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STAT 8010 - Homework 1

1. $n = 30$

a. Hypotheses :-

$H_0 : \mu_1 = \mu_2 = \mu_3$

H_a : Not all means yields are equal

Within-Sample Variance :-

$$S_W^2 = \frac{(6.5)^2 + (7.8)^2 + (7.4)^2}{3} = 52.616$$

Variance of Sample Means :-

$$Var(90.2, 89.3, 85) = \frac{\sum_{i=1}^{t} (\bar{y}_i - \bar{y})^2}{t-1}$$

$$\bar{y} = \frac{\bar{y}_1 + \bar{y}_2 + \bar{y}_3}{3} = \frac{90.2 + 89.3 + 85}{3} = 88.17$$

$$\cancel{Var(90.2} \quad \frac{\sum_{i=1}^{t} (\bar{y}_i - \bar{y})^2}{t-1} = \frac{(90.2 - 88.17)^2 + (89.3 - 88.17)^2 + (85 - 88.17)^2}{2}$$

$$= 7.72335$$

Between-Samples Variance :-

$$S_B^2 = n \times Var(90.2, 89.3, 85)$$

$$= 30 \times 7.72335 = 231.7005$$

Test statistic :-

$$F_{obs} = S_B^2 / S_w^2 = \frac{231.7005}{52.616} = 4.403$$

Rejection Region :- $(F_{obs} > F_{t-1, nt-t, \alpha})$

$$F_{obs} > F_{.05, 2, 87} = 3.10$$

$$F_{obs} > 3.10$$

Decision :-

Reject H_0

Conclusion :-

There is enough evidence at 0.05 significance level to conclude H_a.

b. Assumption 1 :- ~~[illegible]~~ The samples are independent random samples.

The above assumption is satisfied for this problem.

Assumption 2 :- Each sample is selected from a normal population.

The above assumption is also satisfied.

Assumption 3:-

$H_0 : \sigma_1^2 = \sigma_2^2 = \sigma_3^2$

H_a : not all are equal

Test statistic: $F_{max} = \frac{(7.8)^2}{(6.5)^2} = 1.44$

Rejection Region: $F_{max} > F_{3,29,0.05} = 2.40$

$F_{max} \ngtr 2.40$

Decision: Fail to reject H_0

Conclusion: Checked the equal variance condition and the analysis in Part-1 is justified.

Therefore there is no violation for any condition.

2. a. The experimental design is randomized block design.

Statistical model:-

Observations for a Randomized Block Design can be expressed as the sum of three terms.

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$$

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where

μ an overall mean that is an unknown constant

α an effect due to treatment i; α_i is an unknown constant

β_j an effect due to block j; β_j is an unknown constant

ε_{ij} a random error associated with the response on treatment i, block j.

b.

type of music	1	2	3	4	
no music	20	17	24	20	81
hard rock	20	18	23	18	79
classical	24	20	27	22	93
	64	55	74	60	253

$b = 4$

$t = 3$

$$TSS = \sum_{i,j} y_{ij}^2 - \frac{y_{..}^2}{bt}$$

$$= 20^2 + 17^2 + 24^2 + \ldots 22^2 - \frac{253^2}{(4 \times 3)}$$

$$= 5431 - \frac{253^2}{12} = 96.91$$

$$SST = \sum_{i=1}^{t} \frac{y_{i.}^2}{b} - \frac{y_{..}^2}{bt} = \frac{81^2 + 79^2 + 93^2}{4} - \frac{253^2}{4 \times 3}$$

$$= 28.67$$

$$SSB = \sum_{j=1}^{b} \frac{y_{.j}^2}{t} - \frac{y_{..}^2}{bt}$$

$$= \frac{64^2+55^2+74^2+60^2}{3} - \frac{253^2}{12}$$

$$= 64.92$$

$$SSE = TSS - SST - SSB$$

$$= 96.91 - 28.67 - 64.92$$

$$= 3.32$$

Analysis of Variance (ANOVA) for a Randomized Block Design

Source Due to	Sum of Squares (SS)	df	Mean Square (MS)	F
Treatments	28.67	2	14.335	25.92
Blocks	64.92	3	21.64	39.13
Error	3.32	6	0.553	
	96.91	11		

Hypotheses testing for types of music :-

$H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0$

H_a: Not all of the above equal to zero

$F_{obs} = 25.92$

Rejection Region: $F_{obs} > F_{2,6,0.05} = 5.14$

$25.8 > 5.14$

Decision: Reject H_0

Conclusion: There is enough evidence at 0.05 significance level to conclude H_a and hence there is a difference between different types of music (treatments).

Hypotheses testing for different subjects:-

$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$

H_1: Not all of the above are equal to zero

$F_{obs} = 39.13$

Reject Region: $F_{obs} > F_{3,6,0.05} = 4.76$

$39.13 > 4.76$

Decision: Reject H_0

Conclusion: There is enough evidence at 0.05 significance level to conclude H_a and hence there is a difference between different subjects (blocks).

c. Relative efficiency $= \frac{MSE_{CR}}{MSE_{RB}} = \frac{(b-1)MSB + b(t-1)MSE_{RB}}{(bt-1)MSE_{RB}}$

$$= \frac{(4-1)(21.64) + 4(3-1)(0.553)}{((4 \times 3)-1)(0.553)}$$

Relative efficiency $= \underline{\underline{11.39}}$