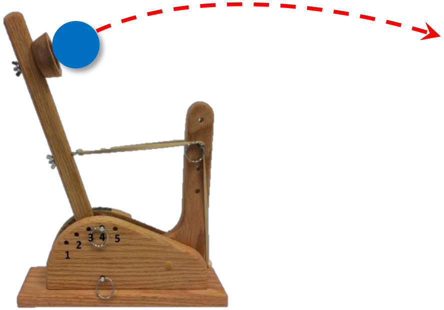
CHULA ENGINEERING
Foundation toward Innovation



Six Sigma Project

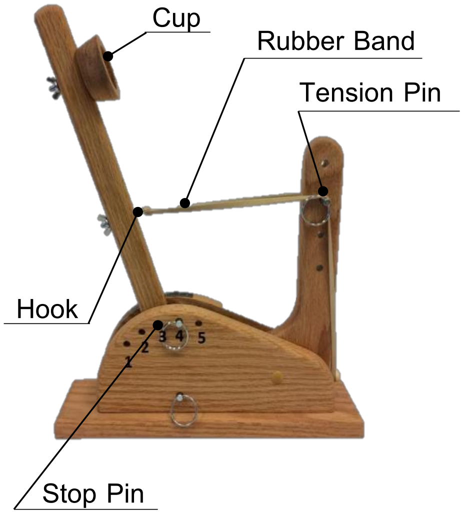
Presented by Group 1:

Thatchakarn	Chariyasethapong	6130239421
Teethavat	Techaphatipong	6130252521

Define Phase

Tools and Equipment

1 Catapult



2 Catapult base

3 Ball(s) and container

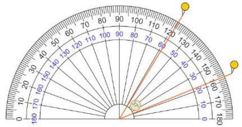
4 Cleaning Equipment

5 Charcoal Powder

6 Tape

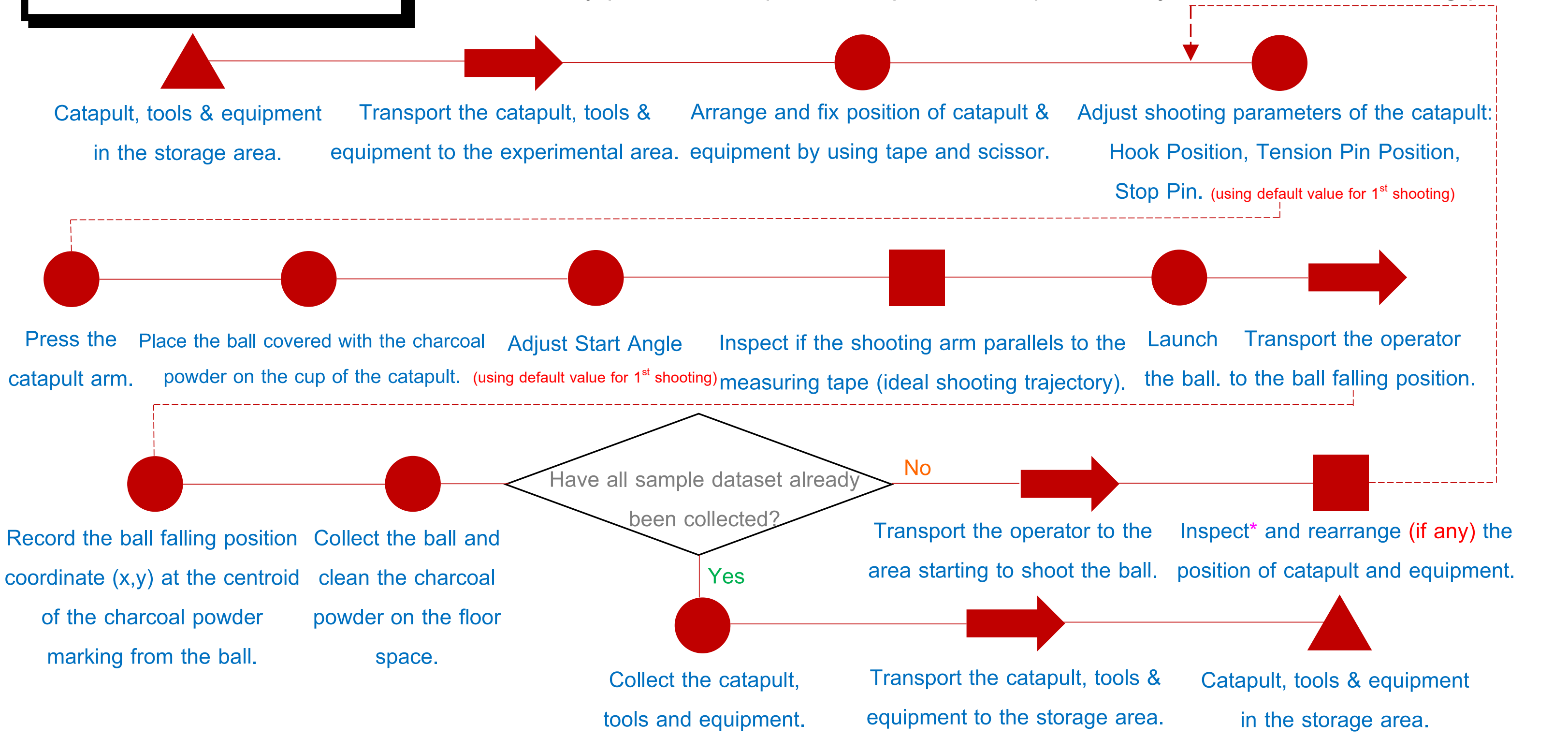
7 Scissor

8 Measurement Tools



Process Flow

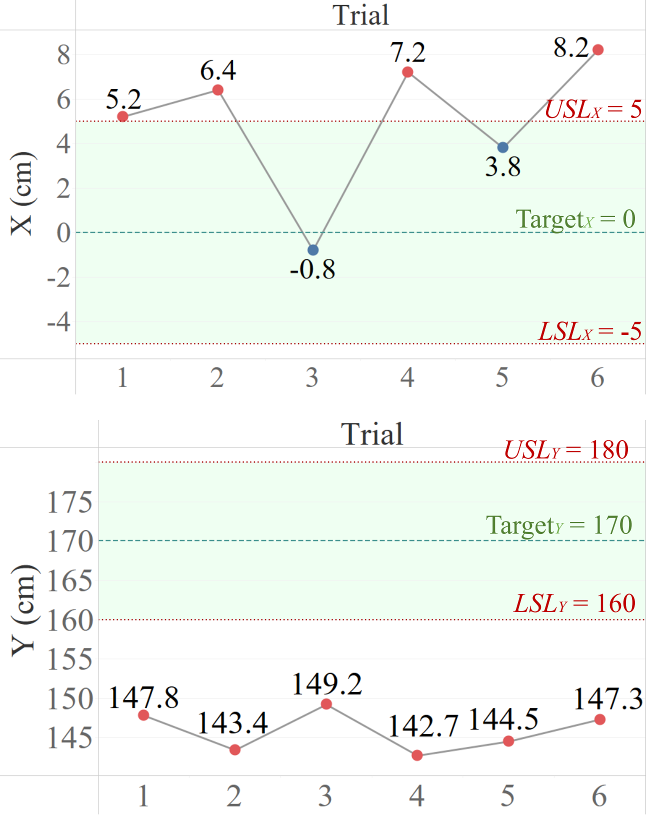
Assumption: The catapult is already finished goods so that no assembly process is required except for some position adjustment before launching process.



Remark: * Inspect the positions of catapult and equipment with the sampling frequency of 2 hr/time.

From 1st data collection:

Trial	X	Y
1	5.2	147.8
2	6.4	143.4
3	-0.8	149.2
4	7.2	142.7
5	3.8	144.5
6	8.2	147.3
Mean	5.00	145.82
SD	3.23	2.64



Out of specification
Within specification

$P(X \notin \text{spec}) = 66.67\%$

$\text{Fine}_{X\text{-bias}}$ = 5,000.00 THB

$\text{Fine}_{X\text{-variation}}$ = 4,172.80 THB

Fine_X = 9,172.80 THB

ΣFine = 128,928.99 THB

Net reward = [- ฿28,928.99] = ฿0

Fine_Y = 119,756.19 THB

$\text{Fine}_{Y\text{-bias}}$ = 116,966.72 THB

$\text{Fine}_{Y\text{-variation}}$ = 2,789.47 THB

$P(Y \notin \text{spec}) = 100\%$

SIX-SIGMA PROJECT CHARTER

Quality improvement of the catapult shooting system for the competition			
Project Sponsor	Master Blackbelt	Blackbelt	
XXXXX X.	Napassavong O.	Thatchakarn C.	Teethavat T.
Executive Leader	Team Members		
YYYYY Y.	AAAAA A.	BBBBB B.	CCCCC C.

Business Case

Compared with the falling distances of the competition's target, the performance from the current settings of the catapult was too low, which may result the team to have high level of fine, leading the team to miss the chance receiving the reward as well as the extra reward if the team had become the first rank of the competition after the team had invested many resources, including money and time, to this competition.

Project Metrics

Business Metric:	Ranking of the competition
Primary Metric:	Mean and standard deviation of falling distances (X,Y)
Secondary Metric:	Knowledge from using Six Sigma methodology and tools
Consequential Metric:	Investment including time consumed in this project and improvement cost
Financial Metric:	Fine and received reward calculated from the falling distances of the ball compared with those of the target

Problem Statement

In February, it was found that the catapult used in the competition can shoot the ball with mean and standard deviation at for 5.00 and 3.23 respectively for X-axis and 145.82 and 2.64 respectively for Y-axis, resulting the team with high amount of fine at 128,928.99 THB and not receiving any money reward.

Objective Statement

The project aims to increase the short-term actual capability index (C_{pk}) of the process from 0.38 in X-axis and -1.42 in Y-axis to 1.33 by 1st May 2022.

Project Constraints

- Total project budget ≤ 100k THB
- Time consuming of team for project consulting ≤ 2 days/week
- Cup position is not allowed for position adjustment.

Project Scope

The project scope covers the study of the given catapult that will be used for the competition.

Project Assumptions

- Workdays of each week range between Friday to Sunday.
- Progress summary of project is required to be reported to master blackbelt and project sponsor in (mostly) Sunday, and the performance as well as the feedback of the project progress will be later discussed on Tuesday morning.

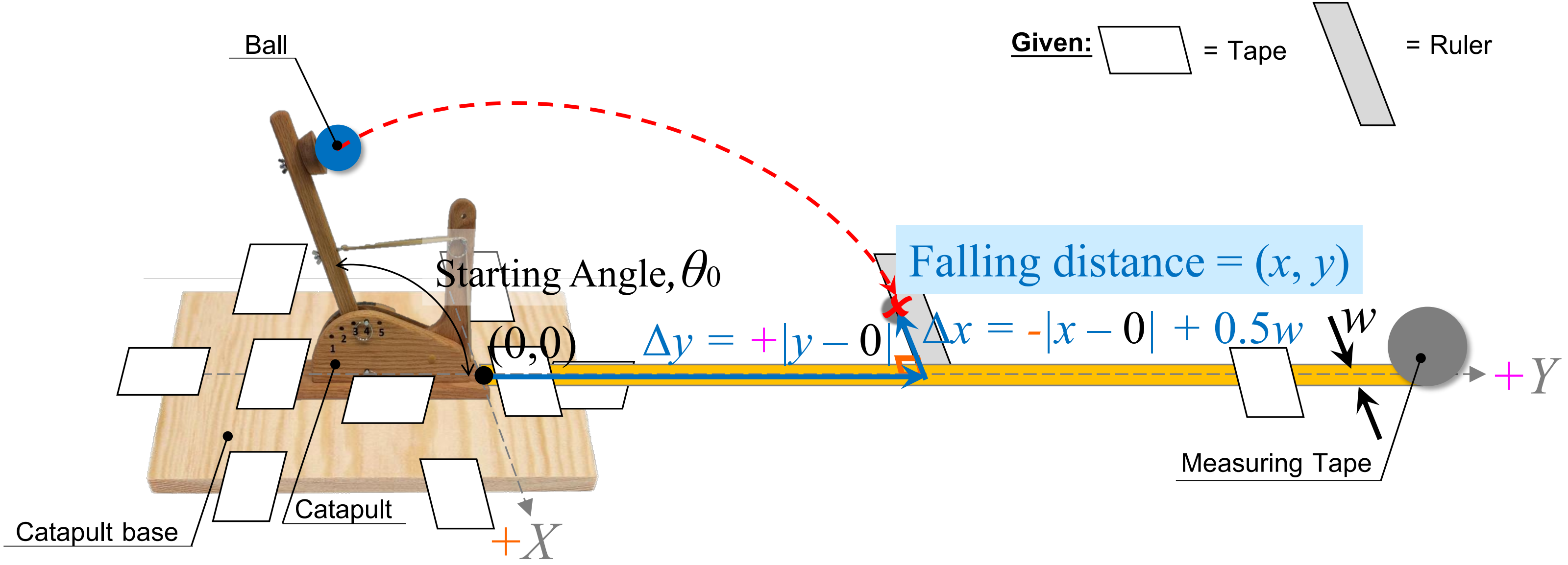
Project Timeline

- Define phase: 01/02/2022 to 06/02/2022
- Measure phase: 08/02/2022 to 06/03/2022
- Analyze phase: 22/02/2022 to 10/04/2022
- Improve phase: 12/04/2022 to 01/05/2022
- Control phase: 26/04/2022 to 01/05/2022

Measure Phase

Measuring the falling distance

Def: Falling distance is the distance from starting point $(0,0)$ at the middle of catapult tip to the falling position of the ball (x, y), measured by using cartesian coordinate.



Sample-size (n) Determination

To estimate population mean

Assumption: Population size is unknown.

X-axis

$$\min(n_{\text{for } \mu_x \approx \bar{x}}) \approx [40.07] = 41$$

Given: Error of sample mean from population mean, $e = 1$ cm [Accept 20% of (|SL - Target| = 5 cm)]

Confidence level, $(1-\alpha) \times 100\% = 95\%$

Population standard deviation, $\sigma_x \approx \hat{\sigma}_x = 3.23$ cm

Y-axis

$$\min(n_{\text{for } \mu_y \approx \bar{y}}) \approx [6.70] = 7$$

Given: Error of sample mean from population mean, $e = 2$ cm [Accept 20% of (|SL - Target| = 10 cm)]

Confidence level, $(1-\alpha) \times 100\% = 95\%$

Population standard deviation, $\sigma_y \approx \hat{\sigma}_y = 2.64$ cm

Sample-size (n) Determination

To estimate population standard deviation

X-axis

$$\min(n_{\text{for } \sigma_x \approx S_x}) \approx 84$$

Given: Width of confidence interval, $w = 1$ cm [Accept 1/3 of Sample Standard deviation]

Confidence level, $(1-\alpha) \times 100\% = 95\%$

Sample standard deviation, $S_x = 3.23$ cm

Y-axis

$$\min(n_{\text{for } \sigma_y \approx S_y}) \approx 58$$

Given: Width of confidence interval, $w = 1$ cm [Accept 1/3 of Sample Standard deviation]

Confidence level, $(1-\alpha) \times 100\% = 95\%$

Sample standard deviation, $S_y = 2.64$ cm

Sample-size (n) Determination

To estimate population proportion

Assumption: Population size is unknown.

X-axis

$$\min(n_{\text{for } p_x \approx \hat{p}_x}) \approx \lceil 85.37 \rceil = 86$$

Given: Error of sample proportion from population proportion, $e = 0.1$ [Accept 10% of proportion deviation]

Confidence level, $(1-\alpha) \times 100\% = 95\%$

Population proportion, $p_x \approx \hat{p}_x = 0.666666\ldots$

Y-axis

$$\min(n_{\text{for } p_y \approx \hat{p}_y}) \approx \lceil 3.8 \times 10^{-11} \rceil = 1$$

Given: Error of sample proportion from population proportion, $e = 0.1$ [Accept 10% of proportion deviation]

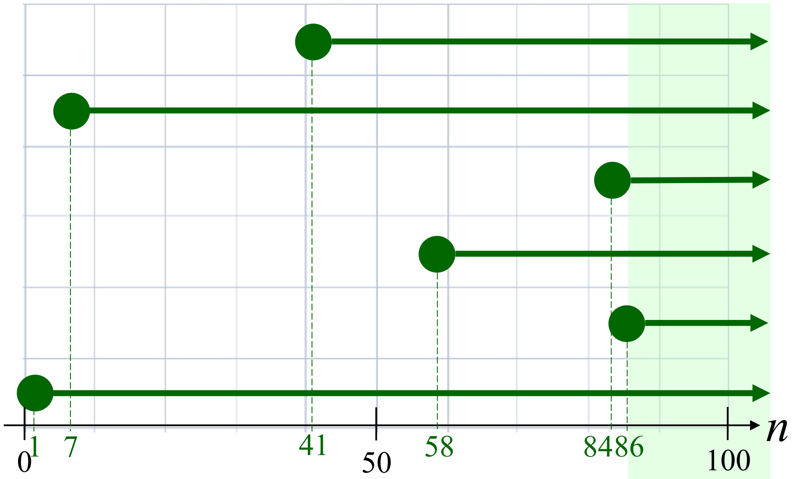
Confidence level, $(1-\alpha) \times 100\% = 95\%$

Population proportion, $p_y \approx \hat{p}_y = 0.999999\ldots$

Sample-size (n) Determination

Given: Sample size from 1st data collection is six.

Parameter Estimation	Required sample size
$\mu_x \approx \hat{\mu}_x$	≥ 41
$\mu_y \approx \hat{\mu}_y$	≥ 7
$\sigma_x \approx \hat{\sigma}_x$	≥ 84
$\sigma_y \approx \hat{\sigma}_y$	≥ 58
$p_x \notin \text{spec} \approx \hat{p}_x \notin \text{spec}$	≥ 86
$p_y \notin \text{spec} \approx \hat{p}_y \notin \text{spec}$	≥ 1



∴ To estimate all three parameters, at least sample size of 86 is required.

Justify: **The additional samples that should be later collected is no less than 80.**

If the sample size was given at 86 and the remaining planning values was set as same as when the sample sizes were estimated,

then either the error (e) for estimating population mean and population proportion

or the width (w) of confidence interval (C.I.) in estimating the population standard deviation will be revised as the following:

Parameter Estimation	Planning values	Given sample size	Revised error/width of C.I.
$\mu_x \approx \hat{\mu}_x$	$\alpha = 0.05$, $\sigma_x = 3.23$ cm	86	$e = 0.6826$
$\mu_y \approx \hat{\mu}_y$	$\alpha = 0.05$, $\sigma_y = 2.64$ cm	86	$e = 0.5581$
$\sigma_x \approx \hat{\sigma}_x$	$\alpha = 0.05$, $S_x = 3.23$ cm	86	$w = 0.9918$
$\sigma_y \approx \hat{\sigma}_y$	$\alpha = 0.05$, $S_y = 2.64$ cm	86	$w = 0.8109$
$p_{x \notin \text{spec}} \approx \hat{p}_{x \notin \text{spec}}$	$\alpha = 0.05$, $p_x = 0.6666\ldots$	86	$e = 0.09963$
$p_{y \notin \text{spec}} \approx \hat{p}_{y \notin \text{spec}}$	$\alpha = 0.05$, $p_y = 0.9999\ldots$	86	$e = 6.684 \times 10^{-8}$

2nd data collection:

- Shooter Order: OP1
- Measurer's Name: AAAAA A.

Subgroup	Observation	*X* (cm)	*Y* (cm)	Subgroup	Observation	*X* (cm)	*Y* (cm)
1	1	8.7	140.4	6	16	4.6	144.5
	2	8.2	140.6		17	4.9	145.7
	3	4.5	148.2		18	0	141
2	4	10.3	135.6	7	19	-5.5	140
	5	5.2	142.3		20	3	135.5
	6	5	144.4		21	4.6	142.5
3	7	4.3	145.7	8	22	3.2	142
	8	6.2	149.4		23	-2	148.7
	9	8.7	139.4		24	4.6	149
4	10	7.7	141.7	9	25	7.4	144.5
	11	-2.5	145.3		26	3.5	140.9
	12	-5	147.4		27	4.5	139.6
5	13	7.8	137.2	10	28	4.1	133.5
	14	-6.5	147		29	2	127.8
	15	1.7	143.3		30	3.5	139.9

2nd data collection:

- Shooter Order: OP2
- Measurer's Name: BBBBB B.

Subgroup	Observation	*X* (cm)	*Y* (cm)	Subgroup	Observation	*X* (cm)	*Y* (cm)
11	31	-0.7	143.7	16	46	-2.5	137.5
	32	-2.6	146.5		47	-1	149.4
	33	-0.6	144.1		48	-1.4	146
12	34	4.6	142.9	17	49	6.5	147.1
	35	0	139.7		50	0	157.8
	36	2.2	141.5		51	-4.7	144
13	37	-6.5	148.7	18	52	-2	147.1
	38	-1.3	144.7		53	-2.9	150.6
	39	2.6	158.6		54	4.8	147.3
14	40	1.1	142	19	55	3.7	150.7
	41	0	144.3		56	0	146.4
	42	-1.3	139.6		57	4.2	147.3
15	43	2.7	141	20	58	1.5	142.9
	44	-1	144		59	1.5	150.3
	45	2.1	139.6		60	4.3	152.9

2nd data collection:

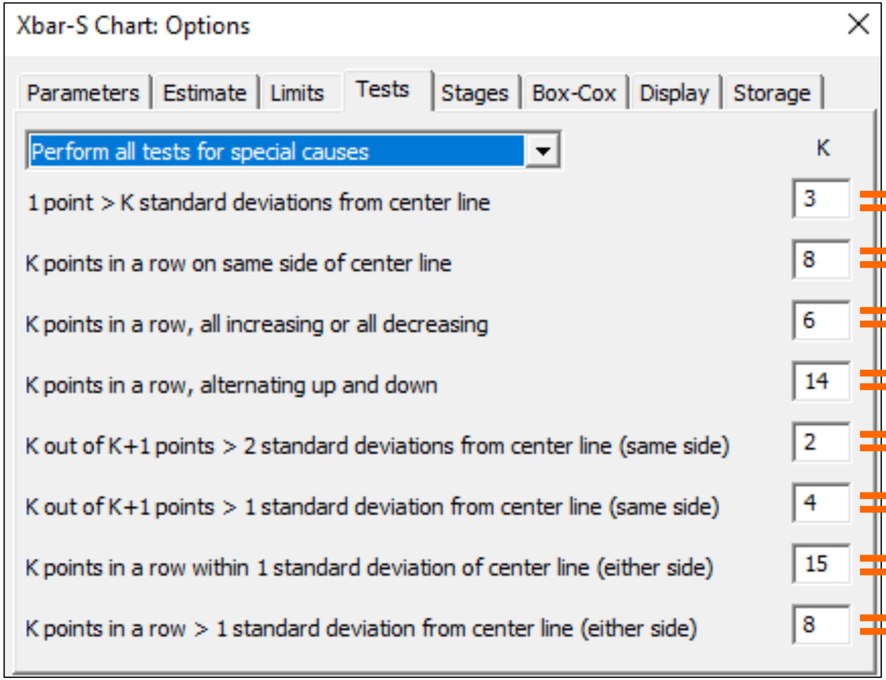
- Shooter Order: OP3
- Measurer's Name: CCCCC C.

Subgroup	Observation	*X* (cm)	*Y* (cm)	Subgroup	Observation	*X* (cm)	*Y* (cm)
21	61	-1.3	142.1	26	76	0	146.2
	62	-2.5	141.4		77	4.3	144.8
	63	7.2	129.3		78	1.9	145.9
22	64	-0.5	140.3	27	79	-1.7	144.3
	65	-1	143.6		80	-1.3	142.4
	66	-1.4	137.9		81	-0.3	142.2
23	67	1.9	140.6	28	82	0	139
	68	1.7	138.3		83	-3.2	140.9
	69	1.8	145.3		84	-1.1	143.2
24	70	2.3	143.2	29	85	-1.9	141.5
	71	-1.3	141.6		86	-3.6	136.8
	72	-0.9	144		87	-1.8	141.4
25	73	3.1	144.6	30	88	-2.5	139.4
	74	-0.5	141.3		89	1.8	138.9
	75	3.4	147.3		90	-1	138

Measuring stability of shooting process

"Control chart"

Given:



Rule #1: Extreme point

Rule #2: Run above/below center line

Rule #3: Trend

Rule #4: Cycle

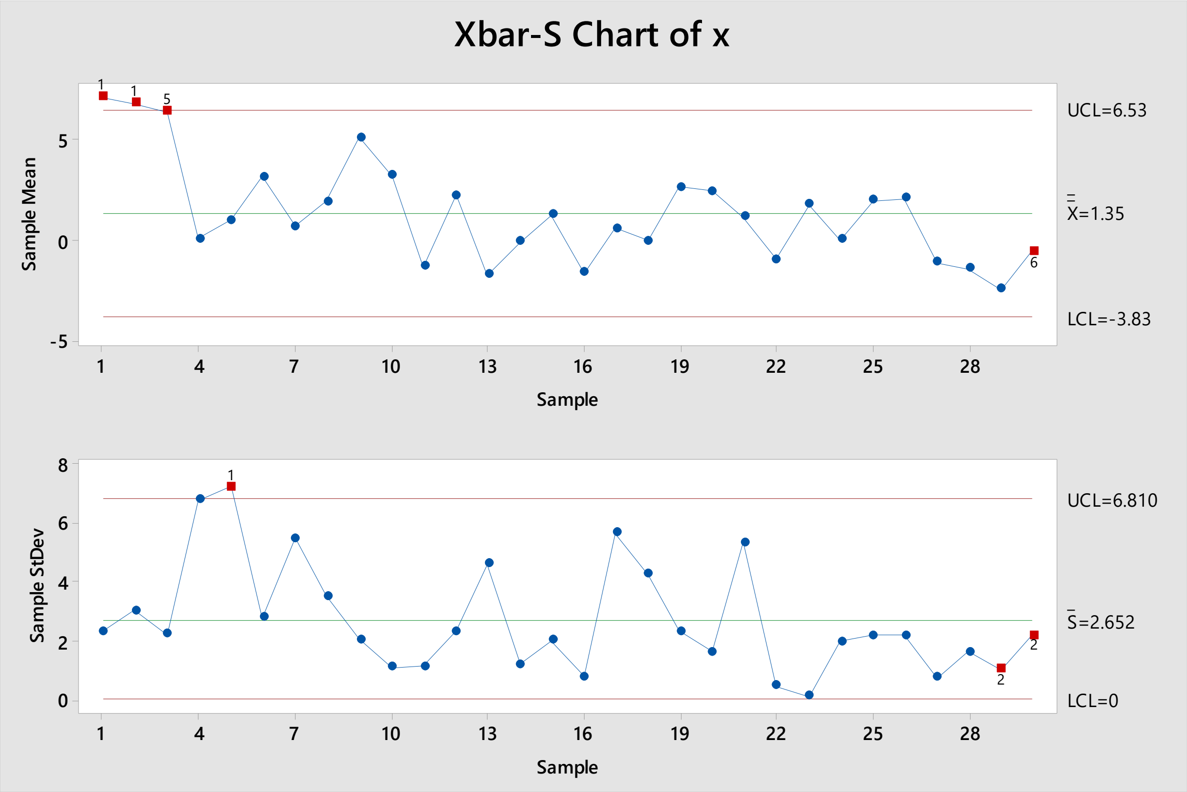
Rule #5: 2/3 in or beyond zone A

Rule #6: 4/5 in or beyond zone B

Rule #7: Consecutive runs in zone C

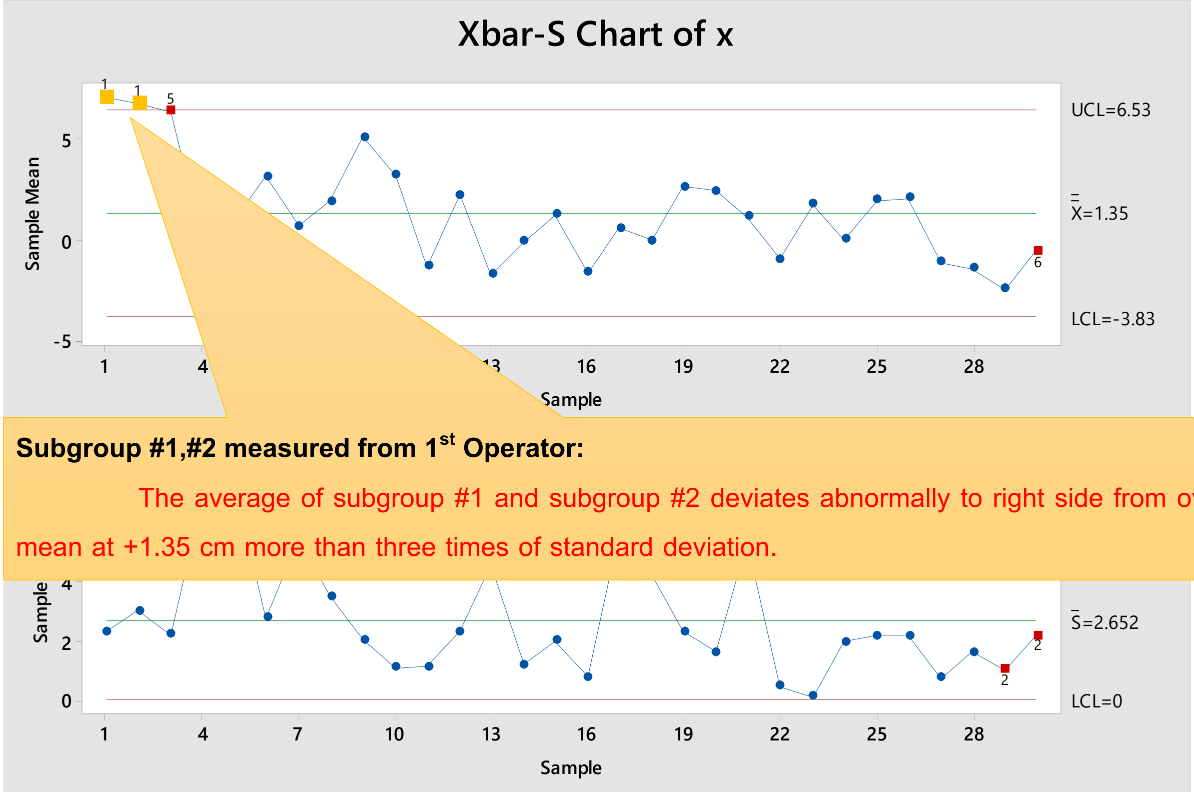
Rule #8: Avoidance of zone C

X-axis falling distance



X-axis falling distance

Violate Rule #1: "Extreme point"

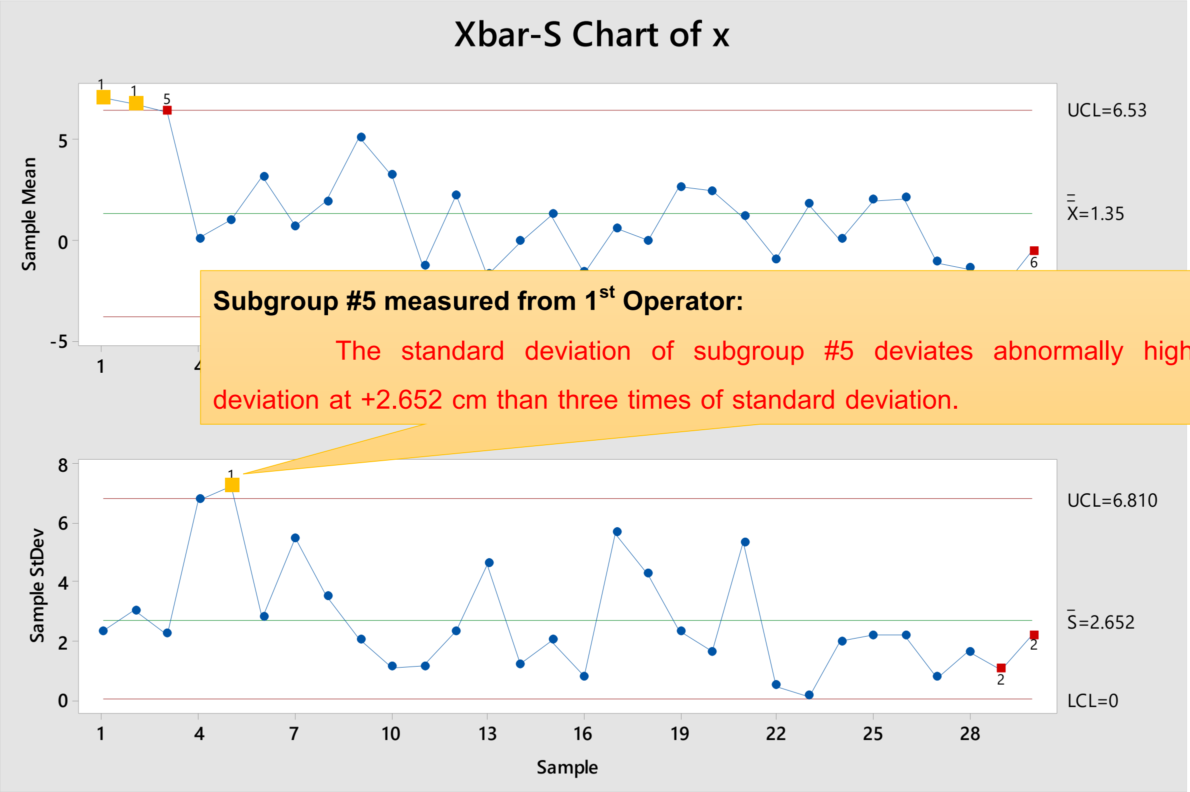


Subgroup #1,#2 measured from 1st Operator:

The average of subgroup #1 and subgroup #2 deviates abnormally to right side from overall grand mean at +1.35 cm more than three times of standard deviation.

X-axis falling distance

Violate Rule #1: “Extreme point”

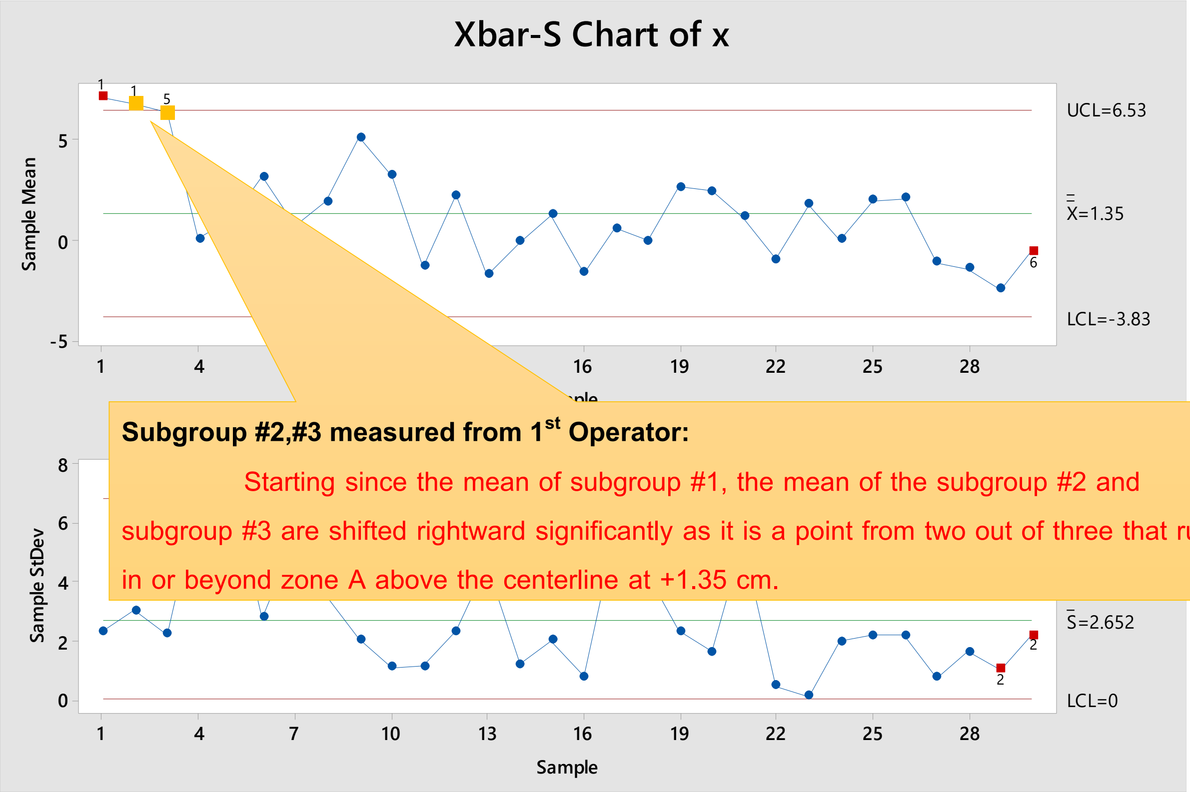


Subgroup #5 measured from 1st Operator:

The standard deviation of subgroup #5 deviates abnormally higher from average standard deviation at +2.652 cm than three times of standard deviation.

X-axis falling distance

Violate Rule #5: "2/3 in or beyond zone A"

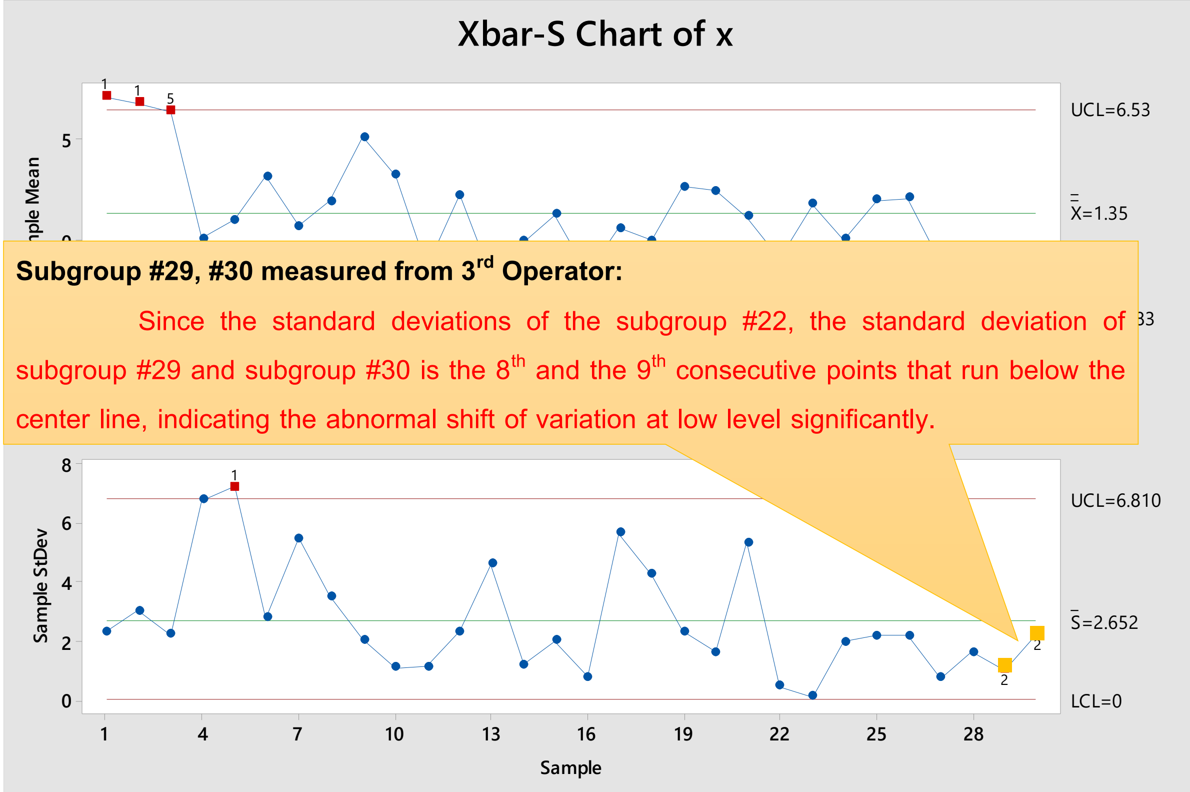


Subgroup #2,#3 measured from 1st Operator:

Starting since the mean of subgroup #1, the mean of the subgroup #2 and subgroup #3 are shifted rightward significantly as it is a point from two out of three that run in or beyond zone A above the centerline at +1.35 cm.

X-axis falling distance

Violate Rule #2: "Run above/below center line"

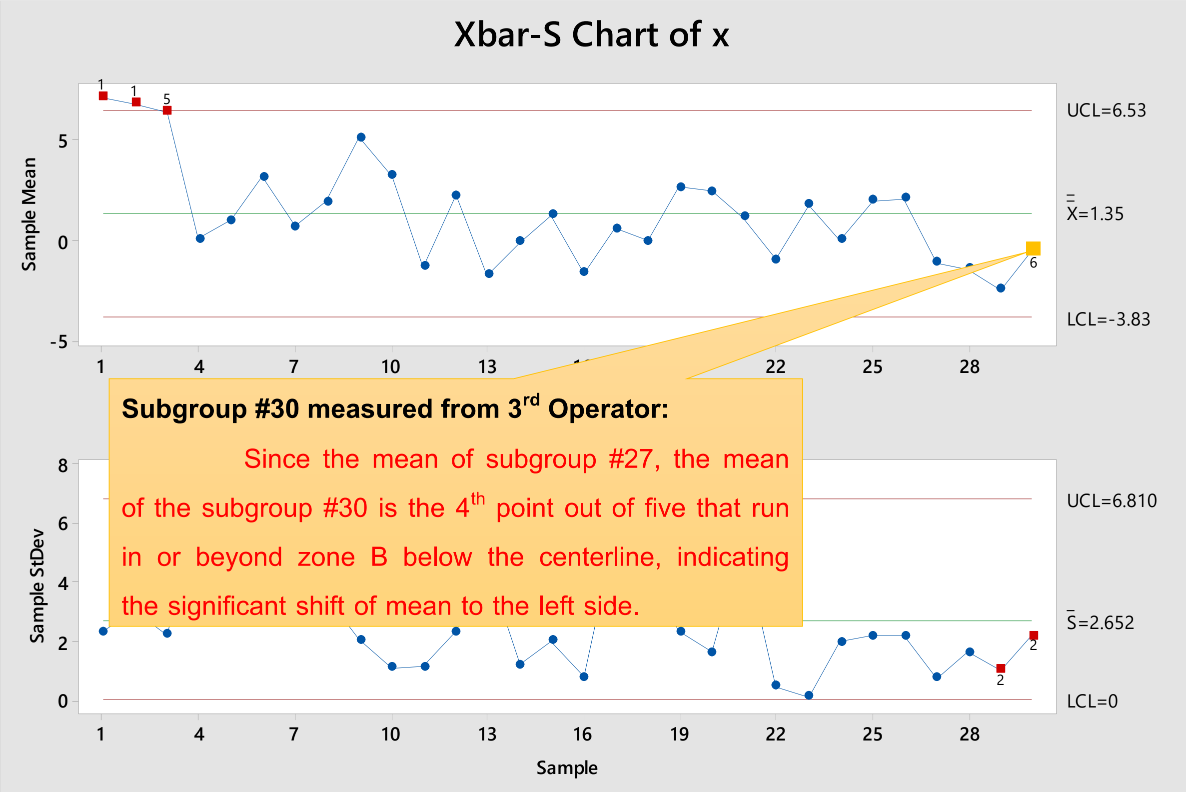


Subgroup #29, #30 measured from 3rd Operator:

Since the standard deviations of the subgroup #22, the standard deviation of subgroup #29 and subgroup #30 is the 8th and the 9th consecutive points that run below the center line, indicating the abnormal shift of variation at low level significantly.

