2. Zonal model and the role of the polar regions

In this part, we want to investigate the role of the polar regions. Polar regions are covered by ice so that they have a high capacity to reflect solar radiations. For this reason, they play an essential role in the energy balance of the Earth. The ice appears in these regions because of the low temperature. In the context of global warming, we face an amplification feedback mechanism: If the temperature increases, ice disappears; then, less radiation is reflected so the temperature increases more and this induces the disappearance of more ice. Let us set up a model that attempts to determine the extend of the ice sheet by computing the temperature distribution with respect to the latitude, denoted ϕ.

In general, the heat equation is given by

$$\frac{\partial}{\partial t}(c\rho T) = \nabla \cdot (k \nabla T) + q \tag{4}$$

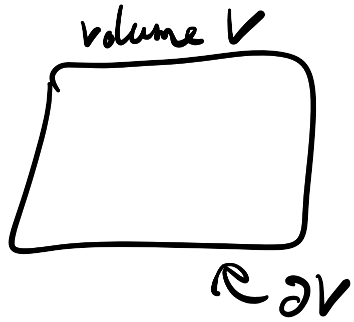
where c is the specific heat capacity, ρ the density, k the thermal conductivity and q a heat source.

Question 5: ***Derive this equation as the expression of the conservation of energy.***

Let

- $u(\vec{r},t)$ be heat energy density (i.e. $E(t) = \iiint_V u \, dV$) $\quad [u] = J/m^3$
- $Q(\vec{r})$ heat source $[Q] = J/m^3$
- $\vec{J}(\vec{r},t)$ heat flux density $\quad [\vec{J}] = J/m^2$

Universal conservation law:



$$\frac{d}{dt}\iiint_V \rho \, dV + \oint\!\!\oint_{\partial V} \vec{J}\cdot\vec{n}\, ds = \iiint_V Q \, dV$$

Denote

- c = specific heat capacity $\quad [c] = J/kg\,K$
- ρ = density $\quad [\rho] = kg/m^3$

Then

$$u = c\rho T$$

By Fourier's law (empirical...)

$$\vec{J} = -k\,\vec{\nabla} T$$

We obtain: $\vec{J}\cdot\vec{n}$

dom conv thm...

$$\frac{d}{dt}\iiint_V c\rho T\,dV = \oint_{\partial V} k\vec{\nabla}T\cdot\vec{n}\,dS + \iiint_V Q\,dV$$

If k cont. diffeable, div theorem gives

$$\oint_{\partial V} k\vec{\nabla}T\cdot\vec{n}\,dS = \iiint_V \vec{\nabla}\cdot(k\vec{\nabla}T)\,dV$$

This must hold for any volume V, hence

$$\frac{d}{dt}(c\rho T) = \vec{\nabla}\cdot(k\vec{\nabla}T) + Q$$