CHAPTER 7

Study Questions

1. Describe the sampling distribution for the random variable $(n-1)s^2/\sigma^2$.

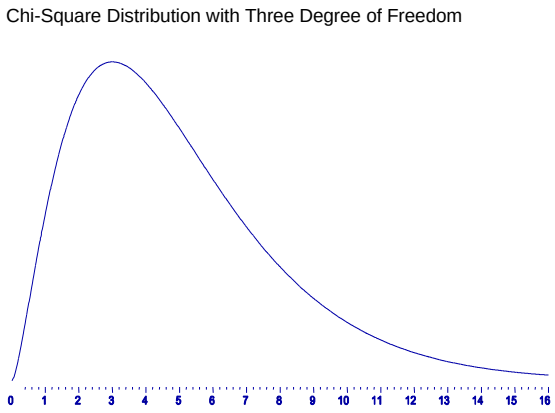
2. How is the sampling distribution for the function of s^2 different from the sampling distribution for $\bar{y}$?

3. What assumptions must be met in order to use the Chi-square and F-distributions for hypothesis testing?

The Chi-Square Distribution

Properties of the Chi-Square Distribution

1. The distribution is on the nonnegative side of the real line.
2. It is a non-symmetric continuous distribution.
3. The mean of the distribution is equal to the degrees of freedom.
4. It is the sampling distribution for a function of s^2.



Finding Values of the Chi-square Distribution

$\chi^2_{.05,5}$ =

$\chi^2_{.01,10}$ =

For 8 degrees of freedom, find the Chi-square value such that 5% of the area is below it.

Small-Sample Hypothesis Test for a Single Variance

Hypotheses

H_o: $\sigma^2 = \sigma_o^2$	H_o: $\sigma^2 = \sigma_o^2$	H_0: $\sigma^2 = \sigma_o^2$
H_a: $\sigma^2 < \sigma_o^2$	H_a: $\sigma^2 > \sigma_o^2$	H_a: $\sigma^2 \neq \sigma_o^2$

Test Statistic

$$\chi^2_{obs} = \frac{(n-1)s^2}{\sigma_0^2}$$

Distribution for the Rejection Region

The rejection region is found using a chi-square distribution with df=(n-1).

Small-Sample Confidence Intervals for a Single Variance

$$\frac{(n-1)s^2}{\chi^2_{(\alpha/2),(n-1)}} \quad , \quad \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2),(n-1)}}$$

Example: A medical supplies manufacturer claims that its new thermometers are so precise that the standard deviation in its measurements is smaller than .25°F. To test this claim, a hospital took 10 measurements in an incubator. Below are the results of the experiment measured in degrees Fahrenheit.

98.82 98.84 98.90 98.87 98.79 98.83 98.85 98.84 98.62 98.72

Conduct a hypothesis test to determine if the manufacturer's claim is true. Use a significance level of .05. {Notes: 1) Sampling Distribution of S2 is right-skewed (Chi-squared, χ^2);
2) Chi-square practice on page 59; 3) Formulas p. 60}

$H_0: \sigma = .25$ $H_A: \sigma < .25$

Standardized Scale Method

$\chi^2_{obs} = (n-1)S^2/\sigma_0^2 = (9)(.08176)^2/(.25)^2 = 0.9626$

$\chi^2_{crit} = 3.325$

R H_0

P-Value Method

$\chi^2_{obs} = 0.9626$

p-value < 0.001

R H_0

Construct a 95% confidence interval for the standard deviation.

$\sqrt{(n-1)S^2/\chi^2_{\alpha/2,n-1}}$ $\sqrt{(n-1)S^2/\chi^2_{\alpha/2,n-1}}$

$\sqrt{(9)(.08176)^2/19.02}$ $\sqrt{(9)(.08176)^2/2.70}$

0.056, 0.149

Problems p. 351 & 352 7.5 - 7.11