

FEATURES: HIGHER ORDER & CURRYING

PARTIALLY APPLIED FUNCTION

```
object FunctionPartiallyApplied {  
  def mul(x:Double, y:Double): Double = {  
    x*y  
  }  
  def partialMul(y:Double):Double = {  
    mul(3, y)  
  }  
}  
  
def main(args: Array[String]): Unit = {  
  val sum = (x: Double, y: Double, z: Double) => x + y + z //fully applied function  
  val f = sum(3, 5, _: Double)  
  
  println(f(2))  
  println(partialMul(3))  
}
```

} logic

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PARTIALLY APPLIED FUNCTION (APPLICATION)

→ Java

```
import java.util.Date

object FunctionPartiallyAppliedApplication {
  def dateMessage(date: Date, s: String): Unit = {
    println(date + ", " + s)
  }

  def main(args: Array[String]): Unit = {
    var date = new Date
    var newMessage = dateMessage(date, _:String)
    for(i: Int <- 0 ≤ .to(≤ 5)) {
      Thread.sleep( millis = 300)
      date = new Date
      newMessage("message " + i)
    }
  }
}
```

→ Java

```
Mon Feb 14 18:35:57 ICT 2022, message 0
Mon Feb 14 18:35:57 ICT 2022, message 1
Mon Feb 14 18:35:57 ICT 2022, message 2
Mon Feb 14 18:35:58 ICT 2022, message 3
Mon Feb 14 18:35:58 ICT 2022, message 4
Mon Feb 14 18:35:58 ICT 2022, message 5
```

CLOSURE

- A function that uses variable(s) declared outside the function.

```
object Closure {  
  var n = 5  
  val add = (x:Int) => x+n    //closure with n coming from outside  
  
  def main(args: Array[String]): Unit = {  
    println(add(2)) → 7      //closure with add coming from outside  
    n = 100  
    println(add(2)) → 102  
  }  
}
```

CLOSURE – WITH SIDE EFFECT ALLOWED ON VARIABLE (IMPURE CLOSURE)

```
object ClosureSideEffect {  
  var n = 5  
  val add = (x:Int) => {  
    n = x+n  
    n  
  } //closure with n coming from outside  
  
  def main(args: Array[String]): Unit = {  
    println(add(2)) //closure with add coming from outside  
    n = 100  
    println(add(2)) → 102  
    println(add(2)) → 104  
  }  
}
```

WHAT IS FUNCTIONAL PROGRAMMING?

- No changing variable.
- No assignment
- No loop
- Just focusing on functions.
- Functions can be defined anywhere, including in other functions.
- Functions can be passed as parameters and returned as results.
- There are operators that can compose functions.

WHAT ARE GOOD ABOUT FUNCTIONAL PROGRAMMING?

- Simpler reasoning.
- Good for multicore and cloud computing.
 - Avoid modifying variables by different parts of the program.
- Places to use (where we want scalable solutions)
 - Web
 - Trading platforms
 - Simulation

EVALUATING FUNCTION == EVALUATING EXPRESSION

- This substitution model (evaluating until getting a value) can be used as long as the function has no side effect.
 - `square(square(2))`
 - `square(4)`
 - `16`
- Example of side effect (cannot be expressed in a substitution model)
 - `x++`

RECURSION IS IMPORTANT IN THIS PARADIGM.

- Need to be able to think of it instead of loop.
- Recursion can be optimized to use only 1 stack frame (if you convert it to tail-recursion)
- But first, you must be more familiar with recursion.

