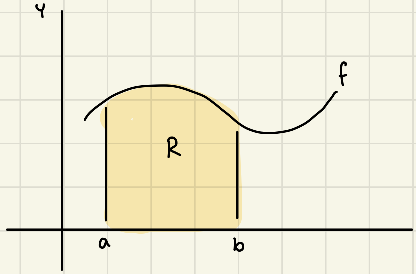
2. ประยุกต์ปริพันธ์

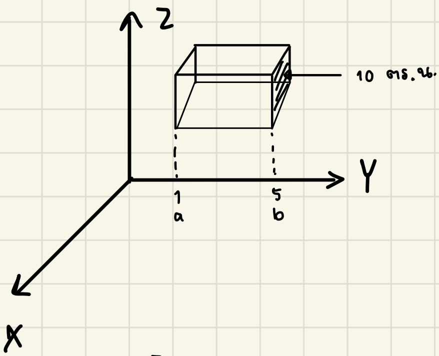
การประยุกต์ปริพันธ์



$$R = \int_a^b f(x)\,dx$$

$$= F(b) - F(a)$$

เมื่อ $\frac{d}{dx}F(x) = f(x)$



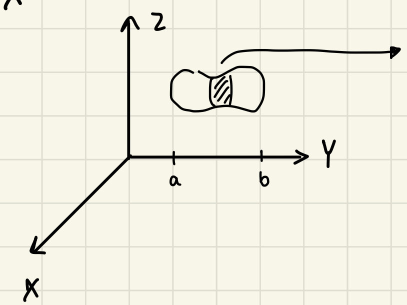
$$V = (5-1)\,10$$

$$= \int_1^5 10\,dx$$

$$= \int_1^5 \text{พท. หน้าตัด}\,dx$$

ตั้งฉากแกน x
เลยเทียบ x

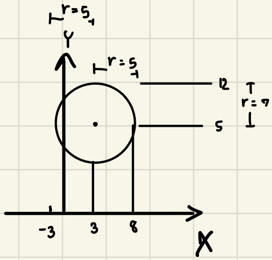
$$= \int_a^b \text{พท. หน้าตัด}\,dx$$

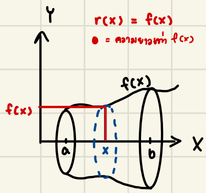


ปริมาตร $(V) = \int_a^b A(x)\,dx$

$A(x)$ → พท. หน้าตัด ที่สนใจ

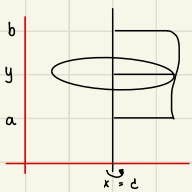
πr^2 ติดแค่ตัวแปร r





$$A(x) = \pi r^2(x)$$

$$\therefore \boxed{\begin{aligned} V &= \int_a^b A(x)\,dx \\ V &= \int_a^b \pi f(x)\,dx \end{aligned}}$$

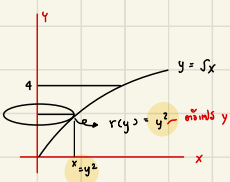


$x = g(y)$

$r(x) = g(y) - c$

$$V = \int_a^b \pi\,(g(y)-c)^2\,dy$$

<u>Ex</u> Δ $y = \sqrt{x}$ $y = 4$ และ แกน y หมุนรอบแกน y

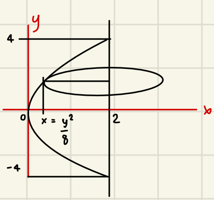


$$V = \int_0^4 \pi (y^2)^2\,dy$$

$$= \frac{\pi y^5}{5}\Big|_0^4$$

$$= \frac{4^5}{5}\pi$$

<u>Ex</u> Δ $y^2 = 8x$ $-4 \le y \le 4$ และ $x = 2$ หมุนรอบแกน y



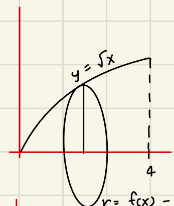
$$V = \int_{-4}^{4} \pi \left(2 - \frac{y^2}{8}\right)^2 dy$$

