# 基礎数理演習課題 13

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## 1

次の関数を微分して下さい。

(1) $f(x) = x^2 + 2^x + log_2 x$

$$f(x) = x^2 + 2^x + \frac{logx}{log2}$$
$$f'(x) = 2x + 2^x log2 + \frac{1}{xlog2}$$

(2) $f(x) = \tan x \sin^{-1} x$

$$\begin{aligned} f'(x) &= (\sin^{-1} x)' \cdot \tan x + \sin^{-1} x \cdot (\tan x)' \\ &= \frac{1}{\sqrt{1-x^2}} \cdot \tan x + \sin^{-1} x \cdot \frac{1}{\cos^2 x} \\ &= \frac{\tan x}{\sqrt{1-x^2}} + \frac{\sin^{-1} x}{\cos^2 x} \end{aligned}$$

(3) $f(x) = \dfrac{1-\cos x}{x^2}$

$$f(x) = \sin x \cdot \frac{1}{x^2} + (1-\cos x) \cdot \left(-\frac{2}{x^3}\right)$$
$$f'(x) = \frac{x \sin x + 2\cos x - 2}{x^3}$$

(4) $f(x) = \sqrt{2e^x + 1}$

$$\begin{aligned} f'(x) &= \frac{(2e^x+1)'}{2\sqrt{2e^x+1}} \\ &= \frac{e^x}{\sqrt{2e^x+1}} \end{aligned}$$

(5) $f(x) = \dfrac{1-\cos x}{x^2}$

$$\begin{aligned} f(x) &= -\frac{(x^2)'}{\sqrt{1-x^4}} \\ &= -\frac{2x}{\sqrt{1-x^4}} \end{aligned}$$

(6) $f(x) = e^{2x} \sin 3x$

$$\begin{aligned} f'(x) &= \left(e^{2x}\right)' \cdot \sin 3x + e^{2x} \cdot (\sin 3x)' \\ &= e^{2x} (2\sin 3x + 3\cos 3x) \end{aligned}$$

(7) $f(x) = x^{\cos x} \quad (x > 0)$

両辺の自然対数をとる

$logf(x) = logx^{\cos x}$

$logf(x) = \cos x logx$

両辺を微分する

$$\frac{f'(x)}{f(x)} = -\sin x logx + \frac{\cos x}{x}$$

$$f'(x) = x^{\cos x}\left(-\sin x logx + \frac{\cos x}{x}\right)$$
$$= x^{\cos x - 1}(-x\sin x logx + \cos x)$$

## 2

次の値を求めて下さい。

(1) $\sin^{-1}\frac{\sqrt{3}}{2}$

$$\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

(2) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

(3) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$

## 3

次の不定積分を求めて下さい。(積分定数として $C, C_1, C_2, \cdots$ を断らずに用いてよい)

(1)

$$\int \sqrt{x}\left(x - 1 + \frac{1}{x}\right)\, dx \cdots (*)$$

$u = \sqrt{x}$とすると

$$(*) = 2\int (u^4 - u^2 + 1)\, du$$
$$= \frac{2u^5}{5} - \frac{2u^3}{3} + 2u + C$$
$$= \frac{2x^2\sqrt{x}}{5} - \frac{2x\sqrt{x}}{3} + 2\sqrt{x} + C$$
$$= \frac{2}{15}\sqrt{x}(3x^2 - 5x + 15) + C$$

(2)

$$\int \frac{1}{x} logx\, dx \cdots (*)$$

$u = logx$ とすると

$$(*) = \int udu$$
$$= \frac{u^2}{2} + C_1$$
$$= \frac{log^2 x}{2} + C_1$$

(3)

$$\int x^2 \sin x \ dx$$

$$= -x^2 \cos x + 2\int x \cos x \ dx + C_2$$

$$= -x^2 \cos x + 2x \sin x - 2\int \sin x \ dx + C_2$$

$$= -x^2 \cos x + 2x \sin x + 2\cos x + C_2$$

$$= 2x \sin x + (2 - x^2)\cos x + C_2$$

(4)

$$\int \frac{e^x}{\sqrt{4 - e^{2x}}} \, dx \cdots (*)$$

$u = e^x$とすると

$$(*) = \int \frac{1}{\sqrt{4 - u^2}} \, du$$

$$= \int \frac{1}{2\sqrt{1 - \dfrac{u^2}{4}}} \, du$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{1 - \dfrac{u^2}{4}}} \, du \cdots (**)$$

$s = \dfrac{u}{2}$とすると

$$(**) = \int \frac{1}{\sqrt{1 - s^2}} \, ds$$

$$=$$

(5)

$$\int \tan^{-1} x \ dx$$

$$= x \tan^{-1} x - \int \frac{x}{x^2 + 1} \, dx \cdots (*)$$

$u = x^2 + 1$ とすると

$$(*) = x \tan^{-1} x - \frac{1}{2}\int \frac{1}{u} \, du$$

$$= x \tan^{-1} x - \frac{logu}{2} + C_4$$

$$= x \tan^{-1} x - \frac{log(x^2 + 1)}{2} + C_4$$

## 1

次の不定積分を求めて下さい。(積分定数として $C, C_1, C_2, \cdots$ を断らずに用いてよい)

