefore \ \phi = 45^\circ$$

7. $\tau = \sigma_n \tan\phi = 1000 \times 0.5774 = 577.4\,kgf/cm^2$.

8.




$\sigma_n = -100\,MN/m^2$ , $\sigma_n' = -60\,MN/m^2$

$$\sigma_n + \sigma_n' = \frac{P}{A} = -160\,MN/m^2$$

$$\cos^2\phi = \sigma_n / \tfrac{P}{A} = \frac{-100}{-160} = 0.625 , \quad \cos\phi = 0.7906$$

$$\therefore \ \phi = 37^\circ 45'$$

9. $l = 30^{cm}$, $\tau_{max} = 140^{kgf/cm^2}$, $d = 15^{cm}$

$$\tau_{max} = \frac{P}{A}\times\frac{1}{2} = \frac{P}{\frac{\pi}{4}\cdot 15^2\cdot 2} = 140$$

∴ $P = 49480$ kgf.

10. 


∵ $\sigma_w = 7\ MN/m^2$, $\tau_w = 4.2\ MN/m^2$

$A = 5^{cm}\times 5^{cm} = 25\ cm^2 = 25\times10^{-4}\ m^2$

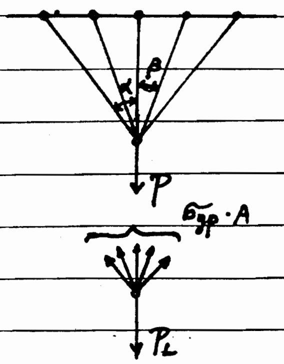
$$\sigma_n = \frac{P}{A}\cos^2\phi = 7 \qquad \tau_n = \frac{1}{2}\cdot\frac{P}{A}\sin 2\phi = 4.2$$

$$\therefore \frac{\tau}{\sigma_n} = \frac{(1/2)(P/A)\sin 2\phi}{(P/A)\cos^2\phi} = \frac{(1/2)\,2\sin\phi\cos\phi}{\cos^2\phi} = \tan\phi = \frac{4.2}{7} = 0.6$$

$$\therefore \phi = 30°58', \quad P = \frac{7\cdot A}{\cos^2\phi} = \frac{7\times 25\times10^{-4}}{0.7353} = 0.0238\ MN$$

$= 23.8$ KN.

## 問題 2·3

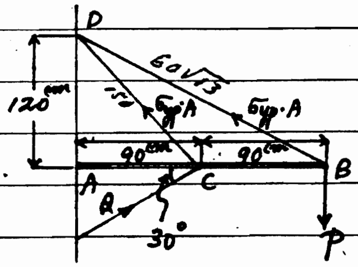
1. 


$A = 0.5\ cm^2$, $\alpha = 45°$, $\beta = 30°$

$\sigma_{yp} = 270\ MN/m^2 = 27\ KN/cm^2$

Equilibrium:

$$P_L = \sigma_{yp}\cdot A(1 + 2\times\cos 30° + 2\times\cos 45°)$$
$$= 27\times 0.5\times(1 + 2\times 0.866 + 2\times 0.7071)$$
$$= 55.97\ KN.$$

2. 


당김줄의 面積: $0.5\ cm^2$

〃 : $\sigma_{yp} = 2800\ kgf/cm^2$

버팀 棒: $Q = 1500\ kgf$

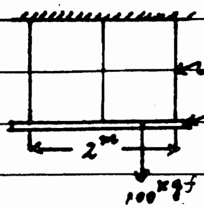
Equilibrium: $\Sigma M = 0$

$$P_L\times 180 = \sigma_{yp}\cdot A\left(90\times\frac{4}{5} + 180\times\frac{2}{\sqrt{13}}\right) + Q\times 90\times\frac{1}{2}$$

$$\therefore P_L = 2800\times 0.5\times\left(\frac{2}{5} + \frac{2}{\sqrt{13}}\right) + \frac{1}{2}\times\frac{1}{2}\times 1500$$

$= 1911.58$ kgf.

3.

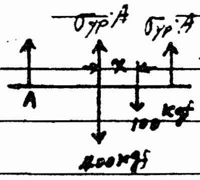



$\sigma_{yp} = 28^{kgf/mm^2}$, $d = 3^{mm}$, $A = \dfrac{\pi \cdot (0.3)^2}{4}$ $cm^2$

$200^{kgf/m}$

$\sigma_{yp} \cdot A = 28 \times 10^2 \times \dfrac{\pi \times (0.3)^2}{4}$

$= 197.92$ kgf



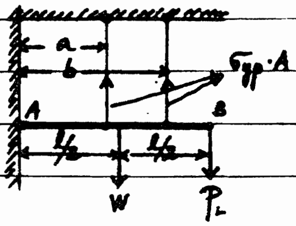

Equilibrium: $\Sigma M = 0$

$400 \times 100 + (100 + x) 100$

$= 197.92 \times (100 + 200)$

$\therefore x = 93.76$ cm

4.




$a = 60^{cm}$, $b = 150^{cm}$, $l = 240^{cm}$

支持鋼線 의 $d = 3^{mm}$

$W = 1000\ N$, $\sigma_{yp} = 250\ MN/m^2 = 250 \times 10^2\ N/cm^2$

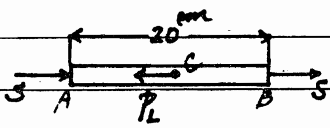
Equilibrium : $\Sigma M = 0$

$\sigma_{yp} \cdot A (60 + 150) = W \times 120 + P_L \times 240$

$\therefore P_L = \dfrac{1}{240}\left(25000 \times \dfrac{\pi (0.3)^2}{4} \times 210 - 1000 \times 120\right)$

$= 1046\ N.$

5.




$d = 12^{mm}$

Prestressed $\sigma_{yp} = 250\ MN/m^2 = 25\ KN/cm^2$

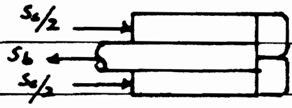
$S = \sigma_{yp} \cdot A$

$A = \dfrac{\pi}{4} \cdot (1.2)^2 = 1.131\ cm^2$

Equilibrium

$\therefore P_L = 2S = 2 \times \sigma_{yp} \cdot A$

$= 56.55\ KN.$

6. 


$\sigma_{yp} = 2500\ ^{kgf/cm^2},\ A_s = 5\ ^{cm^2},\ A_b = 3\ ^{cm^2}$

$(S_b)_L = 2500 \times 3 = 7500\ ^{kgf}$

$(S_b)_L = S_s + P_L = 7500\ ^{kgf}$

$S_s = 0$ 일때

$\therefore (S_b)_L = P = 7500\ ^{kgf}.$

## 問題 2·4

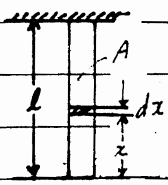
1. $l = 20\ ^{cm},\quad P = 20000\ N,\quad E = 20 \times 10^6\ N/cm^2$

(a) $A = 20\ ^{cm^2}$

$$U = \frac{P^2 l}{2AE} = \frac{20000^2 \times 20}{2 \times 20 \times 20 \times 10^6} = 10\ N\text{-}cm = 0.1\ N\text{-}m\ (J)$$

(b) $A = 10\ ^{cm^2}$

$$U = \frac{20000^2 \times 20}{2 \times 10 \times 20 \times 10^6} = 20\ N\text{-}cm = 0.2\ N\text{-}m\ (J)$$

2. 


單位体積當 質量: $\gamma$

斷面 $x$ 에 걸리는 荷重: $P_x = A x \gamma g$

微小要素 $dx$ 에 저장된 에너지: $dU_x$

$$dU_x = \frac{P_x^2\, dx}{2AE} = \frac{A^2 \gamma^2 g^2 x^2}{2AE}$$

$$\therefore U = \int_0^l \frac{A^2 \gamma^2 g^2}{2AE} x^2\, dx$$

$$= \frac{A^2 \gamma^2 g^2}{2AE} \left[ \frac{x^3}{3} \right]_0^l = \frac{\gamma^2 g^2 A l^3}{6E}$$

3. (1) $w = 7850\ ^{kgf/m^3} = 7.85 \times 10^{-3}\ ^{kgf/cm^3}$

$E = 2.1 \times 10^6\ ^{kgf/cm^2},\quad \sigma_{pl} = 2100\ ^{kgf/cm^2}$

$$u = \frac{\sigma^2}{2E} = \frac{2100^2}{2 \times 2.1 \times 10^6} = 1.05\ ^{kgf\cdot cm/cm^3}$$

$$u' = \frac{u}{w} = 133.758\ ^{kgf\cdot cm/kgf}$$

(3 - Conti.)

(2) $w = 7.85 \times 10^{-3}\ kgf/cm^3,\ E = 2.1 \times 10^6\ kgf/cm^2,\ \sigma_{pl} = 8400\ kgf/cm^2$

$$u = \frac{\sigma^2}{2E} = \frac{8400^2}{2 \times 2.1 \times 10^6} = 16.8\ kgf\text{-}cm/cm^3.$$

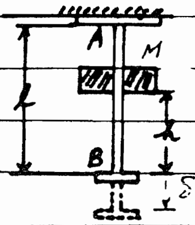
$$u' = \frac{16.8}{7.85 \times 10^{-3}} = 2140.1\ kgf\text{-}cm/kgf$$

(3) $w = 0.93 \times 10^{-3}\ kgf/cm^3,\ E = 21\ kgf/cm^2,\ \sigma_{pl} = 21\ kgf/cm^2$

$$u = \frac{\sigma^2}{2E} = \frac{21^2}{2 \times 21} = 10.5\ kgf\text{-}cm/cm^3$$

$$u' = \frac{10.5}{0.93 \times 10^{-3}} = 11290.3226\ kgf\text{-}cm/kgf$$

4.




$\sigma = 140\ MN/m^2,\quad M = 10\ kg,\ l = 2\ m,\ A = 3\ cm^2$

$E = 70\ GN/m^2 = 70 \times 10^3\ MN/m^2 = 70 \times 10^5\ N/cm^2$

$$Mg(h+\delta) = \frac{AE\delta^2}{2l},\quad \delta_{st} = \frac{Mgl}{AE}$$

$$\delta = \delta_{st} + \sqrt{\delta_{st}^2 + 2\delta_{st} h}$$

$$\delta_{st} = \frac{Mgl}{AE} = \frac{(10 \times 9.807) \times 200}{3 \times 70 \times 10^5} = 9.333 \times 10^{-4}\ cm$$

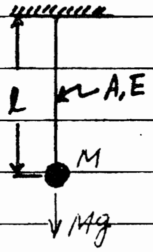
$$\therefore \delta \doteqdot \sqrt{2\delta_{st} h}$$

$$\sigma = \frac{\delta E}{l} = \frac{E\sqrt{2\delta_{st} h}}{l}$$

$$\therefore h = \frac{1}{2\delta_{st}}\left(\frac{\sigma l}{E}\right)^2 = \frac{1}{2 \times 9.333 \times 10^{-4}}\left(\frac{140 \times 200}{70 \times 10^3}\right)^2$$

$$= 85.71\ cm.$$

5.



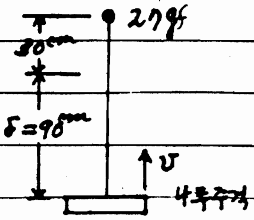

$$Mg(l+\delta) = \frac{AE}{2l}\delta^2,\quad \delta_{st} = \frac{Mg \cdot l}{AE}$$

$$\delta = \delta_{st} + \sqrt{\delta_{st}^2 + 2\delta_{st} \cdot l}$$

$$\sigma = \frac{\delta E}{l} = \frac{E}{l}\left[\frac{Mgl}{AE}\left(1 + \sqrt{1 + 2 \cdot \frac{AE}{Mgl} \cdot l}\right)\right]$$

$$= \frac{Mg}{A}\left(1 + \sqrt{1 + \frac{2AE}{Mg}}\right)$$

6.




$A = 0.01\,cm^2$, $E = 20\,kgf/cm^2$, $\mu = 0.5$, $l = 30\,cm$

$\delta = 3l$ 까지 直線的인 荷重-變形度曲線.

4후주기 初期運動 Energy

$$\frac{mv^2}{2} = \frac{wv^2}{2g}$$

Strain energy $\dfrac{P\delta}{2} = \dfrac{AE}{2l}\delta^2$

Equilibrium $\dfrac{wv^2}{2g} = \dfrac{AE}{2l}\delta^2$

$$v^2 = \frac{AEg}{wl}\delta^2 = \frac{AEg(3l)^2}{wl}$$

$$= \frac{0.01 \times 20 \times 1000 \times 980 \times 9 \times 30}{27}$$

$$= 1460000$$

$$\therefore v = 1400\,cm/sec$$

$$= 14\,m/sec$$

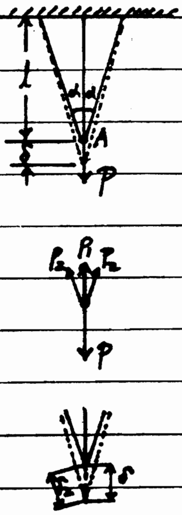
Stretched condition

$$A' = A(1-\mu)^2 = 0.0025\,cm^2$$

$$\therefore P = \frac{AE}{l}\delta = \frac{AE}{l}(3l) = 3AE$$

$$\sigma = \frac{P}{A'} = \frac{3 \times 0.01 \times 20}{0.0025} = 240\,kg$$

7.




A·E. for each string

Equilibrium:

$$P_1 + 2P_2 \cos\alpha = P$$

Compat. $\delta_2 = \delta\cos\alpha$

$$l_2 = l/\cos\alpha$$

$$\therefore \frac{P\delta}{2} = \frac{AE\delta^2}{2l} + \frac{AE\delta_2^2}{2l_1} \times 2$$

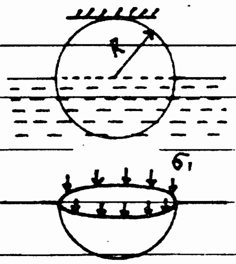
$$\therefore \delta = \frac{Pl}{AE}\left(\frac{1}{1+2\cos^3\alpha}\right)$$

## 問題 3.1

1. $t = 3^{mm}$, $D = 6^{m}$, $\sigma = 90\ MN/m^2$

$$\sigma = \frac{pr}{2t}, \quad \therefore p = \frac{2t\sigma}{r} = \frac{2\times 0.003\times 90}{3} = 0.18\ MN/m^2\ (MPa)$$

2.




F.B.D.

두께 : $t$, 물의 單位体積当 質量 : $\gamma$

부력 $= \frac{1}{2}\left(\frac{4}{3}\pi R^3\right)\gamma g = W$

Equilibrium : $2\pi R\cdot\sigma_1\cdot t = W$

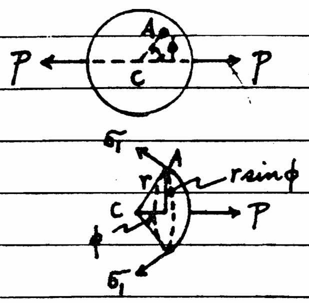
$$\therefore \sigma_1 = \frac{1}{2\pi R t}\cdot\frac{4}{3}\pi R^3\gamma g = \frac{\gamma g R^2}{3t}\ (\text{comp.})$$

Membrane equation : $\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = \frac{p}{t}$

$r_1 = r_2 = R$, $p = 0$ 이므로

$$\therefore \sigma_2 = -\sigma_1 = \frac{\gamma g R^2}{3t}$$

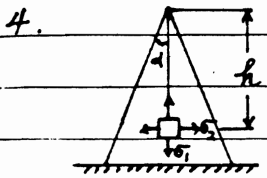
3.




Equilibrium :

$$P = \{\sigma_1(2\pi r\sin\phi)t\}\cos(90-\phi)$$
$$= 2\pi r t\sigma_1\sin^2\phi$$
$$\therefore \sigma_1 = \frac{P}{2\pi r t\sin^2\phi}$$
$$\frac{\sigma_2}{r} + \frac{\sigma_1}{r} = \frac{p}{t} = 0$$
$$\therefore \sigma_2 = -\frac{P}{2\pi r t\sin^2\phi}$$

4.

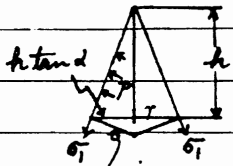



깊이 $h$까지의 表面積을 計算해 보면

表面積: $S=\int_0^r 2\pi x\sqrt{dx^2+dy^2}=\int_0^r 2\pi x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$=\int_0^r 2\pi x\sqrt{1+\frac{h^2}{r^2}}\,dx=2\pi\sqrt{1+\frac{h^2}{r^2}}\cdot\frac{1}{2}r^2$

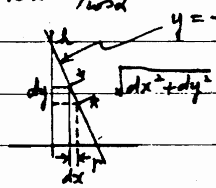
$=\pi r h\sqrt{1+\tan^2\alpha}=\pi r h\sec\theta$




Equilibrium:

$(2\pi r t\sigma_1)\cos\alpha=(\pi r h\sec\alpha)p\sin\alpha$

$r_2=h\tan\alpha/\cos\alpha$



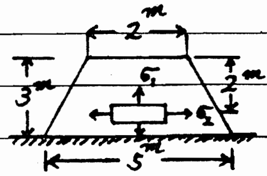

$\therefore\ \sigma_1=\frac{\pi(h\tan\alpha)h\tan\alpha\cdot p}{2\pi h\tan\alpha\cdot t\cdot\cos\alpha}$

$=\frac{h\tan\alpha\cdot p}{2t\cos\alpha}$

$r_1=\infty$

$\therefore\ \frac{\sigma_2}{r_2}=\frac{p}{t}\qquad\therefore\ \sigma_2=\frac{p}{t}\cdot\frac{h\tan\alpha}{\cos\alpha}$

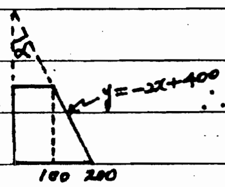
5.




$t=0.3^{mm}$, 물의 比重量: $w=1\ gf/cm^3$

$r_2=(2+2)\times100\times\frac{1}{2}\times\frac{\sqrt{5}}{2}=100\sqrt{5}\ ^{cm}$

$r_1=\infty$



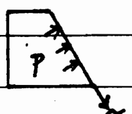

$p=wh=1\times10^{-3}\times200=0.2\ ^{kgf}/cm^2$

$\therefore\ \sigma_2=\frac{0.2}{0.03}\times100\sqrt{5}=1490.712\ ^{kgf}/cm^2$

$dP_x=w(2x-200)\times10^{-3}\cdot2\pi x\,ds$

$=10^{-3}w(2x-200)2\pi x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$=\sqrt{5}\times10^{-3}w(2x-200)\,dx$

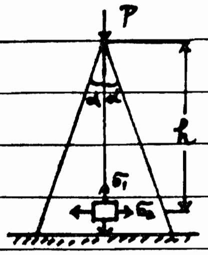



$\therefore$ Equilibrium: $\int_{100}^{200}\sqrt{5}\times10^{-3}\times w\times(2x-200)\cdot2\pi x\,dx\cdot\frac{1}{\sqrt{5}}$

$=2\pi\times200\times0.03\,\sigma_1\cos\alpha$

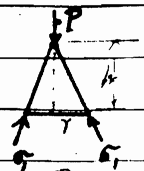
$\therefore\ \sigma_1=\frac{\sqrt{5}\times\frac{5}{6}\times10^3}{200\times0.03}=310.5650\ ^{kgf}/cm^2$

6.




内压: $p = 0$

$\therefore\ \sigma_2 = \dfrac{p}{t}\cdot r_2 = 0$

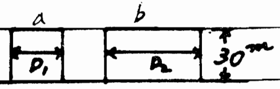



$r = h\tan\alpha$

Equilibrium $\sigma_1\, 2\pi r t\cos\alpha = P$

$\therefore\ \sigma_1 = \dfrac{P}{2\pi r t\cos\alpha} = \dfrac{P}{2\pi h t\sin\alpha}$

7.




조건 $D_1/D_2 = 1/2$

$p_a = w\times 30,\quad p_b = w\times 30$ $\quad w$: 比重量

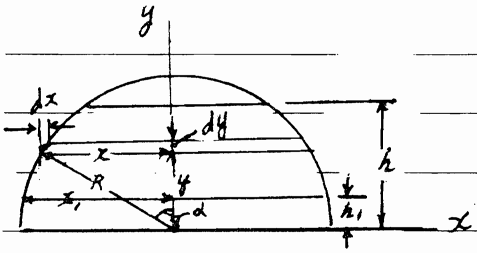
$(\sigma_2)_a = \dfrac{p_a(r_2)_a}{t_a}\quad (\sigma_2)_b = \dfrac{p_b(r_2)_b}{t_b}$

$(\sigma_2)_a = (\sigma_2)_b$ 로 부터

$\dfrac{p_a r_a}{t_a} = \dfrac{p_b r_b}{t_b}$

$\therefore\ \dfrac{t_a}{t_b} = \dfrac{p_a r_a}{p_b r_b} = \dfrac{30w\cdot r_a}{30w\cdot 2r_a} = \dfrac{1}{2}$

8.




