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STAT 8010-2 HOME WORK 1

1)

	Sample mean	Sample sd	Sample size
Herbicide 1	90.2	6.5	30
Herbicide 2	89.3	7.8	30

Herbicide 1 = 1 $\quad \bar{y}_1 = 90.2$, $s_1^2 = (6.5)^2$, $n_1 = 30$

Herbicide 2 = 2 $\quad \bar{y}_2 = 89.3$, $s_2^2 = (7.8)^2$, $n_2 = 30$

$\alpha = 0.05$

this is a Independent Samples Method problem

a) Hypothesis statement for two means

$$H_0: \mu_1 = \mu_2 \quad , \quad \mu_1 - \mu_2 = 0$$

$$H_a: \mu_1 \neq \mu_2 \quad , \quad \mu_1 - \mu_2 \neq 0$$

we need to check the Variances are equal or unequal, so that we can use right method to test the hypothesis of two means of Independent Samples

① $H_0: \sigma_1^2 = \sigma_2^2$

$H_a: \sigma_1^2 \neq \sigma_2^2$

② Test Statistic : $F_{obs} = \frac{\text{bigger } s^2}{\text{smaller } s^2}$

if $s_1^2 < s_2^2$ Test statistic, $F_{obs} = \frac{s_2^2}{s_1^2}$ $\rightarrow df_1 = n_2 - 1$, $\rightarrow df_2 = n_1 - 1$

$$F_{obs} = \frac{s_2^2}{s_1^2} = \frac{(7.8)^2}{(6.5)^2} = 1.44$$

③ Rejection Region

upper tail

$F_{obs} > F_{df_1, df_2, \alpha/2}$ or $F < F_{df_1, df_1, 1-\alpha/2}$

$df_1 = n_2 - 1 = 29$
$df_2 = n_1 - 1 = 29$

$F_{df_1, df_2, \alpha/2} \Rightarrow F_{29, 29, \frac{0.05}{2}}$

$F_{29, 29, 0.025} \approx 2.07$ (taking closest ~~[illegible]~~ df in table)

$F_{obs} > F_{df_1, df_2, \alpha/2}$

$1.44 > 2.07$ (false)

lower tail

$F_{obs} < F_{df_1, df_2, 1-\alpha/2}$

$F_{df_1, df_2, 1-\alpha/2} \Rightarrow F_{29, 29, 0.975}$

$$\boxed{F_{1-\alpha, df_1, df_2} = \frac{1}{F_{\alpha, df_2, df_1}}}$$

$$F_{29, 29, 0.975} = \frac{1}{F_{29, 29, 0.025}}$$

$$= \frac{1}{2.07} = 0.483$$

$F_{obs} < F_{df_1, df_2, 1-\alpha/2}$

$1.44 < 0.483$ (false)

(ii) Decision:
Fail to Reject H_0

$\therefore$ Variances are equal H_0 is accepted.

<u>Hypothesis for $\mu_1 - \mu_2$</u>

Now we can use the Independent samples and equal variances test hypothesis method.

<u>Hypothesis statement</u>

(1) $H_0: \mu_1 = \mu_2$, $\mu_1 - \mu_2 = 0$

$H_a: \mu_1 \neq \mu_2$, $\mu_1 - \mu_2 \neq 0$
$\hookrightarrow D_0$

② Test Statistic for $\mu_1 - \mu_2$

$$t_{obs} = \frac{(\bar{y}_1 - \bar{y}_2) - D_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$\rightarrow$ pooled sample s_d

where $$s_p^2 = \frac{(n_1 - 1) s_1^2 + (n_2 - 1) s_2^2}{n_1 + n_2 - 2}$$ Pooled Variance

Here $D_0 = 0 \Rightarrow \mu_1 - \mu_2 \neq 0$

$$s_p = \sqrt{\frac{(n_1 - 1) s_1^2 + (n_2 - 1) s_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{(30-1)(6.5)^2 + (30-1)(7.8)^2}{30 + 30 - 2}}$$

$$= \sqrt{\frac{(29)(6.5)^2 + (29)(7.8)^2}{58}}$$

$$= \sqrt{\frac{2989.61}{58}} = \sqrt{51.545} = 7.18$$

$\therefore s_p = 7.18$

$$t_{obs} = \frac{(\bar{y}_1 - \bar{y}_2) - D_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{(90.2 - 89.3) - 0}{(7.18) \times \sqrt{\frac{1}{30} + \frac{1}{30}}} = \frac{0.9}{(7.18) \times (0.26)}$$

$$= \frac{0.9}{1.8668} = 0.482$$

$$t_{obs} = 0.48$$

(3) Rejection Rejection

degree of freedom $df = n_1 + n_2 - 2$

$$|t| > t_{n_1+n_2-2,\, \alpha/2}$$

$$t_{n_1+n_2-2,\, \alpha/2} \Rightarrow t_{58,\, 0.025} \approx 2.0$$

$0.48 > 2.0$ (false)

