

$$D_\phi \left( K_a \frac{\partial T}{\partial \phi} \right) \mapsto D \frac{\partial}{\partial x} \left( \overbrace{(1-x^2)}^{= \cos^2 \phi} \frac{\partial T}{\partial x} \right)$$

We have  $x = \sin \phi$ , so  $\frac{\partial T}{\partial x} = \frac{\partial T}{\partial \phi} \frac{d\phi}{dx} = \frac{1}{\cos \phi} \frac{\partial T}{\partial \phi}$

$$\frac{d\phi}{dx} = \cos \phi$$

$$\begin{aligned}
 & \text{Starting from cylindrical form:} \\
 & \int_0^{2\pi} \int_0^R \int_0^h K \frac{\partial T}{\partial \phi} d\phi dz dr \\
 & = \int_0^{2\pi} \int_0^R \int_0^h K \frac{\partial T}{\partial \phi} r \cos(\phi) r \sin(\phi) dr d\phi dz \\
 & = \int_0^{2\pi} \int_0^R \int_0^h K r^2 \cos(\phi) \sin(\phi) dr d\phi dz \\
 & = \frac{1}{2} \int_0^{2\pi} \int_0^R K r^2 \cos^2(\phi) dr d\phi = \frac{1}{2} \cos^2 \phi \int_0^R K r^2 dr
 \end{aligned}$$

Heat eqn on a sphere

$$\frac{\partial}{\partial t} (cST) = \nabla \cdot (k \nabla T) + Q$$

This is the general form on diff form.

We assume  $T$  only depends on  $\phi$  and consider the thin layer at the surface  $R$ . i.e. we want to solve for  $T = T(\phi)$ .

Goal: Express  $\nabla \cdot (k \nabla T) = D_\phi (K_a \frac{\partial T}{\partial \phi})$  for some differential operator  $D_\phi$ . Want to use a control volume approach.

**PROBLEM**

Volume energy equation

$$\begin{aligned}
 & \int_V \left[ K(\phi_0 + \Delta\phi) \frac{\partial T}{\partial \phi} (\phi_0 + \Delta\phi) - T(\phi_0 - \Delta\phi) K(\phi_0 - \Delta\phi) \right] \frac{\partial}{\partial \phi} (\frac{1}{2} R^2 \cos^2 \phi) dV \\
 & = \int_V \Delta\phi R^2 dV
 \end{aligned}$$

Surface source

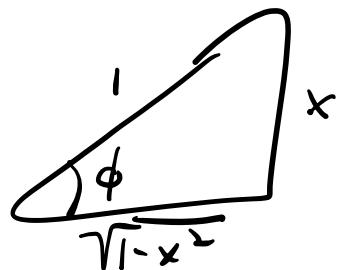
$$\begin{aligned}
 & \int_S \int_Q dA = \int_S \int_{Q \cap V} dA \\
 & Q \text{ is the thing st given into domain to source into } V \\
 & \int_S \int_Q dA = \int_{Q \cap V} dV
 \end{aligned}$$

**KLAUD**

$$\frac{1}{r^2 \cos \phi} \frac{d}{d\phi} \left( k \cos(\phi) \frac{dT}{d\phi} \right)$$

$$\frac{1}{r^2 \cos \phi} \frac{d}{d\phi} \left( k \cos \phi \frac{dT}{d\phi} \right)$$

Bytte  $\sin \phi = x$ .  $\frac{dx}{d\phi} = \cos \phi$



$$\frac{d}{d\phi} = \frac{dx}{d\phi} \frac{d}{dx} = \cos \phi \frac{d}{dx}$$

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$$\frac{1}{r^2} \frac{d}{dx} \left( k \cos^2 \phi \frac{dT}{dx} \right)$$

$$= \frac{1}{r^2} \frac{d}{dx} \left( k (1-x^2) \frac{dT}{dx} \right)$$