The Normal Distribution

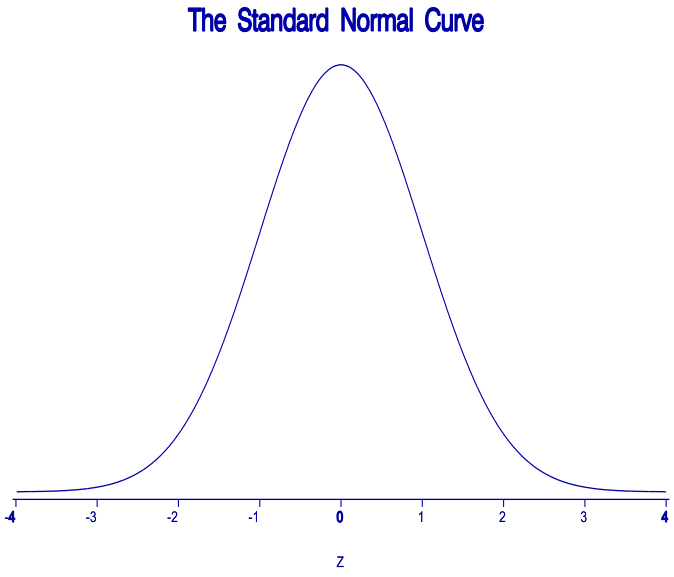
One of the most important continuous probability distributions in statistics is the normal distribution. It has been used to model many situations that occur in nature. Some examples are heart rates, blood cholesterol levels, and SAT scores. The normal distribution is also used to construct some of the quality control charts that are used to monitor manufacturing processes.

The normal distribution is a bell-shaped continuous distribution that has a mean of μ and a variance of σ^2. The equation used to generate the normal curve is

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$$

Below is an example of the normal curve for $\mu = 0$ and $\sigma^2 = 1$.

Important Terms and Definitions



We denote that a random variable Y has a normal distribution with a mean μ and a variance σ^2 by the expression $Y \sim N(\mu,\sigma^2)$.

A **standard normal distribution** is a normal distribution which has a mean $\mu=0$ and a variance $\sigma^2=1$.

If Z has a standard normal distribution, then we can denote this by

$Z \sim N(\_\_\_,\_\_\_)$

Some Properties of the Normal Distribution

1. The area under the entire curve is always 1.
2. The distribution is symmetric about the mean μ.
3. The mean and median are equal.
4. Probabilities for events, which come from a normal distribution, may be found by determining the appropriate area under the normal curve.

Finding Probabilities for the Standard Normal Distribution

Graph

$P(-\infty < Z < \infty) =$

$P(0 < Z < \infty) =$

$P(-\infty < Z < 0) =$

$P(0 < Z < 1) =$

$P(-1 < Z < 0) =$

$P(-1 < Z < 1) =$

$P(Z > 1.56) =$

$P(1 < Z < 2.43) =$

$P(-1 < Z < 2.43) =$

$P(Z < a) = .8790$ Find a. Another name for a is what?

$P(a < Z < 1) = .3321$ Find a.

Suppose we want the area of the upper tail of the standard normal distribution to be equal to .05. Find the value of z which will do this.

Suppose we want the area of the upper tail of the standard normal distribution to be equal to .1. Find the value of z which will do this.

Finding Probabilities for a General Normal Distribution

The table we used is for standard normal distributions. What if we want to find the probabilities for a general normal distribution?

Suppose $X \sim N(\mu,\sigma^2)$. Transform X using the following formula:

$$Z = \frac{X - \mu}{\sigma}$$

The random variable Z will now have a standard normal distribution.

Suppose $X \sim N(100,4)$. Find the following:

$P(100 < X < 105) =$

$P(98 < X < 102) =$

$P(102 < X < 105) =$

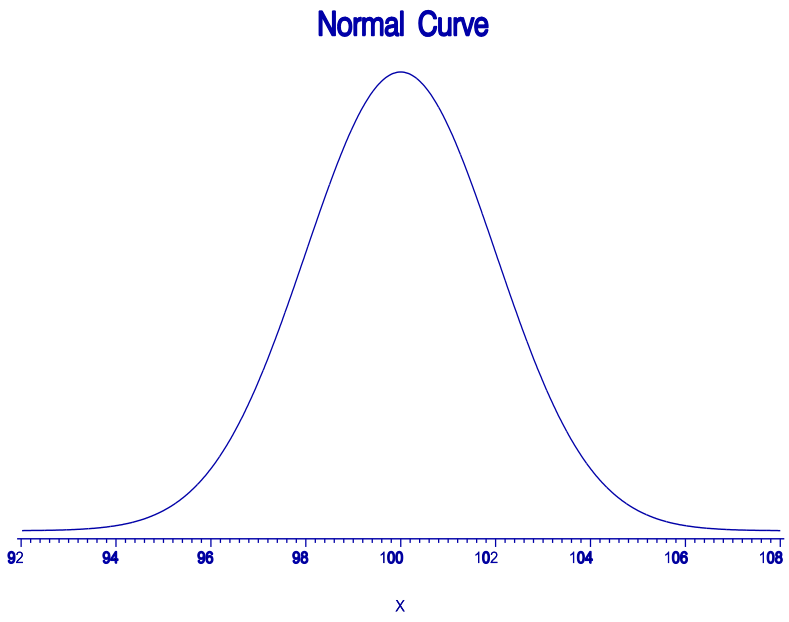
$P(a < X < 102) = .5328$ Find a.

Locating Values on a Normal Curve

$P(X < a) = .025$ Find a.

Find the third quartile.

For the graph below, label the mean, 2.5^{th} percentile, and the third quartile.



Some Word Problems

1. The average length of stay in the hospital for someone who has a tonsillectomy is one day with a standard deviation of one-half day. Suppose we assume that the length of stay is normally distributed. Find the probability that a tonsillectomy patient stays in the hospital for more than two days.

2. A machine produces bolts that have an average diameter of .5 inch and a standard deviation of .1 inch. Bolts which have a diameter less than .4 or greater than .6 are scrapped. Suppose we assume that the diameter of the bolts is normally distributed. Find the proportion of bolts that will have to be scrapped.