Vertical Equilibrium

$$\int p_y\cdot 2\pi x\cdot ds\cdot\cos\alpha = \sigma_1\, 2\pi x_1 t\sin\alpha_1$$

$$\int_{h_1}^{h}\gamma g(h-y)\,2\pi\sqrt{R^2-y^2}\cdot\frac{R\,dy}{\sqrt{R^2-y^2}}\cdot\frac{y}{R}$$

$$=\sigma_1\,2\pi\sqrt{R^2-h_1^2}\cdot t\cdot\frac{\sqrt{R^2-h_1^2}}{R}$$

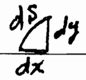
$x^2+y^2=R^2$

$x=\sqrt{R^2-y^2}$

$\sin\alpha = \dfrac{x}{R} = \dfrac{\sqrt{R^2-y^2}}{R}$

$\cos\alpha = \dfrac{y}{R}$

$p_y = \gamma g(h-y)$




$ds = \dfrac{dy}{\sin\alpha} = \dfrac{R\,dy}{\sqrt{R^2-y^2}}$

$$\gamma g\int_{h_1}^{h}(h-y)\,dy = \sigma_1\cdot\frac{t}{R}(R^2-h_1^2)$$

$$\gamma g\left[\frac{h}{2}(h^2-h_1^2)-\frac{1}{3}(h^3-h_1^3)\right]=\sigma_1\frac{t}{R}(r^2-h_1^2)$$

$$\therefore\ \sigma_1=\frac{\gamma g R}{t(R^2-h_1^2)}\left[\frac{h}{2}(h^2-h_1^2)-\frac{1}{3}(h^3-h_1^3)\right]$$

$$=\frac{9.807\times10^{-6}\times500}{0.25(500^2-100^2)}\left[\frac{400}{2}(400^2-100^2)-\frac{1}{3}(400^3-100^3)\right]$$

$$=0.7355\ \mathrm{kN/cm^2} = 7.355\ \mathrm{MN/m^2}$$

$r_1 = r = R = 5\,m = 500\,cm$

$t = 0.25\ cm$

$\gamma g = 9.807\ \mathrm{N/m^3}$

$\quad = 9.807\times10^{-6}\ \mathrm{kN/cm^3}$

Membrane Equation

$$\frac{\sigma_1}{r_1}+\frac{\sigma_2}{r_2}=\frac{p_1}{t}$$

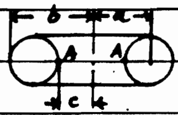
$r_1 = r_2 = R,\quad p_1 = \gamma g(h-h_1)$ 이므로

$$\sigma_1+\sigma_2=\frac{\gamma g R(h-h_1)}{t}$$

$$\therefore\ \sigma_2=\frac{\gamma g R(h-h_1)}{t}-\sigma_1$$

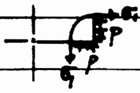
$$=5.1487\ \mathrm{kN/cm^2} = 51.487\ \mathrm{MN/m^2}$$

9.




$a = 36^{cm}$, $b = 45^{cm}$, $c = 27^{cm}$, $t = 3^{mm}$

$p = 0.6\ kgf/cm^2$




$\Sigma F_y = 0$

$\therefore\ 2\pi c \sigma_1 t = p\pi(a^2 - c^2)$

$\therefore\ \sigma_1 = \frac{p\pi(a^2-c^2)}{2\pi c t} = \frac{0.6\times(36^2-27^2)}{2\times27\times0.3} = 21\ kgf/cm^2$

Membrane equation

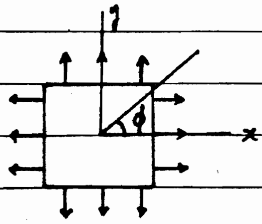
$\frac{\sigma_1}{r_1}+\frac{\sigma_2}{r_2}=\frac{p}{t}$

$r_1 = a - c = 9^{cm}$, $r_2 = -c = -27^{cm}$

$\therefore\ \sigma_2 = \left(\frac{0.6}{0.3} - \frac{21}{9}\right)\times(-27) = 8.999\ kgf/cm^2$

## 問題 3.2

1.




$\sigma_x = 80\ MN/m^2$, $\sigma_y = -40\ MN/m^2$, $\phi = 30°$

$\sigma_n = \frac{1}{2}(80-40) + \frac{1}{2}(80+40)\cos 60°$

$= 20 + 30 = 50\ MN/m^2$

$\sigma_n' = 20 + 60\cdot\cos 2(30+90) = -10\ MN/m^2$

$\tau_n = \frac{1}{2}(80+40)\sin 60° = 51.96\ MN/m^2$

$\tau_n' = \frac{1}{2}(80+40)\sin 2(120) = -51.96\ MN/m^2$

2. $\sigma_x = -500\ kgf/cm^2$, $\sigma_y = -800\ kgf/cm^2$, $\phi = 22°30'$

$\sigma_n = \frac{1}{2}(-500-800) + \frac{1}{2}(-500+800)\cos 45°$

$= -650 + 150\cdot\frac{1}{\sqrt{2}} = -543.9\ kgf/cm^2$

$\sigma_n' = \frac{1}{2}(-500-800)+\frac{1}{2}(-500+800)\cos 2\cdot(112°30')$

$= -650 + 150\left(-\frac{1}{\sqrt{2}}\right) = -756.1\ kgf/cm^2$

$\tau_n = \frac{1}{2}(-500+800)\sin 45° = 106.1\ kgf/cm^2$

$\tau_n' = \frac{1}{2}(300)\sin 2\cdot(112.5) = -106.1\ kgf/cm^2$

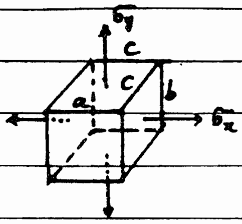
3. $\sigma_x = 50\ MN/m^2$, $\sigma_y = 30\ MN/m^2$

$\tau = \frac{1}{2}(50-30)\sin 2\theta = 10\cdot\sin 2\theta$

$\frac{d\tau}{d\theta} = 20\cdot\cos 2\theta = 0$ $\therefore 2\theta = 90^\circ$ $\therefore \theta = 45^\circ$

$\tau_{max} = 10\cdot\sin 90^\circ = 10\ MN/m^2$.

4.




$a' = a(1+\varepsilon_x)$, $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$

$b' = b(1+\varepsilon_y)$, $\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$

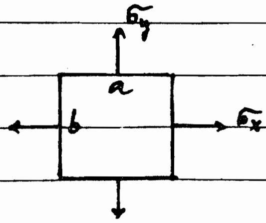
$c' = c(1+\varepsilon_z)$, $\varepsilon_z = \frac{-\nu}{E}(\sigma_x + \sigma_y)$

$a'b'c' = abc(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$

$\fallingdotseq abc(1+\varepsilon_x+\varepsilon_y+\varepsilon_z)$

$\therefore \frac{\Delta V}{V} = \frac{a'b'c'-abc}{abc} = \varepsilon_x+\varepsilon_y+\varepsilon_z$

5.




$t = 1\ cm$

$\sigma_x = 1400\ kgf/cm^2$, $\sigma_y = -1400\ kgf/cm^2$,

$\therefore \varepsilon_x = -\varepsilon_y$

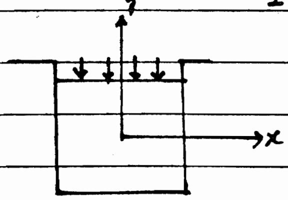
$\therefore a' = a(1+\varepsilon_x)$, $b' = b(1+\varepsilon_y)$, $t=1$

$\therefore V' = ab(1+\varepsilon_x)(1+\varepsilon_y)\cdot 1$

$\fallingdotseq ab(1+\varepsilon_x+\varepsilon_y) = ab$

$\therefore \Delta V = V' - V = ab - ab = 0$

6.




조건 : $\varepsilon_x = 0$, $\sigma_z = 0$

$\sigma_y = -2\ MN/m^2$, $E = 2\ MN/m^2$, $\nu = 1/2$

$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = 0$

$\therefore \sigma_x = \nu(\sigma_y + \sigma_z) = \frac{1}{2}\times(-2) = -1\ MN/m^2$

$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = 0$

$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x - \nu\sigma_z) = \frac{1}{2}\times(-2+\frac{1}{2}\times 1)$

$= -\frac{3}{2}$

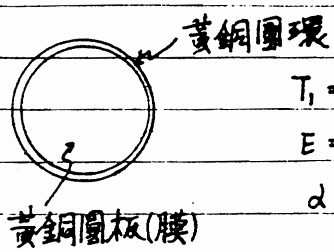
(3-2-6 conti.)

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_y - \nu\sigma_x) = \frac{1}{2}\left\{-\frac{1}{2}\times(-2)+\frac{1}{2}\times 1\right\} = \frac{3}{2}$$

$$\therefore \Delta V = V(\varepsilon_x + \varepsilon_y + \varepsilon_z) = V\left(-\frac{3}{2}+\frac{3}{2}\right)$$

$$= 0$$

7.




$T_1 = 20^\circ C$, $T_2 = -20^\circ C$

$E = 0.95\times 10^6\ kgf/cm^2$, $\nu = 0.34$

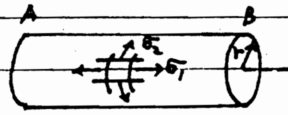
$\alpha = 19.8\times 10^{-6}\ cm/cm/^\circ C$

$$\Delta T = (-20)-20 = -40^\circ C$$

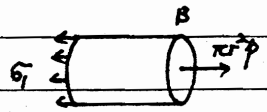
$$\varepsilon_x = \varepsilon_y = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{\sigma}{E}(1-\nu) = \alpha\cdot\Delta T$$

$$\sigma_x = \sigma_y = \sigma = \frac{\alpha\cdot E\cdot\Delta T}{1-\nu} = \frac{19.8\times 0.95\times 40}{1-0.34} = 1140\ kgf/cm^2.$$

8.




$r = 50^{cm}$, $t = 4^{mm}$, $p = 0.8\ MN/m^2$, $\phi = 45^\circ$




from equilibrium:

$$2\pi r\cdot t\,\sigma_1 = \pi r^2 p$$

$$\therefore \sigma_1 = \frac{pr}{2t} = \frac{0.8\times 50\times 10^{-3}}{2\times 4\times 10^{-3}} = 50\ MN/m^2,$$

$$\sigma_2 = \frac{pr}{t} = 2\sigma_1 = 100\ MN/m^2,$$

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_2)\sin 90^\circ = -25\ MN/m^2.$$

9. 例題 3 으로 부터

$$\sigma_1 = \frac{\gamma g\tan\alpha}{2t\cos\alpha}\left(hy - \frac{2}{3}y^2\right),\quad \sigma_2 = \frac{\gamma g(h-y)}{t}\cdot\frac{y\tan\alpha}{\cos\alpha}$$

$$\tau_{max} = \frac{1}{2}\cdot\frac{\gamma g\tan\alpha}{t\cos\alpha}\left(\frac{1}{2}hy - \frac{1}{3}y^2 - hy + y^2\right)\sin 90^\circ$$

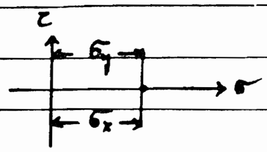
$$= \frac{1}{2}\cdot\frac{\gamma g\tan\alpha}{t\cos\alpha}\left(\frac{2}{3}y^2 - \frac{1}{2}hy\right)$$

$$\frac{d\tau_{max}}{dy} = \frac{1}{2}\cdot\frac{\gamma g\tan\alpha}{t\cos\alpha}\left(\frac{4}{3}y - \frac{1}{2}h\right) = 0 \quad \therefore y = \frac{1}{2}h\cdot\frac{3}{4} = \frac{3}{8}h$$

$$\therefore \tau_{max} = \frac{1}{2}\cdot\frac{\gamma g\tan\alpha}{t\cos\alpha}\left(\frac{2}{3}\cdot\frac{9}{64}h^2 - \frac{3}{16}h^2\right) = -\frac{3\gamma g h^2\tan\alpha}{64t\cos\alpha}$$

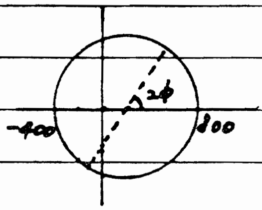
## 問題 3·3

1.




$\therefore \tau = 0$

2.




$\sigma_x = 800\ kgf/cm^2,\ \sigma_y = -400\ kgf/cm^2;\ \phi = 30^\circ$

$\therefore r = 600$

$\sigma_n = 200 + 600\cos 60^\circ = 500\ kgf/cm^2$

$\sigma_n' = 200 - 600\cos 60^\circ = -100\ kgf/cm^2$

$\tau = -\tau' = 600\frac{\sqrt{3}}{2} = 300\sqrt{3} = 519.6\ kgf/cm^2$

3. $h = 3^{cm}$, $\alpha = 22^\circ 30'$, $t = 0.25^{mm}$, $y = h/3$

§ 3.1의 例題3 에서 (물 : $\gamma = 1000\ kg/m^3$, $g = 9.807\ m/sec^2$)

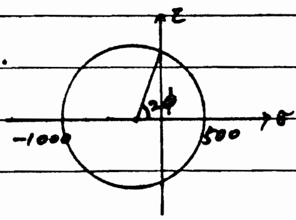
$\sigma_1 = \frac{\gamma g \tan\alpha}{2t\cos\alpha}\left(\frac{1}{3}h^2 - \frac{2}{27}h^2\right) = \frac{7}{54}\cdot\frac{\gamma g \tan\alpha}{t\cos\alpha}\cdot h^2$

$\sigma_2 = \frac{\gamma g (h - \frac{1}{3}h)}{t}\cdot\frac{\tan\alpha}{\cos\alpha}\cdot\frac{h}{3} = \frac{2}{9}\cdot\frac{\gamma g \tan\alpha}{t\cos\alpha}\cdot h^2$

$\tau_{max} = \frac{1}{2}\left(\frac{7}{54} - \frac{2}{9}\right)\frac{\gamma g \tan\alpha}{t\cos\alpha}h^2 = -\frac{-5}{2\times 54}\cdot\frac{9807\times\tan\alpha}{0.25\times 10^{-3}\times\cos\alpha}\cdot(3)^2$

$= 7328\times 10^3\ N/m^2 = 7.328\ MN/m^2$

4.



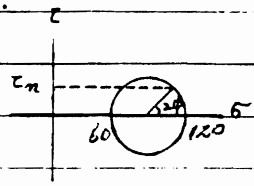

$\sigma_x = 500\ kgf/cm^2,\ \sigma_y = -1000\ kgf/cm^2$

$r = 750 \quad \therefore 750\cos 2\phi = 250$

$\cos 2\phi = \frac{1}{3} \therefore \phi = 35.26^\circ$

$\tau = 750\sin 2\phi = 707.1\ kgf/cm^2$

5.




$$\sigma_x = 120\ MN/m^2, \quad \sigma_y = 60\ MN/m^2.$$

$$\sigma_n = \frac{120+60}{2} + \frac{120-60}{2}\cos 2\phi$$

$$= 90 + 30\cdot\cos 2\phi$$

$$\tau = 30\cdot\sin 2\phi$$

$$\therefore \sigma_n/\tau = \frac{90+30\cdot\cos 2\phi}{30\cdot\sin 2\phi} = \frac{3+\cos 2\phi}{\sin 2\phi}$$

$$\therefore \frac{d}{d\theta}(\sigma_x/\tau) = \frac{-(3+\cos 2\phi)2\cos 2\phi - 2\sin^2 2\phi}{\sin^2 2\phi}$$

$$= \frac{-6\cos 2\phi - 2}{\sin^2 2\phi} = 0$$

$$\therefore \cos 2\phi = -\frac{2}{6} = -\frac{1}{3} \qquad \therefore 2\phi = 109.4712^\circ$$

$$\phi = 54.7356^\circ$$

## 問題 3·4

1. $\sigma_x = -\sigma_y = 200\ MN/m^2, \quad E = 200\ GN/m^2, \quad \mu = 0.25$

$$\tau = \frac{1}{2}(200+200)\sin 90^\circ = 200\ MN/m^2$$

$$G = \frac{E}{2(1+\mu)} = \frac{200}{2(1+0.25)} = 80\ GN/m^2$$

$$\therefore \gamma = \frac{\tau}{G} = \frac{200}{80\times 10^3} = 0.0025$$

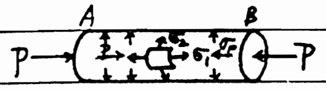
2. 고무: $E_r = 21\ kgf/cm^2, \quad \mu_r = 0.5$

Concrete: $E_c = 0.22\times 10^6\ kgf/cm^2, \quad \mu_c = 0.1$

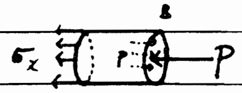
$$G_r = \frac{21}{2(1+0.5)} = 7\ kgf/cm^2$$

$$G_c = \frac{0.22\times 10^6}{2(1+0.1)} = 0.1\times 10^6\ kgf/cm^2.$$

3.



$D = 60^{mm}$, $t = 0.5^{mm}$, $p = 2.5\ MN/m^2$



$r_1 = \infty$, $r_2 = D/2 = 30\ mm$

$\therefore \sigma_2 = \dfrac{p r_2}{t} = \dfrac{2.5 \times 30}{0.5} = 150\ MN/m^2$

$\sigma_x = \sigma_1 - \sigma_p$

$\sigma_1 = \dfrac{p r_2}{2t} = 75\ MN/m^2$

순수전단 상태 : $\sigma_x = -150\ MN/m^2$

Equilibrium :

$\sigma_x \cdot 2\pi r_2 t = \sigma_1 \cdot 2\pi r_2 t - P$

$\therefore P = (\sigma_1 - \sigma_x)\, 2\pi r_2 t$

$= (75 + 150) \cdot 2\pi \cdot (30 \times 10^{-3})(0.5 \times 10^{-3})$

$= 21206 \times 10^{-6}\ MN = 21.21$

## 問題 3·5

1. $100^{mm} \times 25^{mm}$, $d = 22^{mm}$

$\tau_w = 50\ MN/m^2$, $\sigma_w = 100\ MN/m^2$

Mode 1) 第1열에서의 母板의 切断 $= (10-2.2) \times 2.5 \times (100 \times 10^{-1})$

$= 195\ KN$

Mode 2) 3개의 리벳트의 二重剪断 $= (3.80 \times 2) \times (50 \times 10^{-1}) \times 3$

$= 114\ KN$

Mode 3) 第1열의 리베트剪断 + 第2열에서의 母板切断

$= (3.80 \times 2) \times (50 \times 10^{-1})$

$+ (10 - 2.2 \times 2) \times 2.5 \times (100 \times 10^{-1})$

$= 38 + 140$

$= 178\ KN$

따라서 許容荷重 $P = 114\ KN$.

무損傷部의 強度 $= 10 \times 2.5 \times (100 \times 10^{-1}) = 250 kN$

effcy $\eta = \dfrac{114}{250} = 0.456$ or $45.6\%$.

2. 各問題 解예시

$$\text{1st Mode} = (10-d)\times 2.5\times(100\times 10^{-1})$$

$$\text{2nd Mode} = \frac{\pi d^2}{4}\times 2\times(50\times 10^{-1})\times 3$$

$$\text{3rd Mode} = \frac{\pi d^2}{4}\times 2\times(50\times 10^{-1}) + (10-2d)\times 2.5\times(100\times 10^{-1})$$

$$\text{2nd Mode} = \text{3rd Mode}$$

$$\frac{\pi d^2}{4}\times 30 = \frac{\pi d^2}{4}\times 10 + (5-d)\times 50$$

$$\pi d^2(3-1) - 20(5-d) = 0$$

$$\pi d^2 + 10d - 50 = 0$$

$$d = \frac{-10\pm\sqrt{10^2+4\pi\times 50}}{2\pi} = \frac{-5\pm 5\sqrt{1+2\pi}}{\pi}$$

$$= \frac{5}{\pi}(-1\pm\sqrt{1+2\pi}) = \frac{5}{\pi}(-1\pm\sqrt{7.2832})$$

$$= \frac{5}{\pi}(-1\pm 2.6988) = \frac{5}{\pi}\times 1.6988 = 2.704^{cm}$$

$$\text{2nd Mode} = 5.7425\times 30 = 172.28\ KN$$

$$\eta_{max} = \frac{172.28}{250} = 0.689 \text{ or } 68.9\%$$

$$\text{1st Mode} = (10-2.704)\times 25 = 182.4\ KN > 172.28^{KN}(\text{2nd Mode})$$

$$\text{3rd Mode} = 5.7425\times 10 + (10-5.408)\times 25 = 172.23\ KN(\doteqdot \text{2nd Mode})$$

3. $$\text{1st Mode} = (10-2.5)\times 2.5\times(250\times 10^{-1}) = 468.75\ KN$$

$$\text{2nd Mode} = \frac{\pi\times 2.5^2}{4}\times 2\times 3\times(125\times 10^{-1}) = 368.16\ KN$$

$$\text{3rd Mode} = \frac{\pi\times 2.5^2}{4}\times 2\times(125\times 10^{-1}) + (10-5)\times 2.5\times(250\times 10^{-1})$$

$$= 122.72 + 312.5 = 435.22\ KN$$

$$\text{Intact Strength} = 10\times 2.5\times(250\times 10^{-1}) = 625\ KN$$

$$\eta = \frac{368.16}{625} = 0.589 \text{ or } 58.9\%$$

$$P_c = 368.16\ KN.$$

4. $t = 15^{mm},\ d = 25^{mm},\ p_1 = 120^{mm},\ p_2 = 60^{mm}$

$D = 1500^{mm} = 1.5^{m}$

$\sigma_w = 1200^{kgf/cm^2};\ \tau_w = 900^{kgf/cm^2}$

Repeated Section ($12^{cm}$)

1) 2nd row tearing $= (12 - 2\times2.5)\times1.5\times1200$
$= 12600^{kgf}$

2) All rivets (3) Shearing $= \frac{\pi(2.5)^2}{4}\times3\times900$
$= 13.254^{kgf}$

$\therefore$ Strength $= 12.600^{kgf}$

Intact strength $= 12\times1.5\times1200 = 21600^{kgf}$

$\therefore\ \eta = 0.583$ or $58.3\%$

Hoop tension in a repeated section due to internal pressure $p$

$12t\cdot\sigma = 12t\times\frac{pr}{t} = 12\times p\cdot75 = 900p$

$900p = 12600 \quad \therefore\ p = 14^{kgf/cm^2}$

5.

$L\ 75^{mm}\times55^{mm}\times5^{mm},\ A = 6.303^{cm^2},\ \sigma = 1000^{kgf/cm^2}$

$d = 22^{mm},\ A_1 = \frac{\pi d^2}{4} = 3.801^{cm^2},\ \tau_w = 560^{kgf/cm^2}$

i) $S_1 = S_2 = A\sigma = 6300^{kgf}$

Number of rivets required

$\frac{\pi d^2}{4}\cdot n\times560 = 6303$

$\therefore\ n = \frac{6303}{3.801\times560} = 2.96 \quad \therefore$ 3 rivets

ii) $S_3 - S_4 = 2S_1\cos\alpha$

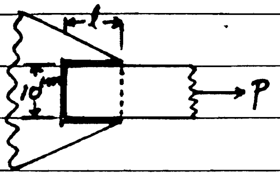
$= 2\times6303\times\frac{1}{\sqrt{2}} = 8914^{kgf}$

(3-5-5 conti.)

$$\frac{\pi d^2}{4}\cdot n \times 560 = 8914 \quad \therefore n = \frac{8914}{3.801\times 560} = 4.188$$

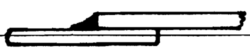
$\therefore$ 5 rivets

6.



