Simulation of 2D Superfluid Helium in the Finite Size Scaling Regime

By Tejan Shah

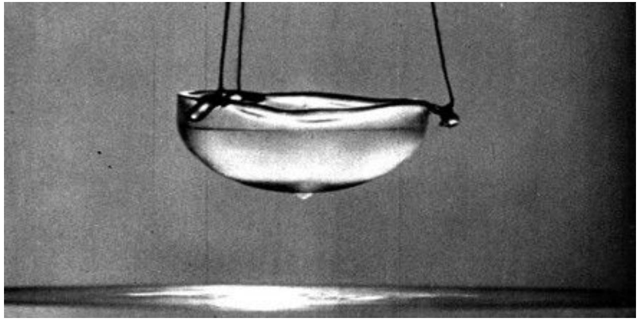
The What? Title Breakdown

Helium: liquid at T -> 0K

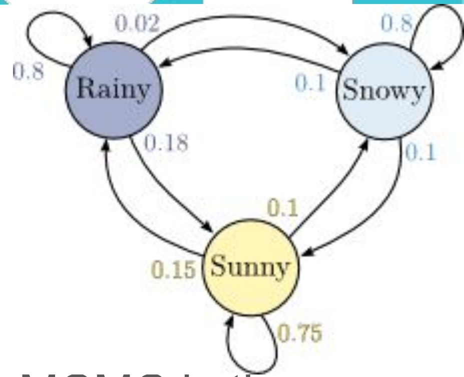
Superfluidity: High thermal conduction, zero viscosity, zero entropy

Films: Effectively 2D. Topological ordering. Kosterlitz and Thouless 2016 Nobel prize in Physics

Finite Size Scaling: Equations give exact solutions in Thermodynamic Limit. Real systems are finite.



The How? Monte Carlo



Analytic solutions are impossible

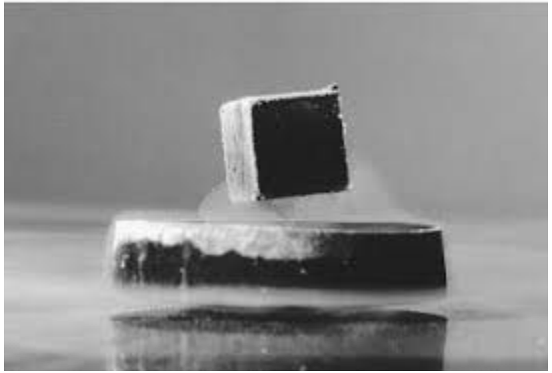
Monte Carlo: Random Sampling gives numerical results

Markov Chain: Movements through statespace are probabilistic

Markov Chain Monte Carlo: Expectation value is a sample mean. MCMC is the sampler

Irreversible MCMC: Directed path through statespace. Avoids diffusive nature of MCMC

The Why? Helium and Magnets



Superfluid Helium described by the XY lattice model

Superconductors also described by XY model

Superconductors ≈ Superfluids ? Yes!

Superconductors will revolutionise electronics

The Who?

Bishop & Reppy: Carried out helium experiments. Derived the scaling relation

Komura et al: Simulated massived system size

Hsieh et al: New fitting method. New estimate for critical transition temperature.

Bramwell & Holdsworth: 1993 Scaling relation approximation.

Hsieh ruined things with $T_{KT} = 0.8935K$

Aims

1. Simulate as big a system as possible
2. Compare data to results from Hsieh et al
3. Test Hsieh's intersection fit method
4. Investigate the behaviour of the scaling relation when applied to simulation data

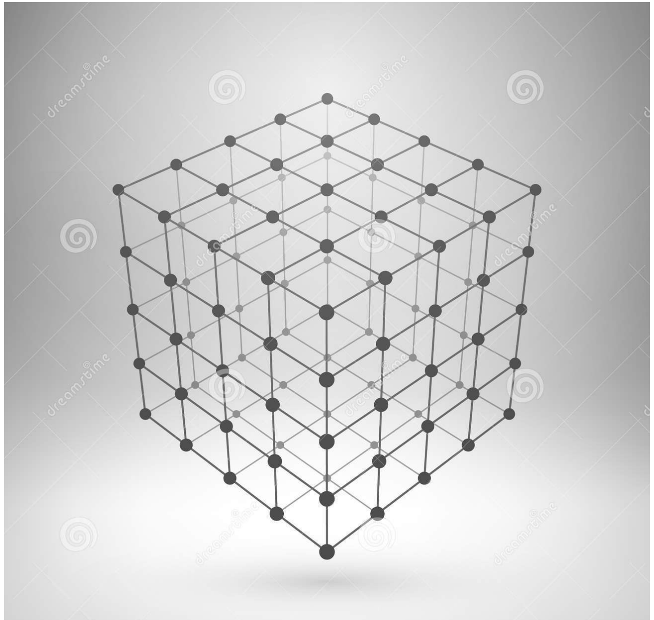
$$L \approx b = exp\left[\frac{1}{\sqrt{c(T-T_{KT})}}\left\{\arctan\left(\frac{\sqrt{c(T-T_{KT})}}{x_f}\right) - \arctan\left(\frac{\sqrt{c(T-T_{KT})}}{x_i}\right)\right\}\right]$$

Method

Simulations run on my own PC (Note to self: ask UCL to pay for my electricity bill)

Analysis carried out using Python

- NK Crossing: interpolate and intersect
- Bramwell-Holdsworth Scaling: 3D grid search
- Magnetisation collapse

