

$$\begin{array}{c}
 1. \quad \frac{\frac{\text{Int } 2 \quad (\tau\text{-Int})}{\Gamma[x \triangleq \text{Int}] \vdash 1 : \text{Int}} \quad \frac{\text{Ident } x \quad \Gamma[x \triangleq \text{Int}](x) = \text{Int} \quad (\tau\text{-Ident})}{\Gamma[x \triangleq \text{Int}] \vdash x : \text{Int}} \quad (\tau\text{-Plus})}{\Gamma[x \triangleq \text{Ident}] \vdash 2 + x : \text{Int}} \quad (\tau\text{-Abs}) \\
 \Gamma \vdash \text{lambda } x : \text{Int}. 2 + x : \text{Int} \Rightarrow \text{Int}
 \end{array}$$

$$\begin{array}{c}
 2. \quad \frac{\frac{\text{Ident } x \quad \Gamma[\$ \triangleq \text{Int} \Rightarrow \text{Int}](\$) = \text{Int} \quad (\tau\text{-Ident})}{\Gamma[\$ \triangleq \text{Int} \Rightarrow \text{Int}](x \triangleq \text{Int}) \vdash x : \text{Int}} \quad (\tau\text{-Abs}) \quad \frac{\text{Ident } \$ \quad \Gamma[\$ \triangleq \text{Int} \Rightarrow \text{Int}](\$) = \text{Int} \Rightarrow \text{Int} \quad (\tau\text{-Ident})}{\Gamma[\$ \triangleq \text{Int} \Rightarrow \text{Int}] \vdash \$ : \text{Int} \Rightarrow \text{Int}} \quad \frac{\text{Int } 1 \quad (\tau\text{-Int})}{\Gamma[\$ \triangleq \text{Int} \Rightarrow \text{Int}] \vdash 1 : \text{Int}} \quad (\tau\text{-App})}{\Gamma[\$ \triangleq \text{Int} \Rightarrow \text{Int}] \vdash \text{lambda } x : \text{Int}. x \text{ in app } \$ : \text{Int} \Rightarrow \text{Int}} \quad (\tau\text{-Let}) \\
 \Gamma \vdash \text{let } f : \text{Int} \Rightarrow \text{Int} = \text{lambda } x : \text{Int}. x \text{ in app } \$ : \text{Int}
 \end{array}$$

$$3. \{x_1 = x_2 \Rightarrow x_3, x_2 = x_3 \Rightarrow x_4, x_3 = \text{Int}\}$$

$$[x_1 \mapsto (\text{Int} \Rightarrow x_4) \Rightarrow x_3, x_2 \mapsto \text{Int} \Rightarrow x_4, x_3 \mapsto \text{Int}]$$

$$\begin{array}{c}
 4. \quad \frac{\text{Ident } x \quad x \in \text{dom}(\Gamma[\$ \triangleq X_1][x \triangleq X_2])}{\Gamma[\$ \triangleq X_1][x \triangleq X_2](x) = X_2} \quad (\tau\text{-Ident1}) \quad \frac{\text{Ident } \$ \quad \$ \in \text{dom}(\Gamma[\$ \triangleq X_1])}{\Gamma[\$ \triangleq X_1](\$) = X_1} \quad (\tau\text{-Ident2}) \quad \frac{\text{Int } 1 \quad (\tau\text{-Int})}{\Gamma[\$ \triangleq X_1] \vdash 1 : \text{Int}} \quad (\tau\text{-App})}{\text{fresh } x_2 \quad \Gamma[\$ \triangleq X_1][x \triangleq X_2] \vdash x : X_2 \mid \{ \} \quad (\tau\text{-Abs}) \quad \text{fresh } x_2, x_4 \quad \Gamma[\$ \triangleq X_1] \vdash f : x_1 \mid \{ \} \quad \Gamma[\$ \triangleq X_1] \vdash \text{app } f 1 : x_4 \mid \{ x_1 = x_3 \Rightarrow x_4, \text{Int} = x_3 \} \quad (\tau\text{-Let})}{\Gamma \vdash \text{let } f = \text{lambda } x. x \text{ in app } \$ : x_4 \mid \{ x_1 = x_3 \Rightarrow x_4, \text{Int} = x_3, x_1 = x_2 \Rightarrow x_2 \}}
 \end{array}$$

$$5. [x_1 \mapsto \text{Int} \Rightarrow \text{Int}, x_2 \mapsto \text{Int}, x_3 \mapsto \text{Int}, x_4 \mapsto \text{Int}]$$