폭 $= 10^{cm}$, 두께 $= 12^{mm}$, throat $= \frac{12}{\sqrt{2}}$ mm

$P = 200^{KN}$, $\begin{cases} \sigma_w = 110\ MN/m^2 \\ \tau_w = 94\ MN/m^2 \end{cases}$



Strenth of end weld : $\frac{1.2}{\sqrt{2}}\times 10 \times (110\times 10^{-1})$ KN

Strength of side weld : $\frac{1.2}{\sqrt{2}}\times 2l \times (94\times 10^{-1})$ KN

Equilibrium

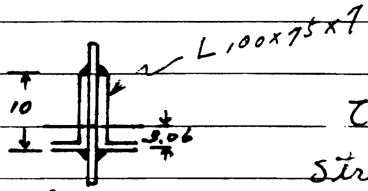
$$\frac{1.2}{\sqrt{2}}\times 10 \times (110\times 10^{-1}) + \frac{1.2}{\sqrt{2}}\times 2l\times(94\times 10^{-1}) = 200$$

$$\therefore l = 6.686\ cm$$

7.

$$\frac{1.2}{\sqrt{2}}\times 2l \times (94\times 10^{-1}) = 200 \quad \therefore l = \frac{200\times\sqrt{2}}{2\times 1.2\times 9.4} = 12.537\ cm$$

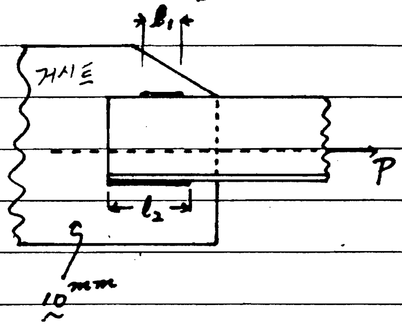
8.



$\tau_w = 960^{\ kgf/cm^2}$

Strenth per Unit lenth of weld :

$\frac{0.7}{\sqrt{2}}\times 960 = 475.2^{\ kgf/cm}$



Total required lenth of weld

$\frac{30000}{475.2} = 63.13^{\ cm}$

$2l_1 + 2l_2 = 63.13^{\ cm}$

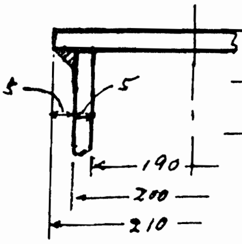
$3.06\times 30000 = 2l_1 \times 475.2 \times 10$

$\therefore l_1 = \frac{3.06\times 30000}{2\times 475.2\times 10} = 9.6596$ cm,

$2l_2 = 63.13 - 2l_1 \quad \therefore l_2 = \frac{63.13 - 2\times 9.659}{2} = 21.9^{cm}$

$\therefore l_1 = 9.66^{cm}, \quad l_2 = 21.9^{cm}$

9.




$$\begin{cases} D_0 = 210^{mm} \\ D_1 = 200^{mm} \\ D_2 = 190^{mm} \end{cases} \qquad t = \frac{D_1 - D_2}{2} = 5^{mm} \qquad \sigma_w = 1200^{kgf/cm^2}$$

i) Weld bead 가 받을수 있는 上向力 = Head 에 걸리는 流體壓 의 合力

$$\frac{0.5}{\sqrt{2}} \times 1200 \times 20\pi = \frac{\pi}{4}(20)^2 \times p$$

$$\therefore p = 84.85\ kgf/cm^2$$

ii) Cylinder wall 에 걸리는 Hoop Stress = $\sigma_w$ (or $\sigma_w = \frac{pr}{t}$)

$$\frac{p \times (9.5 + 0.25)}{0.5} = 1200$$

$$\therefore p = \frac{1200 \times 0.5}{9.75} = 61.54^{kgf/cm^2} \ (\text{Smaller})$$

$$\therefore p_w = 61.5\ kgf/cm^2.$$

## 問題 4 ~ 1

1. $d = 6^{mm}$, $n = 10.000\ rpm$, $\tau_w = 35\ MN/m^2$

$$J = \frac{\pi d^4}{32} \qquad \tau_w = \frac{Tr}{J} = \frac{16T}{\pi d^3}$$

$$\therefore T = \frac{\pi d^3 \tau_w}{16}$$

$$\therefore H = \frac{2\pi n T}{60} = \frac{2\pi n}{60} \cdot \frac{\pi d^3 \tau_w}{16}$$

$$= \frac{2\pi^2 \times 10{,}000 \times (6 \times 10^{-3})^3 \times 35 \times 10^3}{60 \times 16} = 1.554\ kW$$

2. DATA: $n = 105\ rpm$ $H = 200\ PS$

$\tau_w = 400\ kgf/cm^2$

$$d = 71.5 \sqrt[3]{\frac{H}{n \tau_w}}$$

$$= 71.5 \sqrt[3]{\frac{200}{105 \times 400}} = 120.21\ mm.$$

3. DATA: $n = 3600\ rpm$, $H = 300\ kW$, $\tau_w = 40\ MN/m^2$

$$d = 365 \sqrt[3]{\frac{300}{3600 \times 40}} = 46.62\ mm$$

4. $d_o = d$ $d_i = d/2$ $n = 105\ rpm$ $H = 200\ PS$

$\tau_w = 400\ kgf/cm^2$

$$J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32} d^4 \left(1 - \frac{1}{16}\right) = \frac{15 \pi d^4}{32 \times 16}$$

$$\tau_w = \frac{Tr}{J} \quad \therefore T = \frac{\tau_w \cdot J}{r} \quad (\text{where } r = d/2)$$

又 $T = \frac{71620\,H}{n}$

(4 - Cont. )

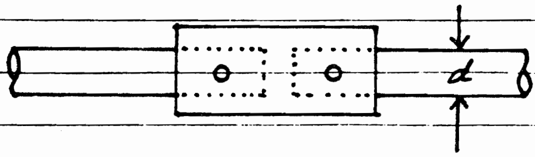
$$\therefore \tau_w = \frac{71620\ H \cdot r}{nJ} = \frac{71620\ H \times (32 \times 16)}{n\,(15\pi d^4)} \cdot \frac{d}{2}$$

$$\therefore d^3 = \frac{71620 \times 32 \times 16 \times H}{n\,(30\pi)\,\tau_w} = \frac{71620 \times 32 \times 16 \times 200}{105 \times 30\pi \times 400}$$

$$= 1852.739$$

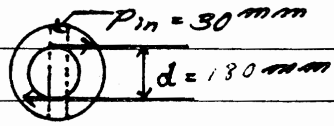
$$\therefore d = 12.282\ cm$$

5.




$n = 120\ rpm$

$\tau_w = 40\ MN/m^2$




Shearing strength of pin

$$\frac{\pi (3 \times 10^{-2})^2}{4} \times (40 \times 10^3) = 9\pi\ (KN)$$

Allowable Torque of the Joint

$$9\pi \times (18 \times 10^{-2}) = 1.62\pi\ (KN\text{-}m)$$

$$\therefore H = \frac{2\pi n T}{60} = \frac{2\pi \times 120 \times 1.62\pi}{60} = 63.955\ KW$$

Allowable Torque of Intact Section

$$\tau_w = \frac{16 T_0}{\pi d^3}$$

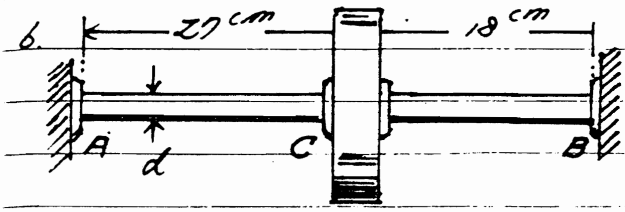
$$T_0 = \frac{\pi d^3 \tau_w}{16} = \frac{\pi \cdot (18 \times 10^{-2})^3 \cdot (40 \times 10^3)}{16} = 14.58\ (KN\text{-}m)$$

(5 - Cont.)

$$\therefore \text{Efficiency} = \frac{1.62\pi}{14.58\pi} = 0.1111$$

$\therefore$ 11.1 %

6.




$d = 12^{mm}$  $\tau_w = 700\ Kgf/cm^2$

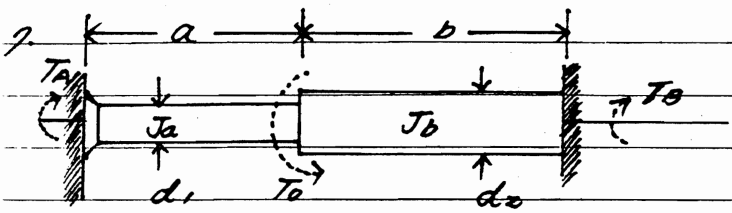
$G = 0.84 \times 10^6\ Kgf/cm^2$

$$\tau_w = \frac{T_w \cdot r}{J} \qquad \therefore T_w = \frac{\tau_w \cdot J}{r}$$

$$\phi = \frac{T \cdot l}{G \cdot J} = \frac{\tau_w \cdot J \cdot l}{G \cdot J \cdot r} = \frac{700 \times 18}{0.84 \times 10^6 \times 0.6} = 0.025\ rad.$$

$$= 1.4324^\circ = 1^\circ 26'$$

7.




DATA : $a = 20^{cm}$  $b = 40^{cm}$  $d_1 = 3^{cm}$

$d_2 = 4^{cm}$  $\tau_w = 55\ MN/m^2$

$\Sigma T = 0 \quad \therefore T_0 = T_A + T_B$

Compatibility  $\phi_a = \phi_b = \phi_0$

$$\therefore \phi_0 = \frac{J_A \cdot a}{G \cdot J_A} = \frac{T_B \cdot b}{G \cdot J_B}$$

(7 - Cont.)

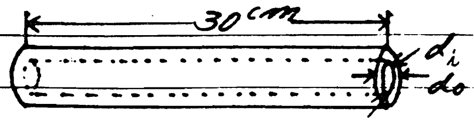
$$\therefore \frac{T_A}{T_B} = \frac{J_A \cdot b}{J_B \cdot a} = \frac{d_1^4 \cdot b}{d_2^4 \cdot a} = \frac{3^4 \cdot 40}{4^4 \cdot 20} = 0.6328$$

$\tau = \frac{T \cdot r}{J}$ 로 부터

$$T_A = \frac{\tau \cdot J_A}{(d_1/2)} = \frac{(55 \times 10^3) \times \pi \times (3 \times 10^{-2})^3}{16} = 0.2916 \ (KN\text{-}m)$$

$$\therefore T_B = \frac{0.2916}{0.6328} = 0.4608 \ (KN\text{-}m)$$

$$\therefore T_0 = 0.2916 + 0.4608 = 0.7524 \ KN\text{-}m$$

8. 


$T = 300 \ kgf\cdot cm$

$\phi = 1°$

$\tau_w = 400 \ kgf/cm^2$

$J = \frac{\pi}{32}(d_o^4 - d_i^4)$

$G = 0.84 \times 10^6 \ kgf/cm^2$

$$\phi = \frac{T \cdot l}{GJ} = 1° = 0.01745 \ rad/cm$$

$$\tau_w = \frac{T \cdot r}{J} = 400 = \frac{T(\frac{d_o}{2})}{J}$$

$$\therefore \frac{T \cdot l}{G \cdot J} \cdot \frac{2J}{T \cdot d_o} = \frac{0.01745}{400}$$

$$\therefore d_o = \frac{30 \cdot 400}{0.01745} \cdot \frac{2}{G} = \frac{800 \times 30}{0.01745 \times 0.84 \times 10^6}$$

$$= 1.63734 \ cm$$

(8- Cont.)

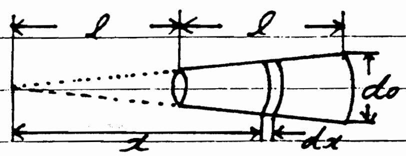
$$J = \frac{T \cdot l}{0.01745 G} = \frac{300 \times 30}{0.01745 \times 0.84 \times 10^6} = 0.613999$$

$$= \frac{\pi}{32} (d_o^4 - d_i^4)$$

$$\therefore d_i^4 = -0.613999 \times \frac{32}{\pi} + (1.63734)^4 = 0.93298$$

$$\therefore d_i = 0.9828^{cm}$$

9.




$$d_x = \frac{x}{2l} d_o$$

$$d\phi = \frac{T dx}{G J_x}$$

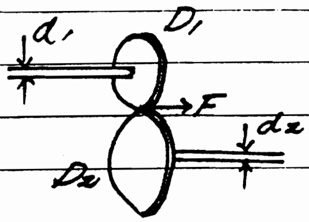
$$\text{where } J_x = \frac{\pi d_x^4}{32} = \frac{\pi d_o^4}{32} \cdot \frac{x^4}{16 l^4} = J_o \frac{x^4}{16 l^4}$$

$$\text{where } J_o = \frac{d_o^4}{32\pi}$$

$$\phi = \int_l^{2l} \frac{T}{G J_o} \cdot \frac{16 l^4}{x^4} dx$$

$$= \frac{T}{G J_o} \cdot 16 l^4 \left[ -\frac{x^{-3}}{3} \right]_l^{2l} = \frac{14 T l}{3 G J_o}$$

10.



(10 — Cont.)

$T_1 = F \cdot \frac{D_1}{2}$ , $T_2 = F \cdot \frac{D_2}{2}$

$\frac{T_1}{T_2} = \frac{D_1}{D_2} = \frac{1}{2}$

$D_1 / D_2 = 1/2$

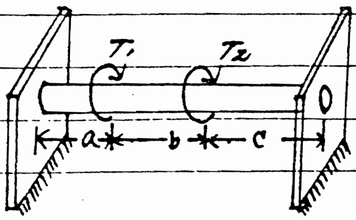
최대 전단응력 같아 계산

$\tau_{max} = \frac{16T_1}{\pi d_1^3} = \frac{16T_2}{\pi d_2^3}$

$\therefore \left(\frac{d_1}{d_2}\right)^3 = \frac{T_1}{T_2} = \frac{1}{2}$

$\therefore \frac{d_1}{d_2} = 0.7937$

11.

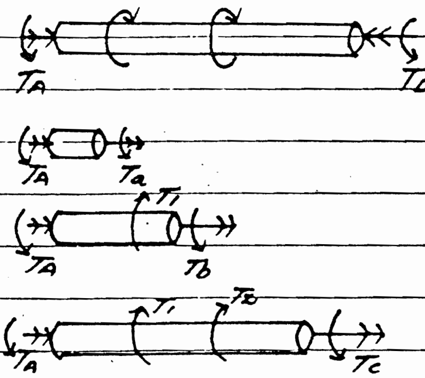



DATA

$a = 60\,cm$ $b = 100^{cm}$ $c = 80^{cm}$

$T_1 = 1.2\ KN\text{-}m$

$T_2 = 2.4\ KN\text{-}m$




구간 a:

$T_A + T_a = 0 \quad \therefore T_A = -T_a \cdots ①$

구간 b:

$T_A - T_1 + T_b = 0 \quad \therefore T_b = T_1 + T_a \cdots ②$

구간 c:

$T_A - T_1 - T_2 + T_c = 0$

$\therefore T_c = T_1 + T_2 + T_a \cdots\cdots ③$

(11 – Cont.)

Compatibility

$\phi a + \phi b + \phi c = 0$ ······ ④

Hooke's law

$\phi_a = \frac{T_a \cdot a}{G J_a} \quad \phi_b = \frac{T_b \cdot b}{G J_b} \quad \phi_c = \frac{T_c \cdot c}{G \cdot J_c}$ ··· ⑤

⑤ → ④

$T_a \cdot a + T_b \cdot b + T_c \cdot c = 0 \quad (\because J_a = J_b = J_c)$ ···· ⑥

②, ③ → ⑥

$T_a \cdot a + (T_1 + T_a) b + (T_1 + T_2 + T_a) c = 0$

$\therefore T_a = -\frac{T_1(b+c) + T_2 c}{a+b+c}$ ······ ⑦

$= -\frac{1.2(1+0.8) + 2.4 \times 0.8}{0.6 + 1 + 0.8}$

$= -1.7$ KN-m

⑦ → ②, ③

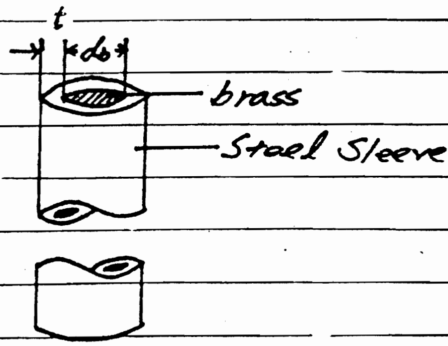
$T_b = T_1 + T_a$

$= 1.2 - 1.7 = -0.5$ KN-m

$T_c = T_1 + T_2 + T_a$

$= 1.2 + 2.4 - 1.7 = 1.9$ KN-m

12.




DATA

$G_s = 0.84 \times 10^6$ kgf/cm²

$G_b = 0.42 \times 10^6$ kgf/cm²

$(\tau_w)_s = 840$ kgf/cm²

$(\tau_w)_b = 640$ kgf/cm²

$d_b = 60^{mm} \quad t = 5^{mm}$

( 12 – Cont. )

Equilibrium $T = T_s + T_b$

Compatibility $\phi_b = \phi_s$ $\quad \therefore \dfrac{T_b\, l}{G_b J_b} = \dfrac{T_s\, l}{G_s J_s}$

$\therefore T_b / T_s = \dfrac{G_b J_b}{G_s J_s}$

Stress Condition

$\tau = \dfrac{T \cdot r}{J}$

$(T_b)_w = \dfrac{\pi d_b^3 (\tau_b)_w}{16}$ $\quad (T_s)_w = \dfrac{\pi [(d_b+2t)^4 - d_b^4](\tau_s)_w}{16(d_b+2t)}$

$$T_b/T_s = \frac{G_b \cdot J_b}{G_s \cdot J_s} = \frac{G_b \frac{\pi}{32}(d)^4}{G_s \cdot \frac{\pi}{32}[(d+2t)^4 - d^4]}$$

$$= \frac{0.42 \times 10^6 \times 6^4}{0.84 \times 10^6 \times (7^4 - 6^4)}$$

$$= 0.5864$$

Based on Brass Stress

$\tau = \dfrac{16T}{\pi d^3}$

$(T_b)_w = \dfrac{\pi \times 6^3 \times 640}{16} = 27143,338$ Kgf-cm

$\therefore T_s = \dfrac{27143.338}{0.5864} = 46,288.093$ Kgf-cm

Kgf-cm

$\therefore T = 27143,338 + 46,288.093 = 73431.431$

Based on Steel

$(T_s)_w = \dfrac{\pi(7^4 - 6^4)}{16 \times 7} \times 840 = 26035.927$ Kgf-cm

$\therefore (T_b) = 26035.927 \times 0.5864 = 15267.4674$ Kgf-cm

$\therefore T = (T_s)_w + T_b = \underline{41303.395 \text{ Kgf-cm}}$

↑ smaller $\quad \therefore$ Ans

$\dfrac{T}{T_b} = \dfrac{41303.395}{27143.338} = 1.5217$

## 問題 4～2

1. Spring 常数 (4・11) 式

$$k = \frac{Gd^4}{64R^3n}$$

$$\therefore k_1/k_2 = \frac{\dfrac{Gd^4}{64R_1^3n}}{\dfrac{Gd^4}{64R_2^3n}} = \frac{R_1^3}{R_2^3} = \frac{D_1^3}{D_2^3}$$

$D_1 = 2^{cm}$ $D_2 = 1^{cm}$

$$k_1/k_2 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

2. DATA $R = 30^{mm}$ $d = 3^{mm}$ $n = 30$ $\tau_w = 120\ MN/m^2 (N/mm^2)$

$G = 80\ GN/m^2 = 80 \times 10^3\ N/mm^2$

$$\tau_{max} = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)$$

$m = 2R/d = 2 \times 3 / 0.3 = 20$

$$(\quad) = \frac{80-1}{80-4} + \frac{0.615}{80} = 1.070224$$

$$\therefore P_w = \frac{\pi d^3 \tau_w}{16R} \times \frac{1}{1.0702} = \frac{\pi \times 3^3 \times 120}{16 \times 30 \times 1.0702} = 19.814\ N$$

$$\delta = \frac{64nPR^3}{d^4G} = \frac{64 \times 30 \times 19.814 \times 30^3}{3^4 \times 80 \times 10^3} = 158.5\ mm$$

3. 問題 1 と 同 $P_1 = 89.0\ N$, $P_2 = 211\ N$, $G = 80 \times 10^3\ N/mm^2$

$$\delta = \frac{64nP_1R_1^3}{d^4G} = \frac{64 \times 8 \times 89 \times 40^3}{10^4 \times 80 \times 10^3} = 3.645\ mm$$

(3- Cont.)

$$k = \frac{P}{\delta} = \frac{300}{3.645} = 82.30\ N/mm$$

4. $R_1 = 30^{mm}$, $a = 0.01$, $d = 3^{mm}$, $n = 10$

$G = 0.84 \times 10^6\ kgf/cm^2$

$R = R_1 e^{a\alpha}$

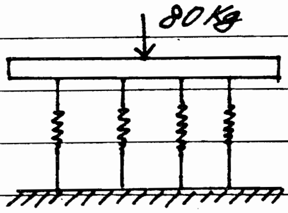
$$\delta = \int_0^{2\pi n} \frac{PR^3}{JG} d\alpha = \frac{32PR_1^3}{\pi d^4 G} \int_0^{2\pi n} e^{3a\alpha} d\alpha = \frac{32PR_1^3}{\pi d^4 G} \left[ \frac{1}{3a} e^{3a\alpha} \right]_0^{2\pi n}$$

$$= \frac{32PR_1^3}{\pi d^4 G} \cdot \frac{1}{3a} \left( e^{6\pi a n} - 1 \right)$$

$$\therefore k = \frac{P}{\delta} = \frac{\pi d^4 G \cdot 3a}{32 R_1^3 (e^{6\pi a n} - 1)} = \frac{\pi \times (0.3)^4 \times 0.84 \times 10^6 \times 0.03}{32 \times 3^3 \times (e^{60 \times 0.01\pi} - 1)}$$

$$= 0.1329\ kgf/cm$$

5.