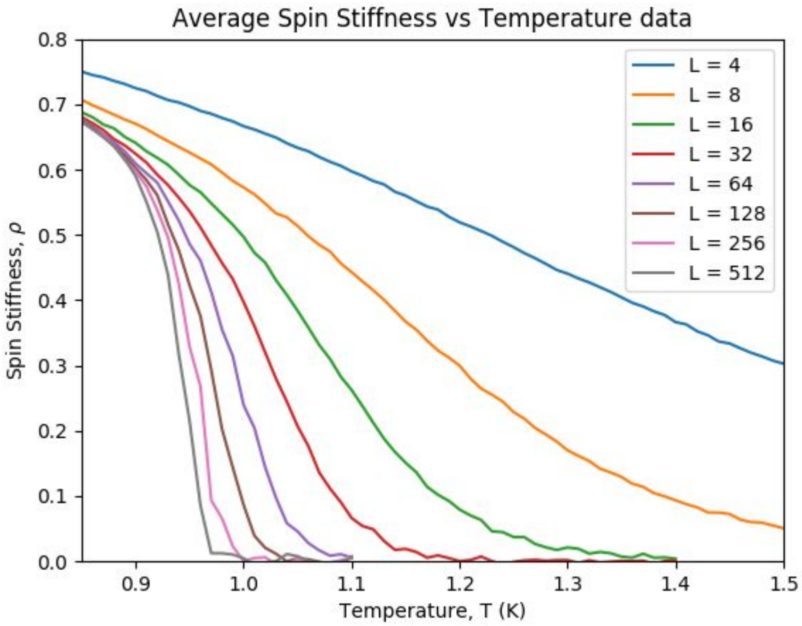


Results

Size dependence of spin stiffness

Simulated up to L = 2048

Data looked nicest up to L = 512

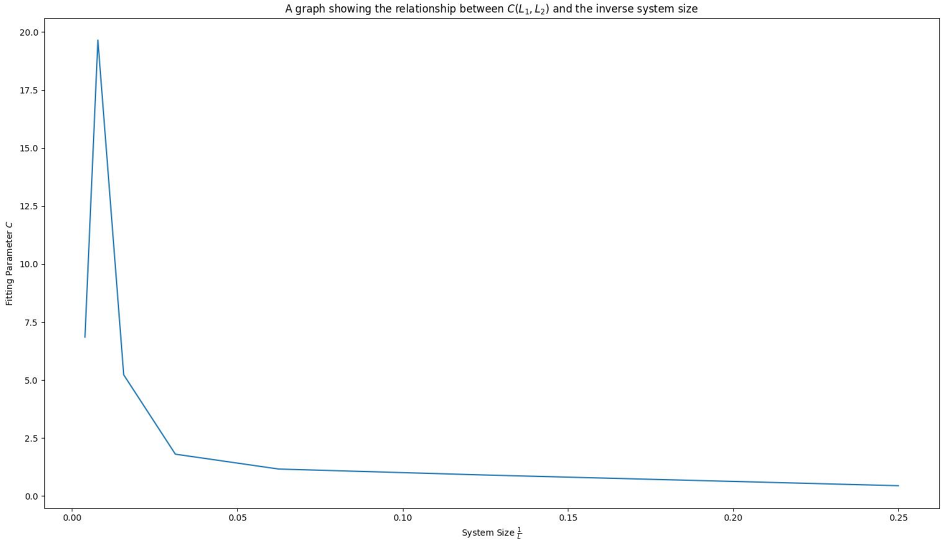


Results

Reproduction of Hsieh’s data failed

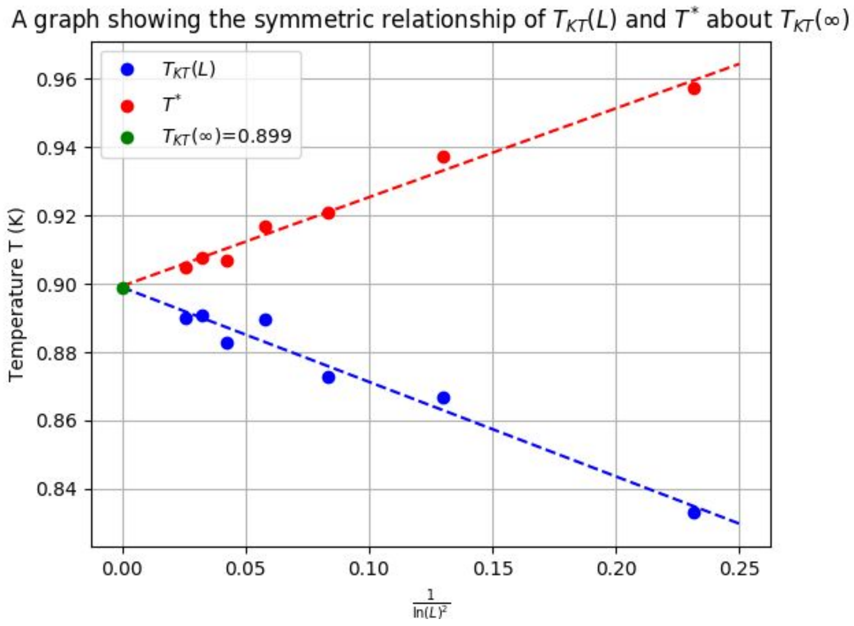
Sharp spike at L = 128 because no intersection

Temperamental method



Results

Beautiful symmetry and elegant scaling relation. Goodbye ugliness.



$$T^*(L) = T_{KT} + \frac{\pi^2}{8c\ln(L)^2}$$

$$T_{KT}(L) = T_{KT} - \frac{\pi^2}{8c\ln(L)^2}$$