基礎数理演習課題 12

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次の不定積分を求めて下さい。(積分定数として $C, C_1, C_2, \cdots$ を断らずに用いてよい)

(1)

$$\int x^2 \cos x \ dx$$

$$= x^2 \sin x - 2\int x \sin x \ dx$$
$$= x^2 \sin x - 2\sin x + 2x\cos x$$
$$= (x^2-2)\sin x + 2x\cos x + C$$

(2)

$$\int (logx)^2 \ \ dx$$

$$= xlog^2x - 2\int logx \ dx$$
$$= xlog^2x - 2\left(xlogx - x\right)$$
$$= x\left(log^2x - 2logx + 2\right) + C_1$$

(3)

$$\int \frac{x}{\sqrt{1-x^2}}\,dx \cdots (*)$$

$u = 1 - x^2$とすると

$$(*) = -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$$
$$= -\sqrt{u} + C_2$$
$$= -\sqrt{1-x^2} + C_2$$

(4)

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx \cdots (*)$$

$u = \sin^{-1} x$ とすると

$$(*) = \int u \ du$$
$$= \frac{u^2}{2} + C_3$$
$$= \frac{\sin^{-1} x^2}{2} + C_3$$

(5)

$$\int \tan^{-1} x \ dx$$

$$= x\tan^{-1} x - \int \frac{x}{x^2+1}\,dx \cdots (*)$$

$u = x^2 + 1$ とすると

$$(*) = x\tan^{-1} x - \frac{1}{2}\int \frac{1}{u}\,du$$
$$= x\tan^{-1} x - \frac{logu}{2} + C_4$$
$$= x\tan^{-1} x - \frac{log(x^2+1)}{2} + C_4$$

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次の不定積分を求めて下さい。(積分定数として $C, C_1, C_2, \cdots$ を断らずに用いてよい)

(1)

$$\int \frac{x^3+2x^2+4x+6}{x^2+4}\,dx$$

$$= \int \left(-\frac{2}{x^2+4}+x+2\right)\,dx$$

$$= -2\int \frac{1}{x^2+4}\,dx + \int x\,dx + 2\int 1\,dx$$

$$= -\tan^{-1}\frac{x}{2}+\frac{x^2}{2}+2x+C$$

$$= \frac{x}{2}(x+4)-\tan^{-1}\frac{x}{2}+C$$

(2)

$$\int \frac{x^2+2x+1}{x^2-2x+2}\,dx$$

$$= \int \left(\frac{4x-1}{x^2-2x+2}+1\right)\,dx$$

$$= \int \frac{4x-1}{x^2-2x+2}\,dx + \int 1\,dx$$

$$= \int \left(\frac{2(2x-2)}{x^2-2x+2}+\frac{3}{x^2-2x+2}\right)\,dx + x + C_1$$

$$= 2\int \frac{2x-2}{x^2-2x+2}\,dx + 3\int \frac{1}{x^2-2x+2}\,dx + x + C_1 \cdots ($$

$u = x^2-2x+2$ とすると

$$(*) = 2logu + 3\int \frac{1}{(x-1)^2+1}\,dx + x + C_1 \cdots (**)$$

$t = x-1$ とすると

$$(**) = 2log(x^2-2x+2) + 3\int \frac{1}{t^2+1}\,dt + x + C_1$$

$$= 2log(x^2-2x+2) - 3\tan^{-1} t + x + C_1$$

$$= 2log(x^2-2x+2) - 3\tan^{-1}(1-x) + x + C_1$$

(3)

$$\int \frac{2}{(x-1)(x+1)^2}\,dx$$

$$= 2\int \left(-\frac{1}{4(x+1)}-\frac{1}{2(x+1)^2}+\frac{1}{4(x-1)}\right)\,dx$$

$$= -\frac{1}{2}\int \frac{1}{x+1}\,dx - \int \frac{1}{(x+1)^2}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx \cdots (*)$$

$u = x+1, t = x-1$ とすると

$$(*) = -\frac{1}{2}\int \frac{1}{u}\,du - \int \frac{1}{u^2}\,du + \frac{1}{2}\int \frac{1}{t}\,dt$$

$$= -\frac{logu}{2} + \frac{1}{u} + \frac{logt}{2} + C_2$$

$$= -\frac{log(x+1)}{2} + \frac{1}{x+1} + \frac{log(x-1)}{2} + C_2$$

$$= \frac{1}{2}\left(\frac{2}{x+1} + log(x-1) - log(x+1)\right) + C_2$$

(4)

$$\int \frac{1}{2x^2 - x}\, dx$$

$$= \int \frac{1}{\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)^2 - \frac{1}{8}}\, dx \cdots (*)$$

$u = \sqrt{2}x - \dfrac{1}{2\sqrt{2}}$とすると

$$(*) = \frac{1}{\sqrt{2}} \int \frac{1}{u^2 - \frac{1}{8}}\, du$$

$$= \frac{1}{\sqrt{2}} \int \frac{8}{8u^2 - 1}\, du$$

$$= 4\sqrt{2} \int \frac{1}{8u^2 - 1}\, du$$

$$= -4\sqrt{2} \int \frac{1}{1 - 8u^2}\, du \cdots (**)$$

$t = 2\sqrt{2}u$ とすると

$$(**) = -2 \int \frac{1}{1 - t^2}\, dt$$

$$= -2 \tanh^{-1} t + C_3$$

$$= log(1-t) - log(1+t) + C_3$$

$$= log\left(1 - 2\sqrt{2}u\right) - log\left(1 + 2\sqrt{2}u\right) + C_3$$

$$= log\left(1 - 2\sqrt{2}\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)\right) - log\left(1 + 2\sqrt{2}\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)\right) + C_3$$

$$= log(2 - 4x) - log 4x + C_3$$

$$= log(1 - 2x) - log 2x + C_3$$

(5)

$$
\begin{aligned}
&\int \frac{4}{(x-1)(x^2+1)}\,dx \\
&\qquad = 4\int \left(\frac{-x-1}{2(x^2+1)} + \frac{1}{2(x-1)} \right)\,dx \\
&\qquad = 2\int \frac{-x-1}{x^2+1}\,dx + 2\int \frac{1}{x-1}\,dx \\
&\qquad = 2\int \left(-\frac{x}{x^2+1} - \frac{1}{x^2+1} \right)\,dx + 2log(x-1) + C_4 \\
&\qquad = -2\int \frac{x}{x^2+1}\,dx - 2\int \frac{1}{x^2+1}\,dx + 2log(x-1) + C_4 \cdots (*)
\end{aligned}
$$

$u = x^2 + 1$ とすると

$$
\begin{aligned}
(*) &= -\int \frac{1}{u}\,du - 2\tan^{-1} x + 2log(x-1) + C_4 \\
&= -logu - 2\tan^{-1} x + 2log(x-1) + C_4 \\
&= -log(x^2+1) - 2\tan^{-1} x + 2log(x-1) + C_4
\end{aligned}
$$