

Multilevel Urban Tree Allometric equations

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Introduction

Multilevel models have been used for decades in tree growth equations (Lappi and Bailey 1988). Multilevel modeling is an attractive approach because it provides a coherent framework to account for the many levels of observation or of groupings in data and to pool information across groups. This paper has two main contributions. First, we demonstrate the use of Stan via the "brms" package in R to fit bayesian nonlinear multilevel models to predict tree diameter growth from age Carpenter et al. (2017); Bürkner (2017). Second, we apply the method to the Urban Tree Database McPherson et al. (2016a,b). This dataset is the result an over a decade long effort to collect age and size data on thousands of trees in 17 cities across the US. Multilevel modeling has the potential to extract more information from the data and improve predictions compared to the existing modeling approach. Improving predictions of tree size from tree age will improve our ability to predict the important ecosystem services these trees provide urban dwellers.

Stan is a probabilistic programming language for bayesian inference Carpenter et al. (2017). It uses No-U-Turn sampler, an adaptive form of Hamiltonian Monte Carlo sampling, to effectively draw samples from the specified log posterior density. Here, we access Stan

via the R package `brms` Bürkner (2017). `brms` allows the user to specify the likelihood and priors in syntax similar to the R package `lme4` commonly used for frequentist? multilevel (mixed effects) models. This makes harnessing the power of Stan much simpler and concise because it doesn't require the user to know how to write efficient Stan code and can convert a few lines of R code into many lines of Stan. `brms` is not as flexible as `stan`, but still can be used to fit many types of models including nonlinear multilevel regression models, such as ours here. Some of the key advantages of fitting a model in Stan via `brms` include relatively simple syntax and efficient posterior sampling for multilevel non-linear models. The bayesian approach gives better estimates of parameter uncertainty and provides a formal way to include prior information.

The existing approach to modeling the diameter growth of trees in the urban tree database (UTD) was to fit a separate model for each tree species in each city and test several model forms with different weights and then select the model with the lowest Akaike Information Criterion (AIC). This approach has several limitations, many highlighted in the report. First, while the model form selected provided the lowest AIC, many of the estimates are not biologically realistic (for example they begin to increase sharply at old ages, cubic fits, or decrease at old ages, quadratic fits). Therefore, the researchers discouraged applying the models beyond the range of the data, or sometimes even within the range of data if the estimates were unrealistic. These unrealistic estimates and the inability to extrapolate severely limits managers' ability to predict growth over meaningful time scales (a century rather than a few decades). A second limitations is that some models predict negative diameters, an impossibility. Third, models are only provided for the cities and the species sampled. If a manager wants to predict the diameter growth of a tree species in an unsampled city, the researchers recommend using the model from the reference city in the same climate region. However many of the reference cities are on the border of climate regions and there is known large variability in growth within regions (see figure XX in utd report which is from McPherson... comparing Cheyenne to Ft Collins). Furthermore, if a manager wants to

predict growth for an unsampled species or a species that was sampled in a different city it is not obvious which equation/model to use and the additional uncertainty that this introduces is not quantified.

Our approach addresses the above limitations. First, we use a weibull curve, commonly used in forestry growth equations and biologically realistic, which makes extrapolation to ages outside the data range less fraught. Second, using this sigmoidal curve and modeling diameter with a gamma distribution ensures our estimates of diameter are positive. Third, by modeling the weibull curve parameters as functions of species, city, and climate, we are able to borrow information across cities and across species to provide predictions an associated uncertainty of diameter growth even in even in cities or species with very little or no data.

Sigmoidal curves very similar to the weibull we use here have been used before in modeling urban tree diameter growth as a function of age. Frelich (1992) use the Chapman-Richards growth curve of form $y = B_0(1 - \exp(-B_1x))^{B_2}$ to predict DBH from age for healthy trees (12 species, 221 trees total) in Minneapolis and St. Paul, Minnesota. This equation form worked very well (8 out of 12 species had an R^2 over .9), but the trees used were only healthy open grown trees, which is not representative of urban trees generally. Following Frelich (1992), in an early version of the urban tree database, McPherson and Simpson (1999) fit the same curve to a small number of observations and adjusted parameters for different locations based on the number of frost free days based on expert opinion. Peper et al. (2001a) and Peper et al. (2001b) compared this modified weibull curve to logarithm regression and selected the logarithm regression based on a higher R^2 . Subsequent urban tree growth equations did not use sigmoidal curves McPherson et al. (2016b).

Multilevel modeling was first introduced to forestry by Lappi and Bailey (1988) and has since been widely used to account for multiple levels of variability (observations correlated within groups) in allometric and growth equations. Hall and Bailey (2001) give a general overview of the approach and an example of loblolly pine height growth in the Southeastern US. Levels in their model are plot and tree nested within plot, and they use tree density

as a plot level covariate. Nothdurft et al. (2006) provide another example of modeling tree height growth using norway spruce in Germany. They use the Sloboda function and levels in the model are plot and trees nested in plot with elevation as a plot level covariate. Li et al. (2011) provide a bayesian example modeling balsam fir height growth in Maine using the Chapman-Richards equation, also with tree nested within plot. They found the bayesian approach has similar parameter point estimates to the frequentist approach. In an urban tree context, Peper et al. (2014) model DBH growth with repeat measures data on individual trees and test a varying coefficients model because they have repeat measures on individual trees.

Indeed the test dataset, "orange", in the statistical programming language, R, is used to demonstrate the fitting of nonlinear multilevel (mixed effects) models R Core Team (2016) orange.

Compared to past work, our approach is new from a modeling standpoint in the use of a bayesian approach with both nested and non-nested groupings and group level predictors. The Bayesian multilevel/hierarchical modeling framework has many strengths as discussed in Li et al. (2011) (and others) and includes the ability to sample parameter values from the entire posterior, rather than maximum likelihood estimates. Nonlinear multilevel models in the frequentist approach depend on linear approximations such as first-order taylor expansion, which is not required in the bayesian framework. Summary statistics of parameters (mean, median, quantiles) can be easily calculated from posterior samples and the common assumption in the frequentist paradigm that parameters are normally distributed can be relaxed. The ability to incorporate prior information from experts or past studies on parameters is another strength of the bayesian approach.

A paragraph on the impact of these equations for managing forests to predict/forecast ecosystem services.

Methods

Data

The urban tree database (UTD) consists of measurements on 14487 trees of 170 species in 17 cities. However, largely because of the difficulty is measuring tree age, there are only 12687 trees with complete age and diameter data (161 species, 17 cities, 309 species by city combinations 1).

