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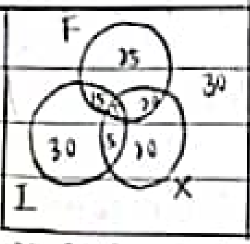
No.: OOE JIE YING A23CS0224 Date:

Assignment 1

1. (a)(i)



ξ
F: The students that using Facebook
I: The students that using Instagram
X: The students that using Twitter.

(ii) $150-25-20-20-15-5-5-30=30$ $\therefore (F \cup I \cup X)' = 30$ students

(iii) $15+20+5=40$ $\therefore (F \cap I \cap X)' \cap ((F \cap I) \cup (F \cap X) \cup (X \cap I)) = 40$ students

(iv) $30+5+20=55$ $\therefore (I \cup X) \cap F' = 55$ students

(b) $A=\{3,5,7,9\}$; $B=\{2,3,5,7\}$; $C=\{3,6,9\}$

(i) $|A|=4$
$|B|=4$
$|C|=3$

(ii) $|P(A)|=2^n=2^4=16$
$\therefore$ Proper subsets of $A=16-1$
$=15$

(iii) $C \times B=\{(3,2),(3,3),(3,5),(3,7),(6,2),(6,3),(6,5),(6,7),$
$(9,2),(9,3),(9,5),(9,7)\}$

2. (a)

p	q	$(p \vee q)$	$\sim(p \vee q)$	$\sim p$	$\sim p \wedge q$	$\sim(p \vee q) \vee (\sim p \wedge q)$
T	T	T	F	F	F	F
T	F	T	F	F	F	F
F	T	T	F	T	T	T
F	F	F	T	T	F	T

$\therefore \sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$ (verified)

$\sim(p \vee q) \vee (\sim p \wedge q)$
$= (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ → De-Morgan's laws
$= \sim p \wedge (\sim q \vee q)$ → Distributive laws
$= \sim p \wedge U$ → Complement laws
$= \sim p$ (shown)

(b)(i) $(r \wedge q) \to p$

(ii) $\neg(r \vee q) \to \neg p$
$\equiv \neg r \wedge \neg q \to \neg p$

(iii) $\neg p \to \neg(r \vee q)$
$\equiv \neg p \to \neg r \wedge \neg q$

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(c) $\forall x\,(x^2+2x-3=0)$

Negation: $\sim(\forall x\,(x^2+2x-3=0)) \equiv \exists x\,\neg(x^2+2x-3=0)$

There is some $x^2+2x-3\neq 0$

When $x=6$, $6^2+2(6)-3=45\ (\neq 0)$ $\therefore$ The resulting proposition is TRUE.

(d) $R(x)$: Students who can speak Russian

$C(x)$: Students who know C++

x: Students at school

(i) $\exists x\,(R(x)\wedge \sim C(x))$

(ii) $\forall x\,(R(x)\vee C(x))$

(iii) $\forall x\,(\sim R(x)\wedge \sim C(x))$

3. (a) For all integers, if a^2-3b is even, then a is even and b is even.

$P(x)$: a^2-3b is even; $Q(x)$: a and b is even

$\forall x\,(P(x)\to Q(x))$

$\forall x\,\neg(P(x)\to Q(x)) \equiv \forall x\,\neg Q(x)\to\neg P(x)$

For all integers, if a or b is odd, then a^2-3b is odd.

Case 1 (a is odd, b is even) let $a=2n+1$, $b=2k$

$a^2-3b=(2n+1)^2-3(2k)$

$=4n^2+4n+1-6k$

$=2(2n^2+2n-3k)+1$ let $m=2n^2+2n-3k$

$=2m+1$ (odd)

Case 2 (a is even, b is odd) let $a=2n$, $b=2n+1$

$a^2-3b=(2n)^2-3(2n+1)$

$=4n^2-6n-3$

$=2(2n^2-3n)-3$ let $m=2n^2-3n$

$=2m-3$ (odd)

Case 3 (a is odd, b is odd) let $a=2n+1$, $b=2n+1$

$a^2-3b=(2n+1)^2-3(2n+1)$

$=4n^2+2n+1-6n-3$

$=4n^2-4n-2$

$=2(2n^2-2n-1)$ let $m=2n^2-2n-1$

$=2m$ (even)

$\therefore$ Since $\neg P(x)$ is false in case 3, thus the statement "$\forall x\,(P(x)\to Q(x))$" is false.

End