X-axis falling distance

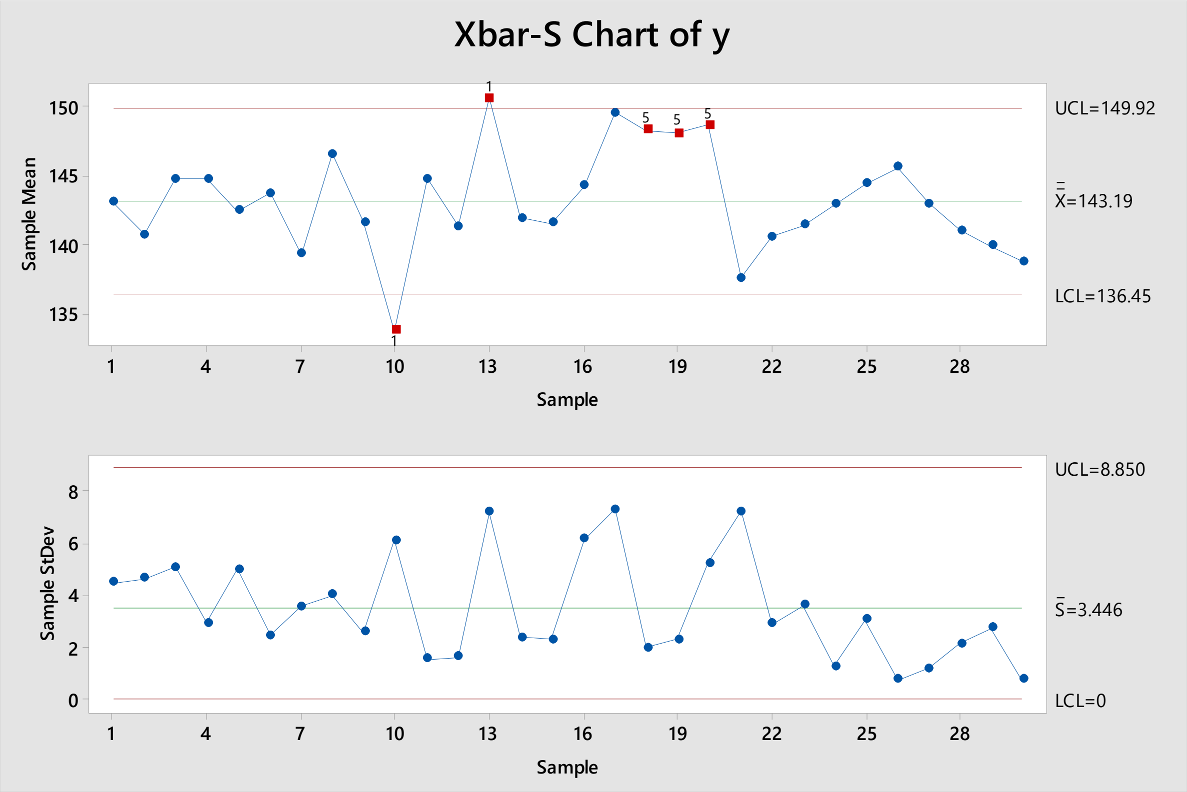
Violate Rule #6: “4/5 in or beyond zone B”



Subgroup #30 measured from 3rd Operator:

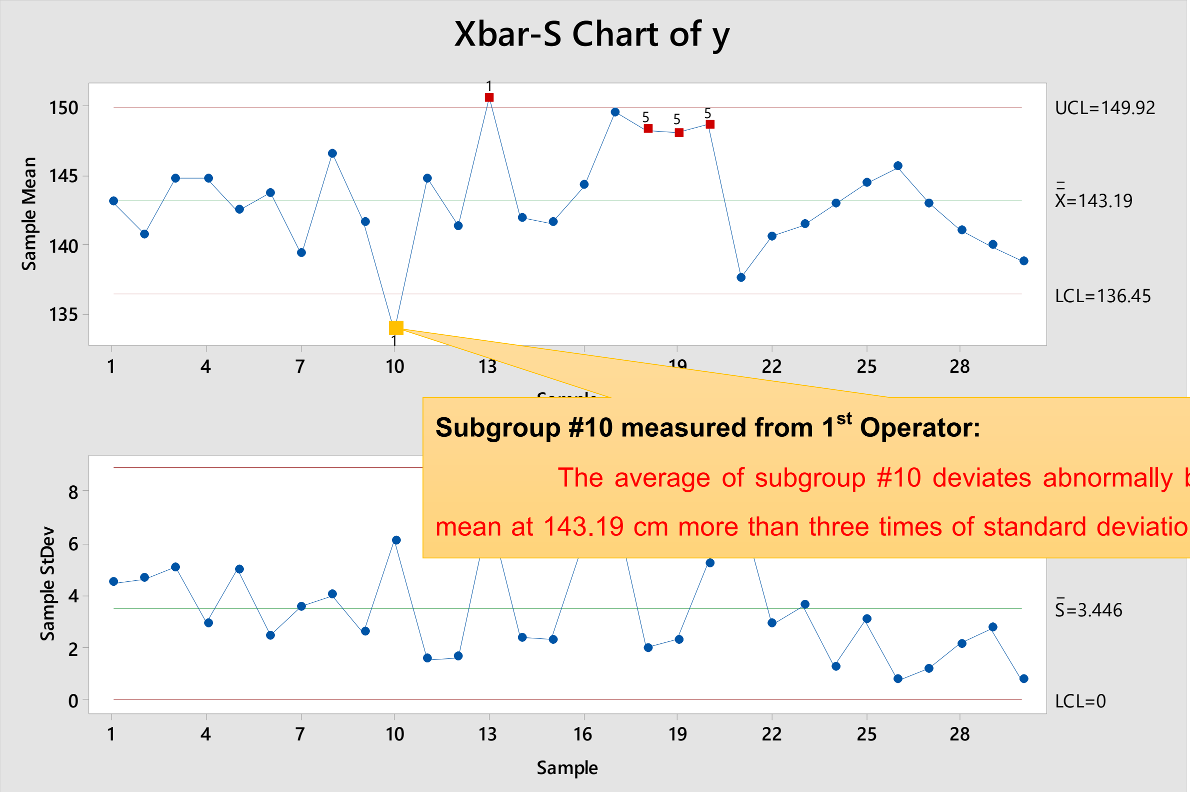
Since the mean of subgroup #27, the mean of the subgroup #30 is the 4th point out of five that run in or beyond zone B below the centerline, indicating the significant shift of mean to the left side.

Y-axis falling distance



Y-axis falling distance

Violate Rule #1: "Extreme point"

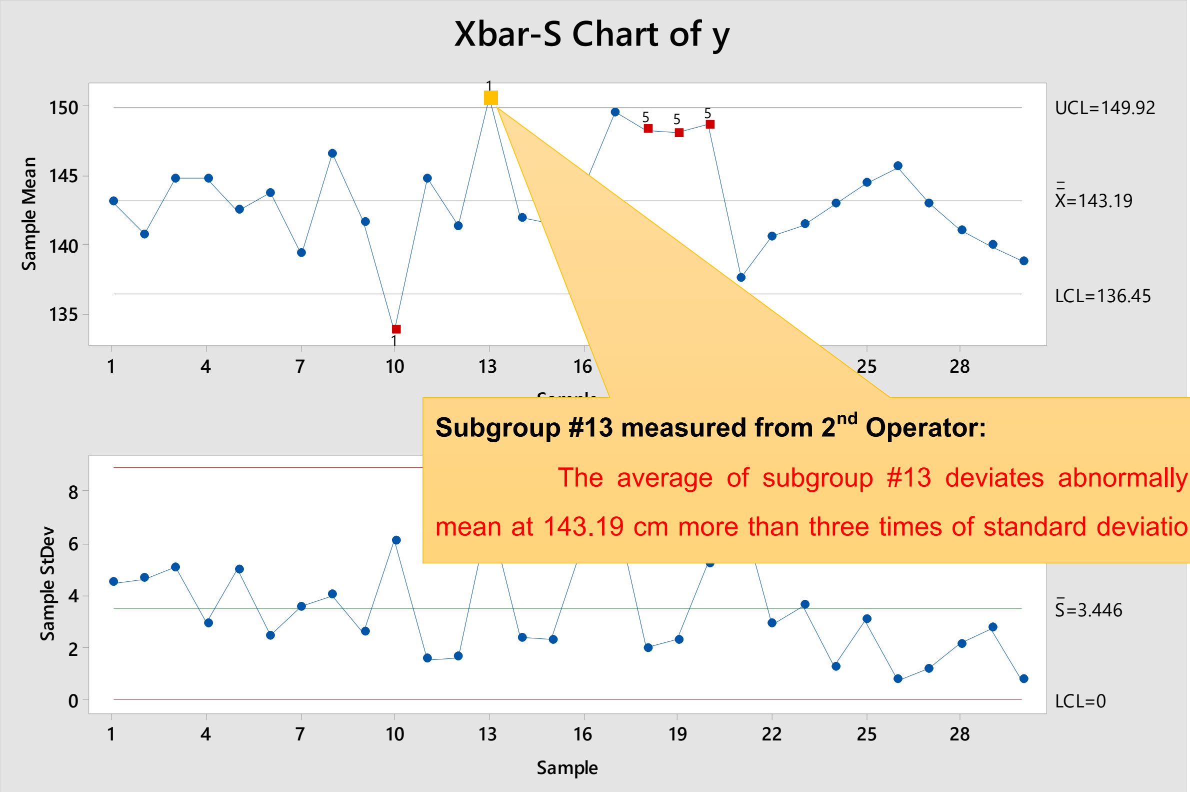


Subgroup #10 measured from 1st Operator:

The average of subgroup #10 deviates abnormally backward from overall grand mean at 143.19 cm more than three times of standard deviation.

Y-axis falling distance

Violate Rule #1: "Extreme point"

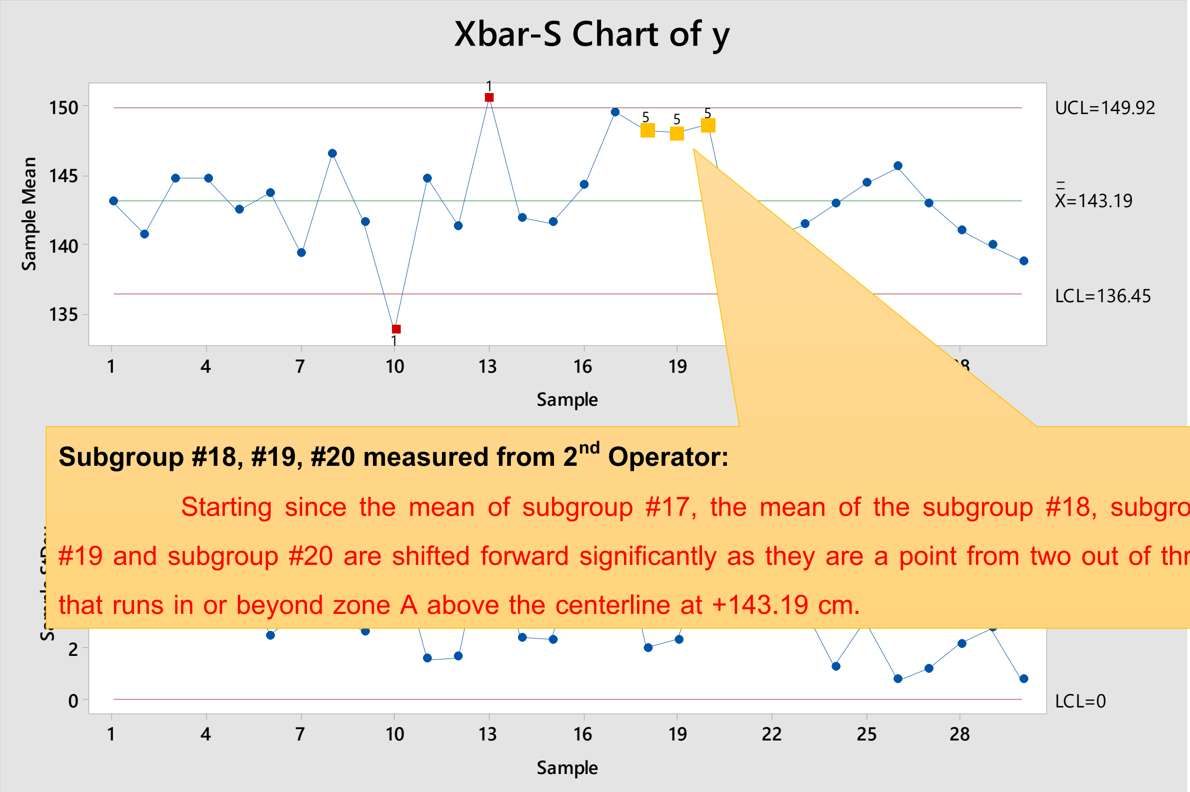


Subgroup #13 measured from 2nd Operator:

The average of subgroup #13 deviates abnormally forward from overall grand mean at 143.19 cm more than three times of standard deviation.

Y-axis falling distance

Violate Rule #5: "2/3 in or beyond zone A"

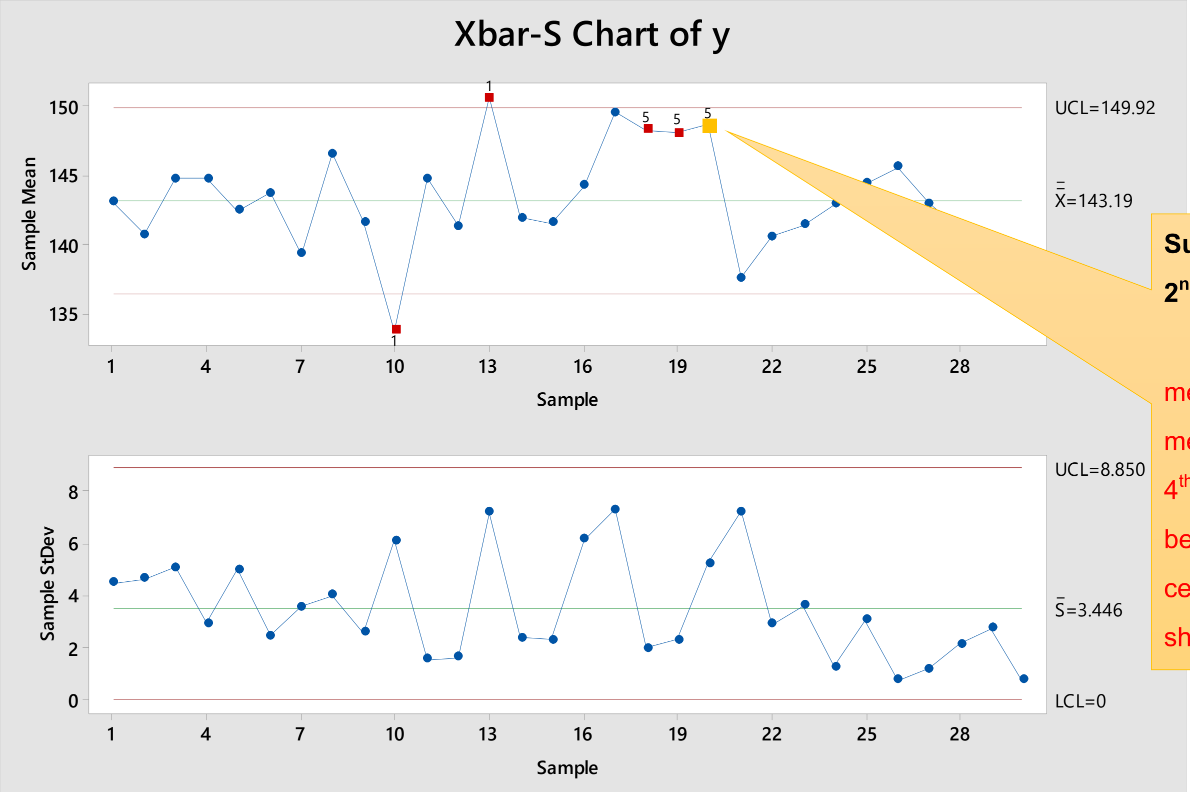


Subgroup #18, #19, #20 measured from 2nd Operator:

Starting since the mean of subgroup #17, the mean of the subgroup #18, subgroup #19 and subgroup #20 are shifted forward significantly as they are a point from two out of three that runs in or beyond zone A above the centerline at +143.19 cm.

Y-axis falling distance

Violate Rule #6: “4/5 in or beyond zone B”



Subgroup #20 measured from 2nd Operator:

Starting since the mean of subgroup #17, the mean of the subgroup #20 is the 4th point out of five that run in or beyond zone B above the centerline , indicating the mean shift to forward significantly.

In conclusion,

Shooter	Mean-X	Mean-Y	Overall variation-X	Overall variation-Y
OP1	3.557	142.100	4.284	4.939
OP2	0.443	145.940	2.979	5.011
OP3	0.053	141.523	2.424	3.535

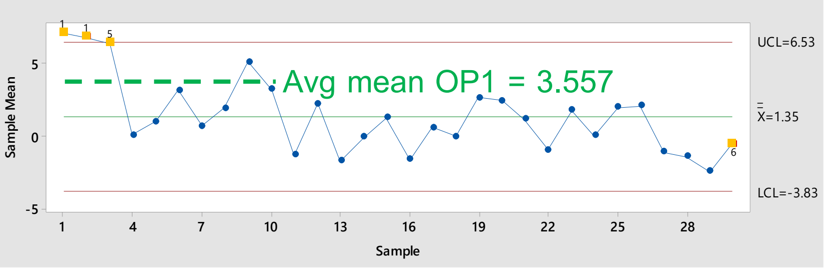
OP1 > OP2 > OP3

4/5 in or beyond zone B: #30 ⇒ **Left deviation**

Extreme Point: #1, #2

2/3 in or beyond zone A: #2, #3

Mean shift rightward



In conclusion,

Shooter	Mean-X	Mean-Y	Overall variation-X	Overall variation-Y
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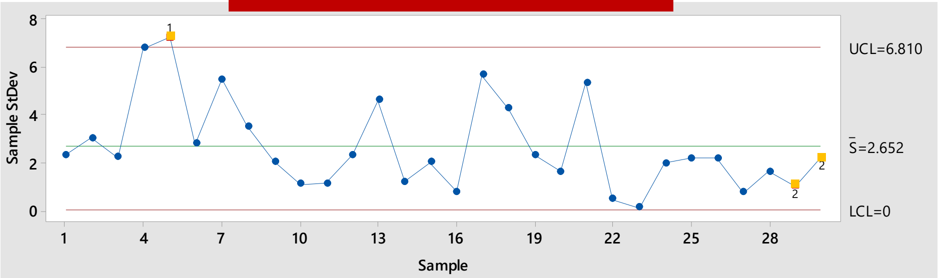
OP3	0.053	141.523	2.424	3.535

Extreme Point: #5

OP1 > OP2 > OP3

Abnormal shift at high level



Run below center line: #29, #30

Abnormal shift at low level

In conclusion,

Shooter	Mean-X	Mean-Y	Overall variation-X	Overall variation-Y
OP1	3.557	142.100	4.284	4.939
OP2	0.443	145.940	2.979	5.011
OP3	0.053	141.523	2.424	3.535

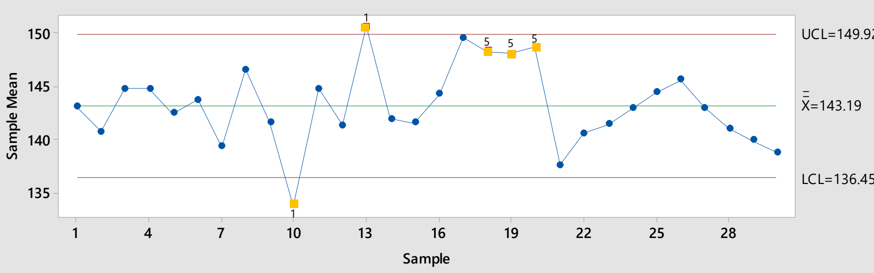
OP2 > OP1 > OP3

Extreme Point: #10 → Abnormally backward

Extreme Point: #13 → Abnormally frontward

2/3 in or beyond zone A: #18, #19, #20

4/5 in or beyond zone B: #20 → Mean shift frontward



In conclusion,

Shooter	Mean-X	Mean-Y	Overall variation-X	Overall variation-Y
OP1	3.557	142.100	4.284	4.939
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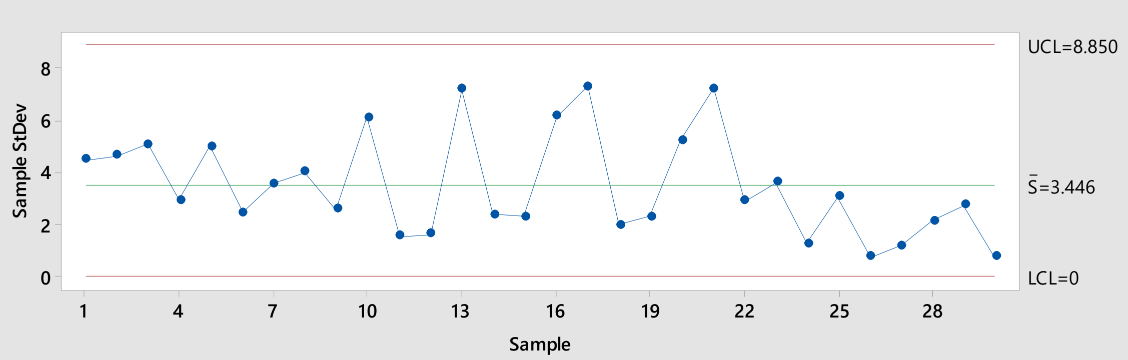
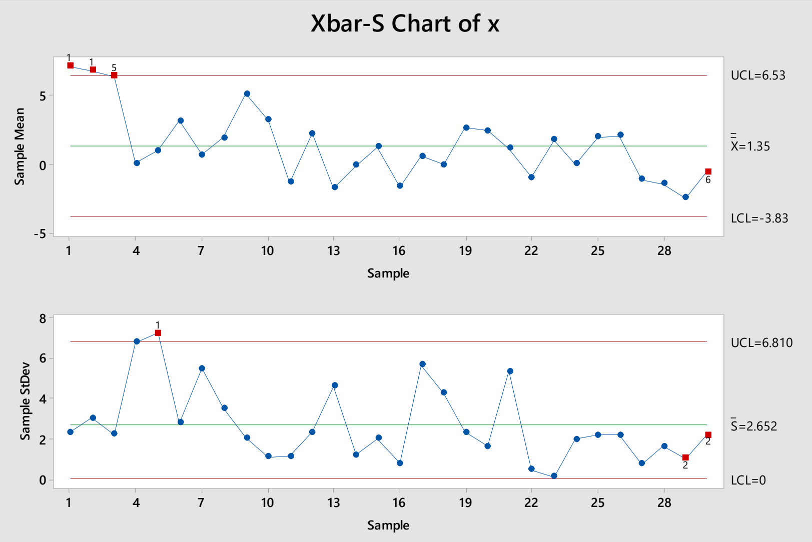
OP3	0.053	141.523	2.424	3.535

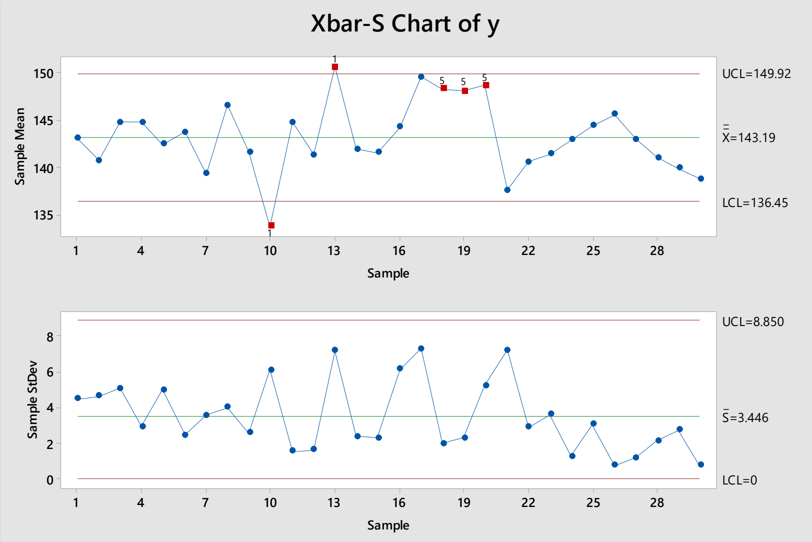
OP2 > OP1 > OP3



"In control"; ∀ subgroup

In conclusion,



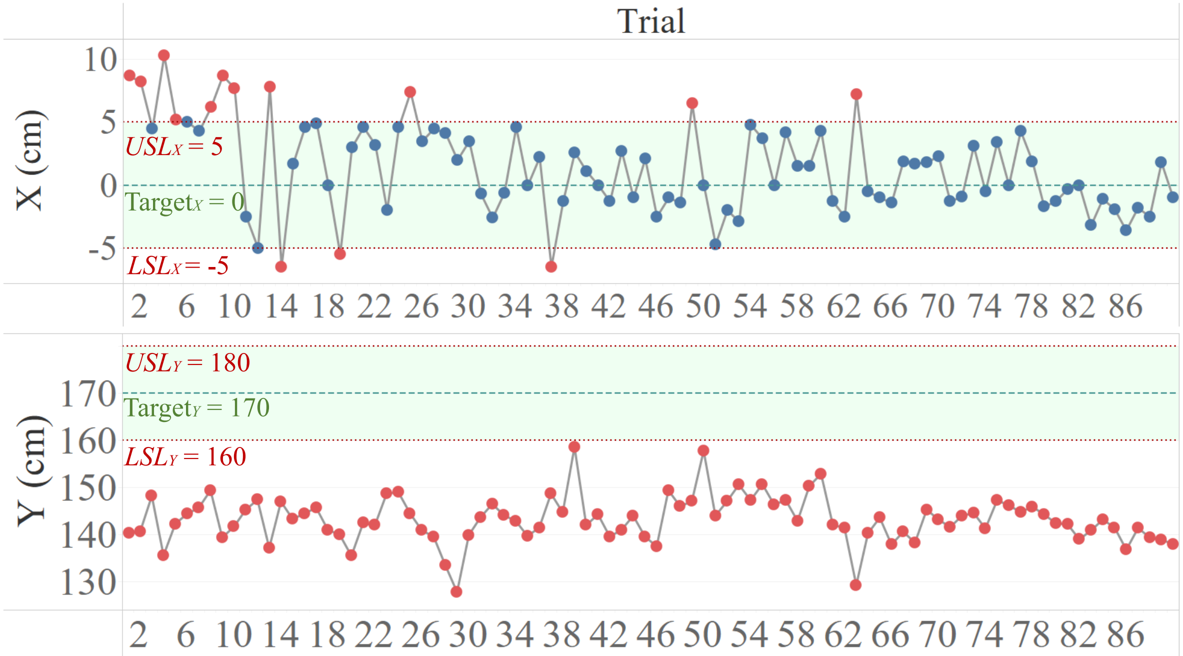


Stability-X: OP2 > OP3 > OP1

Stability-Y: OP3 > OP1 > OP2

From 2nd data collection

Trial	*X*	*Y*
1	8.7	140.4
2	8.2	140.6
3	4.5	148.2
⋮	⋮	⋮
89	1.8	138.9
90	-1	138
Mean	1.35	143.19
SD	3.64	4.91



■ Out of specification
■ Within specification

$Fine_X = 5{,}664.34$ THB
- $Fine_{X\text{-}bias} = 364.50$ THB
- $Fine_{X\text{-}variation} = 5{,}299.84$ THB

$Fine_Y = 153{,}398.46$ THB
- $Fine_{Y\text{-}bias} = 143{,}755.22$ THB
- $Fine_{Y\text{-}variation} = 9{,}643.24$ THB

$\Sigma Fine = 159{,}062.80$ THB

Net reward = [- ฿59,062.80] = ฿0

Potential causes affecting the variation

Causes of short-term variation

- Wrong measuring method
- Random error from wrong setting of the machine such as randomly setting a wrong angle (≠140°) [affecting in obtaining the extreme backward position of y-axis in subgroup #10 and subgroup #13]
- Wear off from a measuring tool such as the distortion (≠90°) at the tip of the steel ruler
- Inconstancy from the operator to measure the falling position by finding the centroid of the charcoal dust occurred after the ball hit the floor (Sometimes it may be hard to find the centroid as it may not be the perfect circle but the freeform oval or the other strange shapes)
- Drag force from the air compression (a.k.a. change from external environment factors) when the ball was shooting.
- The tape may start not to hold the equipment as well as the catapult firmly [affecting in uncertainty during either the shooting process or the measuring process in that trial].
- Fabrication of number from operator’s record to achieve good performance.

Causes of long-term variation

- Unclear work instruction
- Fatigue of an operator over time affecting an inattention of the operator
- Loss of rubber band’s elasticity after it has been used for a period of time
- Sitting position and posture of operator in shooting process
- Lack of effectively inspection in the position of the tape due to each individual laziness
- Wear off from the catapult machine [affecting in a decrease of shooting efficiency]
- Low quality of the rubber ball
- Insufficient quality of charcoal dust that not covered with the ball effectively
- Wrong adjusting default position values such as start angle [Considering with the same launching velocity, if the start angle close to 135° the y position will fall longer otherwise the y-position will fall shorter], tension pin position and hook position [affecting the launching velocity due to the rubber band tension]
- Sound noise from external environment for a period of time such as air conditioner, vacuum cleaner, construction site etc.
- Inappropriate level of luminance from the light source(s) and discomfort from glare

3rd data collection:

- Shooter Order: OP1
- Measurer's Name: AAAAA A.

Subgroup	Observation	*X* (cm)	*Y* (cm)
1	1	1.7	160
	2	1.7	160
	3	2.4	161
2	4	3	162.8
	5	2.3	160
	6	0.6	161.9
3	7	2.1	160.5
	8	1.2	160.2
	9	2.3	162.5
4	10	2.6	164.7
	11	-0.9	156.5
	12	4.1	162.4
5	13	2.8	165.2
	14	-2.7	160.3
	15	3.9	157.3

3rd data collection:

- Shooter Order: OP2
- Measurer's Name: BBBBB B.

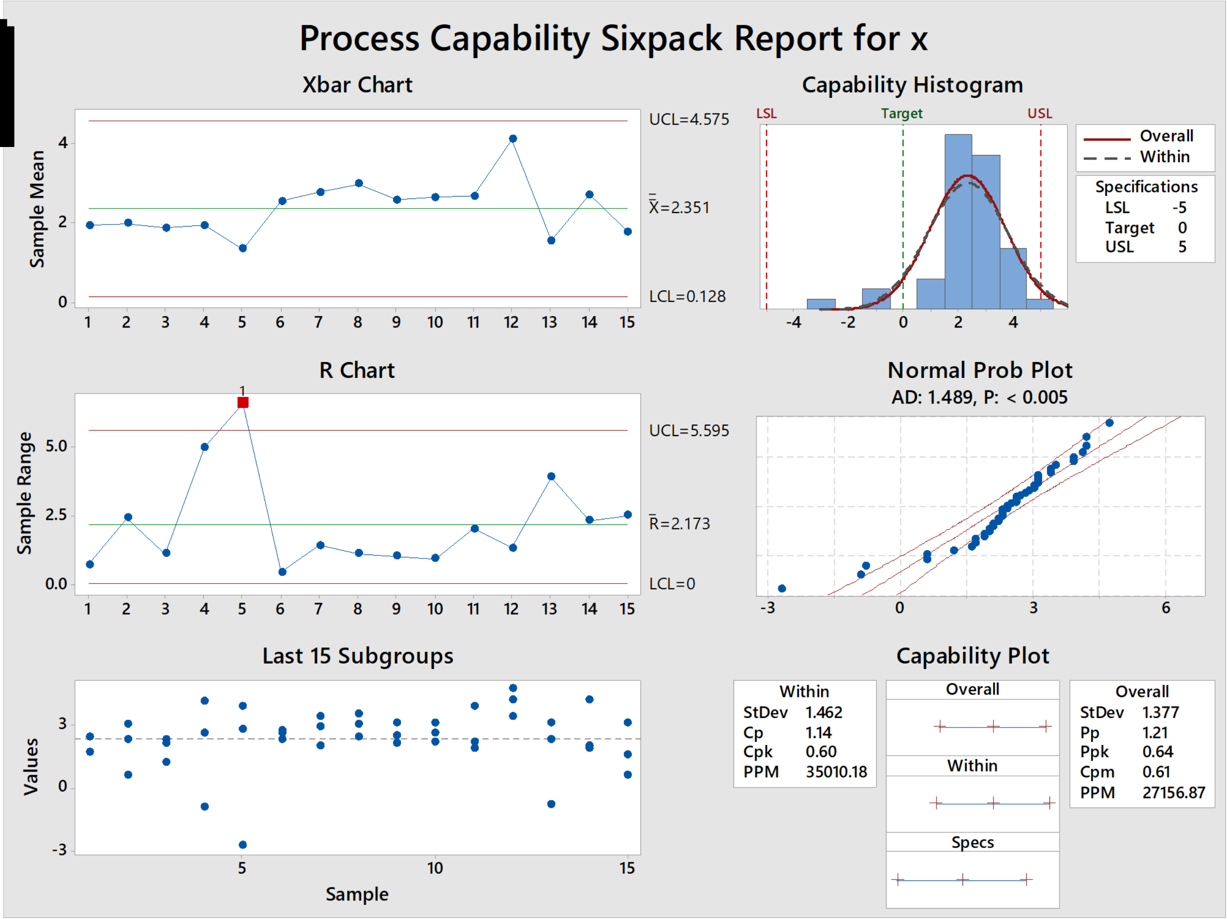
Subgroup	Observation	*X* (cm)	*Y* (cm)
6	16	2.3	167.4
	17	2.7	160.5
	18	2.6	167.5
7	19	2.9	164.5
	20	3.4	165.9
	21	2	164.3
8	22	3.5	169.8
	23	2.4	161.2
	24	3	164.5
9	25	2.1	167.5
	26	2.5	167.9
	27	3.1	168.2
10	28	2.6	168.4
	29	3.1	167.9
	30	2.2	163.3

3rd data collection:

- Shooter Order: OP3
- Measurer's Name: CCCCC C.

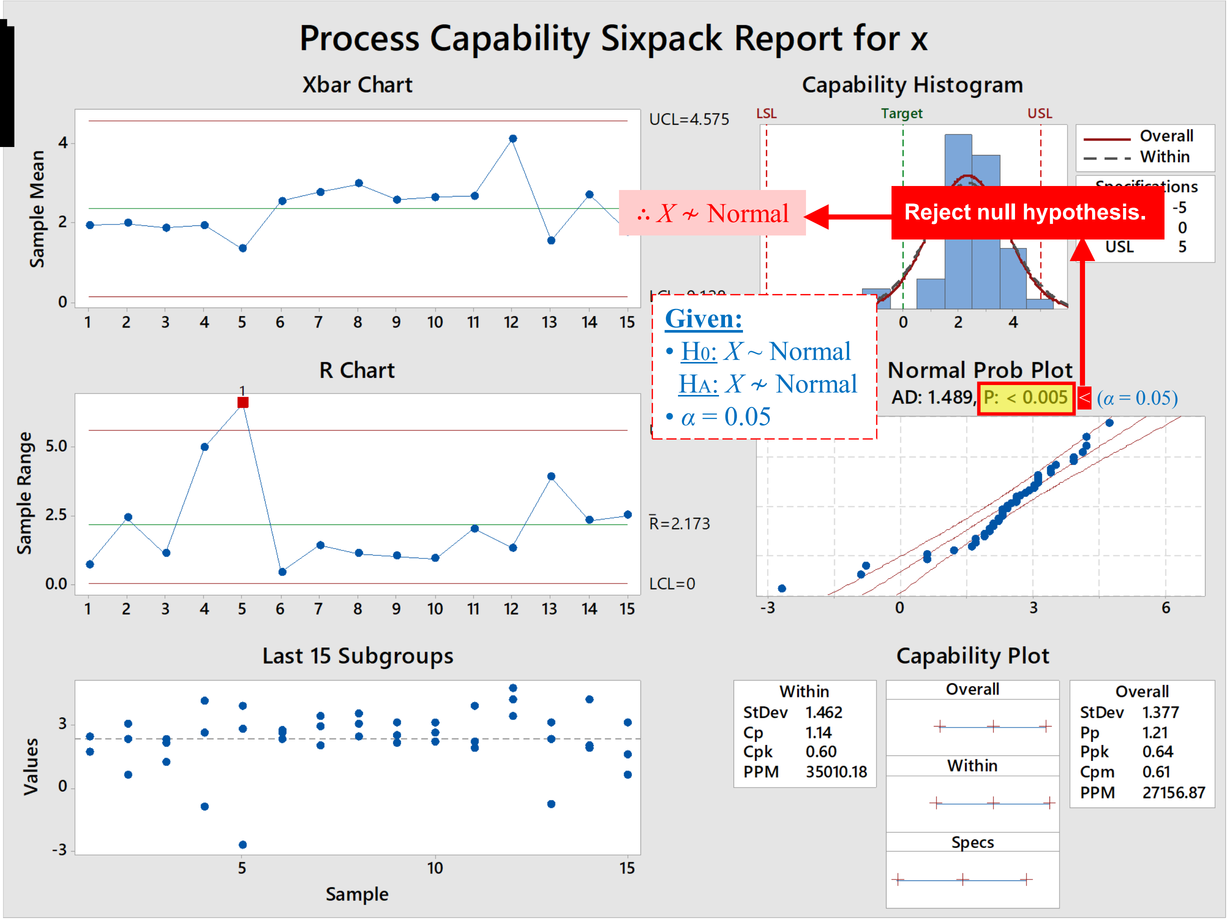
Subgroup	Observation	*X* (cm)	*Y* (cm)
11	31	1.9	159
	32	2.2	160
	33	3.9	165.4
12	34	4.7	166.2
	35	4.2	163.2
	36	3.4	167.2
13	37	2.3	161.1
	38	-0.8	164.2
	39	3.1	161.4
14	40	2	162.1
	41	4.2	155.4
	42	1.9	156
15	43	3.1	160.4
	44	0.6	163
	45	1.6	162

PCA in X-axis



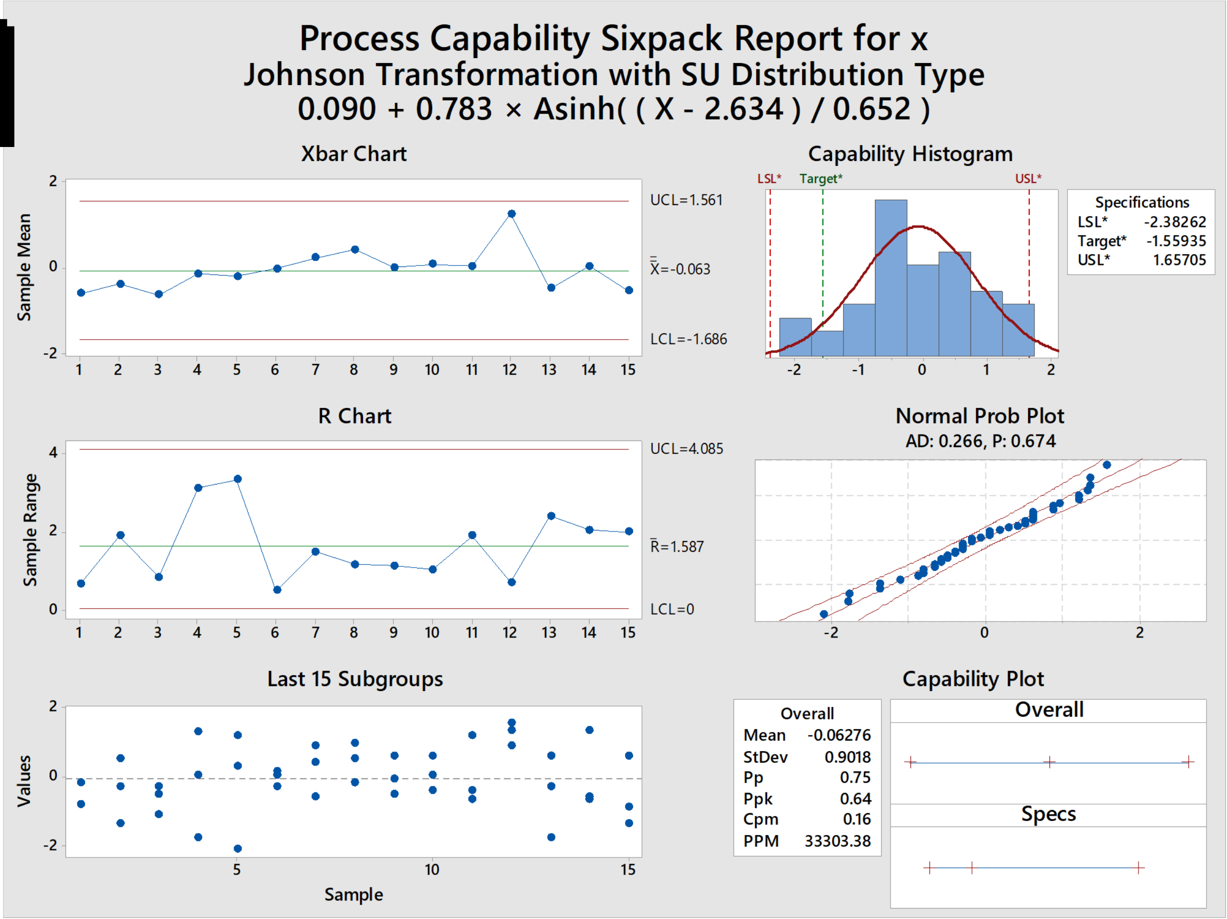
Process Capability Sixpack Report for x
Xbar Chart
Sample Mean
UCL=4.575
X̄=2.351
LCL=0.128
R Chart
Sample Range
UCL=5.595
R̄=2.173
LCL=0
Last 15 Subgroups
Values
Sample
Capability Histogram
LSL
Target
USL
Overall
Within
Specifications
LSL -5
Target 0
USL 5
Normal Prob Plot
AD: 1.489, P: < 0.005
Capability Plot
Within
StDev 1.462
Cp 1.14
Cpk 0.60
PPM 35010.18
Overall
Within
Specs
Overall
StDev 1.377
Pp 1.21
Ppk 0.64
Cpm 0.61
PPM 27156.87

PCA in X-axis

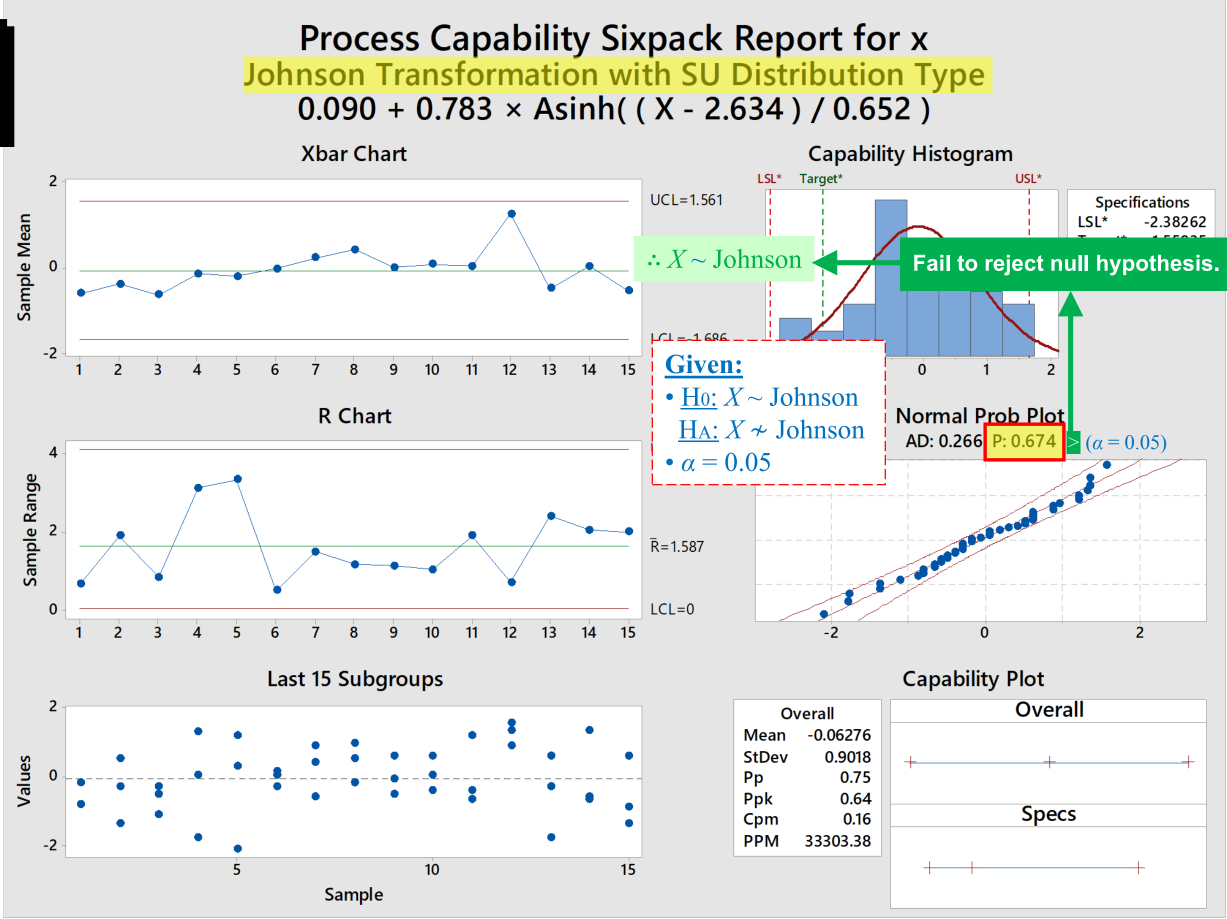


Process Capability Sixpack Report for x
Xbar Chart
Sample Mean
UCL=4.575
R Chart
Sample Range
R̄=2.173
LCL=0
Last 15 Subgroups
Values
Sample
Capability Histogram
LSL
Target
USL
Overall
Within
Specifications
LSL -5
Target 0
USL 5
∴ X ≁ Normal
Reject null hypothesis.
Given:
• H0: X ~ Normal
HA: X ≁ Normal
• α = 0.05
Normal Prob Plot
AD: 1.489, P: < 0.005 < (α = 0.05)
Capability Plot
Within
StDev 1.462
Cp 1.14
Cpk 0.60
PPM 35010.18
Overall
Within
Specs
Overall
StDev 1.377
Pp 1.21
Ppk 0.64
Cpm 0.61
PPM 27156.87

PCA in X-axis


Process Capability Sixpack Report for x
Johnson Transformation with SU Distribution Type
0.090 + 0.783 × Asinh((X - 2.634) / 0.652)
Xbar Chart
Sample Mean
UCL=1.561
X̿=-0.063
LCL=-1.686
R Chart
Sample Range
UCL=4.085
R̄=1.587
LCL=0
Last 15 Subgroups
Values
Sample
Capability Histogram
LSL*
Target*
USL*
Specifications
LSL* -2.38262
Target* -1.55935
USL* 1.65705
Normal Prob Plot
AD: 0.266, P: 0.674
Capability Plot
Overall
Mean -0.06276
StDev 0.9018
Pp 0.75
Ppk 0.64
Cpm 0.16
PPM 33303.38
Overall
Specs