$R = 30^{mm}$, $d = 10^{mm}$, $n = 9$

$G = 80\ GN/m^2 = 80 \times 10^3\ N/mm^2$

$\tau_{max} = 60\ MN/m^2\ (N/mm^2)$

$$\tau_{max} = \frac{16PR}{\pi d^3} \left( \frac{4m-1}{4m-4} + \frac{0.615}{m} \right)$$

$m = 2R/d = 2 \times 30/10 = 6$

$$\therefore (\quad) = \frac{23}{20} + \frac{0.615}{6} = 1.2525$$

$$P_{max} = \frac{\pi d^3 \tau_{max}}{16R} \cdot \frac{1}{1.2525} = \frac{\pi \times 10^3 \times 60}{16 \times 30 \times 1.2525} = 313.5\ N$$

1 spring의 Static load $= 80 \times 9.807/4 = 196.1\ N$

$\therefore P_w = P_{max} - P_{static} = 313.5 - 196.1 = 117.4\ N$

(5 - Cont.)

$$\therefore \delta = \frac{64\,n P R^3}{d^4 G} = \frac{64 \times P \times 11.74 \times 30^3}{10^4 \times 80 \times 10^3} = 2.282 \text{ mm.}$$

6. $R_1 = 40^{mm}$ $R_2 = 160^{mm}$ $\tau_w = 3200\ \text{Kgf/cm}^2$

$d = 20^{mm}$

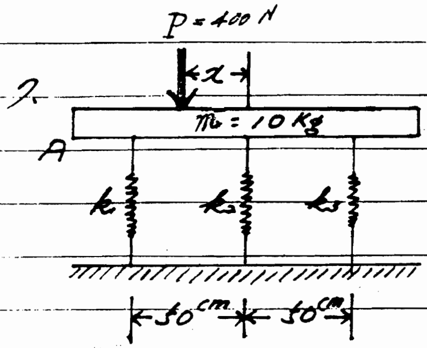
$$\tau_w = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)$$

$$m = \frac{2R}{d} = \frac{2 \times 16}{2} = 16$$

$$\therefore (\quad) = \frac{16 \times 4 - 1}{16 \times 4 - 4} + \frac{0.615}{16} = 1.0884$$

$$\therefore P_w = \frac{\pi d^3 \tau_w}{16R} \times \frac{1}{1.0884} = \frac{\pi \times 2^3 \times 3200}{16 \times 16 \times 1.0884} = 288.633 \text{ Kgf.}$$

7.




$P = 400$ N, $m = 10$ Kg,
$k_1 = 20$ N/mm, $k_2 = 12$ N/mm,
$k_3 = 8$ N/mm,

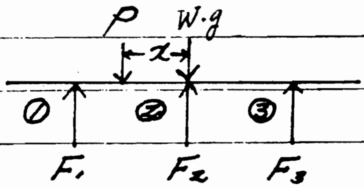
From Static Equilibrium

$P + m \cdot g = F_1 + F_2 + F_3 = 498.07$ N $\cdots$ ①

Compatibility

$\delta_1 = \delta_2 = \delta_3 = \delta$

$$\therefore \frac{F_1}{k_1} = \frac{F_2}{k_2} = \frac{F_3}{k_3} \cdots\cdots ②$$




$\Sigma M_3 = 0$

$F_1 \times 1000 + F_2 \times 500 = 400(500 + x) + 98.07 \times 500 \cdots$ ③

② → ①

$$F_1 + F_1\left(\frac{k_2}{k_1}\right) + F_1\left(\frac{k_3}{k_1}\right) = 498.07$$

(7- Cont.)

$$\therefore F_1 = \frac{498.07}{1+\frac{12}{20}+\frac{8}{20}} = \frac{498.07\times 20}{20+12+8} = 249.033\ N$$

$$\therefore F_2 = F_1\frac{k_2}{k_1} = 249.03\times\frac{12}{20} = 149.42\ N$$

$$\therefore F_3 = F_1\frac{k_3}{k_1} = 249.03\times\frac{8}{20} = 99.612\ N$$

From ③

$$249.03\times 1000 + 149.42\times 500 = 400(500+x) + 98.07\times 500$$

$$\therefore x = 186.76\ mm = 18.676\ cm.$$

## 問題 4-3

1. $U = \frac{T\phi}{2} = \frac{T^2 l}{2GJ}$

$$U_1 = \frac{T^2 l}{2GJ_1} \qquad U_2 = \frac{T^2 l}{2GJ_2}$$

$$\therefore U_1/U_2 = J_2/J_1 = d_2^4/d_1^4$$

2. 단위 체적 당의 질량 $\gamma$

중실 원형 단면축의 질량 $= \pi r^2 l\gamma$

중공 원형 축의 질량 $= \pi(r_o^2 - r_i^2)l\gamma$

질량이 같은 조건으로 부터

$$r^2 = r_o^2 - r_i^2 = (2r_o - t)t$$

$$U_{중실} = \int_0^r \frac{\tau_{max}^2\rho^2}{2Gr^2} l\cdot 2\pi\rho\cdot d\rho = \frac{1}{2}(\pi r^2 l)\frac{\tau_{max}^2}{2G}$$

$$U_{중공} = \int_{r_i}^{r_o} \frac{\tau_{max}^2\rho^2}{2Gr_o^2} l\cdot 2\pi\rho\cdot d\rho = \frac{\tau_{max}^2 l\cdot 2\pi}{2Gr_o^2}\left[\frac{1}{4}\rho^4\right]_{r_i}^{r_o}$$

$$= \frac{\tau_{max}^2 l\cdot 2\pi}{Gr_o^2}(r_o^3 - r_i^3)$$

(2 - cont.)

$$U_{중실}/U_{중공} = \frac{1}{2}\pi r^2 l \frac{\tau_{max}^2}{2G} \Big/ \frac{\tau_{max}^2 l \cdot 2\pi}{8G r_o^2}(r_o^4 - r_i^4)$$

$$= \frac{3}{4}\,\frac{r_o^2 r^2}{r_o^4 - r_i^4} = \frac{r_o^2 r^2}{(r_o^2 - r_i^2)(r_o^2 + r_i^2)}$$

$$= \frac{r_o^2 r^2}{r^2(r_o^2 + r_o^2 - 2r_o t + t^2)}$$

$t \ll r_o$

$$\therefore U_{중실}/U_{중공} \doteqdot \frac{r_o^2}{2r_o^2} = 1/2$$

3. Hard-drawn Alminum : $\sigma_{pl} = 700\ \mathrm{kgf/cm^2}$, $w = 2.68\ \mathrm{gf/cm^3}$

Cold-rolled Steel : $\sigma_{pl} = 2500\ \mathrm{kgf/cm^2}$, $w = 7.83\ \mathrm{gf/cm^3}$

$G_{Al} = 0.27 \times 10^6\ \mathrm{kgf/cm^2}$ $\quad G_{st} = 0.84 \times 10^6\ \mathrm{kgf/cm^2}$

$$U_A = \frac{\tau^2}{2G}\cdot\frac{1}{w} = \frac{700^2}{2\times 0.27\times 10^6 \times 2.68\times 10^{-3}} = 338.585\ \mathrm{kgf\text{-}cm/kgf}$$

$$U_S = \frac{\tau^2}{2G}\cdot\frac{1}{w} = \frac{2500^2}{2\times 0.84\times 10^6\times 7.83\times 10^{-3}} = 475.126\ \mathrm{kgf\text{-}cm/kgf}$$

4. 강제 중실축: 길이 $l$, 직경 $d$

〃 중공축: 길이 $l$, 외경 $d$, 내경 $d/2$

최대 허용 전단 응력: $\tau_w$

$$U_{중실} = \frac{1}{2}(\pi r^2 l)\frac{\tau_w^2}{2G} = \frac{1}{16}(\pi d^2 l)\frac{\tau_w^2}{G}$$

$$U_{중공} = \int_{d/4}^{d/2} \frac{\tau_w^2 \rho^2}{2G(d/2)^2}\cdot l\cdot 2\pi\rho\, d\rho = \frac{\tau_w^2}{G}\cdot\frac{4\pi l}{d^2}\left[\frac{1}{4}\rho^4\right]_{d/4}^{d/2}$$

$$= \left(\frac{1}{16} - \frac{1}{256}\right)\frac{\tau_w^2}{G}\cdot\pi d^2 l$$

$$= \frac{15}{256}(\pi d^2 l)\frac{\tau_w^2}{G}$$

$$\therefore U_{중실}/U_{중공} = 1/16 \times 256/15 = 16/15$$

5. $P = 500\ N$, $G = 80\ GN/m^2 = 80 \times 10^9\ N/m^2$

$$\left.\begin{array}{l} P = P_1 + P_2 = 500 \\ P_1/P_2 = R_2^3/R_1^3 = 27/64 \end{array}\right\} \therefore \begin{array}{l} P_1 = 148.35\ N \\ P_2 = 351.65\ N \end{array}$$

$$U = \frac{(PR)^2 2\pi R n}{2GJ} \ ; \ J = \frac{\pi d^4}{32}.$$

$$U = U_1 + U_2 = \frac{2\pi n}{2GJ}\left[ P_1^2 \cdot R_1^3 + P_2^2 \cdot R_2^3 \right]$$

$$= \frac{2\pi \times 8 \times 32}{2 \times 80 \times 10^9 \times \pi \times (1 \times 10^{-2})^4}\left[ 148.35^2 \times (4 \times 10^{-2})^3 + 351.65^2 \times (3 \times 10^{-2})^3 \right]$$

$$= 1.519\ N\text{-}m$$

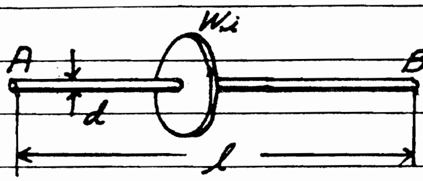
6. $d_o = 6^{cm}$, $d_i = 3^{cm}$, $m = 50\ kg$, $i = 25\ cm$

$$U = \frac{m i^2 \omega^2}{2} = \frac{50 \times (25 \times 10^{-2})^2 (4\pi)^2}{2} = 246.74\ N\text{-}m$$

or

$$U = \int_{d_i/2}^{d_o/2} \frac{\tau_{max}^2 \rho^2}{2G(d_o/2)^2}\, l \cdot 2\pi\rho \cdot d\rho = \frac{15}{256}(\pi d_o^2 l)\frac{\tau_{max}^2}{G}$$

$$\therefore \tau_{max} = \sqrt{\frac{256 \times (80 \times 10^3) \times (246.74 \times 10^{-6})}{15 \times \pi \times (6 \times 10^{-2})^2 \times 1.2}} = 157.55\ MN/m^2.$$

7. 


$d = 5\ cm$
$W = 21\ ^{kgf}$ $i = 25\ ^{cm}$
$n = 120\ ^{rpm}$
$\tau_w = 840\ ^{kgf}/_{cm^2}$

解 $\omega = 2\pi f = 2\pi \times 120/60 = 4\pi\ ^{rad}/_{sec}$

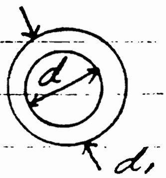
$$K.E. = \frac{1}{2}\frac{W}{g} \cdot i^2 \cdot \omega^2 = \frac{1}{2}\frac{21}{980} \cdot 25^2 \cdot (4\pi)^2 = 108.247\pi^2\ ^{kgf\text{-}cm}$$

$$U = \frac{1}{2}(\pi r^2 l)\frac{\tau_w^2}{2G} \quad \therefore l = \frac{4GU}{\pi r^2 \tau_w^2}$$

$$= \frac{4 \times 0.84 \times 10^6 \times 108.247\pi^2}{\pi \times (\frac{5}{2})^2 \times 840^2}$$

$$= 259.7002\ ^{cm}$$

8.




$d = 7^{mm}$

$d_1 = 10^{mm}$

$l = 250^{cm}$

$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(0.7)^4 \quad cm^4$

$J_1 = \frac{\pi}{32}(1^4 - 0.7^4) \quad cm^4$

$J/J_1 = \frac{0.7^4}{1-0.7^4} = \frac{0.2401}{0.7599} = 0.316$

$U = \frac{GJ\phi^2}{2l} \qquad U_1 = \frac{GJ_1\phi_1^2}{2l}$

$\phi = \frac{Tl}{GJ} \qquad \phi_1 = \frac{Tl}{GJ_1}$

$\therefore \frac{\phi}{\phi_1} = \frac{J_1}{J} \qquad \phi + \phi_1 = \frac{\pi}{2}$

$\therefore \phi = \frac{\frac{\pi}{2}}{1.316} = 0.3799\pi \quad rad$

$\phi_1 = \frac{\pi}{2} - \phi = 0.1201\pi \quad rad$

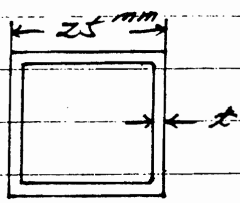
$U_T = U + U_1 = \frac{GJ\phi^2}{2l} + \frac{GJ_1\phi_1^2}{2l} = \frac{GJ\phi^2}{2l}\left\{1 + \frac{J_1}{J}\left(\frac{\phi_1}{\phi}\right)^2\right\}$

$= \frac{0.84\times10^6 \times \frac{\pi}{32}(0.7)^4 \times (0.3799\pi)^2}{2\times250}(1+0.316)$

$= 74.2326^{kgf\text{-}cm}$

## 問題 4-4

1.




$T = 68 \ N\text{-}m = 68\times10^2 \ N\text{-}cm$

$\tau_w = 41 \ MN/m^2 = 41\times10^2 \ N/cm^2$

$\tau = \frac{T}{2A_0 t}$

$A_0 = (2.5 - t)^2$

(1 - Cont.)

$$41 \times 10^2 = \frac{68 \times 10^2}{2(2.5-t)^2 t}$$

$$t(2.5-t)^2 = \frac{68}{2 \times 41} = 0.82927$$

$$\therefore t \approx 0.15^{cm} = 1.5^{mm}$$

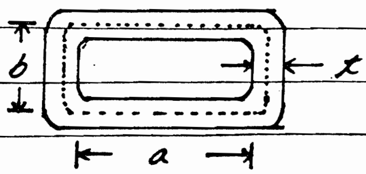
2. $G = 27\ GN/m^2 = 27 \times 10^5\ N/cm^2$, $A_0 = (2.5-0.15)^2 = 5.5225\ cm^2$

$$\phi = \frac{\tau s l}{2A_0 G} \qquad s = (2.5-0.15) \times 4 = 9.4\ cm$$

$$\therefore \theta = \frac{\phi}{l} = \frac{\tau s}{2A_0 G} = \frac{41 \times 10^2 \times 9.4}{2 \times 5.5225 \times 27 \times 10^5}$$

$$= 0.001292\ rad/cm$$

3.



$S = 2(a+b) \qquad T = const.$

$\alpha = a/b$

$$\theta = \frac{\phi}{l} = \frac{\tau S}{2A_0 G} = \frac{S}{2A_0 G}\ \frac{T}{2A_0 t}$$

$$= \frac{TS}{4Gt}\ \frac{1}{(ab)^2}$$

$a = b\alpha$ 를 대입

$$\theta = \frac{TS}{4Gt}\ \frac{1}{\alpha^2 b^4}$$

$a = b\alpha$ 를 대입

$$\theta = \frac{TS}{4Gt}\ \frac{1}{\alpha^2 b^4}$$

$a = (S-2b)/2$

$\therefore S - 2b = 2b\alpha \qquad \therefore (2\alpha+2)b = S \qquad \therefore b = \dfrac{S}{2(1+\alpha)}$

$$\therefore \theta = \frac{TS}{4Gt}\ \frac{1}{\alpha^2}\ \frac{16(1+\alpha)^4}{S^4} = \frac{4T}{GtS^3}\ \frac{(1+\alpha)^4}{\alpha^2}$$

$$\therefore \theta \propto \frac{(1+\alpha)^4}{\alpha^2}$$

4. $f(\alpha) = \dfrac{(1+\alpha)^4}{\alpha^2}$

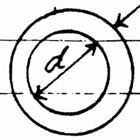
$$f'(\alpha) = \frac{4(1+\alpha)^3\alpha^2 - 2\alpha(1+\alpha)^4}{\alpha^4} = \frac{\alpha(1+\alpha)^3\{4\alpha-2\alpha-2\}}{\alpha^4}$$

$$= \frac{2(1+\alpha)^3\{2\alpha-1\}}{\alpha^3} = 0$$

$\therefore \alpha = 1$

$\therefore a = b$ 일때 最大

5.




$d = 100^{mm}$ $t = 3^{mm}$

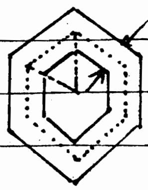
$\tau_w = 420\,kgf/cm^2$ $H = 50\ PS$

$$T = \frac{71620\,H}{n} \qquad \tau = \frac{T}{2A_0 t} = \frac{71620\,H}{2A_0\,t\,n} \quad (kgf/cm^2)$$

$$A_0 = \frac{\pi(10)^2}{4} = 25\pi\ (cm^2)$$

$$\therefore n = \frac{71620\times 50}{2\times 25\pi\times 0.3\times 420} = 180.93\ rpm$$

6.




$S = 6a = 12^{cm}$

$l = 1^{m}$

$\tau_w = 600\ kg/cm^2$

$G = 0.84\times 10^6\,kgf/cm^2$

$$A_0 = \frac{1}{2}\times 2\times\sqrt{3}\times 6 = 10.3923\ cm^2$$

$$T = (2A_0 t)\tau = 2\times 10.3923\times 0.1\times 600$$

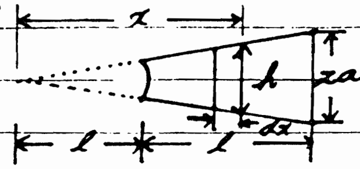
$$= 1247.076\ kgfcm$$

$$\phi = \frac{\tau S l}{2A_0 G} = \frac{600\times 12\times 100}{2\times 10.3923\times 0.84\times 10^6}$$

$$= 0.0412\ rad$$

$$= 2^\circ 22'$$

7.




$$h = \frac{a}{l}x$$

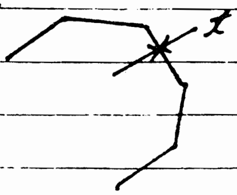
$$A_0 = h^2 = \frac{a^2}{l^2}x^2$$

$$S = 4h = \frac{4ax}{l}$$

$$d\phi = \frac{T \cdot 4ax \cdot dx}{2A_0 t \cdot l \cdot 2A_0 G} = \frac{T \cdot ax \cdot l^4 \, dx}{t \cdot l \cdot G \cdot a^4 x^4} = \frac{T a l^3}{G t a^4 x^3} dx$$

$$\phi = \int_l^{2l} d\phi = \frac{T l^3}{G t a^3}\left[-\frac{1}{2}x^{-2}\right]_l^{2l} = \frac{3Tl}{8ta^3}$$

8.




長さ：$l$　肉厚：$t$　$S = na$

$T = \text{const.}$

$$\phi = \left(\frac{T}{2A_0 t}\right)\frac{S \cdot l}{2A_0 G} = \frac{TSl}{4A_0^2 tG}$$

$n$角形の内角の和 $= \frac{n-2}{2}\pi$

∴ 正多角形 角度の半 $= \frac{n-2}{2n}\pi$

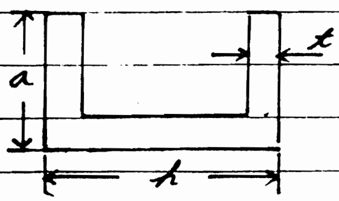
$$\therefore A_0 = \left\{\frac{1}{2} \times a \times \frac{a}{2}\tan\left(\frac{n-2}{2n}\right)\pi\right\} n$$

$$= \frac{n}{4}a^2 \tan\left(\frac{\pi}{2} - \frac{\pi}{n}\right) = \frac{n}{4}a^2 \cot\frac{\pi}{n}$$

$$\therefore \phi = \frac{T \cdot na \cdot l}{4 \cdot \frac{n^2}{16} \cdot a^4 \cdot \cot^2\frac{\pi}{n} Gt} = \frac{4Tl}{G n a^3 \cot^2\frac{\pi}{n} t}$$

## 問題 4-5

1.




$a = 2^{cm}$　$h = 8^{cm}$　$t = 1.6^{mm}$

$T = 0.8^{mm}$

$\tau_a = 60\ MN/m^2 = 60 \times 10^2\ N/cm^2$

$G = 26\ GN/m^2 = 26 \times 10^5\ N/cm^2$

(1- Cont.)

$r/t = \frac{0.8}{1.6} = 0.5$ ∴ Stress Concent. factor $= 2$

$\therefore \tau_{max} = \frac{60}{2} = 30\ MN/m^2 = 30 \times 10^2\ N/cm^2$

$b = 2a + h - 2t = 12 - 0.32 = 11.68\ cm$

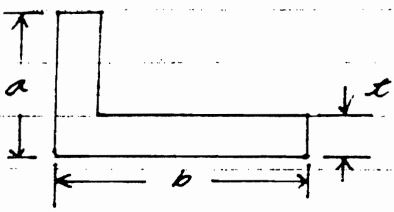
$c = t = 0.16\ ^{cm}$

$$\therefore T_w = \frac{\tau_{max} \cdot bc^2}{3} = \frac{30\times10^2 \times 11.68 \times 0.16^2}{3} = 299.0\ N\text{-}cm = 2.99\ N\text{-}m.$$

$$\therefore \theta = \frac{3T}{bc^3G} = \frac{3\times 299.0}{11.68 \times (0.16)^3 \times 26 \times 10^5}$$

$$= 0.007212\ rad/cm.$$

2.




$a = 8\ ^{cm}$, $b = 18\ ^{cm}$, $t = 1\ ^{cm}$, $r = 1\ ^{cm}$, $\tau_w = 840\ kgf/cm^2$, $G = 0.84 \times 10^6\ kgf/cm^2$.

$r/t = 1/1 = 1$ ∴ Stress Concent. factor $= 1.75$

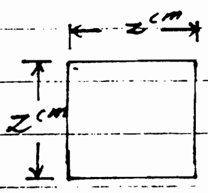
$$\tau_{max} = \frac{840}{1.75} = 480\ kgf/cm^2$$

$b = 8 + 18 - 1 = 25\ cm$, $c = t = 1\ cm$

$$\therefore T = \frac{480 \times 25 \times 1^2}{3} = 4000\ kgf\text{-}cm$$

$$\theta = \frac{3 \times 4000}{25 \times 1^3 \times 0.84 \times 10^6} = 0.000571\ rad/cm$$

3.




