Sampling Distributions and the Central Limit Theorem

Important Terms and Definitions

The **sampling distribution** of a statistic is the probability distribution for the values of the statistic that results when random samples of size n are repeatedly drawn from the population.

The **standard error** is the standard deviation of the sampling distribution of a statistic.

The Central Limit Theorem

If random samples of n observations are drawn from a population with a finite mean, μ, and a finite variance σ^2, then, when n is large (usually greater than 30), the sample mean, $\bar{y}$, will be approximately normally distributed with mean μ and variance σ^2/n. The approximation becomes more and more accurate as n becomes large.

Using the Central Limit Theorem, the approximate sampling distribution of the mean will be

$$\bar{Y} \sim N(\mu_{\bar{Y}}, \sigma^2_{\bar{Y}}) \text{ with } \mu_{\bar{Y}} = \mu \text{ and } \sigma^2_{\bar{y}} = \sigma^2 / n$$

Example: Suppose a sample of 49 observations is taken from a continuous distribution that has a mean of 10 and a variance of 196.

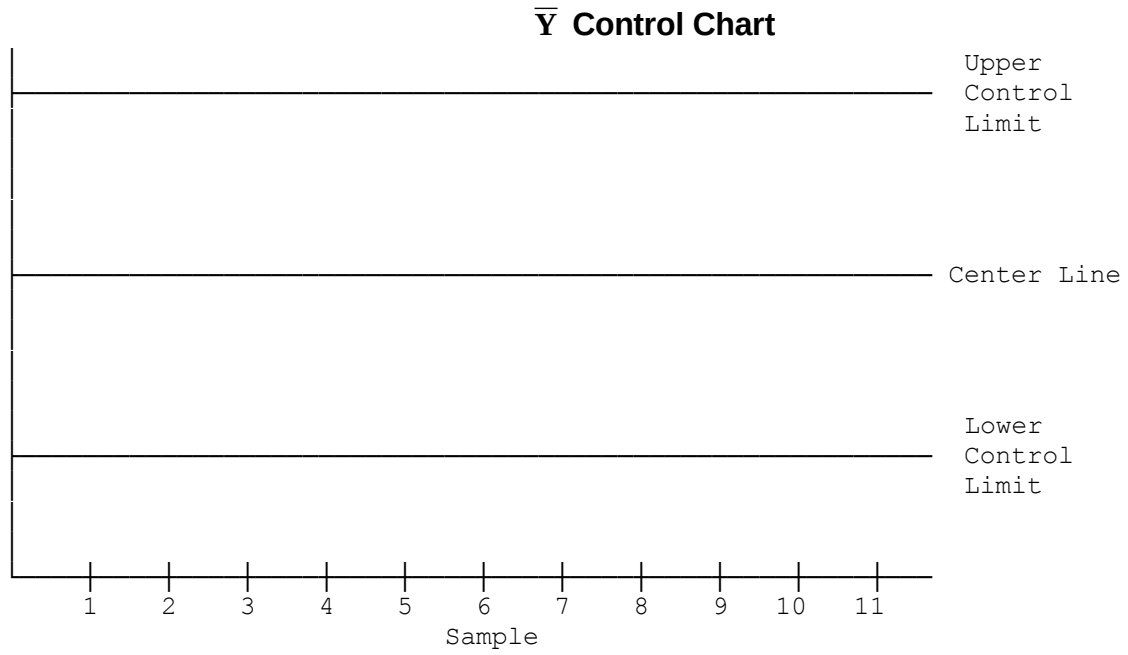
1. Find the approximate sampling distribution of the sample mean.

2. Find the probability that the sample mean will be between 8 and 12.

A Look at Quality Control

Statistical Process Control (SPC) involves the use of statistical methods to detect changes in processes.

The $\overline{Y}$ **Control Chart** can be used to detect shifts in the mean of a process. The chart looks at a sequence of sample means and the process is assumed to be "in control" as long as the sample means are within the control limits. (Below is a diagram of a $\overline{Y}$ Control Chart.)



Example: A glass-bottle manufacturing company wants to maintain a mean bursting strength of 260 psi. Past experience has shown that the standard deviation for the bursting strength is 36 psi. The company periodically pulls 36 bottles off the production line to determine if the mean bursting strength has changed. Construct a control chart so that 95% of the sample means will fall within the control limits when the process is "in control."

Below are the sample means for bottles that have been pulled off of the production line. Plot them on the control chart and determine if the process is "in control."

Sample	Mean Bursting Strength (PSI)	Sample	Mean Bursting Strength (PSI)
1	262	6	255
2	258	7	251
3	260	8	249
4	265	9	244
5	259	10	247