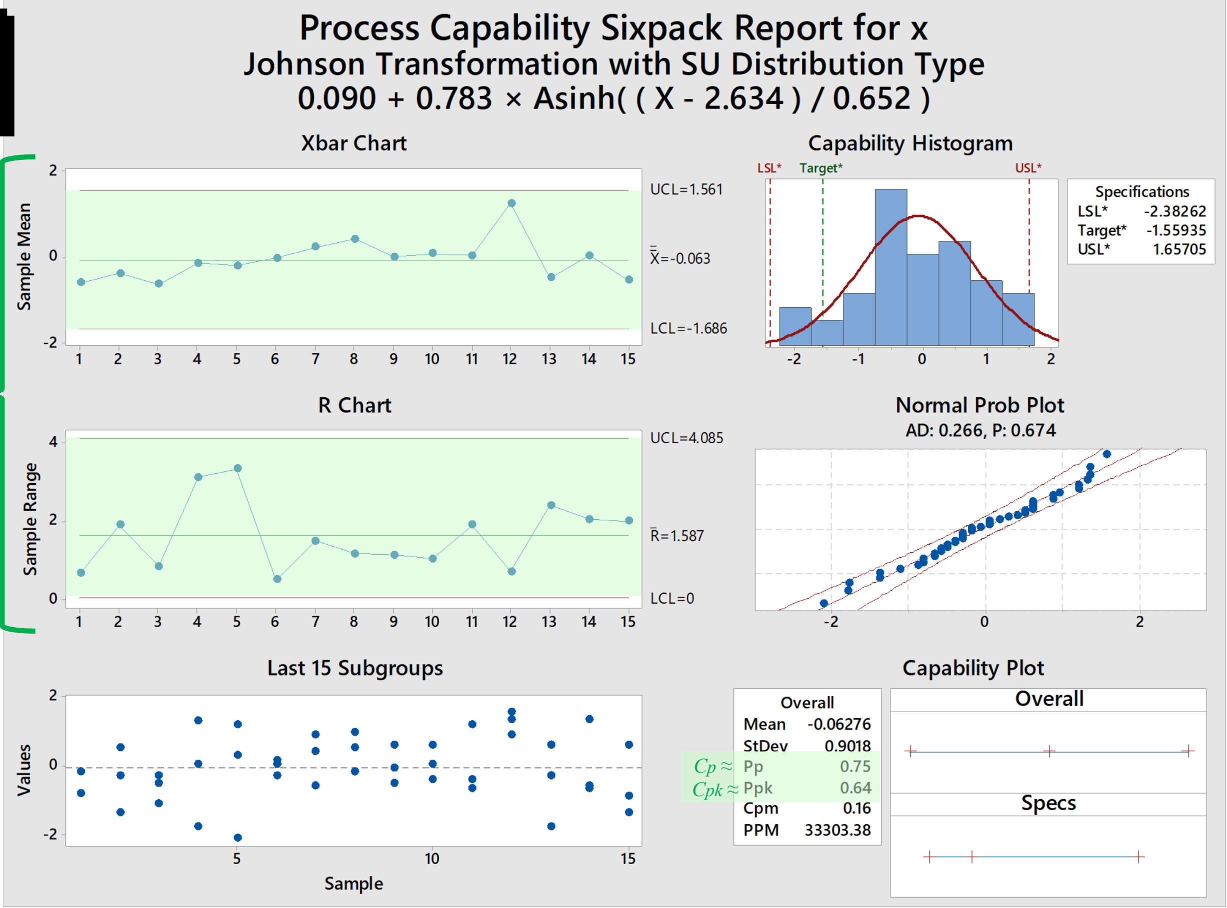
PCA in X-axis



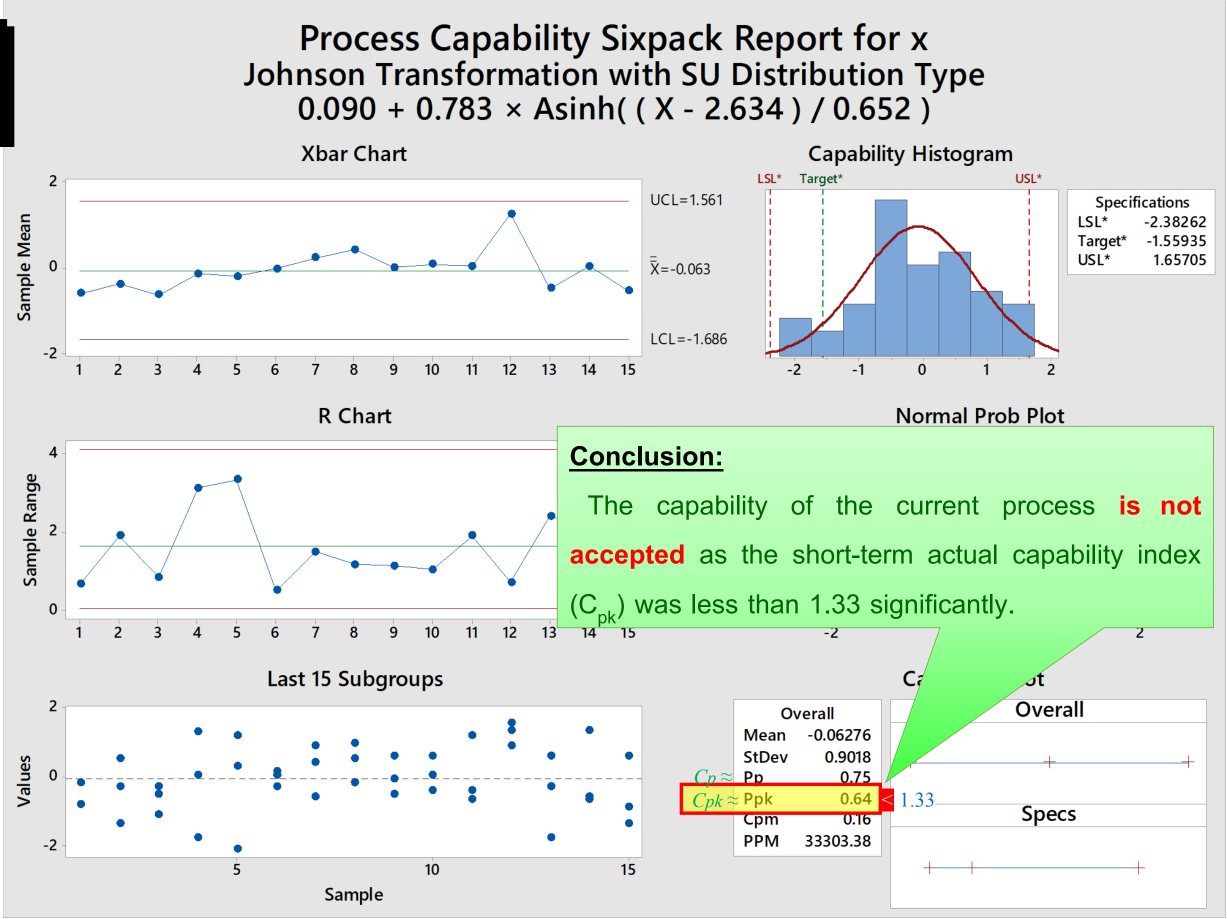
Process Capability Sixpack Report for x
Johnson Transformation with SU Distribution Type
0.090 + 0.783 × Asinh((X - 2.634) / 0.652)
Xbar Chart
Sample Mean
UCL=1.561
LCL=-1.686
R Chart
Sample Range
R̄=1.587
LCL=0
Last 15 Subgroups
Values
Sample
Capability Histogram
LSL*
Target*
USL*
Specifications
LSL* -2.38262
Normal Prob Plot
AD: 0.266
P: 0.674
> (α = 0.05)
∴ X ~ Johnson
Fail to reject null hypothesis.
Given:
• H0: X ~ Johnson
HA: X ≁ Johnson
• α = 0.05
Capability Plot
Overall
Mean -0.06276
StDev 0.9018
Pp 0.75
Ppk 0.64
Cpm 0.16
PPM 33303.38
Overall
Specs

PCA in X-axis

∀ Subgroups are in control



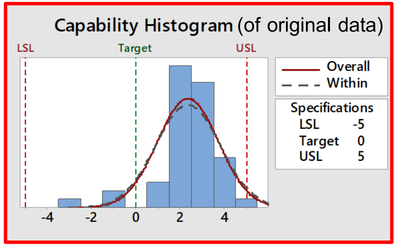
PCA in X-axis



Conclusion:

The capability of the current process **is not accepted** as the short-term actual capability index (C_{pk}) was less than 1.33 significantly.

PCA in X-axis

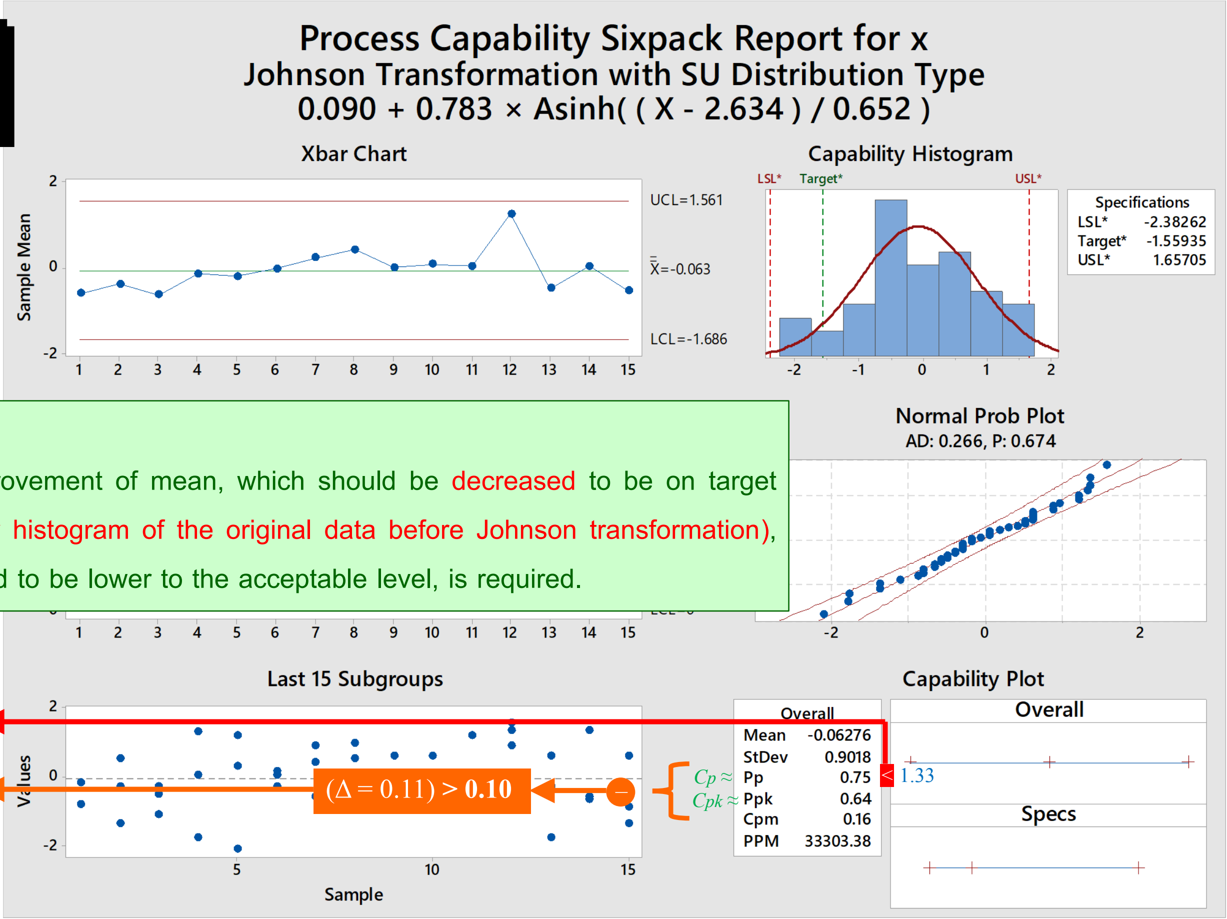


Improvement Direction:

For X-axis distance, the improvement of mean, which should be decreased to be on target (considered from the capability histogram of the original data before Johnson transformation), and variation, which is expected to be lower to the acceptable level, is required.

Improvement for σ is required.

Improvement for μ is required.



Evaluation if the sample size of current X-axis data points is adequate for PCA

Assumption:

From the control charts in Process Capability Sixpack report in X transformed with Johnson technique, it was found that **the overall process was in control**.

Then, $\hat{C}_{pk(X)} \approx \hat{P}_{pk(X)} = 0.64$

Given:

- Confidence Interval $= 95\%$ $\longrightarrow$ Type I error, $\alpha = 0.05 \longrightarrow Z_\alpha \approx -1.645$
- $\dfrac{C_{pk}}{\hat{c}_{pk}} = 0.90$

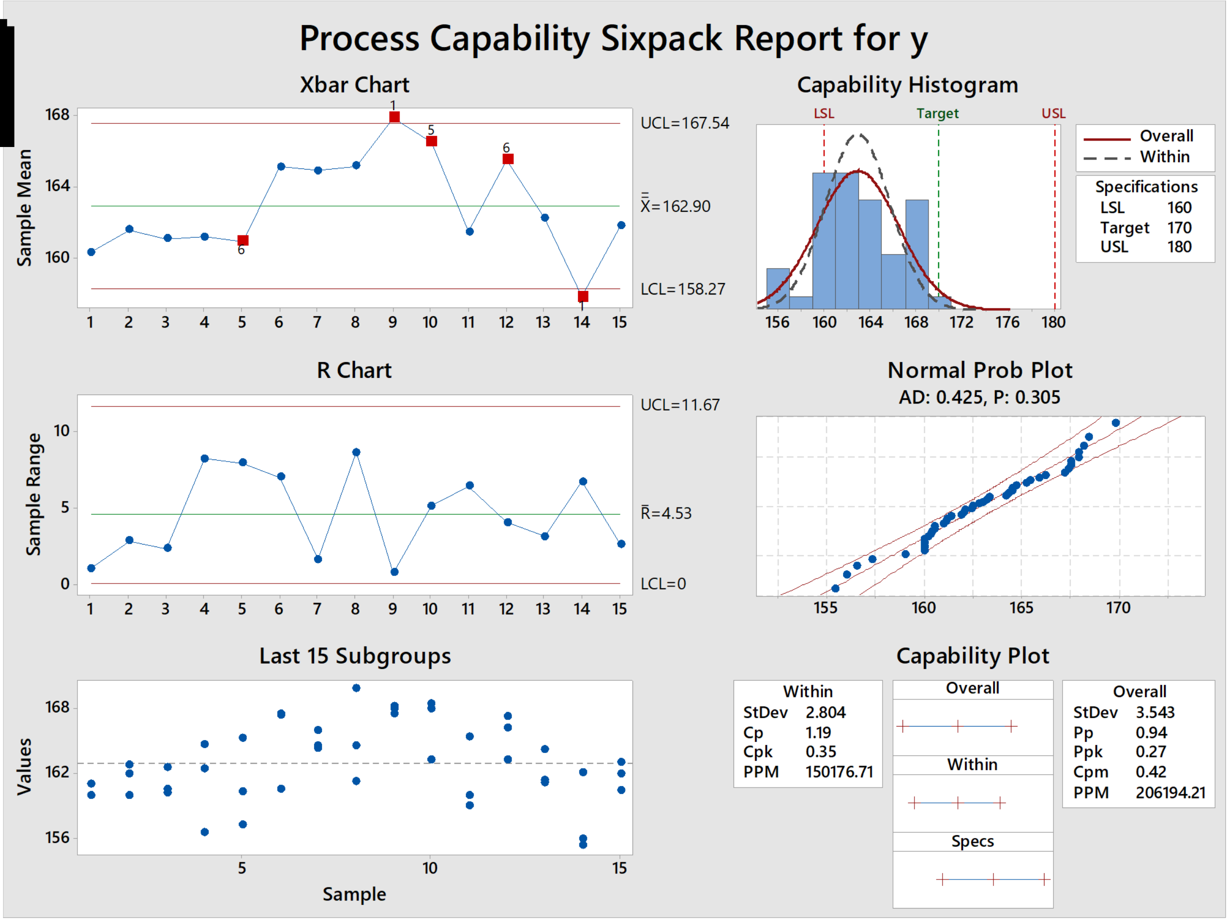
Solution:

$$n = (Z_\alpha)^2 \frac{\left(\dfrac{1}{9\left(\hat{C}_{pk}\right)^2} + \dfrac{1}{2}\right)}{\left(1 - \dfrac{C_{pk}}{\hat{C}_{pk}}\right)^2} = (-1.645)^2 \times \frac{\left(\dfrac{1}{9 \times 0.64^2} + \dfrac{1}{2}\right)}{(1-0.90)^2} = \lceil 208.7 \rceil = 209$$

Conclusion: ∴ To estimate short-term actual capability index (C_{pk}) for X-axis distance, at least sample size of 209 is required.

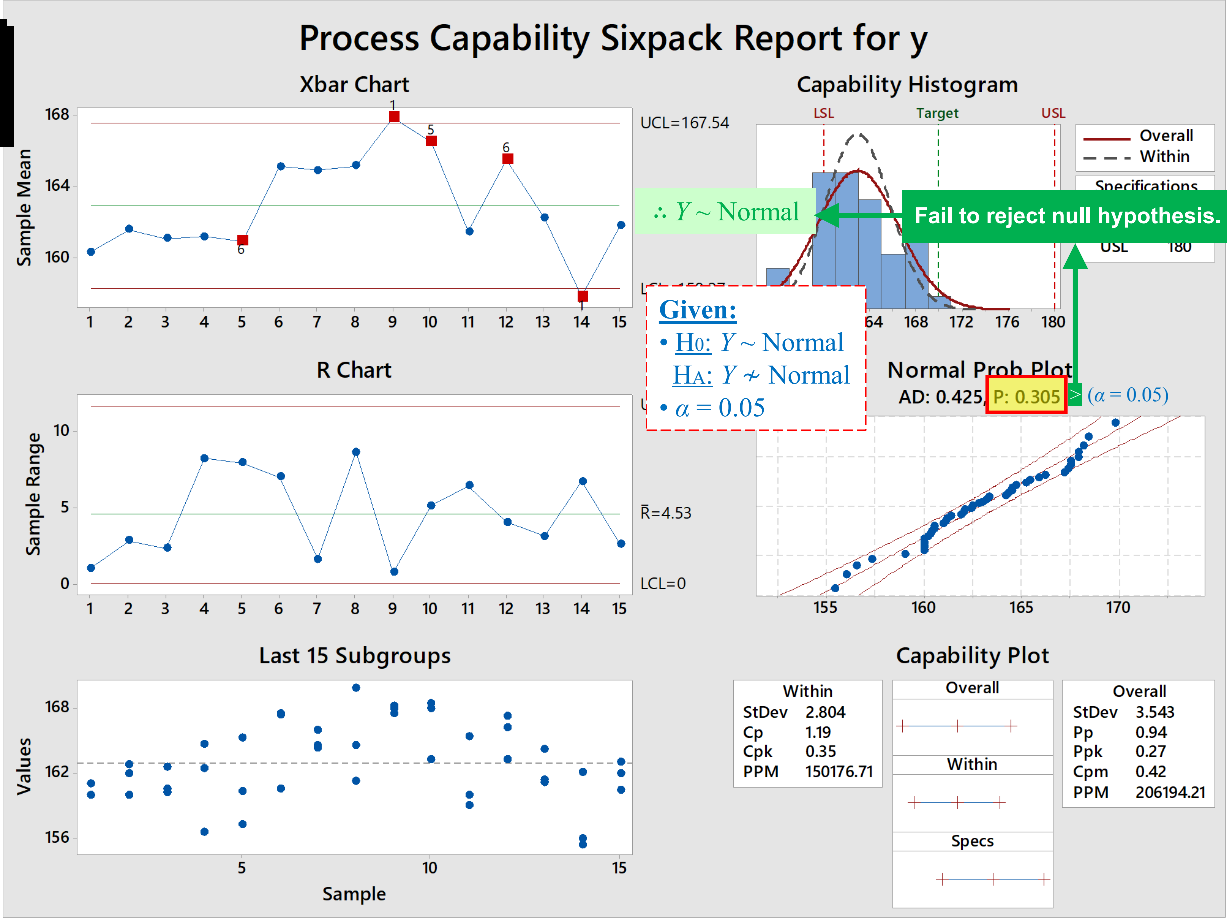
Justify: **The additional samples that should be later collected is no less than 164.**

PCA in Y-axis



Process Capability Sixpack Report for y
Xbar Chart
Sample Mean
UCL=167.54
X̄=162.90
LCL=158.27
R Chart
Sample Range
UCL=11.67
R̄=4.53
LCL=0
Last 15 Subgroups
Values
Sample
Capability Histogram
LSL
Target
USL
Overall
Within
Specifications
LSL 160
Target 170
USL 180
Normal Prob Plot
AD: 0.425, P: 0.305
Capability Plot
Within
StDev 2.804
Cp 1.19
Cpk 0.35
PPM 150176.71
Overall
Within
Specs
Overall
StDev 3.543
Pp 0.94
Ppk 0.27
Cpm 0.42
PPM 206194.21

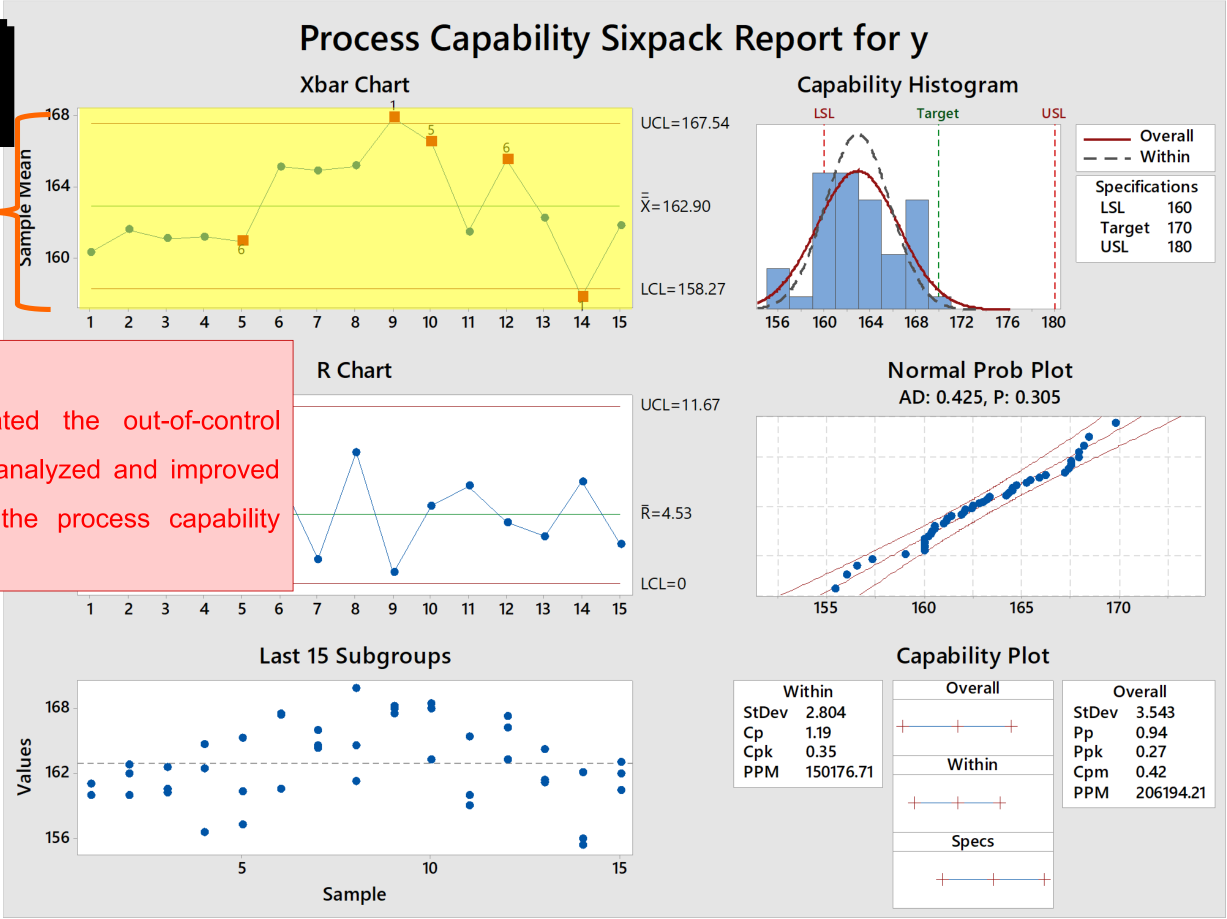
PCA in Y-axis



Process Capability Sixpack Report for y
Xbar Chart
Sample Mean
UCL=167.54
R Chart
Sample Range
R̄=4.53
LCL=0
Last 15 Subgroups
Values
Sample
Capability Histogram
LSL
Target
USL
Overall
Within
Specifications
USL 180
∴ Y ~ Normal
Fail to reject null hypothesis.
Given:
• H0: Y ~ Normal
HA: Y ≁ Normal
• α = 0.05
Normal Prob Plot
AD: 0.425, P: 0.305 > (α = 0.05)
Capability Plot
Within
StDev 2.804
Cp 1.19
Cpk 0.35
PPM 150176.71
Overall
Within
Specs
Overall
StDev 3.543
Pp 0.94
Ppk 0.27
Cpm 0.42
PPM 206194.21

PCA in Y-axis

∃ Subgroups are out of control.

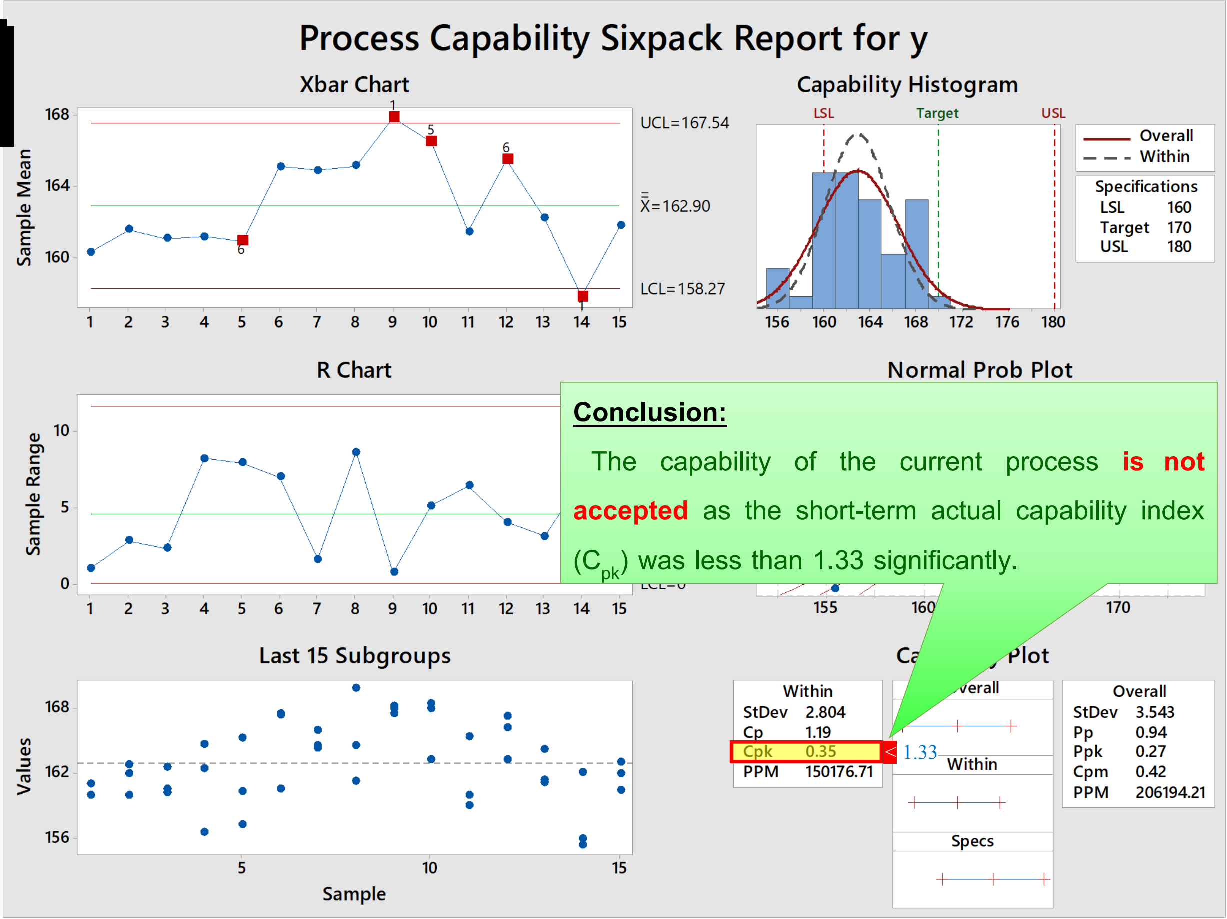


In practical:

The process that generated the out-of-control datasets should have been analyzed and improved to be more stable before the process capability analysis was performed.

In this project due to the limitation of time to perform the study, the mentioned improvement of the process to be stabilized will be neglected.

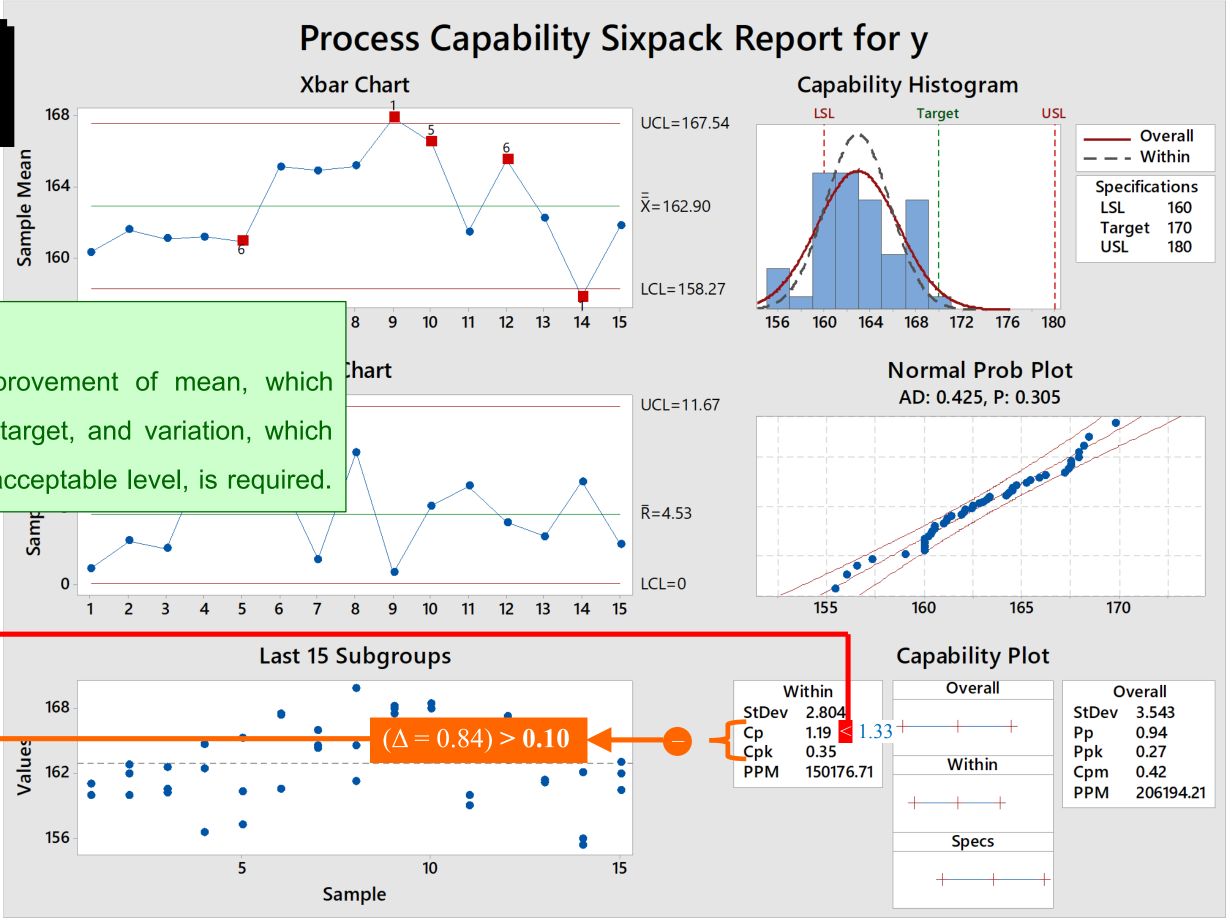
PCA in Y-axis



Conclusion:

The capability of the current process **is not accepted** as the short-term actual capability index (C_{pk}) was less than 1.33 significantly.

PCA in Y-axis



Improvement Direction:

For Y-axis distance, the improvement of mean, which should be increased to be on target, and variation, which is expected to be lower to the acceptable level, is required.

Improvement for σ is required.

Improvement for μ is required.

Evaluation if the sample size of current Y-axis data points is adequate for PCA

Given:

- Confidence Interval $= 95\%$ $\longrightarrow$ Type I error, $\alpha = 0.05$ $\longrightarrow$ $Z_\alpha \approx -1.645$
- $\frac{C_{pk}}{\hat{c}_{pk}} = 0.90$
- $\hat{C}_{pk(Y)} = 0.35$

Solution:

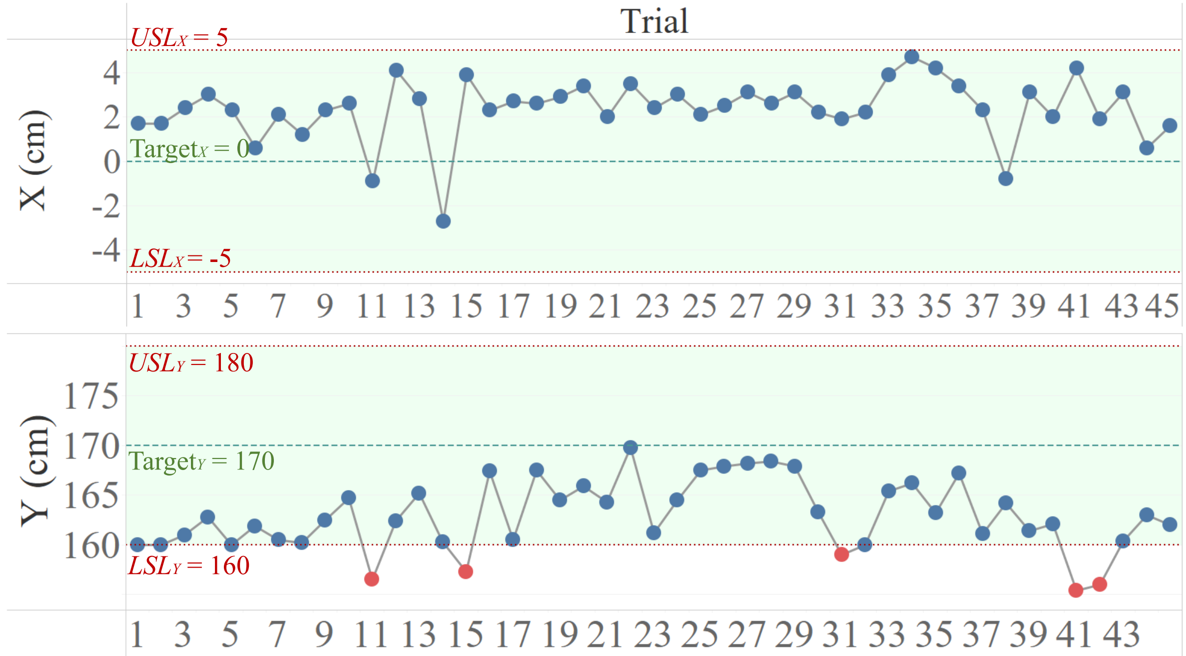
$$n = (Z_\alpha)^2 \frac{\left(\frac{1}{9\left(\hat{C}_{pk}\right)^2} + \frac{1}{2}\right)}{\left(1 - \frac{C_{pk}}{\hat{C}_{pk}}\right)^2} = (-1.645)^2 \times \frac{\left(\frac{1}{9 \times 0.35^2} + \frac{1}{2}\right)}{(1 - 0.90)^2} = \lceil 380.7 \rceil = 381$$

Conclusion: ∴ To estimate short-term actual capability index (C_{pk}) for Y-axis distance, at least sample size of 381 is required.

Justify: **The additional samples that should be later collected is no less than 336.**

From 3rd data collection:

Trial	X	Y
1	1.7	160
2	1.7	160
3	2.4	161
⋮	⋮	⋮
44	0.6	163
45	1.6	162
Mean	2.35	162.90
SD	1.38	3.54



Out of specification
Within specification

$Fine_X$ = 1,863.48 THB

$Fine_{X\text{-bias}}$ = 1,105.54 THB

$Fine_{X\text{-variation}}$ = 757.93 THB

$Fine_Y$ = 15,091.37 THB

$Fine_{Y\text{-bias}}$ = 10,069.38 THB

$Fine_{Y\text{-variation}}$ = 5,021.99 THB

ΣFine = 16,954.85 THB

Net reward = ฿83,045.15

REVISED

SIX-SIGMA PROJECT CHARTER

Quality improvement of the catapult shooting system for the competition

Project Sponsor	Master Blackbelt	Blackbelt	
XXXXX X.	Napassavong O.	Thatchakarn C.	Teethavat T.
Executive Leader	**Team Members**		
YYYYY Y.	AAAAA A.	BBBBB B.	CCCCC C.

Business Case

Compared with the falling distances of the competition's target, the performance from the current settings of the catapult was lower than expectation, which may result the team to have some amount of fine, leading the team to miss the chance receiving the extra reward if the team had become the first rank of the competition after the team had invested many resources, including money and time, to this competition.

Project Metrics

<u>Business Metric:</u>	Ranking of the competition
<u>Primary Metric:</u>	The short-term actual capability index (C_{pk}) in both X axis and Y axis
<u>Secondary Metric:</u>	Knowledge from using Six Sigma methodology and tools
<u>Consequential Metric:</u>	Investment including time consumed in this project and improvement cost
<u>Financial Metric:</u>	Fine and received reward calculated from the falling distances of the ball compared with those of the target

Problem Statement

In February, it was found that the catapult used in the competition can shoot the ball with the short-term actual capability index (C_{pk}) at 0.64 in X-axis and 0.35 in Y-axis, resulting the team with high amount of fine at 16,954.85 THB and receiving only the amount of money reward at 83,045.15 THB.

Objective Statement

The project aims to increase the short-term actual capability index (C_{pk}) of the process from 0.64 in X-axis and 0.35 in Y-axis to 1.33 by 1st May 2022.

Project Constraints

- Total project budget ≤ 100k THB
- Time consuming of team for project consulting ≤ 2 days/week
- Cup position is not allowed for position adjustment.

Project Scope

The project scope covers the study of the given catapult that will be used for the competition.

Project Assumptions

- Workdays of each week range between Friday to Sunday.
- Progress summary of project is required to be reported to master blackbelt and project sponsor in (mostly) Sunday, and the performance as well as the feedback of the project progress will be later discussed on Tuesday morning.

Project Timeline

- ➢ <u>Define phase:</u> 01/02/2022 to 06/02/2022
- ➢ <u>Measure phase:</u> 08/02/2022 to 06/03/2022
- ➢ <u>Analyze phase:</u> 22/02/2022 to 10/04/2022
- ➢ <u>Improve phase:</u> 12/04/2022 to 01/05/2022
- ➢ <u>Control phase:</u> 26/04/2022 to 01/05/2022

Designing Data Collection Plan for GR&R

Given:

- Three operators
- One measurement tool for each axis distance

Then:

∴ The number of round for catapult shooting and replications for measuring a distances of each operator are ten and two respectively.

If the given data collection was performed at **ten rounds** of catapult shooting and **two replications** for measuring a distances of each operator,

Justify:

The additional samples that should be later collected is not required as the sample size in the data collection plan is already adequate.

Designing order of shooting and measuring

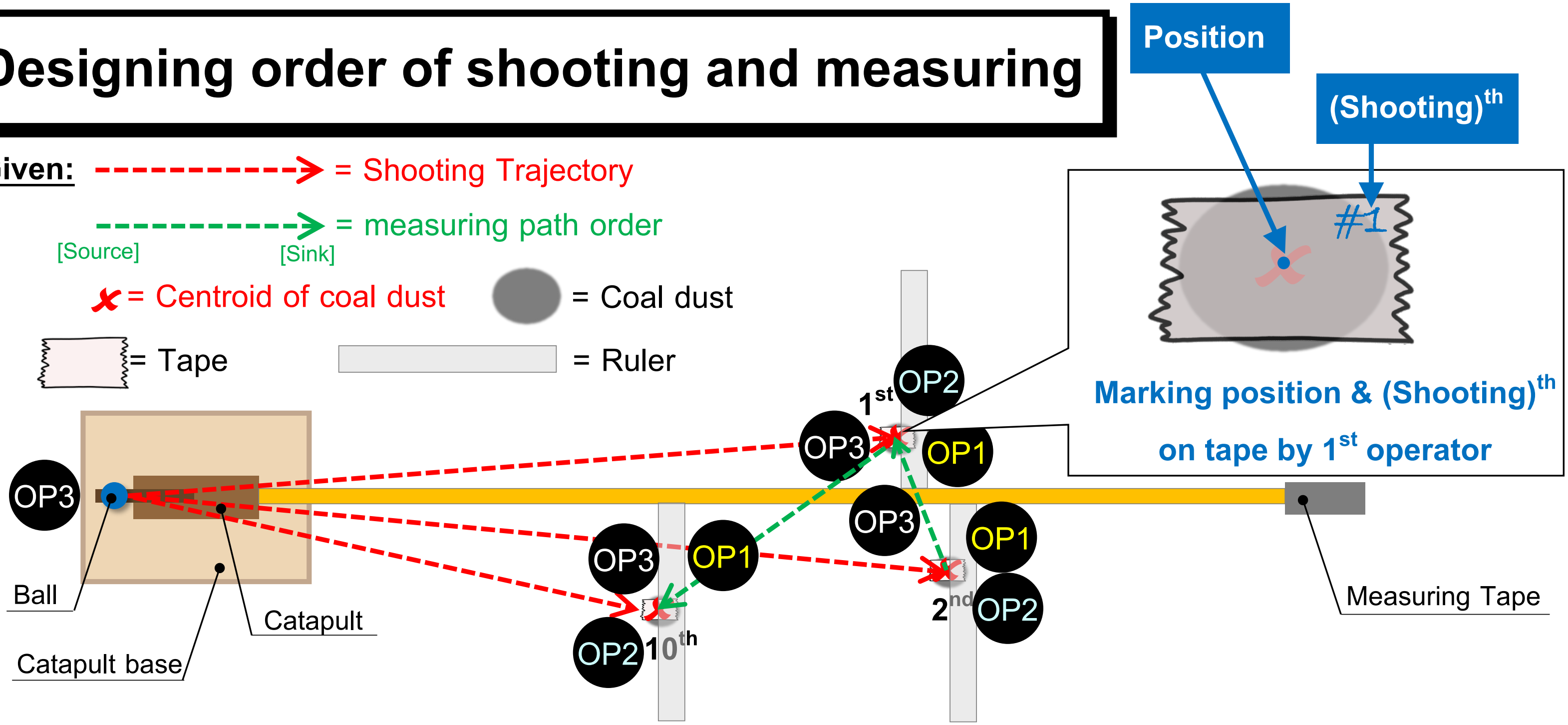
Given: = Shooting Trajectory

= measuring path order

[Source] [Sink]

= Centroid of coal dust = Coal dust

= Tape = Ruler



4th data collection:

- Shooter: CCCCC C. (OP3) [∵ Highest performance in maintaining stability from overall control charts]
- Measurer's Name: AAAAA A. (OP1)

(Shooting)th	Measurement			
	X (cm)		Y (cm)	
	#1	#2	#1	#2
1	1.7	1.8	150	150.2
2	1.7	1.6	150	149.9
3	2.4	2.4	151	151
4	3	3	152.8	152.8
5	2.3	2.4	150	149.8
6	0.6	0.6	151.9	151.9
7	2.1	2.1	150.5	150.5
8	1.2	1.4	150.2	150.2
9	2.3	2.4	152.5	152.6
10	2.6	2.5	154.7	154.6

4^{th} data collection:

- Shooter: CCCCC C. (OP3) [∵ Highest performance in maintaining stability from overall control charts]
- Measurer's Name: BBBBB B. (OP2)

(Shooting)th	Measurement			
	X (cm)		*Y* (cm)	
	#1	#2	#1	#2
1	1.7	1.7	150.1	150.1
2	1.7	1.7	150.1	149.9
3	2.3	2.4	151.2	151.2
4	3	3	152.7	152.7
5	2.6	2.5	150	150.2
6	0.5	0.5	151.6	151.6
7	2.2	2.2	150.4	150.4
8	1.5	1.5	150.3	150.4
9	1.8	1.8	152.5	152.6
10	2.6	2.6	154.7	154.7

4th data collection:

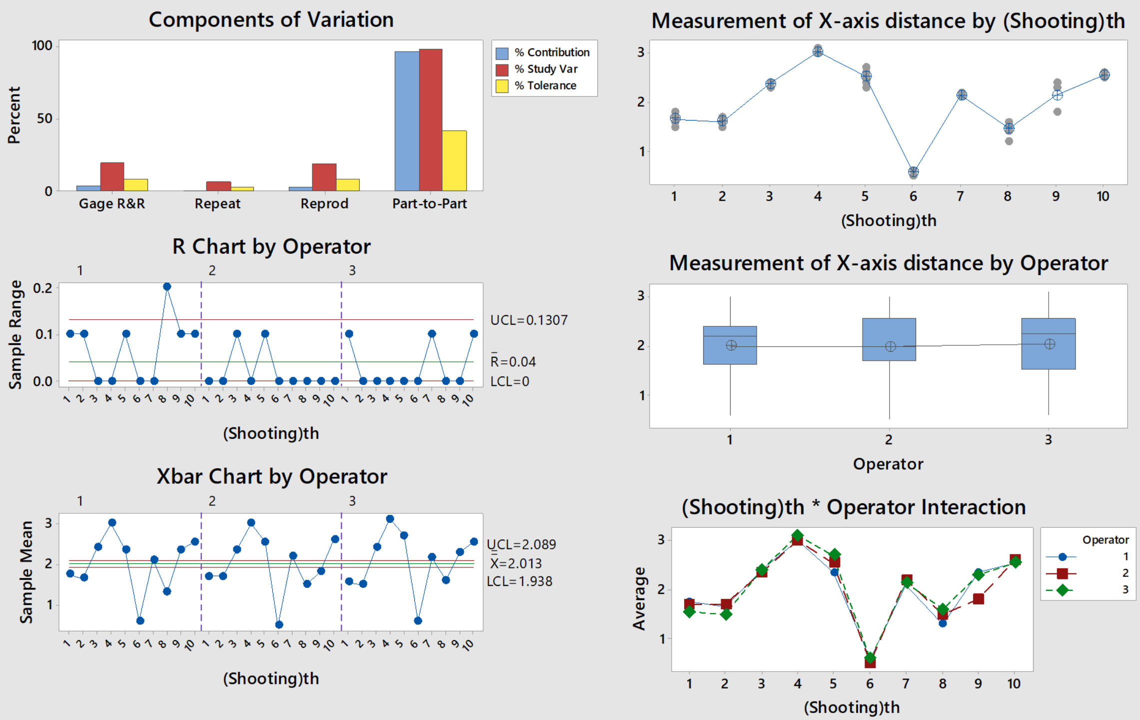
- Shooter: CCCCC C. (OP3) [∵ Highest performance in maintaining stability from overall control charts]
- Measurer's Name: CCCCC C. (OP3)

(Shooting)th	Measurement			
	X (cm)		*Y* (cm)	
	#1	#2	#1	#2
1	1.5	1.6	150.1	150.1
2	1.5	1.5	150.1	150
3	2.4	2.4	151.1	151
4	3.1	3.1	152.6	152.7
5	2.7	2.7	151	151
6	0.6	0.6	151.6	151.6
7	2.1	2.2	150.4	150.4
8	1.6	1.6	150.1	149.8
9	2.3	2.3	152.6	152.6
10	2.6	2.5	154.6	154.8

Gage R&R (ANOVA) Report for Measurement of X-axis distance

Gage name: Ruler
Date of study: March 2022

Reported by: Group 1
Tolerance: 10
Misc:



Gage R&R Study - ANOVA Method

Gage R&R for Measurement of X-axis distance

Gage name: Ruler
Date of study: March 2022
Reported by: Group 1
Tolerance: 10
Misc:

Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
(Shooting)th	9	26.7627	2.97363	78.2153	0.000
Operator	2	0.0323	0.01617	0.4252	0.660
(Shooting)th * Operator	18	0.6843	0.03802	16.2937	0.000
Repeatability	30	0.0700	0.00233		
Total	59	27.5493			

α to remove interaction term = 0.05

Gage R&R

Variance Components

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.020176	3.96
Repeatability	0.002333	0.46
Reproducibility	0.017843	3.50
Operator	0.000000	0.00
Operator*(Shooting)th	0.017843	3.50
Part-To-Part	0.489269	96.04
Total Variation	0.509444	100.00

Process tolerance = 10

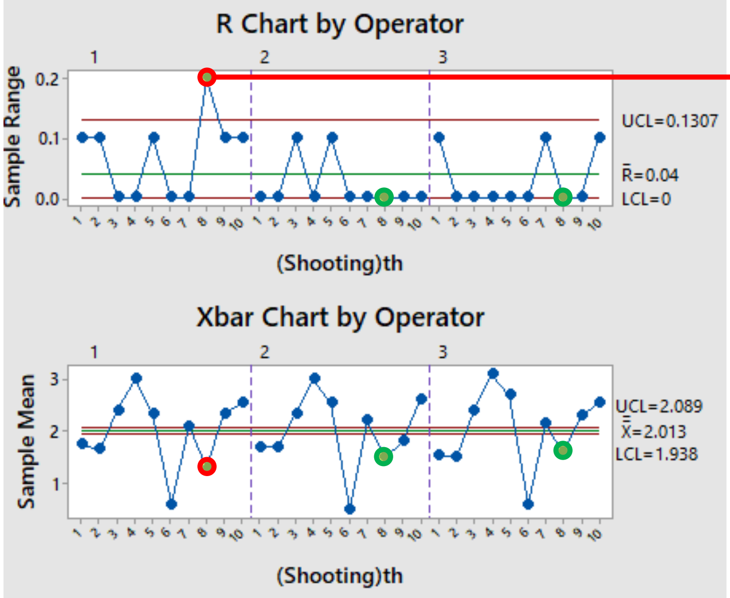
Gage Evaluation

Source	StdDev (SD)	Study Var (6 × SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.142042	0.85225	19.90	8.52
Repeatability	0.048305	0.28983	6.77	2.90
Reproducibility	0.133576	0.80146	18.71	8.01
Operator	0.000000	0.00000	0.00	0.00
Operator*(Shooting)th	0.133576	0.80146	18.71	8.01
Part-To-Part	0.699477	4.19686	98.00	41.97
Total Variation	0.713754	4.28252	100.00	42.83

Number of Distinct Categories = 6

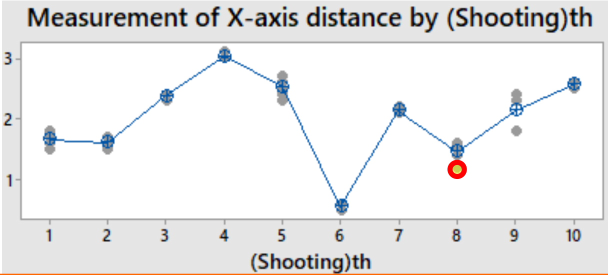
Identification for data abnormality in X-axis measurement

Solution:



There is one point from 1st operator measurement in 8th shooting that is out of control in R-chart by operator.

