H/W 3:

1.

a) $f(n) = 9n$ and $g(n) = 5n^3$.

$f \in \Theta(g)$.

For, Θ we need to check.

- $f \in O(g)$

$\lim_{n \to \infty} \frac{f(n)}{g(n)}$.

$= \frac{9\cancel{n}}{5n^{\cancel{3}}}$

$= \frac{9}{5n^2}$.

$= 0. < \infty$ $\boxed{\therefore f \in O(g) \text{ is true}}$

- $f \in o(g)$.
$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$ $\boxed{\therefore f \in o(g) \text{ is true}}$

- $f \in \Omega(g)$
$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 < 0$ Not true $\boxed{\therefore \cancel{f \in \Theta(g)}\ f \in \Omega \text{ is not true}}$.

- $f \in \Theta(g)$
~~f(n)~~ $O(g) \cap \Omega(g)$ gives us ~~an~~ relation which is false.

- $f \in \omega(g)$.
$\lim_{n \to \infty} \frac{f(n)}{g(n)} \neq \infty$.
$\therefore$ relation not true

- $g \in O(f)$.
$\lim_{n \to \infty} \frac{g(n)}{f(n)} = \lim_{n \to \infty} \frac{5n^3}{9n}$
$= \frac{5}{9} n^2$
$= \infty$
$\boxed{\therefore \text{ relation not true}}$

- $g \in \Omega(f)$ ~~is t~~
$\lim_{n \to \infty} \frac{g(n)}{f(n)} = \infty > 0$ $\boxed{\therefore \text{ relation is true}}$

- $g \in \Theta(f)$ ~~is fal~~
$\lim_{n \to \infty} \frac{g(n)}{f(n)} < \infty$ is false $\boxed{\therefore \text{ relation is false}}$

- $g \in o(f)$:
$\lim_{n \to \infty} \frac{g(n)}{f(n)} \neq 0$ $\boxed{\therefore \text{ relation is false}}$

- $g \in \omega(f)$.
$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$ $\boxed{\therefore \text{ relation is true}}$

b). $f(n) = 9n^{0.8} + 2n^{0.3} + 14\log n$.

$g(n) = \sqrt{n}$.

i).- $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{9n^{0.8} + 2n^{0.3} + 14\log n}{n^{1/2}}$.

$= \lim_{n\to\infty} 9\frac{n^{0.8}}{n^{0.5}} + 2\frac{n^{0.3}}{n^{0.5}} + 14 \cdot \frac{\log n}{n^{0.5}}$.

$= \lim_{n\to\infty} 9n^{0.3} + \frac{2}{n^{0.2}} + \frac{14\log n}{n^{0.8}}$.

$= \infty$.

ii). $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \frac{n^{0.5}}{9n^{0.8} + 2n^{0.3} + 14\log n}$.

$= \frac{\frac{n^{0.5}}{n^{0.8}}}{9\frac{n^{0.8}}{n^{0.8}} + 2\frac{n^{0.3}}{n^{0.8}} + \frac{14\log n}{n^{0.8}}}$.

$= \frac{\frac{1}{n^{0.3}}}{9 + \frac{2}{n^{0.5}} + \frac{14\log n}{n^{0.8}}}$.

$= 0$.

- $f \in o(g)$ is false because bi) $\neq 0$.
- $f \in O(g)$ is false because eq^n bi) ~~bi show lim~~ is not $< \infty$.
- $f \in \Omega(g)$ is true because eq^n bi) is $>$ 0.
- $f \in \Theta(g)$ is false because eq^n bi) ~~states~~ is not $< \infty$.
- $f \in \omega(g)$ is true because eq^n bi) $= \infty$.

