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FINAL EXAM

1. $H_0: \mu_d = \mu_{Before} - \mu_{after} = 0$

$H_a: \mu_d = \mu_{before} - \mu_{after} > 0$

Patient	Before	After	Difference (d_i)
1	22.86	16.11	6.75
2	7.74	-4.02	11.76
3	15.49	8.04	7.45
4	9.97	3.29	6.68
5	1.44	-0.77	2.21

Paired design with Patients being extraneous variable.

$$\bar{d} = \frac{6.75 + 11.76 + 7.45 + 6.68 + 2.21}{5} = 6.97$$

$$S_d^2 = \frac{\Sigma(d_i - \bar{d})^2}{n-1} = 11.49 \Rightarrow S_d = 3.39$$

Test statistic:

$$t_{obs} = \frac{\bar{d} - D_0}{S_d/\sqrt{n}}$$

$$t_{obs} = \frac{6.97 - 0}{3.39/\sqrt{5}} = 4.58$$

Rejection Region: $t_{obs} > t_{n-1,\alpha}$

$t_{n-1,\alpha} = t_{4,0.05} = 2.132$

$t_{obs} > t_{4,0.05}$

$\therefore$ Reject H_0. We have sufficient evidence to conclude that mean salt sensitivity value decreased after the Patient received antihypertensive treatment i.e., μ_{before} is greater than μ_{after}.

2a)

Brand	Sample size	Sample mean	Sample s.d
low Tar	100	8	0.3
A	100	10	0.4
B	100	10	0.4
C	100	11	0.5

$H_0: \mu_{low} = \mu_A = \mu_B = \mu_c$

H_a: Not all the above are equal

As all the sample sizes are equal, we can conduct a-f method.

Test statistic: $F_{obs} = \dfrac{S_B^2}{S_W^2}$

$$S_w^2 = \frac{S_{law}^2 + S_A^2 + S_B^2 + S_c^2}{t}$$

$$S_w^2 = \frac{(0.3)^2 + (0.4)^2 + (0.4)^2 + (0.5)^2}{4}$$

$$S_w^2 = 0.165$$

$S_B^2 = n * (\text{variance of sample means})$

$= 100 * 1.5833$

$= 158.333$

$$F_{obs} = \frac{S_B^2}{S_w^2} = \frac{158.333}{0.165} = 959.59$$

Rejection Region: $F_{obs} > F_{df_1, df_2, \alpha}$

$df_1 = t - 1 = 3$

$df_2 = nt - t = 100(4) - 4 = 396$

$F_{3, 396, 0.05} = 2.6$

$F_{obs} > F_{3, 396, 0.05}$

We reject H_0. We have sufficient evidence to conclude that there is significant difference in average tar contents from the above four brands of cigarettes.

b) check for equal variance condition

$H_0: \sigma^2_{low} = \sigma^2_A = \sigma^2_B = \sigma^2_c$

H_a: Not all the above are equal

Test statistic: $F_{max} = \dfrac{S^2_{max}}{S^2_{min}}$

$$F_{max} = \frac{(0.5)^2}{(0.3)^2} = 2.78$$

Rejection Region: $F_{max} > F_{max, df_1, df_2, \alpha}$

$F_{4,99,0.05}$; $F_{max} > 1$

$df_1 = t = 4$

$df_2 = n-1 = 99$

$\therefore F_{max} > F_{4,99,0.05}$

$\therefore$ Reject H_0. We have sufficient evidence to conclude that not all variances are equal. Hence the condition of equal population variances for test in part (a) is violated.

3a) Randomized Block Design

b)

Gasoline	Car model 1	2	3	4	Sum
A	15	33	13	29.2	90.2 $y_{1.}$
B	16.3	26.4	19.1	22.5	84.3 $y_{2.}$
C	10.5	31.5	17.5	30.1	89.6 $y_{3.}$
D	14.0	34.5	19.7	21.6	89.8 $y_{4.}$
Sum	55.8 $y_{.1}$	125.4 $y_{.2}$	69.3 $y_{.3}$	103.4 $y_{.4}$	353.9 $y_{..}$

$$TSS = \sum_{i,j} y_{ij}^2 - \frac{y_{..}^2}{bt} = 897.18 \text{ (given)} \qquad [\because t=4, b=4]$$

$$SST = \left(\sum_i \frac{y_{i.}^2}{b}\right) - \frac{y_{..}^2}{bt} = 7833.68 - 7827.82$$

$$= 5.86$$

$$SSB = \left(\sum_j \frac{y_{.j}^2}{t}\right) - \frac{y_{..}^2}{bt} = 8583.21 - 7827.82$$