Fail to Reject H_0

<u>Pvalue</u> : $2 \cdot P(t_{n_1+n_2-2} > |t_{obs}|)$

P-value $= 2 \cdot P\left(t_{58} > 0.48\right)$ (taking close df in table)

$= 2 \cdot \left(0.25 < P\left(t_{58} > 0.48\right) < 0.4\right)$

$= 0.5 <$ p-value < 0.8

p-value $< \alpha$ (false)

Fail to reject Ho

(4) Decision

Fail to Reject Ho

$\therefore$ we don't have sufficient evidence to

Conclude $H_a: \mu_1 - \mu_2 \neq 0$

1\. b) 90% Confidence interval for difference of two means

$1-\alpha = 0.9$

$\alpha = 0.1$

Confidence Interval (CI) for difference of two Population means (Independent Samples & Equal variance)

$$(\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2} \cdot (s_p) \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$\bar{y}_1 = 90.2$, $n_1 = 30$ $s_p = 7.18$

$\bar{y}_2 = 89.3$, $n_2 = 30$ $df = n_1 + n_2 - 2 = 58$

$$= (90.2 - 89.3) \pm t_{\frac{0.1}{2}, 58} \; (7.18)\left(\sqrt{\frac{1}{30} + \frac{1}{30}}\right)$$

$$= (0.9) \pm t_{0.05, 58} \times (7.18) \times (0.26)$$

$$= (0.9) \pm (1.671)(7.18)(0.26)$$

$$= (0.9) \pm (3.12)$$

Confidence Interval $CI = (0.9 - 3.12, \; 0.9 + 3.12)$

$= (-2.22, \; 4.02)$

2.

a)

type of music	Subject 1	2	3	4	5
hard rock	20	18	23	18	20
classical	24	20	27	22	21

This is the Paired Data Experimental design with the pairs of hardrock and classical music. In this we have five paired subject values for the type of music hard rock and classical.

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2 b) Hypothesis Testing

$\alpha = 0.05$, $n = 5$

① Hypothesis statement

$H_0 : \mu_d = D_0$

$H_a : \mu_d \neq D_0$

$\boxed{\mu_d = \mu_1 - \mu_2}$

1 = hardrock
2 = classical

$D_0 = 0$

② Test statistic for difference of means μ_d

$$TS = \frac{\bar{d} - D_0}{s_d / \sqrt{n}}$$

we have to find $\bar{d}$ and s_d

$\boxed{d = x_1 - y_1}$

$x_1 \rightarrow$ subject values of hardrock
$y_1 \rightarrow$ subject values of classical

$$\boxed{s_d = \sqrt{s_d^2} = \sqrt{\frac{\sum_{i=1}^{n} (d_i - d)^2}{n-1}}}$$

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$$\bar{d} = \frac{d_1 + d_2 + \cdots + d_n}{n}$$

	type of music		
Subject	hard rock	Classical	difference (d_i)
1	20	24	-4
2	18	20	-2
3	23	27	-4
4	18	22	-4
5	20	21	-1

$$\bar{d} = \frac{(-4) + (-2) + (-4) + (-4) + (-1)}{5} = \frac{-15}{5}$$

$$\bar{d} = -3$$

$$s_d = \sqrt{s_d^2} = \sqrt{\frac{(-4+3)^2 + (-2+3)^2 + (-4+3)^2 + (-4+3)^2 + (-1+3)^2}{5-1}}$$

$$= \sqrt{\frac{(-1)^2 + (1)^2 + (-1)^2 + (-1)^2 + (2)^2}{4}} = \sqrt{\frac{8}{4}}$$

$$s_d = 1.41$$

$$T_S = \frac{\bar{d} - D_0}{s_d/\sqrt{n}} = \frac{-3-0}{(1.41)/\sqrt{5}}$$

$$= \frac{-3}{0.6306} = -4.76$$

(3) Rejection Region

$t_{critical}$ value $= t_{n-1, \alpha/2} = t_{5-1, \frac{0.05}{2}}$

$$t_{4, 0.025} = 2.776$$

$$|t_{obs}| > t_{n-1, \alpha/2}$$

$$|-4.76| > 2.776$$

$$4.76 > 2.776 \quad (\text{true})$$

(4) Decision

Reject H_0

<u>P-value Method</u>:

$$= 2 \cdot P(t_{n-1} > |t_{obs}|)$$

$$= 2 \cdot P(t_4 > 4.76)$$

P-Value $= 2 \cdot (0.001 < P(t_4 > 4.76) < 0.005)$

Pvalue

$\Rightarrow 0.002 < \text{P-value} < 0.01$

$\text{P-value} < \alpha \; (0.05)$ (true)

Reject Ho

(4) Decision

Reject Ho

we have sufficient evidence to conclude

Ha.

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