<u>Inferences for Multiple Regression</u>

Let the prediction equation for the multiple regression be given by

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \ldots + \hat{\beta}_k x_k$$

Hypothesis Test for All Partial Slopes

Hypotheses
H_0: $\beta_1 = \beta_2 = \ldots = \beta_k = 0$
H_a: Not all β_i's are equal to zero

Test Statistic
It is the F value from the following ANOVA table:

Analysis of Variance (ANOVA)
for Multiple Linear Regression

Source Due to	Sum of Squares (SS)	df	Mean Square(MS)	F
Regression	SSREG	k	MSREG = SSREG/k	F = MSREG/MSE
Error	SSE	n-k-1	MSE = SSE/(n-k-1)	
Total	SSTotal	n-1		

Rejection Region
It is an F-distribution with $df_1 = k$ and $df_2 = n-k-1$.

Hypothesis Test for a Single Partial Slope

Hypotheses

H_0: $\beta_i = \beta_{i0}$	H_0: $\beta_i = \beta_{i0}$	H_0: $\beta_i = \beta_{i0}$
H_a: $\beta_i < \beta_{i0}$	H_a: $\beta_i > \beta_{i0}$	H_a: $\beta_i \neq \beta_{i0}$

Test Statistic

$$t_{obs} = \frac{\hat{\beta}_i - \beta_{io}}{s_{\hat{\beta}_i}}$$

Distribution for the Rejection Region

The rejection region is found by using a t-distribution with n-k-1 degrees of freedom.

Hypothesis Test that a Subgroup of Partial Slopes Have Values of Zero

Complete (Full) Model

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \dots + \hat{\beta}_k x_k$$

Reduced Model (g<k):

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \dots + \hat{\beta}_g x_g$$

Hypotheses

H_0: $\beta_{g+1} = \beta_{g+2} = \dots = \beta_k = 0$

H_a: Not all β_i's above are equal to zero

Testing Procedure

Step 1: Find the SSE for both the Full (SSE_{Full}) and reduced ($SSE_{Reduced}$) models.

Step 2: Calculate the drop in the sum of squares and the mean square drop.

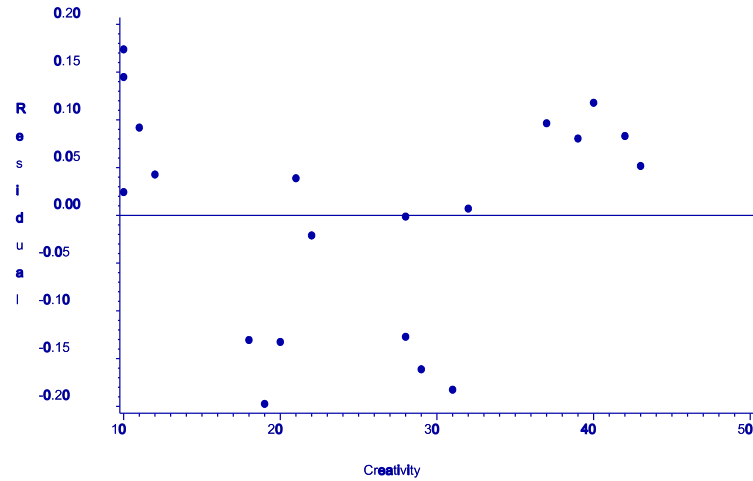$$MS_{Drop} = \frac{SSE_{Reduced} - SSE_{Full}}{k - g}$$

Step 3: Calculate the F statistic as
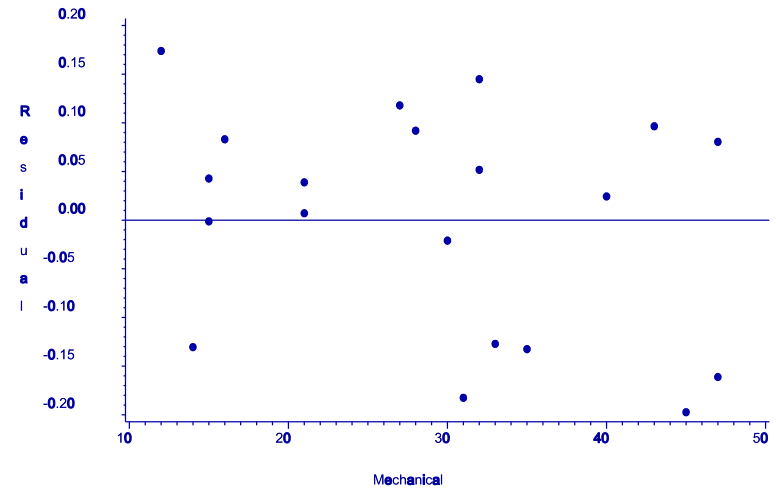
$$F_{obs} = \frac{MS_{Drop}}{MSE_{Full}}$$

Step 4: Reject H_0 if F_{obs} is greater than the table F with $df_1 = k-g$ and $df_2 = n-k-1$.