(Shooting)th	Measurement from OP1 X (cm) #1	#2
8	1.2	1.4

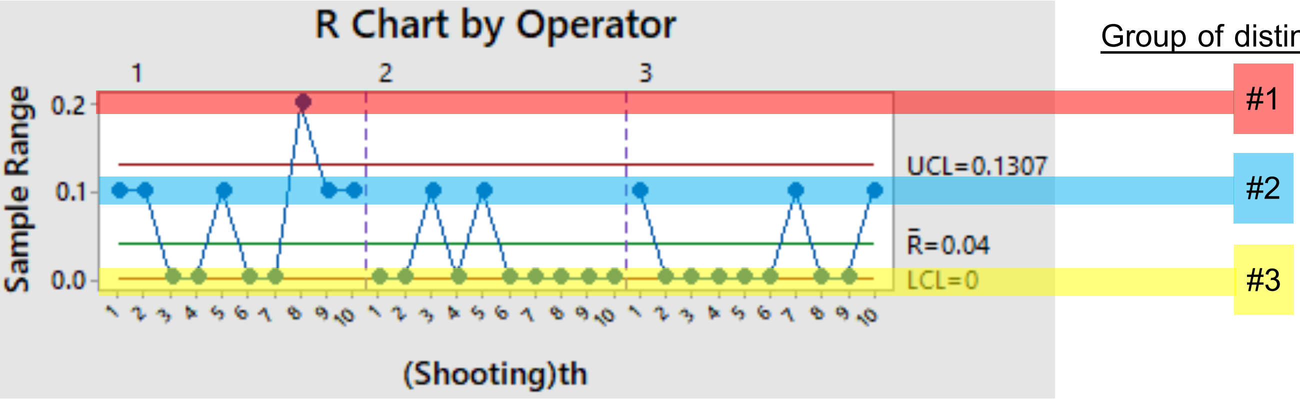


Justify:

There might be a significant abnormality in 1st operator's measuring record of 8th shooting. Thus, the investigation of this abnormality should be conducted. Nevertheless, one of the cause may come from recording the data via computer keyboard before GR&R analysis.

Justification for adequate resolution of ruler in X-axis measurement

Solution:



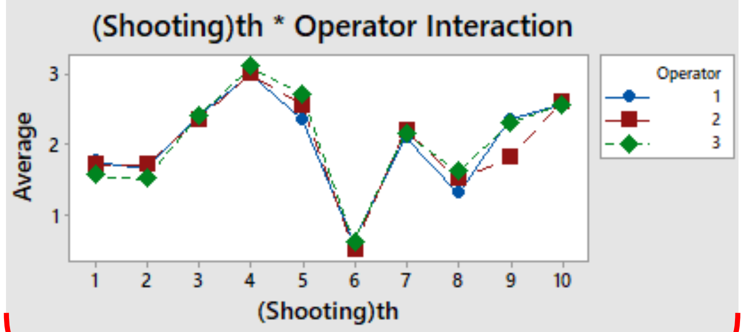
From R-chart by operator, **#Group of distinct R value = 3 < 4**

Justify:

∴ Ruler as the tool for measuring X-axis distances had inadequate resolution. Thus, the ruler did not possess the ability to discriminate the process variation.

Identification for the interaction effect between operator and the shooting round in X-axis

Solution:



Given:

- H0: There is no interaction between operator and shooting round in X axis.

  HA: There is interaction between operator and shooting round in X axis.
- $\alpha = 0.05$

Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
(Shooting)th	9	26.7627	2.97363	78.2153	0.000
Operator	2	0.0323	0.01617	0.4252	0.660
(Shooting)th * Operator	18	0.6843	0.03802	16.2937	$\boxed{0.000}$ < ($\alpha = 0.05$)
Repeatability	30	0.0700	0.00233		
Total	59	27.5493			

α to remove interaction term = 0.05

Reject null hypothesis.

Each connecting line seems to be parallel and have same direction except the measurement of 9th shooting from OP1 of which value differs significantly.

∴ There is interaction between operator and shooting round in X axis.

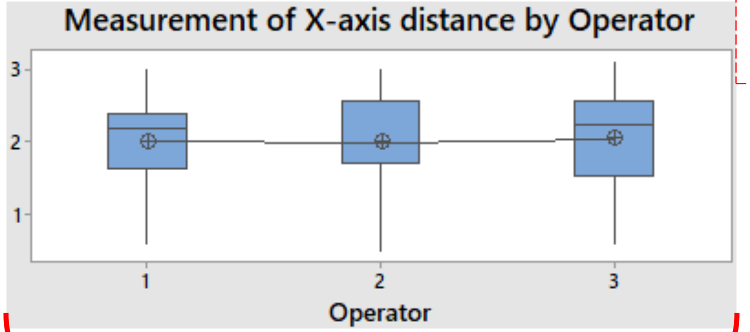
∴ There is interaction between operator and shooting round in X axis.

Justify:

From both statistical and graphical analysis, the result interpretation are consistent that the measured values obtained from the ruler can be significantly affected by the variation interacting between each operator and shooting round.

Identification for the effect from operation in measuring X-axis distances

Solution:



Mean of measuring distance from each operator doesn't differ significantly.

∴ There is no enough evidence to conclude that the effect from operator can differ the mean of measured distances in X-axis.

Given:

- H_0: The operator variation doesn't differ the mean of measuring X distances.
  H_A: The operator variation differs the mean of measuring X distances.
- $\alpha = 0.05$

Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
(Shooting)th	9	26.7627	2.97363	78.2153	0.000
Operator	2	0.0323	0.01617	0.4252	0.660
(Shooting)th * Operator	18	0.6843	0.03802	16.2937	0.000
Repeatability	30	0.0700	0.00233		
Total	59	27.5493			

α to remove interaction term = 0.05

$0.660 > (\alpha = 0.05)$

Fail to reject null hypothesis.

∴ There is no enough evidence to conclude that the effect from operator can differ the mean of measured distances in X-axis.

Justify: **From both statistical and graphical analysis, the result interpretation are consistent that the measured values obtained from the ruler may not be significantly affected by the variation from each operator.**

Justification if the X-axis measurement system should be accepted otherwise find improvement issue(s)

Solution:

Variance Components

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.020176	3.96
Repeatability	0.002333	0.46
Reproducibility	0.017843	3.50
Operator	0.000000	0.00
Operator*(Shooting)th	0.017843	3.50
Part-To-Part	0.489269	96.04
Total Variation	0.509444	100.00

From variance component, the total variation at 100% comprise of variation from shooting round at 96.04% and variation from gage at 3.96% consisting of repeatability and reproducibility at 0.46% and 3.50% respectively.

Gage Evaluation

Source	StdDev (SD)	Study Var (6 × SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.142042	0.85225	P/TV = 19.90	P/T = 8.52
Repeatability	0.048305	0.28983	6.77	2.90
Reproducibility	0.133576	0.80146	18.71	8.01
Operator	0.000000	0.00000	0.00	0.00
Operator*(Shooting)th	0.133576	0.80146	18.71	8.01
Part-To-Part	0.699477	4.19686	98.00	41.97
Total Variation	0.713754	4.28252	100.00	42.83

P/TV = 19.90 < 30%

Justify: This measurement system is reliable as the process variation is within acceptable level at 30% based on AIAG criteria. Thus, there is no need for improving this measurement system.

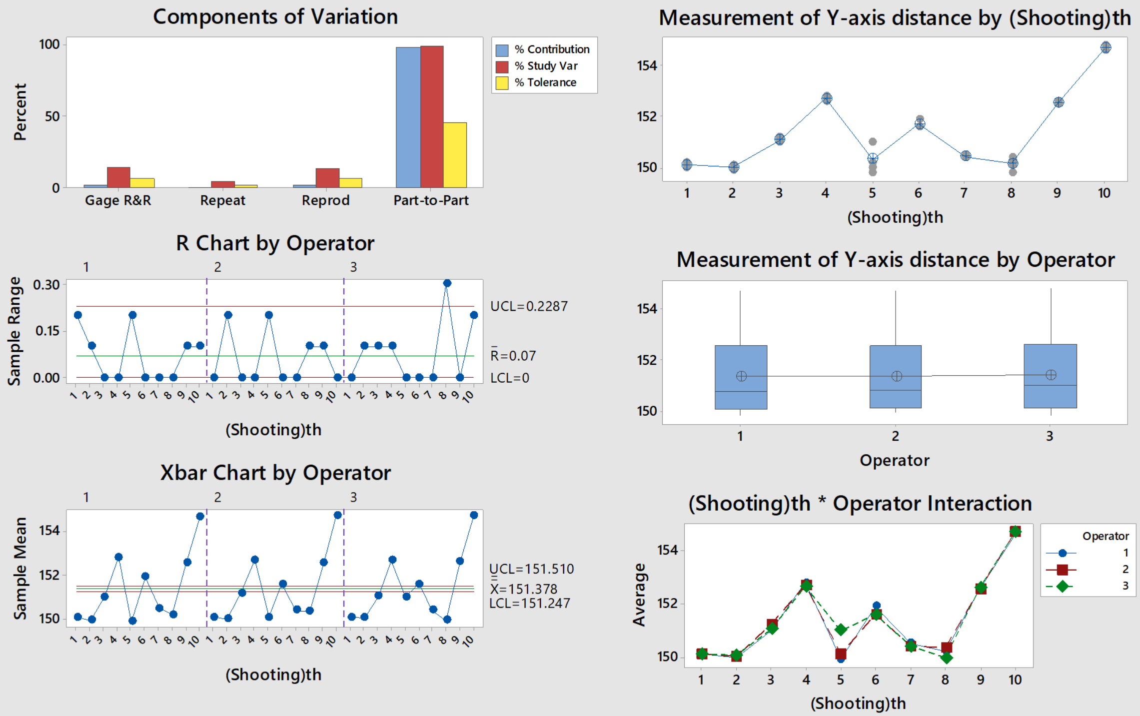
What if: The future process variation decreases and affects the ratio between the process and total variation to exceed the acceptable level at 30%.

Then: Improvement in reducing reproducibility variation should be conducted as the variance component of reproducibility might be greater than that of repeatability.

Gage R&R (ANOVA) Report for Measurement of Y-axis distance

Gage name: Measuring tape
Date of study: March 2022

Reported by: Group 1
Tolerance: 20
Misc:



Gage R&R Study - ANOVA Method

Gage R&R for Measurement of Y-axis distance

Gage name:	Measuring tape
Date of study:	March 2022
Reported by:	Group 1
Tolerance:	20
Misc:	

Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
(Shooting)th	9	127.823	14.2026	148.546	0.000
Operator	2	0.032	0.0162	0.169	0.846
(Shooting)th * Operator	18	1.721	0.0956	15.505	0.000
Repeatability	30	0.185	0.0062		
Total	59	129.762			

α to remove interaction term = 0.05

Gage R&R

Variance Components

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.05089	2.12
Repeatability	0.00617	0.26
Reproducibility	0.04472	1.86
Operator	0.00000	0.00
Operator*(Shooting)th	0.04472	1.86
Part-To-Part	2.35117	97.88
Total Variation	2.40206	100.00

Process tolerance = 20

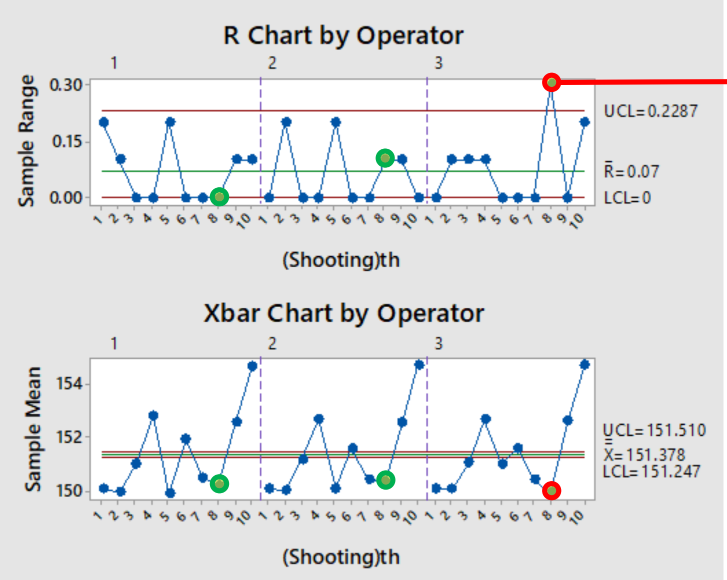
Gage Evaluation

Source	StdDev (SD)	Study Var (6 × SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.22559	1.35351	14.56	6.77
Repeatability	0.07853	0.47117	5.07	2.36
Reproducibility	0.21148	1.26886	13.64	6.34
Operator	0.00000	0.00000	0.00	0.00
Operator*(Shooting)th	0.21148	1.26886	13.64	6.34
Part-To-Part	1.53335	9.20011	98.94	46.00
Total Variation	1.54986	9.29914	100.00	46.50

Number of Distinct Categories = 9

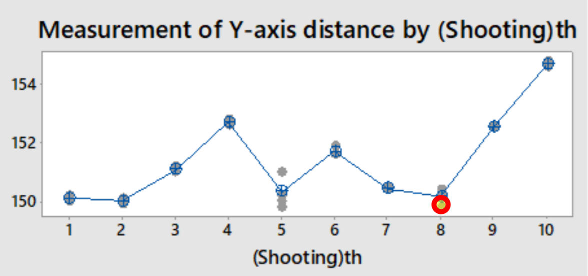
Identification for data abnormality in Y-axis measurement

Solution:



There is one point from 3rd operator measurement in 8th shooting that is out of control in R-chart by operator.

(Shooting)th	Measurement from OP3 Y (cm) #1	Measurement from OP3 Y (cm) #2
8	150.1	149.8

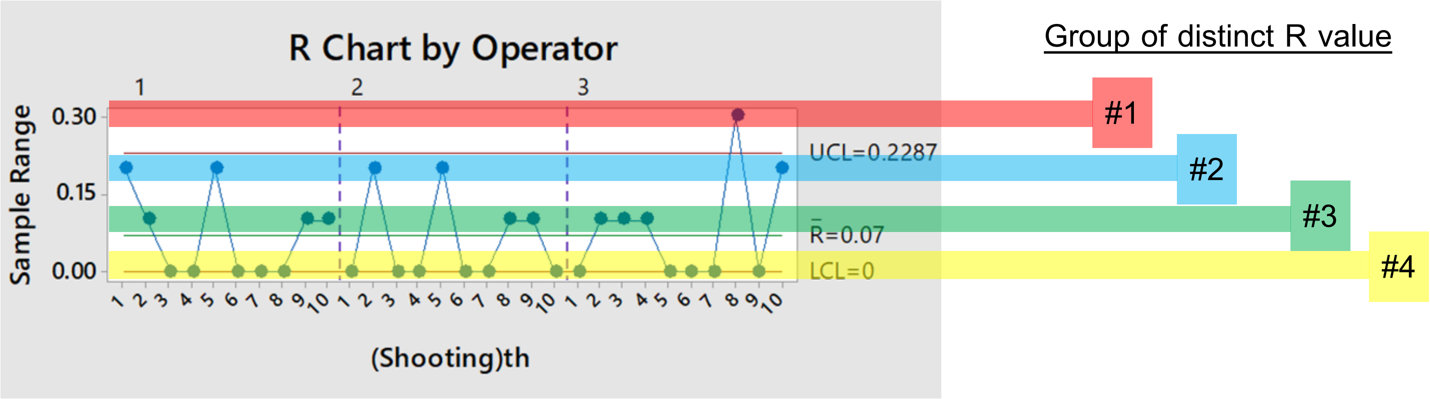


Justify:

There might be a significant abnormality in 3rd operator's measuring record of 8th shooting. Thus, the investigation of this abnormality should be conducted.

Justification for adequate resolution of measuring tape in Y-axis measurement

Solution:

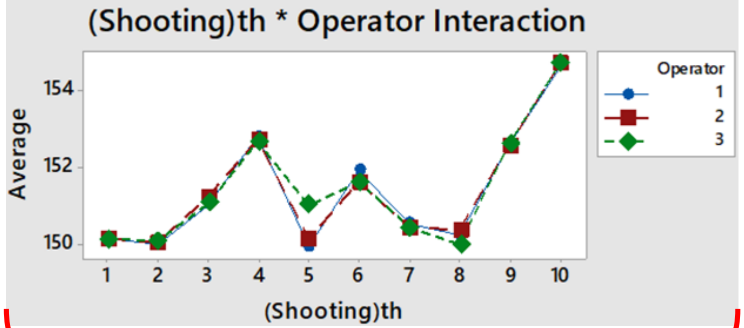


From R-chart by operator, **#Group of distinct R value = 4 ≥ 4**

Justify: **∴ Measuring tape as the tool for measuring Y-axis distances has adequate resolution.**

Identification for the interaction effect between operator and the shooting round in Y-axis

Solution:



Given:

- H_0: There is no interaction between operator and shooting round in Y axis.
  H_A: There is interaction between operator and shooting round in Y axis.
- $\alpha = 0.05$

Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
(Shooting)th	9	127.823	14.2026	148.546	0.000
Operator	2	0.032	0.0162	0.169	0.846
(Shooting)th * Operator	18	1.721	0.0956	15.505	0.000
Repeatability	30	0.185	0.0062		
Total	59	129.762			

to remove interaction term = 0.05

$0.000 < (\alpha = 0.05)$

Reject null hypothesis.

Each connecting line seems to be parallel and have same direction except the measurement of 5th shooting from OP3 of which value differs significantly.

∴ There is interaction between operator and shooting round in Y axis.

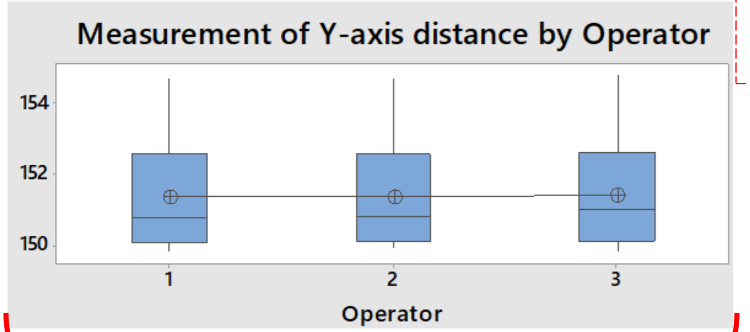
∴ There is interaction between operator and shooting round in Y axis.

Justify:

From both statistical and graphical analysis, the result interpretation are consistent that the measured values obtained from the measuring tape can be significantly affected by the variation from each operator and shooting round interaction.

Identification for the effect from operation in measuring Y-axis distances

Solution:



Given:

- H_0: The operator variation doesn't differ the mean of measuring Y distances.
  H_A: The operator variation differs the mean of measuring Y distances.
- $\alpha = 0.05$

Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
(Shooting)th	9	127.823	14.2026	148.546	0.000
Operator	2	0.032	0.0162	0.169	0.846
(Shooting)th * Operator	18	1.721	0.0956	15.505	0.000
Repeatability	30	0.185	0.0062		
Total	59	129.762			

α to remove interaction term = 0.05

$0.846 > (\alpha = 0.05)$

Fail to reject null hypothesis.

Mean of measuring distance from each operator doesn't differ significantly.

∴ There is no enough evidence to conclude that the effect from operator can differ the mean of measured distances in Y-axis.

∴ There is no enough evidence to conclude that the effect from operator can differ the mean of measured distances in Y-axis.

Justify: **From both statistical and graphical analysis, the result interpretation are consistent that the measured values obtained from the measuring tape may not be significantly affected by the variation from each operator.**

Justification if the Y-axis measurement system should be accepted otherwise find improvement issue(s).

Solution:

Variance Components

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.05089	2.12
Repeatability	0.00617	0.26
Reproducibility	0.04472	1.86
Operator	0.00000	0.00
Operator*(Shooting)th	0.04472	1.86
Part-To-Part	2.35117	97.88
Total Variation	2.40206	100.00

From variance component, the total variation at 100% comprise of variation from shooting round at 97.88% and variation from gage at 2.12% consisting of repeatability and reproducibility at 0.26% and 1.86% respectively.

Gage Evaluation

Source	StdDev (SD)	Study Var (6 × SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.22559	1.35351	P/TV = 14.56	P/T = 6.77
Repeatability	0.07853	0.47117	5.07	2.36
Reproducibility	0.21148	1.26886	13.64	6.34
Operator	0.00000	0.00000	0.00	0.00
Operator*(Shooting)th	0.21148	1.26886	13.64	6.34
Part-To-Part	1.53335	9.20011	98.94	46.00
Total Variation	1.54986	9.29914	100.00	46.50

P/TV = 14.56 < 30%

Justify: This measurement system is reliable as the process variation is within acceptable level at 30% based on AIAG criteria. Thus, there is no need for improving this measurement system.

What if: The future process variation will decrease and affect the ratio between the process and total variation to exceed the acceptable level at 30%

Then: Improvement in reducing reproducibility variation should be conducted as the variance component of reproducibility might be greater than that of repeatability.

Causes of variation in GR&R

In gage repeatability variation

- Unclear scale of measurement tools for measuring both X-axis and Y-axis.
- Wear off from a measuring tool such as the distortion (≠90°) at the tip of the steel ruler.
- Inconstancy from the operator to measure the falling position by finding the centroid of the charcoal dust occurred after the ball hit the floor (Sometimes it may be hard to find the centroid as it may not be the perfect circle but the freeform oval or the other strange shapes)
- The tape may start not to hold the equipment as well as the catapult firmly [affecting in uncertainty during either the shooting process or the measuring process in that trial].
- Fabrication of number from operator’s record to achieve good performance.

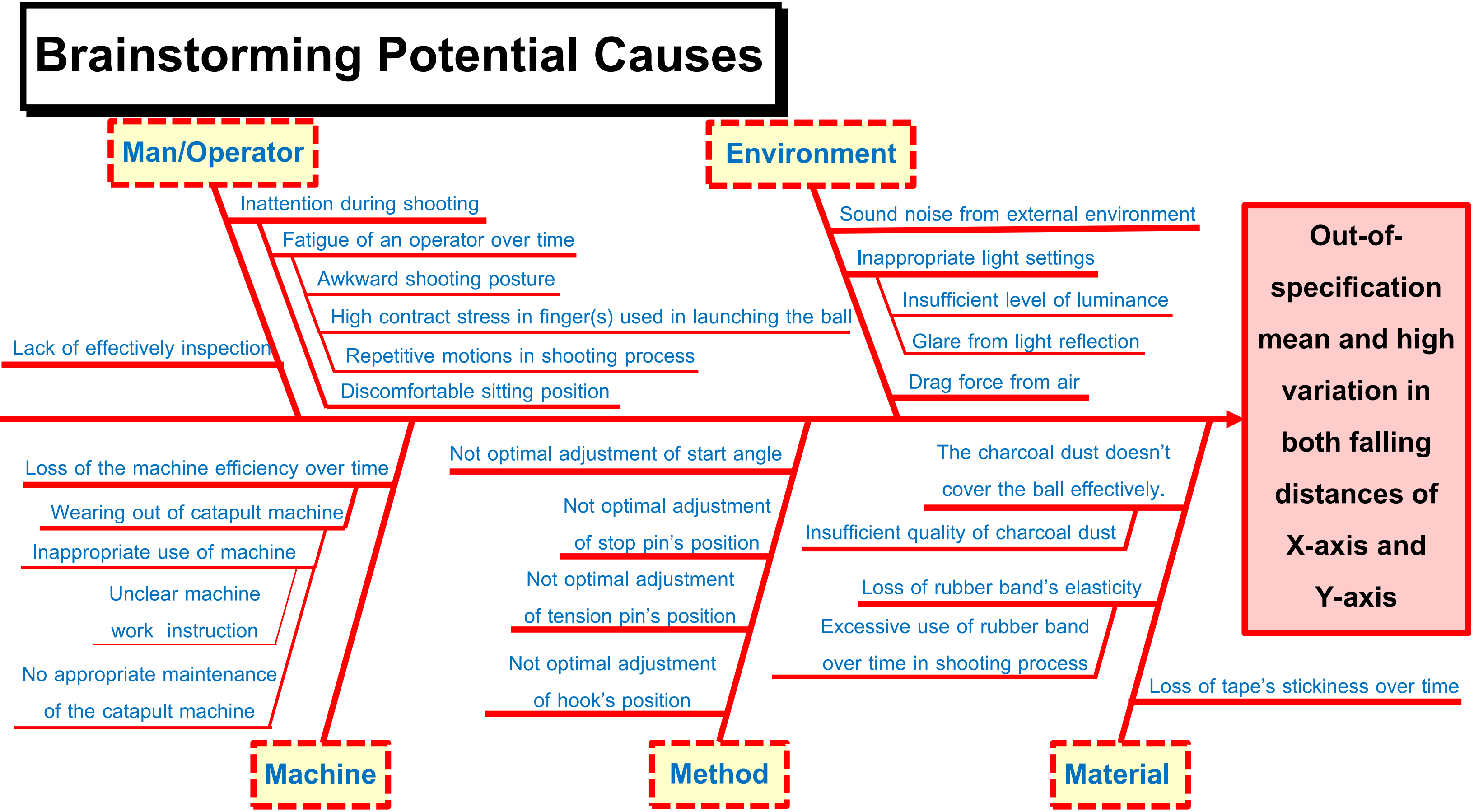
In gage reproducibility

- Non standardized measuring method for each operator (maybe from lack of practice or low learning rate)
- Unclear work instruction for measuring falling distances

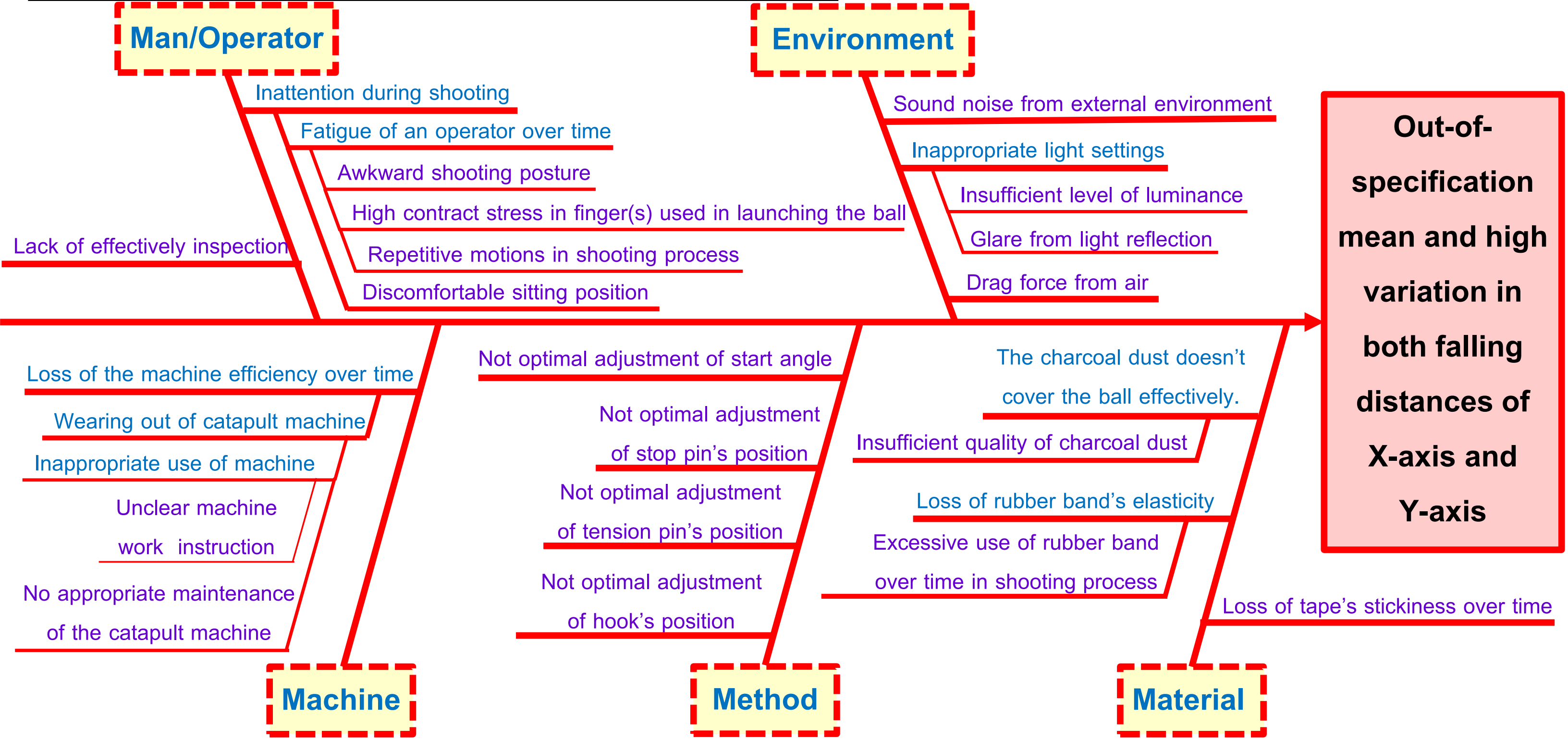
In variation from each shooting round

- Random error from wrong setting of the machine such as randomly setting a wrong angle
- The tape may start not to hold the equipment as well as the catapult firmly [affecting in uncertainty during either the shooting process or the measuring process in that trial].
- Drag force from the air compression
- Unclear work instruction for shooting
- Sound noise from external environment for a period of time such as air conditioner, vacuum cleaner, construction site etc.
- Inappropriate level of luminance from the light source(s) and discomfort from glare
- Fatigue of an operator over time affecting an inattention of the operator
- Loss of rubber band’s elasticity after it has been used for a period of time
- Sitting position and posture of operator in shooting process
- Lack of effectively inspection in the position of the tape due to each individual laziness

Analyze Phase


Brainstorming Potential Causes
Man/Operator
Inattention during shooting
Fatigue of an operator over time
Awkward shooting posture
High contract stress in finger(s) used in launching the ball
Repetitive motions in shooting process
Discomfortable sitting position
Lack of effectively inspection
Environment
Sound noise from external environment
Inappropriate light settings
Insufficient level of luminance
Glare from light reflection
Drag force from air
Out-of-specification mean and high variation in both falling distances of X-axis and Y-axis
Loss of the machine efficiency over time
Wearing out of catapult machine
Inappropriate use of machine
Unclear machine work instruction
No appropriate maintenance of the catapult machine
Machine
Not optimal adjustment of start angle
Not optimal adjustment of stop pin's position
Not optimal adjustment of tension pin's position
Not optimal adjustment of hook's position
Method
The charcoal dust doesn't cover the ball effectively.
Insufficient quality of charcoal dust
Loss of rubber band's elasticity
Excessive use of rubber band over time in shooting process
Loss of tape's stickiness over time
Material

Brainstorming Potential Causes



Prioritizing Causes Affecting Responses

Given the criteria score:

Relationship	Relationship between factor/cause and response	Score
Strong	The factor has strong association with response and can cause the occurrence of the response significantly.	9
Medium	The factor has moderate association with response and could cause the occurrence of the response.	3
Weak	The factor has weak association with response and may cause the occurrence of the response.	1
Very weak	The factor has very weak association with response and might rarely cause the occurrence of the response.	0

Then,

No.	Category	Factor\Response	$\overline{y}$	$\widehat{\sigma}_y$	$\overline{x}$	$\widehat{\sigma}_x$	Total Score
1	Man						
2							
3							
4							
5							
6	Machine						
7							
8	Material						
9							
10							
11	Method						
12							
13							
14							
15	Environment						
16							
17							
18							

Then,

No.	Category	Factor\Response	$\overline{y}$	$\hat{\sigma}_y$	$\overline{x}$	$\hat{\sigma}_x$	Total Score
1	Man	Lack of effectively inspection					
2		Awkward shooting posture					
3		High contract stress in finger(s) used in launching the ball					
4		Repetitive motions in shooting process					
5		Discomfortable sitting position					
6	Machine	Unclear machine work instruction					
7		No appropriate maintenance of the catapult machine					
8	Material	Insufficient quality of charcoal dust					
9		Excessive use of rubber band over time in shooting process					
10		Loss of tape's stickiness over time					
11	Method	Not optimal adjustment of start angle					
12		Not optimal adjustment of stop pin's position					
13		Not optimal adjustment of tension pin's position					
14		Not optimal adjustment of hook's position					
15	Environment	Sound noise from external environment					
16		Insufficient level of luminance					
17		Glare from light reflection					
18		Drag force from air					

Then,

No.	Category	Factor\Response	$\bar{y}$	$\hat{\sigma}_y$	$\bar{x}$	$\hat{\sigma}_x$	Total Score
		Response Importance weight					
1	Man	Lack of effectively inspection					
2		Awkward shooting posture					
3		High contract stress in finger(s) used in launching the ball					
4		Repetitive motions in shooting process					
5		Discomfortable sitting position					
6	Machine	Unclear machine work instruction					
7		No appropriate maintenance of the catapult machine					
8	Material	Insufficient quality of charcoal dust					
9		Excessive use of rubber band over time in shooting process					
10		Loss of tape's stickiness over time					
11	Method	Not optimal adjustment of start angle					
12		Not optimal adjustment of stop pin's position					
13		Not optimal adjustment of tension pin's position					
14		Not optimal adjustment of hook's position					
15	Environment	Sound noise from external environment					
16		Insufficient level of luminance					
17		Glare from light reflection					
18		Drag force from air					

Then,

No.	Category	Factor\Response	$\bar{y}$	$\hat{\sigma}_y$	$\bar{x}$	$\hat{\sigma}_x$	Total Score
		Response Importance weight	10	0.7	0.0	0.4	
1	Man	Lack of effectively inspection					
2		Awkward shooting posture					
3		High contract stress in finger(s) used in launching the ball					
4		Repetitive motions in shooting process					
5		Discomfortable sitting position					
6	Machine	Unclear machine work instruction					
7		No appropriate maintenance of the catapult machine					
8	Material	Insufficient quality of charcoal dust					
9		Excessive use of rubber band over time in shooting process					
10		Loss of tape's stickiness over time					
11	Method	Not optimal adjustment of start angle					
12		Not optimal adjustment of stop pin's position					
13		Not optimal adjustment of tension pin's position					
14		Not optimal adjustment of hook's position					
15	Environment	Sound noise from external environment					
16		Insufficient level of luminance					
17		Glare from light reflection					
18		Drag force from air					

Then,

No.	Category	Factor\Response	$\bar{y}$	$\hat{\sigma}_y$	$\bar{x}$	$\hat{\sigma}_x$	Total Score
		Response Importance weight	10	0.7	0.0	0.4	
1	Man	Lack of effectively inspection	1	0	9	0	
2		Awkward shooting posture	3	3	9	3	
3		High contract stress in finger(s) used in launching the ball	3	3	3	3	
4		Repetitive motions in shooting process	3	3	9	3	
5		Discomfortable sitting position	3	3	9	3	
6	Machine	Unclear machine work instruction	1	1	3	1	
7		No appropriate maintenance of the catapult machine	1	1	1	1	
8	Material	Insufficient quality of charcoal dust	0	1	0	1	
9		Excessive use of rubber band over time in shooting process	3	1	0	0	
10		Loss of tape's stickiness over time	1	0	9	0	
11	Method	Not optimal adjustment of start angle	9	9	0	0	
12		Not optimal adjustment of stop pin's position	9	9	0	0	
13		Not optimal adjustment of tension pin's position	9	9	0	0	
14		Not optimal adjustment of hook's position	9	9	0	0	
15	Environment	Sound noise from external environment	1	1	1	1	
16		Insufficient level of luminance	3	3	3	3	
17		Glare from light reflection	1	1	1	1	
18		Drag force from air	1	1	1	1	

Then,

No.	Category	Factor\Response	$\bar{y}$	$\hat{\sigma}_y$	$\bar{x}$	$\hat{\sigma}_x$	Total Score
		Response Importance weight	10	0.7	0.0	0.4	
1	Man	Lack of effectively inspection	1	0	9	0	10
2		Awkward shooting posture	3	3	9	3	33
3		High contract stress in finger(s) used in launching the ball	3	3	3	3	33
4		Repetitive motions in shooting process	3	3	9	3	33
5		Discomfortable sitting position	3	3	9	3	33
6	Machine	Unclear machine work instruction	1	1	3	1	11
7		No appropriate maintenance of the catapult machine	1	1	1	1	11
8	Material	Insufficient quality of charcoal dust	0	1	0	1	1
9		Excessive use of rubber band over time in shooting process	3	1	0	0	31
10		Loss of tape’s stickiness over time	1	0	9	0	10
11	Method	Not optimal adjustment of start angle	9	9	0	0	96
12		Not optimal adjustment of stop pin’s position	9	9	0	0	96
13		Not optimal adjustment of tension pin’s position	9	9	0	0	96
14		Not optimal adjustment of hook’s position	9	9	0	0	96
15	Environment	Sound noise from external environment	1	1	1	1	11
16		Insufficient level of luminance	3	3	3	3	33
17		Glare from light reflection	1	1	1	1	11
18		Drag force from air	1	1	1	1	11

Then,

∴ The response variables can be optimized with "Response Surface Design"

No.	Category	Factor\Response	$\bar{y}$	$\hat{\sigma}_y$	$\bar{x}$	$\hat{\sigma}_x$	Total Score
		Response Importance weight	10	0.7	0.0	0.4	
1	Man	Lack of effectively inspection	1	0	9	0	10
2		Awkward shooting posture	3	3	9	3	33
3		High contract stress in finger(s) used in launching the ball	3	3	3	3	33
4		Repetitive motions in shooting process	3	3	9	3	33
5		Discomfortable sitting position	3	3	9	3	33
6	Machine	Unclear machine work instruction	1	1	3	1	11
7		No appropriate maintenance of the catapult machine	1	1	1	1	11
8	Material	Insufficient quality of charcoal dust	0	1	0	1	1
9		Excessive use of rubber band over time in shooting process	3	1	0	0	31
10		Loss of tape's stickiness over time	1	0	9	0	10
11	Method	Not optimal adjustment of start angle	9	9	0	0	96
12		Not optimal adjustment of stop pin's position	9	9	0	0	96
13		Not optimal adjustment of tension pin's position	9	9	0	0	96
14		Not optimal adjustment of hook's position	9	9	0	0	96
15	Environment	Sound noise from external environment	1	1	1	1	11
16		Insufficient level of luminance	3	3	3	3	33
17		Glare from light reflection	1	1	1	1	11
18		Drag force from air	1	1	1	1	11

"Selected Factors"

Designing Experiment

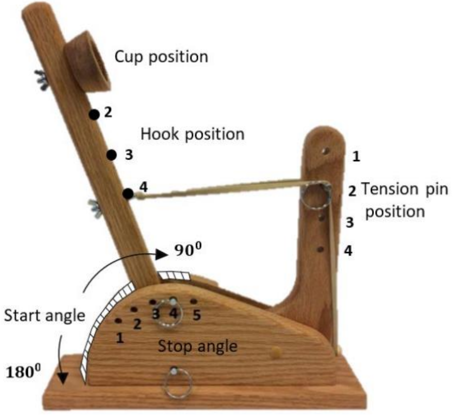
Given:

- Responses: Falling distances in X-axis (X) and Y-axis (Y)
- Objective: To achieve X-axis and Y-axis distances at 0 cm and 170 cm respectively

Assumption:

- Due to the limitation that the operator cannot perform the real data observation, then, given that there are five considerable treatment factors that affect the responses:

1) Hook Position = {2,3,4}
2) Tension Pin Position = {1,2,3,4}
3) Start Angle ∊ [90, 180]
4) Stop Pin Position = {1,2,3,4,5}
5) Hand Position = {-1, 0, 1}



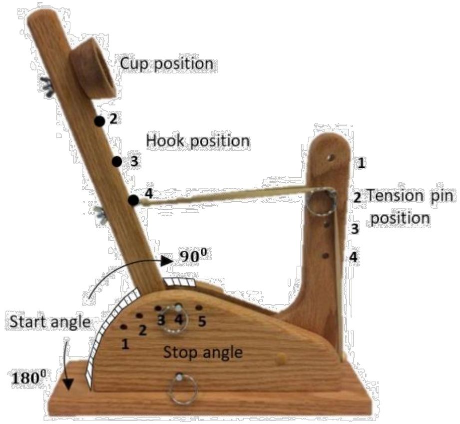
Choosing appropriate design

To optimize the current operating conditions for each factors to get the falling distances at target.

