Final paper.

1.) 7Marks

patient	Before	After	Before-After	$\bar{d}-x_i$
1	22.86	16.11	6.75	0.22
2	7.74	-4.02	11.76	-4.79
3	15.49	8.04	7.45	-0.48
4	9.97	3.29	6.68	0.29
5	1.44	-0.77	2.21	4.76

$$\bar{d} \Rightarrow \frac{6.75+11.76+7.45+6.68+2.21}{5} = 6.97$$

$$S_d^2 \Rightarrow \frac{(0.22)^2+(-4.79)^2+(-0.48)^2+(0.29)^2+(4.76)^2}{4}$$

$$= 11.49.$$

$$\Rightarrow S_d = \sqrt{11.49} = 3.38$$

Given $n=5$ μ_B = Befor
μ_A = After

~~μ_A~~ $\mu_B > \mu_A$.

~~Hypothesis~~ $H_0: \mu_B - \mu_A = 0$.
$H_a: \mu_B - \mu_A > 0$

Test statistic $\rightarrow \dfrac{\bar{d} - D_0}{sd/\sqrt{n}} \Rightarrow \dfrac{6.97 - 0}{3.38/\sqrt{5}} \rightarrow \dfrac{6.97}{3.38/2.23}$

$= \dfrac{6.97}{1.51} \rightarrow 4.61.$

RR:- tcritical $\rightarrow t_{0.05, 4} = 2.132$.

$t_{obs} > t_{critical} \rightarrow 4.61 > 2.132$.

Decision $\rightarrow$ Reject H_0.

p value $\rightarrow p(t_{n-1} > t_{obs})$
$p(t_4 > 4.61)$
lies between 0.001 & 0.001.
$\rightarrow$ pvalue $< \alpha$.
$\therefore$ Reject H_0.

Conclusion :- There is sufficient Evidence to conclude H_a.

2) 8 Marks.

Given

Brand	Sample size	Sample Mean	Sample SD
low Tar	100	8	0.3
A	100	10	0.4
B	100	10	0.4
C	100	11	0.5
D	100	11	0.5

a) Step 1 : Hypothesis : $H_0: \mu_{lowstar} = \mu_A = \mu_B = \mu_C = \mu_D$

H_a: Not all above are equal.

Step 2 $S_w^2 \Rightarrow$ ~~99(~~

$$\frac{\cancel{99}\,(0.3^2 + 0.4^2 + 0.4^2 + 0.5^2 + 0.5^2)}{\cancel{495}\ 5} = 0.182$$

Variance of Sample Means

$\Rightarrow Var(8, 10, 10, 11, 11)$

$$\bar{y} = \frac{8+10+10+11+11}{5} = 10$$

$$Sd^2 \Rightarrow \frac{(10-8)^2 + (10-10)^2 + (10-10)^2 + (10-11)^2 + (10-11)^2}{4}$$

$$\Rightarrow \frac{4+0+0+1+1}{4} = \frac{6}{4} = 1.5$$

$S_B^2 \Rightarrow n \times$ Var of Means $\Rightarrow 100 \times 1.5 \Rightarrow 150$

Test statistic $\Rightarrow$ $F_{obs} = \frac{S_B^2}{S_w^2}$

$$= \frac{150}{0.182} = 824.17$$

RR : $F_{critical} \Rightarrow F_{\alpha, df_1, df_2}$ where $df_1 = t-1$, $df_2 = nt-t$

$= F_{0.05, 4, 495} = 2.37$

$F_{obs} > F_{\alpha, df_1, df_2}$

$\Rightarrow 824.17 > 2.37$ (True)

pvalue: $p(F_{4, 495} > 824.17)$
this is less than 0.001
$\Rightarrow$ pvalue $< \alpha$.

Decision $\Rightarrow$ Reject Ho.
Conclusion: There is sufficient Evidence to Conclude Ha.

b.) 5 Marks.
Violation of Required Conditions $\rightarrow$ check for equal Variances.
Hypothesis: $H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2 = \sigma_5^2$
H_a: Not all of those are equal.
Test-statistic: $F_{max} = \dfrac{S^2_{max}}{S^2_{min}} \Rightarrow \dfrac{(0.5)^2}{(0.3)^2} = 2.77$.

RR: $F_{crit} \Rightarrow F_{max, \alpha, df_1, df_2}$ $\qquad df_1 = t$.
$= F_{max, 0.05, 5, 99} = 2.04$ $\qquad df_2 = n-1$.
or less than that.