Some species were measured in multiple cities, but not most. The number of trees of each city by species combination sampled ranged from 1 (both *Liquidambar styraciflu* and *Prunus serrulata* in Queens, NY) to 79 (*Quercus laurifolia* in Charleston, SC). The median number of trees in each species-city combination was 37.

Age is defined in this dataset as time since planting, since this is the record kept by cities. Actual age of the trees may be several years more. Diameter (cm) of the trees is measured at breast height (1.37m above ground).

In the UTD, trees are classified taxonomically down to cultivar for some individuals, but here we aggregate cultivars up to the species level. Species are then nested within Genera.

The 17 cities in the UTD cover much of the US geographically, 2, and much of the variation in climate, 3. However, New York City only has a few observations and the data for Indianapolis is missing too.

Rather than using the aggregated sunset zones as done in UTD, we used growing degree days (GDD) and precipitation data from climate NOAA’s climate normals to continuously vary equation parameters across climate. Figure 3 shows each census tract centroid in the conterminous US plotted in GDD and precipitation space. We appoximated the GDD and precipitation for each tract by assigning the values of the weather station closest to the centroid. This allows us to vary our model continouously across geographic space in a way that better captures the natural gradients of climate.



Figure 1: Number of trees sampled of each species and city combination in the urban tree database.

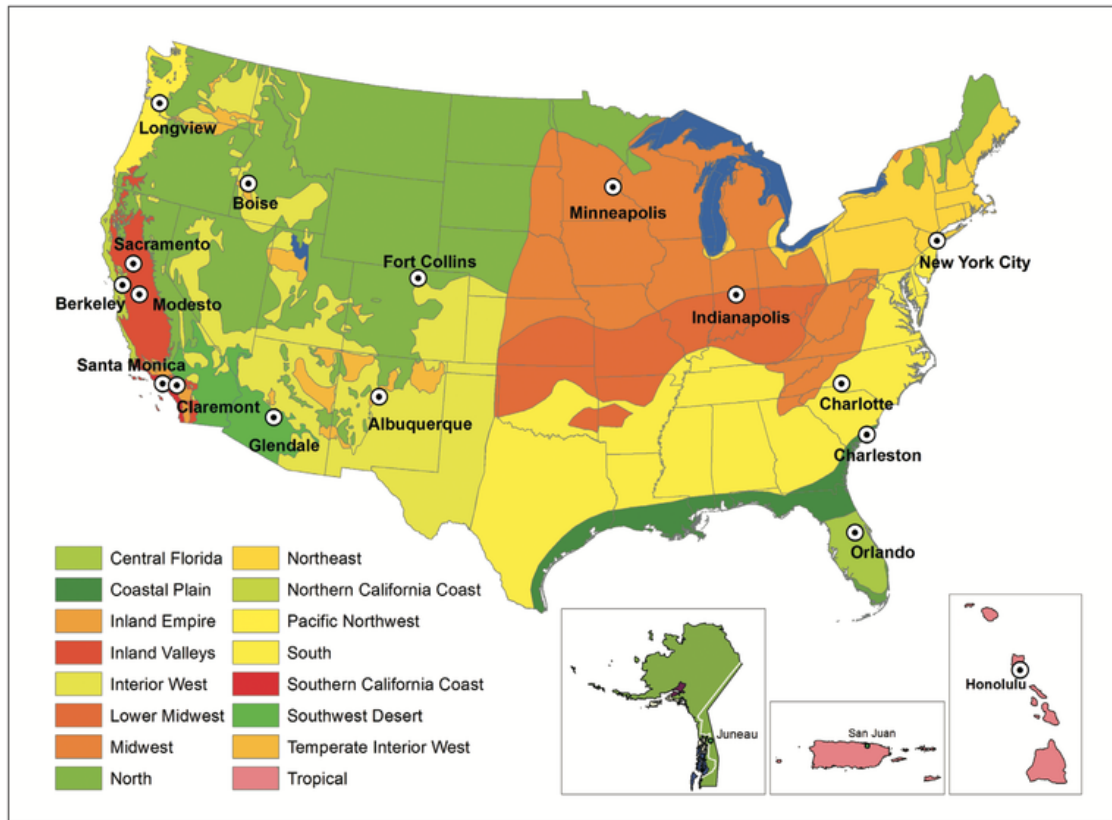


Figure 9—Climate zones were aggregated from 45 Sunset climate zones into 16 zones. Each zone has a reference city where tree growth data were collected. Sacramento, California, was added as a second reference city (with Modesto) to the Inland Valleys zone.

Figure 2: 16 climate regions and 17 representative cities in the UTD (McPherson et al., 2016b).