$$= 755.39$$

$SSE = TSS - SST - SSB$

$SSE = 135.93$

$df_1 = t-1 = 3$

$df_2 = b-1 = 3$

$$MST = \frac{SST}{(t-1)} = \frac{5.86}{3} = 1.95$$

$$MSB = \frac{SSB}{(b-1)} = \frac{755.39}{3} = 251.80$$

$$MSE = \frac{SSE}{(b-1)(t-1)} = \frac{135.93}{9} = 15.10$$

$$F_{obs\,1} = \frac{MST}{MSE} = 0.13$$

$$F_{obs\,2} = \frac{MSB}{MSE} = 16.67$$

ANOVA for Randomized Block Design

Source due to	Sum of Squares (SS)	df	Mean Square (MS)	F
Treatments	5.86	3	1.95	0.13
Blocks	755.39	3	251.80	16.67
Error	135.93	9	15.10	
Total	897.18	15		

c) $H_0: \alpha_A = \alpha_B = \alpha_C = \alpha_D = 0$

H_a: Not all the above are equal to 0.

Test statistic : $F_{obs} = \frac{MST}{MSE} = 0.13$

Rejection Region: $F_{obs} > F_{(t-1),(t-1)(b-1),\alpha}$

$P\text{-value} = P(F_{3,9,0.05} > F_{obs})$

$= P(F_{3,9,0.05} > 0.13)$

$P\text{-value} > 0.25$

$\alpha > P\text{-value}$ (False) $[\because \alpha = 0.05]$

Fail to reject H_0. We do not have sofficient evidence to conclude H_a.

d) $H_0: \beta_A = \beta_B = \beta_C = \beta_D = 0$

H_a: Not all the above are equal to 0.

Test statistic : $F_{obs} = \frac{MSB}{MSE} = 16.67$

Rejection Region : $F_{obs} > F_{(b-1),(t-1)(b-1),\alpha}$

$P\text{-value} = (F_{3,9,0.05} > F_{obs})$

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$P\text{-value} = (F_{3,9,0.05} > 16.67)$

$P\text{-value} < 0.001$

$\alpha > P\text{-value}$ (true) $[\because \alpha = 0.05]$

Reject H_0. We have sufficient evidence to conclude H_a:

e) Relative efficiency $= \dfrac{MSE_{CR}}{MSE_{RB}}$

$$= \frac{(b-1)MSB + b(t-1)MSE_{RB}}{(bt-1)MSE_{RB}}$$

$$= \frac{3(251.80) + 4(3)(15.10)}{15(15.10)}$$

$$= 4.135$$

4.

					Sum
A	2.2	3	2.7	2.7	10.6
B	3.6	3.9	4.1	4.3	15.9
C	4.3	4.4	4.5	4.1	17.3
Sum	10.1	11.3	11.3	11.1	43.8

4a) Completely Randomized Design

b) $H_0: \mu_A = \mu_B = \mu_C$

H_a: Not all of the above are equal

Test statistic: $F_{obs} = \dfrac{S_B^2}{S_\omega^2}$

$$TSS = \sum_{ij} y_{ij}^2 - \frac{y_{..}^2}{n_T} \qquad [\because n_T = 4+4+4 = 12]$$

TSS = 6.93 (given)

$$SSB = \sum_i \frac{y_{i}^2}{n_i} - \frac{y_{..}^2}{n_T} = 166.115 - 159.87 = 6.245$$

$$SSW = TSS - SSB = 0.685$$

$$S_B^2 = \frac{SSB}{t-1} = \frac{6.245}{3-1} = 3.12$$

$$S_\omega^2 = \frac{SSW}{n_T - t} = \frac{0.685}{9} = 0.076$$

$$F_{obs} = \frac{S_B^2}{S_\omega^2} = 41.05$$

ANOVA for completely randomized design

Source due to	sum of squares (SS)	df	mean square (MS)	F
Between samples	6.245	2	3.12	$F_{obs} = 41.05$
within samples	0.685	9	0.076	
Totals	6.93	11		

c) $H_0: \mu_A = \mu_B = \mu_C$

H_a: Not all the above are equal

Test-statistic: $F_{obs} = 41.05$

Rejection Region: $F_{obs} > F_{df_1, df_2, \alpha}$

$df_1 = t - 1 = 2$

$F_{df_1, df_2, \alpha} = F_{2,9,0.05} = 4.26$

$df_2 = n_T - t = 9$

$F_{obs} > F_{2,9,0.05}$

P-value $= P(F_{2,9,0.05} > 41.05)$

P-value < 0.001

$\alpha >$ P-value (true) $[\because \alpha = 0.05]$

Reject H_0. We have sufficient evidence to conclude H_a.

