

1D Heat Eq. Solver

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

FDM:

Taylor series expansions:

$$u(x+\Delta x) = u(x) + \Delta x u'(x) + \frac{\Delta x^2}{2} u''(x) + \frac{\Delta x^3}{6} u'''(x) \dots$$

$$u(x-\Delta x) = u(x) - \Delta x u'(x) + \frac{\Delta x^2}{2} u''(x) - \frac{\Delta x^3}{6} u'''(x) \dots$$

Δx^3 and up are negligible error

$$\begin{aligned} u(x+\Delta x) &= u(x) + \Delta x u' + \frac{\Delta x^2}{2} u'' + \dots + \frac{\Delta x^4}{24} u^{(4)} \\ u(x-\Delta x) &= u(x) - \Delta x u' + \frac{\Delta x^2}{2} u'' - \dots + \frac{\Delta x^4}{24} u^{(4)} \end{aligned}$$

$$= 2u(x) + \Delta x^2 u'' + \underbrace{\frac{\Delta x^4}{12} u^{(4)} + \dots}_{\text{negligible error}}$$

$$u(x+\Delta x) + u(x-\Delta x) = 2u(x) + \Delta x^2 u''(x)$$

$$u''(x) = \frac{u(x+\Delta x) + u(x-\Delta x) - 2u(x)}{\Delta x^2}$$

$$\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2}$$

from Taylor series exp.

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u(x+\Delta x) + u(x-\Delta x) - 2u(x)}{\Delta x^2}$$

$$\text{and } \frac{\partial u}{\partial t} \approx \frac{u(t+\Delta t) - u(t)}{\Delta t}$$

$$\Rightarrow u(t+\Delta t) \approx \Delta t \frac{\partial u}{\partial t} + u(t)$$

$$\Rightarrow u(t+\Delta t) \approx \Delta t \left(a \frac{u(x+\Delta x) + u(x-\Delta x) - 2u(x)}{\Delta x^2} \right) + u(t)$$

$$\Rightarrow u_i^{n+1} \approx u_i^n + \frac{a\Delta t}{\Delta x^2} (u_{i+1}^n + u_{i-1}^n - 2u_i^n)$$

↳ in my program \Rightarrow

$$a = \frac{a\Delta t}{\Delta x^2}, \quad \text{surrounding} = (u_{i+1}^n + u_{i-1}^n - 2u_i^n)$$

$$\text{so } u_{\text{next}}[i] = u[i] - (\text{surrounding} * a)$$

$$== u_i^{n+1} = u_i^n + \frac{a\Delta t}{\Delta x^2} (u_{i+1}^n + u_{i-1}^n - 2u_i^n)$$

$$\text{surrounding} = \text{diff_left} + \text{diff_right}$$

$$= u[i] - u_{\text{left}}[i] + u[i] - u_{\text{right}}[i]$$

$$= 2u[i] - u_{\text{left}}[i] - u_{\text{right}}[i]$$

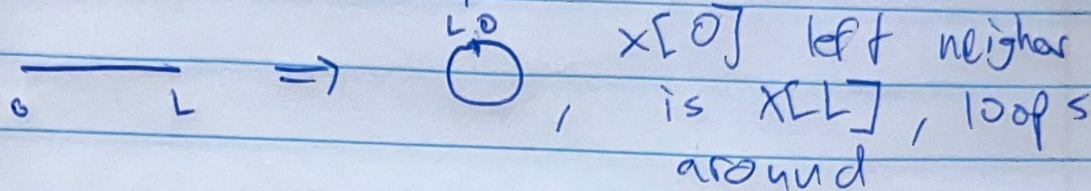
$$\Rightarrow -(2u_i^n + u_{i+1}^n + u_{i-1}^n)$$

20m
20/10 = 2

1D Heat Flow chart

use up, roll (k)
k=-1: shift array elements left
one entry, wraps

Periodic Boundary:



$\frac{L}{k}$
 $\frac{L}{k}$
 $= k$

Domain: $x \in [0, L]$

$u[0]$ left is $u[L]$
 $u[L]$ right is $u[0]$

$x = \text{array} [k]$ ~~entries~~

k entries, $\Delta x = \frac{L}{k}$

$$\frac{\partial u}{\partial t}(x, t_n) = \frac{u_i^{n+1} - u_i^n}{\Delta t}$$

$u(x)$ = heat at a point during i timestep

goal: $t: i \rightarrow i+1$

$\hookrightarrow u_i[k] \rightarrow u_{i+1}[k]$

where u at each $x[k]$ is updated based on its neighbors

$$\text{Surrounding} = u(x) - u(x+\Delta x) + u(x) - u(x-\Delta x) \\ = 2u(x) - u(x+\Delta x) - u(x-\Delta x)$$

debugging:

after 60 iterations, u starts increasing

$u = [.5, 1, .5, 0, -.5, -1, -.5, 0]$ $L=20$
 $u_{\text{right}} = [1, .5, 0, -.5, -1, -.5, 0, .5]$ $k=20$
 $u_{\text{left}} = [0, .5, 1, .5, 0, -.5, -1, -.5]$ $\Delta x = 1$

diff-right = $.5 - 1$ diff-left = $.5 - 0$

surrounding = 0

$u[1] = .5$ $u[2] = 1$ diff-left = $1 - .5 = .5$ right = $.5$
surrounding = 1
 $u[2] = 1 - (1 - .1) = .9$