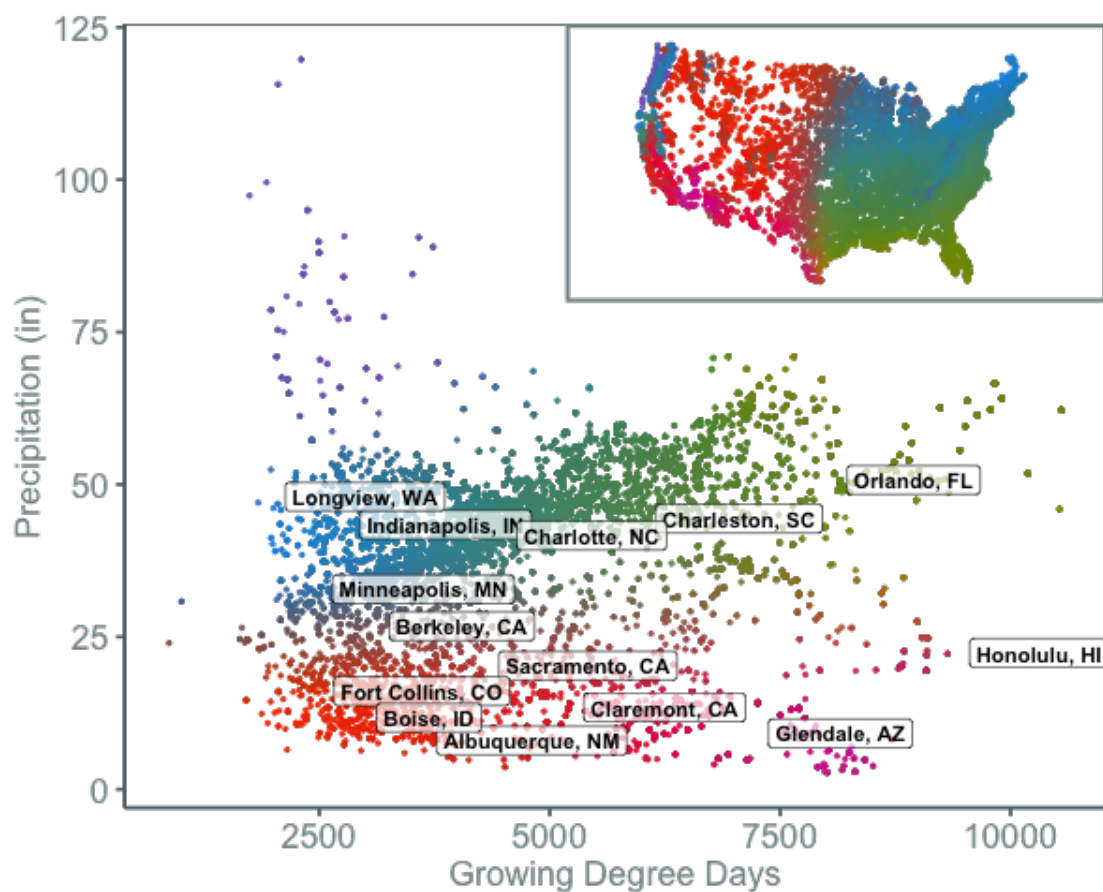


Figure 3: US census tract centroids with UTD reference cities overlaid in growing degree day (GDD) and precipitation climate space and matching color gradient in geographic space. The reference cities cover climate space well, and variation in precipitation and growing degree days is continuous.

Modelling

Model requirements

We sought a model of tree growth that would adequately represent the known biological dynamics of tree growth, namely that diameter growth rate starts slow, reaches a maximum at a young age, and then gradually declines to nearly zero. Diameter for trees much always increase however slightly because the growth of new wood is essential for proper function. This is different than tree height which often reaches its asymptote. Instead the asymptote in our curve could be considered a pragmatic way to constrain diameter growth, or a practical asymptote. While trees could theoretically continue to increase in diameter indefinitely, they don't in reality. The asymptote represents this practical maximum diameter. An additional feature to the data is that age is time since transplanting. This means trees can have substantial diameter at age 0.

The type of curve that meets these criteria would be an asymmetric sigmoidal curve with an intercept. A modified weibull is such a curve that has worked well in forestry and is the one we use here Weiskittel et al. (2011). However, there are many other curves such as the Chapman-Richards that meet these criteria and could be used.

Another characteristic of tree growth curves is heteroskedasticity, namely that as the age of a tree increases, so does the variability around the mean. Often past modellers controlled this using log - log transformations Troxel et al. (2013), but we wanted to keep units in their original scale. We tested fitting models where the variance was a linear function or a smoothed spline function of age. However, this still could yield negative predictions at low ages. Instead we adopted the approach of modeling DBH from a gamma distribution, which yielded more realistic posterior predictions.

We fit models of generally increasing complexity starting a single weibull curve for all trees and then varying the curve parameters by city, by genus and species, and by climate. Using approximate leave-one-out cross validation we selected the model with the highest estimated

log posterior density Vehtari et al. (2017). The following section details this model, and other models tested are in supplementary materials.

Model

- Bayesian Multilevel Modeling a bit of background?
- likelihood

$$y_{igsc} \sim \text{Gamma}(\mu_{igsc}, \alpha_y)$$

$$\mu_{igsc} = \beta_{igsc}^0 + \beta_{igsc}^1 (1 - \exp(-\beta_{igsc}^2 x_{igsc}^{\beta_{igsc}^3}))$$

where:

y_{igsc} is the diameter at breast height of tree i of genus, g , species, s , and city, c . y_{igsc} has a gamma distribution with mean, μ_{igsc} , and shape, α_y .

$i = 1, \dots, n_{sc}$; where n_{sc} is the number of trees sampled for species, s , and city, c .

$g = 1, \dots, G$; where G is the number of genera (G)

$s = 1, \dots, S_g$; where S_g is the number of species in genus g .

x_{igsc} is the transplant age in years of tree $igsc$ (i.e. years since transplanting).

β_{igsc}^0 is the intercept, or the diameter of a tree at time of transplanting.

β_{igsc}^1 (plus β_{igsc}^0) is the asymptote of the sigmoidal weibull curve.

β_{igsc}^2 and β_{igsc}^3 affect the rate of growth. β_{igsc}^2 provides flexibility to have slow or fast growth at young ages.

For each β_{igsc}^j , $j = 0,1,2$:

$$\beta_{igsc}^j = \beta_0^j + \gamma_{gs}^j + \delta_c^j$$

for β_{igsc}^3 :

$$\beta_{igsc}^3 = \beta_0^3 + \tau_1 * \text{precip}_c + \tau_2 * \text{gdd}_c + \tau_3 * (\text{precip}_c * \text{gdd}_c) + \gamma_{gs}^3 + \delta_c^3$$

where β_0^j is the mean for β_{igsc}^j . γ_{gs}^j is genetic (genus and species) effect on β^j . δ_c^j is the city effect on β^j

The species effect is nested within the genus effect. Both are normally distributed, such that:

$$\gamma_{gs}^j \sim N(\gamma_g^j, \sigma_{genus:species}^j)$$

$$\gamma_g^j \sim N(0, \sigma_{genus}^j)$$

The effect of city is normally distributed:

$$\delta_c^j \sim N(0, \sigma_{city}^j)$$

- Priors

The priors were selected to make biologically unrealistic parameters highly improbable, but they have a small effect on the posterior estimates. The β 's and α_y are gamma distributed, while the variance parameters are half-normal. More details on the selection of priors are available in supplementary materials and code. Many priors that might appear to be narrow (e.g Half-Normal(0,.1)) are actually fairly wide given the scale of the predictors and the reasonable range of some parameters.

Results

Model Comparisons

Short descriptions of the models tested and the brms syntax are in table ??

Note, I removed the scaling of the parameters (multiplying and dividing by 100) in the formula for clarity. In the code they are scaled so that the parameters are on the same order of magnitude and HMC sampling is improved.

