5.1

a). In zip.

b). .txt in zip, did not make table

c). No, because of overflow that can occure and rounding error occuring in closed form.

d). did not make plot.

5 1.

a) $a \to n$, $b \to n+$

Each position (ones, tens, hundreds, etc.) is multiplied with all the positions of the number we are multiplying with

For example, $a = 123$, $b = 456$

$$\begin{array}{r} 1\,2\,3 \\ \times\ 4\,5\,6 \\ \hline \end{array}$$

we would multiple 6 with 3, 2, 1
5 with 3, 2, 1
4 with 3, 2, 1

Then we ~~we~~ add the values

The multiplication is $\Theta(n^2)$ and addition is $\Theta(n)$.

$\therefore \Theta(n^2) + \Theta(n)$

Hence, the overall time complexity is $\Theta(n^2)$.

b).

First, consider, $x = 12$, which can be split into $x = 1 \cdot 10^{2/2} + 2$.

consider $z = 1234$

$z = 12 \cdot 10^{4/2} + 34$

$= 12 \cdot 10^{4/2} + 3 \cdot 10^{2/2} + 4$.

So, our general formula would be

$A = a \cdot 10^{n/2} + b$, where be ~~could~~ would be recursively solved.

~~So, B =~~

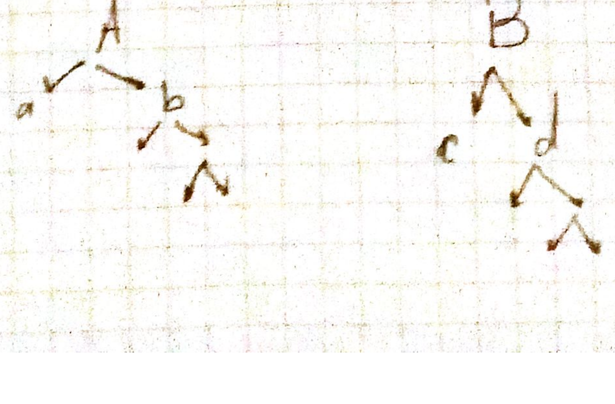
Let $B = c \cdot 10^{n/2} + d$

$A \times B = (a\,10^{n/2} + b)(c\,10^{n/2} + d)$

$= a \cdot c\,10^{n} + ad\,10^{n/2} + cb\,10^{n/2} + b \cdot d$

$= (a \cdot c\,10^{n}) + 10^{n/2}(a \cdot d + c \cdot b) + b \cdot d$

Diagram.



$a(a+b)(c+d)$
$ad + ac + bc + bd$

$\Rightarrow ac+bc = (a+b)(c+d) - ad - bd$
$ad+bc = (a+b)(c+d) - ac - bd$

Algorithm:

multiply (A, B)

n = max (A, B)

if (n == 1) :
    return A*B
else :
    temp = A
    temp2 = B
    a = temp / ($10^{n/2}$) //left side of A//
    b = temp % ($10^{n/2}$) //right side of A//

    c = temp2 / ($10^{n/2}$) //left side of B//
    d = temp2 % ($10^{n/2}$) //right side of B//

    ~~W = multiply (a, c)~~
    ~~X = multiply (a, d)~~
    ~~Y = multiply (b, c)~~
    ~~Z = multiply (b, d)~~

    ~~Y = Y + multiply (a, d) +~~

    W = multiply (a, c)

    X = multiply (b, d)

    Y = multiply (a + b, c + d)

return (W · (10 ^ n) + ((10 ^ (n/2)) · (Y - W - X)) + X)

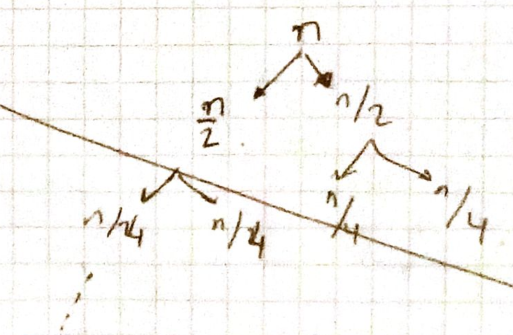
c).

$$T(n) = \underbrace{3\,T\left(\frac{n}{2}\right)}_{\text{divide}} + \underbrace{\Theta(n)}_{\text{conquer}}$$

divide (come from recursion)
$\Rightarrow$ 3 times because W, X, Y.

conquer (comes from the addition)

d).



n
n/2
n/4

$\Rightarrow n + \frac{n}{2} + \frac{n}{4} + \frac{n}{8}$

$\Rightarrow n\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right)$

$\Rightarrow 2n$.

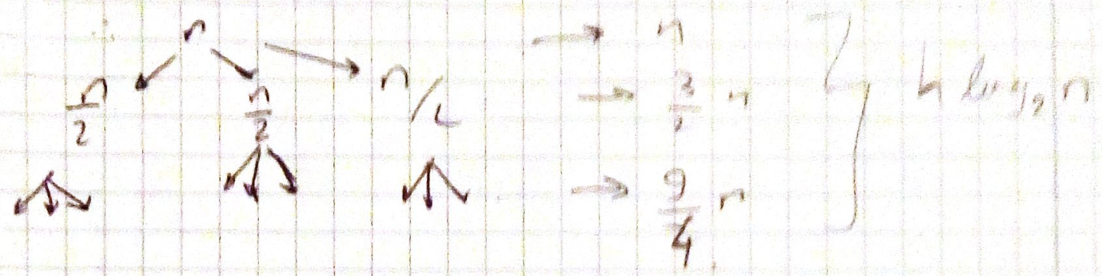
c) $a = 3$, $b = 2$.

$n^{\log_2 3} = n^{1.58}$.

$f(n) = n$.

$= O(n^{1.58 - \epsilon})$ for $\epsilon = 0.58$.

d)



$$\sum_{k=0}^{h} \left(\frac{3}{2}\right)^{k} \cdot n = n \sum^{\log_2 n} \left(\frac{3}{2}\right)^{k}.$$

$$= n\left[\frac{\left(1 - 3/2\right)^{\log_2 n}}{1 - \frac{3}{2}}\right]$$

$$= n\left[\frac{1 - n^{\log_2 3/2}}{-1/2}\right].$$

$$= 2n^{1.58} - 2n.$$

$= \Theta(n^{1.58}$.