# Hierarchical model: SBC

## Prior SBC can't recommend model parameterisation

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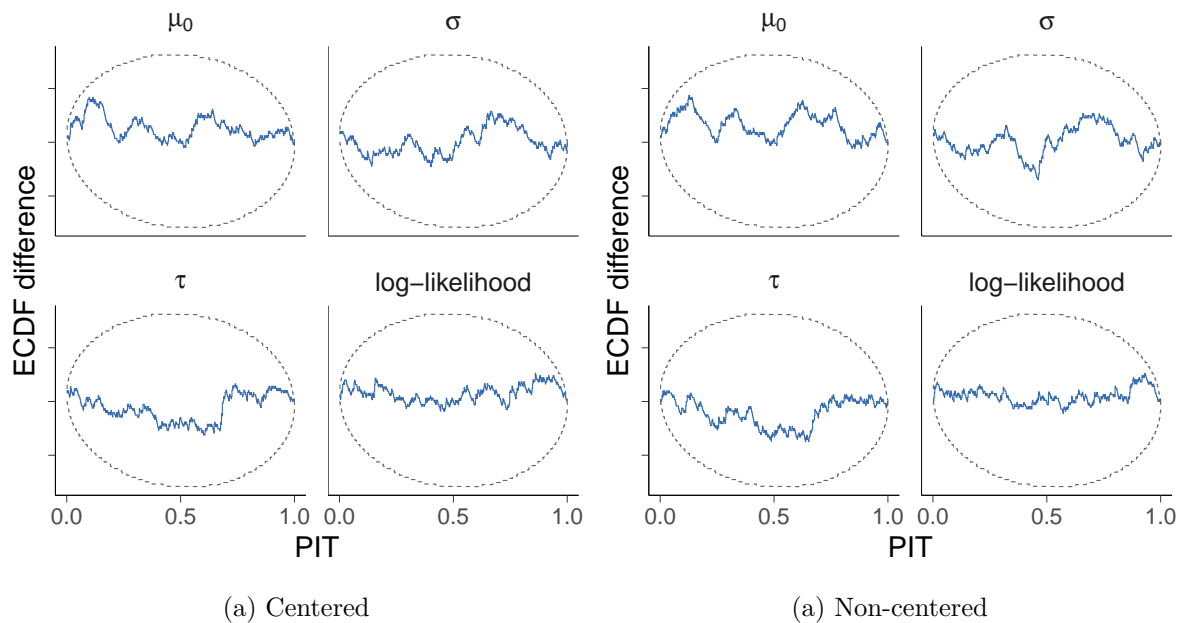2024-11-08

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Below, we have 50 groups of 5 observations with

$$\mu_0 \sim \mathcal{N}(0, 1),$$
$$\tau, \sigma \sim \mathcal{N}^+(0, 1),$$
$$\mu_j \sim \mathcal{N}(\mu_0, \tau),$$
$$y_{j,i} \sim \mathcal{N}(\mu_j, \sigma).$$

# 1 Prior SBC



(a) Centered          (a) Non-centered

Prior SBC for both CP and NCP looks good. No noticeable miscalibration.

## 1.1 Centered parametrisation

**SBC diagnostic messages**

```
SBC_results with 500 total fits.
 - No fits had errors.
 - No fits gave warnings.
 - 127 (25%) fits had at least one Rhat > 1.01. Largest Rhat was 1.605.
 - 115 (23%) fits had tail ESS undefined or less than half of the maximum
 rank, potentially skewing
the rank statistics. The lowest tail ESS was 11.
 If the fits look good otherwise, increasing `thin_ranks` (via
 recompute_SBC_statistics)
or number of posterior draws (by refitting) might help.
 - The lowest bulk ESS was 7
 - No fits had failed chains.
```

```
- 59 (12%) fits had divergent transitions. Maximum number of divergences
was 359.
- 1 (0%) fits had iterations that saturated max treedepth. Maximum
number of max treedepth was 319.
- 434 (87%) fits had some steps rejected. Maximum number of rejections
was 5.
- Maximum time per chain was 9.315 sec.
```