Model	Description	brms formula syntax
1	No varying parameters	$DBH \sim b0 + b1 * (1 - \exp(-b2 * AGE^{b3}))$ $b0 \sim 1$ $b1 \sim 1$ $b2 \sim 1$ $b3 \sim 1$
2	Parameters vary by city	$DBH \sim b0 + b1 * (1 - \exp(-b2 * AGE^{b3}))$ $b0 \sim (1 City)$ $b1 \sim (1 City)$ $b2 \sim (1 City)$ $b3 \sim (1 City)$
3	Parameters vary by genus and species Species is nested in genus	$DBH \sim b0 + b1 * (1 - \exp(-b2 * AGE^{b3}))$ $b0 \sim (1 Genus / Species)$ $b1 \sim (1 Genus / Species)$ $b2 \sim (1 Genus / Species)$ $b3 \sim (1 Genus / Species)$
4	Asymptote (β_1) varies by climate	$DBH \sim b0 + b1 * (1 - \exp(-b2 * AGE^{b3}))$ $b0 \sim 1$ $b1 \sim gdd * precip$ $b2 \sim 1$ $b3 \sim 1$
5	Growth rate (β_3) varies by climate	$DBH \sim b0 + b1 * (1 - \exp(-b2 * AGE^{b3}))$ $b0 \sim 1$ $b1 \sim 1$ $b2 \sim 1$ $b3 \sim gdd * precip$
6	Parameters vary by city, genus, and species (but asymptote does not vary by city). Growth rate varies by climate.	$DBH \sim b0 + b1 * (1 - \exp(-b2 * AGE^{b3}))$ $b0 \sim (1 City) + (1 Genus/Species)$ $b1 \sim (1 Genus/Species)$ $b2 \sim (1 City) + (1 Genus/Species)$ $b3 \sim precip * gdd + (1 City) + (1 Genus/Species)$
7	Parameters vary by city, genus, and species. Growth rate varies by climate.	$DBH \sim b0 + b1 * (1 - \exp(-b2 * AGE^{b3}))$ $b0 \sim (1 City) + (1 Genus/Species)$ $b1 \sim (1 City) + (1 Genus/Species)$ $b2 \sim (1 City) + (1 Genus/Species)$ $b3 \sim precip * gdd + (1 City) + (1 Genus/Species)$

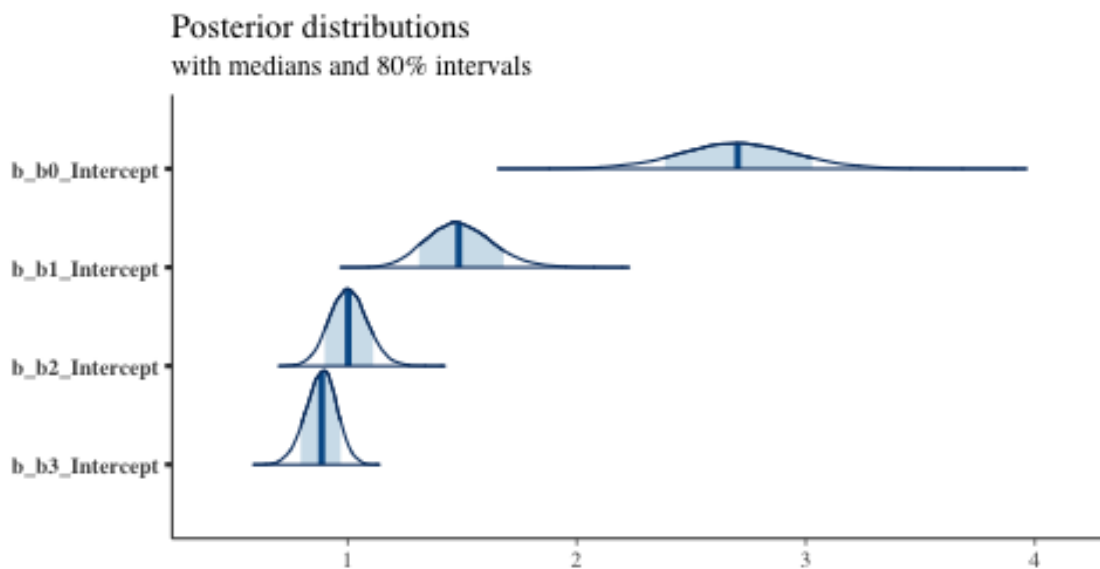
Table 1: \widehat{elpd}_{loo} is the estimated expected log pointwise predictive density. elpd diff is the difference from the \widehat{elpd}_{loo} of the top model. se elpd loo is standard error of Vehtari et al. (2017) for descriptions

