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STAT 8010 - ~~EXAM~~ TEST 1

100

1.

a. Arranging the observations in ascending order:

1, 1.5, 1.5, 2, 3, 4

Since the number of observations is equal to 6, which is an even number, the sample median is the average of middle two observations.

$\therefore$ Sample median $= \frac{1.5+2}{2} = \frac{3.5}{2} = \underline{\underline{1.75}}$

The sample median is the preferred measure of central tendency as it is a robust statistic and it is not affected or influenced by outliers.

b. The parameter of interest is the average price of 64 oz orange juice bottles, i.e, the population mean. The symbol for the population mean is μ.

c. Confidence coefficient (CC) = 95% = 0.95

$\therefore \alpha = 1 - CC = 1 - 0.95 = 0.05$

Confidence Interval for sample size = 6 < 30,

$$\bar{y} \pm t_{\alpha/2, n-1} \cdot S/\sqrt{n}$$

$$\bar{y} = \frac{1.5 + 3 + 4 + 1.5 + 2 + 1}{6} = 2.16$$

$$S = \sqrt{\frac{(2.16-1.5)^2 + (2.16-3)^2 + (2.16-4)^2 + (2.16-1.5)^2 + (2.16-2)^2 + (2.16-1)^2}{6-1}}$$

$$= 1.12$$

$n = 6$

$$t_{0.05/2, 6-1} = t_{0.025, 5} = 2.571$$

$$\therefore CI : \quad 2.16 \pm 2.571 \times \frac{1.12}{\sqrt{6}}$$

d. Margin of error(E) = $\frac{Z_{\alpha/2} \cdot S}{\sqrt{n}}$

If the confidence coefficient (CC) decreases, then α would increase as $\alpha = 1 - CC$

If α increases, then value of $\alpha/2$ also increases.

As $\alpha/2$ increases, the value $Z_{\alpha/2}$ decreases.

Therefore, when S and n are constant, as confidence coefficient decreases, margin of error also decreases.

2.

a. Y is a discrete random variable since it can assume a countable numbers of values.

b. The numbers of females selected to the delegation can either be 1, 2 (or) 3, i.e, Y can assume the values of 1, 2 (or) 3. Probability distribution for each can be given as below:

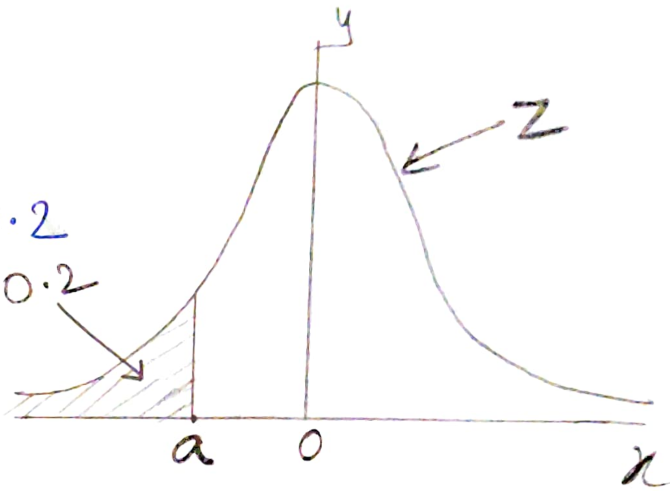
$P(Y=1) = \frac{3}{10}$

$P(Y=2) = \frac{6}{10}$

$P(Y=3) = \frac{1}{10}$

Y	P(y)
1	3/10
2	6/10
3	1/10

3. $P(Z<a) = 0.2$



$P(-\infty<Z<0) - P(a<Z<0) = 0.2$

$0.5 - P(a<Z<0) = 0.2$

$P(a<Z<0) = 0.3$

$\boxed{a = -0.84}$ (from Z-table)