$2 > \frac{y^2}{8}$

<u>Ex</u> จงหาปริมาตรของทรงตันที่เกิดจากทรงตัน ที่เกิดจากการหมุนบริเวณที่ล้อมรอบด้วยเส้นโค้ง

$y = \sqrt{x}$ $x = 4$ และแกน x รอบแกน x

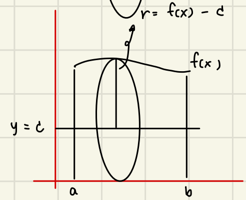
$$V = \int_0^4 \pi r^2(x)\,dx$$



$$= \int_0^4 \pi (\sqrt{x}^2)\,dx$$

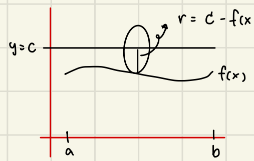
$$= \frac{\pi x^2}{2}\Big|_0^4$$

$$= 8\pi$$

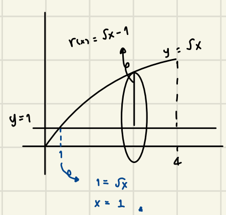


$$= \int_0^4 \pi r^2(x)\,dx$$

$$= \pi \int_0^4 (f(x) - c)^2\,dx$$

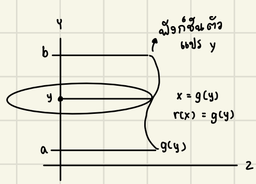


<u>Ex</u> Δ $y = \sqrt{x}$ $x = 4$ $y = 1$ รอบแกน $y = 1$



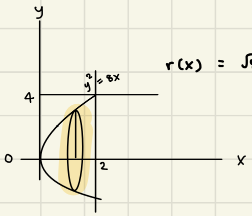
$$V = \int_1^4 \pi r^2(x)\,dx$$

$$= \pi \int_1^4 (\sqrt{x} - 1)^2\,dx$$



$$A(y) = \int \pi r^2(y)\,dy$$

Ex Δ $y^2 = 8x$ $0 \le y \le 4$ $x = 2$ หมุนรอบแกน x



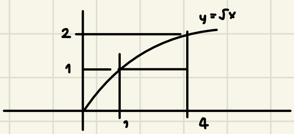
$$r(x) = \sqrt{8x}\,dx \quad V = \int_0^2 \pi r^2(x)$$

$$= \int_0^2 \pi (8x)\ dx$$

$$= \pi 4x^2 \Big|_0^2$$

$$= 16\pi$$

$y = \sqrt{x}$, $x = 1$, $x = 4$ แกน x [illegible]

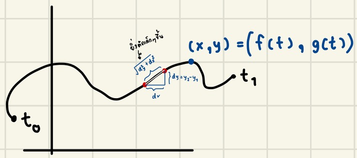


$$\int_0^2$$

11/01

ความยาวส่วนโค้ง

แต่ละจุดเป็นอนุภาค
ที่เคลื่อนที่ ขึ้นกับ เวลา
↳ มองเป็น ฟังก์ชันได้



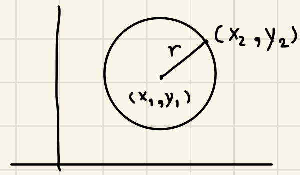
① ตัดเป็นเส้นเล็กๆ

$$S = \int_{t_0}^{t_1} ds$$

$$S = \int_{t_0}^{t_1} \sqrt{(dy)^2 + (dx)^2}$$

$$S = \int_{t_0}^{t_1} \sqrt{(g'(t)\,dt)^2 + (f'(t)\,dt)^2}$$

$$S = \int_{t_0}^{t_1} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\; dt$$



$$(x-h)^2 + (y-k)^2 = r^2$$

$$\left(\frac{x-h}{r}\right)^2 + \left(\frac{y-k}{r}\right)^2 = 1$$

$$\frac{x-h}{r} = \cos t \qquad \frac{y-k}{r} = \sin t$$

↳ $2\pi R$

$$\boxed{|\sin x| = \sqrt{\sin^2 x}}$$

$\int_0^{2\pi} |\sin t \cos t|$ แบ่งคิด 4 ช่วง

$$= \int_0^{\frac{\pi}{2}} (\sin t)(\cos t) + \int_{\frac{\pi}{2}}^{\pi} (\sin t)(-\cos t)$$

$$+ \int_{\pi}^{\frac{3\pi}{2}} (-\sin t)(-\cos t) + \int_{\frac{3\pi}{2}}^{2\pi} (-\sin t)(\cos t)$$