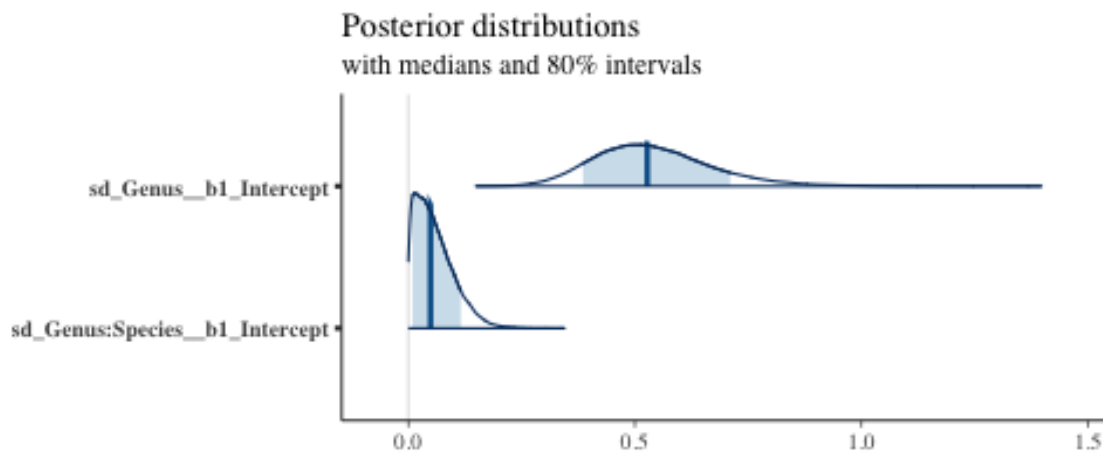
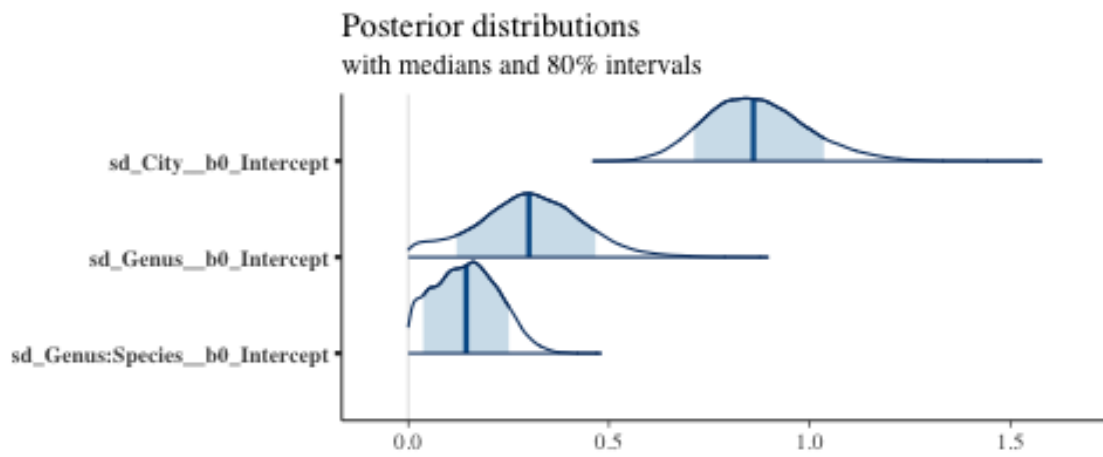
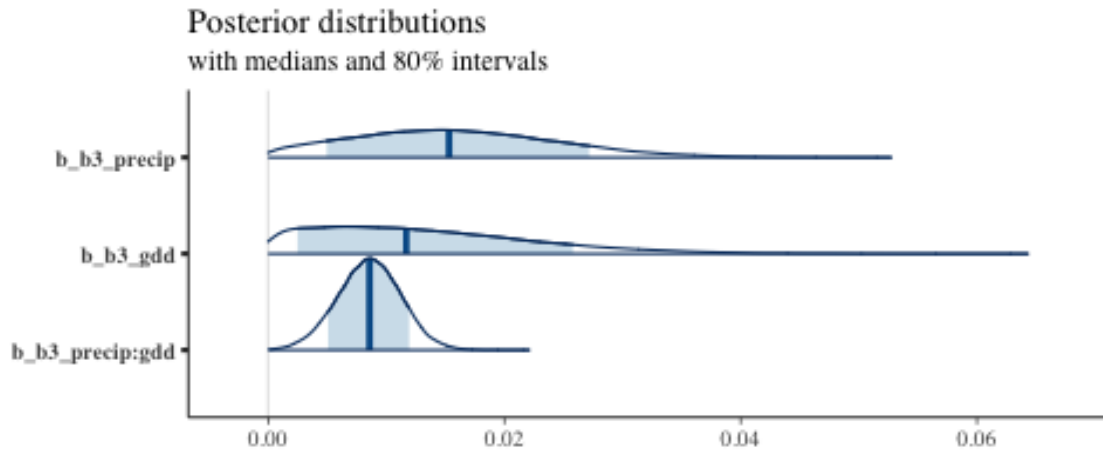
	Model	\widehat{elpd}_{loo}	difference
Best	6	-18845.41	0.00
	7	-18976.38	-130.97
	3	-18989.24	-143.83
	2	-19764.48	-919.06
	5	-20180.41	-1334.99
	4	-20195.21	-1349.80
Worst	1	-20513.12	-1667.70

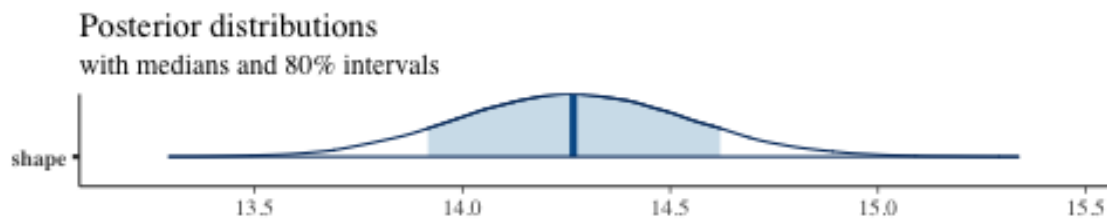
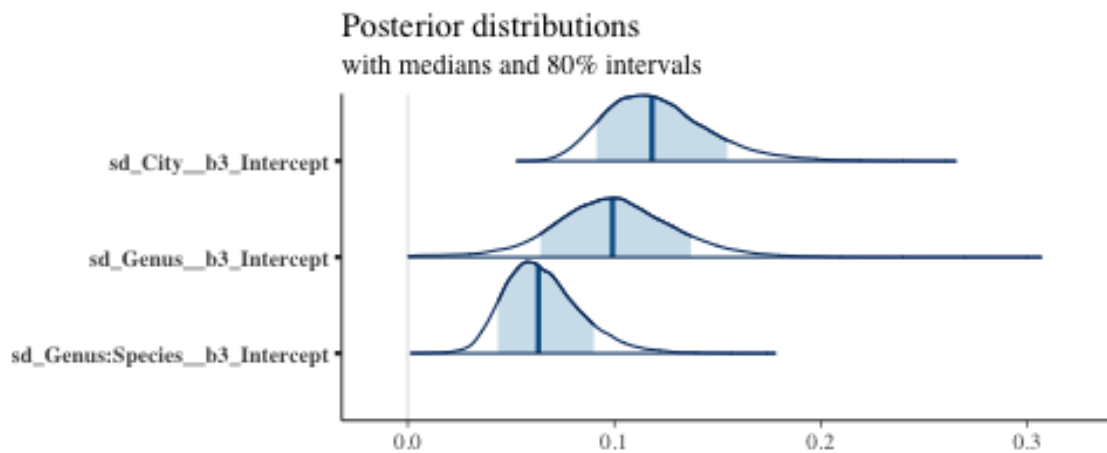
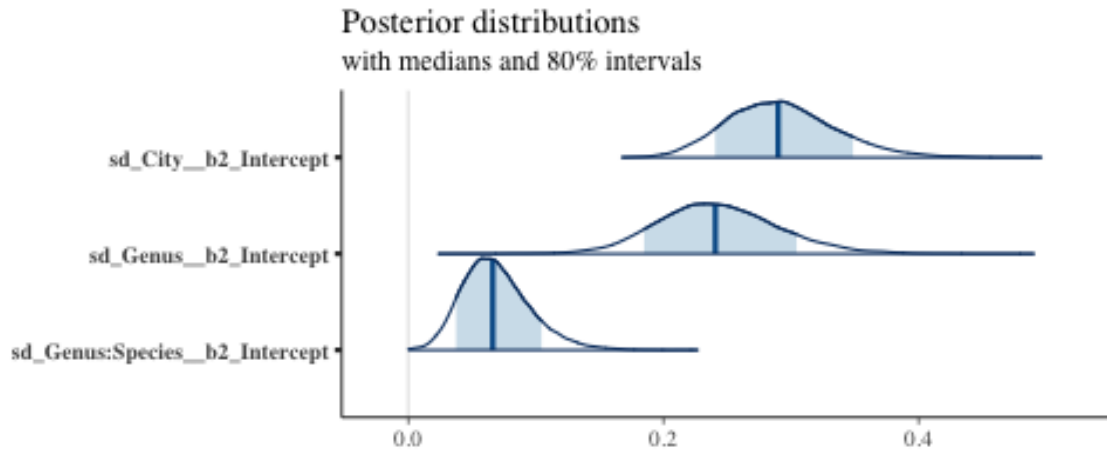
The standard error for the elpd difference of 131 between model 6 and model 7 is 21.4. Therefore, there is strong evidence that model 6 has higher out of sample predictive accuracy than model 7.