$F_{obs} > F_{max, \alpha, df_1, df_2} \Rightarrow 2.77 > 2.04$.
Decision: Reject Ho.
Conclusion: There is sufficient Evidence to conclude Ha

3. Given

Carmodel

Gasoline	1	2	3	4	Sum
A	15	33	13	29.2	90.2
B	16.3	26.4	19.1	22.5	84.3
C	10.5	31.5	17.5	30.1	89.6
D	14.0	34.5	19.7	21.6	89.8
Sum	55.8	125.4	69.3	103.4	353.9

a) 5 Marks

Selected 5 Car Models:

Here Car Model is Extraneous Variable

It is a Randomized block design.

b) 3 Marks

Statistical Model:

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$$

μ = Overall mean that is an unknown const

α_i = an effect due to treatment, unknown const

β_j = an effect due to Block, unknown const

ε_{ij} = a Random Error associated with response on treatment i, block j.

c.) 10 Marks. Here $t = 4$.

$b = 4$.

ANOVA Table:

Source Due to	Sum of Squares	df	Mean Square (MS)	F
Treatments	5.7	3	1.9 = MST	0.125
Blocks	755.2	3	251.7 = MSB	16.5
Error	137.1	9	15.2 = MSE	
Total	898	15		

$$TSS \Rightarrow \sum_{i,j} y_{ij}^2 - \frac{y_{..}^2}{bt}$$

$$(15)^2 + 33^2 + 13^2 + \cdots 21.6^2 - \frac{(353.9)^2}{4\times4}.$$

$$8725 - 7827 \Rightarrow 898$$

$$SST \Rightarrow \sum_{i=1}^{t} \frac{y_{i.}^2}{b} - \frac{y_{..}^2}{bt} \Rightarrow \frac{90.2^2 + 84.3^2 + 89.6^2 + 89.8^2}{4} - 7827.8$$

$$= 7833.5 - 7827.8$$

$$= 5.7$$

$$SSB \Rightarrow \sum_{j=} \frac{y_{.j}^2}{t} - \frac{y_{..}^2}{bt} \Rightarrow \frac{55.8^2 + 125.4^2 + 67.3^2 + 103.4^2}{4} - 7827.8$$

$$= 8583 - 7827.8$$

$$= 755.2$$

$SSE \Rightarrow TSS - SST - SSB$

$= 898 - 5.7 - 755.2$

$= 137.1$

d) 5 Marks

Hypothesis:- $H_0: \alpha_A = \alpha_B = \alpha_C = \alpha_D = 0$

H_a: Not all of the above are eq to zero

Test Statistic $\Rightarrow$ $F_{obs1} = \dfrac{MST}{MSE}$

$= \dfrac{1.9}{15.2} \Rightarrow 0.125$

RR:- $F_{obs}^{①} > F_{t-1,(b-1)(t-1),\alpha} \Rightarrow F_{3,9,0.05}$

$= 3.86$

pvalue:- $F_{obs}^{①}$

$P\left(F_{t-1,(b-1)(t-1)} > F_{obs}^{①}\right)$

$\Rightarrow P(F_{3,9} > 0.125)$ which is < 0.25

$\Rightarrow$ pValue $> \alpha$

Decision:- FTR H_0

Conclusion:- Insufficient Evidence to Conclude H_a

e) 5 Marks:

"Car Model" is the Extraneous Variable.

Hypothesis: $H_0: \beta_A = \beta_B = \beta_C = \beta_D = 0$

H_a: Not all of the above are equal to zero

Test statistic $\Rightarrow F_{obs}$ ③ $\Rightarrow \frac{MSB}{MSE} \Rightarrow \frac{251.7}{15.2} = 16.5$

RR: $F_{crit} \Rightarrow F_{b-1,\,(t-1)(b-1),\,\alpha} \Rightarrow F_{3,9,0.05} = 3.86$

p Value: $P(F_{b-1,\,(t-1)(b-1)} > F_{obs}$ ②$)$

$P(F_{3,9} > 16.5)$ less than 0.001

pValue $< \alpha$

Decision: Reject H_0.

Conclusion: There is Sufficient Evidence to Conclude H_a.

f) 3 Marks.

$RE = \frac{MSE_{CR}}{MSE_{RB}} \Rightarrow \frac{(b-1)MSB + b(t-1)MSE_{RB}}{(bt-1)MSE_{RB}}$

$\Rightarrow \frac{3 \times 251.7 + (4)(3) \times 15.2}{15 \times 15.2} \Rightarrow \frac{755.1 + 182.4}{228}$

$= 4.11$

4) Given

					Sum
A	2.2	3	2.7	2.7	10.6
B	3.6	3.9	4.1	4.3	15.9
C	4.3	4.4	4.5	4.1	17.3
Sum	10.1	11.3	11.3	11.1	43.8

a.) 5 Marks.