∴ The response surface method (RSM) should be applied.

Because of the limitations:

- The level of some factors can be only set as discrete variables:
  - Hook Position
  - Tension Pin Position
  - Stop Pin Position
  - Hand Position
- Some factors had less than five levels:
  - Three levels in “Hook Position”
  - Four levels in “Tension Pin Position”
  - Three levels in “Hand Position”



∴ Traditional Central Composite Design (CCD) which requires five levels for each factor can’t be performed.

Remaining RSM candidate: Face-center Central Composite Design (CCF) **Vs.** Box-Behnken Design (BB)

Prediction Quality

CCD > CCF > BB

#Runs

Available Response Surface Designs

Design		Continuous Factors								
		2	3	4	5	6	7	8	9	10
Central composite full	unblocked	13	20	31	52	90	152			
	blocked	14	20	30	54	90	160			
Central composite half	unblocked				32	53	88	154		
	blocked				33	54	90	160		
Central composite quarter	unblocked							90	156	
	blocked							90	160	
Central composite eighth	unblocked									158
	blocked									160
Box-Behnken	unblocked		15	27	46	54	62		130	170
	blocked			27	46	54	62		130	170

Resolution

Available Factorial Designs (with Resolution)

Factorial

	Factors													
Run	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	Full	III												
8		Full	IV	III	III	III								
16			Full	V	IV	IV	IV	III	III	III	III	III	III	III
32				Full	VI	IV	IV	IV	IV	IV	IV	IV	IV	IV
64					Full	VII	V	IV	IV	IV	IV	IV	IV	IV
128						Full	VIII	VI	V	V	IV	IV	IV	IV

∴ 2^{5-1} CCF is the most appropriate to perform:

The smallest total runs with resolution V

Design Generator: E = ABCD

Defining Relation: I = ABCDE

Alias: **Each main effect is aliased with a single 4-factor interaction.**

[A] ⇨ A + ~~BCDE~~
[B] ⇨ B + ~~ACDE~~
[C] ⇨ C + ~~ABDE~~
[D] ⇨ D + ~~ABCE~~
[E] ⇨ E + ~~ABCD~~

Each 2-factor interaction effect is aliased with a 3-factor interaction.

[AB] ⇨ AB + ~~CDE~~
[AC] ⇨ AC + ~~BDE~~
[AD] ⇨ AD + ~~BCE~~
[AE] ⇨ AE + ~~BCD~~
[BC] ⇨ BC + ~~ADE~~
[BD] ⇨ BD + ~~ACE~~
[BE] ⇨ BE + ~~ACD~~
[CD] ⇨ CD + ~~ABE~~
[CE] ⇨ CE + ~~ABD~~
[DE] ⇨ DE + ~~ABC~~

Assumption: By the sparsity of effects principle, the high-order (k ≥ 3) interaction effects are neglected.

The unique interpretation of the results from main effects and 2-factor interaction term can be obtained clearly.

Central Composite Design **(CCF)**

Design Summary

Factors:	5	Replicates:	1
Base runs:	32	Total runs:	32
Base blocks:	1	Total blocks:	1

α = 1

Two-level factorial: Half fraction

Point Types

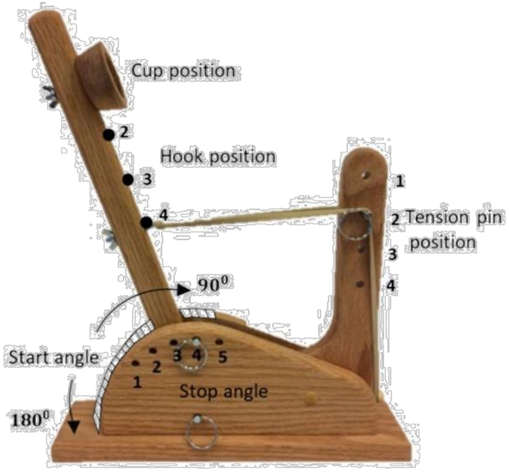
Cube points: (n_F)	16
Center points in cube: (n_C)	6
Axial points: (n_α)	10
Center points in axial:	0

	Hook	Tension	Start	Stop	Hand
$n_F = 2^{5-1} = 16$	-	-	-	-	+
	+	-	-	-	-
	-	+	-	-	-
	+	+	-	-	+
	-	-	+	-	-
	+	-	+	-	+
	-	+	+	-	+
	+	+	+	-	-
	-	-	-	+	-
	+	-	-	+	+
	-	+	-	+	+
	+	+	-	+	-
	-	-	+	+	+
	+	-	+	+	-
	-	+	+	+	-
	+	+	+	+	+

	Hook	Tension	Start	Stop	Hand
$n_\alpha = 2(5) = 10$	-	0	0	0	0
	+	0	0	0	0
	0	-	0	0	0
	0	+	0	0	0
	0	0	-	0	0
	0	0	+	0	0
	0	0	0	-	0
	0	0	0	+	0
	0	0	0	0	-
	0	0	0	0	+
$n_C = 6$ (rep)	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0

Coded Unit ⇨ Uncoded Unit

Treatment Factor	Level		
	Low (-)	Mid (0)	High (+)
Hook Position	2	3	4
Tension Pin Position	1	2	3
Start Angle	143	145	147
Stop Pin Position	2	3	4
Hand Position	-1	0	1



Design Matrix

Factorial Runs
$= 2^{k-p}$
$= 2^{5-1}$
$= 16$ runs

StdOrder	RunOrder	PtType	Blocks	Hook	Tension	Start	Stop	Hand
1	1	1	1	2	1	143	2	1
2	2	1	1	4	1	143	2	-1
3	3	1	1	2	3	143	2	-1
4	4	1	1	4	3	143	2	1
5	5	1	1	2	1	147	2	-1
6	6	1	1	4	1	147	2	1
7	7	1	1	2	3	147	2	1
8	8	1	1	4	3	147	2	-1
9	9	1	1	2	1	143	4	-1
10	10	1	1	4	1	143	4	1
11	11	1	1	2	3	143	4	1
12	12	1	1	4	3	143	4	-1
13	13	1	1	2	1	147	4	1
14	14	1	1	4	1	147	4	-1
15	15	1	1	2	3	147	4	-1
16	16	1	1	4	3	147	4	1

Design Matrix

Axial Runs
= 2k
= 2(5)
= 10 runs

Center Runs
= 6 runs

StdOrder	RunOrder	PtType	Blocks	Hook	Tension	Start	Stop	Hand
17	17	-1	1	2	2	145	3	0
18	18	-1	1	4	2	145	3	0
19	19	-1	1	3	1	145	3	0
20	20	-1	1	3	3	145	3	0
21	21	-1	1	3	2	143	3	0
22	22	-1	1	3	2	147	3	0
23	23	-1	1	3	2	145	2	0
24	24	-1	1	3	2	145	4	0
25	25	-1	1	3	2	145	3	-1
26	26	-1	1	3	2	145	3	1
27	27	0	1	3	2	145	3	0
28	28	0	1	3	2	145	3	0
29	29	0	1	3	2	145	3	0
30	30	0	1	3	2	145	3	0
31	31	0	1	3	2	145	3	0
32	32	0	1	3	2	145	3	0

Improve Phase

5^{th} data collection:

Factorial Runs
$= 2^{k-p}$
$= 2^{5-1}$
= 16 runs

StdOrder	RunOrder	PtType	Blocks	Hook	Tension	Start	Stop	Hand	*X*(cm)	*Y*(cm)
1	1	1	1	2	1	143	2	1	5.2	189
2	2	1	1	4	1	143	2	-1	-2.1	139
3	3	1	1	2	3	143	2	-1	-2.6	129
4	4	1	1	4	3	143	2	1	2.2	104
5	5	1	1	2	1	147	2	-1	-6.4	205
6	6	1	1	4	1	147	2	1	-3.3	171
7	7	1	1	2	3	147	2	1	4.1	153.5
8	8	1	1	4	3	147	2	-1	-5.5	137
9	9	1	1	2	1	143	4	-1	9.6	210.5
10	10	1	1	4	1	143	4	1	-4.9	165.5
11	11	1	1	2	3	143	4	1	-6.5	144
12	12	1	1	4	3	143	4	-1	-3.9	96.5
13	13	1	1	2	1	147	4	1	2.1	247.5
14	14	1	1	4	1	147	4	-1	-7.6	180
15	15	1	1	2	3	147	4	-1	2.5	160
16	16	1	1	4	3	147	4	1	2.2	110

5th data collection:

Axial Runs
= 2k
= 2(5)
= 10 runs

Center Runs
= 6 runs

StdOrder	RunOrder	PtType	Blocks	Hook	Tension	Start	Stop	Hand	X(cm)	Y(cm)
17	17	-1	1	2	2	145	3	0	-5.6	187.5
18	18	-1	1	4	2	145	3	0	2.2	154
19	19	-1	1	3	1	145	3	0	-10.5	194
20	20	-1	1	3	3	145	3	0	3.8	142.3
21	21	-1	1	3	2	143	3	0	-3.6	152
22	22	-1	1	3	2	147	3	0	-5.9	177
23	23	-1	1	3	2	145	2	0	-5	135
24	24	-1	1	3	2	145	4	0	5.8	167.4
25	25	-1	1	3	2	145	3	-1	0	164.5
26	26	-1	1	3	2	145	3	1	-5.9	170
27	27	0	1	3	2	145	3	0	-3.8	163
28	28	0	1	3	2	145	3	0	5.7	161
29	29	0	1	3	2	145	3	0	2.2	176
30	30	0	1	3	2	145	3	0	-0.8	171.5
31	31	0	1	3	2	145	3	0	-2.8	166.5
32	32	0	1	3	2	145	3	0	0	174

Analyzing Experimental Result #1

Response Surface Regression: X versus Hook, Tension, ... t, Stop, Hand

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	20	386.397	19.3199	0.67	0.790
Linear	5	63.746	12.7491	0.44	0.810
Hook	1	29.645	29.6450	1.03	0.332
Tension	1	11.202	11.2022	0.39	0.546
Start	1	6.969	6.9689	0.24	0.633
Stop	1	8.961	8.9606	0.31	0.588
Hand	1	6.969	6.9689	0.24	0.633
Square	5	30.805	6.1610	0.21	0.949
Hook*Hook	1	1.731	1.7307	0.06	0.811
Tension*Tension	1	1.620	1.6199	0.06	0.817
Start*Start	1	12.033	12.0335	0.42	0.531
Stop*Stop	1	21.250	21.2504	0.74	0.409
Hand*Hand	1	0.416	0.4164	0.01	0.906
2-Way Interaction	10	291.846	29.1846	1.01	0.488
Hook*Tension	1	41.926	41.9256	1.45	0.253
Hook*Start	1	0.276	0.2756	0.01	0.924
Hook*Stop	1	10.401	10.4006	0.36	0.560
Hook*Hand	1	11.391	11.3906	0.40	0.542
Tension*Start	1	86.026	86.0256	2.98	0.112
Tension*Stop	1	5.881	5.8806	0.20	0.660
Tension*Hand	1	2.176	2.1756	0.08	0.789
Start*Stop	1	21.856	21.8556	0.76	0.402
Start*Hand	1	45.901	45.9006	1.59	0.233
Stop*Hand	1	66.016	66.0156	2.29	0.158
Error	11	317.038	28.8216		
Lack-of-Fit	6	256.829	42.8049	3.55	0.093
Pure Error	5	60.208	12.0417		
Total	31	703.435			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
5.36858	54.93%	0.00%	0.00%

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-1.41	1.53	-0.92	0.376	
Hook	-1.28	1.27	-1.01	0.332	1.00
Tension	0.79	1.27	0.62	0.546	1.00
Start	-0.62	1.27	-0.49	0.633	1.00
Stop	0.71	1.27	0.56	0.588	1.00
Hand	0.62	1.27	0.49	0.633	1.00
Hook*Hook	0.84	3.42	0.25	0.811	3.20
Tension*Tension	-0.81	3.42	-0.24	0.817	3.20
Start*Start	-2.21	3.42	-0.65	0.531	3.20
Stop*Stop	2.94	3.42	0.86	0.409	3.20
Hand*Hand	-0.41	3.42	-0.12	0.906	3.20
Hook*Tension	1.62	1.34	1.21	0.253	1.00
Hook*Start	-0.13	1.34	-0.10	0.924	1.00
Hook*Stop	-0.81	1.34	-0.60	0.560	1.00
Hook*Hand	0.84	1.34	0.63	0.542	1.00
Tension*Start	2.32	1.34	1.73	0.112	1.00
Tension*Stop	-0.61	1.34	-0.45	0.660	1.00
Tension*Hand	0.37	1.34	0.27	0.789	1.00
Start*Stop	1.17	1.34	0.87	0.402	1.00
Start*Hand	1.69	1.34	1.26	0.233	1.00
Stop*Hand	-2.03	1.34	-1.51	0.158	1.00

Regression Equation in Uncoded Units

X = -10988 + 2.4 Hook - 167.1 Tension + 156 Start - 98.0 Stop - 119.3 Hand + 0.84 Hook*Hook
- 0.81 Tension*Tension - 0.553 Start*Start + 2.94 Stop*Stop - 0.41 Hand*Hand
+ 1.62 Hook*Tension - 0.066 Hook*Start - 0.81 Hook*Stop + 0.84 Hook*Hand
+ 1.159 Tension*Start - 0.61 Tension*Stop + 0.37 Tension*Hand + 0.584 Start*Stop
+ 0.847 Start*Hand - 2.03 Stop*Hand

Fits and Diagnostics for Unusual Observations

Obs	X	Fit	Resid	Std Resid	
1	5.20	2.65	2.55	2.52	R
12	-3.90	-1.50	-2.40	-2.38	R

R Large residual

Response Surface Regression: Y versus Hook, Tension, ... t, Stop, Hand

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	20	28907.4	1445.4	33.22	0.000
Linear	5	26219.7	5243.9	120.51	0.000
Hook	1	7564.5	7564.5	173.84	0.000
Tension	1	15324.2	15324.2	352.16	0.000
Start	1	2485.1	2485.1	57.11	0.000
Stop	1	785.4	785.4	18.05	0.001
Hand	1	60.5	60.5	1.39	0.263
Square	5	924.3	184.9	4.25	0.021
Hook*Hook	1	54.0	54.0	1.24	0.289
Tension*Tension	1	10.7	10.7	0.25	0.630
Start*Start	1	6.0	6.0	0.14	0.717
Stop*Stop	1	543.8	543.8	12.50	0.005
Hand*Hand	1	3.5	3.5	0.08	0.783
2-Way Interaction	10	1763.4	176.3	4.05	0.015
Hook*Tension	1	206.6	206.6	4.75	0.052
Hook*Start	1	0.0	0.0	0.00	0.985
Hook*Stop	1	446.3	446.3	10.26	0.008
Hook*Hand	1	62.0	62.0	1.43	0.258
Tension*Start	1	9.8	9.8	0.22	0.645
Tension*Stop	1	791.0	791.0	18.18	0.001
Tension*Hand	1	153.1	153.1	3.52	0.087
Start*Stop	1	37.5	37.5	0.86	0.373
Start*Hand	1	47.3	47.3	1.09	0.320
Stop*Hand	1	9.8	9.8	0.22	0.645
Error	11	478.7	43.5		
Lack-of-Fit	6	292.8	48.8	1.31	0.391
Pure Error	5	185.8	37.2		
Total	31	29386.1			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
6.59661	98.37%	95.41%	14.65%

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	167.18	1.89	88.68	0.000	
Hook	-20.50	1.55	-13.18	0.000	1.00
Tension	-29.18	1.55	-18.77	0.000	1.00
Start	11.75	1.55	7.56	0.000	1.00
Stop	6.61	1.55	4.25	0.001	1.00
Hand	1.83	1.55	1.18	0.263	1.00
Hook*Hook	4.68	4.21	1.11	0.289	3.20
Tension*Tension	2.08	4.21	0.50	0.630	3.20
Start*Start	-1.57	4.21	-0.37	0.717	3.20
Stop*Stop	-14.87	4.21	-3.53	0.005	3.20
Hand*Hand	1.18	4.21	0.28	0.783	3.20
Hook*Tension	3.59	1.65	2.18	0.052	1.00
Hook*Start	-0.03	1.65	-0.02	0.985	1.00
Hook*Stop	-5.28	1.65	-3.20	0.008	1.00
Hook*Hand	-1.97	1.65	-1.19	0.258	1.00
Tension*Start	-0.78	1.65	-0.47	0.645	1.00
Tension*Stop	-7.03	1.65	-4.26	0.001	1.00
Tension*Hand	-3.09	1.65	-1.88	0.087	1.00
Start*Stop	-1.53	1.65	-0.93	0.373	1.00
Start*Hand	-1.72	1.65	-1.04	0.320	1.00
Stop*Hand	0.78	1.65	0.47	0.645	1.00

Regression Equation in Uncoded Units

Y = -9416 - 38 Hook + 29 Tension + 122 Start + 237 Stop + 136 Hand + 4.68 Hook*Hook
+ 2.08 Tension*Tension - 0.39 Start*Start - 14.87 Stop*Stop + 1.18 Hand*Hand
+ 3.59 Hook*Tension - 0.016 Hook*Start - 5.28 Hook*Stop - 1.97 Hook*Hand
- 0.391 Tension*Start - 7.03 Tension*Stop - 3.09 Tension*Hand - 0.766 Start*Stop
- 0.859 Start*Hand + 0.78 Stop*Hand

Fits and Diagnostics for Unusual Observations

Obs	Y	Fit	Resid	Std Resid	
23	135.00	145.71	-10.71	-2.31	R

R Large residual

ANOVA Assumption Validation for X-axis

$\epsilon_X \sim NID(0, \sigma^2)$

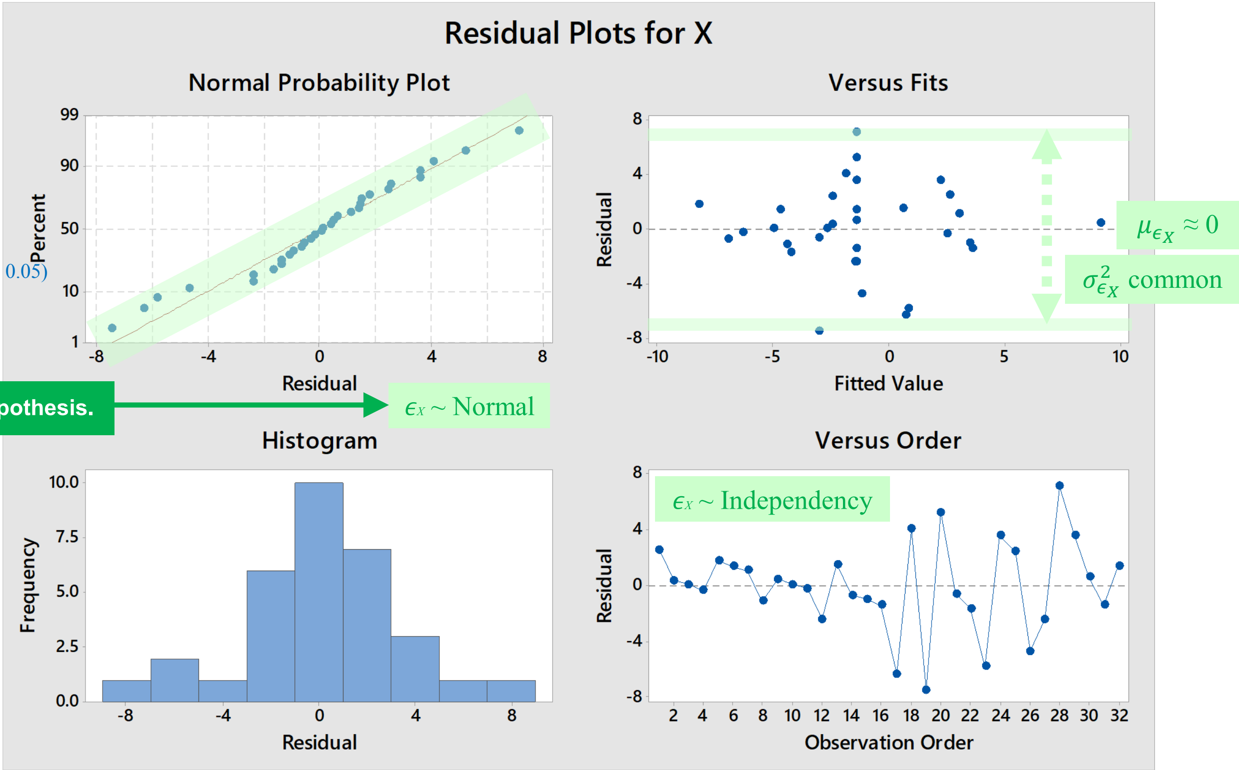
Given:
- H0: $\epsilon_X \sim$ Normal
  HA: $\epsilon_X \nsim$ Normal
- $\alpha = 0.05$

Mean	-4.78784E-16
StDev	3.198
N	32
AD	0.420
P-Value	0.306

$0.306 > (\alpha = 0.05)$

Fail to reject null hypothesis.

$\epsilon_X \sim$ Normal



ANOVA Assumption Validation for Y-axis

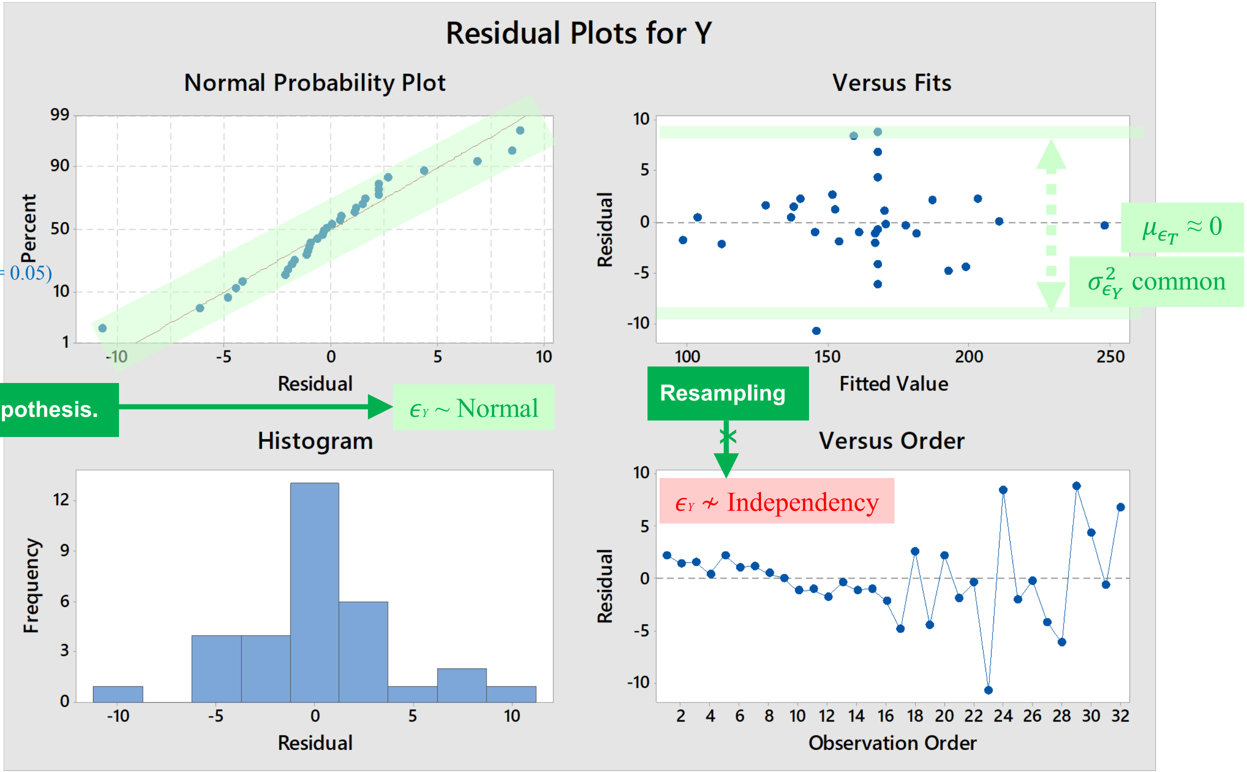
$\epsilon_Y \sim NID(0, \sigma^2)$

Given:
- H_0: $\epsilon_Y \sim$ Normal
  H_A: $\epsilon_Y \nsim$ Normal
- $\alpha = 0.05$

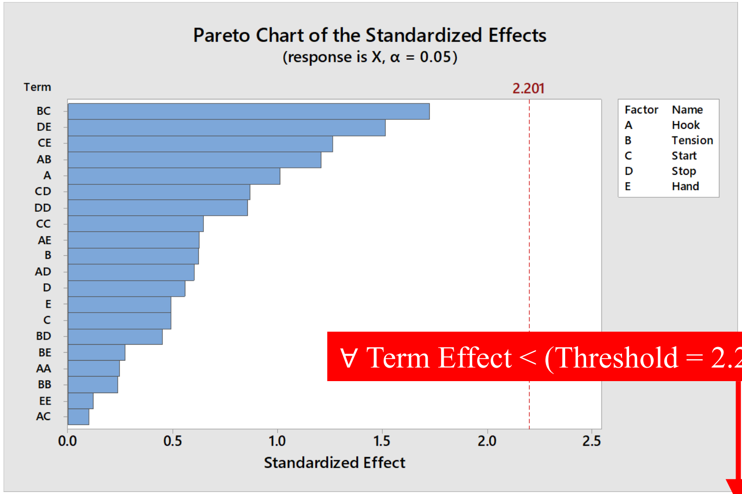
Mean	-8.88178E-15
StDev	3.929
N	32
AD	0.672
P-Value	0.072

$0.072 > (\alpha = 0.05)$

Fail to reject null hypothesis. → $\epsilon_Y \sim$ Normal



Significant Factors for X-axis distance



$\forall$ Term Effect $<$ (Threshold = 2.201)

∴All term effects may not be significant for mean of X-axis falling distance.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	20	386.397	19.3199	0.67	0.790
Linear	5	63.746	12.7491	0.44	0.810
Hook	1	29.645	29.6450	1.03	0.332
Tension	1	11.202	11.2022	0.39	0.546
Start	1	6.969	6.9689	0.24	0.633
Stop	1	8.961	8.9606	0.31	0.588
Hand	1	6.969	6.9689	0.24	0.633
Square	5	30.805	6.1610	0.21	0.949
Hook*Hook	1	1.731	1.7307	0.06	0.811
Tension*Tension	1	1.620	1.6199	0.06	0.817
Start*Start	1	12.033	12.0335	0.42	0.531
Stop*Stop	1	21.250	21.2504	0.74	0.409
Hand*Hand	1	0.416	0.4164	0.01	0.906
2-Way Interaction	10	291.846	29.1846	1.01	0.488
Hook*Tension	1	41.926	41.9256	1.45	0.253
Hook*Start	1	0.276	0.2756	0.01	0.924
Hook*Stop	1	10.401	10.4006	0.36	0.560
Hook*Hand	1	11.391	11.3906	0.40	0.542
Tension*Start	1	86.026	86.0256	2.98	0.112
Tension*Stop	1	5.881	5.8806	0.20	0.660
Tension*Hand	1	2.176	2.1756	0.08	0.789
Start*Stop	1	21.856	21.8556	0.76	0.402
Start*Hand	1	45.901	45.9006	1.59	0.233
Stop*Hand	1	66.016	66.0156	2.29	0.158
Error	11	317.038	28.8216		
Lack-of-Fit	6	256.829	42.8049	3.55	0.093
Pure Error	5	60.208	12.0417		
Total	31	703.435			

Given:

- H_0: Effect ? may not differ μ_x
  H_A: Effect ? differ μ_x
- $\alpha = 0.05$

$>$ $(\alpha = 0.05)$

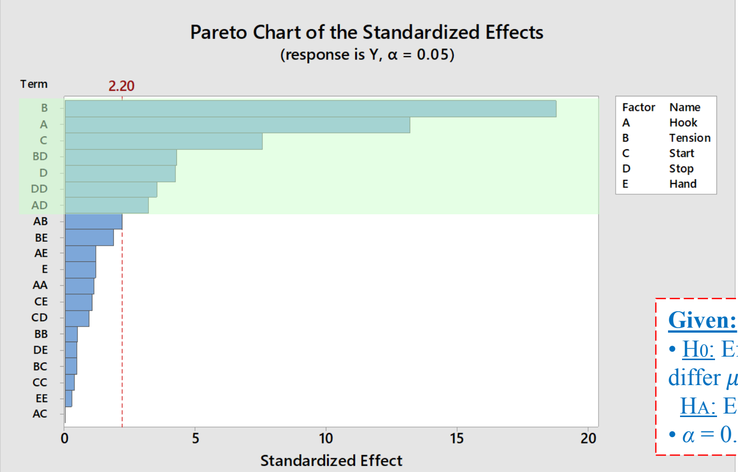
Fail to reject null hypothesis.

∴ With 95% confidence interval, there is not sufficient evidence to conclude that all term effects can differ the mean of X-axis falling distance significantly.

Justify:

From both graphical and statistical analysis, the result interpretation are consistent that not enough evidence can be concluded that X-axis falling distance can be differed from any term effects significantly, corresponding to the prioritization matrix of factors in the analyze phase.

Significant Factors for Y-axis distance



Given:

- H0: Effect ? may not differ μ_Y

  HA: Effect ? differs μ_Y
- $\alpha = 0.05$

∴ All main effects except "Hand" (E), two interaction terms including "Tension*Stop" (BD) and "Hook*Stop" (AD), along with the quadratic term of factor "Stop" (DD) is significant for mean of Y-axis falling distance.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value	
Model	20	28907.4	1445.4	33.22	0.000	
Linear	5	26219.7	5243.9	120.51	0.000	
Hook	1	7564.5	7564.5	173.84	0.000	< $(\alpha = 0.05)$
Tension	1	15324.2	15324.2	352.16	0.000	< $(\alpha = 0.05)$
Start	1	2485.1	2485.1	57.11	0.000	< $(\alpha = 0.05)$
Stop	1	785.4	785.4	18.05	0.001	< $(\alpha = 0.05)$
Hand	1	60.5	60.5	1.39	0.263	
Square	5	924.3	184.9	4.25	0.021	
Hook*Hook	1	54.0	54.0	1.24	0.289	
Tension*Tension	1	10.7	10.7	0.25	0.630	
Start*Start	1	6.0	6.0	0.14	0.717	
Stop*Stop	1	543.8	543.8	12.50	0.005	< $(\alpha = 0.05)$
Hand*Hand	1	3.5	3.5	0.08	0.783	
2-Way Interaction	10	1763.4	176.3	4.05	0.015	
Hook*Tension	1	206.6	206.6	4.75	0.052	
Hook*Start	1	0.0	0.0	0.00	0.985	
Hook*Stop	1	446.3	446.3	10.26	0.008	< $(\alpha = 0.05)$
Hook*Hand	1	62.0	62.0	1.43	0.258	
Tension*Start	1	9.8	9.8	0.22	0.645	
Tension*Stop	1	791.0	791.0	18.18	0.001	< $(\alpha = 0.05)$
Tension*Hand	1	153.1	153.1	3.52	0.087	
Start*Stop	1	37.5	37.5	0.86	0.373	
Start*Hand	1	47.3	47.3	1.09	0.320	
Stop*Hand	1	9.8	9.8	0.22	0.645	
Error	11	478.7	43.5			
Lack-of-Fit	6	292.8	48.8	1.31	0.391	
Pure Error	5	185.8	37.2			
Total	31	29386.1				

Reject null hypothesis.

∴ With 95% confidence interval, there is sufficient evidence to conclude that all main effects except "Hand" (E), two interaction terms including "Tension*Stop" (BD) and "Hook*Stop" (AD), as well as the quadratic term of factor "Stop" (DD) can differ the mean of Y-axis falling distance significantly.

Justify:

From both graphical and statistical analysis, the result interpretation are consistent that enough evidence can be concluded that Y-axis falling distance can be differed significantly from all main effects except "Hand" (E), two interaction terms including "Tension*Stop" (BD) and "Hook*Stop" (AD), as well as the quadratic term of factor "Stop" (DD) .

Full Regression Model of predicting X-axis falling distance

Regression Equation in Uncoded Units

$$
\begin{aligned}
X \quad = \quad & -10988 + 2.4\ \text{Hook} - 167.1\ \text{Tension} + 156\ \text{Start} - 98.0\ \text{Stop} - 119.3\ \text{Hand} + 0.84\ \text{Hook*Hook} \\
& - 0.81\ \text{Tension*Tension} - 0.553\ \text{Start*Start} + 2.94\ \text{Stop*Stop} - 0.41\ \text{Hand*Hand} \\
& + 1.62\ \text{Hook*Tension} - 0.066\ \text{Hook*Start} - 0.81\ \text{Hook*Stop} + 0.84\ \text{Hook*Hand} \\
& + 1.159\ \text{Tension*Start} - 0.61\ \text{Tension*Stop} + 0.37\ \text{Tension*Hand} + 0.584\ \text{Start*Stop} \\
& + 0.847\ \text{Start*Hand} - 2.03\ \text{Stop*Hand}
\end{aligned}
$$

Full Regression Model of predicting X-axis falling distance

Tests on Individual Regression Coefficients

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-1.41	1.53	-0.92	0.376
Hook	-1.28	1.27	-1.01	0.332
Tension	0.79	1.27	0.62	0.546
Start	-0.62	1.27	-0.49	0.633
Stop	0.71	1.27	0.56	0.588
Hand	0.62	1.27	0.49	0.633
Hook*Hook	0.84	3.42	0.25	0.811
Tension*Tension	-0.81	3.42	-0.24	0.817
Start*Start	-2.21	3.42	-0.65	0.531
Stop*Stop	2.94	3.42	0.86	0.409
Hand*Hand	-0.41	3.42	-0.12	0.906
Hook*Tension	1.62	1.34	1.21	0.253
Hook*Start	-0.13	1.34	-0.10	0.924
Hook*Stop	-0.81	1.34	-0.60	0.560
Hook*Hand	0.84	1.34	0.63	0.542
Tension*Start	2.32	1.34	1.73	0.112
Tension*Stop	-0.61	1.34	-0.45	0.660
Tension*Hand	0.37	1.34	0.27	0.789
Start*Stop	1.17	1.34	0.87	0.402
Start*Hand	1.69	1.34	1.26	0.233
Stop*Hand	-2.03	1.34	-1.51	0.158

$>$ ($\alpha = 0.05$)

Fail to reject null hypothesis.

Given:

- H_0: Term ? may not be useful for predicting X-axis distance.
  H_A: Term ? is useful for predicting X-axis distance.
- $\alpha = 0.05$

∴ With 95% confidential interval, there is not enough evidence to conclude that all terms are useful for predicting X-axis distance.

Full Regression Model of predicting X-axis falling distance

Multicollinearity Identification

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-1.41	1.53	-0.92	0.376	
Hook	-1.28	1.27	-1.01	0.332	1.00
Tension	0.79	1.27	0.62	0.546	1.00
Start	-0.62	1.27	-0.49	0.633	1.00
Stop	0.71	1.27	0.56	0.588	1.00
Hand	0.62	1.27	0.49	0.633	1.00
Hook*Hook	0.84	3.42	0.25	0.811	3.20
Tension*Tension	-0.81	3.42	-0.24	0.817	3.20
Start*Start	-2.21	3.42	-0.65	0.531	3.20
Stop*Stop	2.94	3.42	0.86	0.409	3.20
Hand*Hand	-0.41	3.42	-0.12	0.906	3.20
Hook*Tension	1.62	1.34	1.21	0.253	1.00
Hook*Start	-0.13	1.34	-0.10	0.924	1.00
Hook*Stop	-0.81	1.34	-0.60	0.560	1.00
Hook*Hand	0.84	1.34	0.63	0.542	1.00
Tension*Start	2.32	1.34	1.73	0.112	1.00
Tension*Stop	-0.61	1.34	-0.45	0.660	1.00
Tension*Hand	0.37	1.34	0.27	0.789	1.00
Start*Stop	1.17	1.34	0.87	0.402	1.00
Start*Hand	1.69	1.34	1.26	0.233	1.00
Stop*Hand	-2.03	1.34	-1.51	0.158	1.00

< 10

From Montgomery:

- Multicollinearity is a problem if any variance inflation factor (VIF) exceeds 10.

∴ The multicollinearity issues were not found in each term of the full regression model used for predicting X-axis falling distance.

Full Regression Model of predicting X-axis falling distance

Model Utility Test

Given:

- H_0: The model may not be useful for predicting X-axis distance.
  H_A: The model is useful for predicting X-axis distance.
- $\alpha = 0.05$

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	20	386.397	19.3199	0.67	0.790
Linear	5	63.746	12.7491	0.44	0.810
Square	5	30.805	6.1610	0.21	0.949
2-Way Interaction	10	291.846	29.1846	1.01	0.488
Error	11	317.038	28.8216		
Lack-of-Fit	6	256.829	42.8049	3.55	0.093
Pure Error	5	60.208	12.0417		
Total	31	703.435			

$0.790 > (\alpha = 0.05)$

Fail to reject null hypothesis.

∴ There is not enough evidence to conclude that the model is useful for predicting X-axis distance at significant level of 5%.

Full Regression Model of predicting Y-axis falling distance

Regression Equation in Uncoded Units

$$
\begin{aligned}
Y = {} & -9416 - 38\ \text{Hook} + 29\ \text{Tension} + 122\ \text{Start} + 237\ \text{Stop} + 136\ \text{Hand} + 4.68\ \text{Hook*Hook} \\
& + 2.08\ \text{Tension*Tension} - 0.39\ \text{Start*Start} - 14.87\ \text{Stop*Stop} + 1.18\ \text{Hand*Hand} \\
& + 3.59\ \text{Hook*Tension} - 0.016\ \text{Hook*Start} - 5.28\ \text{Hook*Stop} - 1.97\ \text{Hook*Hand} \\
& - 0.391\ \text{Tension*Start} - 7.03\ \text{Tension*Stop} - 3.09\ \text{Tension*Hand} - 0.766\ \text{Start*Stop} \\
& - 0.859\ \text{Start*Hand} + 0.78\ \text{Stop*Hand}
\end{aligned}
$$

Full Regression Model of predicting Y-axis falling distance

Tests on Individual Regression Coefficients

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	
Constant	167.18	1.89	88.68	0.000	
Hook	-20.50	1.55	-13.18	0.000	< ($\alpha = 0.05$)
Tension	-29.18	1.55	-18.77	0.000	< ($\alpha = 0.05$)
Start	11.75	1.55	7.56	0.000	< ($\alpha = 0.05$)
Stop	6.61	1.55	4.25	0.001	< ($\alpha = 0.05$)
Hand	1.83	1.55	1.18	0.263	
Hook*Hook	4.68	4.21	1.11	0.289	
Tension*Tension	2.08	4.21	0.50	0.630	
Start*Start	-1.57	4.21	-0.37	0.717	
Stop*Stop	-14.87	4.21	-3.53	0.005	< ($\alpha = 0.05$)
Hand*Hand	1.18	4.21	0.28	0.783	
Hook*Tension	3.59	1.65	2.18	0.052	
Hook*Start	-0.03	1.65	-0.02	0.985	
Hook*Stop	-5.28	1.65	-3.20	0.008	< ($\alpha = 0.05$)
Hook*Hand	-1.97	1.65	-1.19	0.258	
Tension*Start	-0.78	1.65	-0.47	0.645	
Tension*Stop	-7.03	1.65	-4.26	0.001	< ($\alpha = 0.05$)
Tension*Hand	-3.09	1.65	-1.88	0.087	
Start*Stop	-1.53	1.65	-0.93	0.373	
Start*Hand	-1.72	1.65	-1.04	0.320	
Stop*Hand	0.78	1.65	0.47	0.645	

Reject null hypothesis.

Given:

- H_0: Term ? may not be useful for predicting Y-axis distance.
  H_A: Term ? is useful for predicting Y-axis distance.
- $\alpha = 0.05$

∴ With 95% confidential interval, there is enough evidence to conclude that all main effects expect "Hand", two interaction terms including "Tension*Stop" and "Hook*Stop", as well as the quadratic term of factor "Stop" are useful for predicting Y-axis distance.

Full Regression Model of predicting Y-axis falling distance

Multicollinearity Identification

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	167.18	1.89	88.68	0.000	
Hook	-20.50	1.55	-13.18	0.000	1.00
Tension	-29.18	1.55	-18.77	0.000	1.00
Start	11.75	1.55	7.56	0.000	1.00
Stop	6.61	1.55	4.25	0.001	1.00
Hand	1.83	1.55	1.18	0.263	1.00
Hook*Hook	4.68	4.21	1.11	0.289	3.20
Tension*Tension	2.08	4.21	0.50	0.630	3.20
Start*Start	-1.57	4.21	-0.37	0.717	3.20
Stop*Stop	-14.87	4.21	-3.53	0.005	3.20
Hand*Hand	1.18	4.21	0.28	0.783	3.20
Hook*Tension	3.59	1.65	2.18	0.052	1.00
Hook*Start	-0.03	1.65	-0.02	0.985	1.00
Hook*Stop	-5.28	1.65	-3.20	0.008	1.00
Hook*Hand	-1.97	1.65	-1.19	0.258	1.00
Tension*Start	-0.78	1.65	-0.47	0.645	1.00
Tension*Stop	-7.03	1.65	-4.26	0.001	1.00
Tension*Hand	-3.09	1.65	-1.88	0.087	1.00
Start*Stop	-1.53	1.65	-0.93	0.373	1.00
Start*Hand	-1.72	1.65	-1.04	0.320	1.00
Stop*Hand	0.78	1.65	0.47	0.645	1.00

< 10

From Montgomery:

- Multicollinearity is a problem if any variance inflation factor (VIF) exceeds 10.

Justify:

The multicollinearity issues were not found in each term of the full regression model used for predicting Y-axis falling distance.

Full Regression Model of predicting Y-axis falling distance

Model Utility Test

Given:
- H_0: The model may not be useful for predicting Y-axis distance.
  H_A: The model is useful for predicting Y-axis distance.
- $\alpha = 0.05$

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	20	28907.4	1445.4	33.22	0.000
Linear	5	26219.7	5243.9	120.51	0.000
Square	5	924.3	184.9	4.25	0.021
2-Way Interaction	10	1763.4	176.3	4.05	0.015
Error	11	478.7	43.5		
Lack-of-Fit	6	292.8	48.8	1.31	0.391
Pure Error	5	185.8	37.2		
Total	31	29386.1			

$< (\alpha = 0.05)$

Reject null hypothesis.

There is enough evidence to conclude that the model is useful for predicting Y-axis distance at significant level of 5%.