ความยาวเส้นโค้งในรูปสมการอิงตัวแปรเสริม

Ex $y = \frac{4\sqrt{2}}{3} x^{\frac{3}{2}} \qquad 0 \le x \le 1$

ให้ $x = t$, $y = \frac{4\sqrt{2}t^{3/2}}{3}$

$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = 2\sqrt{2}t^{\frac{1}{2}}$

$$S = \int_0^1 \sqrt{1^2 + (2\sqrt{2}t^{\frac{1}{2}})^2}\, dt$$

Ex $x = \frac{y^4}{4} + \frac{1}{8y^2} \qquad 1 \le y \le 2$

ให้ $y = t \qquad x = \frac{t^4}{4} + \frac{1}{8t^2}$ $8t^{-1}$

$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = t^3 - \frac{2}{8t^3}$

$$S = \int_1^2 \sqrt{1^2 + \left(t^3 - \frac{1}{4t^3}\right)^2}\, dt$$

$$= \int_1^2 \sqrt{1 + t^6 - \cancel{2}\left(\frac{1}{\cancel{2}4t^3}\right)(\cancel{t^3}) + \frac{1}{16t^6}}\, dt$$

$$= \int_1^2 \sqrt{1 + t^6 - \frac{1}{2} + \frac{1}{16t^6}}\, dt$$

$$= \int_1^2 \sqrt{\left(t^3 + \frac{1}{4t^3}\right)^2} \rightarrow t^6 + \frac{1}{2} + \frac{1}{16t^6}$$

$$= \int_1^2 \left(t^3 + \frac{1}{4t^3}\right) dt$$

Ex $y = \left(\frac{x}{2}\right)^{2/3} \qquad 0 \le x \le 2$

ให้ $x = t \qquad y = \left(\frac{t}{2}\right)^{\frac{2}{3}}$

$\frac{dx}{dt} = 1 \qquad \frac{dy}{dx} = \frac{\cancel{2}}{3}\left(\frac{t}{2}\right)^{-\frac{1}{3}}\left(\frac{1}{\cancel{2}}\right)$

$$S = \int_0^2 \sqrt{1^2 + \left[\left(\frac{1}{3}\right)\left(\frac{t}{2}\right)^{-\frac{1}{3}}\right]^2}$$

$$= \int_0^2 \sqrt{1 + \frac{1}{9}\left(\frac{t}{2}\right)^{-\frac{2}{3}}}\, dt \rightarrow$$

ไม่ออก

$=$

$x = 2y^{\frac{3}{2}}$

$y = t \qquad x = 2t^{3/2}$

$0 \le x \le 2$

$\therefore\ 0 \le 2t^{3/2} \le 2$

$0 \le t^{3/2} \le 1$

$0 \le t \le 1$

$\frac{dy}{dt} = 1$, $\frac{dx}{dt} = \frac{3}{2} \cdot 2 \cdot (t)^{-\frac{1}{2}}$

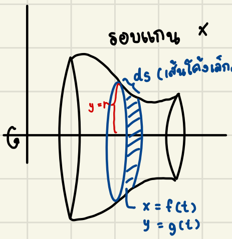
$$S = \int_0^1 \sqrt{1 + 9t}\, dt$$

$u = 1 + 9t$

พื้นผิวเกิดจากการหมุนเส้นโค้ง

↳ ทาสีแจกัน

รอบแกน x

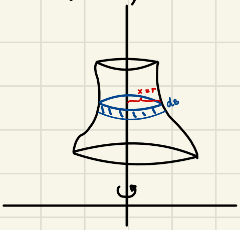


→ ถูกประมาณด้วยความสูงทรงกระบอกตัดให้เล็กมากๆ

$$\text{พ.ท.ผิว} = \int_{t_0}^{t_1} 2\pi \underbrace{\overset{y}{g(t)}}_{r} \underbrace{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}}_{ds}\, dt$$