$b = c = 2\ ^{cm}$ $T = 120\ N\text{-}m = 120 \times 10^2\ N\text{-}cm$,

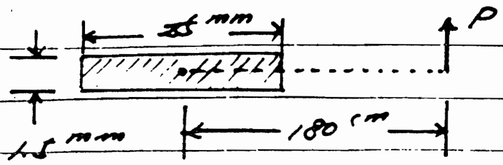
$G = 80\ GN/m^2 = 26 \times 10^5\ N/cm^2$.

$\therefore b/c = 1 \rightarrow \alpha = 0.208$ $\beta = 0.141$.

$$\therefore \tau_{max} = \frac{120 \times 10^2}{0.208 \times 2 \times 2^2} = 7211.5\ N/cm^2 = 72.115\ MN/m^2,$$

$$\therefore \theta = \frac{120 \times 10^2}{0.141 \times 2 \times 2^3 \times 80 \times 10^5} = 0.0006649\ rad/cm.$$

4




$b = 25^{mm}$ $c = 1.5^{mm}$ $l = 180^{cm}$

$T_0 = 20 kgf\text{-}cm$,

$$T_1 = \frac{\phi b c^3 G}{2l} = \frac{\pi \times 2.5 \times 0.15^3 \times 0.84 \times 10^6}{2 \times 3 \times 180}$$

$$= 20.62 \ kgf\text{-}cm$$

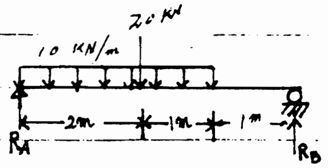
$$T = T_1 + T_0 = 40.62 \ kgf\text{-}cm$$

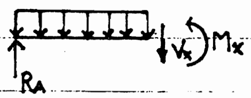
Required $P = \frac{40.62}{80} = 0.508 \ kgf$,

$$\tau_{max} = \frac{3 \times 40.62}{2.5 \times 0.15^2} = 2166 \ kgf/cm^2$$

## 問題 5.1

1.




$\Sigma F_y = 0$

$R_A + R_B = (10 \times 3) + 20$ (KN)

$\Sigma M_B = 0$

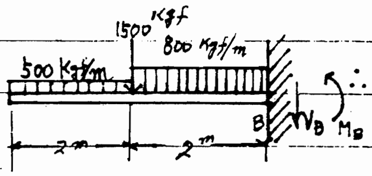
$20 \times 2 + (10 \times 3) \times 2.5 = R_A \times 4$ (KN-m)

$\therefore R_A = 28.75$ KN, $R_B = 21.25$ KN.

$M_{x=2m} = 28.75 \times 2 - (10 \times 2) \times 1 = 37.5$ KN-m,

$V_{x=2m} = 28.75 - (10 \times 2) = 8.75$ KN.

2.




$\Sigma F_y = 0$

$\therefore V_B = -500 \times 2 - 1500 - 800 \times 2$

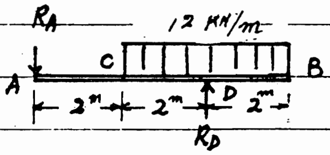
$= -4100$ kgf,

$\Sigma M = 0$

$\therefore M_B = -500 \times 2 \times 3 - 1500 \times 2 - 800 \times 2 \times 1$

$= -7600$ kgf-m.

3.




$\Sigma M_A = 0$ :

$4R_D - 48 \times 4 = 0$

$\therefore R_D = 48$ KN

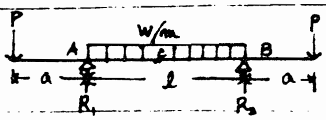
$\Sigma Y_i = 0$ ; $R_A + 48 - R_D = 0$

$\therefore R_A = 0$

At C : $M_C = 2R_A = 0$

At D : $M_D = 4R_A - 24 \times 1 = -24$ KN-m.

4.

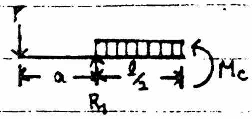



$$wl = P$$

$$\Sigma F = 0$$

$$R_1 + R_2 = 3P$$

그림으로 부터 대칭조건을 만족하므로

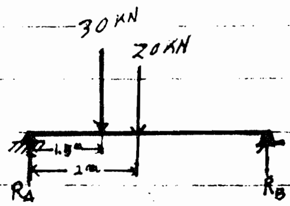



$$R_1 = R_2 = \frac{3}{2}P$$

$$\therefore\ M_c = -P\left(\frac{l}{2}+a\right) - \frac{P}{2}\cdot\frac{l}{4} + \frac{3}{2}P\cdot\frac{l}{2} = 0$$

$$\therefore\ a/l = 1/8$$

5.



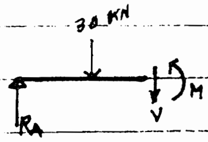

Equilibrium 으로부터

$$\therefore\ R_A = 28.75\ \text{KN},\quad R_B = 21.25\ \text{KN}.$$

$$\Sigma F_y = 0$$

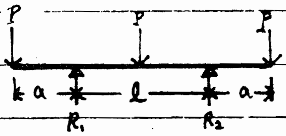
$$V = R_A - 30 = -1.25\ \text{KN}.$$

$$\Sigma M = 0$$




$$\therefore\ M = 28.75\times 2 - 30\times 0.5 = 42.5\ \text{KN-m}.$$

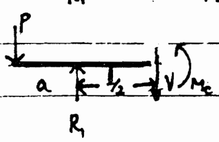
6.




$\Sigma F_y = 0$ , Symmetry condition

$$\therefore\ R_1 = R_2 = 3P/2$$

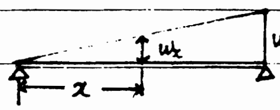
$$M_{\text{中央点}} = 0$$




$$\therefore\ -P\left(\frac{l}{2}+a\right) + \frac{3P}{2}\cdot\frac{l}{2} = 0$$

$$\therefore\ a/l = 1/4$$

7.




$$w_x = \frac{w_0}{l}x$$

$$\frac{dV_x}{dx} = -w_x = -\frac{w_0}{l}x$$

$$V_x = -\frac{w_0}{2l}x^2 + C_1$$

$$\frac{dM_x}{dx} = V_x = -\frac{w_0}{2l}x^2 + C_1$$

$$\therefore\ M_x = -\frac{w_0}{6l}x^3 + C_1 x + C_2$$

(5-1-7 Conti.)

Boundary conditions.

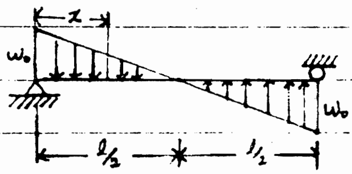
$M_x = 0$ at $x = 0, l$

$\therefore C_2 = 0$ , $C_1 = \frac{w_0 l}{6}$

$\therefore V_x = -\frac{w_0}{2l}x^2 + \frac{w_0 l}{6}$

$M_x = -\frac{w_0}{6l}x^3 + \frac{w_0 l}{6}x$

8.




$w_x = \frac{w_0}{l}(l - 2x)$

$\frac{dV_x}{dx} = -w_x = \frac{w_0}{l}(2x - l)$

$V_x = \frac{w_0}{l}x^2 - w_0 x + C_1$

$\frac{dM_x}{dx} = V_x$

$\therefore M_x = \frac{w_0}{3l}x^3 - \frac{w_0}{2}x^2 + C_1 x + C_2$

Boundary Conditions

$M_x = 0$, at $x = 0, l$ $\therefore C_2 = 0$

$C_1 = \frac{1}{6}w_0 l$

$\therefore V_x = \frac{w_0}{l}x^2 - w_0 x + \frac{1}{6}w_0 l$

$M_x = \frac{w_0}{3l}x^3 - \frac{w_0}{2}x^2 + \frac{1}{6}w_0 l x$

for maximum moment

$\therefore x = \frac{l \pm \sqrt{l^2 - 4l^2/6}}{2} = \frac{l}{2} \pm \frac{l}{2\sqrt{3}}$

$= \begin{cases} 0.7887\,l \\ 0.2113\,l \end{cases}$

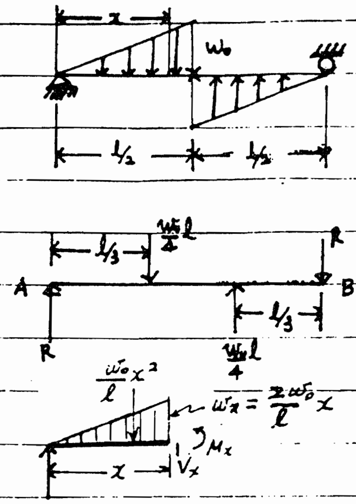
for maximum shear force : $w_x = 0$

$\therefore x = \frac{l}{2}$

$V_{max} = V_{x=l/2} = \frac{w_0}{l}\left(\frac{l}{2}\right)^2 - w_0\left(\frac{l}{2}\right) + \frac{1}{6}w_0 l$

$= -\frac{w_0 l}{12}$

9.




$\Sigma M_B = 0$

$\therefore\ Rl = \frac{w_0 l}{4}\left(\frac{2}{3}l - \frac{1}{3}l\right) = \frac{w_0 l}{4}\cdot\frac{l}{3}$

$\therefore\ R = \frac{w_0 l}{12}$

$\therefore\ V_x = \frac{w_0 l}{12} - \frac{w_0 x^2}{l}$

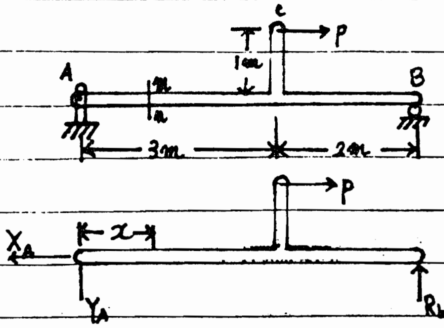
$V_x = 0 \ \rightarrow\ x = \frac{l}{2\sqrt{3}}$

中央点에서의 剪断力

$(V_x)_{x=l/2} = \frac{w_0 l}{12} - \frac{w_0}{l}\left(\frac{l}{2}\right)^2 = \left(\frac{1}{12} - \frac{1}{4}\right)w_0 l$

$= -\frac{w_0 l}{6}$

10.



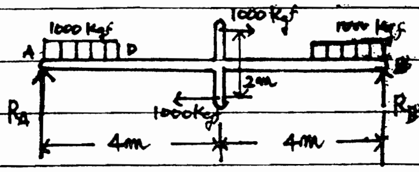

$\Sigma F_x = 0$

$\therefore\ X_A = P$, $\Sigma M_B = 0$, $\therefore\ 5Y_A = P$

$\therefore\ Y_A = \frac{P}{5} = 0.2P$

$\therefore \begin{cases} N_x = X_A = P \\ V_x = -Y_A = -0.2P \\ M_x = -Y_A \cdot x = -0.2Px \end{cases}$

11.



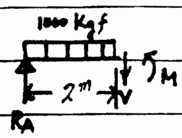

$\Sigma M_B = 0$

$8R_A + 1000\times2 - 1000\times7 - 1000\times1 = 0$

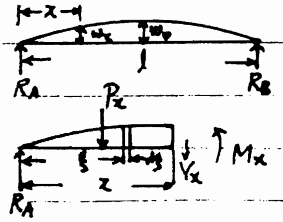
$\therefore\ R_A = 750\ \text{kgf}$

$M_D = 750\times2 - 1000\times1 = 500\ \text{kgf-m}$,

$V_D = 750 - 1000 = -250\ \text{kgf}$.



12.




$$w_x = -\frac{4w_0}{l^2}\left(x-\frac{l}{2}\right)^2 + w_0 = -\frac{4w_0}{l^2}x^2 + \frac{4w_0}{l}x$$

$$P = \int_0^l \left[-\frac{4w_0}{l^2}\left(x-\frac{l}{2}\right)^2 + w_0\right]dx$$

$$= \int_0^l \left[-\frac{4w_0}{l^2}x^2 + \frac{4w_0}{l}x\right]dx$$

$$= \left[-\frac{4w_0}{3l^2}x^2 + \frac{2w_0}{l}x^2\right]_0^l$$

$$= \frac{2}{3}w_0 l$$

from symmetry; $\quad R_A = R_B = \frac{P}{2} = \frac{1}{3}w_0 l$

$$\bar{x} = \frac{\int_0^x w_\xi\, d\xi \cdot \xi}{\int_0^x w_\xi\, d\xi} = \frac{\int_0^x \left(-\frac{4w_0}{l^2}\xi^3 + \frac{4w_0}{l^2}\xi^2\right)d\xi}{\int_0^x \left(-\frac{4w_0}{l^2}\xi^2 + \frac{4w_0}{l}\xi\right)d\xi}$$

$$= \frac{\left[-\frac{w_0}{l^2}x^4 + \frac{4w_0}{3l}x^3\right]}{\left[-\frac{4w_0}{3l^2}x^3 + \frac{2w_0}{l}x^2\right]}$$

$$= \frac{-\frac{w_0}{l^2}x^3\left(x-\frac{4}{3}l\right)}{-\frac{4w_0}{3l^2}x^2\left(x-\frac{6}{4}l\right)}$$

$$= \frac{3x\left(x-\frac{4}{3}l\right)}{4\cdot\left(x-\frac{6}{4}l\right)}$$

$\Sigma M_x = 0$ ;

$$M_x = R_A \cdot x - P_x(x-\bar{x})$$

$$= \frac{1}{3}w_0 l x + \frac{4w_0}{3l^2}x^2\left(x-\frac{6}{4}l\right)\left\{x - \frac{3x\left(x-\frac{4}{3}l\right)}{4\left(x-\frac{6}{4}l\right)}\right\}$$

$$= \frac{w_0 l x}{3} + \frac{4w_0}{3l^2}x^2\left(x-\frac{6}{4}l\right)\frac{x(x-2l)}{4\left(x-\frac{6}{4}l\right)}$$

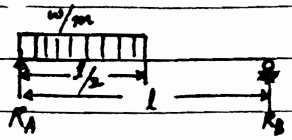
$$= \frac{w_0 x}{3l^2}\left\{x^2(x-2l) + l^3\right\}$$

$$V_x = R_A - P_x = \frac{w_0 l}{3} + \frac{4w_0}{3l^2}x^2\left(x-\frac{3}{2}l\right)$$

$$= \frac{4w_0}{3l^2}\left(x^3 - \frac{3}{2}lx^2 + \frac{1}{4}l^3\right)$$

$$\therefore \quad M_{x=\frac{l}{2}} = \frac{5}{48}w_0 l^2 , \qquad V_{x=\frac{l}{2}} = 0$$

13.



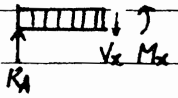

$$\Sigma M_B = 0$$

$$\therefore R_A l = \frac{wl}{2} \times \frac{3}{4} l$$

$$\therefore R_A = \frac{3}{8} wl$$

$$\Sigma Y = 0 \ \therefore R_A + R_B = \frac{wl}{2}$$

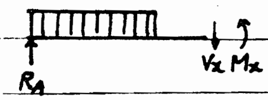
$$\therefore R_B = \frac{wl}{2} - \frac{3}{8} wl = \frac{wl}{8}$$




① if $x < \frac{l}{2}$

$$M_x = R_A \cdot x - wx \cdot \frac{x}{2}$$

$$= \frac{3}{8} wlx - \frac{wx^2}{2}$$




② if $x > \frac{l}{2}$

$$M_x = R_A \cdot x - \frac{wl}{2} \cdot \left(x - \frac{l}{4}\right)$$

$$= \frac{3wlx}{8} - \frac{wlx}{2} + \frac{wl^2}{8}$$

$$= -\frac{wlx}{8} + \frac{wl^2}{8}$$

①의 경우

$$\frac{dM_x}{dx} = \frac{3}{8} wl - wx = 0 \quad \therefore x = \frac{3l}{8}$$

$$\therefore M_{max} = \frac{3}{8} wl \cdot \frac{3}{8} l - \frac{w}{2} \left( \frac{9l^2}{64} \right)$$

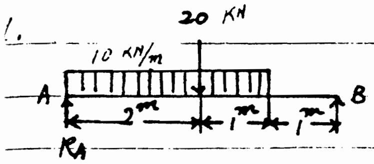
$$= \left( \frac{9}{64} - \frac{9}{128} \right) wl^2$$

$$= \frac{9}{128} wl^2$$

②의 경우 $\frac{dM_x}{dx} = -\frac{wl}{8} \neq 0$ $\therefore$ Maximum 이 없.

## 問題 5.2

1.

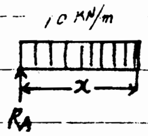



$$\Sigma M_B = 0$$

$$R_A \times 4 - 10 \times 3 \times 2.5 - 20 \times 2 = 0$$

$$\therefore \ R_A = 28.75 \text{ KN}.$$

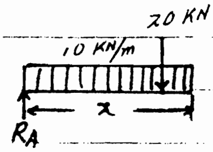
$$\Sigma Y = 0 \ \therefore R_B = 50 - 28.75 = 21.25 \text{ KN}.$$




i) $x < 2$

$$V_x = 28.75 - 10x$$

$$M_x = 28.75x - 5x^2$$



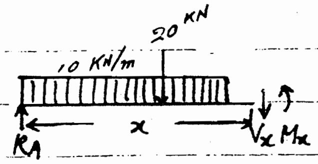

ii) $2 \leq x < 3$

$$V_x = 28.75 - 10x - 20$$

$$= 8.75 - 10x$$

$$M_x = 28.75x - 5x^2 - 20(x-2)$$

$$= 40 + 8.75x - 5x^2$$

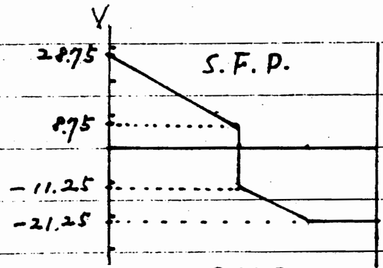


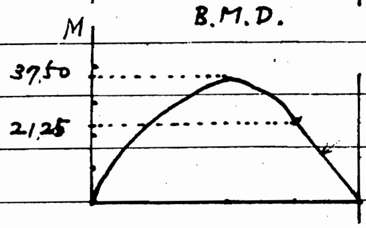

iii) $3 \leq x$

$$V_x = 28.75 - 30 - 20 = -21.25$$

$$M_x = 28.75x - 30(x-1.5) - 20(x-2)$$

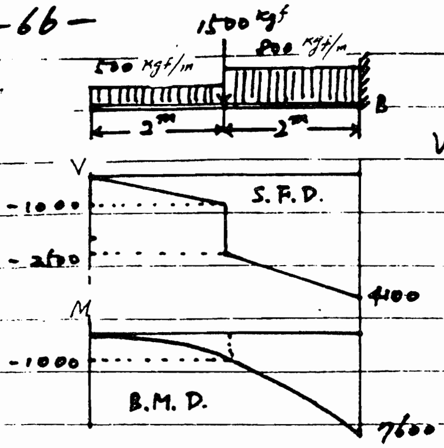
$$= -21.25x + 85$$



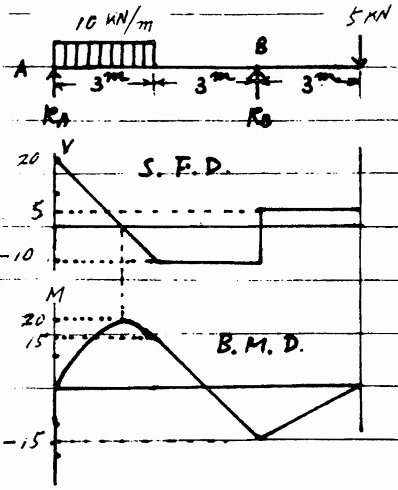

2直線

2.




$V_B = -4100^{kgf}$ $\quad$ $M_B = -7600^{kgf\text{-}m}$

3.




$\Sigma M_B = 0$

$R_A \times 6 + 5 \times 3 - 30 \times 4.5 = 0$

$R_A = 20$ KN.

$\Sigma Y = 0$

$\therefore R_A + R_B = 30 + 5$ $\quad$ $\therefore R_B = 15$ KN.

$0 \le x \le 3$

$V_x = 20 - 10x$ (KN)

$M_x = 20x - 5x^2$ (KN-m)

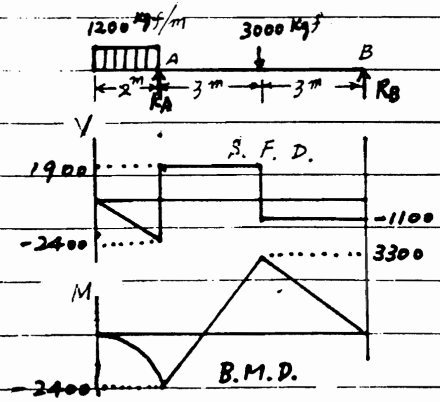
$3 \le x \le 6$

$V_x = 20 - 30 = -10$ (KN)

$M_x = 20x - 30(x - 1.5)$

$= -10x + 45$ (KN-m)

4.




$\Sigma M_B = 0$

$6R_A - 1200 \times 2 \times 7 - 3000 \times 3 = 0$

$\therefore R_A = 4300^{kgf}$, $\quad$ $\therefore R_B = 1100^{kgf}$

$0 \le x \le 2^m$

$V_x = -1200x$, $V_A = -2400$ kgf,

$M_x = -600x^2$, $M_A = -2400^{kgf\text{-}m}$

(5-2-4 conti.)

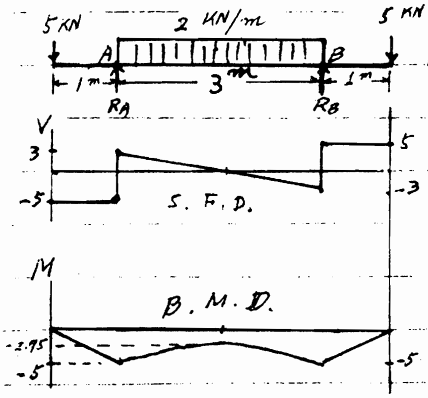
$2 \leq x \leq 5$

$V_x = 4300 - 2400 = 1900^{kgf}$,

$M_x = -2400(x-1) + 4300(x-2)$

$= 1900x - 6200$, $\therefore$ $M_D = 3300^{kgf\text{-}m}$.

