Reconstruction of Undersampled Super High Frequency Signals using Recurrent Nonuniform Sampling

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Abstract ─ Sampling devices form an interface between front ends of radio receivers and signal processing units and therefore play a vital role in digital radio and radar communications. To avoid any loss of information, effective signal reconstruction is essential. In general, a signal can be faithfully reconstructed if it is sampled at a rate greater than the Nyquist rate. In super high frequency applications, however, it is not possible to sample the signal at the Nyquist rate. In this paper, we deal with undersampling and effective reconstruction of super high frequency signals. Here, we first further the discussion of accurately determining the center frequency using the main lobe to side lobe ratio, as discussed in paper [1], and then develop a practical implementation for the same. Next, we tackle the problem of frequency detection when folded frequency of the signal coincides with or close to the frequency where side lobe occurs. The aim is to develop a practical solution that comes within 3% of the theoretical results. Finally, we reconstruct the signal using band pass filtering (for removal of side lobes) and interpolation.

Keywords ─ Nyquist frequency, nonuniform sampling, interpolation

I. Introduction

The most common sampling methods used in Digital Signal Processing (DSP) employ uniform sampling. But in many practical cases, uniform sampling of data is not possible. And since most of the traditional reconstruction methods are for uniformly sampled signals, the effective reconstruction of nonuniformly sampled signals still remains a problem [2]. Furthermore, the requirement of sampling at the Nyquist rate may exceed the capacity of currently available analog-to-digital converters. This shows that, it is not always possible to sample signals above the Nyquist rate, in which case under sampling is the only option. But effective reconstruction of nonuniformly under sampled signals is essential in many fields like image processing, speech processing, medical imaging, radar communication, geophysics, communication theory and astronomy.

Recently, it has been found that reconstruction of a narrow band signal is possible even when it is sampled at a rate lower than the Nyquist rate, provided the center frequency is known. Recurrent nonuniform sampling results in the generation of side lobes which in turn can be used to obtain the center frequency. This can be done by first obtaining the Discrete Fourier Transform of the nonuniformly sampled signal and then estimating the frequency of the original signal using the side lobe to main lobe ratio. So, even if the main lobe folds down due to aliasing, the center frequency of the signal can be found with the help of the side lobes [1].

In this paper, we discuss the practical sampling and reconstruction of a narrow band nonuniformly undersampled signal. This is done by first nonuniformly undersampling the signal which results in side lobes. These help in estimating the center frequency about which the signal can be reconstructed. Finally, the undersampled signal is passed through a band pass filter to eliminate the side lobes and provide a reconstructed output. If the center frequency estimate is higher than half the sampling frequency, the signal is folded down below the Nyquist frequency and must therefore be interpolated. This method is feasible and economical and can be used in frequency hopping and radar communications, bringing down the cost of expensive high speed digital systems.

This paper is organized as follows. In section II, the estimated frequency for various input signals is calculated using the side lobe to main lobe ratio. The test ratios are compared with theoretically calculated reference ratios and the error percentages in different cases are presented. Section III deals with a problem that arises when the folded frequency of the input signal coincides with of close to the side lobe frequencies. A solution is proposed and an equation is derived, to calculate the center frequency in these cases. Section IV, discusses the reconstruction of the original signal using the processes of bandpass filtering and interpolation. In section V, experimental results are shown. Finally, section VI presents the conclusion.

II. Center frequency calculation

For super high frequency applications, the criteria of sampling at the Nyquist rate is hard to fulfill. However, the recent concept of compressive sensing states that the level of information contained in a signal is often much lower than the actual bandwidth [3] – [6]. With this as its basis, paper [1] comes up with a method of estimating the frequency of the original signal by nonuniformly undersampling it. This type of sampling results in the formation of side lobes along with the main lobe [7] – [10]. The ratio of the side lobes to the main lobe, as shown below, provides an estimate of the frequency of the original signal [1].

$$\textit{Side Lobe to Main lobe ratio} = \frac{[R(1)+R(2)]/2}{R(0)} \qquad (1)$$

where R(1) and R(2) are the amplitudes of the two side lobes and R(0) is the amplitude of the main lobe if we use three ADCs for recurrent nonuniform sampling.

In this paper, we perform the practical implementation of the aforementioned method. The first step in obtaining the main lobe to side lobe ratio is to perform nonuniform sampling of the input signal, followed by zero padding followed by DFT. The zero padding is done to improve the resolution of the result. Next, the main lobe and side lobes are located in the result and their amplitudes are measured. Finally, the reference ratio can then be obtained by using the formula in equation (1). This process is implemented in Matlab and a few results have been shown in the two tables below. The test ratios obtained experimentally are compared with the theoretically calculated reference ratios, and the error percentage is calculated which tells us the deviation of the test ratios from their reference ratios. The Tables1 and Table 2 show the test and reference ratios for different frequencies of pure sinusoidal and FM signals.

