Kibble Topology code

March 8, 2023

Abstract

This is to document the logic and structure of the code used to find the monopole and string distribution in a lattice where the Higgs field assigned at each lattice site has random Hopf angular coordinates assigned. Furthermore, the algorithm to transform the initially random Higgs field distribution in way that the monopoles and antimonopoles are brought in to a single tetrahedral lattice is described as well.

The setup of the procedure and the method of calculating the various quantities are described in the paper. The overall summary entails that a regular cubic lattice is transformed into a tetrahedral lattice. Each tetrahedral cell consists of triangular faces or plaquettes as illustrated in Fig .1. An example of the distribution of monopole-antimonopole pairs (*or dumbbells*) is shown in Fig .2.

The main quantities of interest here are the total topological winding for a given cell n_M and the overall phase resulting from parallel transporting the vector $\vec{n}$ around a plqauette δ. If $n_M = \pm 1$, there is a monopole(antimonopole) inside the cell and if $\delta = \pm 2\pi$, there is a string(antistring) passing through the plqautte as seen from a cell.

The sign of δ is important to understand for understanding the algorithm. Suppose we have a string passing from cell A into cell B another via a common face C. The value of δ as computed from cell A would be 2π and it would be -2π when computed from cell B. Thus, a positive sign for δ implies a string leaving the cell and a negative means it is entering the cell. This is illustrated in Fig .3

1 Tracing pairs and loops

Refer to flowcharts 4 and 5

2 Interpolating a cubic lattice magnetic field from a tetrahedral lattice

The topological analysis of the presence of monopoles and strings is most conveniently done on a tetrahedral lattice. Such a lattice is obtained by using a regular cubic lattice and discarding the points along the edges of a 3^3 cell as can be inferred from Figure 1. The differentiation scheme for obtaining derivatives at the points in a tetrahedral cell is not trivial since the nearest neighbors for points on the faces of the cell would differ from from the ones on the vertices as well as the one in the center. These derivatives would then be used to compute the magnetic field at the center, face centers and the vertices. The magnetic fields at the centers of edges would then be interpolated from the nearest neighbors. Additional attention needs to paid at any points on the boundaries, depending on the choice of boundary conditions.

3 Magnetic field spectrum

4 Hosking integral

5 Collapsing

Here, the methodology and algorithm to move monopoles along strings and into tetrahedral cells with antimonopoles resulting in a pair annihilation is described. These are strictly transformations of the Higgs doublet Φ based on topological criteria and, at best, mimic annihilation of dumbbells.

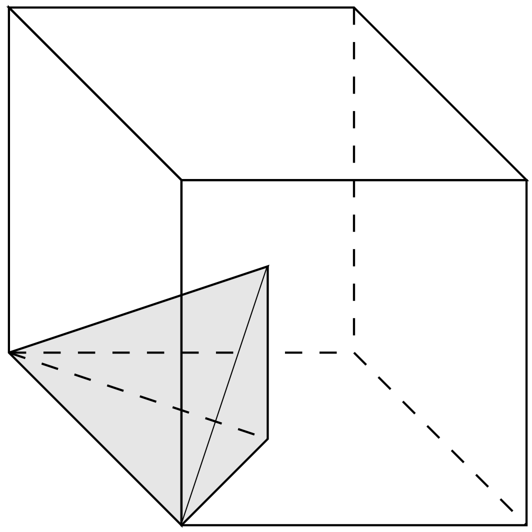

Figure 1:

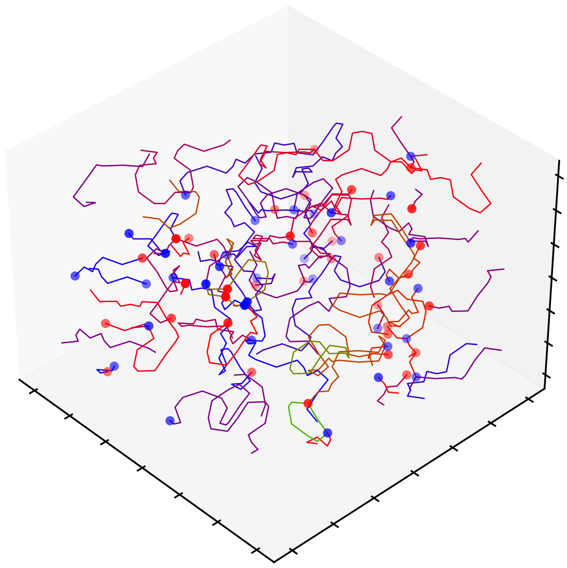

Figure 2:

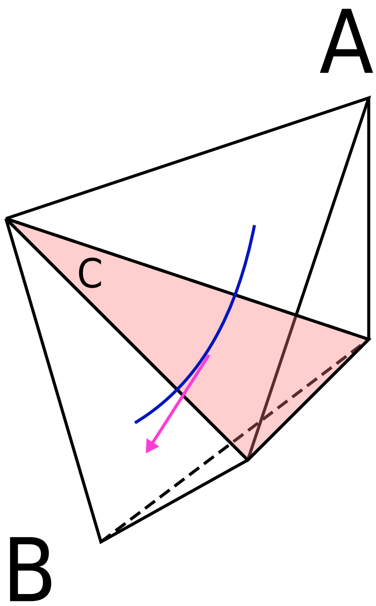

Figure 3:

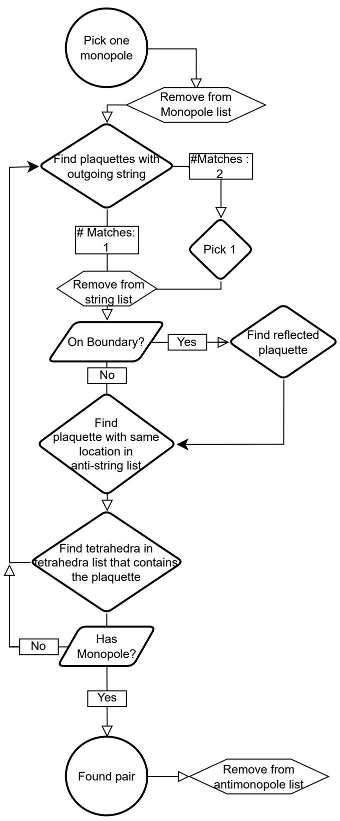

Figure 4: Figure illustrating algorithm to tracking a pair of monopole and antimonopole, with the string that connects them.

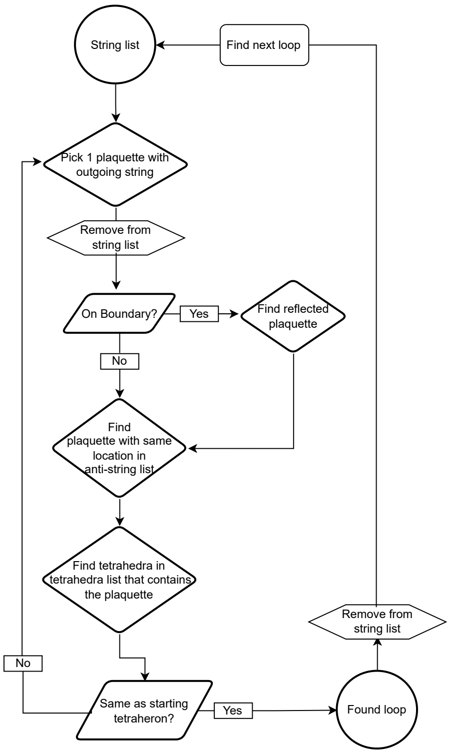

Figure 5: This is a flowchart that describes how to find loops after all the monopole-antimonopole pairs have been tracked