parameter estimates

Posterior distributions with 80% interval and median for parameters in



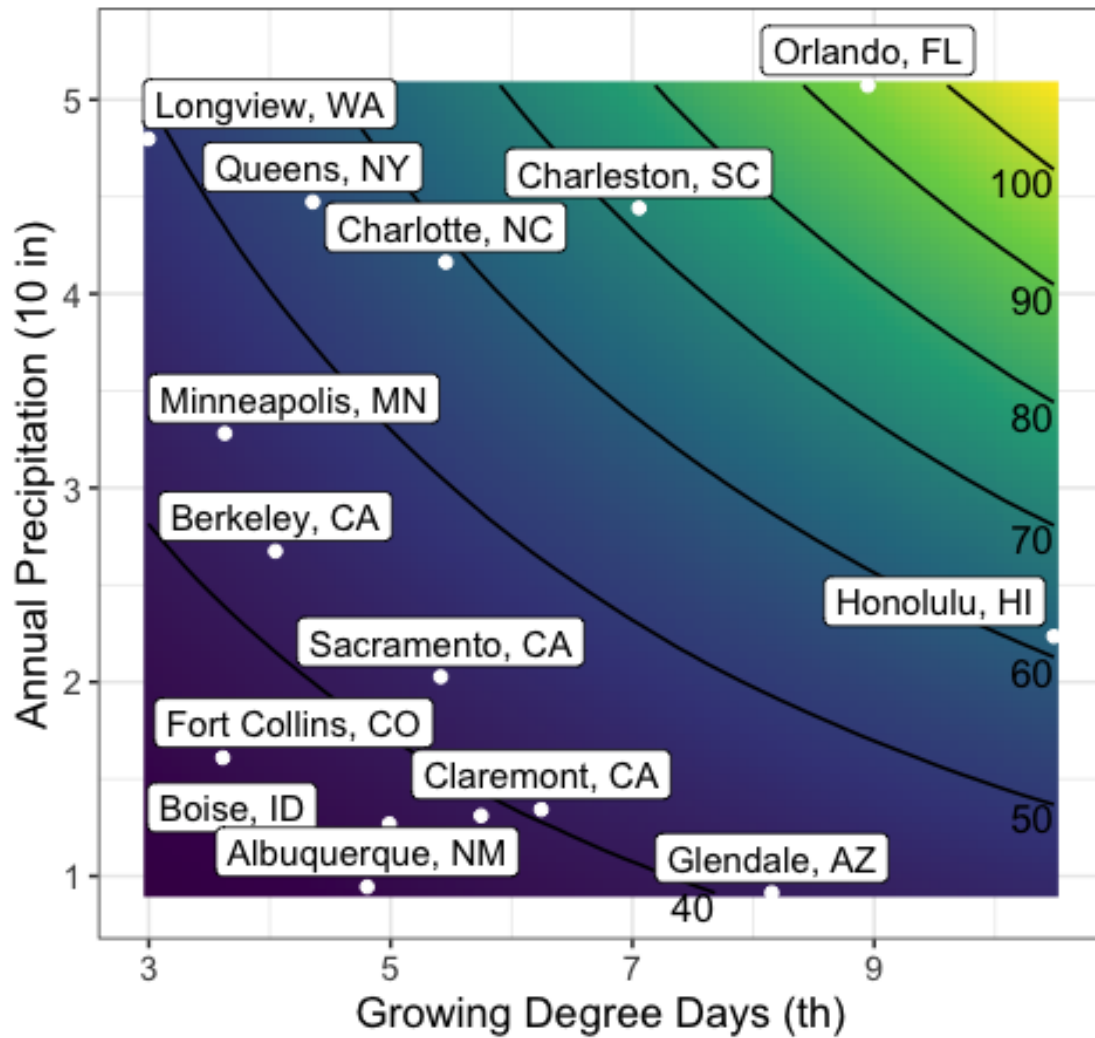




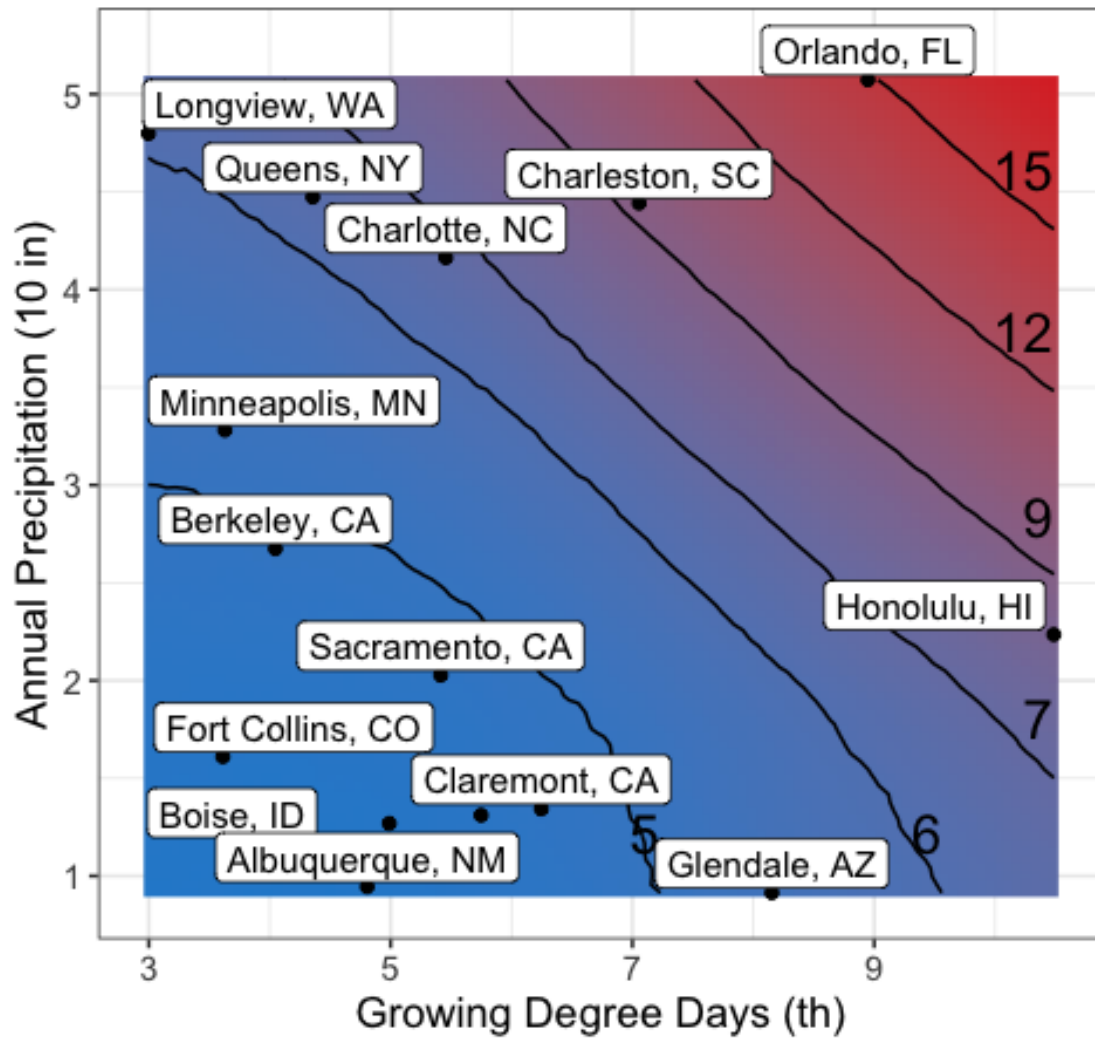
Climate Effects on Growth Rate

There is a positive effect of growing degree days (gdd) and annual precipitation (precip) on tree diameter (dbh), and a positive interaction between the two. Marginal effects