TABLE I
Test and reference ratios for different frequencies of pure sinusoidal signal

Center Frequency $(\times 2\pi)$	Test Ratio	Reference Ratio	Error Percentage
0.2	0.020950	0.021665	3.412778
0.4	0.041934	0.042198	0.630106
0.6	0.063132	0.063132	0.229063
0.8	0.084145	0.084362	0.257554
0.9	0.094774	0.094774	0.000000

TABLE II
Test and reference ratios for different frequencies of FM signal

Center Frequency $(\times 2\pi)$	Test Ratio	Reference Ratio	Error Percentage
0.4	0.043146	0.042902	2.890972
1.8	0.196598	0.192783	1.978819
3.4	0.387961	0.386821	0.294803
4.4	0.533540	0.531670	0.351715
4.8	0.602610	0.597456	0.862806

III. PROBLEM

From Tables 2 and 3, it is clear that for most values of normalized frequencies, the error percentage does not exceed 10%. This, however, does not hold true when the main lobe appears at the 1/3 or 2/3 of the sampling frequency. In these cases, the error percentages tend to be greater than 10%, which is too high. This occurs because we take 3-point nonuniform sampling. At these frequencies we see that the main lobe merges with one of the side lobes which results in a peak of unusual high amplitude. It then becomes difficult to find the true side lobe to main lobe ratio.

That said, on observing the varying amplitudes of the side lobes for a variety of such frequencies, it is seen that a method of estimation can be set up. In Table 3, we observe that the magnitude of the side lobe varies in a certain range for input frequencies of a certain range. For example, for a frequency of 0.333 the side lobe amplitude is about 2.6 and for frequencies from 0.3333 to 0.333333 the side lobe amplitude ranges between 1 and 2. This range of amplitudes is placed in category 0. Similarly, for a frequency of 2.333 the side lobe amplitude is about 41 and for frequencies from 2.3333 to 2.333333 the side lobe amplitude ranges between 9 and 12. This range of amplitudes is placed in category 2. In this way the range of amplitudes for all such frequencies is calculated and each range is assigned specific categories. Now, whenever an amplitude from one of these ranges is obtained, the center frequency (CF) can simply be calculated using the formula given below:

$$CF = C + MLF \qquad (2)$$

where *C* is the category to which the amplitude range belongs and, *MLF* is the main lobe frequency

For example, let's say we come across a sampled output with the main lobe located at a normalized frequency of 0.3334 with a side lobe of amplitude 10.9. From Table 3, it is clear that this falls in category 2. Therefore, the center frequency (CF) can be calculated as follows:

$$CF = C + MLF = 2 + 0.3334 = 2.3334 \qquad (3)$$

TABLE III
Amplitude of the side lobe for 1/3rd and 2/3rd sampling frequencies

Center Frequency $(\times 2\pi)$ (CF)	Main Lobe Frequency (MLF)	Side Lobe Amplitude	Category (C)
0.333	0.3330	2.6379	0
0.3333	0.3333	1.5200	0
0.33333	0.3334	1.1921	0
0.333333	0.3334	1.2158	0
1.333	0.3330	8.2772	1
1.3333	0.3333	2.7637	1
1.33333	0.3334	3.7113	1
1.333333	0.3334	3.8094	1
2.333	0.3330	41.4342	2
2.3333	0.3333	9.3006	2
2.33333	0.3334	10.9831	2
2.333333	0.3334	11.1482	2
3.333	0.3330	53.7589	3
3.3333	0.3333	19.2738	3
3.33333	0.3334	21.4827	3
3.333333	0.3334	21.6966	3
4.333	0.3330	67.0041	4
4.3333	0.3333	31.6080	4
4.33333	0.3334	34.1399	4
4.333333	0.3334	34.3817	4

Therefore, we can say that if the error percentage is observed to be greater than 10% and if the side lobe amplitude falls within one of the ranges seen in the Table 3, then the center frequency is simply the sum of the range and the main lobe frequency as shown in equation (2). The table also shows how the amplitude of the sidelobe increases with an increase in the center frequency.

IV. RECONSTRUCTION

Once the center frequency of the signal is determined, the output is passed through a band pass filter to remove the side lobes introduced during nonuniform sampling [11] – [14]. Now, if the signal frequency is lower than half the sampling frequency, it means the signal is not folded, and the output of the filtering process gives the reconstructed signal without any side lobes. However, if the signal frequency turns out to be higher than half the sampling frequency, then the signal appears folded and must undergo upsampling before bandpass filtering for interpolation or reconstruction .