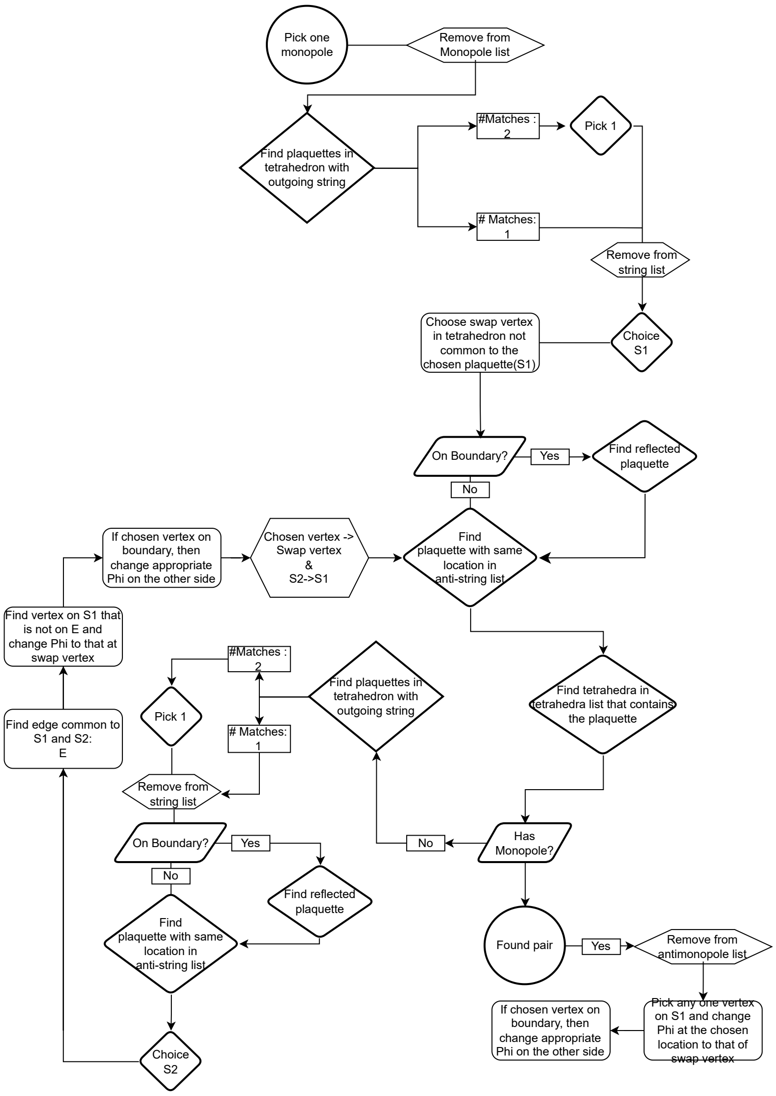

Figure 6: Algorithm flowchart illustrating the process of moving the monopole along the string until it reaches a tetrahedron containing an antimonopole.

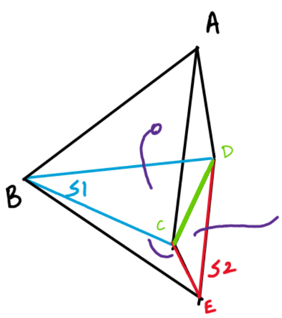

Figure 7: 2 adjacent tetrahedra cells noted by their respective vertices ABCD and BCDE. Here a monopole is illustrated in the tetrahedra ABCD and a string attached to it is passing through the triangular plaquette S1:BCD and into the tetrahedra BCDE and finally through the plaquette S2:CDE

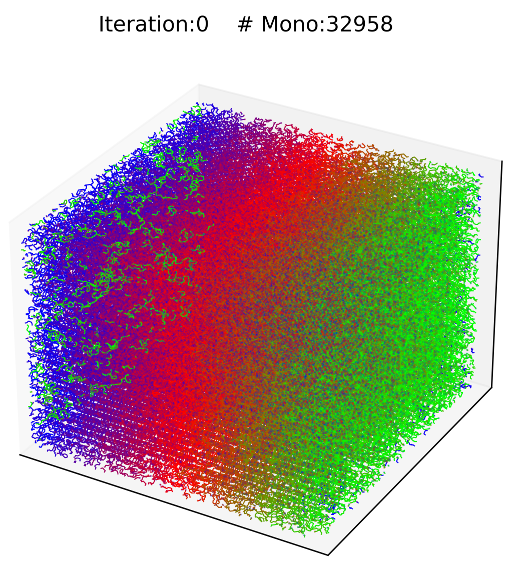

Figure 8: Initial distribution of monopoles and strings on a randomly populated 57^3 lattice

5.1 Swap tranformation

Figure 6 describes a simple algorithm. Figure 7 is an illustration of the transformation. Here, a monopole is in one of the cells and there is a string is passing through the adjacent cell. In this case, the common edge(CD) between the first(S1) and the second(S2) plaquettes through which the string passes through is found. The Higgs field at vertex(B) in the plaquette S1, that is not on the common edge (CD), is then switched to the Higgs at the vertex (A), that is in the starting cell and not in the plaquette (S1), or $\Phi(B) \to \Phi(A)$. **Outcomes** This process does guarantee that the monopole in the cell ABCD moves into an adjacent cell. The drawback is that the cell that it moves to is not the cell in which the string is passing into. Additionally, if there were no strings passing through any of the plaquettes except for S1, then this guarantees that the string passing through S1 disappears. Thus doing this transformation along every dumbbell, could lead to new monopoles and antimonopoles created. However, repeatedly performing this transformation eventually leads to a Higgs distribution that only has dumbbells within cells and thus there are no divergence terms in the corresponding magnetic field.

Simple collapse from swap Collapsing a distribution 57^3 with the simple swapping method required 361 transformation before all monopole-antimonopole pairs disappeared. The initial string distribution and after the 150th transformation are shown in figures 8 and 9

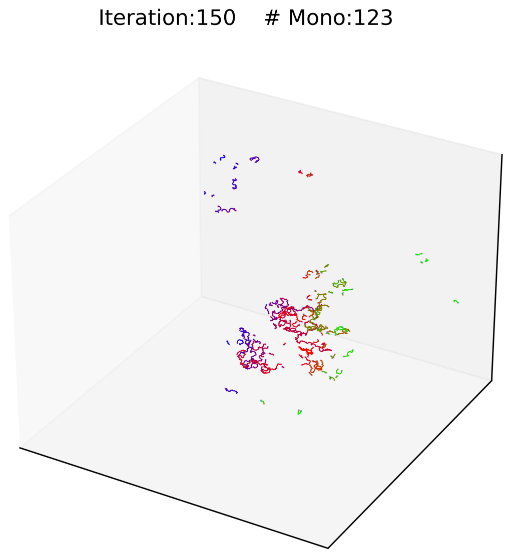

Figure 9: Distribution of monopoles after 150 simple swap transformations