$t \in [t_0, t_1]$

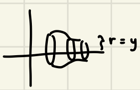
รอบแกน y



$$\text{พท.ผิว} = \int_{t_0}^{t_1} 2\pi \underbrace{\overset{x}{f(t)}}_{r} \underbrace{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}}_{ds}\, dt$$

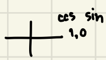
Ex หาพท.ผิวจากการ ↻ แกน x

$x = \cos t \qquad y = 1 + \sin t \qquad t \in [0, 2\pi]$



$$\text{พท.ผิว} = \int_0^{2\pi} 2\pi(1+\sin t)\sqrt{(-\sin t)^2 + (\cos t)^2}\, dt$$

$\frac{dx}{dt} = -\sin t$



$$= \int_0^{2\pi} 2\pi(1+\sin t)(1)\, dt$$

$\frac{dy}{dt} = \cos t$

$$= 2\pi(1+0) + 2\pi(1+0)$$

$$= 4\pi + c \quad \#$$

Ex □ ↻ แกน y

$x = t + \sqrt{2} \qquad y = \frac{t^2}{2} + \sqrt{2}t \ , \quad -\sqrt{2} \le t \le \sqrt{2}$

$$\text{พท.ผิว} = \int_{-\sqrt{2}}^{\sqrt{2}} 2\pi(t+\sqrt{2})\sqrt{1 + (t+\sqrt{2})^2}\, dt \to 2\pi(\cancel{t+\sqrt{2}})\sqrt{u}\cdot\frac{du}{2(\cancel{t+\sqrt{2}})}$$

$$u = 1 + (t+\sqrt{2})^2$$

$$\to \pi\int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{u}^{\,\frac{1}{2}+\frac{2}{2}}\, du$$

$\frac{dx}{dt} = 1 \ , \ \frac{dy}{dt} = t + \sqrt{2}$

$$du = 2(t+\sqrt{2})\, dt$$

$$\pi\left(\frac{2}{3}u^{\frac{3}{2}}\right)\Big|_{\sqrt{2}}^{-\sqrt{2}} \to \frac{2}{3}\left(1 + (t+\sqrt{2})\right)^{2\cdot\frac{3}{2}} \cancel{2}\pi \Big|_{\sqrt{2}}^{-\sqrt{2}}$$

Ex □ ↻ แกน x

$y = 2\sqrt{x}$ $\quad 1 \le x \le 2$

ให้ $x = t \qquad y = 2\sqrt{t}$

$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = t^{-\frac{1}{2}}$

$\sqrt{1 + \frac{1}{t}}$ } ติด x

พท.ผิว $= \int_{1}^{2} 2\pi(t)\sqrt{1 + \frac{t^2}{4}}$

$x = \frac{y^2}{4}$, $y = t$

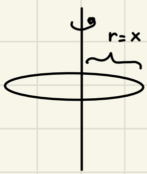
$\frac{dy}{dt} = 1$, $\frac{dx}{dt} = \frac{1}{2}t$

Ex □

$x + y = 1 \qquad 0 \le y \le 1$

$y = t \qquad \frac{dy}{dt} = 1$

$x = 1 - t \qquad \frac{dx}{dt} = 1$



พท.ผิว $= \int_{0}^{1} 2\pi(1-t)\sqrt{1^2+1^2}\ dt$

(ไม่ออก)

• ครึ่งมุม → $u = \tan\left(\frac{x}{2}\right)$

$\sin x = \frac{2u}{1+u^2}$

$\cos x = \frac{1-u^2}{1+u^2}$