V. SIMULATION RESULTS

Let us consider a few examples with different input frequencies. Figure 1 shows the nonuniform sampling and reconstruction of a pure sinusoidal signal, while Figures 2 and 3 represent the sampling and reconstruction of a FM signal. In Figure 1, an input with a normalized frequency of 0.4 is used. It is seen that the recurrent nonuniform sampling of this signal yields a main lobe at 0.4 and two small side lobes. Now, since its frequency is lower than half the sampling frequency (0.5), this signal remains unfolded and is reconstructed by simply passing the output of the sampling process through a band pass filter. The reconstructed signal is located at 0.4.

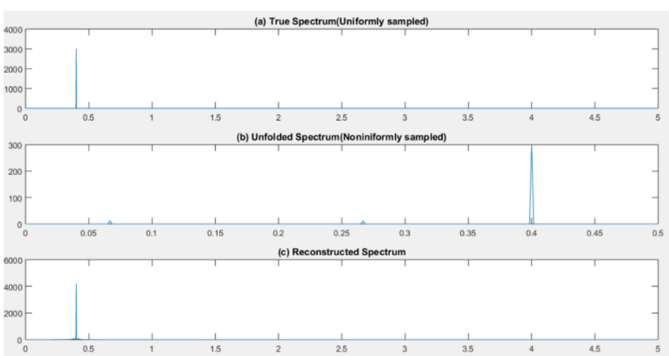

Figure 1. Nonuniform sampling and reconstruction of an unfolded input sinusoidal signal (0.4)

In Figure 2, an input FM signal with a normalized frequency of 2.7 is used. The nonuniformly sampled output of this signal is seen to have its main lobe at 0.3. This occurs due to folding of the signal, which takes place because its frequency is higher than half the sampling frequency (0.5). The sampled output signal is a folded mirror image of what it should have been. In this case, the main lobe should have been at 0.2 (2.7 - highest deductible multiple of 0.5), but due to folding, its mirror image places the main lobe at 0.3. The reconstructed signal is located at 2.7.

In Figure 3, an input FM signal with a normalized frequency of 4.6 is used. The nonuniformly sampled output of this signal is seen to have its main lobe at 0.4 due to folding. 0.4 is a mirror image of 0.1 (4.6 – highest deductible multiple of 0.5). The reconstructed signal is located at 4.6.

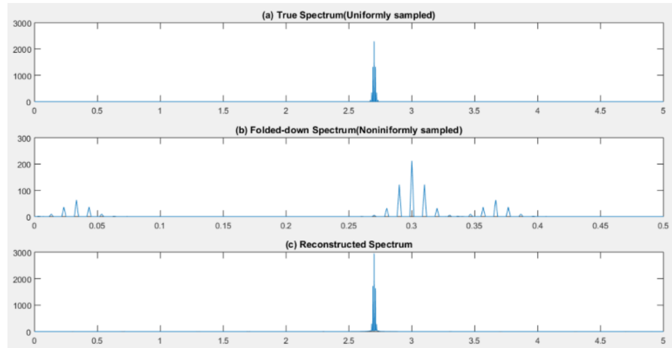

Figure 2. Nonuniform sampling and reconstruction of a folded input FM signal (2.7)

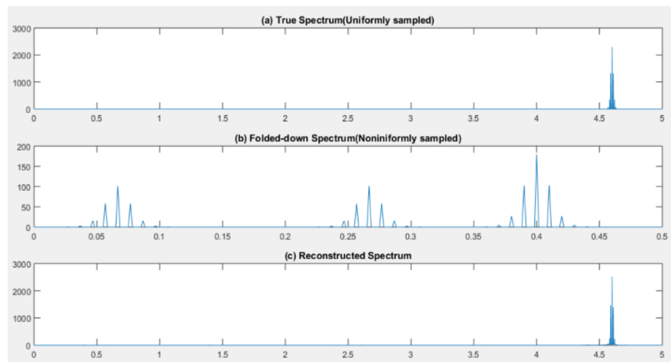

Figure 3. Nonuniform sampling and reconstruction of a folded input FM signal (4.6)

In each case, after the input signal is reconstructed, it is compared with the spectrum of the same signal sampled uniformly so as to observe any inconsistencies. The code for this method has been implemented in Matlab. It accepts different values for - number of ADCs, number of samples taken per ADC and order of the bandpass filter.

VI. CONCLUSION

We use super high frequency radio signals in a large variety of applications today, especially in the communication and radar sectors. The sampling of these high frequency signals at the Nyquist rate puts a lot of stress on present day ADCs and memory locations. However, it has been observed that the level of information contained in a signal is often much lower than the actual bandwidth, which means that better schemes can be used to overcome the stress. One such scheme was developed in paper [1] which put forth the idea of using nonuniform undersampling to estimate the frequency of the original signal. This paper practically implements the idea put forth in that paper and also implements a method for reconstruction of those signals. This method employs the use of recurrent nonuniform sampling coupled with band pass filtering and interpolation to effectively sample and reconstruct super high frequency signals.

VII. References

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