5. a)

Price (X)	Quantity (Y)
58.7	20
59	15
60.1	17
61.3	16
63.2	13
64	11

independent variable (X) → Price

dependent variable (Y) → Quantity

b) $H_0: \beta_1 = 0$

$H_a: \beta_1 \neq 0$

Test-Statistic: $t_{obs} = \dfrac{\hat{\beta}_1 - \beta_{10}}{S_\varepsilon / \sqrt{S_{xx}}}$

$\hat{\beta}_1 = \dfrac{S_{xy}}{S_{xx}} = \dfrac{-29.5}{24} = -1.23$

[∵ given $S_{xx} = 24$
$S_{xy} = -29.5$
$S_\varepsilon = 1.8$]

$t_{obs} = \dfrac{-1.23 - 0}{1.8 / \sqrt{24}} = -3.35$

Rejection Region : $|t_{obs}| > t_{n-2, \frac{\alpha}{2}}$

$t_{4, 0.025} = 2.776$

$|t_{obs}| > t_{4, 0.025}$

$\therefore$ Reject H_0. We have sufficient to say that independent variable is useful to Predict dependent variable as slope is not equal to 0.

6a) check the variance, in order to choose between Pooled t-test and separate t-test

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_a : \sigma_1^2 \neq \sigma_2^2$

Test Statistic : $F_{obs} = \frac{S_i^2}{S_j^2} = \frac{(480)^2}{(350)^2} = 1.88$

$S_1 < S_2$

$df_1 = n_2 - 1 = 8$

$df_2 = n_1 - 1 = 9$

Rejection Region : $F_{obs} > F_{df_1, df_2, \frac{\alpha}{2}}$

(or)

$F_{obs} < \frac{1}{F_{df_2, df_1, \frac{\alpha}{2}}}$

$F_{8,9,0.025} = 4.10$

$$\frac{1}{F_{9,8,0.025}} = \frac{1}{4.36} = 0.23$$

$F_{obs} > F_{8,9,0.025}$ (or) $F_{obs} < \frac{1}{F_{9,8,0.025}}$

(False) (False)

$\therefore$ Failed to Reject H_0. We conduct Pooled t-test

b) $H_0: \mu_c - \mu_M = 0$

$H_a: \mu_c - \mu_m > 0$

① California

② Michigan

Test-statistic : $t_{obs} = \dfrac{(\bar{y}_1 - \bar{y}_2) - D_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

$$S_p^2 = \frac{8(480)^2 + 9(350)^2}{17} = 173276.47$$

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$S_p = 416.26$

$$t_{obs} = \frac{(8540 - 7358) - 0}{416.26 \sqrt{\frac{1}{9} + \frac{1}{10}}} = \frac{1182}{191.26} = 6.18$$

Rejection Region: $t_{obs} > t_{n_1+n_2-2, \alpha}$

$t_{17, 0.05} = 1.74$

$\therefore t_{obs} > t_{17, 0.05}$

$\therefore$ Reject H_0. We have sufficient evidence to conclude that California has higher mean hysterectomy cost than Michigan.

7.

School/Rating	outstanding	Average	Poor	
undesirable	10	4	2	$R_1 = 16$
Adequate	12	8	2	$R_2 = 22$
	$C_1 = 22$	$C_2 = 12$	$C_3 = 4$	

$R_1 = 16$ $\qquad$ $C_1 = 22$

$R_2 = 22$ $\qquad$ $C_2 = 12$

$C_3 = 4$

Total = 38

X = School type

Y = Rating

Ho: X and Y are not contingent

Ha: X and Y are contingent

$$E_{11} = \frac{R_1 C_1}{n} = \frac{16(22)}{38} = 9.26$$

$$E_{12} = \frac{R_1 C_2}{n} = \frac{16(12)}{38} = 5.05$$

$$E_{13} = \frac{R_1 C_3}{n} = \frac{16(4)}{38} = 1.68$$

$$E_{21} = \frac{R_2 C_1}{n} = \frac{22(22)}{38} = 12.74$$

$$E_{22} = \frac{R_2 C_2}{n} = \frac{22(12)}{38} = 6.95$$

$$E_{23} = \frac{R_2 C_3}{n} = \frac{22(4)}{38} = 2.31$$

Test statistic: $\chi^2_{obs} = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(n_{ij} - E_{ij})^2}{E_{ij}}$

$$\chi^2_{obs} = \frac{(10-9.26)^2}{9.26} + \frac{(4-5.05)^2}{5.05} + \frac{(2-1.68)^2}{1.68} + \frac{(12-12.74)^2}{12.74}$$

$$+ \frac{(8-6.95)^2}{6.95} + \frac{(2-2.31)^2}{2.31}$$

$$\chi^2_{obs} = 0.06 + 0.22 + 0.06 + 0.04 + 0.16 + 0.04$$

$$\chi^2_{obs} = 0.58$$

Rejection Region: $\chi^2_{obs} > \chi^2_{(r-1)(c-1),\alpha}$

$$\chi^2_{(1)(2),\,0.05} = \chi^2_{2,\,0.05} = 5.99$$

$\chi^2_{obs} > \chi^2_{2,0.05}$ (False)

Fail to Reject Ho. ~~...~~

There is no sufficient evidence to conclude that rating is contingent on school type.