This is a Completely Randomized design.

$t = 3$

$n_T = 12$

b.) 10 Marks

ANOVA

Source Due to	S.S	df	MS	F
Between Samples	SSB = 6.245	2	$S_B^2 = 3.125$	41.11
Within Samples	SSW = 0.685	9	$S_w^2 = 0.076$	
Totals	TSS = 6.93	11		

$\cancel{SSB} \rightarrow TSS \rightarrow \sum_{ij}^{o} y_{ij}^2 - \frac{y_{..}^2}{n_T} \rightarrow 2.2^2 + 3^2 + \dots 4.1^2 - \frac{(43.8)^2}{12}$

$166.8 - 159.87 \rightarrow 6.93$

$SSB \rightarrow \frac{10.6^2}{4} + \frac{15.9^2}{4} + \frac{17.3^2}{4} - 159.87 \rightarrow 6.245$

$SSW \rightarrow TSS - SSB \rightarrow 6.93 - 6.245 = 0.685$

c.) Hypothesis :- $H_0: \mu_A = \mu_B = \mu_C$

H_a: Not all the above are Equal.

TS → $\frac{S_B^2}{S_W^2}$ → $\frac{3.125}{0.076} = 41.11 = F_{obs}$.

RR →

$df_1 = t-1$

$df_2 = n_T - T$.

$F_{\alpha, t-1, n_T - T}$ → $F_{0.05, 2, 9}$

$= 4.26$.

p Value → $p(F_{t-1, n_T - T} > F_{obs})$

$= p(F_{2,9} > 41.11)$ less than 0.001

→ p Value < α.

& $F_{obs} > F_{\alpha, t-1, n_T - 1}$

Decision :- Reject H_0.

Conclusion :- There is sufficient evidence to conclude H_a.

d.) 3 Marks.

Not Answering this, Since it is CR design.

e) 4 Marks:-

Hypothesis:- $H_0: \mu_A - \mu_B = 0$

$H_a: \mu_A - \mu_B \neq 0$

Fischer's LSD Rule:-

$$LSD = t_{\alpha/2,\, n_T - t} \sqrt{S_w^2\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}$$

$$= t_{0.025,\, 9} \sqrt{0.076\left(\frac{1}{4} + \frac{1}{4}\right)}$$

$$= 2.26 \sqrt{0.076 \times 1/2}$$

$$LSD = 0.44$$

Mean difference between A & B

$$|\bar{y}_A - \bar{y}_B| \rightarrow \left|\frac{10.6}{4} - \frac{15.9}{4}\right| \rightarrow |2.65 - 3.975| = 1.325$$

Step 3:- $|\bar{y}_A - \bar{y}_B| = 1.325$

$|\bar{y}_A - \bar{y}_B| > LSD$

$1.325 > 0.44$

~~Con~~ Decision:- Reject H_0

Conclusion:- There is sufficient Evidence to Conclude the

574 + 626 +
580 + 665 +
602 + 682

5) a) 3 Marks.

price = Independent Variable $\longrightarrow X$

Quantity sold = ~~In~~dependent Variable $\longrightarrow Y$

Given.

price (x)	Qty sold (y)	x^2	y^2	xy
58.7	20	3445	400	1174
59	15	3481	225	885
60.1	17	3612	289	1021.7
61.3	16	3757	256	980.8
63.2	13	3994	169	821.6
64	11	4096	121	704

b.) 8 Marks.

Hypothesis :- $H_0: \beta_1 = 0$

$H_a: \beta_1 \neq 0$

$S_{xx} = \sum_{i=1}^{n}$

TS:- $t_{obs} = \frac{\hat{\beta}_1 - \beta_{10}}{S_\varepsilon / \sqrt{S_{xx}}}$

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 \rightarrow \sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}$$