PASCAL'S TRIANGLE (RECURSION EXERCISE – 5 MINS)

```
      1
    1 1
  1 2 1
1 3 3 1
  1 4 6 4 1
    1 5 10 10 5 1
      1 6 15 20 15 6 1
        1 7 21 35 35 21 7 1
```

```
def pascal(c: Int, r: Int): Int
```

Returns the number at column *c* in row *r*, where *c* and *r* start at 0, and value of *c* never exceeds value of *r*.

```
object PascalTriangle {
  def pascal(c:Int, r:Int):Int = {
    if (c==0) 1
    else if (c==r) 1
    else pascal(c-1,r-1)+pascal(c,r-1)
  }

  def main(args: Array[String]): Unit = {
    println(pascal(3,7))
  }
}
```

PARENTHESIS BALANCING EXERCISE (RECURSIVE 15 MINS)

- `def`
- `()()`
- `c`
- `c`
- `c`
- `l`

```
object Parenthesis {  
  def balance(chars: List[Char]): Boolean = {  
    balance(chars, acc = 0);  
  }  
  
  def balance(chars: List[Char], acc: Int): Boolean = {  
    if(chars.isEmpty && acc == 0) true  
    else if(chars.isEmpty && acc != 0) false  
    else if (acc < 0) false  
    else if (chars.head != '(' && chars.head != ')') balance(chars.tail, acc)  
    else if (chars.head == '(') balance(chars.tail, acc+1)  
    else balance(chars.tail, acc-1)  
  }  
  
  def main(args: Array[String]): Unit = {  
    println(balance("(if(zero?x) max(/1 x)).toList"))  
  }  
}
```

TAIL RECURSION


- If a function just calls another or call itself without any extra work, the language runtime system can optimize the function to use only one stack frame, just like using a loop.
- If you see a recursive function that is not tail-recursive, trying to make it tail-recursive will help optimize memory (stack frame) usage.

FACTORIAL (NON TAIL-RECURSIVE)

```
object Factorial {  
  def factorial(x: Int): Int = {  
    if (x == 0) return 1  
    x * factorial(x-1)  
  }  
  
  def main(args: Array[String]): Unit = {  
    println(factorial(4))  
  }  
}
```

FACTORIAL (TAIL-RECURSIVE)

-EXERCISE 5 MINS

```
object FactorialTail {  
  def factorial(x: Int, acc: Int): Int = {  
    if (x == 0) return acc  
    return factorial(x-1, x*acc)  
  }  
    
  def main(args: Array[String]): Unit = {  
    println(factorial(4, acc = 1))  
  }  
}
```

HIGHER ORDER FUNCTION

Take
functions as
arguments.

Can return
function.

```
object FunctionHigherOrder {                                Function as parameter
  def calculate(x: Double, y: Double, myF: (Double, Double) => Double): Double = {
    myF(x, y)
  }

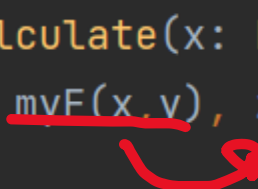
  def mul(x: Double, y: Double): Double = x * y

  def main(args: Array[String]): Unit = {
    println(calculate(3, 5, (a, b) => a + b))
    println(calculate(3, 5, mul))
  }
}
```

```
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```


CHAINING FUNCTIONS

```
object FunctionChain {  
  def calculate(x: Double, y: Double, z: Double, myF: (Double, Double) => Double): Double = {  
    myF(myF(x, y), z)  
  }  
}
```



```
def mul(x: Double, y: Double): Double = x * y
```

```
def main(args: Array[String]): Unit = {  
  println(calculate(3, 5, 7, (a, b) => a + b)) } same  
  println(calculate(3, 5, 7, _+_))  
  println(calculate(3, 5, 7, mul))  
  println(calculate(3, 5, 7, (a, b) => a min b)) } same  
  println(calculate(3, 5, 7, _ min _))  
}
```

LET'S DEFINE $\sum_{n=a}^b f(n)$ WHERE F CAN BE ANY FUNCTION

f {

```
object FunctionHigherOrderSum {  
  def sum(f: Int => Int, a: Int, b: Int): Int = {  
    if (a > b) 0  
    else f(a) + sum(f, a+1, b)  
  }  
  
  def id(a: Int): Int = a  
  def square(a: Int): Int = a * a  
  def factorial(x: Int, acc: Int): Int = {  
    if (x == 0) return acc  
    return factorial(x-1, x*acc)  
  }  
  def fac(a: Int): Int = factorial(a, acc = 1)  
  
  def main(args: Array[String]): Unit = {  
    println(sum(id, 2, 4)) // 2+3+4  
    println(sum(square, 2, 4)) // 2^2 + 3^2 + 4^2  
    println(sum(fac, 2, 4)) // 2! + 3! + 4!  
  }  
}
```

$\sum_{n=a}^b f(n)$ CAN BE WRITTEN USING TAIL RECURSION TOO (EXERCISE – 5 MINS)

- Write only the definition of function sum