4. $X \sim N(5,16)$

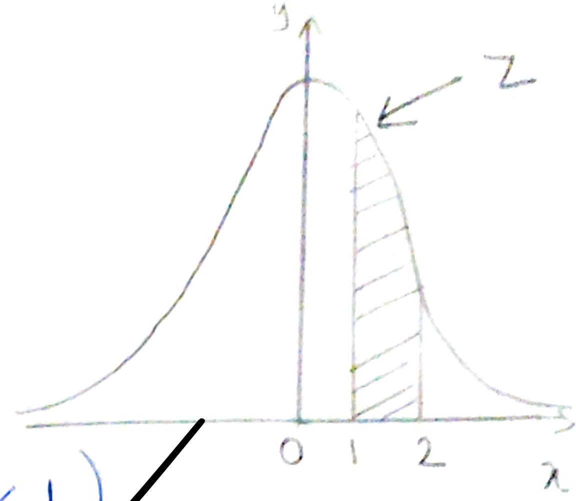
a. $P(9<X<13)$

X follows a general normal distribution with mean 5 and variance 16

$\mu = 5$; $\sigma^2 = 16$

To transform X to standard normal distribution, we apply $Z = \dfrac{X-\mu}{\sigma}$

$\Rightarrow P\left(\frac{9-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{13-\mu}{\sigma}\right)$



$\Rightarrow P\left(\frac{9-5}{4} < Z < \frac{13-5}{4}\right)$

$\Rightarrow P(1 < Z < 2)$

$\Rightarrow P(0 < Z < 2) - P(0 < Z < 1)$

$\Rightarrow .4772 - .3413 = 0.1359$

$\therefore P(9 < X < 13) = 0.1359$

b. $P(x_0 < X < 13) = 0.6328$

X again follows the same distribution

$\mu = 5, \ \sigma^2 = 16$

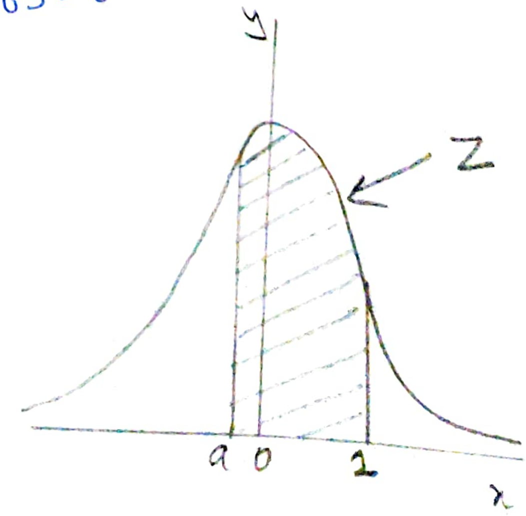
$Z = \frac{X-\mu}{\sigma}$

$P\left(\frac{x_0-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{13-\mu}{\sigma}\right) = 0.6328$

$P\left(\frac{x_0-5}{4} < Z < \frac{13-5}{4}\right) = 0.6328$

$P\left(\frac{x_0-5}{4} < Z < 2\right) = 0.6328$

Let $\frac{x_0-5}{4} = a$



$P(a < Z < 2) = 0.6328$

$P(a < Z < 0) + P(0 < Z < 2) = 0.6328$

$P(a < Z < 0) + 0.4772 = 0.6328$

$P(a < Z < 0) = 0.1556$

$a = -0.4$ (from Z-table)