$\sum x_i^2 \rightarrow 22385$

$\sum x_i^2 = 22385$

$\Sigma x_i \Rightarrow 366.3$

$\frac{(\Sigma x_i)^2}{n} = 22362$

$S_{xx} \Rightarrow 22385 - 22362$

$\boxed{S_{xx} = 23}$

$\Sigma y_i = 92$

$\frac{(\Sigma y_i)^2}{n} = 1410$

$\Sigma y_i^2 \Rightarrow 1460$

$S_{yy} \Rightarrow 1460 - 1410 = 50$

$$S_{xy} \Rightarrow \sum_{i=1}^{n} x_i y_i - \frac{\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n}$$

$$5588 - \frac{366.3 \times 92}{6}$$

$$\Rightarrow 5588 - 5616.6$$

$$\Rightarrow -28.6$$

$S_\varepsilon \rightarrow \sqrt{\dfrac{SSE}{n-2}}$ $\quad\quad$ $SSE = S_{yy} - \dfrac{S_{xy}^2}{S_{xx}}$

$= \sqrt{\dfrac{14.44}{4}}$ $\quad\quad$ $\rightarrow 50 - \dfrac{(-28.6)^2}{23}$

$= \sqrt{3.61} = 1.9$ $\quad\quad$ $\rightarrow 50 - 35.56 = 14.44$

TS:- $t_{obs} \rightarrow \dfrac{\hat{\beta}_1 - \beta_{10}}{S\varepsilon/\sqrt{S_{xx}}}$ $\quad\quad$ $\hat{\beta}_1 = \dfrac{S_{xy}}{S_{xx}}$

$\rightarrow \dfrac{-28.6}{23} \rightarrow -1.24$

$\rightarrow \dfrac{-1.24-0}{1.9/\sqrt{23}}$

$\rightarrow \dfrac{-1.24-0}{1.9/4.79} \rightarrow \dfrac{-1.24}{0.39} \rightarrow -3.17$

RR:- $t_{\alpha/2, n-2} \rightarrow t_{0.025,4} = 2.776$

$|t_{obs}| > t_{\alpha/2, n-2} \rightarrow 3.17 > 2.776$ ✓

Decision:- Reject Ho

Conclusion:- There is sufficient Evidence to conclude Ha

6.) a.) 5 Marks.

Given.

State	n	Mean	SD
Michi	10	8458	250
Cali	9	9690	420

First we need to do F-test to check between pooled t-test & Seperate t-test.

Hypothesis :- $H_0: \sigma_1^2 = \sigma_2^2$

$H_a: \sigma_1^2 \neq \sigma_2^2$

TS :- $F_{obs} \rightarrow \frac{S_i^2}{S_j^2} \rightarrow \frac{420^2}{250^2} \rightarrow 2.82$

$df_1 \rightarrow n_2 - 1 = 8$

$df_2 = n_1 - 1 \rightarrow 9$

RR :- $F_{obs} > F_{\alpha/2, df_1, df_2}$

$F_{0.025, 8, 9} \rightarrow 4.10$

$F_{obs} > F_{crit} \rightarrow 2.82 \not> 4.10$

Decision :- FTR H_0

Conclusion :- No sufficient Evidence to conclude the

$\rightarrow$ Equal Variances.

We conduct pooled t-test.

$C > M$
$\mu_2 > \mu_1$
$\mu_1 - \mu_2 > 0$

b) μ_C = Cali Mean
$\mu_M \rightarrow$ Michi Mean
μ_0 = Diff in Mean $\rightarrow \mu_C - \mu_M$
$\mu_C > \mu_M$
$\mu_C - \mu_M > 0$

Hypothesis :- $H_0: \mu_C - \mu_M = 0$
$H_a: \mu_C - \mu_M > 0$

TS :- $S_p^2 \rightarrow \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} \rightarrow \frac{9(250)^2 + 8(420)^2}{17}$

$\frac{562500 + 1411200}{17} \rightarrow \sqrt{116100} = 340.7$

$t_{obs} \rightarrow \frac{9690 - 8458 - 0}{341\sqrt{\frac{1}{10}+\frac{1}{9}}} \rightarrow \frac{1232}{341 \times 0.45} = 8.02$

RR :- $t_{(n_1+n_2-2),\alpha} \rightarrow t_{17,0.05} = 1.74$

$t_{obs} > t_{crit} \rightarrow 8.02 > 1.74$ True

pvalue
$p(t_{17} > 8.02)$
less than 0.005
p value $< \alpha$

Decision :- Reject H_0
Conclusion :- There is sufficient Evidence to Conclude H_a
i.e California has higher mean hysterectomy cost than michigan

c) $CI \rightarrow (\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2,(n_1+n_2-2)} S_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$

$(9690-8455) \pm t_{0.025,17} \times 340.7 \times 0.45$

$1235 \pm 2.11 \times 341 \times 0.45$

1235 ± 323.7

$\rightarrow (911.3, 1558.7)$