5.




$R_A = R_B = \dfrac{5 + 2 \times 3 + 5}{2} = 8\ KN$

$0 \leq x \leq 1$

$V_x = -5$, $M_x = -5x$.

$1 \leq x \leq 4$

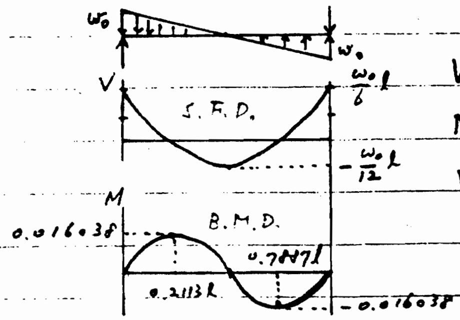
$V_x = -5 + 8 - 2(x-1) = 5 - 2x$

$M_x = -5x + 8(x-1) - 2(x-1) \cdot \left(\dfrac{x-1}{2}\right)$

$= -9 + 5x - x^2$

$M_{x=2.5} = -2.75\ KN\text{-}m$

6.




$V_x = \dfrac{w_0}{l} \cdot x^2 - w_0 x + \dfrac{1}{6} w_0 l$

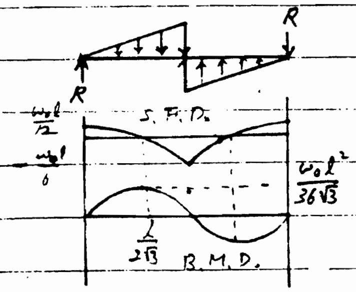
$M_x = \dfrac{w_0}{3l} x^3 - \dfrac{w_0}{2} x^2 + \dfrac{1}{6} w_0 l x$

$V_{min} = V_{x=l/2} = -\dfrac{w_0 l}{12}$, $V_{max} = V_{x=0,\,l} = \dfrac{w_0 l}{6}$

$M_{\substack{max \\ min}} = M_{x=\substack{0.7887l \\ 0.2113l}} = \mp 0.016038\, w_0 l^2$

$M_{x=l/2} = 0$

7.




$R = \dfrac{w_0 l}{12}$

$V_x = \dfrac{w_0 l}{12} - \dfrac{w_0 x^2}{l}$

$x = \dfrac{l}{2\sqrt{3}}$ 일 때, $V = 0$

$x = l/2$ 일 때, $V = -\dfrac{w_0 l}{6}$

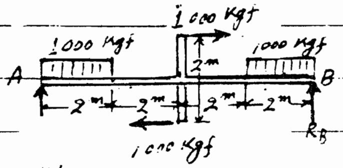
$M_x = \dfrac{w_0 l}{12} x - \dfrac{w_0 x^3}{3l}$

(5-2-7 conti.)

$$M_{max} = \frac{w_0 l}{12}\left(\frac{l}{2\sqrt{3}}\right) - \frac{w_0}{3l}\left(\frac{l^3}{24\sqrt{3}}\right)$$

$$= \frac{w_0 l^2}{3\sqrt{3}}\left(\frac{1}{8} - \frac{1}{24}\right) = \frac{w_0 l^2}{36\sqrt{3}}$$

8.

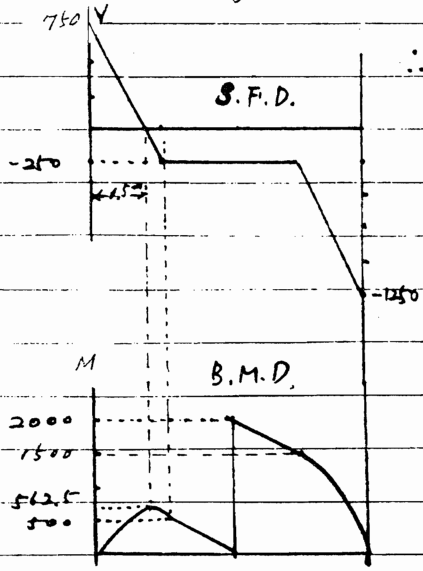



$\Sigma M_B = 0$ , $8R_A + 2000 - 7000 - 1000 = 0$

$R_A = 750^{kgf}$

$\Sigma M_A = 0$ , $8R_B - 2000 - 1000 \times 7 - 1000 = 0$

$\therefore R_B = 1250^{kg}$




$0 \leq x \leq 2^m$

$V_x = 750 - 500x$ , $V_{x=2} = -250^{kgf}$, $V_{x=1.5} = 0$,

$M_x = 750x - 250x^2$ , $M_{x=2} = 500^{kgf\text{-}m}$,

$M_{x=1.} = 562.5^{kgf\text{-}m}$.

$2 \leq x \leq 4$

$V_x = 750 - 1000 = -250$

$M_x = 750x - 1000(x-1) = -250x + 1000$

$M_{x=4} = 0$

$4 \leq x \leq 6$

$V_x = 750 - 1000 = -250$

$M_x = 750x - 1000(x-1) + 2000 = -250x + 3000$

$M_{x=4} = 2000^{kgf\text{-}m}$, $M_{x=6} = 1500^{kgf\text{-}m}$

$6 \leq x \leq 8$

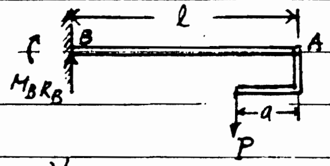
$V_x = -250 - 500(x-6) = -500x + 2750$

$V_{x=6} = -250^{kgf}$, $V_{x=8} = -1250$ kgf

$M_x = -250x + 3000 - 250(x-6)^2$

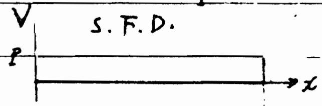
$= -250x^2 + 2750x - 6000$

$M_{x=8} = 0$




$a = l/3$

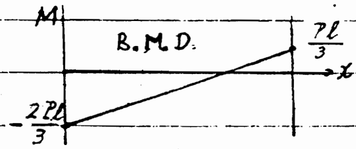
$\Sigma Y = 0 \quad \therefore R_B = P$




$\Sigma M_A = 0:$

$M_B - Pa + Pl = 0 \quad \therefore M_B = -\frac{2}{3}Pl.$

$0 \le x \le l$

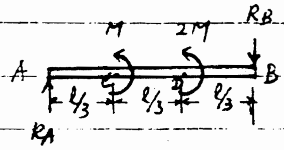



$V_x = R_B = P$

$M_x = M_B + R_B x = -\frac{2}{3}Pl + Px$

$M_{x=0} = -\frac{2}{3}Pl, \quad M_{x=l} = \frac{1}{3}Pl$

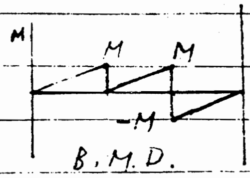
c.




$\Sigma M_B = 0:$

$R_A l = 3M \quad \therefore R_A = \frac{3M}{l}$ up

$\therefore R_B = \frac{3M}{l}$ down



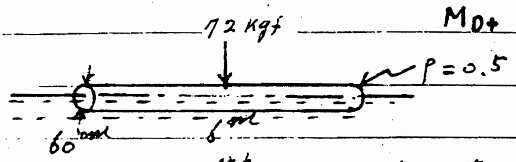

$M_{C-} = \frac{3M}{l} \times \frac{l}{3} = M$

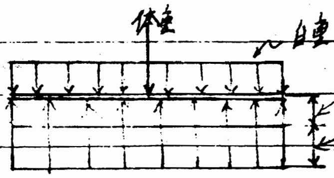
$M_{C+} = \frac{3M}{l} \times \frac{l}{3} - M = 0$

$M_{D-} = \frac{3M}{l} \times \frac{2l}{3} - M = M$

$M_{D+} = \frac{3M}{l} \times \frac{2l}{3} - 3M = -M$

11.



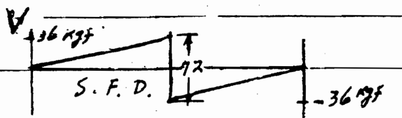




自重 = $45\pi$ kgf/m

自重에 対応하는 浮力 = $45\pi$ kgf/m

体重에 〃 〃 = 12 kgf/m

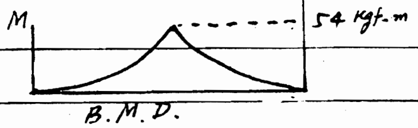
$0 \le x \le 3$




$V_x = 12x, \quad M_x = 12x \cdot \frac{x}{2} = 6x^2$

$M_{x=3} = 54$ kgf-m

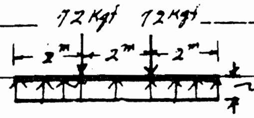
$3 \le x \le 6$



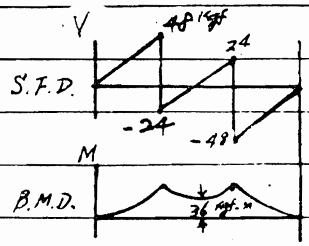

$V_x = 12x - 72, \quad M_x = 6x^2 - 72(x-3)$

$M_{x=3} = 54$ kgf-m

12.




$\frac{72\times 2}{6} = 24$ Kgf/m




$0 \le x \le 2$

$V_x = 24x,\quad M_x = 12x^2$

$V_{x=2} = 48$ Kgf, $M_{x=2} = 48$ Kgf·m.

$2 \le x \le 4$

$V_x = 24x - 72,\quad M_x = 12x^2 - 72(x-2)$

$V_{x=2} = -24,\ V_{x=4} = 24,\quad M_{x=2} = 48\quad M_{x=4} = 48$

$M_{x=3} = 36$ Kgf·m.

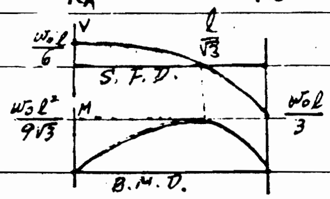
$4 \le x \le 6$

$V_x = 24x - 144,\quad M_x = 12x^2 - 72(x-2) - 72(x-4)$

$V_{x=4} = -48,\ V_{x=6} = 0,\ M_{x=4} = 48,\ M_{x=6} = 0$

13.


from 例題 § 5-1.1




$V_x = \frac{w_0 l}{6} - \frac{w_0 x^2}{2l},\quad R_A = \frac{w_0 l}{6},\ R_B = \frac{w_0 l}{3}$

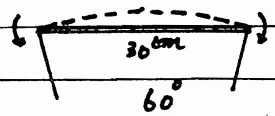
$M_x = \frac{w_0 l x}{6} - \frac{w_0 x^3}{6l}$

$V_x = 0\ ;\quad x = l/\sqrt{3}$

$\therefore\ M_{max} = \frac{w_0 l^2}{6\sqrt{3}} - \frac{w_0 l^2}{18\sqrt{3}} = \frac{w_0 l^2}{9\sqrt{3}}$

## 問題 5·3

1.




$\theta = 60^\circ = \frac{\pi}{3}$ Rad.

$I = \frac{3\times(0.06)^3}{12}$

$= 5.4\times 10^{-5}$ cm$^4$

$E = 200$ GN/m$^2$

$= 200\times 10^5$ N/cm$^2$

$\rho\cdot\theta = 30$ cm

$\rho = 30\cdot\frac{3}{\pi} = 28.6479$ cm $= 0.286479$ m.

$\frac{1}{\rho} = \frac{M}{EI}$

$\therefore\ M = \frac{EI}{\rho} = \frac{200\times 10^5\times 5.4\times 10^{-5}}{28.6479}$

$= 37.699$ N·cm

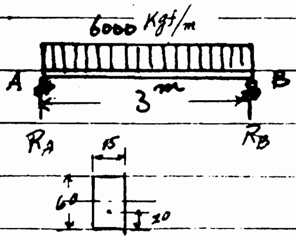
$= 0.37699$ N·m.

$\sigma = \frac{M\left(\frac{t}{2}\right)}{I} = \frac{37.699\times 0.03}{5.4\times 10^{-5}}$

$= 0.20944\times 10^5$ N/cm$^2$

$= 209.44$ MN/m$^2$.

2.




断面 $15^{cm} \times 60^{cm}$

$R_A = R_B = \dfrac{6000 \times 3}{2} = 9000^{kgf}$

$M_x = 9000x - 3000x^2$

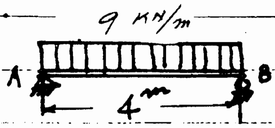
$\therefore M_{x=0.9} = 8100 - 2430 = 5670^{kgf \cdot m}$

$I = \dfrac{bh^3}{12} = \dfrac{15(60)^3}{12} = 270000\ cm^4$

$\therefore \sigma = \dfrac{My}{I} = \dfrac{5670 \times 10^2 \times (30-20)}{270000}$

$= 21\ Kgf/cm^2$

3.




$b = h/2$

$\sigma_w = 8\ MN/m^2$

$R_A = R_B = \dfrac{9 \times 4}{2} = 18\ KN$

$\therefore M_x = 18x - 4.5x^2$

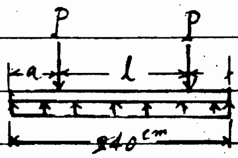
$M_{max} = M_{x=2} = 18\ KN\text{-}m$

$\sigma_w = \dfrac{My}{I} = \dfrac{M \cdot \frac{h}{2}}{\frac{bh^3}{12}} = \dfrac{12Mh}{h^4} = 8000\ KN/m^2$

$\therefore h^3 = \dfrac{12 \times 18}{8000} = 0.027\ m^3$

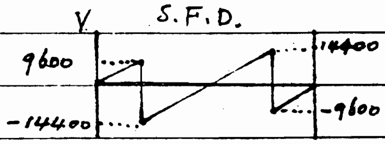
$\therefore h = 0.3\ m = 30\ cm.$

4.




$b = 30^{cm}$, $h = 26^{cm}$, $l = 144^{cm}$

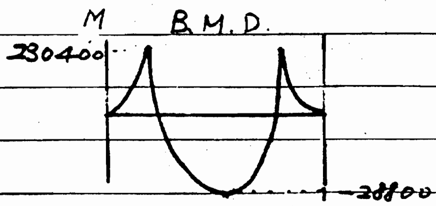
$a = 48^{cm}$, $P = 24000^{kgf}$




$w = \dfrac{24000 \times 2}{240} = 200^{kgf/cm}$

$0 \leq x \leq 48$ ; $V_x = +200x$

$V_{x=48^-} = 9600^{kgf}$




$M_x = 100x^2$

$M_{x=48} = 230400^{kgf \cdot cm}$

$48 \leq x \leq 192$ ; $V_x = 200x - 24000$

$V_{x=48^+} = -14400^{kgf}$

(5-3-4 conti.)

$$V_{x=192} = 14400\ \text{kgf}$$

$$M_x = 100x^2 - 24000(x-48) = 100x^2 - 24000x + 1152000$$

$$M_{x=48} = 230400\ \text{kgf-cm}$$

$$M_{x=192} = 230400\ \text{kgf-cm}$$

$$M_{x=120} = -288000\ \text{kgf-cm}$$

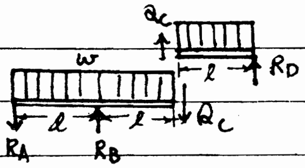
$$\therefore M_{max} = -288000\ \text{kgf-cm}$$

$$I = \frac{bh^3}{12} = \frac{30 \times (26)^3}{12} = 43940\ \text{cm}^4$$

$$h/2 = 13\ \text{cm}$$

$$\therefore \sigma_{max} = \frac{288000 \times 13}{43940} = 85.2071\ \text{kgf/cm}^2.$$

5.



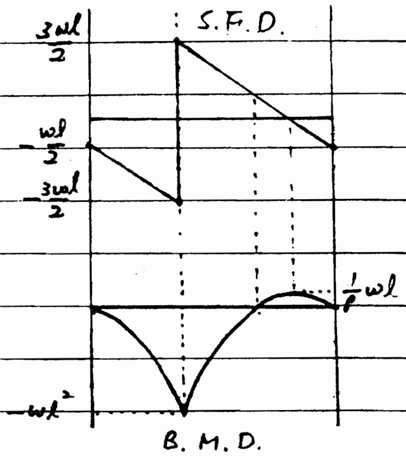

$$-R_A + R_B + R_D = 3wl$$

$$R_D \cdot l - \frac{wl^2}{2} = 0$$

$$\therefore R_D = \frac{wl}{2} \qquad \therefore Q_C = \frac{wl}{2}$$

$$R_B \cdot l - \frac{wl}{2} \cdot 2l - 2wl^2 = 0$$

$$\therefore R_B = 3wl \qquad \therefore R_A = \frac{wl}{2}$$




$0 \leq x \leq l$

$$V_x = -\frac{wl}{2} - wx\ , \quad M_x = -\frac{wl}{2}x - \frac{w}{2}x^2$$

$$x = l\ ; \quad V = -\frac{3}{2}wl\ , \quad M = -wl^2$$

$l \leq x \leq 3l$

$$V_x = -R_A + R_B - wx = -\frac{w}{2}l + 3wl - wx$$

$$= -wx + \frac{5}{2}wl$$

$$M_x = -\frac{wl}{2}x + 3wl(x-l) - \frac{w}{2}x^2$$

$$= -\frac{w}{2}x^2 + \frac{5}{2}wlx - 3wl^2$$

$$x = l\ ; \quad V = \frac{3}{2}wl\ , \quad M = -wl^2$$

$$x = 2l\ ; \quad V = \frac{1}{2}wl\ , \quad M = 0$$

(5-3-5 Conti)

$$x = \frac{5}{2}l \ ; \quad V = 0 \ , \ M = \frac{1}{8}wl^2$$

$$\therefore \ M_{max} = wl^2$$

Numerical data: $Z = 86.0\ cm^3$, $\sigma_w = 100\ MN/m^2 = 10\ KN/cm^2$

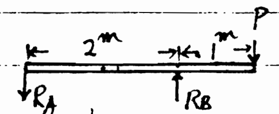
$l = 200\ cm$

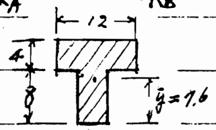
$$\sigma_w = \frac{M_{max}}{Z} = \frac{wl^2}{Z}$$

$$\therefore \quad w = \frac{\sigma_w \cdot Z}{l^2} = \frac{10 \times 86.0}{(200)^2} = 0.0215\ KN/cm$$

$$= 2.15\ KN/m$$

6.




$$\bar{y} = \frac{12 \times 4 \times 10 + 8 \times 4 \times 4}{12 \times 4 + 8 \times 4}$$

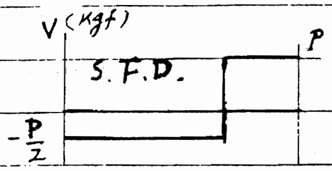
$$= \frac{608}{80} = 7.6\ cm$$

$$I = \left(\frac{12 \times 4^3}{12} + 12 \times 4 \times (2.4)^2\right)$$

$$+ \left(\frac{4 \times 8^3}{12} + 8 \times 4 \times (3.6)^2\right)$$

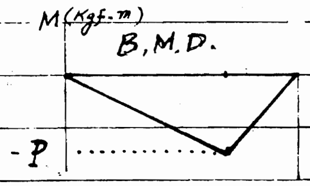
$$= 64 + 276.48 + 170.67 + 414.72$$

$$= 925.87\ cm^4$$




$$Z_1 = \frac{925.87}{4.4} = 210.42\ cm^3$$

$$Z_2 = \frac{925.87}{7.6} = 121.82\ cm^3$$




$\Sigma M_B = 0$ , $\therefore 2R_A = P$ $\quad \therefore R_A = P/2$ down

$R_B = 3/2\,P$ up

$$\therefore \ M_{max} = -P\ kgf\text{-}m$$

$(\sigma_t)_w = 420\ kgf/cm^2$ ; $(\sigma_c)_w = 700\ kgf/cm^2$

$$(\sigma_t)_w = \frac{P \times 100}{Z_1} \quad \therefore P = \frac{\sigma_t Z_1}{100} = \frac{420 \times 210.42}{100}$$

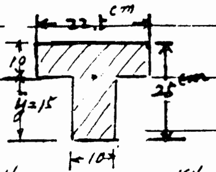
$$= 883.76\ kgf$$

(5-3-6 conti)

$(\sigma_c)_w = \frac{P\times 100}{Z_2}$ $\quad\therefore P = \frac{\sigma_c Z_2}{100} = \frac{700\times 121.85}{100} = 852.94^{kgf}$

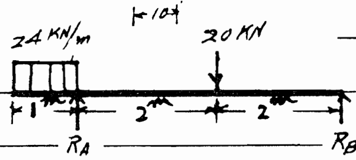
$\therefore P = 852.94^{kg}$ ← Ans.

7.




$\bar{y} = \frac{22.5\times10\times20+15\times10\times7.5}{22.5\times10+15\times10} = 15^{cm}$

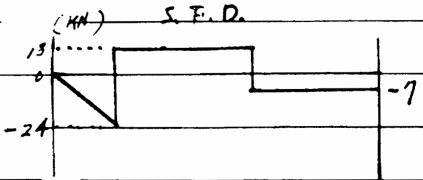
$I = \frac{22.5\times10^3}{12} + 22.5\times10\times5^2 + \frac{10\times15^3}{12} + 10\times15\times7.5^2 = 18750\ cm^4$

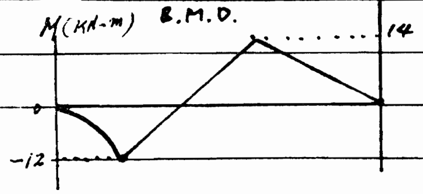



$\sum M_B = 0\ ;$

$-24\times1\times4.5 + R_A\times4 - 20\times2 = 0$

$\therefore R_A = 37\ KN,\quad \therefore R_B = 24+20-37 = 7\ KN.$




$0\leq x\leq 1\ ;\ V_x = -24x,\ M_x = -12x^2,$

$x=1\ ;\ V = -24^{KN},\ M = -12^{KN\text{-}m}$

$1\leq x\leq 3\ ;\ V_x = -24+R_A = 13\ KN,$

$M_x = -24(x-0.5)+R_A(x-1)$

$= 13x - 25$

$x=1;\ M = -12\ KN\text{-}m$

$x=3\ \ M = 14\ KN\text{-}m$

$3\leq x\leq 5\ ;\ V_x = -24+37-20$

$= -7\ KN,$

$M_x = -24(x-0.5)+37(x-1)$

$-20(x-3)$

$\therefore M_x = -7x+35$

$x=5,\ V = -7\ KN,\ M = 0.$

$x=3,\ V = -7^{KN},\ M = 14\ KN\text{-}m.$

$Z_1 = \frac{18950}{10} = 1895\ cm^3$

$Z_2 = \frac{18750}{15} = 1250\ cm^3$

(5-3-7 conti.)