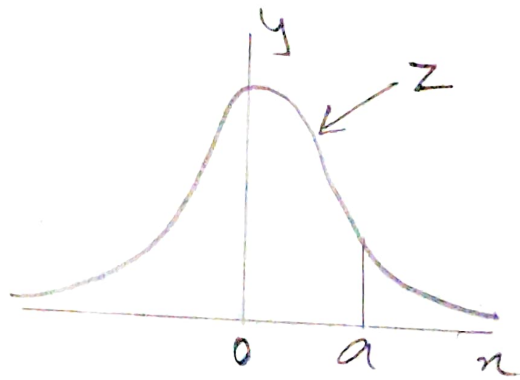
$$\frac{X_0 - 5}{4} = -0.4$$

$X_0 = 5 - (4 \times 0.4)$

$\boxed{X_0 = 3.4}$

5. $\mu = 100$, $\sigma = 14$ $\Big\}$ $X \sim N(100, 14^2)$

a. $P(X > a) = 0.1$



$$P\left(\frac{X-\mu}{\sigma} > \frac{a-\mu}{\sigma}\right) = 0.1$$

$$P\left(Z > \frac{a-100}{14}\right) = 0.1$$

$$P(0 < Z < \infty) - P\left(0 < Z < \frac{a-100}{14}\right) = 0.1$$

$$P\left(0 < Z < \frac{a-100}{14}\right) = 0.5 - 0.1 = 0.4$$

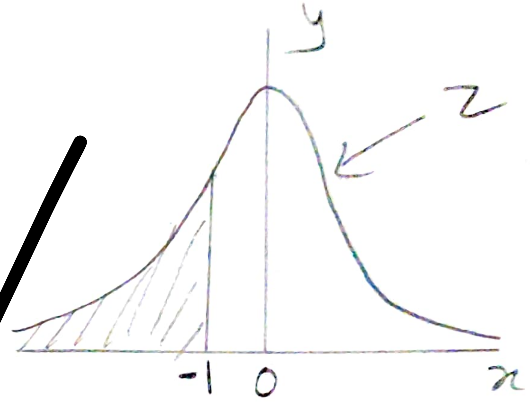
$$\frac{a-100}{14} = 1.28 \quad \text{(from Z-table)}$$

$\therefore a = 100 + (14 \times 1.28)$

$$\boxed{a = 117.92}$$

The lowest IQ score that would qualify a person for membership in Mensa is 117.92.

b. $P(X < 86)$



$P\left(\frac{X-\mu}{\sigma} < \frac{86-100}{14}\right)$

$P(Z < -1)$

$P(-\infty < Z < 0) - P(-1 < Z < 0)$

$0.5 - 0.3413 = 0.1587$

$\therefore P(X < 86) = 0.1587$

c. $n = 49 > 30$

~~Since, we are~~ sampling ~~from a large~~ Since, the sample size is large and also the population is given to be normally distributed, the sampling distribution of the sample mean also follow a normal distribution with mean μ and variance σ^2/n.

$$\bar{Y} \sim N(\mu, \sigma^2/n)$$

$\bar{Y} \sim N(100, 14^2/49)$

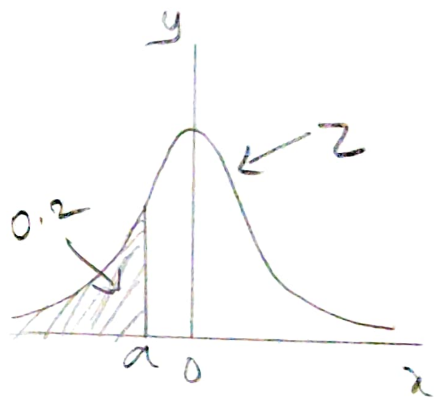
$\bar{Y} \sim N(100, 4)$

d. $P(\bar{Y} < 98) = 0.2$

$$Z = \frac{\bar{Y} - \mu}{\sigma/\sqrt{n}}$$

$$P\left(\frac{\bar{Y} - \mu}{\sigma/\sqrt{n}} < \frac{98 - \mu}{\sigma/\sqrt{n}}\right) = 0.2$$

$$P\left(Z < \frac{98 - 100}{14/\sqrt{n}}\right) = 0.2$$



$$P\left(Z < -\frac{\sqrt{n}}{7}\right) = 0.2$$

$$P(-\infty < Z < 0) - P(a < Z < 0) = 0.2$$

$$\left(\text{let } a = -\frac{\sqrt{n}}{7}\right)$$

$P(a < Z < 0) = 0.5 - 0.2 = 0.3$

$a = -0.84$

$\therefore -0.84 = -\frac{\sqrt{n}}{7}$

$n = (7 \times 0.84)^2 = 34.57$

$$\boxed{n \geq 35}$$

6. a. $n = 64$ (sample size)

$\bar{x} = 42$ (sample mean)

$s = 5$ (sample standard deviation)

$\alpha = 0.05$ (significance level)

Hypothesis Test on the population mean, i.e, μ

$\mu_0 = 40$

~~∴~~ Null hypothesis, $H_0 : \mu = 40$

Alternative hypothesis, $H_a : \mu > 40$

b. Since the sample size is 64 (>30), we need to ~~can~~ use Z_{obs} as our test statistic

$$Z_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{42-40}{5/\sqrt{64}} = \frac{2 \times 8}{5} = 3.2$$