## 1.2 Non-centered parametrisation

```
SBC diagnostic messages


SBC_results with 500 total fits.
 - No fits had errors.
 - No fits gave warnings.
 - 172 (34%) fits had at least one Rhat > 1.01. Largest Rhat was 1.661.
 - 79 (16%) fits had tail ESS undefined or less than half of the maximum
 rank, potentially skewing
the rank statistics. The lowest tail ESS was 12.
 If the fits look good otherwise, increasing `thin_ranks` (via
 recompute_SBC_statistics)
or number of posterior draws (by refitting) might help.
 - The lowest bulk ESS was 7
 - No fits had failed chains.
 - No fits had divergent transitions.
 - 7 (1%) fits had iterations that saturated max treedepth. Maximum
number of max treedepth was 76.
 - 246 (49%) fits had some steps rejected. Maximum number of rejections
was 3.
 - Maximum time per chain was 5.546 sec.
```
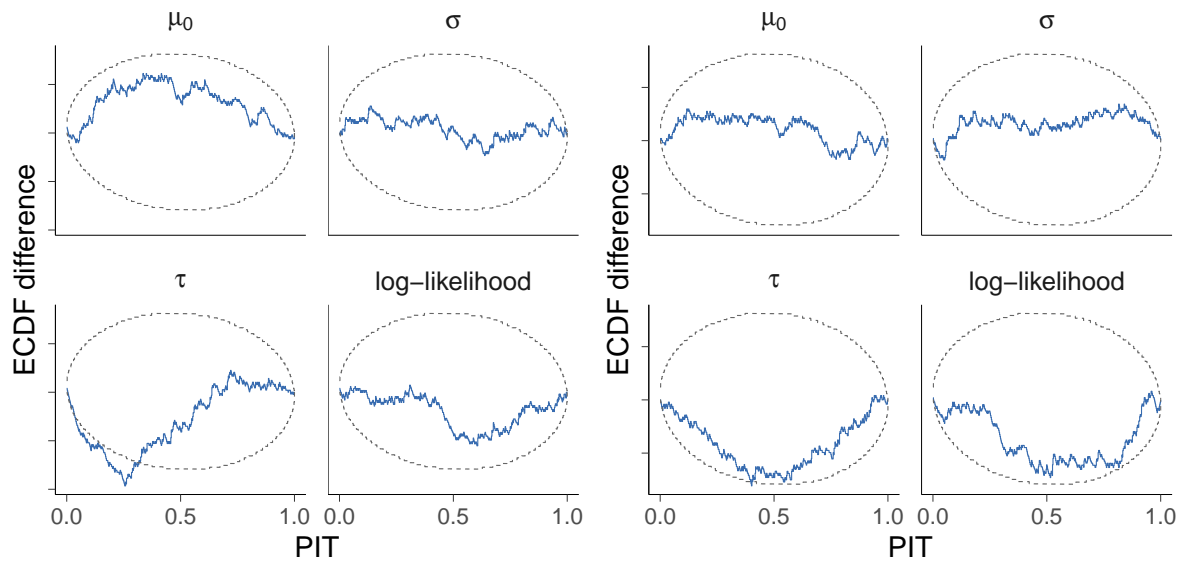
# 2 Posterior SBC

## 2.1 Weak likelihood

Generate data using $\mu_0 = 0$, $\tau_w = 0.06$ (the 5% quantile), and $\sigma = 1.96$ (the 95% quantile).

For the centered parameterisation, we see a dip in the PIT-ECDF of $\tau$. The dip is visible also for the non-centered parameterisation, with also the joint log-likelihood looking suspicious.