At point A, $M = -12$ KN-m

$$\sigma_t = \frac{12 \times 100}{1875} = 0.64\ KN/cm^2 = 6.4\ MN/m^2$$

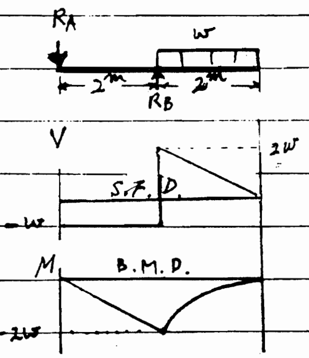
$$\sigma_c = \frac{12 \times 100}{1250} = 0.96\ KN/cm^2 = 9.6\ MN/m^2 \leftarrow Ans.$$

At point c, $M = 14$ KN-m

$$\sigma_t = \frac{14 \times 100}{1250} = 1.12\ KN/cm^2 = 11.2\ MN/m^2 \leftarrow Ans.$$

$$\sigma_c = \frac{14 \times 100}{1875} = 0.7467\ KN/cm^2 = 7.467\ MN/m^2$$

8.




$2R_B = 3 \times 2w \quad \therefore R_B = 3w$ (up)

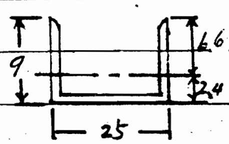
$\therefore R_A = w$ (down)

$M_{max} = -2w$ kgf-m (at B)

$\sigma_t = 1400$ kgf/cm², $\sigma_c = 1100$ kgf/cm²

U-250 × 90 × 9 × 13

$I_y = 294$ cm⁴, $Z_1 = 44.55$ cm³, $Z_2 = 122.5$ cm³

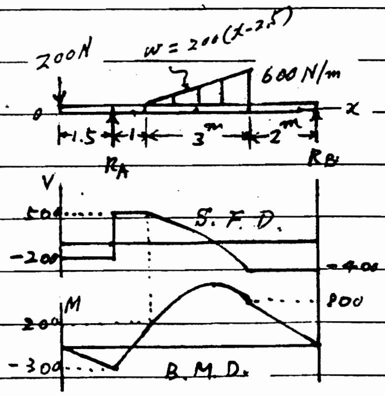



~~Tension side;~~ $1400 = \frac{2w \times 100}{44.55} \quad \therefore w = 311.85$ kgf/m

compression side; $1100 = \frac{2w \times 100}{122.5} \quad \therefore w = 673.75$ kgf/m

$\therefore w = 312$ kgf/m.

9.




$\sum M_B = 0$ :

$-200 \times 7.5 + 6R_A - \frac{1}{2} \times 3 \times 600 \times 3 = 0$

$\therefore R_A = 700$ N, $R_B = 400$ N.

$2.5 \leq x \leq 5.5$,

$V_x = -200 + 700 - \frac{1}{2} \times 200 (x-2.5)^2$

$= 500 - 100(x-2.5)^2$

(5-3-9 conti.)

$$M_x = -200x + 700(x-1.5) - \frac{100}{3}(x-2.5)^3$$

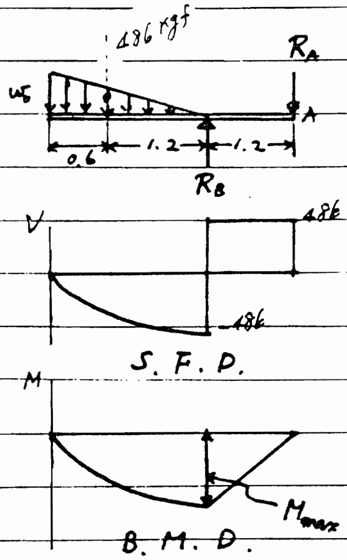
$$V_x = 0 \; ; \quad x = \frac{5+2\sqrt{5}}{2} = 4.736$$

$$\therefore M_{max} = -200\times4.736 + 700\times3.236 - \frac{100}{3}\times(2.236)^3$$

$$= 945.4 \text{ N-m}.$$

$$\therefore Z = \frac{M_{max}}{\sigma_w} = \frac{945.4}{8.4\times10^6} = 112.5\times10^{-6}\,m^3 = 112.5\,cm^3$$

10.




水底에서의 水圧 :

$$p_0 = 1.8\times10^6 = 1800 \text{ kgf/m}^2$$

木板의 下端에서의 荷重의 세기

$$w_0 = 0.3\times1800 = 540 \text{ kgf/m}$$

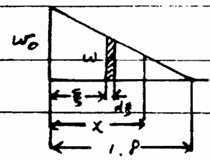
$$\sum M_A = 0 : \frac{540\times1.8}{2}\times2.4 + 1.2R_B = 0$$

$$\therefore R_B = 972 \text{ kgf}$$

Bending Moment

$0 < x < 1.8$

$$M_x = \int_0^x w\,d\xi\cdot\xi$$




$1.8 < x < 3$ :

$$M_x = -486(x-0.6) + 972(x-1.8)$$

$$= 486x - 1458$$

$$M_{max} = M_{x=1.8} = 583.2 \text{ kgf-m}$$

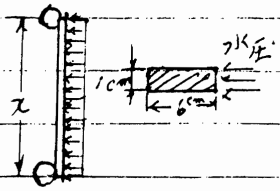
Section Modulus:

$$Z = \frac{bh^2}{6} = \frac{30\times8^2}{6} = 320 \text{ cm}^3$$

Max. Stress:

$$\sigma_{max} = \frac{M_{max}}{Z} = \frac{583.2\times100}{320} = 182 \text{ kgf/cm}^2$$

i) 木板의 強度로 부터 要求되는 杭木의 间隔 x




$w = 28 \times 0.01 = 0.28\ KN/m = 280\ N/m$

$M_{max} = \frac{wl^2}{8} = \frac{280x^2}{8} = 35x^2\ N\text{-}m$

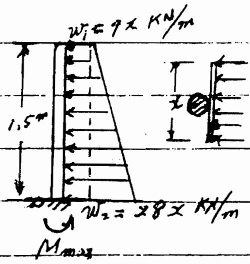
$Z = \frac{1 \times 6^2}{6} = 6\ cm^3 = 6 \times 10^{-6}\ m^3$

$\sigma = \frac{M}{Z}$ ; $(\sigma_w = 8.4\ MN/m^2 = 8.4 \times 10^6\ N/m^2)$

$8.4 \times 10^6 = \frac{35x^2}{6 \times 10^{-6}}$ $\therefore x^2 = \frac{84 \times 6}{350} = 1.44$

$\therefore x = 1.2^m$

ii) 杭木의 強度로 부터 要求되는 杭木의 间隔 x




$M_{max} = 7x \times 1.5 \times \frac{1.5}{2} + \frac{21x \times 1.5}{2} \times \frac{1.5}{3}$

$= 15.75x\ KN\text{-}m$

$\sigma_w = 8.4\ MN/m^2 = 8.4 \times 10^3\ KN/m^2$

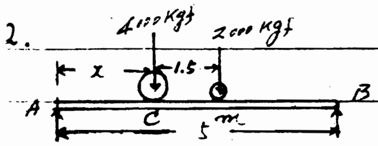
$Z = \frac{\pi r^3}{4} = \frac{\pi \cdot 15^3}{4} = 843.75\pi\ cm^3 = 843.75\pi \times 10^{-6}\ m^3$

$\sigma = \frac{M}{Z}$ ; $8.4 \times 10^3 = \frac{15.75x}{843.75\pi \times 10^{-6}}$

$\therefore x = \frac{8.4 \times 843.75\pi}{15.75}$

i) ii) 中 작은 값을 択하면, $x = 1.2^m$ 로 해야 한다.

2.




$\sigma_w = 1100\ kgf/cm^2$

$5R_A - 4000(5-x) - 2000(5-x-1.5)$

$\therefore R_A = 5400 - 1200x$

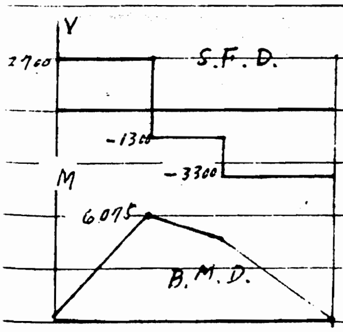
$M_c = R_A x = 5400x - 1200x^2$

$\frac{dM_c}{dx} = 5400 - 2400x$, $x = 2.25^m$

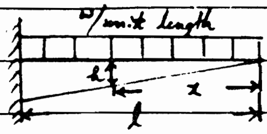
$\therefore (M_c)_{max} = 5400 \times 2.25 - 1200 \times (2.25)^2 = 6075\ kgf\text{-}m$

$\therefore Z = \frac{M_{max}}{\sigma_w} = \frac{6075 \times 100}{1100} = 552.3\ cm^3$

$\therefore$ 한개의 보의 단면 계수 ; $Z/2 = 276.1364\ cm^3$



13.




$M_x = \frac{wx^2}{2}$ , $Z_x = \frac{bh^2}{6}$

$\sigma_x = \frac{M_x}{Z_x} = \frac{6wx^2}{2bh^2} = \frac{3w}{b}\left(\frac{x}{h}\right)^2$

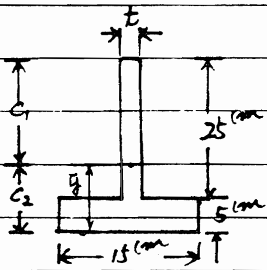
if $h = \alpha x$,

$\sigma = \frac{3w}{\alpha^2 b} = \text{const}$

∴ 直線的으로 変해야 함.

## 問題 5·4

1.




$\sigma_t = 28\ MN/m^2$, $\sigma_c = 56\ MN/m^2$

$\bar{y} = \frac{15\times5\times2.5 + 25t\times17.5}{15\times5 + 25t} = \frac{437.5t + 187.5}{25t + 75}$

$C_1 = 30 - \bar{y}$, $C_2 = \bar{y}$

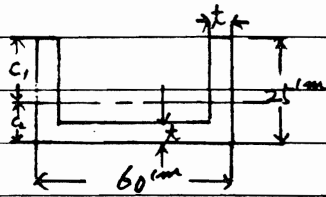
$\sigma_t = \frac{MC_2}{I}$, $\sigma_c = \frac{MC_1}{I}$

$\therefore \frac{C_1}{C_2} = \frac{\sigma_c}{\sigma_t} = \frac{56}{28} = \frac{2}{1}$

$C_1 + C_2 = 30 \quad \therefore C_1 = 20,\ C_2 = 10$

$\therefore C_2 = 10 = \bar{y} = \frac{437.5t + 187.5}{25t + 75} \quad \therefore t = 3^{cm}$

2.




$\sigma_t : \sigma_c = 3 : 7$

$C_2 = \frac{60t\cdot\frac{t}{2} + 2(25-t)t\cdot\left(\frac{25-t}{2} + t\right)}{60t + 2(25-t)t}$

$= \frac{-3t^3 + 30t^2 + 625t}{-2t^2 + 110t}$

$= \frac{-t^2 + 30t + 625}{-2t + 110}$

$\sigma_t = \frac{MC_2}{I}$ , $\sigma_c = \frac{MC_1}{I}$

$\therefore \sigma_t/\sigma_c = \frac{C_2}{C_1} = \frac{3}{7}$

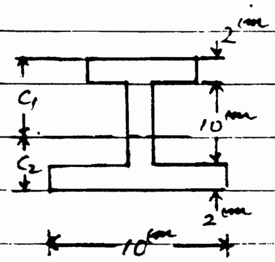
(5-4-2 conti.)

$$C_1 + C_2 = 25 \quad \therefore C_1 = 17.5^{cm},\ C_2 = 7.5^{cm}$$

$$\therefore \frac{-t^2+30t+625}{-2t+110} = 7.5 \quad \therefore t^2 - 45 + 200 = 0$$

$$(t-40)(t-5) = 0$$

$$\therefore t = 5^{cm}$$

3. 

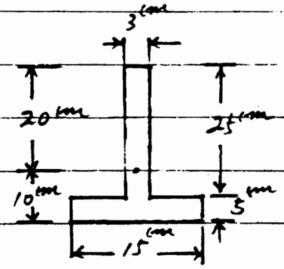

$$\sigma_1 : \sigma_2 = 4 : 3$$

$$\therefore C_1/C_2 = 4/3 \quad C_1 + C_2 = 14$$

$$\therefore C_1 = 8^{cm},\ C_2 = 6^{cm}$$

$$C_2 = \frac{10\times2\times1 + 10\times2\times7 + 2b\times13}{10\times2 + 10\times2 + 2b} = 6$$

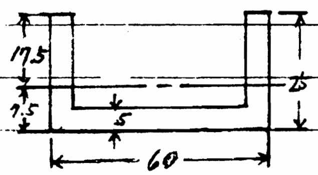
$$\therefore b = 5.7143^{cm}$$

4. 


$$I = \frac{15\times5^3}{12} + 15\times5\times7.5^2 + \frac{3\times25^3}{12} + 25\times3\times7.5^2$$

$$= 12500\ cm^4.$$

$$\therefore Z_1 = \frac{12500}{20} = 625^{cm^3},\ Z_2 = \frac{12500}{10} = 1250^{cm^3}$$

5. 

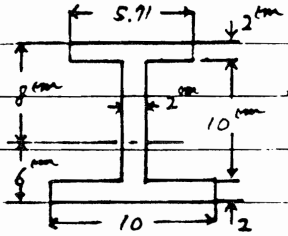

$$I = \frac{60\times5^3}{12} + 60\times5\times5^2 + \frac{2\times5\times20^3}{12} + 2\times20\times5\times7.5^2$$

$$= 26041.67^{cm^4}$$

$$\therefore Z_1 = \frac{26041.69}{17.5} = 1488.095^{cm^3}$$

$$Z_2 = \frac{26041.69}{7.5} = 3472.2^{cm^3}$$

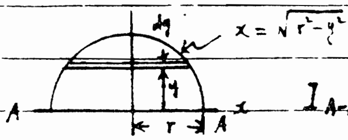
6.




$$I = \frac{10 \times 2^3}{12} + 2 \times 10 \times 5^2 + \frac{2 \times 10^3}{12} + 2 \times 10 \times 1^2 + \frac{5.91 \times 2^3}{12} + 2 \times 5.91 \times 7^2$$

$$= 1257.92 \text{ cm}^4$$

7.




$$I_{A\text{-}A} = \int_0^r y^2 dA = 2\int_0^r y^2 \sqrt{r^2 - y^2}\, dy$$

$$= 2\int_0^{\pi/2} r^2 \sin^2\theta \cdot r\cos\theta \cdot r\cos\theta\, d\theta$$

$$= 2r^4 \int_0^{\pi/2} \sin^2\theta \cdot \cos^2\theta\, d\theta$$

$$= 2r^4 \int_0^{\pi/2} \left(\frac{\sin 2\theta}{2}\right)^2 d\theta$$

$$= \frac{1}{4} r^4 \int_0^{\pi/2} (1 - 2\cos 4\theta)\, d\theta$$

$$= \frac{1}{4} r^4 \left[\theta - \frac{1}{2}\sin 4\theta\right]_0^{\pi/2}$$

$$= \frac{\pi r^4}{8}$$

$$\therefore I_{AA} = \frac{\pi r^4}{8}$$

$$\bar{y} = \frac{2\int_0^r y\sqrt{r^2 - y^2}\, dy}{\frac{1}{2}\pi r^2} = \frac{2\int_0^{\pi/2} r\sin\theta\, r\cos\theta \cdot r\cos\theta\, d\theta}{\frac{1}{2}\pi r^2}$$

$$= \frac{4r^3 \int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta}{\pi r^2}$$

$$= \frac{4r^3}{\pi r^2}\left[-\frac{1}{3}t^3\right]_1^0 = \frac{4r}{3\pi}$$

$$\therefore I_x = \frac{\pi r^4}{8} - \frac{1}{2}\pi r^2 \left(\frac{4r}{3\pi}\right)^2 = \frac{\pi r^4}{8} - \frac{8}{9}\frac{r^4}{\pi}$$

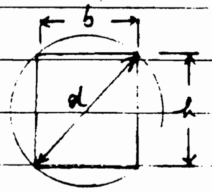
$$= \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$$

$$C_2 = r - \frac{4r}{3\pi} = \left(1 - \frac{4}{3\pi}\right) r = 0.5756r$$

$$C_1 = \frac{4r}{3\pi} = 0.4244r$$

$$\therefore \sigma_1 / \sigma_2 = \frac{0.4244}{0.5756} = 0.736.$$

8.



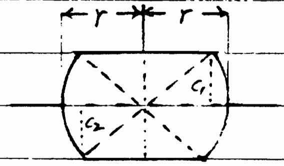

$$b^2+h^2=d^2=\text{const.}\quad b/h=t$$

$$Z=\frac{bh^2}{6}=\frac{b(d^2-b^2)}{6}$$

$$=\frac{dZ}{dh}=\frac{1}{6}(d^2-3b^2)=0 \quad \therefore b=\frac{d}{\sqrt{3}}$$

$$\frac{b}{h}=\frac{b}{\sqrt{d^3-b^2}}=\frac{\frac{d}{\sqrt{3}}}{\sqrt{d^2-\frac{d^2}{3}}}=\frac{1}{\sqrt{2}}$$

9.




$r=15^{cm}$

$$C_1=C_2=r\sin 45^\circ=\frac{r}{\sqrt{2}}$$

$$I=2\int_0^{r/\sqrt{2}}2y^2\cdot\sqrt{r^2-y^2}\,dy=4\int_0^{45^\circ}r^2\sin^2\theta\, r\cos\theta\, r\cos\theta$$

$y=r\sin\theta$

$dy=r\cos\theta\, d\theta$

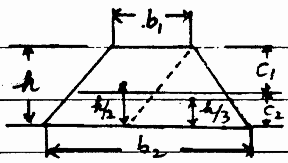
$$=4r^4\int_0^{45^\circ}\sin^2\theta\cos^2\theta\, d\theta$$

$$=4r^4\int_0^{45^\circ}\frac{1}{8}(1-2\cos 4\theta)\,d\theta$$

$$=\frac{1}{2}r^4\left[\theta-\frac{1}{2}\sin 4\theta\right]_0^{\frac{\pi}{4}}$$

$$Z=\frac{I}{C}=\frac{\frac{1}{8}\pi(15)^4}{15/\sqrt{2}}=\frac{\sqrt{2}\pi(15)^4}{15\times 8}=1874.322^{cm^3}$$

10.

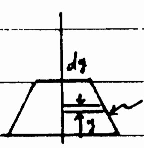



$$h=30^{cm},\ b_2=25^{cm},\ b_1=15^{cm}$$

$$C_2=\frac{b_1h\cdot\frac{h}{2}+\frac{b_2 b_1}{2}h\cdot\frac{h}{3}}{\frac{b_1+b_2}{2}h}$$

$$=\frac{h}{3}\cdot\frac{2b_1+b_2}{b_1+b_2}$$

$$\therefore C_1=\frac{h}{3}\cdot\frac{b_1+2b_2}{b_1+b_2}$$




$$y=\frac{2h}{b_1-b_2}x-\frac{b_2h}{b_2-b_1}$$

$$=\frac{2h}{b_1-b_2}\left(x-\frac{b_2}{2}\right)$$

$$x=\frac{b_1-b_2}{2h}y+\frac{b_2}{2}$$

(5-4-10 conti.)

$$I_{AA} = 2\int_0^h y^2 x\,dy = 2\int_0^h \left(\frac{b_1 - b_2}{2h} y^3 + \frac{b_2}{2} y^2\right) dy$$

$$= 2\left(\frac{b_1 - b_2}{8h} \cdot h^4 + \frac{b_2}{6} h^3\right) = \frac{2h^3}{24}(3b_1 - 3b_2 + 4b_2)$$

$$= \frac{3b_1 + b_2}{12} h^3$$

$$\therefore I_x = \frac{3b_1 + b_2}{12} h^3 - \frac{(b_1 + b_2)h}{2} \left\{\frac{(2b_1 + b_2)h}{3(b_1 + b_2)}\right\}^2$$

$$= \left[\frac{3b_1 + b_2}{12} - \frac{b_1 + b_2}{2} \quad \frac{(2b_1 + b_2)^2}{9(b_1 + b_2)}\right] h^3$$

$$= \frac{(b_1^2 + 4b_1 b_2 + b_2^2)}{36(b_1 + b_2)} h^3$$

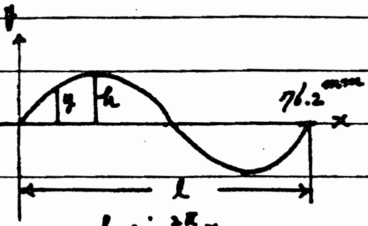
$$\therefore Z_1 = \frac{I}{C_1} = \frac{h^2(b_1^2 + 4b_1 b_2 + b_2^2)}{12(b_1 + 2b_2)} = \frac{30^2(15^2 + 4.15.25 + 25^2)}{12(15 + 50)}$$

$$= 2911.5^{cm^3}$$

$$\therefore Z_2 = \frac{I}{C_2} = \frac{h^2(b_1^2 + 4b_1 b_2 + b_2^2)}{12(2b_1 + b_2)} = \frac{30^2(15^2 + 4.15.25 \times 25^2)}{12(30 + 25)}$$

$$= 3204.5^{cm^3}$$

11.




