Algorithms Homework- week 11

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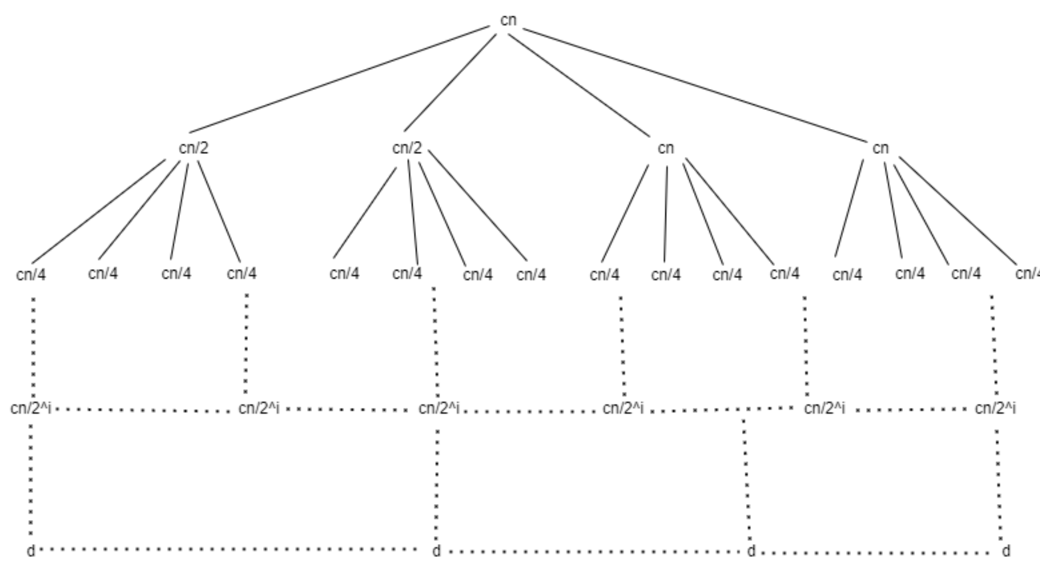
Recursion tree



Level	#nodes	cost
0	1	cn
1	4	$4\frac{cn}{2}$
2	16	$16\frac{cn}{4}$
..	..	..
i	2^{2i}	$2^{2i}\frac{cn}{2^i}$
..	..	..
logn	n^2	$n^2 d$

Substitution method

$$\sum_{i=0}^{logn-1} 2^i cn + n^2 d = cn\frac{2^0 - 2^{logn}}{1-2} + n^2 d$$

$$= -cn + cn^2 + n^2d = n^2(c + d - \frac{c}{n}) = \Theta(n^2)$$

Master theorem

$$T(n) = 4T(\frac{n}{2}) + cn, T(1) = d$$

$a = 4$, $b = 2$, $f(n) = cn$

$$n^{logba} = n^{log_2 4} = n^2$$

We have $f(n) = O(n^{2-\epsilon})$, with $\epsilon = 0.5$. For case 1 of the master theorem:

$$T(n) = \Theta(n^2)$$

4-1

b

$$T(n) = T(\frac{7n}{10}) + n$$

$a = 1$, $b = \frac{10}{7}$, $f(n) = n$.

$$n^{log_b a} = n^0$$

$f(n) = n^{0+\epsilon}$. With $\epsilon = 1$ we have $f(n) = \Omega(n^{0+\epsilon})$. f(n) grows polinomially faster than n^0.
For case 3 of the master theorem:

$$T(n) = \Theta(n)$$

c

$$T(n) = 16T(\frac{n}{4}) + n^2$$

$a = 16$, $b = 4$, $f(n) = n^2$.

$$n^{log_b a} = n^{log_4 16} = n^2$$

We have $f(n) = n^{log_b a}$, they grow at similar rates.
For case 2 of the master theorem:

$$T(n) = \Theta(n^2 logn)$$

d

$$T(n) = 7T(\frac{n}{3}) + n^2$$

$a = 7$, $b = 3$, $f(n) = n^2$.

$$n^{log_b a} = n^{log_3 7}$$

$f(n) = n^{0+\epsilon}$. With $\epsilon = 2 - n^{log_3 7}$ we have $f(n) = \Omega(n^{log_3 7+\epsilon})$. f(n) grows polinomially faster than $n^{log_3 7}$.
For case 3 of the master theorem:

$$T(n) = \Theta(n^2)$$

4-2

b

1

$$T(n) = 2T(\frac{n}{2}) + cn$$

$a = 2,\ b = 2,\ f(n) = cn.$

$$n^{logba} = n$$

We have $fn = cn$, so for the second case of the master theorem:

$$T(n) = \Theta(nlogn)$$

2

We assume N = n.

$$T(n) = 2T(\frac{n}{2}) + cn + 2\Theta(n) = 4n + cn + 2c(\frac{n}{2}) + 4T(\frac{n}{4}) =$$

We can apply the geometric series:

$$= \sum_{i=0}^{logn-1} (cn + 2^i n) = cnlogn + n\frac{1 - 2^{logn}}{1 - 2} = cnlogn + n^2 - n =$$

$$= \Theta(n^2)$$

3

$$T(n) = 2T(\frac{n}{2}) + cn + n = 2T(\frac{n}{2} + (c+1)n$$

$a = 2,\ b = 2,\ f(n) = (c+1)n.$

$$n^{logba} = n$$

We have $fn = (c+1)n$, so for the second case of the master theorem:

$$T(n) = \Theta(nlogn)$$