(1)

$$\int \frac{x^3+2x^2+4x+6}{x^2+4}\,dx$$
$$= \int \left(-\frac{2}{x^2+4}+x+2\right)\,dx$$
$$= -2\int \frac{1}{x^2+4}\,dx + \int x\,dx + 2\int 1\,dx$$
$$= -\tan^{-1}\frac{x}{2} + \frac{x^2}{2} + 2x + C$$
$$= \frac{x}{2}(x+4) - \tan^{-1}\frac{x}{2} + C$$

(2)

$$\int \frac{x^2+2x+1}{x^2-2x+2}\,dx$$
$$= \int \left(\frac{4x-1}{x^2-2x+2}+1\right)\,dx$$
$$= \int \frac{4x-1}{x^2-2x+2}\,dx + \int 1\,dx$$
$$= \int \left(\frac{2(2x-2)}{x^2-2x+2}+\frac{3}{x^2-2x+2}\right)\,dx + x + C_1$$
$$= 2\int \frac{2x-2}{x^2-2x+2}\,dx + 3\int \frac{1}{x^2-2x+2}\,dx + x + C_1 \cdots ($$

$u = x^2-2x+2$ とすると

$$(*) = 2logu + 3\int \frac{1}{(x-1)^2+1}\,dx + x + C_1 \cdots (**)$$

$t = x-1$ とすると

$$(**) = 2log(x^2-2x+2) + 3\int \frac{1}{t^2+1}\,dt + x + C_1$$
$$= 2log(x^2-2x+2) - 3\tan^{-1} t + x + C_1$$
$$= 2log(x^2-2x+2) - 3\tan^{-1}(1-x) + x + C_1$$

(3)

$$\int \frac{2}{(x-1)(x+1)^2}\,dx$$
$$= 2\int \left(-\frac{1}{4(x+1)}-\frac{1}{2(x+1)^2}+\frac{1}{4(x-1)}\right)\,dx$$
$$= -\frac{1}{2}\int \frac{1}{x+1}\,dx - \int \frac{1}{(x+1)^2}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx \cdots (*)$$

$u = x+1, t = x-1$ とすると

$$(*) = -\frac{1}{2}\int \frac{1}{u}\,du - \int \frac{1}{u^2}\,du + \frac{1}{2}\int \frac{1}{t}\,dt$$
$$= -\frac{logu}{2} + \frac{1}{u} + \frac{logt}{2} + C_2$$
$$= -\frac{log(x+1)}{2} + \frac{1}{x+1} + \frac{log(x-1)}{2} + C_2$$
$$= \frac{1}{2}\left(\frac{2}{x+1} + log(x-1) - log(x+1)\right) + C_2$$

(4)

$$\int \frac{1}{2x^2-x}\,dx$$

$$= \int \frac{1}{\left(\sqrt{2}x-\frac{1}{2\sqrt{2}}\right)^2-\frac{1}{8}}\,dx \cdots (*)$$

$u=\sqrt{2}x-\dfrac{1}{2\sqrt{2}}$とすると

$$(*)=\frac{1}{\sqrt{2}}\int \frac{1}{u^2-\frac{1}{8}}\,du$$

$$=\frac{1}{\sqrt{2}}\int \frac{8}{8u^2-1}\,du$$

$$=4\sqrt{2}\int \frac{1}{8u^2-1}\,du$$

$$=-4\sqrt{2}\int \frac{1}{1-8u^2}\,du \cdots (**)$$

$t=2\sqrt{2}u$ とすると

$$(**)=-2\int \frac{1}{1-t^2}\,dt$$

$$=-2\tanh^{-1}t+C_3$$

$$=log(1-t)-log(1+t)+C_3$$

$$=log\left(1-2\sqrt{2}u\right)-log\left(1+2\sqrt{2}u\right)+C_3$$

$$=log\left(1-2\sqrt{2}\left(\sqrt{2}x-\frac{1}{2\sqrt{2}}\right)\right)-log\left(1+2\sqrt{2}\left(\sqrt{2}x-\frac{1}{2\sqrt{2}}\right)\right)+C_3$$

$$=log(2-4x)-log4x+C_3$$

$$=log(1-2x)-log2x+C_3$$

(5)

$$\int \frac{4}{(x-1)(x^2+1)}\,dx$$

$$= 4\int \left(\frac{-x-1}{2(x^2+1)} + \frac{1}{2(x-1)}\right)\,dx$$

$$= 2\int \frac{-x-1}{x^2+1}\,dx + 2\int \frac{1}{x-1}\,dx$$

$$= 2\int \left(-\frac{x}{x^2+1} - \frac{1}{x^2+1}\right)\,dx + 2log(x-1) + C_4$$

$$= -2\int \frac{x}{x^2+1}\,dx - 2\int \frac{1}{x^2+1}\,dx + 2log(x-1) + C_4 \cdots (*)$$

$u = x^2 + 1$ とすると

$$(*) = -\int \frac{1}{u}\,du - 2\tan^{-1}x + 2log(x-1) + C_4$$

$$= -logu - 2\tan^{-1}x + 2log(x-1) + C_4$$

$$= -log(x^2+1) - 2\tan^{-1}x + 2log(x-1) + C_4$$