Analyzing Experimental Result after performing Stepwise Regression

Given: Significance level to remove or enter term at 5%

Response Surface Regression: X versus Hook, Tension, ... t, Stop, Hand

Stepwise Selection of Terms

α to enter = 0.05, α to remove = 0.05
The stepwise procedure added terms during the procedure in order to maintain a hierarchical model at each step.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	104.197	34.732	1.62	0.206
Linear	2	18.171	9.086	0.42	0.658
Tension	1	11.202	11.202	0.52	0.475
Start	1	6.969	6.969	0.33	0.573
2-Way Interaction	1	86.026	86.026	4.02	0.055
Tension*Start	1	86.026	86.026	4.02	0.055
Error	28	599.238	21.401		
Lack-of-Fit	23	539.030	23.436	1.95	0.237
Pure Error	5	60.208	12.042		
Total	31	703.435			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
4.62616	14.81%	5.69%	0.00%

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-1.222	0.818	-1.49	0.146	
Tension	0.79	1.09	0.72	0.475	1.00
Start	-0.62	1.09	-0.57	0.573	1.00
Tension*Start	2.32	1.16	2.00	0.055	1.00

Regression Equation in Uncoded Units

X = 379 - 167.3 Tension - 2.63 Start + 1.159 Tension*Start

Fits and Diagnostics for Unusual Observations

Obs	X	Fit	Resid	Std Resid	
9	9.60	0.93	8.67	2.10	R

R Large residual

Response Surface Regression: Y versus Hook, Tension, ... t, Stop, Hand

Stepwise Selection of Terms

α to enter = 0.05, α to remove = 0.05
The stepwise procedure added terms during the procedure in order to maintain a hierarchical model at each step.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	8	28406.2	3550.8	83.35	0.000
Linear	4	26159.2	6539.8	153.51	0.000
Hook	1	7564.5	7564.5	177.56	0.000
Tension	1	15324.2	15324.2	359.70	0.000
Start	1	2485.1	2485.1	58.33	0.000
Stop	1	785.4	785.4	18.44	0.000
Square	1	803.1	803.1	18.85	0.000
Stop*Stop	1	803.1	803.1	18.85	0.000
2-Way Interaction	3	1443.9	481.3	11.30	0.000
Hook*Tension	1	206.6	206.6	4.85	0.038
Hook*Stop	1	446.3	446.3	10.48	0.004
Tension*Stop	1	791.0	791.0	18.57	0.000
Error	23	979.9	42.6		
Lack-of-Fit	18	794.0	44.1	1.19	0.463
Pure Error	5	185.8	37.2		
Total	31	29386.1			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
6.52708	96.67%	95.51%	92.74%

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	168.09	1.74	96.36	0.000	
Hook	-20.50	1.54	-13.33	0.000	1.00
Tension	-29.18	1.54	-18.97	0.000	1.00
Start	11.75	1.54	7.64	0.000	1.00
Stop	6.61	1.54	4.29	0.000	1.00
Stop*Stop	-10.10	2.33	-4.34	0.000	1.00
Hook*Tension	3.59	1.63	2.20	0.038	1.00
Hook*Stop	-5.28	1.63	-3.24	0.004	1.00
Tension*Stop	-7.03	1.63	-4.31	0.000	1.00

Regression Equation in Uncoded Units

Y = -743 - 11.84 Hook - 18.87 Tension + 5.875 Start + 97.1 Stop - 10.10 Stop*Stop
+ 3.59 Hook*Tension - 5.28 Hook*Stop - 7.03 Tension*Stop

Fits and Diagnostics for Unusual Observations

Obs	Y	Fit	Resid	Std Resid	
23	135.00	151.39	-16.39	-2.66	R

R Large residual

ANOVA Assumption Validation for X-axis

$\epsilon_X \sim NID(0, \sigma^2)$

Given:

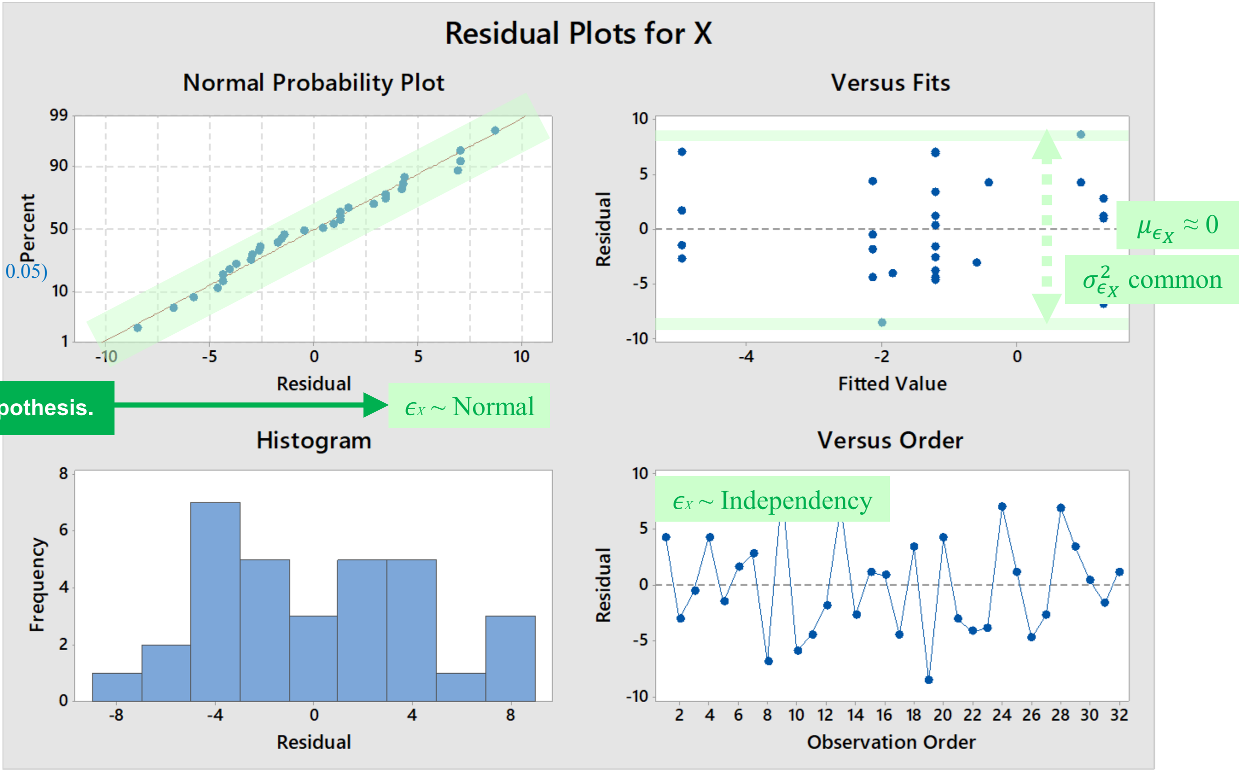
- H0: $\epsilon_X \sim$ Normal
  HA: $\epsilon_X \nsim$ Normal
- $\alpha = 0.05$

Mean	-4.44089E-16
StDev	4.397
N	32
AD	0.292
P-Value	0.583

$> (\alpha = 0.05)$

Fail to reject null hypothesis.

$\epsilon_X \sim$ Normal



ANOVA Assumption Validation for Y-axis

$\epsilon_Y \sim NID(0, \sigma^2)$

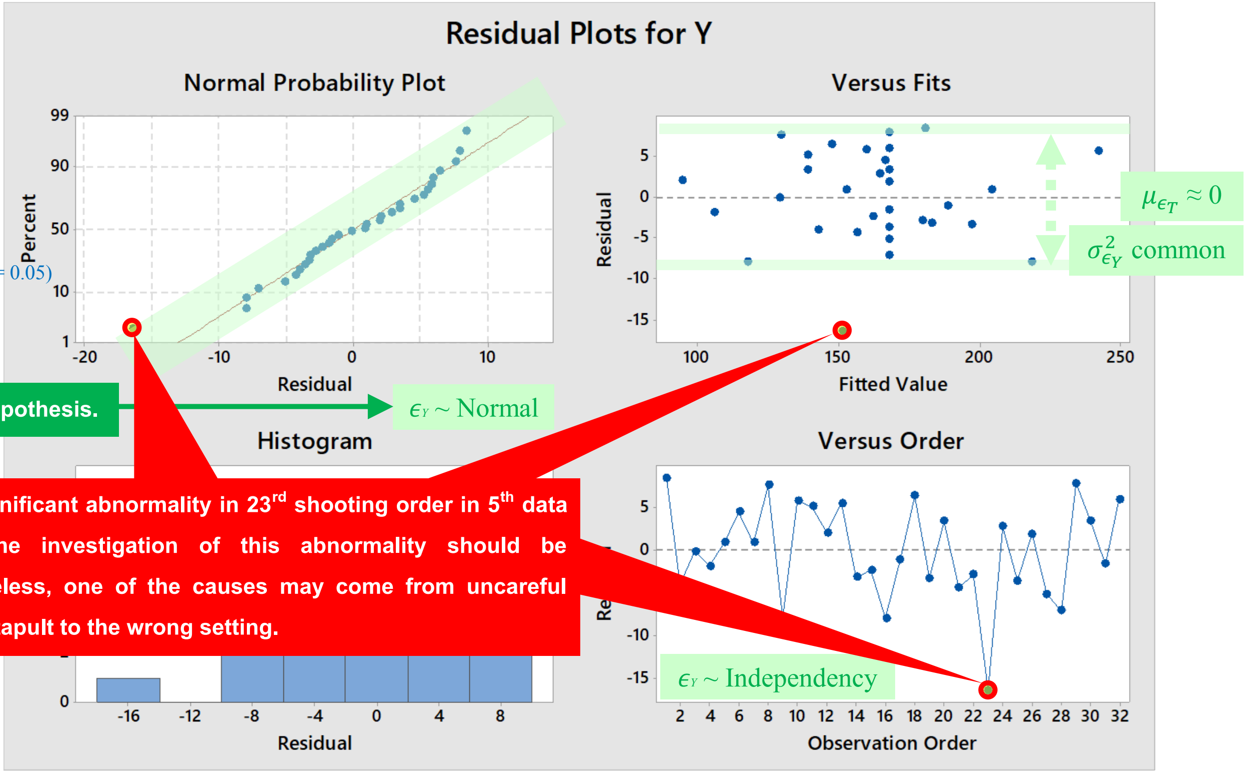
Given:

- H0: $\epsilon_Y \sim$ Normal
  HA: $\epsilon_Y \nsim$ Normal
- $\alpha = 0.05$

Mean	-2.04281E-14
StDev	5.622
N	32
AD	0.332
P-Value	0.496

$> (\alpha = 0.05)$

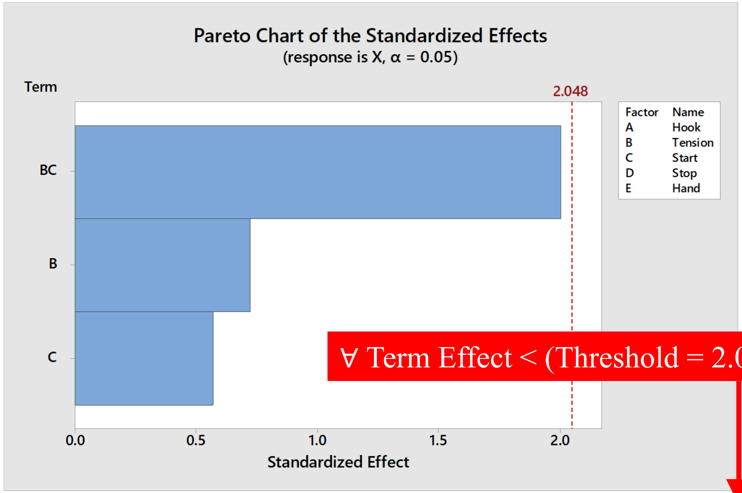
Fail to reject null hypothesis.



There might be a significant abnormality in 23rd shooting order in 5th data collection. Thus, the investigation of this abnormality should be conducted. Nevertheless, one of the causes may come from uncareful adjustment of the catapult to the wrong setting.

Significant Factors for X-axis distance

Given:
- H_0: Effect ? may not differ μ_x
  H_A: Effect ? differ μ_x
- $\alpha = 0.05$



$\forall$ Term Effect < (Threshold = 2.048)

∴All term effects may not be significant for mean of X-axis falling distance.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	104.197	34.732	1.62	0.206
Linear	2	18.171	9.086	0.42	0.658
Tension	1	11.202	11.202	0.52	0.475
Start	1	6.969	6.969	0.33	0.573
2-Way Interaction	1	86.026	86.026	4.02	0.055
Tension*Start	1	86.026	86.026	4.02	0.055
Error	28	599.238	21.401		
Lack-of-Fit	23	539.030	23.436	1.95	0.237
Pure Error	5	60.208	12.042		
Total	31	703.435			

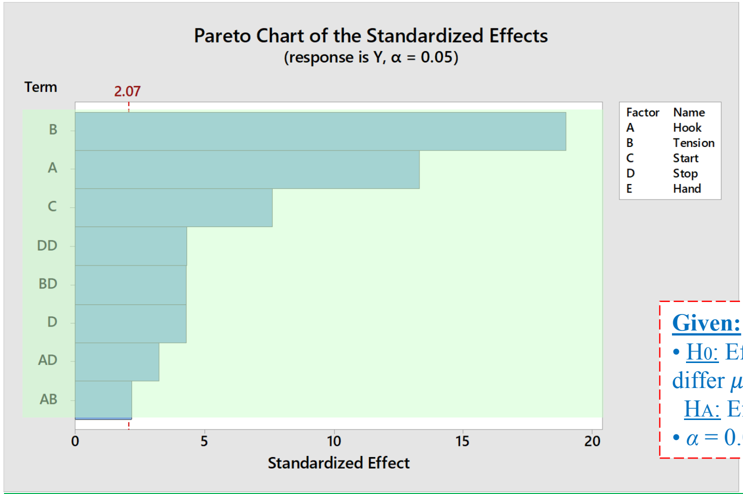
> ($\alpha = 0.05$)

Fail to reject null hypothesis.

∴ With 95% confidence interval, there is not sufficient evidence to conclude that all term effects can differ the mean of X-axis falling distance significantly.

Justify: From both graphical and statistical analysis, the result interpretation are consistent that not enough evidence can be concluded that X-axis falling distance can be differed from any term effects significantly. corresponding to the prioritization matrix of factors in the analyze phase.

Significant Factors for Y-axis distance



Given:
- H0: Effect ? may not differ μ_Y
  HA: Effect ? differs μ_Y
- $\alpha = 0.05$

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value	
Model	8	28406.2	3550.8	83.35	0.000	
Linear	4	26159.2	6539.8	153.51	0.000	
Hook	1	7564.5	7564.5	177.56	0.000	< ($\alpha = 0.05$)
Tension	1	15324.2	15324.2	359.70	0.000	< ($\alpha = 0.05$)
Start	1	2485.1	2485.1	58.33	0.000	< ($\alpha = 0.05$)
Stop	1	785.4	785.4	18.44	0.000	< ($\alpha = 0.05$)
Square	1	803.1	803.1	18.85	0.000	
Stop*Stop	1	803.1	803.1	18.85	0.000	< ($\alpha = 0.05$)
2-Way Interaction	3	1443.9	481.3	11.30	0.000	
Hook*Tension	1	206.6	206.6	4.85	0.038	< ($\alpha = 0.05$)
Hook*Stop	1	446.3	446.3	10.48	0.004	< ($\alpha = 0.05$)
Tension*Stop	1	791.0	791.0	18.57	0.000	< ($\alpha = 0.05$)
Error	23	979.9	42.6			
Lack-of-Fit	18	794.0	44.1	1.19	0.463	
Pure Error	5	185.8	37.2			
Total	31	29386.1				

Reject null hypothesis.

∴ All main effects except "Hand" (E), two interaction terms including "Tension*Stop" (BD), "Hook*Stop" (AD) and "Hook*Tension"(AB) along with the quadratic term of factor "Stop" (DD) is significant for mean of Y-axis falling distance.

∴ With 95% confidence interval, there is sufficient evidence to conclude that all main effects except "Hand" (E), two interaction terms including "Tension*Stop" (BD) , "Hook*Stop" (AD) and "Hook*Tension"(AB), as well as the quadratic term of factor "Stop" (DD) can differ the mean of Y-axis falling distance significantly.

Justify: **From both graphical and statistical analysis, the result interpretation are consistent that enough evidence can be concluded that Y-axis falling distance can be differed significantly from all main effects except "Hand" (E), two interaction terms including "Tension*Stop" (BD), "Hook*Stop" (AD) and "Hook*Tension"(AB) , as well as the quadratic term of factor "Stop" (DD) .**

Reduced Regression Model of predicting X-axis falling distance

Regression Equation in Uncoded Units

$$X = 379 - 167.3\ \text{Tension} - 2.63\ \text{Start} + 1.159\ \text{Tension*Start}$$

Reduced Regression Model of predicting X-axis falling distance

Tests on Individual Regression Coefficients

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-1.222	0.818	-1.49	0.146
Tension	0.79	1.09	0.72	0.475
Start	-0.62	1.09	-0.57	0.573
Tension*Start	2.32	1.16	2.00	0.055

$> (\alpha = 0.05)$

Fail to reject null hypothesis.

Given:
- H0: Term ? may not be useful for predicting X-axis distance.
  HA: Term ? is useful for predicting X-axis distance.
- $\alpha = 0.05$

∴ With 95% confidential interval, there is not enough evidence to conclude that all terms are useful for predicting X-axis distance.

Reduced Regression Model of predicting X-axis falling distance

Multicollinearity Identification

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-1.222	0.818	-1.49	0.146	
Tension	0.79	1.09	0.72	0.475	1.00
Start	-0.62	1.09	-0.57	0.573	1.00
Tension*Start	2.32	1.16	2.00	0.055	1.00

< 10

From Montgomery:

- Multicollinearity is a problem if any variance inflation factor (VIF) exceeds 10.

∴ The multicollinearity issues were not found in each term of the reduced regression model used for predicting X-axis falling distance.

Reduced Regression Model of predicting X-axis falling distance

Model Utility Test

Given:

- H_0: The model may not be useful for predicting X-axis distance.
  H_A: The model is useful for predicting X-axis distance.
- $\alpha = 0.05$

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	104.197	34.732	1.62	0.206
Linear	2	18.171	9.086	0.42	0.658
2-Way Interaction	1	86.026	86.026	4.02	0.055
Error	28	599.238	21.401		
Lack-of-Fit	23	539.030	23.436	1.95	0.237
Pure Error	5	60.208	12.042		
Total	31	703.435			

> $(\alpha = 0.05)$

Fail to reject null hypothesis.

∴ There is not enough evidence to conclude that the model is useful for predicting X-axis distance at significant level of 5%.

Reduced Regression Model of predicting Y-axis falling distance

Regression Equation in Uncoded Units

$$Y = -743 - 11.84\,\text{Hook} - 18.87\,\text{Tension} + 5.875\,\text{Start} + 97.1\,\text{Stop} - 10.10\,\text{Stop*Stop} + 3.59\,\text{Hook*Tension} - 5.28\,\text{Hook*Stop} - 7.03\,\text{Tension*Stop}$$

Reduced Regression Model of predicting Y-axis falling distance

Tests on Individual Regression Coefficients

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	
Constant	168.09	1.74	96.36	0.000	
Hook	-20.50	1.54	-13.33	0.000	$< (\alpha = 0.05)$
Tension	-29.18	1.54	-18.97	0.000	$< (\alpha = 0.05)$
Start	11.75	1.54	7.64	0.000	$< (\alpha = 0.05)$
Stop	6.61	1.54	4.29	0.000	$< (\alpha = 0.05)$
Stop*Stop	-10.10	2.33	-4.34	0.000	$< (\alpha = 0.05)$
Hook*Tension	3.59	1.63	2.20	0.038	$< (\alpha = 0.05)$
Hook*Stop	-5.28	1.63	-3.24	0.004	$< (\alpha = 0.05)$
Tension*Stop	-7.03	1.63	-4.31	0.000	$< (\alpha = 0.05)$

Reject null hypothesis.

Given:

- H_0: Term ? may not be useful for predicting Y-axis distance.
  H_A: Term ? is useful for predicting Y-axis distance.
- $\alpha = 0.05$

∴ With 95% confidential interval, there is enough evidence to conclude that all main effects except "Hand", two interaction terms including "Tension*Stop" (BD), "Hook*Stop" (AD) and "Hook*Tension"(AB), as well as the quadratic term of factor "Stop" are useful for predicting X-axis distance.

Reduced Regression Model of predicting Y-axis falling distance

Multicollinearity Identification

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	168.09	1.74	96.36	0.000	
Hook	-20.50	1.54	-13.33	0.000	1.00
Tension	-29.18	1.54	-18.97	0.000	1.00
Start	11.75	1.54	7.64	0.000	1.00
Stop	6.61	1.54	4.29	0.000	1.00
Stop*Stop	-10.10	2.33	-4.34	0.000	1.00
Hook*Tension	3.59	1.63	2.20	0.038	1.00
Hook*Stop	-5.28	1.63	-3.24	0.004	1.00
Tension*Stop	-7.03	1.63	-4.31	0.000	1.00

< 10

From Montgomery:
- Multicollinearity is a problem if any variance inflation factor (VIF) exceeds 10.

Justify:

The multicollinearity issues were not found in each term of the reduced regression model used for predicting Y-axis falling distance.

Reduced Regression Model of predicting Y-axis falling distance

Model Utility Test

Given:
- H_0: The model may not be useful for predicting Y-axis distance.
  H_A: The model is useful for predicting Y-axis distance.
- $\alpha = 0.05$

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	8	28406.2	3550.8	83.35	0.000
Linear	4	26159.2	6539.8	153.51	0.000
Square	1	803.1	803.1	18.85	0.000
2-Way Interaction	3	1443.9	481.3	11.30	0.000
Error	23	979.9	42.6		
Lack-of-Fit	18	794.0	44.1	1.19	0.463
Pure Error	5	185.8	37.2		
Total	31	29386.1			

$< (\alpha = 0.05)$

Reject null hypothesis.

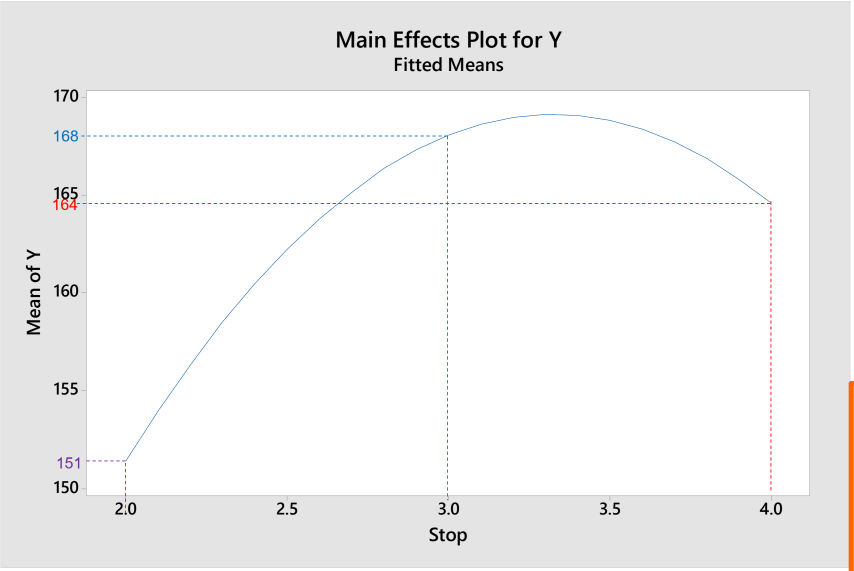
There is enough evidence to conclude that the model is useful for predicting Y-axis distance at significant level of 5%.

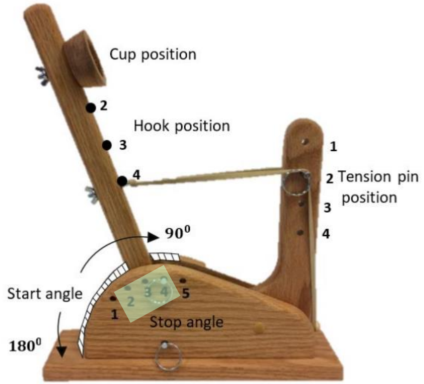
Studying response of quadratic effect via Main Effect Plot in Y-axis

Stop*Stop

$$\hat{Y} = -743 - 11.84\,\text{Hook} - 18.87\,\text{Tension} + 5.875\,\text{Start} + 97.1\,\text{Stop} - 10.10\,\text{Stop*Stop} + 3.59\,\text{Hook*Tension} - 5.28\,\text{Hook*Stop} - 7.03\,\text{Tension*Stop}$$

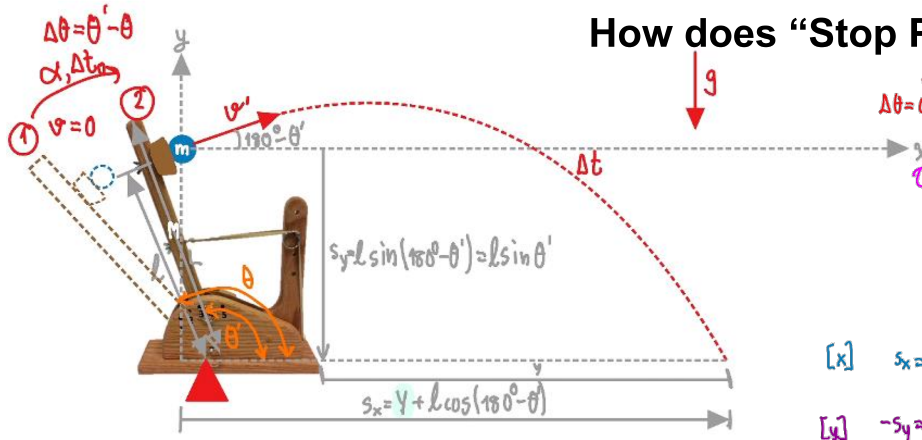
Hook = 3, Tension = 2, Start = 145, Hook*Tension = 3, 2, Hook*Stop = 3, Tension*Stop = 2





It was found that there was a quadratic relationship between treatment factor, "Stop Pin Position". As the level of stop pin position was adjusted in higher position (2 to 3), the Y-axis distance was falling farther but when it was adjusted even higher position (3 to 4), the Y-axis distance was falling nearer instead.

How does “Stop Pin Position” affect Y-axis response?



$$\Delta\theta = \cancel{\omega_0}^{\,0} + \frac{1}{2}\alpha\Delta t^2 \qquad F = k\Delta s$$

$$\tau = I\alpha;\ \sum_{\forall i} F_i \times r_i = \left[I_{arm} + I_{cannonball}\right]\left[\frac{2\Delta\theta}{(\Delta t_0)^2}\right]$$

$$I_{arm} = \frac{1}{3}ML^2, \quad I_{cannonball} = m\ell^2$$

$$v' = \omega'\ell \qquad \omega' = \cancel{\omega_0}^{\,0} + \alpha\Delta t_0 = 2\left(\frac{\Delta\theta}{\Delta t_0}\right), \quad \alpha = \frac{2\Delta\theta}{(\Delta t_0)^2}$$

$$\frac{2}{g}\left[\frac{\Delta\theta}{\Delta t_0}\ell + \sqrt{\left(\frac{\Delta\theta}{\Delta t_0}\ell\right)^2 + \frac{g}{2\sin\theta'}}\right]$$

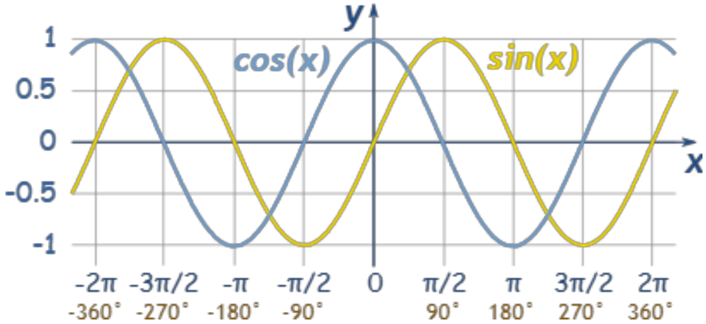
$$[x] \quad s_x = (v'\cos(180^\circ-\theta'))\Delta t \longrightarrow y = -\ell\cos(180^\circ-\theta') + \left[2\left(\frac{\Delta\theta}{\Delta t_0}\right)\ell\cos(180^\circ-\theta')\right]\Delta t$$

$$[y] \quad -s_y = [v'\sin(180^\circ-\theta')]t + \frac{1}{2}at^2 \longrightarrow 0 = \left[2\left(\frac{\Delta\theta}{\Delta t_0}\right)\ell\sin\theta'\right]\Delta t + \frac{1}{2}(-g)\Delta t^2 + \ell\sin\theta'$$

$$\Delta t = \frac{-\left(\frac{\Delta\theta}{\Delta t_0}\right)\ell\cancel{\sin\theta'} \pm \sqrt{\left[\left(\frac{\Delta\theta}{\Delta t_0}\right)\ell\cancel{\sin\theta'}\right]^2 - \left(\frac{-g}{2}\right)(\sin\theta')^{-1}}}{\left(\frac{-g}{2}\right)\cancel{(\sin\theta')}}$$

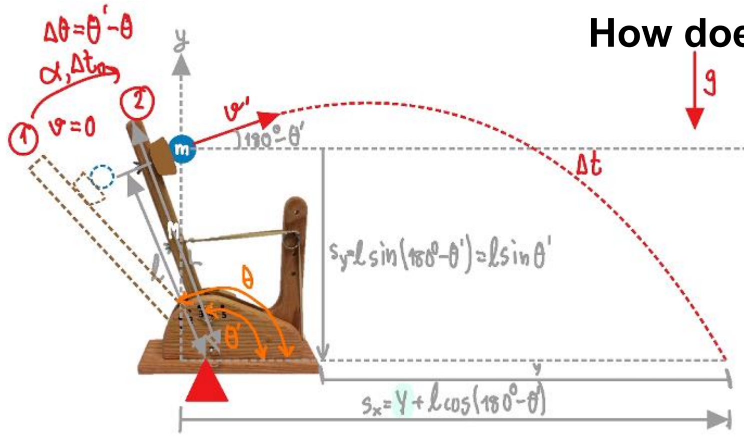
$$\Delta t = \frac{2}{g}\left[\frac{\Delta\theta}{\Delta t_0}\ell + \sqrt{\left(\frac{\Delta\theta}{\Delta t_0}\ell\right)^2 + \frac{g}{2\sin\theta'}}\right]$$

1 Stop angle at “Stop Pin Position”



Y-axis falling distance = $f(\cos(180^\circ - \theta'), \sin\theta')$

How does “Stop Pin Position” affect Y-axis response?



$$\Delta\theta=\omega_0+\tfrac{1}{2}\alpha\Delta t^2 \quad (\omega_0 \to 0)$$

$$F=k\Delta s$$

$$\tau=I\alpha;\ \sum_{\forall i} F_i\times r_i=\left[I_{arm}+I_{cannonball}\right]\left[\frac{2\Delta\theta}{(\Delta t_0)^2}\right]$$

$$I_{arm}=\tfrac{1}{3}ML^2,\quad I_{cannonball}=ml^2$$

$$v'=\omega' l;\quad \omega'=\cancel{\omega_0}+\alpha\Delta t_0=2\left(\frac{\Delta\theta}{\Delta t_0}\right),\quad \alpha=\frac{2\Delta\theta}{(\Delta t_0)^2}$$

[x] $$S_x=\left(v'\cos(180^\circ-\theta')\right)\Delta t \longrightarrow Y=-l\cos(180^\circ-\theta')+\left[2\left(\frac{\Delta\theta}{\Delta t_0}\right)l\cos(180^\circ-\theta')\right]\Delta t$$

[y] $$-S_y=\left[v'\sin(180^\circ-\theta')\right]t+\tfrac{1}{2}at^2 \longrightarrow 0=\left[2\left(\frac{\Delta\theta}{\Delta t_0}\right)l\sin\theta'\right]\Delta t+\tfrac{1}{2}(-g)\Delta t^2+l\sin\theta'$$

$$\Delta t=\frac{-\left(\frac{\Delta\theta}{\Delta t_0}\right)l\cancel{\sin\theta'}\pm\sqrt{\left[\left(\frac{\Delta\theta}{\Delta t_0}\right)l\cancel{\sin\theta'}\right]^2-\left(\frac{-g}{2}\right)(\sin\theta')^{-1}}}{\left(\frac{-g}{2}\right)\cancel{(\sin\theta')}}$$

$$\Delta t=\frac{2}{g}\left[\frac{\Delta\theta}{\Delta t_0}l+\sqrt{\left(\frac{\Delta\theta}{\Delta t_0}l\right)^2+\frac{g}{2\sin\theta'}}\right]$$

$$\frac{2}{g}\left[\frac{\Delta\theta}{\Delta t_0}l+\sqrt{\left(\frac{\Delta\theta}{\Delta t_0}l\right)^2+\frac{g}{2\sin\theta'}}\right]$$

2 **Different of angle between “Start Angle” and “Stop Angle”**

$$\boxed{\Delta\theta\Uparrow : \omega\Uparrow : v'\Uparrow : s_x : Y\Uparrow}$$

3 **Time since pressing to launching**

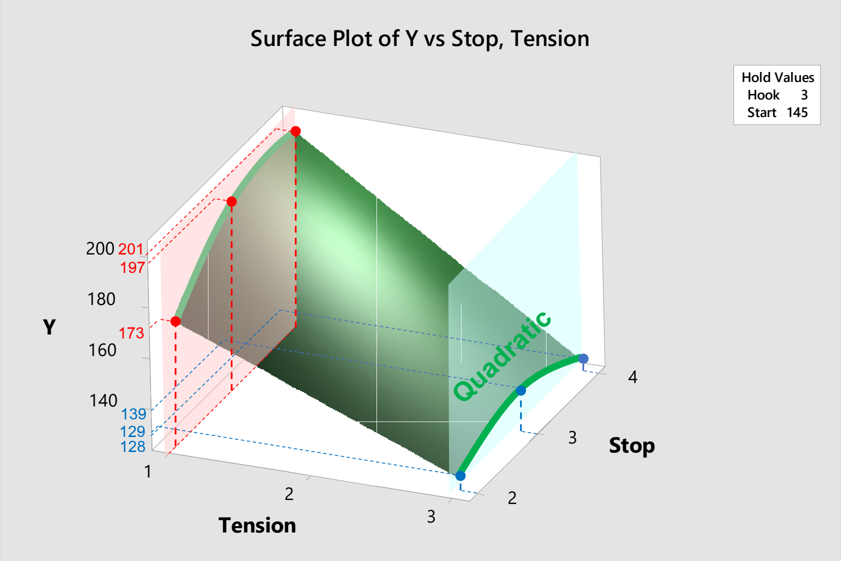
$$\boxed{\Delta\theta\Uparrow : \Delta x\Uparrow : |F|\Uparrow : \Delta t_0\Downarrow : v'\Uparrow : s_x : Y\Uparrow}$$

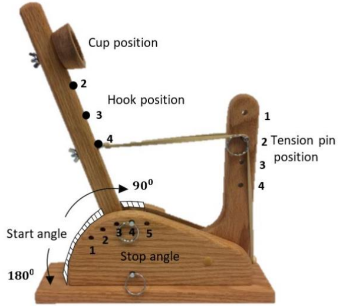
Studying response surface via Surface Plot in Y-axis

Tension*Stop

$$\hat{Y} = -743 - 11.84\,\text{Hook} - 18.87\,\text{Tension} + 5.875\,\text{Start} + 97.1\,\text{Stop} - 10.10\,\text{Stop*Stop} + 3.59\,\text{Hook*Tension} - 5.28\,\text{Hook*Stop} - 7.03\,\text{Tension*Stop}$$

Hook = 3, Start = 145, Hook = 3, Hook = 3





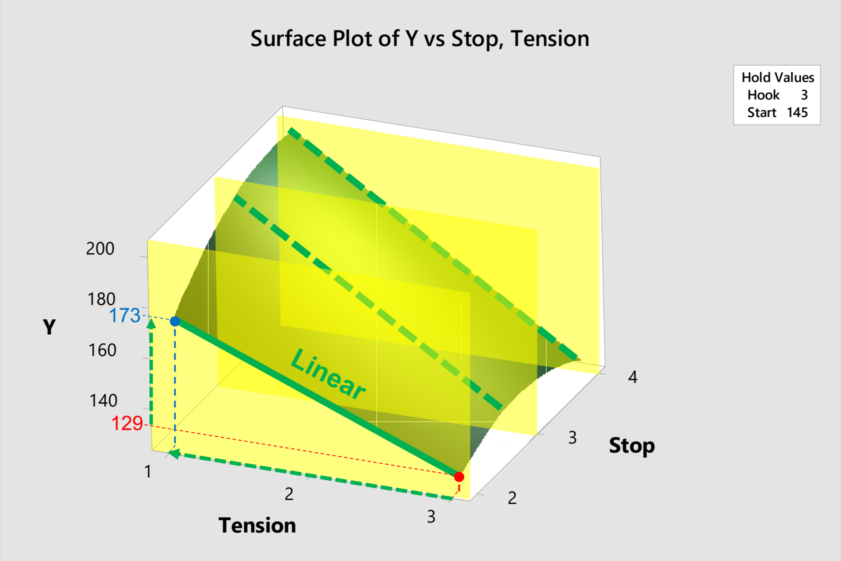
When the level of tension pin position was fixed at position 3, it was found that there was a quadratic relationship between treatment factor, "Stop Pin Position". As the level of stop pin position was adjusted in higher position (2 to 3), the Y-axis distance was falling farther but when it was adjusted even higher position (3 to 4), the Y-axis distance was falling nearer instead. In contrast, when the level of tension pin position was fixed at position 1, it was found that there was a quadratic relationship between treatment factor, "Stop Pin Position". As the level of stop pin position was adjusted in higher position (2 to 3), the Y-axis distance was falling farther but when it was adjusted even higher position (3 to 4), the Y-axis distance was still falling farther but at slower rate.

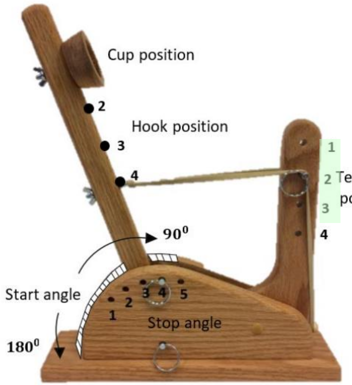
Studying response surface via Surface Plot in Y-axis

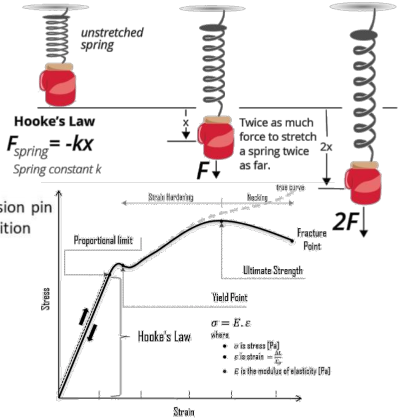
Tension*Stop

$$\hat{Y} = -743 - 11.84\,\text{Hook} - 18.87\,\text{Tension} + 5.875\,\text{Start} + 97.1\,\text{Stop} - 10.10\,\text{Stop*Stop} + 3.59\,\text{Hook*Tension} - 5.28\,\text{Hook*Stop} - 7.03\,\text{Tension*Stop}$$

(Hook = 3, Start = 145, Hook = 3, Hook = 3)





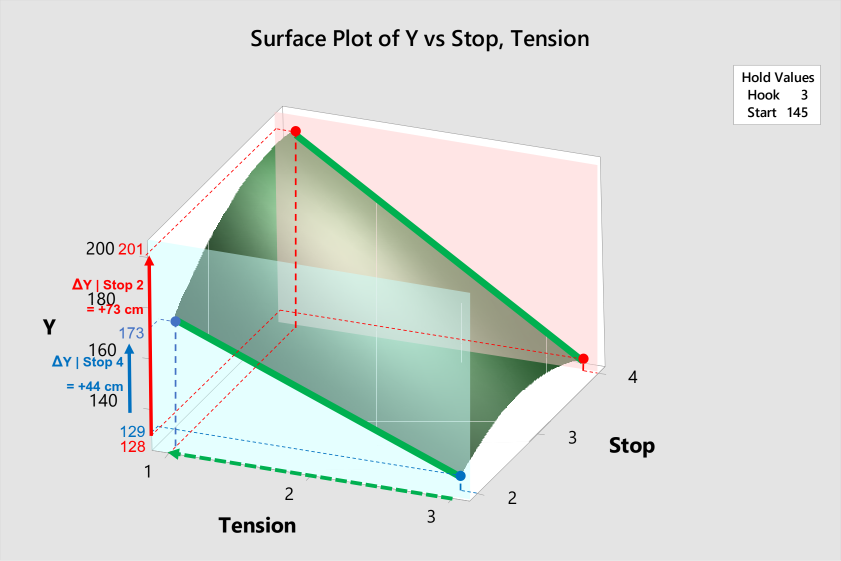


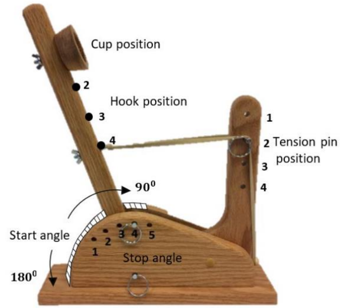
It was found that when the level of tension pin position was adjusted in higher elevation (3 to 1), the Y-axis distance was falling farther in linear relationship for each level of stop pin position, corresponding with Hooke's law stating that the amount of tension force is directly proportional to the stretch or compression displacement of the rubber band.

Studying response surface via Surface Plot in Y-axis

Tension*Stop

$$\hat{Y} = -743 - 11.84\,\overset{3}{\text{Hook}} - 18.87\,\text{Tension} + 5.875\,\overset{145}{\text{Start}} + 97.1\,\text{Stop} - 10.10\,\text{Stop*Stop} + 3.59\,\overset{3}{\text{Hook}}\text{*Tension} - 5.28\,\overset{3}{\text{Hook}}\text{*Stop} - 7.03\,\text{Tension*Stop}$$





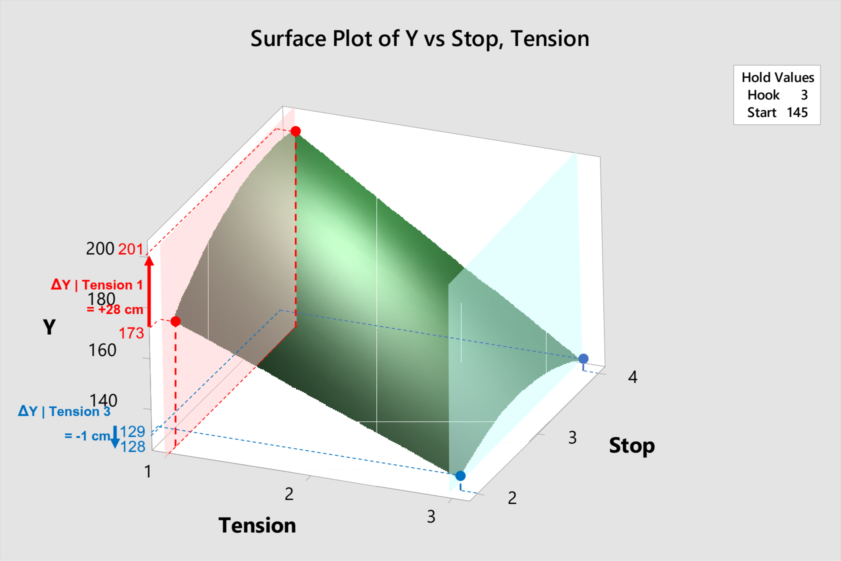
It was found that when the level of stop pin position was fixed at position 4 and tension pin position change from position 3 to 1, the falling distance in the Y-axis was increased by 73 cm (201 – 128). In contrast, if stop pin position was adjusted to position 2 and tension pin position change from position 3 to 1, then the falling distance in Y-axis was increased by 44 cm (173 - 129). Consequently, the amount of change in Y-axis distance when stop pin at position 4 is greater than that at position 2. Additionally, both changes of Y-axis falling distance is in increasing direction with linear relationship.

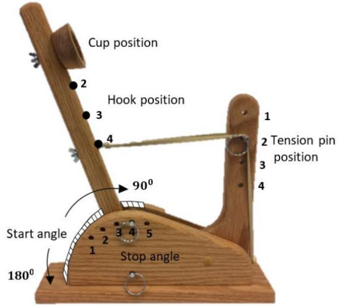
Studying response surface via Surface Plot in Y-axis

Tension*Stop

$$\hat{Y} = -743 - 11.84\,\text{Hook} - 18.87\,\text{Tension} + 5.875\,\text{Start} + 97.1\,\text{Stop} - 10.10\,\text{Stop*Stop} + 3.59\,\text{Hook*Tension} - 5.28\,\text{Hook*Stop} - 7.03\,\text{Tension*Stop}$$

3 145 3 3



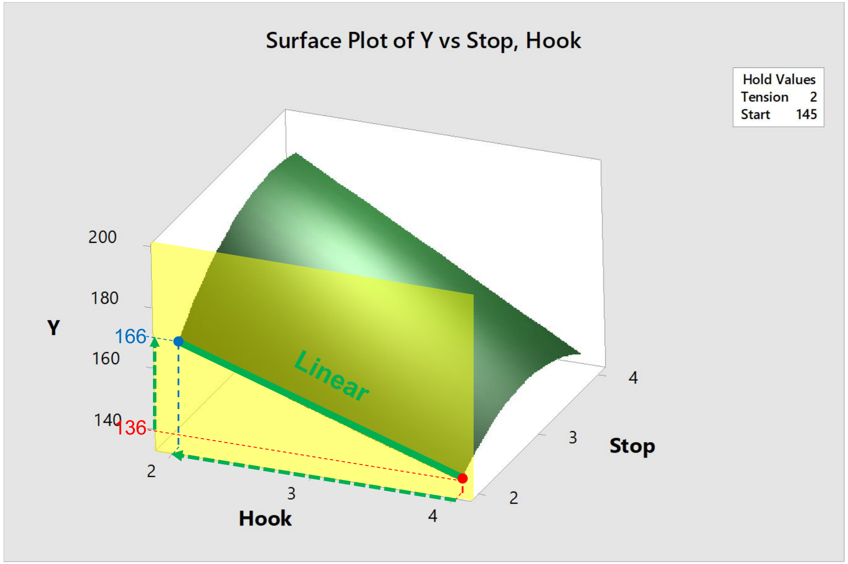


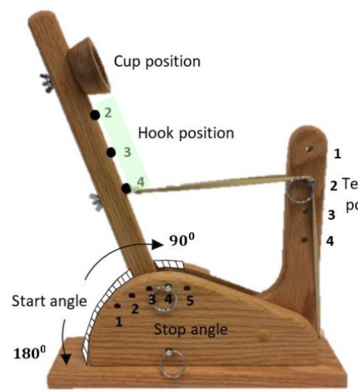
It was found that when the level of tension pin position was fixed at position 3 and stop pin position change from position 2 to 4, the falling distance in the Y-axis was reduced by 1 cm (128 - 129). In contrast, if tension pin position was adjusted to position 1 and stop pin position change from position 2 to 4, then the falling distance in Y-axis was increased by 28 cm (201 - 173). Consequently, the amount of change in Y-axis distance when tension pin at position 1 is greater than that at position 3 with quadratic relationship. Additionally, the change in position 1 is in increasing direction as that in position 3 is in decreasing direction.