$y = h\sin\frac{2\pi}{l}x$


$$\begin{cases} t = 1.6^{mm} \\ h = 9^{mm} \end{cases}$$

$$\frac{dy}{dx} = \frac{2\pi h}{l}\cos\frac{2\pi}{l}x$$

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$\doteqdot \left[1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2\right] dx$$

$$= \left(1 + \frac{1}{2} \cdot \frac{4\pi^2 h^2}{l^2} \cos^2 \frac{2\pi x}{l}\right) dx$$

$$I = \int_0^l y^2 dA = \int_0^l y^2 t\,ds = \int_0^l h^2 \sin^2 \frac{2\pi x}{l} \cdot t \cdot \left(1 + \frac{1}{2} \cdot \frac{4\pi^2 h^2}{l^2} \cos^2 \frac{2\pi x}{l}\right) dx$$

(5-4-11 conti)

Let $\frac{2\pi x}{l}=u$, $x=\frac{lu}{2\pi}$, $dx=\frac{l}{2\pi}du$

$$I=\frac{lh^2t}{2\pi}\int_0^{2\pi}\sin^2u\,du+\frac{lh^2}{2\pi}\cdot\frac{1}{2}\cdot\frac{4\pi^2h^2}{l^2}\int_0^{2\pi}\sin^2u\cos^2u\,du$$

$$=\frac{lh^2t}{2\pi}\left[\frac{u-\frac{1}{2}\sin 2u}{2}\right]_0^{2\pi}+\frac{\pi t h^4}{l}\left[\frac{1-\frac{1}{4}\sin 4u}{8}\right]_0^{2\pi}$$

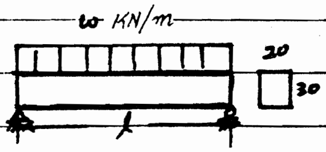
$$=\frac{lh^2t}{2}+\frac{\pi^2th^4}{4l}=\frac{h^2lt}{2}\left(1+\frac{\pi^2}{2}\cdot\frac{h^2}{l^2}\right)$$

수치를 代入

$$\therefore I=\frac{0.9^2\times7.62\times0.16}{2}\left(1+\frac{\pi^2}{2}\times\frac{0.9^2}{7.62^2}\right)=0.52777\ cm^4$$

$$\therefore Z=\frac{0.5277}{0.9}=0.5864\ cm^3$$

## 問題 5.5

1. 


$l=2^m$

$\sigma_w=7\ MN/m^2=0.7\ KN/cm^2$

$\tau_w=1\ MN/m^2=0.1\ KN/cm^2$

$$V_{max}=\frac{wl}{2}=w\ KN$$

$$M_{max}=\frac{wl^2}{8}=\frac{w}{2}\ KN\text{-}m=50w\ KN\text{-}cm$$

$$A=20\times30=600\ cm^2$$

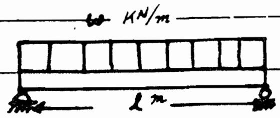
$$Z=\frac{bh^3}{6}=3000\ cm^3$$

$$\tau_w=0.1=\frac{3}{2}\frac{V}{A}=\frac{3w}{2\times600}=\frac{w}{400}\rightarrow w=40\ KN/m,$$

$$\sigma_w=0.7=\frac{M}{Z}=\frac{50w}{3000}=\frac{w}{60}\rightarrow w=42\ KN/m.$$

$\therefore w=40\ KN/m.$

2.




$A = 600\ cm^2, \quad Z = 3000\ cm^3$

$V_{max} = \frac{wl}{2}$ (KN)

$\tau_w = 1\ MN/m^2 = 0.1\ KN/cm^2$

$\sigma_w = 7\ MN/m^2 = 0.7\ KN/cm^2$

$M_{max} = \frac{wl}{2} \cdot \frac{l}{2} - \frac{wl}{2} \cdot \frac{l}{4} = \frac{wl^2}{8}$ (KN-m)

$\tau_{max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2} \cdot \frac{\frac{wl}{2}}{600} = \frac{3wl}{2400} = \frac{wl}{800} = 0.1$ (KN/cm²)

$\sigma_{max} = \frac{M}{Z} = \frac{\frac{wl^2}{8} \times 100}{3000} = \frac{wl^2}{240} = 0.7$ (KN/cm²)

위의 2式으로부터 $w$를 消去하면

$l = \frac{240 \times 0.7}{800 \times 0.1} = 2.1^m$

3.

$l = 3^m, \ b = 20^{cm}, \ h = 30^{cm}$

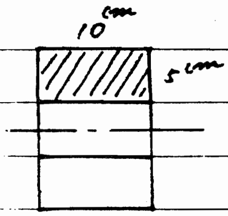
$V_{max} = 3w/2$ (kgf) $\quad M_{max} = \frac{3}{2}w \times \frac{3}{2} - \frac{3w}{2} \times \frac{3}{4} = \frac{9}{8}w$ (kgf-m)

$A = 20 \times 30 = 600\ cm^2, \quad Z = \frac{bh^2}{6} = 3000\ cm^3$

$\sigma_{max} = \frac{M}{Z} = \frac{\frac{9}{8}w \times 100}{3000} = 84\ kgf/cm^2 \quad \therefore w = 2240\ kgf/m$

$\therefore \tau_{max} = \frac{3}{2} \cdot \frac{V}{A} = \frac{3}{2} \cdot \frac{\frac{3}{2}w}{600} = \frac{3 \times 3 \times 2240}{2 \times 2 \times 600} = 8.4\ kgf/cm^2$

4.




$\tau_w = 4\ kgf/cm^2, \quad l = 2^m$

$V_{max} = \frac{P}{2}$ kgf, $M_{max} = \frac{P}{2} \cdot 1 = \frac{P}{2}$ kgf-m

$I = \frac{bh^3}{12} = \frac{10 \times 15^3}{12} = 2812.5\ cm^4$

$Q = 10 \times 5 \times 5 = 250\ cm^3$

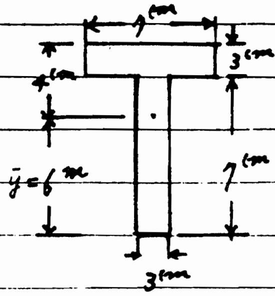
$\tau_{max} = \frac{VQ}{Ib} = \frac{\frac{P}{2} \times 250}{2812.5 \times 10} = 4\ kgf/cm^2$

$\therefore P = 900\ kgf$

$\sigma_{max} = \frac{My}{I} = \frac{\frac{P}{2} \times 100 \times 7.5}{2812.5} = \frac{750 \times 900}{2 \times 2812.5}$

$= 120\ kgf/cm^2$

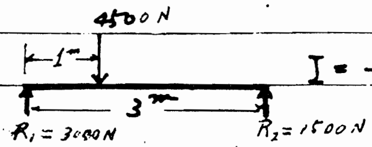
5.




$l = 3^m$

$A = 7\times3 + 7\times3 = 42\,cm^2$

$$\bar{y} = \frac{7\times3\times3.5 + 7\times3\times8.5}{42} = 6^{cm}$$



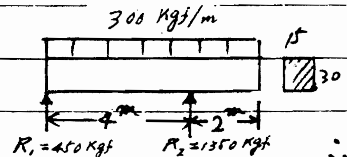

$$I = \frac{7\times3^3}{12} + 7\times3\times2.5^2 + \frac{3\times7^3}{12} + 7\times3\times2.5^2 = 364\,cm^4$$

$V_{max} = 3000N$

$Q = 3\times6\times3 = 54\,cm^3$

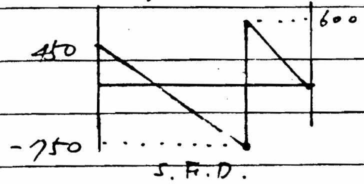
$$\therefore \tau_{max} = \frac{3000\times54}{364\times3} = 148.35\ N/cm^2 = 1.48\ MN/m^2$$

6.




$4R_A - 300\times4\times2 + 300\times2\times1 = 0$

$\therefore R_A = 450^{kgf},\ R_B = 1350^{kgf}\ \therefore V_{max} = 750^{kgf}$

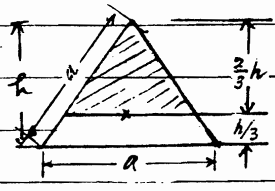



$$I = \frac{bh^3}{12} = \frac{15\times30^3}{12} = 33{,}750^{cm^4}$$

$Q = 15\times15\times7.5 = 1687.5^{cm^3}$

$$\therefore \tau_{max} = \frac{750\times1687.5}{33750\times15} = 2.5^{kgf/cm^2}$$

7.




$a = 30\,cm$

$V = 30\ KN,\quad h = \frac{\sqrt3}{2}a$

$$I = \frac{ah^3}{36} = \frac{a}{36}\cdot\frac{3\sqrt3}{8}a^3 = \frac{\sqrt3}{96}a^4$$

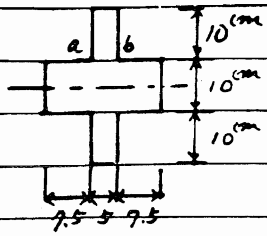
$$Q = \frac{1}{2}\left(\frac{2}{3}a\cdot\frac{2}{3}h\right)\times\frac{1}{3}\left(\frac{2}{3}h\right)$$

$$= \frac{2.2.2\,a}{2\cdot3\cdot3\cdot3\cdot3}\cdot\frac{3}{4}a^2 = \frac{a^3}{27}$$

$$\tau_{max} = \frac{VQ}{Ib} = \frac{V\cdot\frac{a^3}{27}}{\frac{\sqrt3}{96}a^4\cdot\frac{2}{3}a} = \frac{96\cdot3\cdot30}{2\sqrt3\cdot27\cdot30^2}$$

$$= 0.10264\ KN/cm^2 = 1.0264\ MN/m^2$$

8.




$$I = \left(\frac{5\times 10^3}{12} + 5\times 10\times 10^2\right)\times 2 + \frac{20\times 10^3}{12} = 12500\ \mathrm{cm^4}$$

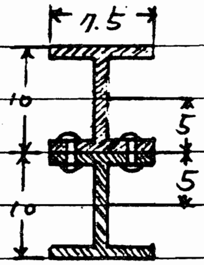
$$Q = 10\times 5\times 10 = 500\ \mathrm{cm^3}$$

$$V = 20000\ \mathrm{kgf}$$

$$\tau = \frac{20000\times 500}{12500\times 5} = 160\ \mathrm{kgf/cm^2}$$

## 問題 5.6

1.




$$I - 100\times 75\times 5\times 8$$

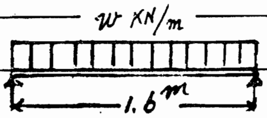
$$I_x = 281\ \mathrm{cm^4},\quad A = 16.43\ \mathrm{cm^2}$$

$$e = 10\ \mathrm{cm},\quad l = 1.6\ \mathrm{m}$$

$$\sigma_w = 100\ \mathrm{MN/m^2} = 10\ \mathrm{KN/cm^2}$$

$$I = 2\left[281 + 16.43(5)^2\right] = 1383.5\ \mathrm{cm^4}$$

$$Z = \frac{1383.5}{10} = 138.35\ \mathrm{cm^3}$$




$$中:\ M_{max} = \frac{wl^2}{8} = 0.32w\ \mathrm{KN\text{-}m}$$

$$\sigma_{max} = \frac{M_{max}}{Z}$$

$$10 = \frac{0.32w\times 100}{138.35}$$

$$\therefore\ w = \frac{10\times 138.35}{0.32\times 100} = 43.234\ \mathrm{KN/m}$$

Rivet: $d = 1.9\ \mathrm{cm}$

$A_r = 2.835\ \mathrm{cm^2}$

$$V_{max} = \frac{wl}{2} = 34.587\ \mathrm{KN}$$

$$Q = 16.43\times 5 = 82.15\ \mathrm{cm^3}$$

$$F = \frac{Ve}{I}Q = \frac{34.587\times 10}{1383.5}\times 82.15 = 20.537\ \mathrm{KN}$$

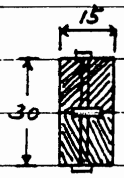
$$\tau_{av} = \frac{F}{2A_r} = \frac{20.537}{2\times 2.835} = 3.622\ \mathrm{KN/c}$$

$$= 36.22\ \mathrm{MN/m^2}$$

2.

15 cm × 15 cm - 斷面 Bolt 구멍 $d = 19^{mm}$

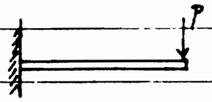
連結用環의 抵抗力 = $3000^{kgf}$, $P = 2500^{kgf}$




$A = 15 \times 30 = 450^{cm^2}$

$V = P = 2500^{kgf}$

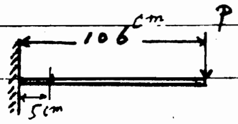
$\tau_{max} = \frac{3}{2}\frac{V}{A} = \frac{3 \times 2500}{2 \times 450} = \frac{25}{3}$ $kgf/cm^2$

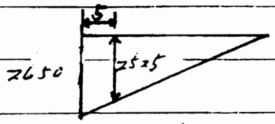



$F = 15e\tau$

$\therefore\ e = \frac{F}{15\tau} = \frac{3000 \times 3}{15 \times 25} = 24^{cm}$

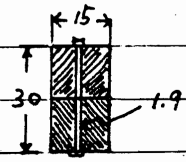
3.




$M_{x=0} = 2650^{kgf\text{-}m}$, $M_{x=5cm} = 2525^{kgf\text{-}m}$

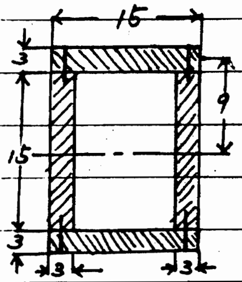
$I = \frac{(15-1.9) \times 30^3}{12} = 29,475^{cm^4}$




$Z = \frac{29475}{15} = 1965^{cm^3}$

$\sigma = \frac{2525 \times 100}{1965} = 128^{kgf/cm^2}$

4.




15cm × 30cm - 木板

나사못 $F = 1200\,N$, $V = 5000\,N$

$I = \frac{6 \times 15^3}{12} + 2\left[\frac{15 \times 3^3}{12} + 45(9)^2\right]$

$= 1687.5 + 2[33.75 + 3645]$

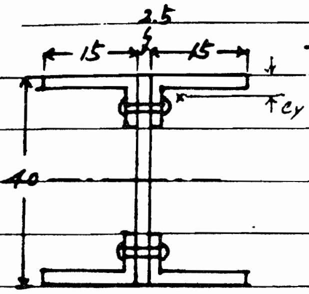
$= 9045^{cm^4}$

$Q_F = 45 \times 9 = 405^{cm^3}$

$\tau = \frac{VQ_F}{Ib} = \frac{5000 \times 405}{9045 \times 6}$, $F = \frac{1}{2}(6e\tau)$

$\therefore\ e = \frac{F}{3\tau} = \frac{9045 \times 6 \times 1200}{3 \times 5000 \times 405} = 10.72$ cm.

5.



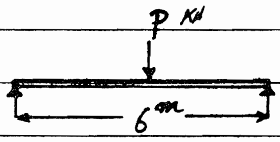

韓工·標準 $L-150\times100\times12$, $l = 6\,m$

Rivet ; $d_r = 2.5^{cm}$, $A_r = 4.909\,cm^2$.

$\sigma_w = 110\ MN/m^2$, $\tau_w = 42\ MN/m^2$.

$A = 28.56^{cm^2}$, $C_y = 2.41^{cm}$

$I = 228^{cm^4}$




斷面 :

$$I = 4\left[228 + 28.56(20-2.41)^2\right] + \frac{2.5\times40^3}{12}$$

$$= 36259 + 13333 = 49592^{cm^4}$$

$$Z = \frac{I}{20} = 2479.6\ cm^3$$

$$V_{max} = \frac{P}{2} \qquad Q_F = 2\times28.56\times(20-2.41) = 1004.7\ cm^3$$

$$M_{max} = \frac{Pl}{4} = 150P^{\ KN\text{-}cm}$$

$$\sigma_{max} = \frac{M_{max}}{Z} \ ; \quad \sigma_w = 110\ MN/m^2 = 11\ KN/cm^2$$

$$11 = \frac{150P}{2479.6} \quad \therefore P = 181.84\ KN, \quad \therefore V = \frac{P}{2} = 90.92\ KN.$$

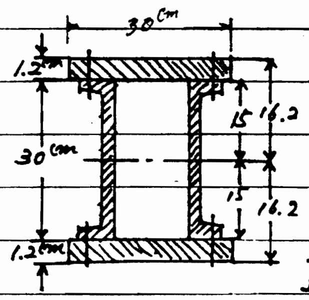
Shear :

$$F = \frac{Ve}{I}Q_F = \frac{90.92\cdot e\cdot 1004.7}{49592} = 1.842\,e$$

한편 $F = 2\times4.909\times \qquad = 41.236$

$$\therefore e = \frac{41.236}{1.842} = 22.39\ cm.$$

6.



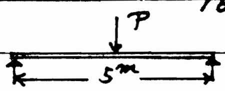

韓工·標準 :

$U-300\times90\times10\times15.5$ $\begin{cases} A = 55.74^{cm^2} \\ I_x = 7410^{cm^4} \end{cases}$

$$I = 2\left[7410 + \frac{30\times1.2^3}{12} + (30\times1.2)(15.6)^2\right]$$

$$= 2\left[7410 + 4.32 + 8761\right] = 32351^{cm^4}$$

(5-6-6 conti.)




$$Z = \frac{32351}{16.2} = 1997^{cm^3}, \quad \sigma_{max} = \frac{125P}{1997} = 1100 \ kgf/cm^2$$

$$\therefore P = 17574^{kgf}.$$

$$V_{max} = \frac{P}{2} = 8787^{kgf}$$

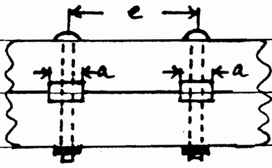
$$Q_F = 30 \times 1.2 \times 15.6 = 561.6^{cm^3}$$

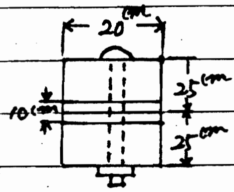
$$F = \frac{Ve}{I} Q_F = \frac{8787 \times e}{32351} \times 561.6$$

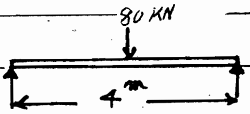
$$\tau_{av} = \frac{F}{2A} = \frac{8787 \times 561.6 \times e}{2 \times 2.835 \times 32351} = 420 \ kgf/cm^2$$

$$\therefore e = \frac{2 \times 2.835 \times 32351 \times 420}{8787 \times 561.6} = 15.61 \ cm.$$

7.




Oak block, (Parallel to grain)

$$\sigma_w = 7 \ MN/m^2 = 0.7 \ KN/cm^2, \quad \tau_w = 1.6 \ MN/m^2 = 0.16 \ KN/cm^2.$$

$$I = \frac{20 \times 50^3}{12} - \frac{20 \times 10^3}{12} = \frac{620000}{3} cm^4$$

$$Q = 20 \times 20 \times 15 = 6000^{cm^3}$$

$$F = \frac{Ve}{I} Q = \frac{3 \times 40}{620000} \times 6000 \times e$$

$$= \frac{36}{31} e$$

for Oak block:

$$20a\,\tau_w = 20 \times 5 \times \sigma_w$$

$$\therefore a = \frac{20 \times 5 \times 0.7}{\cancel{20} \times 0.16} = 21.875 \ cm.$$

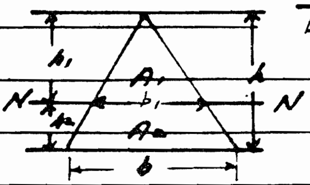
$$20 \times 5 \times \sigma_w = \frac{36}{31} e$$

$$\therefore e = \frac{20 \times 5 \times 0.7 \times 31}{36}$$

$$= 60.28 \ cm.$$

## 問題 6・1

**1.**



$$\frac{b_1}{b} = \frac{h_1}{h} \;;\; b_1 = \frac{bh_1}{h}$$

$$A_1 = \frac{b_1 h_1}{2} = \frac{b h_1^2}{2h} = \frac{1}{2}\left(\frac{bh}{2}\right)$$

$$h_1^2 = \frac{h^2}{2} \;;\; h_1 = \frac{h}{\sqrt{2}} = \underline{0.7071h} \;;\; b_1 = 0.7071b$$

$$h_2 = h - h_1 = 0.2929h$$

$$Z_p = \frac{A}{2}(y_1 + y_2) \;;\; y_1 = \frac{h_1}{3} = \frac{0.7071h}{3} = \underline{0.2357h}$$

$$y_2 = h_2 - \frac{h_2}{3}\left(\frac{b+2b_1}{b+b_1}\right) = h_2\left[1 - \frac{1}{3}\left(\frac{2.4142}{1.7071}\right)\right]$$

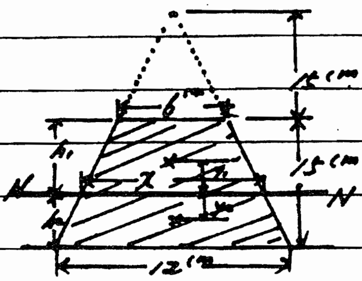
$$= h_2[1 - 0.4714]$$

$$y_2 = 0.2929h(0.5259) = \underline{0.1540h}$$

$$Z_p = \frac{bh^2}{4}(0.2357 + 0.1540) = \frac{0.3897}{4}bh^2$$

$$= \underline{0.0974bh^2}$$

**2.**



$$\frac{x}{12} = \frac{15+h_1}{30} \qquad \therefore x = \frac{12}{30}(15+h_1) = 0.4(15+h_1)$$

$$\left(\frac{x+6}{2}\right)h_1 = \left(\frac{12+x}{2}\right)h_2 = \frac{1}{2}\left(\frac{12+6}{2}\right)\times 15$$

$$= 67.5\ \mathrm{cm}^2$$

$$(x+6)h_1 = 135$$

$$[0.4(15+h_1)+6]h_1 = 135,\ (0.4h_1+12)h_1 - 135 = 0$$

$$0.4h_1^2 + 12h_1 - 135 = 0,\ h_1^2 + 30h_1 - 337.5 = 0$$

$$h_1 = -15 \pm \sqrt{15^2 + 337.5} = -15 \pm \sqrt{562.5}$$

$$\fallingdotseq -15 \pm 23.717 = 8.717\ \mathrm{cm}$$

$$\therefore h_2 = 15 - h_1 = 6.283\ \mathrm{cm}$$

$$\therefore x = 0.4(15 + 8.717) = 9.4868\ \mathrm{cm}$$

$$y_1 = \frac{h_1}{3}\left(\frac{x+2\times 6}{x+6}\right) = \frac{8.717}{3}\left(\frac{21.4868}{15.4868}\right)$$

$$= 4.0314\ \mathrm{cm}$$