```
def sum(f: Int => Int, a: Int, b: Int): Int = {  
  def sumAcc(a: Int, acc: Int): Int = {  
    if (a > b) acc  
    else sumAcc(a + 1, acc + f(a))  
  }  
  sumAcc(a, acc = 0)  
}
```

CURRYING - FUNCTION AS RETURN VALUE

- Function with multiple arguments ->
 - Function with one argument, returning another function.

```
val sum30 = addCurryShort(30) |  
println(sum30(1))
```

```
object Currying000 {  
  def add(x:Int,y:Int): Int = {  
    x+y  
  }  
  
  def addCurry(x:Int): Int => Int = {  
    (y:Int) => x+y  
  }  
  
  def addCurryShort(x:Int)(y:Int):Int = x+y  
  
  def main(args: Array[String]): Unit = {  
    println(addCurry(3)(5))  
  
    val sum20 = addCurry(20) //yes, it's partial execution  
    println(sum20(7))  
    println(addCurryShort(3)(5))  
  }  
}
```

use for partial execution

CURRYING — Example on $\sum_{n=a}^b f(n)$

```
object Currying {  
  def sum(f: Int => Int): (Int, Int) => Int = {  
    def sumF(a: Int, b: Int): Int = {  
      if(a > b) 0  
      else f(a) + sumF(a+1, b)  
    }  
    sumF  
  }  
}
```

```
def main(args: Array[String]): Unit = {  
  println(sum(id)(2, 4)) // 2+3+4  
  println(sum(square)(2, 4)) // 2^2 + 3^2 + 4^2  
  println(sum(fac)(2, 4)) // 2! + 3! + 4!  
}
```

```
var a = sum(square) // can be stored in variable to use later
```

CURRYING – SPECIAL SYNTAX (MULTIPLE PARAMETER LIST)

```
def sum(f: Int => Int)(a: Int, b: Int): Int = {  
    if(a > b) 0  
    else f(a) + sum(f)(a+1, b)  
}
```

The type of this function is

$(\text{Int} \Rightarrow \text{Int}) \Rightarrow ((\text{Int}, \text{Int}) \Rightarrow \text{Int})$ or $(\text{Int} \Rightarrow \text{Int}) \Rightarrow (\text{Int}, \text{Int}) \Rightarrow \text{Int}$

Since function types are right associative, so $\text{Int} \Rightarrow \text{Int} \Rightarrow \text{Int}$ is equivalent to $\text{Int} \Rightarrow (\text{Int} \Rightarrow \text{Int})$

EXERCISE: FACTORIAL IN TERMS OF PRODUCT? – 2 MINS

```
def product(f: Int => Int)(a: Int, b: Int): Int = {  
  if(a > b) 1  
  else f(a) * product(f)(a+1, b)  
}
```

```
def myFac(n: Int): Int = {  
  product(id)(1, n)  
}  
  
def main(args: Array[String]) {  
  println(product(id)(2, 4))  
  println(myFac(4))  
}
```

EXERCISE: WRITE A FUNCTION THAT CAN BE CHANGED TO USE EITHER SUM OR PRODUCT (EACH WITH 2 PARAMETER LIST) – 5 MINS

- Using the new function, in main, calculate $2+3+4$ and $2^2 * 3^2 * 4^2$

```
def general(f:Int => Int, op: (Int,Int) => Int, startValue:Int)(a:Int,b:Int):Int = {  
  if(a>b) startValue  
  else op(f(a),general(f,op,startValue)(a+1,b))  
}  
  
def main(args: Array[String]): Unit = {  
  println(general(id, (x,y) => x+y, startValue = 0)(2,4)) //2+3+4  
  println(general(square, (x,y) => x*y, startValue = 1)(2,4)) //2^2 * 3^2 * 4^2  
}
```