

基礎数理演習課題 11

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1

次の不定積分を求めて下さい。(積分定数として C, C_1, C_2, \dots を断らずに用いてよい)

(1)

$$\begin{aligned}\int \left(\frac{e}{x} - \frac{x}{\sqrt{1-x^2}} \right) dx \\ = e \log x + \sqrt{1-x^2} + C\end{aligned}$$

(2)

$$\begin{aligned}\int \left(x^3 + \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \right) dx \\ = \frac{1}{4} \left(3x^{\frac{4}{3}} + 6x^{\frac{2}{3}} + x^4 \right) + C_1\end{aligned}$$

(3)

$$\begin{aligned}\int \frac{1}{4+x^2} dx \\ = \frac{1}{2} \tan^{-1} \frac{x}{2} + C_2\end{aligned}$$

(4)

$$\begin{aligned}\int \frac{2}{\sqrt{2-x^2}} dx \\ = 2 \sin^{-1} \frac{x}{\sqrt{2}} + C_3\end{aligned}$$

(5)

$$\begin{aligned}\int x \sin x dx \\ = \sin x - x \cos x + C_4\end{aligned}$$

(6)

$$\begin{aligned}\int \sin 4x dx \\ = -\frac{1}{4} \cos 4x + C_5\end{aligned}$$

(7)

$$\begin{aligned}\int x \log x dx \\ = \frac{1}{4} x^2 (2 \log x - 1) + C_6\end{aligned}$$

(8)

$$\begin{aligned}\int x \cos x^2 dx \\ = \frac{\sin x^2}{2} + C_7\end{aligned}$$

(9)

$$\begin{aligned}\int x^2 e^x dx \\ = e^x (x^2 - 2x + 2) + C_8\end{aligned}$$

(10)

$$\begin{aligned}\int x^2 \sqrt{x^3+1} dx \\ = \frac{2}{9} (x^3+1)^{\frac{3}{2}} + C_9\end{aligned}$$

2

次の定積分を求めて下さい。

(1)

$$\begin{aligned}\int_1^3 x^2 \log x dx &= \left[\frac{1}{3} x^3 \log x - \frac{x^3}{9} \right]_1^3 \\ &= (9 \log 3 - 3) - \left(-\frac{1}{9} \right) \\ &= 9 \log 3 - \frac{26}{9}\end{aligned}$$

(2)

$$\begin{aligned}\int_2^3 x e^{x^2} dx &= \left[\frac{e^{x^2}}{2} \right]_2^3 \\ &= \frac{e^9}{2} - \frac{e^4}{2} \\ &= \frac{1}{2} e^4 (e^5 - 1)\end{aligned}$$

3

$x = 0$ と $x = 3$ の間でグラフ $y = x^3 - 4x$ と x 軸 に挟まれた領域の (通常の) 面積を求めて下さい。

$$\begin{aligned}\int x^3 - 4x dx &= \frac{x^4}{4} - 2x^2 + C \\ \frac{x^4}{4} - 2x^2 = 0 &\Leftrightarrow x = 0, 2\sqrt{2}, -2\sqrt{2} \\ S = \int_0^3 x^3 - 4x dx &= - \int_0^{2\sqrt{2}} x^3 - 4x dx + \int_{2\sqrt{2}}^3 x^3 - 4x dx \\ &= - \left[\frac{x^4}{4} - 2x^2 \right]_0^{2\sqrt{2}} + \left[\frac{x^4}{4} - 2x^2 \right]_{2\sqrt{2}}^3 \\ &= -(16 - 16) + \left\{ \left(\frac{81}{4} - 18 \right) - 0 \right\} \\ &= \frac{9}{4}\end{aligned}$$