The F-distribution

<u>Properties of the F-distribution</u>

1. The distribution is on the nonnegative side of the real line.

2. It is a non-symmetric continuous distribution that looks similar to the Chi-square distribution.

3. The distribution has two different degrees of freedom.

4. It is the sampling distribution of the ratio of two sample variances when the variances of the two populations are equal.

Another Property of the F-distribution

$$F_{1-\alpha\ ,\ df\,1, df\,2} = \frac{1}{F_{\alpha\ ,\ df\,2, df\,1}}$$

Finding values for the F-distribution

$F_{.05,\,3,\,4} =$ $\qquad$ $F_{.025,\,7,\,6} =$ $\qquad$ $F_{.95,\,4,\,3} =$

Small-Sample Hypothesis Test for a Ratio of Two Variances

Hypotheses

H_0: $\sigma_1^2 = \sigma_2^2$	H_0: $\sigma_1^2 = \sigma_2^2$	H_0: $\sigma_1^2 = \sigma_2^2$
H_a: $\sigma_1^2 < \sigma_2^2$	H_a: $\sigma_1^2 > \sigma_2^2$	H_a: $\sigma_1^2 \neq \sigma_2^2$

Test Statistic:

$$For \quad Ha:\ \sigma_1^2 > \sigma_2^2 \qquad F_{obs} = \frac{s_1^2}{s_2^2}$$

$$For \quad Ha:\ \sigma_2^2 > \sigma_1^2 \qquad F_{obs} = \frac{s_2^2}{s_1^2}$$

$$For \quad Ha:\ \sigma_1^2 \neq \sigma_2^2 \qquad F_{obs} = \frac{s_i^2}{s_j^2}\ ;\ where\ s_i^2 > s_j^2$$

Rejection Region:

The rejection region is found using an F-distribution with df_1=degrees of freedom of the numerator and df_2=degrees of freedom of the denominator.

Small-Sample Confidence Intervals for the Ratio of Two Variances

$$\frac{s_1^2}{s_2^2} \bullet \frac{1}{F_{(\alpha/2),df_1,df_2}} \quad , \quad \frac{s_1^2}{s_2^2} \bullet F_{(\alpha/2),df_2,df_1}$$

Comparison of Variances

Super alloys used in the aerospace industry require heat treatment to improve their material properties. The heat treatment is most effective if there is very little temperature fluctuation in the furnace. A company is considering buying one of two types of furnaces and have tested the temperature variability for both types of furnaces. The results were that for the first furnace the standard deviation of sixteen observations was 10°C and for the second furnace the standard deviation of twenty-one observations was 16°C. Below, conduct a hypothesis test to determine if there is evidence (at the .05 level of significance) to conclude that the population variances for the two types of furnaces are different. {Notes: 1) F-distribution practice, p.62; 2) Formulas, p. 63}

Hypotheses

H_0: $\sigma_1^2 = \sigma_2^2$
H_a: $\sigma_1^2 \neq \sigma_2^2$

Level of Significance

$\alpha = 0.05$

Rejection Region

Lower $F_{crit} = F_{0.975,20,15} = 1/F_{0.025,15,20} = 1 / 2.57 = 0.389$

Upper $F_{crit} = F_{0.025,\ 20,15} = 2.76$

Test Statistic

$F_{obs} = S^2_i/S^2_j = 16^2 / 10^2 = 2.56$

Decision

FTR H_0 "Barely" Good time to think about Type I and Type II

Conclusion

Construct a 99% confidence interval for the ratio of the two population variances.

Problems p. 363 & 364 7.15, 7.17, 7.18