Multilevel Urban Tree Allometric equations

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Model

likelihood

$$y_{igsc} \sim Gamma(\mu_{igsc}, \alpha_y)$$

$$\mu_{igsc} = \beta^0_{igsc} + \beta^1_{igsc}(1 - \exp(-\beta^2_{igsc} x^{\beta^3_{igsc}}_{igsc}))$$

where:

y_{igsc} is the diameter at breast height of tree i of genus, g, species, s, and city, c. y_{igsc} has a gamma distribution with mean, μ_{igsc}, and shape, α_y.

$i = 1, \ldots., n_{sc}$; where n_{sc} is the number of trees sampled for species, s, and city, c.

$g = 1, \ldots, G$; where G is the number of genera (G)

$s = 1, \ldots, S_g$; where S_g is the number of species in genus g.

x_{igsc} is the transplant age in years of tree $igsc$ (i.e. years since transplanting).

β^0_{igsc} is the intercept, or the diameter of a tree at time of transplanting.

β^1_{igsc} (plus β^0_{igsc}) is the asymptote of the sigmoidal weibull curve.

β^2_{igsc} and β^3_{igsc} affect the rate of growth. β^2_{igsc} provides flexibility to have slow or fast growth at young ages.

For each β^j_{igsc}, j = 0,1,2:

$$\beta^j_{igsc} = \beta^j_0 + \gamma^j_{gs} + \delta^j_c$$

for β^3_{igsc}:

$$\beta^3_{igsc} = \beta^3_0 + \tau_1 * \text{precip}_c + \tau_2 * \text{gdd}_c + \tau_3 * (\text{precip}_c * \text{gdd}_c) + \gamma^3_{gs} + \delta^3_c$$

where β^j_0 is the mean for β^j_{igsc}. γ^j_{gs} is genetic (genus and species) effect on β^j. δ^j_c is the city effect on β^j

The species effect is nested within the genus effect. Both are normally distributed, such that:

$$\gamma^j_{gs} \sim N(\gamma^j_g, \sigma^j_{genus:species})$$

$$\gamma^j_g \sim N(0, \sigma^j_{genus})$$

The effect of city is normally distributed:

$$\delta^j_c \sim N(0, \sigma^j_{city})$$

Priors:

Just describe the priors roughly, gamma, half normal etc. Then put the details in an appendix.

Priors were selected to be slightly informative and make very biologically unreasonable parameters improbable. The quanitity of data overwhelms the priors, but the relatively narrow priors also helps with sampling.