(a) Centered

(a) Non-centered

**SBC diagnostic messages**

```
SBC_results with 500 total fits.
 - No fits had errors.
 - No fits gave warnings.
 - 500 (100%) fits had at least one Rhat > 1.01. Largest Rhat was 1.77.
 - 494 (99%) fits had tail ESS undefined or less than half of the maximum
 rank, potentially skewing
the rank statistics. The lowest tail ESS was 8.
 If the fits look good otherwise, increasing `thin_ranks` (via
 recompute_SBC_statistics)
or number of posterior draws (by refitting) might help.
 - The lowest bulk ESS was 6
 - No fits had failed chains.
 - 344 (69%) fits had divergent transitions. Maximum number of
 divergences was 589.
 - 19 (4%) fits had iterations that saturated max treedepth. Maximum
number of max treedepth was 609.
 - 445 (89%) fits had some steps rejected. Maximum number of rejections
was 4.
 - Maximum time per chain was 24.182 sec.
```
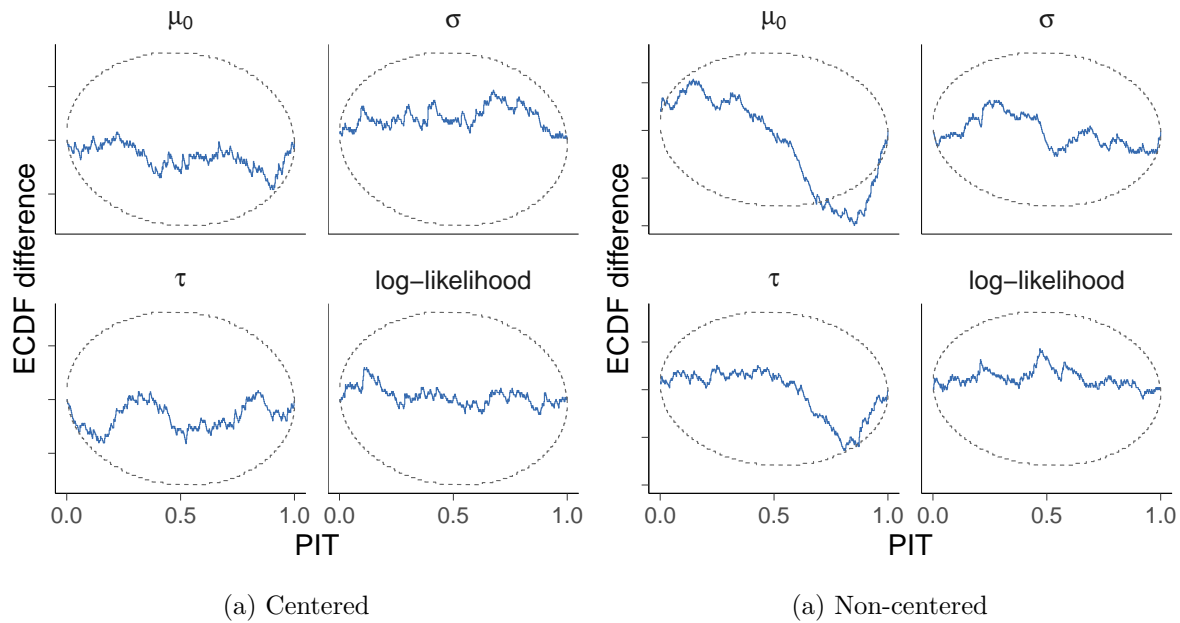
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> **SBC diagnostic messages**
>
> ```
> SBC_results with 500 total fits.
>  - No fits had errors.
>  - No fits gave warnings.
>  - 1 (0%) fits had at least one Rhat > 1.01. Largest Rhat was 1.011.
>  - All fits had tail ESS > half of the maximum rank.
>  - The lowest bulk ESS was 701
>  - No fits had failed chains.
>  - No fits had divergent transitions.
>  - No fits had iterations that saturated max treedepth.
>  - 252 (50%) fits had some steps rejected. Maximum number of rejections
>  was 4.
>  - Maximum time per chain was 2.388 sec.
> ```

## 2.2 Strong likelihood

Generate data using $\mu_0 = 0$, $\tau_w = 1.96$ (the 95% quantile), and $\sigma = 0.06$ (the 5% quantile).



(a) Centered  (a) Non-centered

This looks good.

The centered parameterisation is suitable for the observsation.

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Although not visible in the joint log-likelihood, if calibrated estimation of the population level parameters is important, the non-centered parameterisation would not be a good choice.

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**SBC diagnostic messages**

```
SBC_results with 500 total fits.
 - No fits had errors.
 - No fits gave warnings.
 - 4 (1%) fits had at least one Rhat > 1.01. Largest Rhat was 1.011.
 - All fits had tail ESS > half of the maximum rank.
 - The lowest bulk ESS was 1051
 - No fits had failed chains.
 - No fits had divergent transitions.
 - No fits had iterations that saturated max treedepth.
 - 455 (91%) fits had some steps rejected. Maximum number of rejections
was 5.
 - Maximum time per chain was 0.759 sec.
```

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**SBC diagnostic messages**

```
SBC_results with 500 total fits.
 - No fits had errors.
 - No fits gave warnings.
 - 500 (100%) fits had at least one Rhat > 1.01. Largest Rhat was 2.192.
 - 500 (100%) fits had tail ESS undefined or less than half of the
 maximum rank, potentially skewing
the rank statistics. The lowest tail ESS was 11.
 If the fits look good otherwise, increasing `thin_ranks` (via
 recompute_SBC_statistics)
or number of posterior draws (by refitting) might help.
 - The lowest bulk ESS was 5
 - No fits had failed chains.
 - No fits had divergent transitions.
 - 333 (67%) fits had iterations that saturated max treedepth. Maximum
 number of max treedepth was 111.
 - 280 (56%) fits had some steps rejected. Maximum number of rejections
was 4.
 - Maximum time per chain was 17.125 sec.
```