1.

a) $H_0 : \mu_1 = \mu_2 = \mu_c$

H_a : Not all the above are equal.

As sample sizes are equal, so we can use

$$S_w^2 = \frac{S_1^2 + S_2^2 + S_c^2}{3} = \frac{(6.5)^2 + (7.8)^2 + (7.4)^2}{3}$$

$$S_w^2 = 52.62$$

$$S_B^2 = \frac{n * \Sigma(\bar{y}_i - \bar{y}_{..})^2}{t-1}$$

$$\left[\because \bar{y}_{..} = \frac{90.2+89.3+85}{3} = 88.17\right]$$

$$S_B^2 = 30 * \frac{\left[(90.2-88.2)^2 + (89.3-88.2)^2 + (85-88.2)^2\right]}{2}$$

$$S_B^2 = 15 * 15.45 = 231.75$$

$$F_{obs} = \frac{S_B^2}{S_w^2} = \frac{231.75}{52.62} = 4.40$$

$$F_{2,87,0.05} = 3.10$$

$$F_{obs} > F_{2,87,0.05}$$

$\therefore$ We can reject Ho.

$\therefore$ We have sufficient evidence to conclude that there's significant difference in the mean yields.

1.b) $H_0: \sigma_1^2 = \sigma_2^2 = \sigma_c^2$

H_a: Not all variances are equal.

Test statistic -

$$F_{max} = \frac{S_{max}^2}{S_{min}^2} = \frac{(7.8)^2}{(6.5)^2} = 1.44$$

Rejection Region

$$F_{3,29,0.05} = 2.93$$

$$F_{max} < F_{3,29,0.05}$$

$\therefore$ We cannot reject Ho.

$\therefore$ We do not have sufficient evidence to conclude that not all variances are equal. Hence the condition of equal population variances for test in Part (a) is not violated.

2a. Randomized Block Design.

observations for a Randomized Block Design can be expressed as the sum of three terms.

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$$

where,

μ: an overall mean that is an unknown constant

α: an effect due to treatment i; α_i is an unknown constant.

β_j: an effect due to block j; β_j is an unknown constant.

ε_{ij}: a random error associated with the response on treatment i, block j.

2.b)

	Subject				
Type of music	1	2	3	4	Sum
No music	20	17	24	20	81 $y_{1\cdot}$
Hard Rock	20	18	23	18	79 $y_{2\cdot}$
classical	24	20	27	22	93 $y_{3\cdot}$
sum	64	55	74	60	253 $y_{\cdot\cdot}$
	$y_{\cdot 1}$	$y_{\cdot 2}$	$y_{\cdot 3}$	$y_{\cdot 4}$	

$$TSS = \sum_{i,j} y_{ij}^2 - \frac{y_{\cdot\cdot}^2}{bt} \qquad [\because b = 4, t = 3]$$

$$\sum_{i,j} y_{ij}^2 = (20)^2 + (20)^2 + (24)^2 + (17)^2 + (18)^2 + (20)^2 + (24)^2 + (23)^2 + (27)^2 + (20)^2 + (18)^2 + (22)^2$$

$$= 5431$$

$$TSS = 5431 - \frac{(253)^2}{12} = 5431 - 5334.08$$

$$TSS = 96.92$$

$$SST = \left(\sum_i \frac{y_{i\cdot}^2}{b} \right) - \frac{y_{\cdot\cdot}^2}{bt}$$

$$SST = \frac{(81)^2 + (79)^2 + (93)^2}{4} - \frac{(253)^2}{12} = 28.67$$

$$SSB = \left(\sum_j \frac{y_{.j}^2}{t}\right) - \frac{y_{..}^2}{bt}$$

$$SSB = \frac{(64)^2 + (55)^2 + (74)^2 + (60)^2}{3} - \frac{(253)^2}{12}$$

$SSB = 64.92$

$SSE = TSS - SST - SSB = 96.92 - 28.67 - 64.92$

$SSE = 3.33$

ANOVA for a Randomized Block Design.

Source due to	Sum of squares (SS)	df	Mean square (MS)	F
Treatment	28.67	2	14.33	26.05
Blocks	64.92	3	21.64	39.34
Error	3.33	6	0.55	
Totals	96.92	11		

→ $H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0$

H_a: Not all of the above are equal to zero.

$TS = F_{obs} = \frac{MST}{MSE} = 26.05$

P-value : $P(F_{2,6} > F_{obs}) = P(F_{2,6} > 26.05)$

P-value lies between 0.001 and 0.005

∴ We reject H_0.

∴ We have sufficient evidence to say that there is difference among treatment means

→ $H_0: B_1 = B_2 = B_3 = B_4 = 0$

H_a: Not all of the above are equal to zero.

$TS = F_{obs} = \frac{MSB}{MSE} = 39.34$

P-value : $P(F_{3,6} > F_{obs}) = P(F_{3,6} > 39.34)$

P-value < 0.001

$\therefore$ We reject H_0.

$\therefore$ We have sufficient evidence to say that there is difference among block effects.

2c. Relative efficiency $= \dfrac{MSE_{CR}}{MSE_{RB}}$

$$= \frac{(b-1)\,MSB + b(t-1)\,MSE_{RB}}{(bt-1)\,MSE_{RB}}$$

$$= \frac{(4-1) * 21.64 + 4 * (3-1) * 0.55}{(4*3-1) * 0.55}$$

$$= \frac{64.92 + 4.4}{6.05}$$

$$= 11.46$$