of climate on posterior mean of DBH for a tree 25 years after planting in , standard error of this mean is in . There is an estimated 40cm difference in dbh between an average tree in Orlando, FL and one in Boise, ID.

Posterior median DBH at age 25



Posterior standard error of DBH at age 25



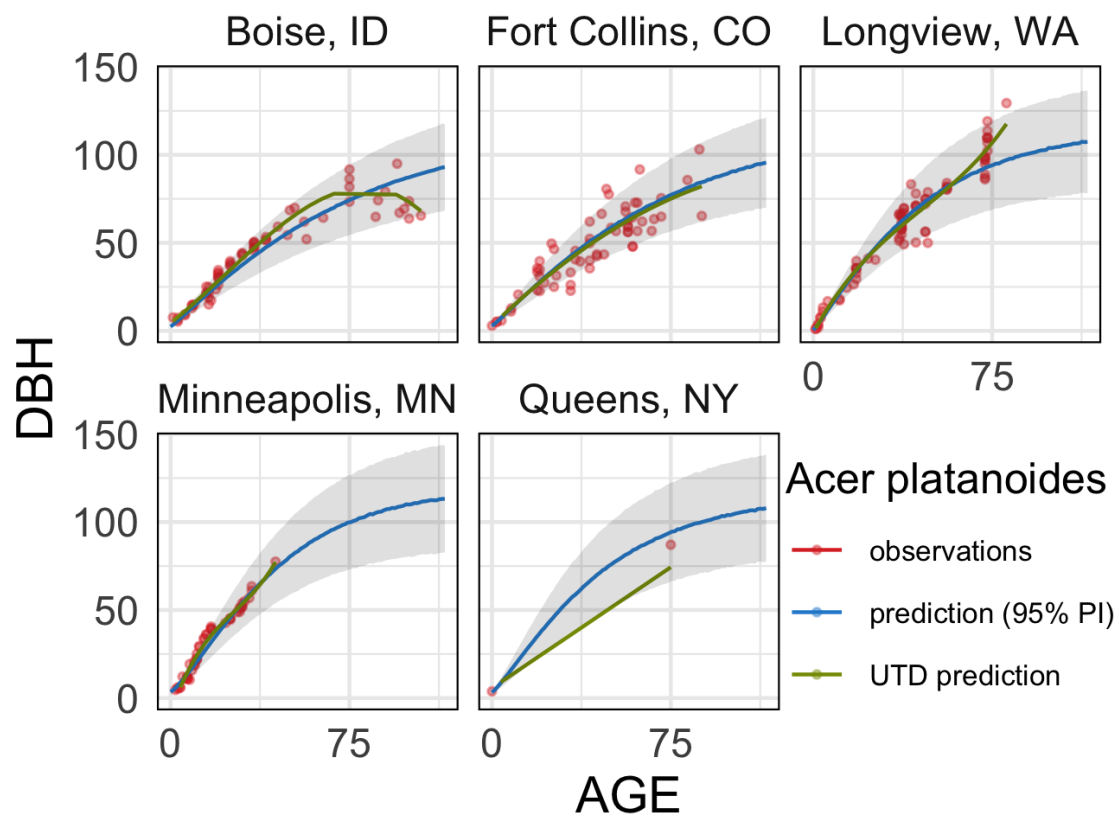
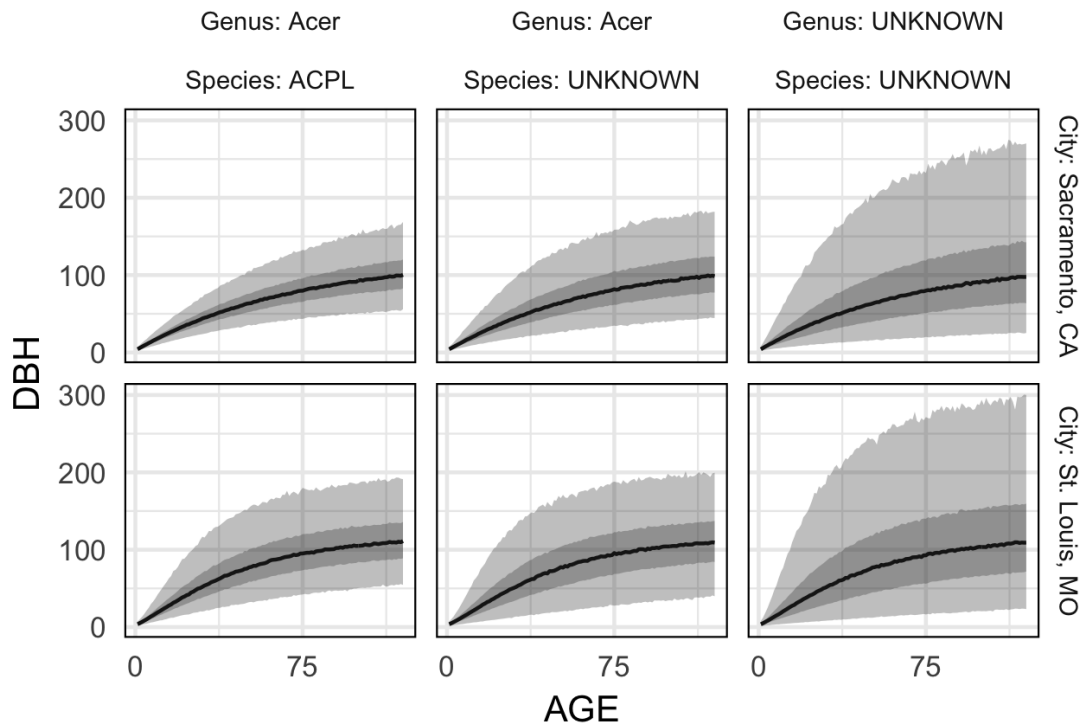