$$\boxed{Z_{obs} = 3.2}$$

c. P-value $= P(Z > Z_{obs})$

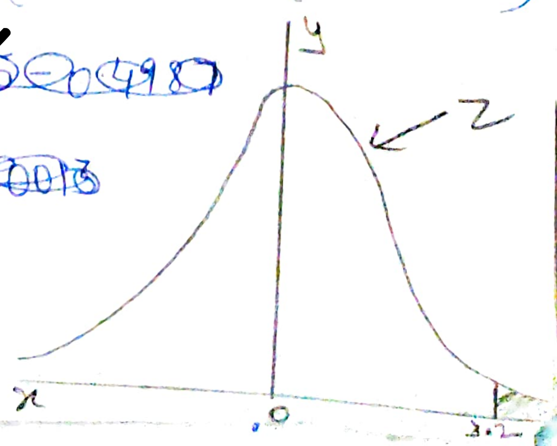
$= P(Z > 3.2) = P(0 < Z < \infty) - P(0 < Z < 3.2)$

~~$= 0.5 - 0.4987$~~

~~P-value $= 0.0013$~~

$P(Z > 3.2) \simeq 0$

∴ p-value $\simeq 0$



d. Rejection Region (RR): $Z_{obs} > Z_{\alpha}$

$Z_{obs} > Z_{0.05} = 1.64$

RR: $Z_{obs} > 1.64$

Decision: $3.2 > 1.64$, this is true

Therefore, we reject H_0.

Conclusion: There is sufficient evidence to conclude H_a.

e. $\mu_a = 42$

$$P(\text{making a type II error}) = \beta = P\left(Z < Z_{\alpha} - \frac{|\mu_0 - \mu_a|}{s/\sqrt{n}}\right)$$

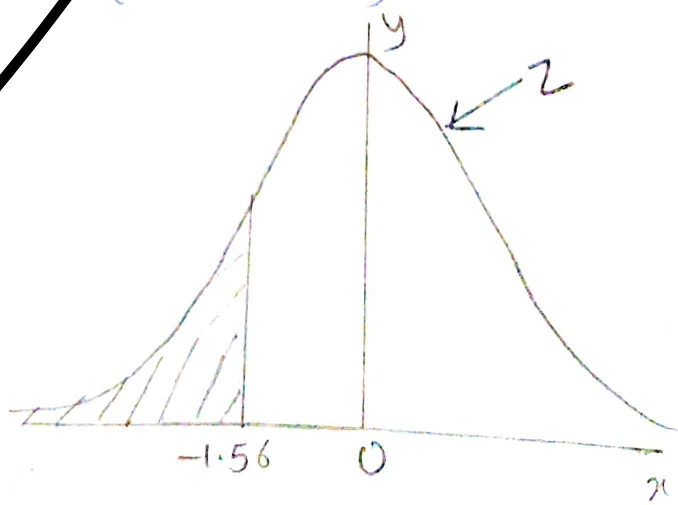
$$\beta = P\left(Z < Z_{0.05} - \frac{|40-42|}{5/\sqrt{64}}\right)$$

$$\beta = P(Z < 1.64 - 3.2) = P(Z < -1.56)$$

$$\beta = P(-\infty < Z < 0) - P(-1.56 < Z < 0)$$

$$= 0.5 - 0.4406$$

$$\boxed{\beta = 0.0594}$$



f. $n = 25 \ (< 30)$

$\bar{y} = 44$

$s = 4$

$\alpha = 0.05$

Hypothesis test on the population mean, i.e, μ

$\mu_0 = 45$

Step 1

Null hypothesis, $H_0: \mu = 45$

Alternative hypothesis, $H_a: \mu \neq 45$

Step 2

Test statistic to be used is t_{obs} as sample size is less than 30

$$t_{obs} = \frac{\bar{y} - x_0}{s/\sqrt{n}} = \frac{44 - 45}{4/\sqrt{25}} = \frac{-5}{4} = -1.25$$

Step 3

$$\text{P-value} = 2 \times P(t_{n-1} > |t_{obs}|)$$

$P(t_{24} > 1.25)$ lies between $.1$ and $.25$

$\therefore \quad 2 \times 0.1 < 2 \times P(t_{24} > 1.25) < 2 \times 0.25$

$0.2 < \text{P-value} < 0.5$

Step 4

Decision:

Since p-value lies between 0.2 and 0.5, it is clearly greater than alpha.

$$p\text{-value} > \alpha$$

Therefore, we fail to reject H_0.

Step 5

Conclusion:

There is no sufficient evidence to conclude H_a.