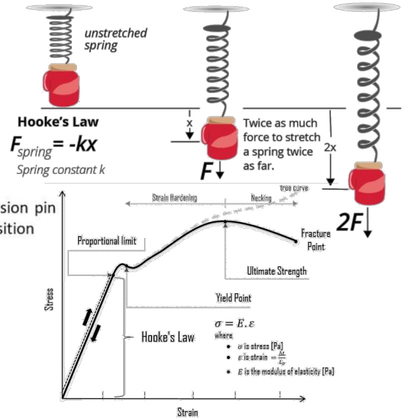
Studying response surface via Surface Plot in Y-axis

Hook*Stop

$$\hat{Y} = -743 - 11.84\,\text{Hook} - 18.87\,\overset{2}{\text{Tension}} + 5.875\,\overset{145}{\text{Start}} + 97.1\,\text{Stop} - 10.10\,\text{Stop*Stop} + 3.59\,\text{Hook*}\overset{2}{\text{Tension}} - 5.28\,\text{Hook*Stop} - 7.03\,\overset{2}{\text{Tension}}\text{*Stop}$$





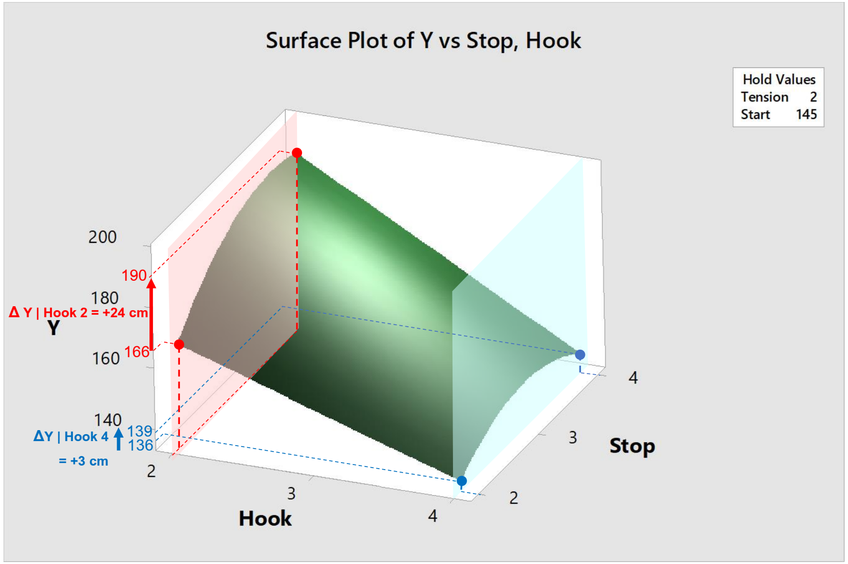


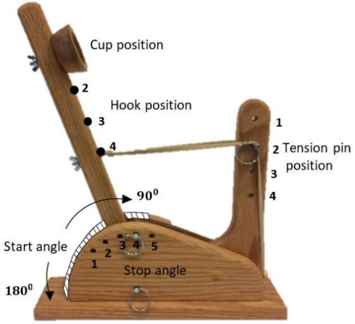
It was found that when the level of hook position was adjusted in higher elevation (4 to 2), the Y-axis distance was falling farther for each level of hook position, corresponding with Hooke's law stating that the amount of tension force is directly proportional to the stretch or compression displacement of the rubber band.

Studying response surface via Surface Plot in Y-axis

Hook*Stop

$$\hat{Y} = -743 - 11.84\,\text{Hook} - 18.87\,\overset{2}{\text{Tension}} + 5.875\,\overset{145}{\text{Start}} + 97.1\,\text{Stop} - 10.10\,\text{Stop*Stop} + 3.59\,\text{Hook*}\overset{2}{\text{Tension}} - 5.28\,\text{Hook*Stop} - 7.03\,\overset{2}{\text{Tension}}\text{*Stop}$$



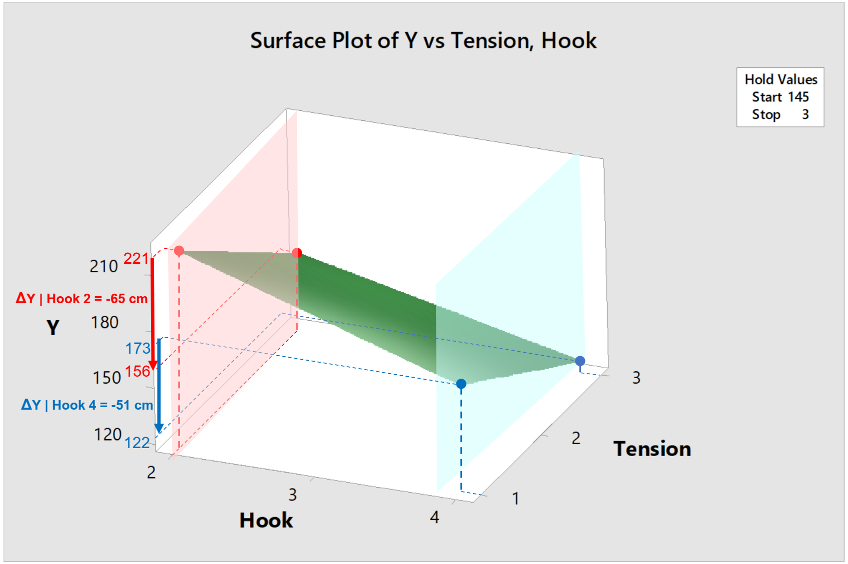


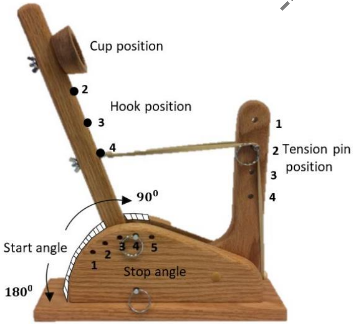
It was found that when the level of hook position was fixed at position 4 and stop pin position change from position 2 to 4, the falling distance in the Y-axis was increased by 3 cm (139 - 136). In contrast, if hook position was adjusted to position 2 and stop pin position change from position 2 to 4, then the falling distance in the Y-axis was increased by 24 cm (190 - 166). Consequently, the increase of Y-axis distance when hook position at position 2 is greater than that at position 4.

Studying response surface via Surface Plot in Y-axis

Hook*Tension

$$\hat{Y} = -743 - 11.84\ \text{Hook} - 18.87\ \text{Tension} + 5.875\ \underbrace{\text{Start}}_{145} + 97.1\ \underbrace{\text{Stop}}_{3} - 10.10\ \underbrace{\text{Stop}}_{3}*\underbrace{\text{Stop}}_{3} + 3.59\ \text{Hook*Tension} - 5.28\ \text{Hook}*\underbrace{\text{Stop}}_{3} - 7.03\ \text{Tension}*\underbrace{\text{Stop}}_{3}$$



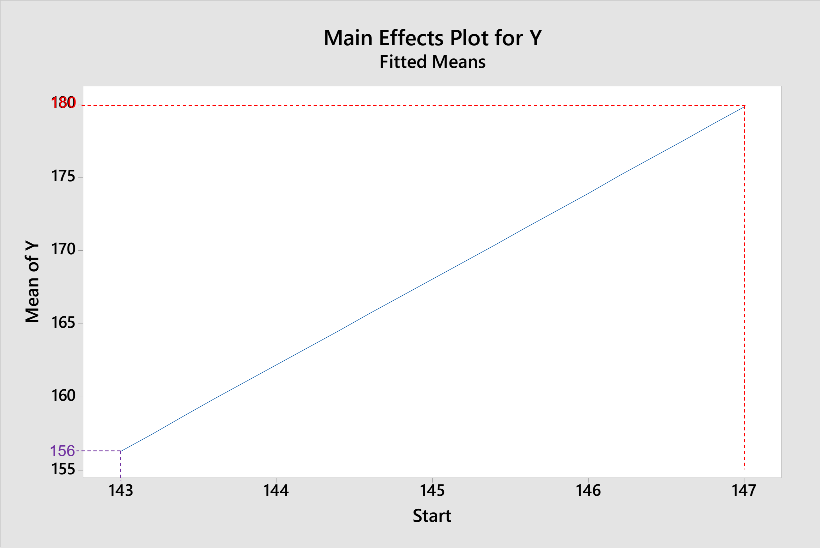


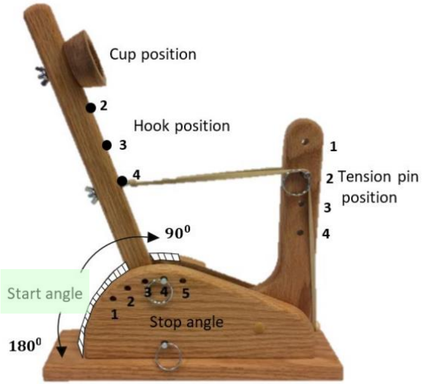
It was found that when the level of hook position was fixed at position 4 and tension pin position change from position 1 to 3, the falling distance in the Y-axis was decreased by 51 cm (122 - 173). In contrast, if hook position was adjusted to position 2 and tension pin position change from position 1 to 3, then the falling distance in the Y-axis was decreased by 65 cm (156 - 221). Consequently, the decrease of Y-axis distance hook position at position 2 is greater than that at position 4.

Studying response of main effect via Main Effect Plot in Y-axis

Start

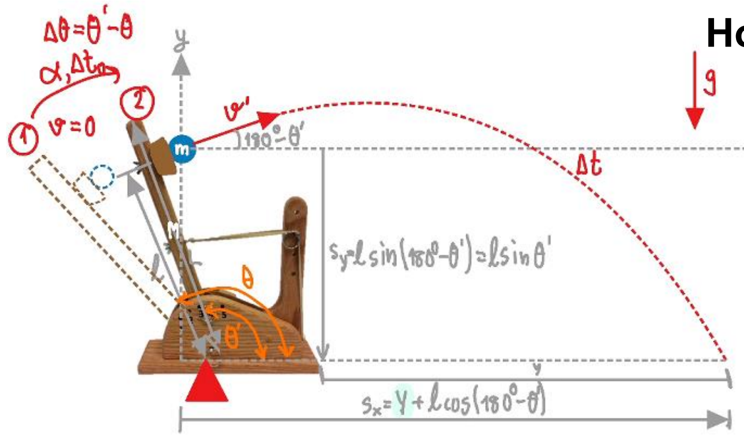
$$\hat{Y} = -743 - 11.84\,Hook - 18.87\,Tension + 5.875\,Start + 97.1\,Stop - 10.10\,Stop*Stop + 3.59\,Hook*Tension - 5.28\,Hook*Stop - 7.03\,Tension*Stop$$





It was found that there was linear relationship between the treatment "Start Angle" and the Y-axis falling distance. When the start angle was adjusted in higher degrees (143° to 147°), the distance was falling farther for each degrees of start angle.

How does "Start Angle" affect Y-axis response?



$$\Delta\theta = \omega_0 + \tfrac{1}{2}\alpha\Delta t^2 \qquad F = k\Delta s$$

$$\tau = I\alpha;\ \sum_{\forall i} F_i \times r_i = \left[I_{arm} + I_{cannonball}\right]\left[\frac{2\Delta\theta}{(\Delta t_0)^2}\right], \quad I_{arm} = \tfrac{1}{3}ML^2, \quad I_{cannonball} = ml^2$$

$$v' = \omega' l, \quad \omega' = \omega_0 + \alpha\Delta t_0 = 2\left(\frac{\Delta\theta}{\Delta t_0}\right), \quad \alpha = \frac{2\Delta\theta}{(\Delta t_0)^2}$$

$$[x] \quad s_x = \left(v'\cos(180^\circ-\theta')\right)\Delta t \longrightarrow Y = -l\cos(180^\circ-\theta') + \left[2\left(\frac{\Delta\theta}{\Delta t_0}\right) l\cos(180^\circ-\theta')\right]\Delta t$$

$$\frac{2}{g}\left[\frac{\Delta\theta}{\Delta t_0} l + \sqrt{\left(\frac{\Delta\theta}{\Delta t_0} l\right)^2 + \frac{g}{2\sin\theta'}}\right]$$

$$[y] \quad -s_y = \left[v'\sin(180^\circ-\theta')\right]t + \tfrac{1}{2}at^2 \longrightarrow 0 = \left[2\left(\frac{\Delta\theta}{\Delta t_0}\right) l\sin\theta'\right]\Delta t + \tfrac{1}{2}(-g)\Delta t^2 + l\sin\theta'$$

$$\Delta t = \frac{-\left(\frac{\Delta\theta}{\Delta t_0}\right) l\cancel{\sin\theta'} \pm \sqrt{\left[\left(\frac{\Delta\theta}{\Delta t_0}\right) l\cancel{\sin\theta'}\right]^2 - \left(\frac{-g}{2}\right)(\sin\theta')^{-1}}}{\left(\frac{-g}{2}\right)\cancel{(\sin\theta')}}$$

$$\Delta t = \frac{2}{g}\left[\frac{\Delta\theta}{\Delta t_0} l + \sqrt{\left(\frac{\Delta\theta}{\Delta t_0} l\right)^2 + \frac{g}{2\sin\theta'}}\right]$$

1. **Different of angle between "Start Angle" and "Stop Angle"**

$$\Delta\theta \Uparrow : \omega \Uparrow : v' \Uparrow : s_x : Y \Uparrow$$

2. **Time since pressing to launching**

$$\Delta\theta \Uparrow : \Delta x \Uparrow : |F| \Uparrow : \Delta t_0 \Downarrow : v' \Uparrow : s_x : Y \Uparrow$$

Optimizing Response

Setting Parameters

Goal → Follow given specification

Response	Goal	Lower	Target	Upper	Weight	Importance
Y	Target	160	170	180	3	1
X	Target	-5	0	5	2	1

Importance → Follow fine calculating equation

The equation for calculating the fine is $F = c_m(\mu - T)^2 + c_s\sigma^2$

where F = Fine

c_m = coefficient of bias = 200

μ = mean falling distance

T = target

c_s = coefficient of variation = 400

σ = standard deviation of falling distance

1. Fine regarding bias and variation along X axis is expressed by

$$F_x = c_m(\mu_x - T_x)^2 + c_s\sigma_x^2$$

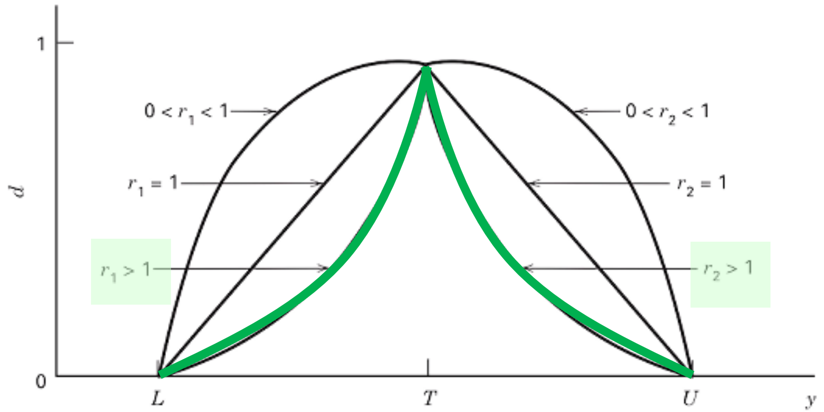
2. Fine regarding bias and variation along Y axis is expressed by

$$F_y = c_m(\mu_y - T_y)^2 + c_s\sigma_y^2$$

Thus, the total fine is $F_{all} = F_x + F_y$

Then, **the reward is** $\mathrm{Re}\,ward = 100{,}000 - F_{all}$

Weights ⟹ More emphasis on Target



X: $\underbrace{-5 \quad 0}_{5}$, $\underbrace{0 \quad +5}_{5}$ ⟹ Weight $X = 2$

Y: $\underbrace{160 \quad 170}_{10}$, $\underbrace{170 \quad 180}_{10}$ ⟹ Weight Y = 4

Choosing Optimal Solution

Response Optimization: Y, X

Parameters

Response	Goal	Lower	Target	Upper	Weight	Importance
Y	Target	160	170	180	4	1
X	Target	-5	0	5	2	1

Solution

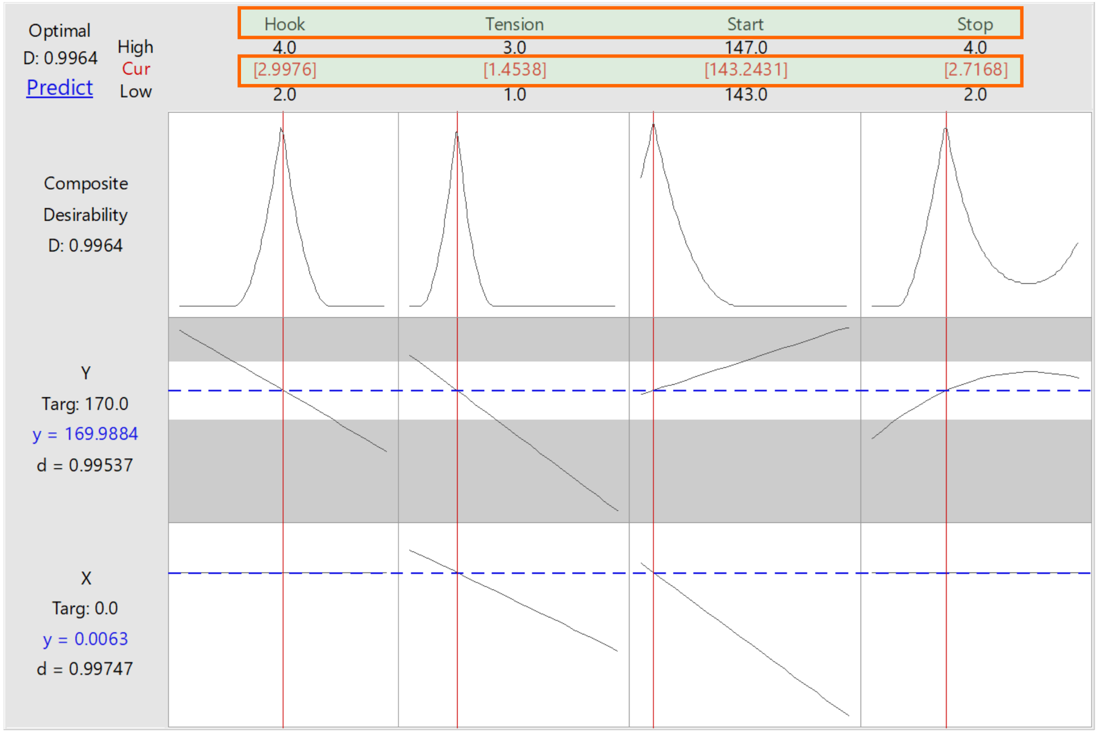
Solution	Hook	Tension	Start	Stop	Y Fit	X Fit	Composite Desirability
1	2.99765	1.45384	143.243	2.71681	169.988	0.0063264	0.996420

Multiple Response Prediction

Variable	Setting
Hook	2.99765
Tension	1.45384
Start	143.243
Stop	2.71681

Response	Fit	SE Fit	95% CI	95% PI
Y	169.99	2.32	(165.19, 174.78)	(155.66, 184.32)
X	0.01	1.50	(-3.07, 3.08)	(-9.96, 9.97)

Optimization Plot



Choosing Optimal Solution

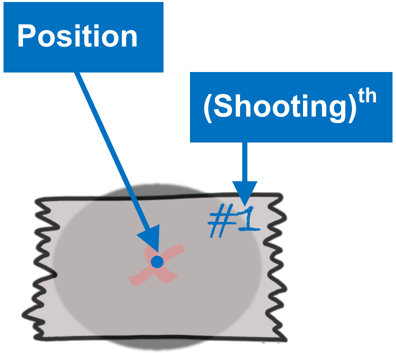
Obj: max composite desirability

Hook	Tension	Stop	Start	Composite Desirability	X(cm)	Y(cm)
3	1	2	143	0.0259	0.93	161.79
3	1	2	144	0.5234	-0.54	167.66
3	1	2	145	0.2498	-2.01	173.53
3	1	3	no suitable value	∀ Adjustments give Y out of spec limit		
3	2	2	145	0.0000	-1.22	151.39
3	2	2	146	0.0000	-1.53	157.26
3	2	2	147	0.0622	-1.84	163.14
3	2	3	144	0.0402	-0.91	162.22
3	2	3	145	0.4949	-1.22	168.09
3	2	3	146	0.2523	-1.53	173.97

The optimal solution corresponding with the limitation of the catapult setting is to set hook position at position 3, tension pin position at position 1, start angle at 144 degrees and stop pin position at position 2.

Recap: #Improvement

Improvement #1: ☑ Make sure that the measurement process is stable by marking the falling position on the tape based on the centroid of charcoal dust which occurs when the ball hit the ground.



Improvement #2: ☑ Make sure that the adjusted levels of settings in catapult machine are optimal by using the principle of design of experiment.

Hook	Tension	Stop	Start	Composite Desirability	X(cm)	Y(cm)
3	1	2	144	0.5234	-0.54	167.66

Improvement #3: Improve additional factors that may affect the responses with intermediate total score around 30s.

No.	Category	Factor\Response	$\bar{y}$	$\hat{\sigma}_y$	$\bar{x}$	$\hat{\sigma}_x$	Total Score
		Response Importance weight	10	0.7	0.0	0.4	
1	Man	Lack of effectively inspection	1	0	9	0	10
2		Awkward shooting posture	3	3	9	3	33
3		High contract stress in finger(s) used in launching the ball	3	3	3	3	33
4		Repetitive motions in shooting process	3	3	9	3	33
5		Discomfortable sitting position	3	3	9	3	33
6	Machine	Unclear machine work instruction	1	1	3	1	11
7		No appropriate maintenance of the catapult machine	1	1	1	1	11
8	Material	Insufficient quality of charcoal dust	0	1	0	1	1
9		Excessive use of rubber band over time in shooting process	3	1	0	0	31
10		Loss of tape's stickiness over time	1	0	9	0	10
~~11~~	~~Method~~	~~Not optimal adjustment of start angle~~	~~9~~	~~9~~	~~0~~	~~0~~	~~96~~
~~12~~		~~Not optimal adjustment of stop pin's position~~	~~9~~	~~9~~	~~0~~	~~0~~	~~96~~
~~13~~		~~Not optimal adjustment of tension pin's position~~	~~9~~	~~9~~	~~0~~	~~0~~	~~96~~
~~14~~		~~Not optimal adjustment of hook's position~~	~~9~~	~~9~~	~~0~~	~~0~~	~~96~~
15	Environment	Sound noise from external environment	1	1	1	1	11
16		Insufficient level of luminance	3	3	3	3	33
17		Glare from light reflection	1	1	1	1	11
18		Drag force from air	1	1	1	1	11

Additional Improvement

Other remaining factors

- Awkward shooting posture
- High contract stress in finger(s) used in launching the ball
- Repetitive motions in shooting process
- Discomfortable sitting position

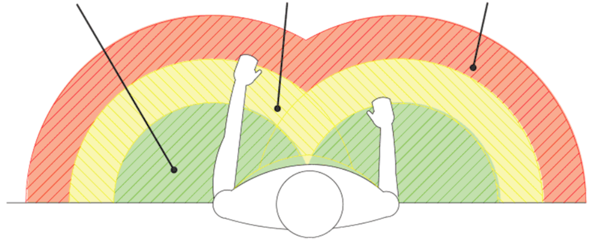
Solution

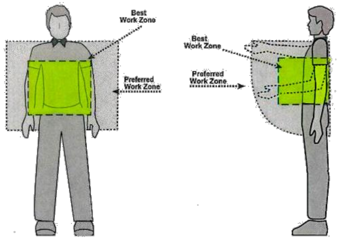
☑ Improve the operation that workers don't have to twist / reach out thing

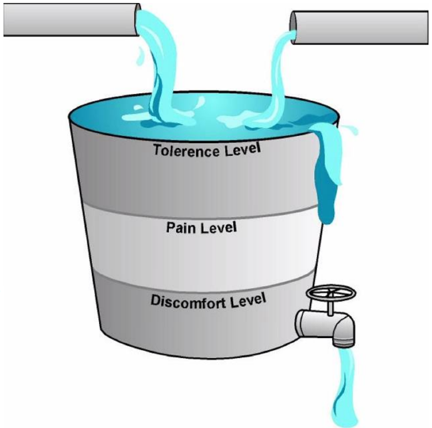
☑ Adjust operation to be done with slow movement and using appropriate range of motion (ROM)

☑ Schedule suitable work-rest cycle to prevent fatigue of worker:

"short, frequent and intermittent"







Work/Rest Cycles Murrell's formula for calculating the total amount of rest, in minutes, required after performing a particular type of work activity for 30 minutes is given by the formula

$$R(w) = \frac{30(w-4)}{w-1.5}$$

where w is the work expended in kilocalories per minute, kcal/min. Source: Human Factors in Engineering and Design.

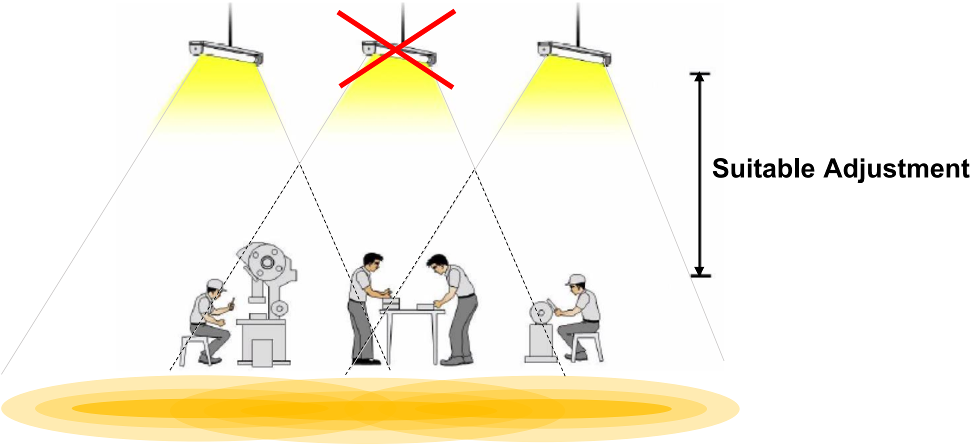
Additional Improvement

Other remaining factors

- Insufficient level of luminance

Solution

☑ Make sure that the current light intensity is suitable for working without gazing



Additional Improvement

Other remaining factors

- Insufficient level of luminance

Solution

☑ Make sure that the current light intensity is suitable for working without gazing

หมวด ๓

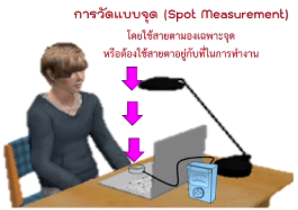
การตรวจวัดความเข้มของแสงสว่างและประเภทกิจการที่ต้องดำเนินก

ข้อ ๑๐ การตรวจวัดความเข้มของแสงสว่างบริเวณที่ลูกจ้างต้องทำงานโดยใช้สายตามองเฉพาะจุดหรือต้องใช้สายตาอยู่กับที่ในการทำงาน ให้ตรวจวัดในจุดที่สายตาตกกระทบชิ้นงานหรือจุดที่ทำงานของลูกจ้าง (Workstation)

นำค่าความเข้มของแสงสว่างทีตรวจวัดได้ตามวรรคหนึ่ง เปรียบเทียบกับความเข้มของแสงสว่างตามที่กำหนดไว้ตามตารางในประกาศกรมสวัสดิการและคุ้มครองแรงงาน เรื่อง มาตรฐานความเข้มของแสงสว่าง ลงวันที่ ๒๗ พฤศจิกายน พ.ศ. ๒๕๖๐

ตารางที่ ๒ มาตรฐานความเข้มของแสงสว่าง ณ บริเวณที่ลูกจ้างต้องทำงาน โดยใช้สายตามองเฉพาะจุดหรือต้องใช้สายตาอยู่กับที่ในการทำงาน

การใช้สายตา	ลักษณะงาน	ตัวอย่างลักษณะงาน	ค่าความเข้มของแสงสว่าง (ลักซ์)
งานหยาบ	งานที่ชิ้นงานมีขนาดใหญ่สามารถมองเห็นได้อย่างชัดเจน มีความแตกต่างของสีชัดเจนมาก	- งานหยาบที่ทำที่โต๊ะหรือเครื่องจักร ชิ้นงานที่มีขนาดใหญ่กว่า ๗๕๐ ไมโครเมตร (๐.๗๕ มิลลิเมตร) - การตรวจงานหยาบด้วยสายตา การประกอบ การนับ การตรวจเช็คสิ่งของที่มีขนาดใหญ่ - การรีดเส้นด้าย - การอัดเบล การผสมเส้นใย หรือการสางเส้นใย - การซักรีด ซักแห้ง การอบ - การปั๊มขึ้นรูปแก้ว เป่าแก้ว และขัดเงาแก้ว - งานตี และเชื่อมเหล็ก	๒๐๐ – ๓๐๐
งานละเอียดเล็กน้อย	งานที่ชิ้นงานมีขนาดปานกกลาง สามารถมองเห็นได้และมีความแตกต่างของสีชัดเจน	- งานรับจ่ายเสื้อผ้า - การทำงานไม้ที่ชิ้นงานมีขนาดปานกลาง - งานบรรจุน้ำลงขวดหรือกระป๋อง - งานเจาะรู ทากาว หรือเย็บเล่มหนังสือ งานบันทึกและคัดลอกข้อมูล - งานเตรียมอาหาร ปรุงอาหาร และล้างจาน - งานผสมและตกแต่งขนมปัง - การทอผ้าดิบ	๓๐๐ – ๔๐๐



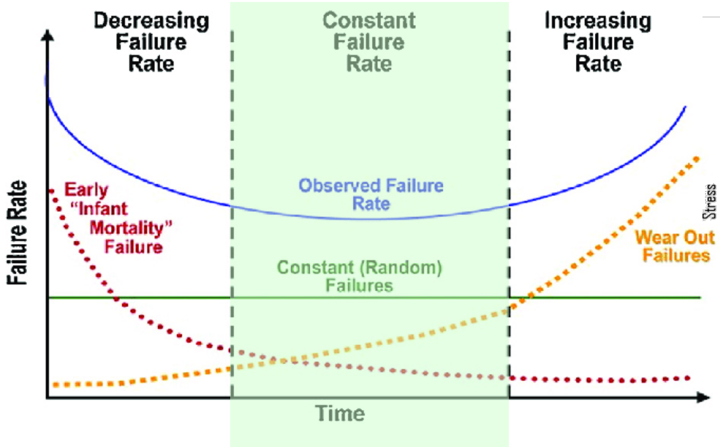
Additional Improvement

Other remaining factors

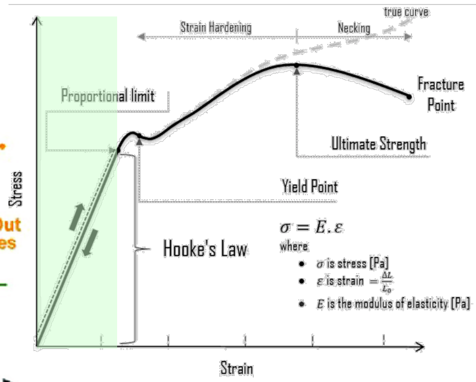
Solution

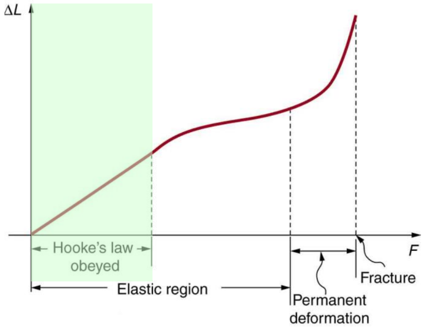
- **Excessive use of rubber band over time in shooting process**

☑ **Make sure that the used rubber band is within the useful life period.**



Useful Life Period





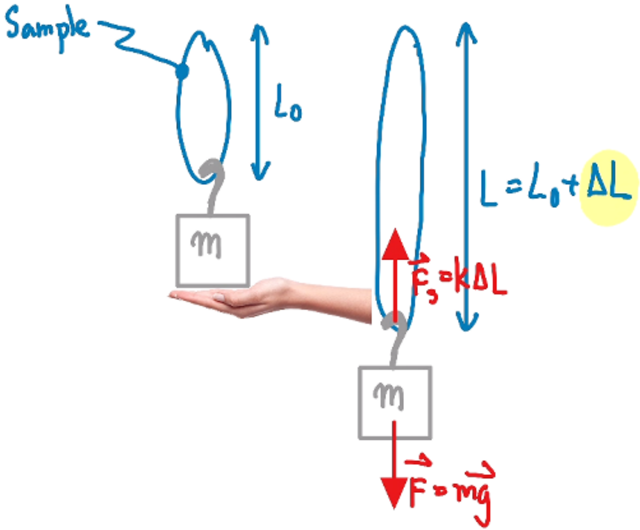
Additional Improvement

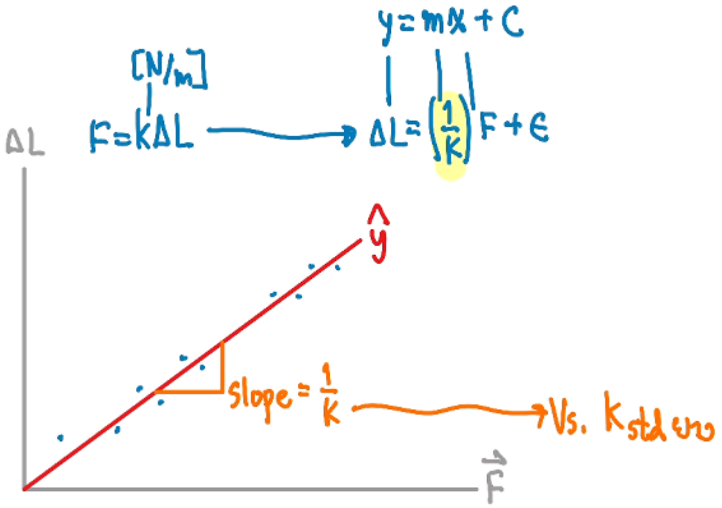
Other remaining factors

- Excessive use of rubber band over time in shooting process

Solution

☑ Make sure that the used rubber band is within the useful life period.





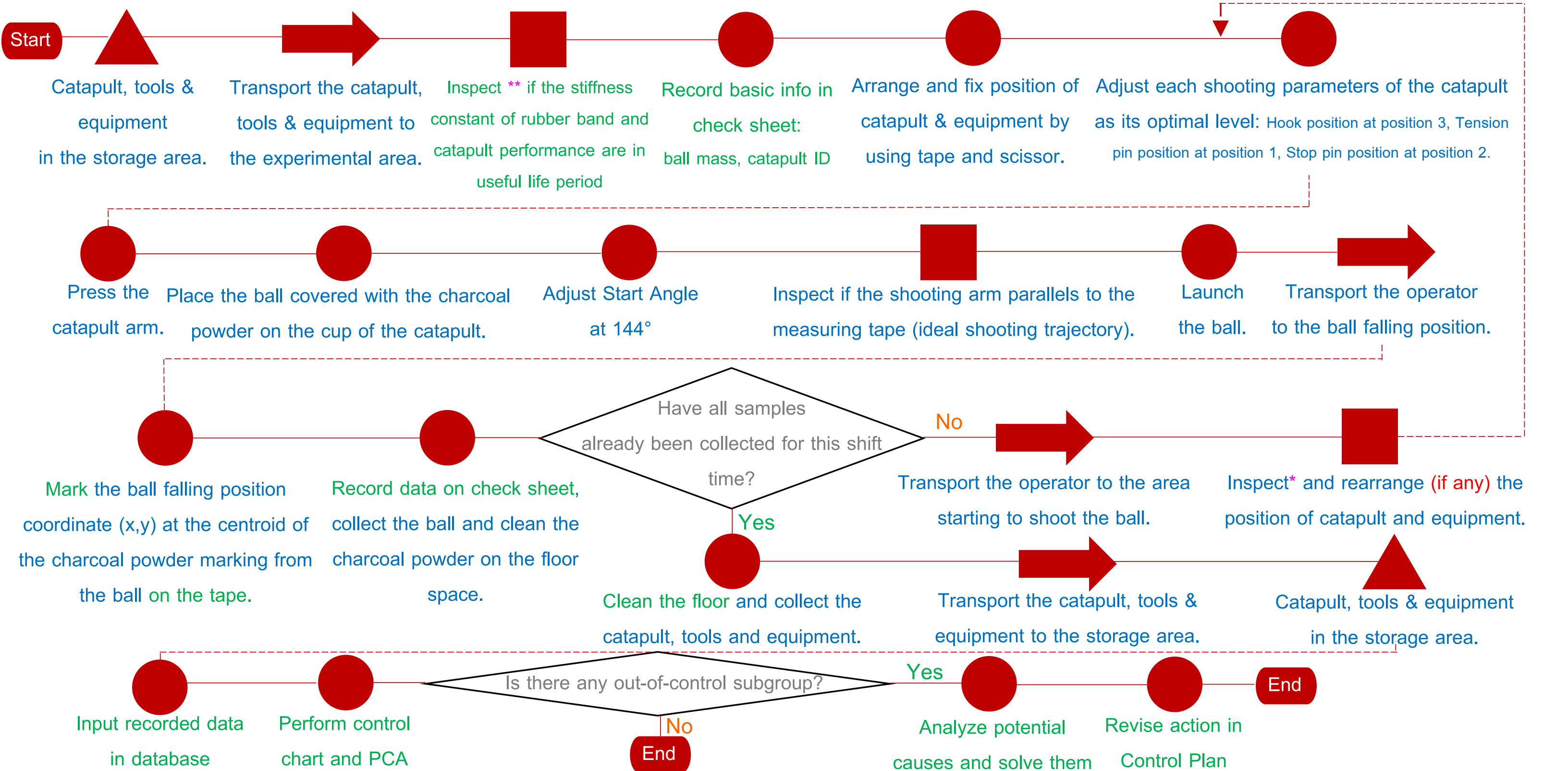
Control Phase

Revised Process Flow

Assumption: The catapult is already finished goods so that no assembly process is required except for some position adjustment before launching process.

Remark: * Inspect the positions of catapult and equipment with the sampling frequency of 2 hr/time.

** Inspect the stiffness and catapult performance with the frequency of 15 days/time.



Work manual for ball shooting with catapult

Title: Standard process for ball shooting with catapult

Objective: Standardized the process of shooting ball with catapult

Procedure for operator in ball shooting with catapult:

1. Take the catapult, tools & equipment from the storage area.
2. Transport the catapult, tools & equipment to the experimental area.
3. Inspect if the stiffness constant of rubber band and catapult performance are in useful life period:
   - Inspection of stiffness constant follows the *Work Instruction for inspecting rubber band's quality* and record data in *Check sheet for inspecting stiffness of a rubber band*
   - Inspection of catapult's performance is performed by the professional skill operator to test the shooting process of catapult [follow step 5-14 similarly] at varied level of setting, record the testing falling distance (X,Y) along with another basic info in *Check sheet for evaluating machine performance* and then compare to standard distance as well as evaluate the performance of shooting if more than 50% of shooting approximately near each of standard distance.
4. Record basic info in *Check sheet for monitoring and controlling the shooting process*: ball mass, catapult ID
5. Arrange and fix position of catapult & equipment by using tape and scissor.
6. Adjust each shooting parameters of the catapult as its optimal level: Hook position at position 3, Tension pin position at position 1, Stop pin position at position 2.
7. Press the catapult arm.
8. Place the ball covered with the charcoal powder on the cup of the catapult.
9. Adjust Start Angle at 144°
10. Inspect if the shooting arm parallels to the measuring tape (ideal shooting trajectory).
11. Launch the ball.
12. Transport the operator to the ball falling position.
13. Mark the ball falling position coordinate (X,Y) at the centroid of the charcoal powder marking from the ball on the tape.
14. Record measuring distance for on *Check sheet for monitoring and controlling the shooting process*, collect the ball and clean the charcoal powder on the floor space.

Have all samples already been collected for all operators?

If yes, skip to process 17. Otherwise, go to process 15,16 and then perform process 6 – 14.

15. Transport the operator to the area starting to shoot the ball.
16. Inspect and rearrange (if any) the positions of catapult and equipment with the sampling frequency of 2 hours/time.
17. Clean the floor and collect the catapult, tools and equipment.
18. Transport the catapult, tools & equipment to the storage area.
19. Input recorded data on *Check sheet for monitoring and controlling the shooting process* in database
20. Perform control chart and PCA

Is there any out-of-control subgroup?

If yes, perform process 21,22. Otherwise, end process.

21. Analyze potential causes and solve them
22. Revise action in Control Plan

References: *Work Instruction for inspecting rubber band's quality*

Forms in used: *Check sheet for inspecting stiffness of a rubber band, Check sheet for monitoring and controlling the shooting process, Check sheet for evaluating machine performance*

Check sheet for monitoring and controlling the shooting process

Ball Mass:

Catapult ID:

Stiffness of rubber band:

Run No.	Subgroup No.	Date (DD/MM/YYYY)	Time (XX:XX AM/PM)	Hook Position	Tension Pin Position	Stop Pin Position	Start Angle (Degree)	X falling distance [cm]	Y falling distance [cm]	Operator ID

Work Instruction for inspecting rubber band’s quality

<u>Title:</u> Inspection of rubber band’s quality
<u>Performer:</u> Shooting operator

<u>Document No.:</u> RB-SQ-001
<u>Editing Edition:</u> 1st ed

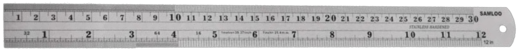
<u>Reviewed by:</u> Thatchakarn C., Teethavat T.
<u>Approved by:</u> Napassavong O.

<u>Equipment:</u>

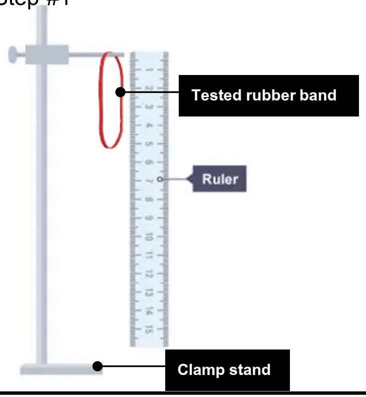
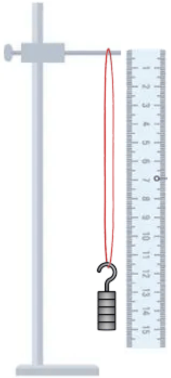
Cylindrical mass set with hook

Tested rubber band

Clamp stand

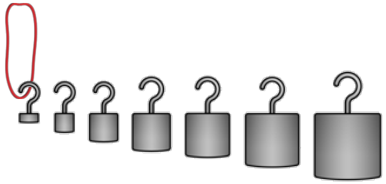


Stainless Steel Metal Ruler 30 cm

Step	Instruction	Step	Instruction
Step #1 	Set up the clamp stand and the tested rubber band as the following figure and then measure the length of rubber band before extension (L_0) in the check sheet for inspecting stiffness of a rubber band.	Step #2 	Place the hook of cylindrical mass to the tested rubber band and then record the mass as well as measure the length of rubber band after the extension in the check sheet for inspecting stiffness of a rubber band.

Work Instruction for inspecting rubber band's quality

Step	Instruction
Step #3	Increase the suspension mass and record the parameters, including mass (m) and the length after extension (L) in the check sheet for inspecting stiffness of a rubber band.



No.	m [kg]	W [N]	L [m]	ΔL [m]
1	0.5	[redacted]	0.055	[redacted]
2	1	[redacted]	0.100	[redacted]
3	1.5	[redacted]	0.145	[redacted]
⋮	⋮	[redacted]	⋮	[redacted]

Step	Instruction
Step #4	Calculate weight (W) from the product of mass (m) and acceleration due to gravity (g) as well as the extension length (ΔL) from the difference between the length after extension (L) and the initial length before extension (L_0) in the check sheet for inspecting stiffness of a rubber band.

$W = mg$ $\qquad$ $\Delta L = L - L_0$

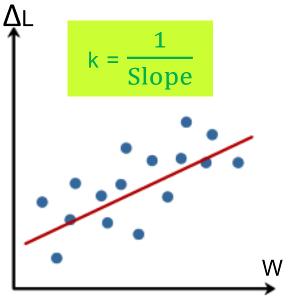
No.	m [kg]	W [N]	L [m]	ΔL [m]
1	0.5	**4.905**	0.055	0.045
2	1	**9.81**	0.100	0.090
3	1.5	**14.715**	0.145	0.135
⋮	⋮	⋮	⋮	⋮

Work Instruction for inspecting rubber band's quality

Step	Instruction
Step #5	Fit the recorded dataset with simple linear regression given that independent variable is weight and dependent variable is extended length. Then, calculate the stiffness of specimen rubber band from the inverse of slope of the fitted regression equation
Step #6	Evaluate the rubber band quality from comparison of stiffness constant between that of standard and that of specimen. If both value estimated to be near, then the specimen can be used for shooting. Otherwise, specimen rubber band should not be used in catapult project and should be replaced with the other one .

Step #5

No.	m [kg]	W [N]	L [m]	ΔL [m]
1	0.5	4.905	0.055	0.045
2	1	9.81	0.100	0.090
3	1.5	14.715	0.145	0.135
⋮	⋮	⋮	⋮	⋮



Step #6

k:
- $\approx k_{std}$ → The specimen rubber band can be used in catapult project.
- $\gg k_{std}$
- $\ll k_{std}$

The specimen rubber band should not be used in catapult project and should be replaced with the other one.

Check sheet for inspecting stiffness of a rubber band

Rubber band ID:

Stiffness constant of standard rubber band (k_{std}): 45 N/m or 4.5 N/cm

Length of rubber band before extension (L_0):

No.	Mass, m [kg]	Weight, W [N] = mass [kg] * 9.81 [m/s^2]	Length after extension, L [m]	Extending length, ΔL [m] = $L - L_0$
1				
2				
3				
⋮				

The estimated simple linear regression equation when independent variable is weight and dependent variable is extending length: ΔL = _____W + ______

Then, the specimen stiffness constant, $k = \frac{1}{\text{Slope}} = \frac{1}{\_\_\_\_\_\_}$ = ______ N/m

Evaluating the stiffness of used rubber band:

- ❑ The specimen rubber band can be used in catapult project ($\because k \approx k_{std}$)
- ❑ The specimen rubber band should not be used in catapult project and should be replaced with the other one ($\because k >> k_{std}$ or $k << k_{std}$)

Check sheet for evaluating machine performance

Catapult ID: Stiffness of rubber band: 45 N/m	Inspection Date: Ball Mass: 500 g	Warranty Period: Step 0: ☐ Catapult is now in warranty period.

