基礎数理演習課題 8

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次の値を求めて下さい。

(1) $\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

$\sin^{-1} -\dfrac{\sqrt{3}}{2} = -\dfrac{\pi}{3} \left(\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right)$

(2) $\cos^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$

$\cos^{-1}\left(-\dfrac{\sqrt{2}}{2}\right) = \dfrac{3\pi}{4} \left(\in [0, \pi]\right)$

(5) $\tan^{-1}\left(-\sqrt{3}\right)$

$\tan^{-1}\left(-\sqrt{3}\right) = -\dfrac{\pi}{3} \left(\in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right)$

1

次の関数の導関数を求めて下さい。

(1) $f(x) = x^3 + x^2 - x - 1$

$f'(x) = 3x^2 + 2x - 1$

(2) $f(x) = x^{-2} + x^{\frac{1}{2}}$

$f'(x) = -2x^{-3} + \dfrac{x^{\left(-\frac{1}{2}\right)}}{2}$

(3) $f(x) = \dfrac{1}{x} - \sqrt[4]{x}$

$f'(x) = -\dfrac{1}{x^2} - \dfrac{1}{4x^{\frac{3}{4}}}$

(4) $f(x) = 3^x + log_3 x$

$f'(x) = \dfrac{1}{xlog3} + 3^x log3$

(5)

$f(x) = \sqrt{x}\cos x$

$f'(x) = \dfrac{1}{2\sqrt{x}} \cdot \cos x + \sqrt{x} \cdot (-\sin x)$

$= \dfrac{\cos x - 2x \sin x}{2\sqrt{x}}$

(6)

$f(x) = x \sin^{-1} x$

$f'(x) = \sin^{-1} x + x \cdot \dfrac{1}{\sqrt{1 - x^2}}$

$= \sin^{-1} x + \dfrac{x}{\sqrt{1 - x^2}}$

(7)

$$
\begin{aligned}
f(x) &= \frac{-x}{x^2-2x+1} \\
f'(x) &= \frac{-(x^2-2x+1)-(-x)\cdot(2x-2)}{(x^2-2x+1)^2} \\
&= \frac{x^2-1}{(x-1)^4} \\
&= \frac{x+1}{(x-1)^3}
\end{aligned}
$$

(8)

$$
\begin{aligned}
f(x) &= \frac{x}{logx} \\
f'(x) &= \frac{logx - x\cdot\frac{1}{x}}{log^2x} \\
&= \frac{logx-1}{log^2x}
\end{aligned}
$$

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次の関数の導関数を求めて下さい。

(1)

$$
\begin{aligned}
f(x) &= (logx)^2 \\
f'(x) &= 2logx\cdot\frac{1}{x} \\
&= \frac{2logx}{x}
\end{aligned}
$$

(2)

$$
\begin{aligned}
f(x) &= e^{\tan^{-1}x} \\
f'(x) &= e^{\tan^{-1}x}\cdot\frac{1}{x^2+1} \\
&= \frac{e^{\tan^{-1}x}}{x^2+1}
\end{aligned}
$$

(3)

$$
\begin{aligned}
f(x) &= \cos^{-1}2x \\
f'(x) &= -\frac{1}{\sqrt{1-(2x)^2}}\cdot 2 \\
&= -\frac{2}{\sqrt{1-4x^2}}
\end{aligned}
$$

(4) $f(x) = x^{\sin x} \quad (x>0)$

$y = x^{\sin x}$ と置く。

両辺の自然対数を取ると、

$$
\begin{aligned}
logy &= logx^{\sin x} \\
&= \sin x\cdot logx
\end{aligned}
$$

両辺を x で微分すると、

$$
\begin{aligned}
\frac{y'}{y} &= \sin x\cdot\tfrac{1}{x} + \cos x\cdot logx \\
&= \frac{\sin x}{x} + \cos x logx \\
y' &= y\cdot\left\{\frac{\sin x}{x} + \cos x logx\right\} \\
&= x^{\sin x}\left\{\frac{\sin x}{x} + \cos x logx\right\}
\end{aligned}
$$