Residual Analysis for GPR=-0.062321+0.042472*Abstract+0.042133*Create

Grade Point Ratio Residual Analysis
(Residual vs Creativity)

Grade Point Ratio Residual Analysis
(Residual vs Mechanical)

Grade Point Ratio Residual Analysis
(Residual vs Abstract)

Grade Point Ratio Residual Analysis
(Residual vs Mathematical)
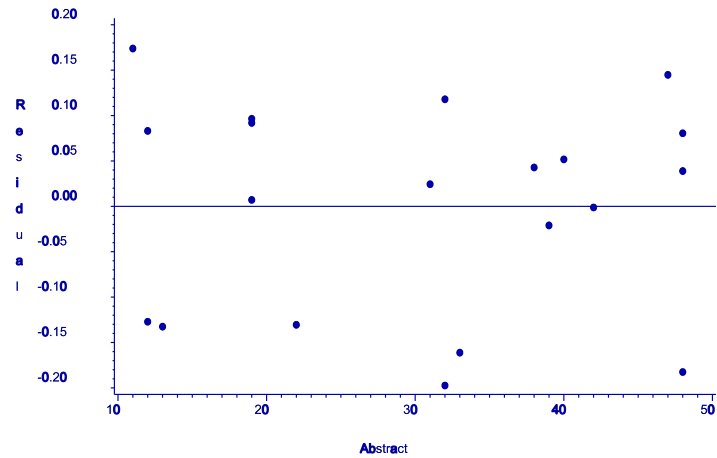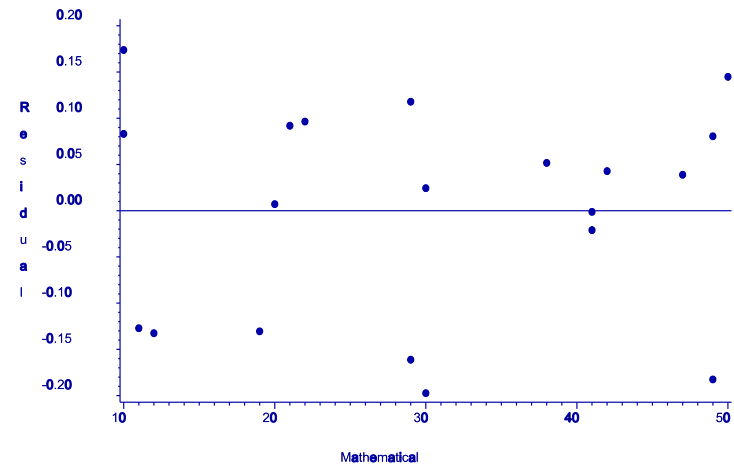
Multiple Regression Models Using Abstract Thinking Score and (Creativity Score)^2 to Predict GPR

```
Model: MODEL1
Dependent Variable: GPR
```

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	11.15876	3.71959	525.548	0.0001
Error	16	0.11324	0.00708		
C Total	19	11.27200			

Root MSE	0.08413	R-square	0.9900	
Dep Mean	2.28000	Adj R-sq	0.9881	
C.V.	3.68983			

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.354861	0.11294579	3.142	0.0063
ABSTRACT	1	0.042869	0.00144909	29.583	0.0001
CREATE	1	0.000922	0.00945397	0.098	0.9235
CREATE2	1	0.000803	0.00018125	4.431	0.0004

```
Model: MODEL2
Dependent Variable: GPR
```

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	11.15869	5.57935	837.089	0.0001
Error	17	0.11331	0.00667		
C Total	19	11.27200			

Root MSE	0.08164	R-square	0.9899	
Dep Mean	2.28000	Adj R-sq	0.9888	
C.V.	3.58072			

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.364599	0.05120856	7.120	0.0001
ABSTRACT	1	0.042879	0.00140282	30.566	0.0001
CREATE2	1	0.000820	0.00003152	26.030	0.0001