- $g \in o(f)$ is true because bii $= 0$.
- $g \in O(f)$ is true because bii $< \infty$.
- $g \in \Omega(f)$ is false because bii is not > 0.
- $g \in \Theta(f)$ is false because bii is not > 0.
- $g \in \omega(f)$ is false because bii $\neq \infty$.

c) $f(n) = \frac{n^2}{\log n}$, $g(n) = n \log n$.

i). $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\frac{n^{\cancel{2}}}{\log n}}{\cancel{n}\log n} = \cancel{\frac{n}{(\log n)^2}}$

$= \lim_{n\to\infty} \frac{n}{(\log n)^2}$

$= \lim_{n\to\infty} \frac{\frac{dn}{dn}}{\frac{d(\log n)^2}{d(\log n)} \times \frac{d \log n}{dn}}$

$= \lim_{n\to\infty} \frac{1}{2\log n \times \frac{1}{n \ln 10}}$

$= \lim_{n\to\infty} \frac{n \ln 10}{2 \log n}$

$= \frac{\ln 10}{2} \lim_{n\to\infty} \frac{1}{\frac{1}{n \ln 10}}$

$= \frac{(\ln 10)^2}{2} \lim_{n\to\infty} n$

$= \infty$

ii) $\lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$.

- $f \in o(g)$ is false as ci $\neq 0$.
- $f \in O(g)$ is false as ci is not $< \infty$.
- $f \in \Omega(g)$ is true as ci is > 0
- $f \in \Theta(g)$ is false as ci is not $< \infty$.
- $f \in \omega(g)$ is true as ci $= \infty$

- $g \in o(f)$ is true as cii $= 0$.
- $g \in O(f)$ is true as cii < 0.
- $g \in \Omega(g)$ is false as cii is not > 0
- $g \in \Theta(g)$ is false as cii is not > 0
- $g \in \omega(g)$ is false as cii $\neq \infty$.

d). $f(n) = (\log(3n))^3 = \left(\frac{\ln(3n)}{\ln(10)}\right)^3$, $g(n) = 9\log n = \frac{9\ln(n)}{(\ln 10)}$

i) $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\frac{1}{(\ln 10)^{\cancel{3}\,2}} \times (\ln 3n)^3}{\frac{9\ln(n)}{(\ln 10)}}$

$= \frac{\cancel{1}}{(\ln 10)^2} \lim_{n\to\infty}$

$= \frac{1}{9(\ln 10)^2} \lim_{n\to\infty} \frac{\frac{d(\ln 3n)^3}{d\ln 3n} \times \frac{d\ln 3n}{d3n} \times \frac{d3n}{dn}}{\frac{d\ln(n)}{dn}}$

$= \cancel{\frac{1}{9(\ln 10)^2}}$

$= \frac{1}{9(\ln 10)^2} \lim_{n\to\infty} \frac{3(\ln 3n)^2 \times \frac{1}{\cancel{3n}} \times \cancel{3}}{\frac{1}{\cancel{n}}}$

$= \infty$

ii) $g(n) = \lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$

- $f \in o(g)$ is false as di $\neq 0$.
- $f \in O(g)$ is false as di is not $< \infty$
- $f \in \Omega(g)$ is true as di > 0
- $f \in \Theta(g)$ is false as di is not $< \infty$
- $f \in \omega(g)$ is ~~false~~ true as di $= \infty$

- $g \in o(g)$ is true as dii $= 0$
- $g \in O(g)$ is true as dii $< \infty$
- $g \in \Omega(g)$ is false as dii is not > 0
- $g \in \Theta(g)$ is false as dii is not > 0
- $g \in \omega(g)$ is false as dii $\neq \infty$

2a). Is in the file sorting.cpp

b) For this algorithm to be correct, it needs to pass 3 conditions: initialization, Maintainence, and Termination.

- Initialization. → The loop invariant must be true before 1st execution of the loop, which for selection sort is that the array is sorted for first i elements. In our case, the sorted array has no values in it, so it can be said that it is sorted.

- Maintainence: → If invariant is true before iteration, it should be true after iteration, which is true in our case as ~~we~~ after each iteration i elements would be sorted.

- Termination → when the loop is terminated the invariant should show something useful. At the end of the loop ~~then~~ all n elements would be sorted.

c) Case A and B are shown in sorting.cpp.

d). increased the sample size to 500 samples and the best, worst & averages cases were stored in its respective .txt files.

e) Looking at the code, its seems ~~O(n)~~ both best & worst case ~~the nub~~ the complexity was ~~the~~ :

$$\frac{n(n+1)}{2}.$$

∴ complexity was n^2; but in graph - there were some varients.