Step 1: Fundamental inspecting for machine condition

No.	Catapult's Right Condition List	Does catapult have right condition? [✓ or ✗]
1	All pin holes fit perfectly for each stop pin and tension pin.	
2	Catapult arm has stability (a.k.a. no vibration) when moving.	
3	Catapult has no broken or nearly wear off parts.	
⋮	⋮	

Step 2: Test for catapult's shooting performance

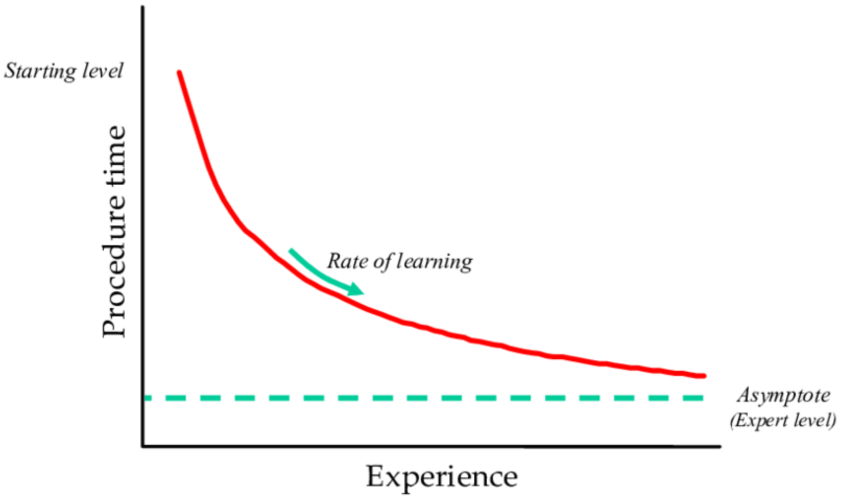
Note: - Step 2 will be performed only if the catapult has all right condition in Step 1.
- Only operator who has steady learning curve can perform the shooting test in Step 2.

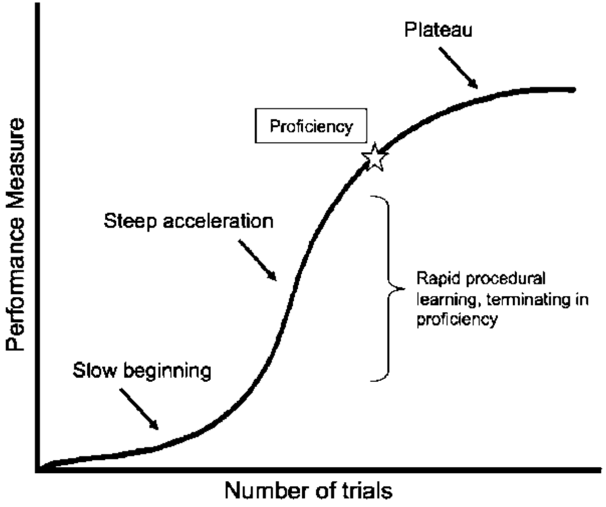
No.	Hook Position	Tension Pin Position	Stop Pin Position	Start Angle	Y standard [cm]	Y test [cm]	Y on std? (Test ≈ std ±2 cm) [✓ or ✗]
1	3	1	2	144	167.7		
2	2	3	4	145	150.6		
3	4	2	3	146	153.5		
⋮	⋮	⋮	⋮	⋮	⋮		

Evaluating catapult performance:

- ☐ Catapult have normal life (Catapult has all right condition & Less than 50% of tests are out of standard)
- ☐ Catapult begin to wear off (Catapult has some abnormality condition / At least 50% of tests are out of standard)

Training operator





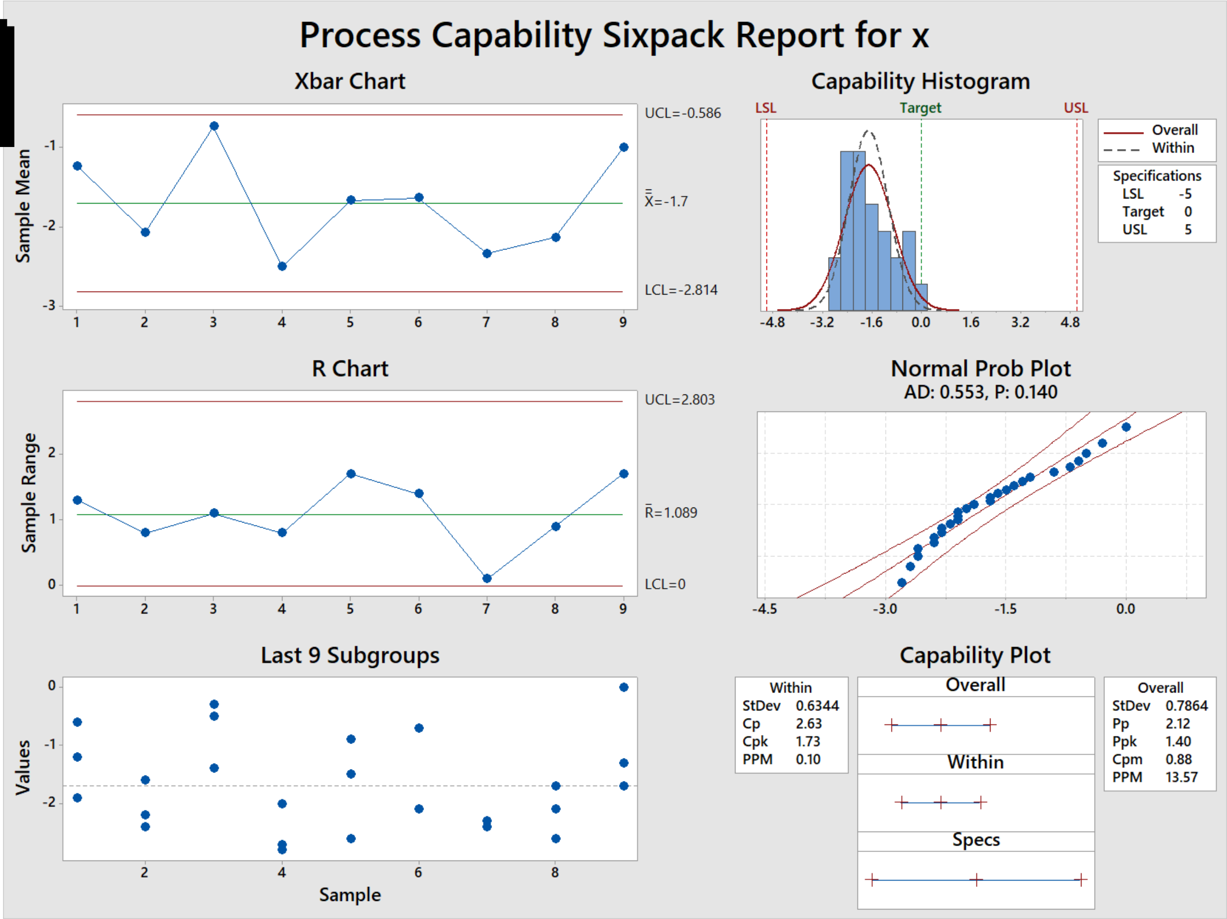
Each operator will train themselves with doing by following work manual and work instruction until they gained expert level experience according to the principle of learning curve.

6^{th} data collection:

- Shooter Order: OP1
- Measurer's Name: AAAAA A.

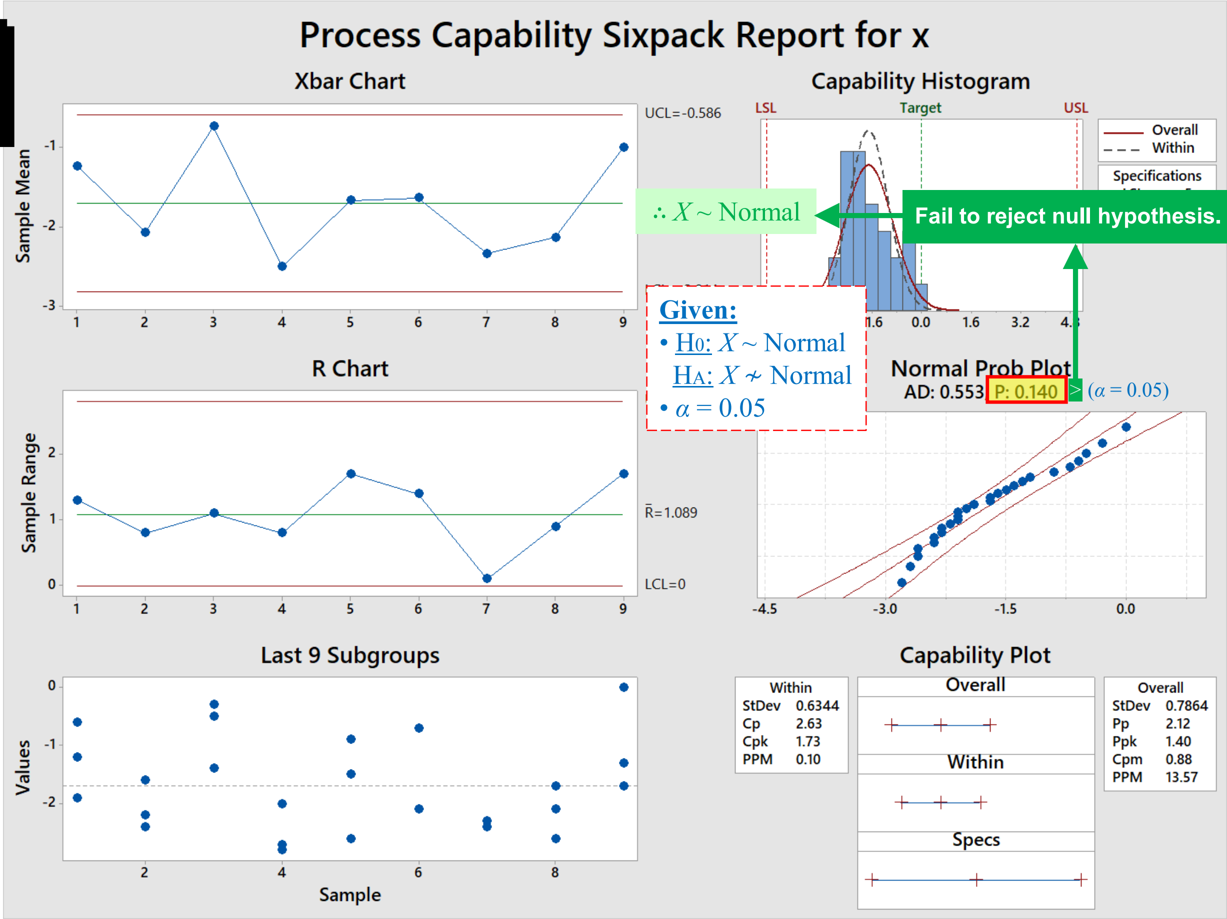
Subgroup	Observation	X (cm)	Y (cm)	Subgroup	Observation	X (cm)	Y (cm)
1	1	-1.9	173.5	6	16	-2.1	174.5
	2	-1.2	168		17	-2.1	169
	3	-0.6	167		18	-0.7	170
2	4	-2.2	170.5	7	19	-2.4	167.4
	5	-2.4	171.4		20	-2.3	170.6
	6	-1.6	170		21	-2.3	169.5
3	7	-1.4	169.5	8	22	-2.1	173.8
	8	-0.3	172		23	-1.7	172.1
	9	-0.5	166.5		24	-2.6	170.5
4	10	-2.8	168	9	25	-1.3	170
	11	-2.7	167		26	0	168.4
	12	-2	166		27	-1.7	172.6
5	13	-0.9	170.5				
	14	-1.5	166.5				
	15	-2.6	168.5				

PCA in X-axis



Process Capability Sixpack Report for x
Xbar Chart
Sample Mean
UCL=-0.586
X̿=-1.7
LCL=-2.814
R Chart
Sample Range
UCL=2.803
R̄=1.089
LCL=0
Last 9 Subgroups
Values
Sample
Capability Histogram
LSL
Target
USL
Overall
Within
Specifications
LSL -5
Target 0
USL 5
Normal Prob Plot
AD: 0.553, P: 0.140
Capability Plot
Within
StDev 0.6344
Cp 2.63
Cpk 1.73
PPM 0.10
Overall
Within
Specs
Overall
StDev 0.7864
Pp 2.12
Ppk 1.40
Cpm 0.88
PPM 13.57

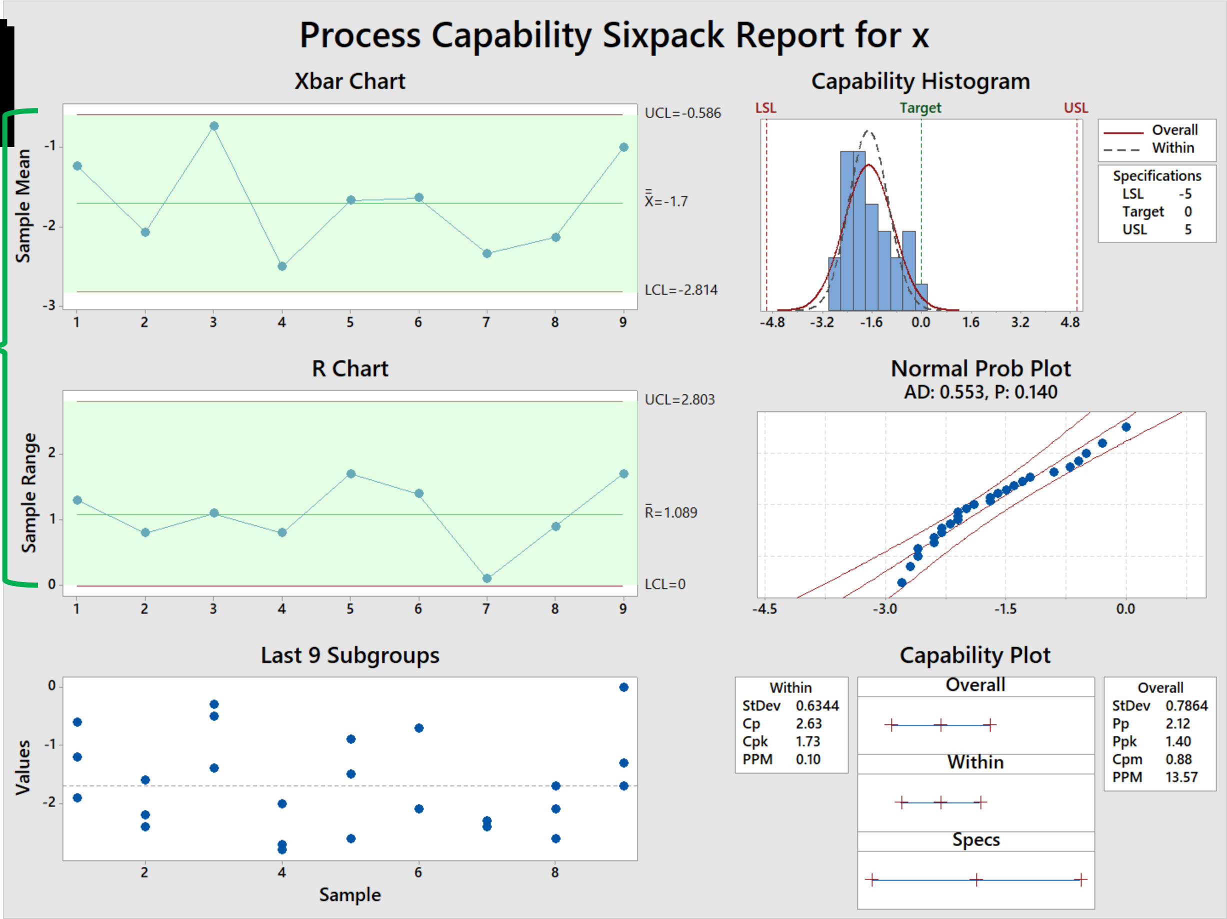
PCA in X-axis



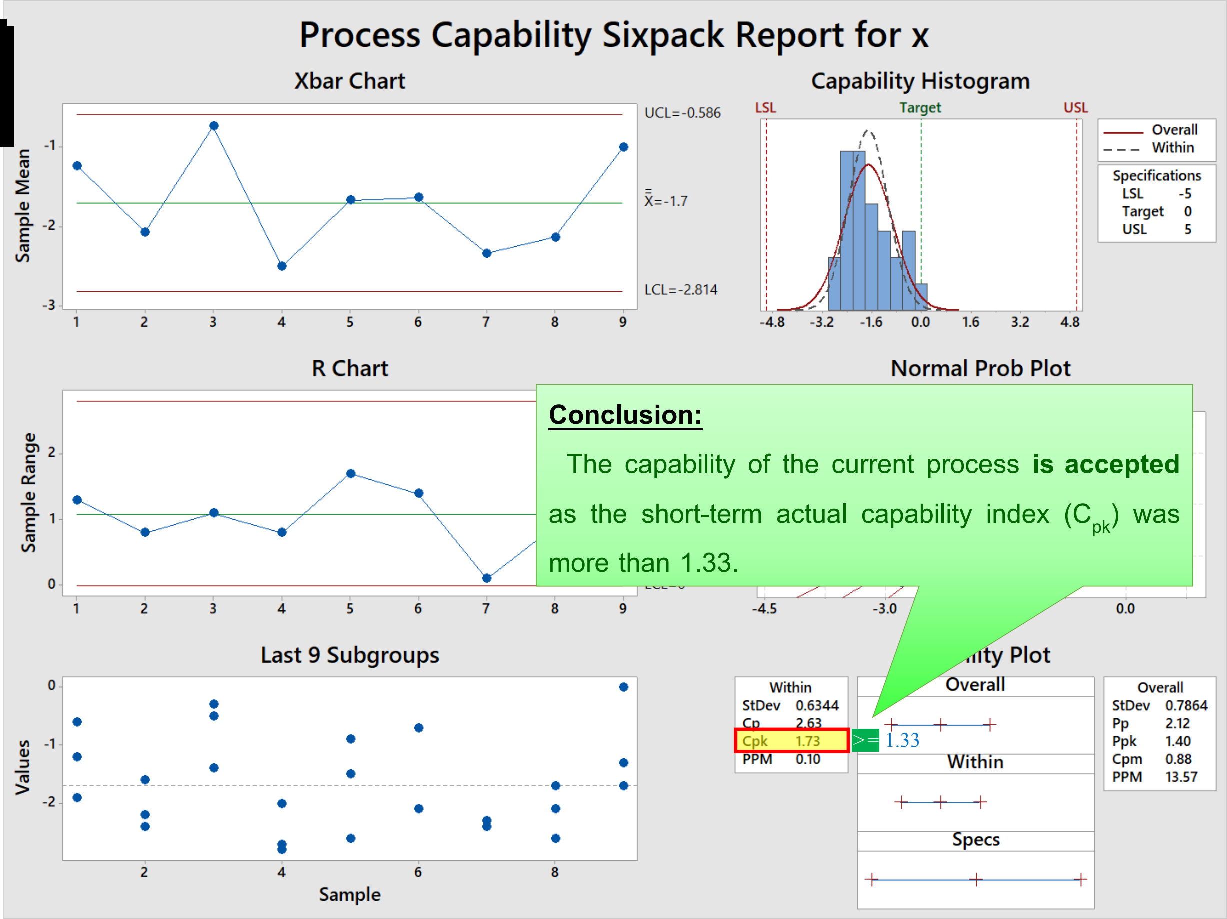
Process Capability Sixpack Report for x
Xbar Chart
Sample Mean
UCL=-0.586
R Chart
Sample Range
R̄=1.089
LCL=0
Last 9 Subgroups
Values
Sample
Capability Histogram
LSL
Target
USL
Overall
Within
Specifications
∴ X ~ Normal
Fail to reject null hypothesis.
Given:
• H0: X ~ Normal
HA: X ≁ Normal
• α = 0.05
Normal Prob Plot
AD: 0.553 P: 0.140 > (α = 0.05)
Capability Plot
Within
StDev 0.6344
Cp 2.63
Cpk 1.73
PPM 0.10
Overall
StDev 0.7864
Pp 2.12
Ppk 1.40
Cpm 0.88
PPM 13.57
Overall
Within
Specs

PCA in X-axis

∀ Subgroups are in control



PCA in X-axis



Conclusion:

The capability of the current process **is accepted** as the short-term actual capability index (C_{pk}) was more than 1.33.

Evaluation if the sample size of current X-axis data points is adequate for PCA

Given:

- Confidence Interval $= 95\%$ $\longrightarrow$ Type I error, $\alpha = 0.05$ $\longrightarrow$ $Z_\alpha \approx -1.645$
- $\dfrac{C_{pk}}{\hat{C}_{pk}} = 0.90$
- $\hat{C}_{pk(X)} = 1.73$

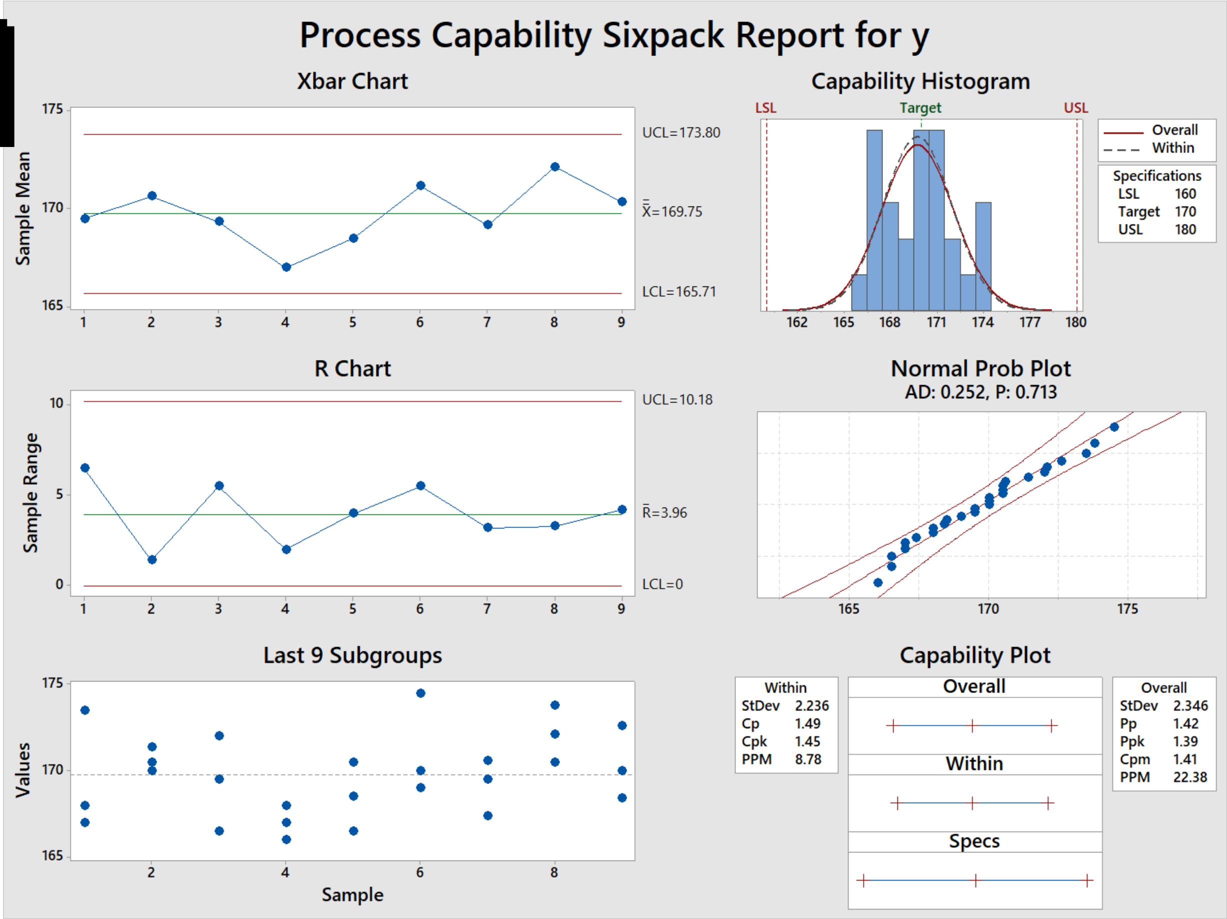
Solution:

$$n = (Z_\alpha)^2 \frac{\left(\dfrac{1}{9\left(\hat{C}_{pk}\right)^2} + \dfrac{1}{2}\right)}{\left(1 - \dfrac{C_{pk}}{\hat{C}_{pk}}\right)^2} = (-1.645)^2 \times \frac{\left(\dfrac{1}{9 \times 1.73^2} + \dfrac{1}{2}\right)}{(1 - 0.90)^2} = \lceil 145.3 \rceil = 145$$

Conclusion: ∴ To estimate short-term actual capability index (C_{pk}) for X-axis distance, at least sample size of 145 is required.

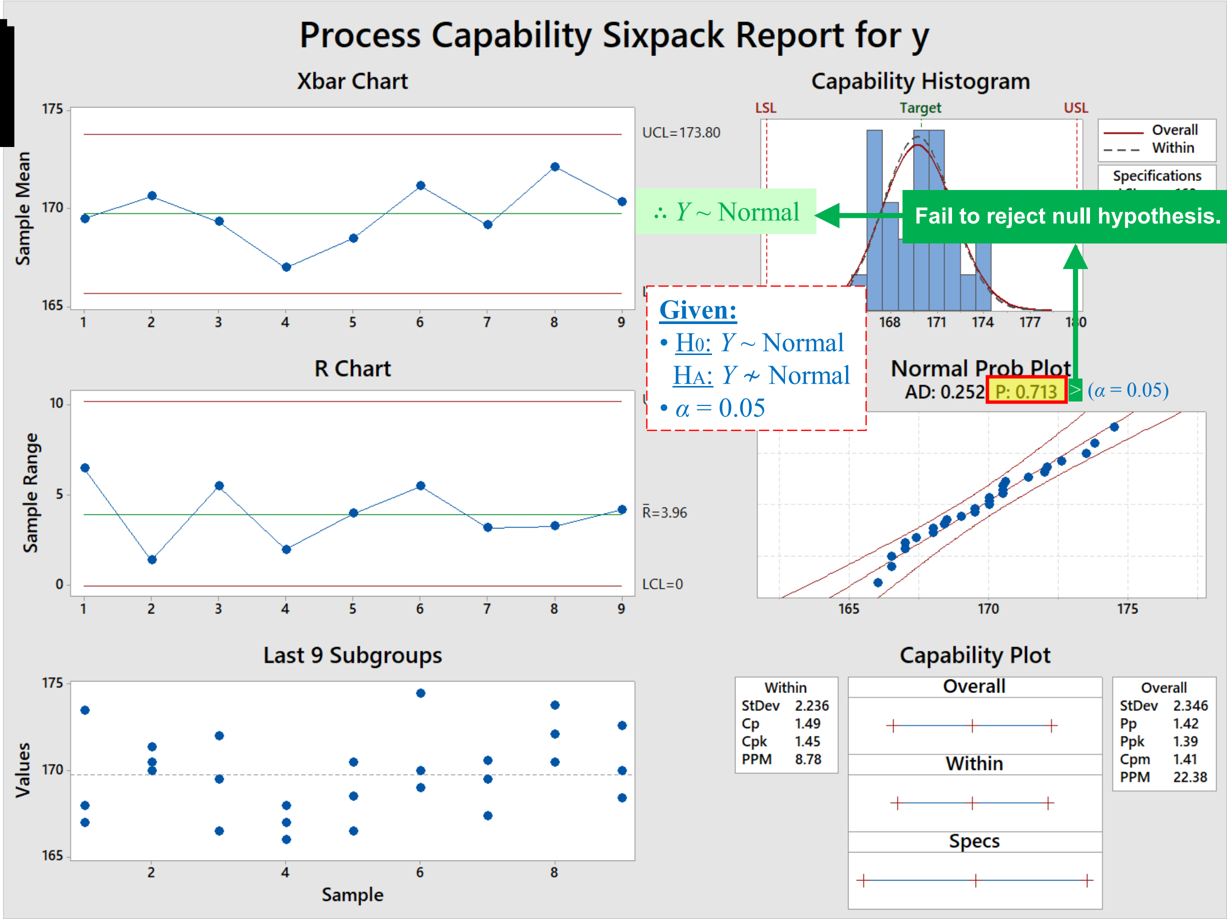
Justify: **The additional samples that should be later collected is no less than 118.**

PCA in Y-axis



Process Capability Sixpack Report for y
Xbar Chart
Sample Mean
UCL=173.80
X̄=169.75
LCL=165.71
R Chart
Sample Range
UCL=10.18
R̄=3.96
LCL=0
Last 9 Subgroups
Values
Sample
Capability Histogram
LSL
Target
USL
Overall
Within
Specifications
LSL 160
Target 170
USL 180
Normal Prob Plot
AD: 0.252, P: 0.713
Capability Plot
Within
StDev 2.236
Cp 1.49
Cpk 1.45
PPM 8.78
Overall
Within
Specs
Overall
StDev 2.346
Pp 1.42
Ppk 1.39
Cpm 1.41
PPM 22.38

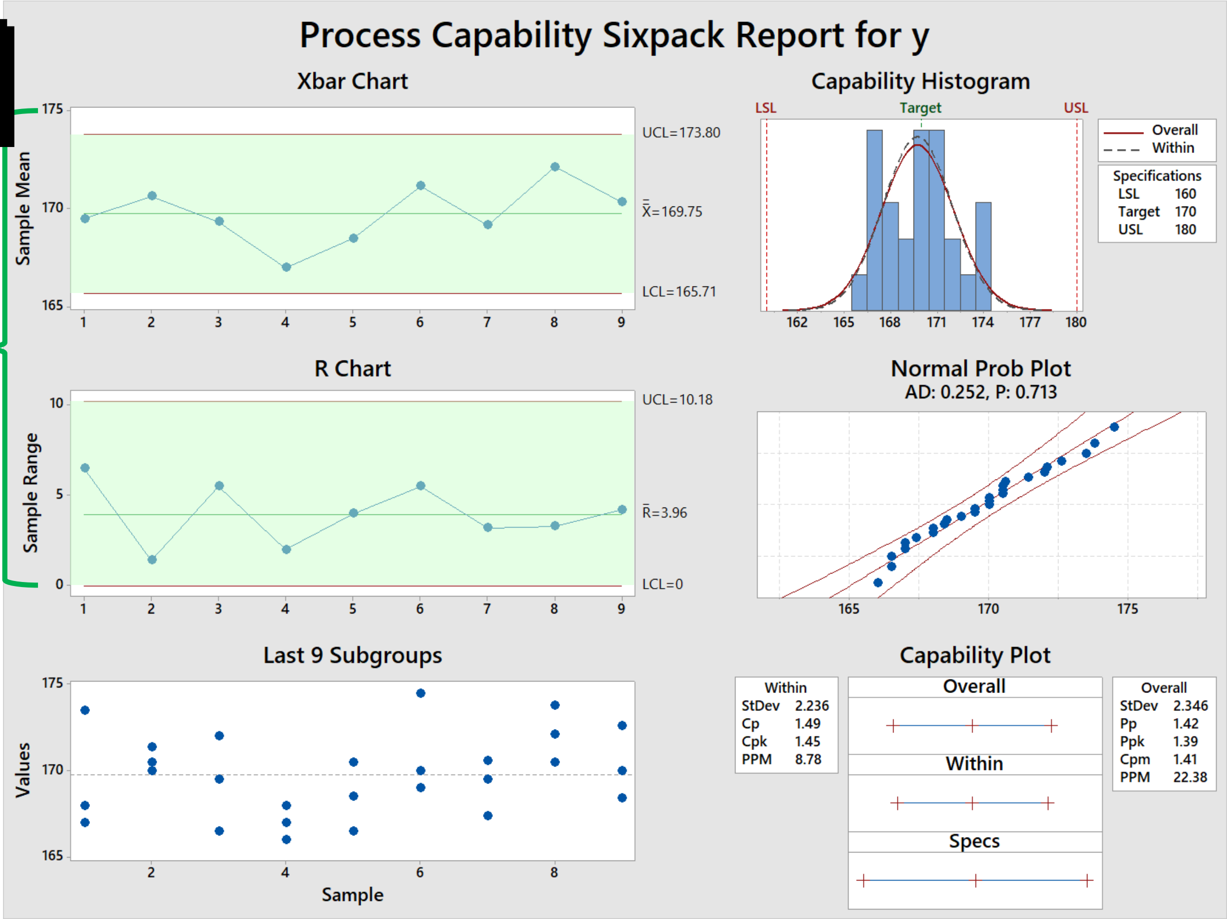
PCA in Y-axis



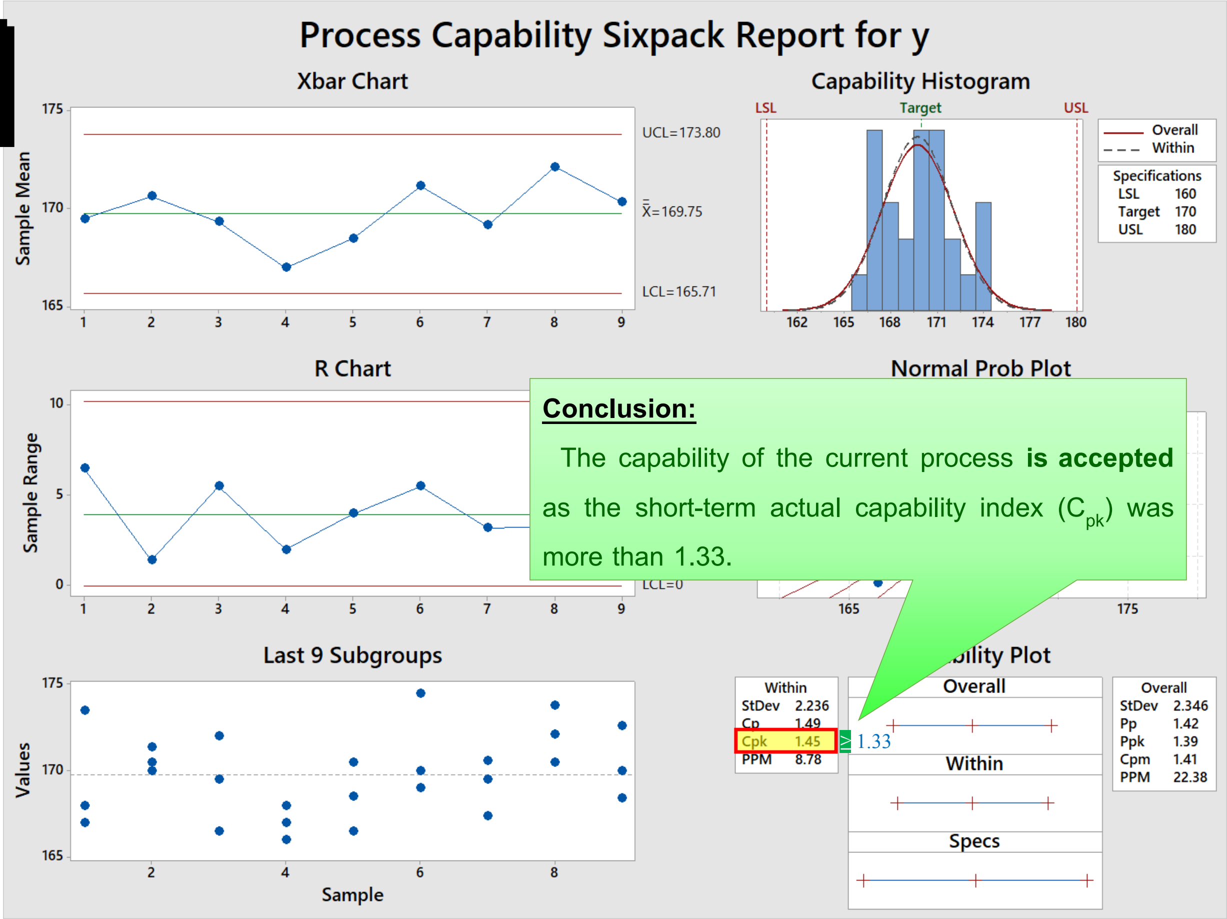
Process Capability Sixpack Report for y
Xbar Chart
UCL=173.80
Sample Mean
R Chart
R̄=3.96
LCL=0
Sample Range
Last 9 Subgroups
Values
Sample
Capability Histogram
LSL
Target
USL
Overall
Within
Specifications
Normal Prob Plot
AD: 0.252 P: 0.713
Capability Plot
Within
StDev 2.236
Cp 1.49
Cpk 1.45
PPM 8.78
Overall
StDev 2.346
Pp 1.42
Ppk 1.39
Cpm 1.41
PPM 22.38
Overall
Within
Specs
∴ Y ~ Normal
Fail to reject null hypothesis.
Given:
• H0: Y ~ Normal
HA: Y ≁ Normal
• α = 0.05
> (α = 0.05)

PCA in Y-axis

∀ Subgroups are in control



PCA in Y-axis



Conclusion:

The capability of the current process **is accepted** as the short-term actual capability index (C_{pk}) was more than 1.33.

Evaluation if the sample size of current Y-axis data points is adequate for PCA

Given:

- Confidence Interval $= 95\%$ $\longrightarrow$ Type I error, $\alpha = 0.05$ $\longrightarrow$ $Z_\alpha \approx -1.645$
- $\dfrac{C_{pk}}{\hat{c}_{pk}} = 0.90$
- $\hat{C}_{pk(Y)} = 1.45$

Solution:

$$n = (Z_\alpha)^2 \frac{\left(\frac{1}{9\left(\hat{C}_{pk}\right)^2} + \frac{1}{2}\right)}{\left(1 - \frac{C_{pk}}{\hat{C}_{pk}}\right)^2} = (-1.645)^2 \times \frac{\left(\frac{1}{9 \times 1.45^2} + \frac{1}{2}\right)}{(1 - 0.90)^2} = \lceil 149.6 \rceil = 150$$

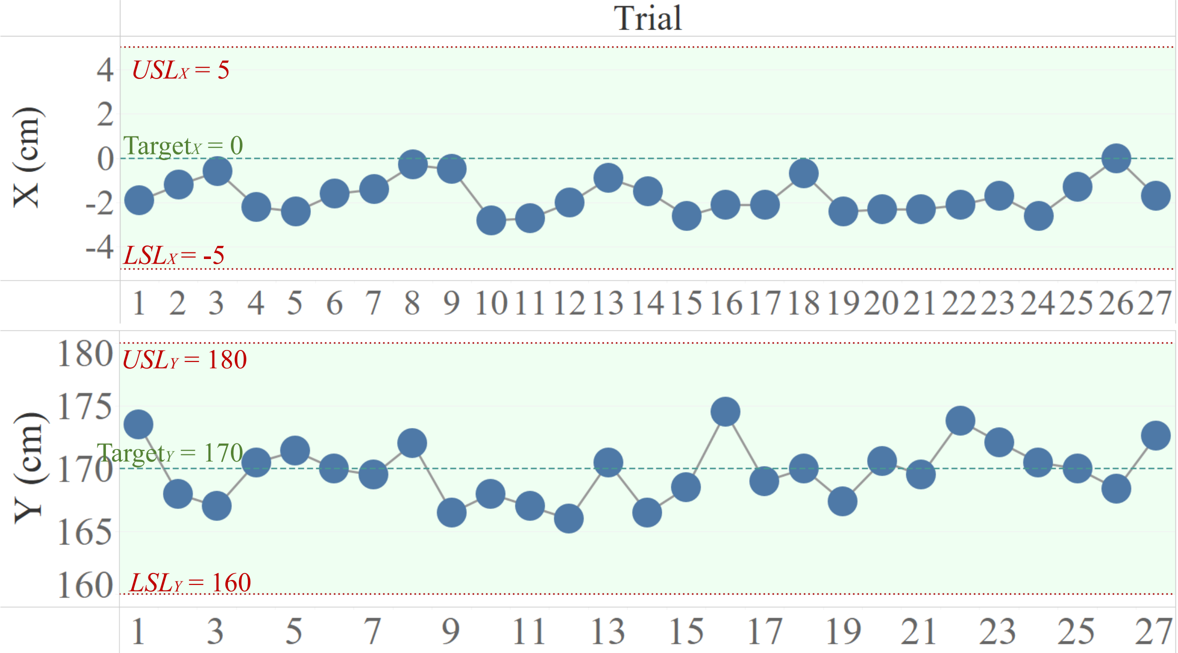
Conclusion: ∴ To estimate short-term actual capability index (C_{pk}) for Y-axis distance, at least sample size of 150 is required.

Justify: **The additional samples that should be later collected is no less than 123.**

From 6th data collection

Trial	*X*	*Y*
1	-1.9	173.5
2	-1.2	168
3	-0.6	167
⋮	⋮	⋮
26	0	168.4
27	-1.7	172.6
Mean	-1.70	169.75
SD	0.79	2.35

Out of specification
Within specification



$Fine_X$ = 825.38 THB

$Fine_{X\text{-}bias}$ = 578.00 THB

$Fine_{X\text{-}variation}$ = 247.38 THB

$Fine_Y$ = 2,213.66 THB

$Fine_{Y\text{-}bias}$ = 12.32 THB

$Fine_{Y\text{-}variation}$ = 2,201.34 THB

ΣFine = 3,039.04 THB

Net reward = ฿96,960.96

Result Comparison between before and after improvement

Response	Index	Expect		After Improvement		Before Improvement
X-axis Distance	MAPE	10%	<	64.33%		
	|Average – Target|	0	←	|-1.70-0|	<	|2.35-0|
	SD	0	←	0.79	<	1.38
	C_{pk}	1.33	<	1.73	>	0.64
	Fine	0		825.38	<	1,863.48
Y-axis Distance	MAPE	10%	>	1.46%		
	|Average – Target|	0	←	|169.75-170|	<	|162.90-170|
	SD	0	←	2.35	<	3.54
	C_{pk}	1.33	<	1.45	>	0.35
	Fine	0		2,213.66	<	15,091.37
Total Fine		0		3,039.04	<	16,954.85
Reward		100,000	≈	96,960.96	>	83,045.15

KSF of project:

- |Average – Target| ⇩
- SD ⇩

Better value of C_{pk}

Overall performance after improvement meets goal ✓

Evaluating ROQI of project

Given:

Capital Investment(I): ฿65,525.33

Improvement cost:

- 1st data collection for initial trial: 6 times
- 2nd data collection for control chart: 90 times
- 3rd data collection for PCA: 45 times
- 4th data collection for MSA: 30 times
- 5th data collection for DOE: 32 runs
- 6th data collection for Improve validation: 27 times

=50(6+90+45+30+27)
+ 500(32)
+ (32+1)
= ฿25,933

Equipment:

- Clamp Stand: ฿342.33
- Cylindrical mass set with hook: ฿250

= ฿592.33

Wage:

- Feb 8 days
- Mar 8 days
- April 10 days

=26 days * 300 baht/day*man * 5 mans
= ฿39,000

Assumption:

- Working days of project for each week are Friday and Saturday, starting from 1st Feb to 1st May 2022.

Given:

Operating Expenses(E): ฿3,953

Operating Cost: ฿3,600

- Wage: 1 day * 300baht/day*man * 3 mans = ฿900 per 3 months * 12 months per year = ฿3,600 per year

Assumption:

- Catapult competition contest can be participated in every quarter of each year as the final assessment of project including final testing of launching the ball via the catapult with improved adjustment of settings takes duration for one day.

Maintenance Cost: ฿353

- Replacement parts(rubber band + tape + charcoal powder) = (฿300+฿28+฿25)per year = ฿353 per year

Benefits(B): ฿55,663.24

Reduction in Fine: ฿ 55,663.24

Total Fine $_{\text{before improvement}}$ - Total Fine $_{\text{after improvement}}$

= (฿16,954.85 - ฿3,039.04)per 3 months * 12 months per year

= ฿ 55,663.24 per year

Lifetime of Investment(N): **2 years** (∵Consider from warranty period of catapult machine)

Given:

Capital Investment(I):	65,525.33	THB
Lifetime of Investment(N):	2	years
Operating Expenses(E):	3,953	THB/year
Benefits(B):	55,663.24	THB/year

Solution:

Benefit to Cost Ratio (B/C Ratio)

$$\frac{B}{C} = \frac{B}{(\frac{I}{N} + E)} = \frac{55{,}663.24}{(\frac{65{,}525.33}{2} + 3{,}953)} = 1.52 > 1$$

Justify:

In this improvement project, the benefit was obtained greater than the total of capital investment and operating expense at 1.52 times when being considered annually.

Given:

Capital Investment(I):	65,525.33	THB
Lifetime of Investment(N):	2	years
Operating Expenses(E):	3,953	THB/year
Benefits(B):	55,663.24	THB/year

Solution:

Net Benefit Value (NBV)

$$NBV = B - \frac{I}{N} - E = 55{,}663.24 - \frac{65{,}525.33}{2} - 3{,}953 = 18947.58\ THB/year$$ **> 0**

Justify:

In this improvement project, the benefit was obtained greater than the total of investment cost and expense along the lifetime of investment by 18,947.58 THB/year.

Given:

Capital Investment(I):	65,525.33	THB
Lifetime of Investment(N):	2	years
Operating Expenses(E):	3,953	THB/year
Benefits(B):	55,663.24	THB/year

Solution:

Payback Period

$$Payback\ period = \frac{I}{(B-E)} = \frac{65{,}525.33}{(55{,}663.24 - 3{,}953)} = 1.27\ years$$ **< (N = 2 years)**

Justify:

The investment of this improvement project can be paid back around 3rd to 4th month after the first year that the project has been initiated which is before the lifetime period of capital investment at second year after initiating the project.

Worthiness assessment from ROQI of this project

Index	Calculated index from this project		Target
Benefit to Cost Ratio	1.52	>	1
Net Benefit Value	18947.58	>	0
Payback Period	1.27	<	2

Justify: **This improvement project is worthwhile for investment.**