Obs	Dep Var GPR	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	1.0000	0.9183	0.038	0.7283	1.1083	0.0817
2	2.9000	2.7846	0.033	2.5991	2.9702	0.1154
3	2.5000	2.4619	0.037	2.2731	2.6508	0.0381
4	2.4000	2.4696	0.019	2.2929	2.6464	-0.0696
5	3.7000	3.6707	0.039	3.4802	3.8613	0.0293
6	3.1000	3.0495	0.032	2.8642	3.2348	0.0505
7	2.1000	2.1121	0.029	1.9294	2.2949	-0.0121
8	2.3000	2.3265	0.045	2.1294	2.5236	-0.0265
9	1.3000	1.2786	0.031	1.0944	1.4628	0.0214

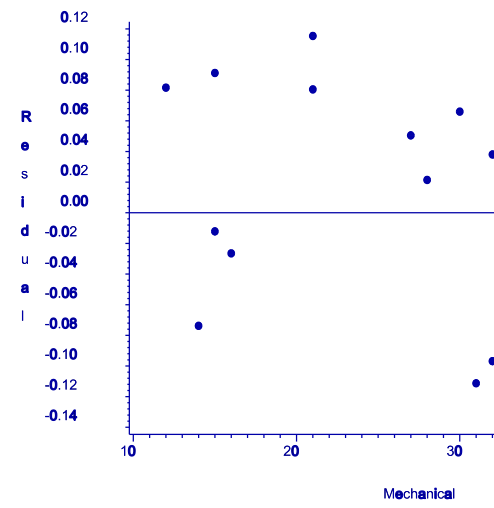| 10 | 2.9000 | 2.8088 | 0.025 | 2.6289 | 2.9887 | 0.0912 |
11	1.9000	2.0329	0.022	1.8544	2.2114	-0.1329
12	3.1000	3.2113	0.031	3.0267	3.3958	-0.1113
13	1.7000	1.7759	0.028	1.5941	1.9577	-0.0759
14	2.1000	2.0195	0.026	1.8388	2.2001	0.0805
15	1.2000	1.2502	0.032	1.0652	1.4352	-0.0502
16	2.4000	2.3025	0.031	2.1180	2.4870	0.0975
17	2.5000	2.4340	0.024	2.2546	2.6134	0.0660
18	1.5000	1.5738	0.025	1.3935	1.7541	-0.0738
19	3.5000	3.5968	0.041	3.4041	3.7895	-0.0968
20	1.5000	1.5224	0.031	1.3378	1.7070	-0.0224

Sum of Residuals   0
Sum of Squared Residuals   0.1133
Predicted Resid SS (Press)   0.1537

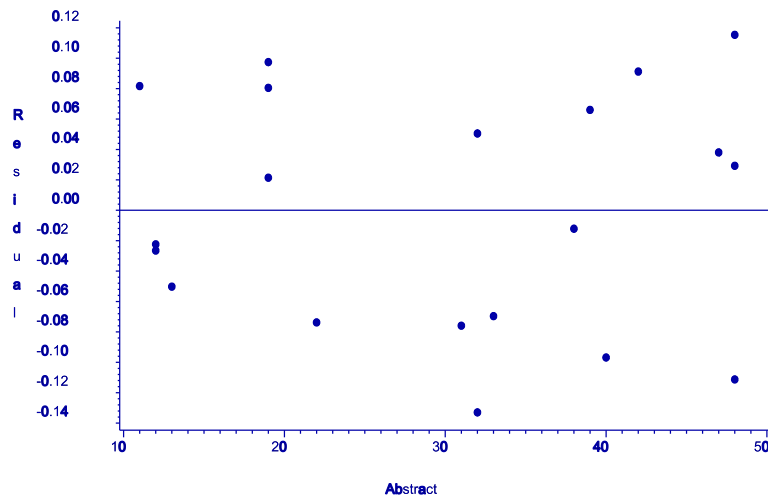# Residual Analysis for GPR=0.364599+0.042879*Abstract+0.000820*Create^2

Grade Point Ratio Residual Analysis

Creativity

Grade Point Ratio Residu

Mechanical

Grade Point Ratio Residual Analysis

Abstract

Grade Point Ratio Residu

Mathematical