Figure 4: red points are observations, blue lines are predictions of the model and shading indicates 95% prediction interval, green lines show UTD equations

Comparing to existing equations

illustrative comparisons

Uncertainty increases when predicting out of sample cities, genera, and species



Discussions

Our selected model provides biologically realistic estimates of tree diameter growth for all species and cities observed, and also a way to make predictions for species with little or no data while appropriately reporting uncertainty. Our model will make better out of sample predictions than the existing UTC equations which are overfit to single species by cities combinations. We found the growth curve parameters, β_0 , β_2 , β_3 , vary more across cities, than they do across genera, and there is much less variation due to species within genera. This suggests that management practices and other factors that vary across cities have a stronger

impact on growth than the genetic potential of trees. This result is not surprising for β_0 , the diameter of trees when they are transplanted, since it is almost completely determined by management decisions about when a tree is large enough to withstand the urban environment. Differences in other growth parameters across cities have been found before. As noted by Peper et al. (2001a), differences in the dimensions of sweetgum and camphor in Modesto and in Santa Monica were due to different pruning regimes and cultural practices. While including more information at the genus level such as leaf foliar traits or wood characteristics (e.g. Dietze et al. (2008)) could certainly help explain variation in growth, our results show that an information on city-level management practices would do more to improve predictive performance, especially in unsampled cities.

There is growth rate (β_3) varies with climatic differences across cities, but there is still considerable uncertainty in these effects due largely to the relatively small sample size of cities and their uneven distribution in climate space. 25 years after planting, the model predicts that trees in warm and wet climates (e.g. central florida) will be about 60 cm greater in DBH than trees grown in cool dry climates (e.g. Fort Collins). While this is physically possible, the high uncertainty qualifies this estimate. Orlando is the only city sampled from a very warm and wet climate, it's large influence in the model and the high uncertainty suggests more cities need to be sampled in places like Louisiana and eastern Texas.

The mean asymptote for diameter growth of common urban species (β_1) is about 150 cm (95% CI: 131 - 171). This varies considerably by genus and at times in unrealistic ways due to data limitations. For example, the estimated mean asymptotic DBH for *Prunus* (cherry), a small tree is 194 cm, while for *Robinia pseudoacacia*, a larger tree, it is 107 cm. *Prunus* has such a large estimated asymptote because there are several trees younger than 30 with a DBH greater than 80 cm, but the oldest tree sampled was just 30 years old. Extrapolation and the interpretation of β_1 as the asymptotic dbh should be used with caution depending on the presence of observations at large ages. Half of the species observed have an oldest tree of less than 55 years. To make better predictions of diameter at large ages,

older trees are needed. Another alternative could be to create constraints on maximum DBH based on the largest individuals (called champion trees) of the species found anywhere (not just urban).

Estimates of uncertainty reveal where new observations are most needed. Sampling older trees would improve extrapolation to century scales, new cities in warm and wet climates will improve extrapolation to more regions across the country, sampling less common species will improve estimates of how all (not just common) urban trees grow, and repeated measures on individual tree would improve predictions for trees that are already planted (not just those at age 0).

We have modeling extensions phylo distance uncertainty in age individual tree level covariates like utility poles, sidewalk etc model more than just diameter, also height and crown.

new data: more cities less common species older trees management practices within individuals

Peper et al. (2001a) - tested modified weibull following Frelich (1992), but went with logarithm regression model because it had the best in-sample fit. We think weibull would have the best out of sample fit.

age several trees

- what is the distribution of maximum age by species? many have very young
- or the distribution of apps max?
- We need to be able to predict to older ages if we want to make realistic predictions.

bayesian approach where age can be treated as a random variable.

make better

- more data, duh, perhaps used results to identify where to sample
- more cities, this is important for interpolation across climate space. Could allow for nonlinear relationships and for more variables.

- better climate predictors
- interactions between climate and species.
- use phylogenetic distance, gaussian process, instead of multiple levels of taxonomy.
- an easy one is to nest genus with tree functional type (broad leaf deciduous, evergreen conifer, etc.)
- extend species with species level predictors (leaf morphology, wood characteristics, shade tolerance, etc).
- smarter priors (e.g. max dbh based on champion trees?) is this possible? I think it would be a very neat extension, but need to think about how these champions are not urban trees most the time. They provide the upper limit on the asymptote, but for urban trees the asymptote could be quite lower.
- incorporate uncertainty in AGE
- There were only 4 trees in Queens NY sampled.
- repeat measures on the same individuals would help much.
- Get more trees in the database, UFIA effort?
- add varying intercept for location of tree in sidewalk/underpowerlines etc.
- There are some funny things with minneapolis data. is it from the same individual for each species?
- challenges of separating β_1 and β_3 without old trees.
- urban trees don't live very long
- our sample of species is not representative.

this different? Well the urban part, management practices like topping, pollarding, pruning, drastically alter growth and tree dimensions. Others have discussed the difference between forest and urban/open grown trees a fair bit.

because of the richness of the dataset, the geographic range of that these equations can be applied to is larger than many other equations which are usually for a particular species in a particular region.

Unlike many past studies we do not have repeat measures on individual trees, but it would be straightforward to incorporate such data in the model.

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to address

- fitting multilevel models in the frequentist way (lme4) seems quite complex and is much over my head. Talk to Jun about this concern.
- why model correlations of parameters within groups?