H/W 3:

1.

a) $f(n) = 9n$ and $g(n) = 5n^3$.

$f \in \Theta(g)$.

For Θ we need to check.

- $f \in O(g)$
- $\lim_{n \to \infty} \frac{f(n)}{g(n)}$.

$= \frac{9n}{5n^3}$

$= \frac{9}{5n^2}$.

$= 0. < \infty$ ∴ $f \in O(g)$ is true

- $f \in o(g)$. $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$ ∴ $f \in o(g)$ is true
- $f \in \Omega(g)$

$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 < 0$ Not true ∴ $f \in \Omega$ but $f \notin \Theta(g)$ is not true.

- $f \in \Theta(g)$ ~~$f(n)$~~ $\Theta(g) \cap \Omega(g)$ gives us a relation which is false.
- $g \in O(f)$.

$\lim_{n \to \infty} \frac{g(n)}{f(n)} = \frac{5n^3}{9n}$

$= \frac{5}{9} n^2$

$= \infty$ ∴ relation not true.

- $g \in \Omega(f)$ ~~is t~~

$\lim_{n \to \infty} \frac{g(n)}{f(n)} = \infty > 0$ ∴ relation is true

- $g \in \Theta(f)$ ~~is fal~~

$\lim_{n \to \infty} \frac{g(n)}{f(n)} < \infty$ is false ∴ relation is false

- $g \in o(f)$.

$\lim_{n \to \infty} \frac{g(n)}{f(n)} \neq 0$ ∴ relation is false

- $g \in \omega(f)$.

$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$ ∴ relation is true

$f \in \omega(g)$.

$\lim_{n \to \infty} \frac{f(n)}{g(n)} \neq \infty$.

∴ relation not true

b). $f(n) = 9n^{0.8} + 2n^{0.3} + 14\log n$.

$g(n) = \sqrt{n}$.

i). $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{9n^{0.8} + 2n^{0.3} + 14\log n}{n^{1/2}}$.

$= \lim_{n\to\infty} \cdot 9\frac{n^{0.8}}{n^{0.5}} + 2\frac{n^{0.3}}{n^{0.5}} + 14 \cdot \frac{\log n}{n^{0.5}}$.

$= \lim_{n\to\infty} 9n^{0.3} + \frac{2}{n^{0.2}} + \frac{14\log n}{n^{0.8}}$.

$= \infty$.

ii). $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \frac{n^{0.5}}{9n^{0.8} + 2n^{0.3} + 14\log n}$.

$= \frac{\frac{n^{0.5}}{n^{0.8}}}{\frac{9n^{0.8}}{n^{0.8}} + \frac{2n^{0.3}}{n^{0.8}} + \frac{14\log n}{n^{0.8}}}$.

$= \frac{\frac{1}{n^{0.3}}}{9 + \frac{2}{n^{0.5}} + \frac{14\log n}{n^{0.8}}}$.

$= 0$.

- $f \in o(g)$ is false because bii $\neq 0$.
- $f \in O(g)$ is false because $\overset{eq^n}{=}$ bi> ~~bi shows lim~~ is not $< \infty$.
- $f \in \Omega(g)$ is true because eq^n bi> is > 0.
- $f \in \Theta(g)$ is false because eq^n bi> ~~states~~ is not $< \infty$.
- $f \in \omega(g)$ is true because eq^n bi> $= \infty$.

- $g \in o(f)$ is true because bii $= 0$.
- $g \in O(f)$ is true because bii $< \infty$.
- $g \in \Omega(f)$ is false because bii is not > 0.
- $g \in \Theta(f)$ is false because bii is not > 0.
- $g \in \omega(f)$ is false because bii $\neq \infty$.

c) $f(n) = \frac{n^2}{\log n}$ & $g(n) = n \log n$.

i) $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\frac{n^2}{\log n}}{n \log n} = \frac{n}{(\log n)^2}$

$= \lim_{n\to\infty} \frac{n}{(\log n)^2}$

$= \lim_{n\to\infty} \frac{\frac{dn}{dn}}{\frac{d(\log n)^2}{d(\log n)} \times \frac{d\log n}{dn}}$

$= \lim_{n\to\infty} \frac{1}{2\log n \times \frac{1}{n \ln 10}}$

$= \lim_{n\to\infty} \frac{n \ln 10}{2 \log n}$

$= \frac{\ln 10}{2} \lim_{n\to\infty} \frac{1}{\frac{1}{n \ln 10}}$

$= \frac{(\ln 10)^2}{2} \lim_{n\to\infty} n$

$= \infty$

ii) $\lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$.

- $f \in O(g)$ is false as $c_i \neq 0$.
- $f \in O(g)$ is false as c_i is not $< \infty$.
- $f \in \Omega(g)$ is true as c_i is > 0
- $f \in \Theta(g)$ is false as c_i is not $< \infty$.
- $f \in \omega(g)$ is true as $c_i = \infty$.

- $g \in o(f)$ is true as $c_{ii} = 0$.
- $g \in O(f)$ is true as $c_{ii} > 0$.
- $g \in \Omega(g)$ is false as c_{ii} is not > 0
- $g \in \Theta(g)$ is false as c_{ii} is not > 0.
- $g \in \omega(g)$ is false as $c_{ii} \neq \infty$.

d). $f(n) = (\log(3n))^3 = \left(\frac{\ln(3n)}{\ln(10)}\right)^3$, $g(n) = g\log n = \frac{g\ln(n)}{(\ln 10)}$.

i). $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\frac{1}{(\ln 10)^3} \times (\ln 3n)^3}{\frac{g\ln(n)}{(\ln 10)}}$.

$= \frac{1}{(\ln 10)^2} \lim_{n\to\infty}$

$= \frac{1}{g(\ln 10)^2} \lim_{n\to\infty}$.

$= \frac{1}{g(\ln 10)^2}$

$= \frac{1}{g(\ln 10)^2} \lim_{n\to\infty}$

$= \infty$.

$\frac{d(\ln 3n)^3}{d\ln 3n} \times \frac{d\ln 3n}{d3n} \times \frac{d3n}{dn}$

$\frac{d\ln(n)}{dn}$.

$\frac{3(\ln 3n)^2 \times \frac{1}{3n} \times 3}{\frac{1}{n}}$.

ii) $g(n) = \lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$.

- $f \in o(g)$ is false as di $\neq 0$.
- $f \in O(g)$ is false as di is not $< \infty$
- $f \in \Omega(g)$ is true as di > 0.
- $f \in \Theta(g)$ is false as di is not $< \infty$.
- $f \in \omega(g)$ is ~~false~~ true as di ~~=~~ ∞.

- $g \in o(g)$ is true as dii $= 0$.
- $g \in O(g)$ is true as dii $< \infty$.
- $g \in \Omega(g)$ is false as dii is not > 0.
- $g \in \Theta(g)$ is false as dii is not > 0.
- $g \in \omega(g)$ is false as dii $\neq \infty$.