AI1110
Assignment 11

U.S.M.M TEJA
CS21BTECH11059

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Outline

question 6.49

The random variables x and y are N(0, σ) are independent. Show that if z = $|x - y|$, $thenE(z) = 2\sigma\frac{}{\sqrt{\pi}}$, E(Z^2) = $2\sigma^2$

Theory

given that z = $|x - y|$ $and E(w) = E(x) - E(y) = 0$,
$\sigma_w^2 = \sigma_x^2 + \sigma_y^2$ = $2\sigma^2$
now we come to a conclusion that w is a random variable is N(0, $\sqrt{2}\sigma$)
we know that

$$E(\left|x^{2k+1}\right|) = 2\int_0^\infty x^{2k+1} f(x)\, dx$$

and E(x^{2k}) = 1.3.5...(n-1)σ^n

Calculation

So

$$E(\left|x^{2k+1}\right|) = 2\int_0^\infty x^{2k+1} f(x)dx \tag{1}$$

$$E(\left|x^{1}\right|) = 2\int_0^\infty x^{1} f(x)dx \tag{2}$$

$$= \frac{2}{\sigma\sqrt{2\pi}}\int_0^\infty x^{1} e^{\frac{-x^2}{2\sigma^2}} dx \tag{3}$$

$$= -\sqrt{2}\sigma\sqrt{\frac{2}{\pi}}(e^{\frac{-\infty^2}{2\sigma^2}} - e^{\frac{-0^2}{2\sigma^2}}) \tag{4}$$

$$= \frac{2\sigma}{\pi} \tag{5}$$

and for

$$E(x^2) = 1 \times \sigma^2 \tag{6}$$

$$= (\sqrt{2}\sigma)^2 = 2\sigma^2 \tag{7}$$